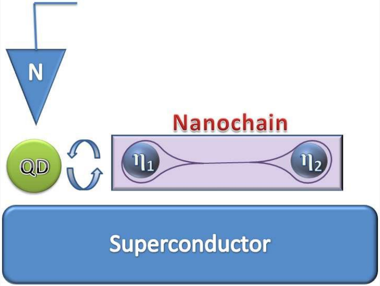
Are pairs of Majorana modes distantly cross-correlated?

Tadeusz Domański

M. Curie-Skłodowska University

Lublin, Poland



XXI Krajowa Konferencja Nadprzewodnictwa

Kraków, 25/09/2024

OUTLINE

1. **Topological superconductivity**

   (pairs of boundary modes)

2. **Stationary phenomena**

   (in topological hybrid structures)

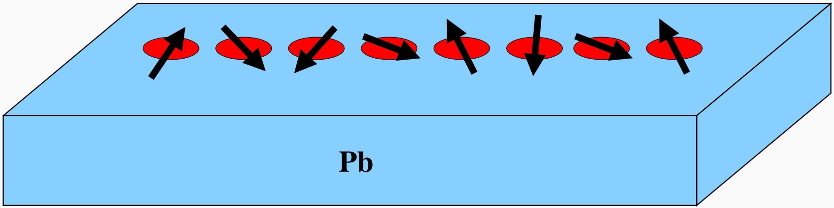
3. **Dynamical effects**

   (transmitted via Majorana modes)

Part 1. Topological superconductivity

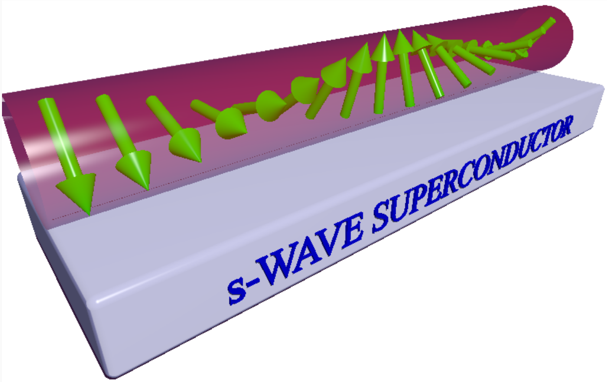
MAGNETIC IMPURITIES + SUPERCONDUCTOR

Magnetic atoms (like Fe) on a surface of s-wave superconductor (for example Pb or Al) arrange themselves into the spiral order, promoting the topological superconducting state (topofilia).



MAGNETIC IMPURITIES + SUPERCONDUCTOR

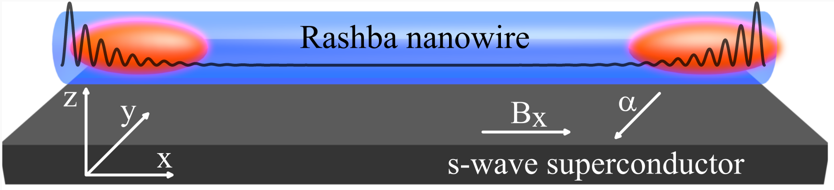
Such helix magnetic texture, supporting the topological superconducting state survives at finite temperatures (up to a few K).



A. Gorczyca-Goraj, T. Domański, M.M. Maśka, PRB 99, 235430 (2019).

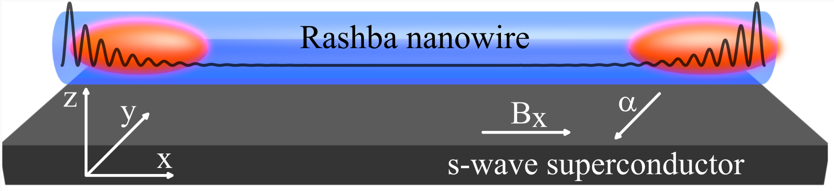
PAIRING & MAGNETISM IN NANOWIRES

Spin-orbit (Rashba) interaction in presence of magnetic field applied to semiconducting nanowire proximitized to s-wave superconductor can also induce triplet pairing of electrons between neighboring sites.



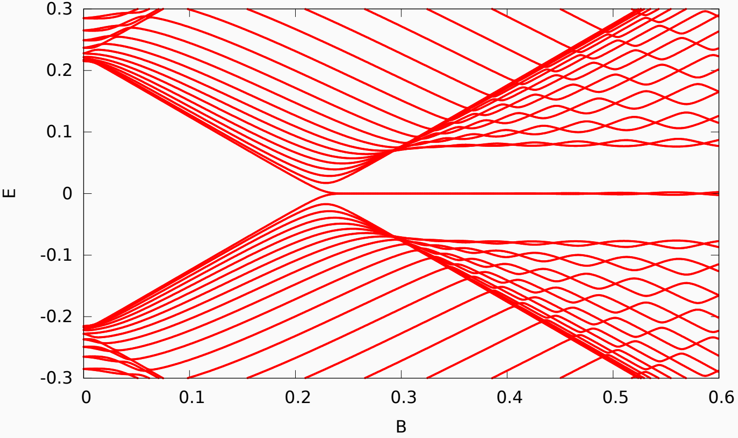
PAIRING & MAGNETISM IN NANOWIRES

Spin-orbit (Rashba) interaction in presence of magnetic field applied to semiconducting nanowire proximitized to s-wave superconductor can also induce triplet pairing of electrons between neighboring sites.

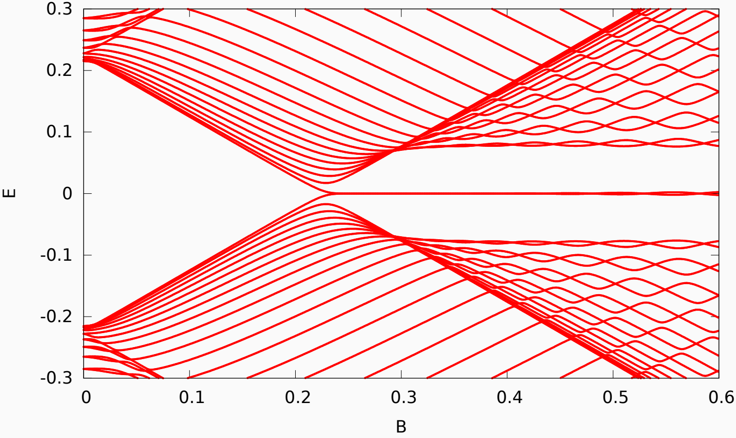


Such intersite triplet pairing of mobile electrons in 1-dimensional chains has been predicted to host the Majorana boundary modes.

Effective quasiparticle states of the Rashba nanowire



Effective quasiparticle states of the Rashba nanowire



Closing / reopening of a gap $\Longleftrightarrow$ band-invertion of topological insulators

M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB <u>95</u>, 045429 (2017).

BONDARY MODES IN TRIPLET SUPERCONDUCTOR

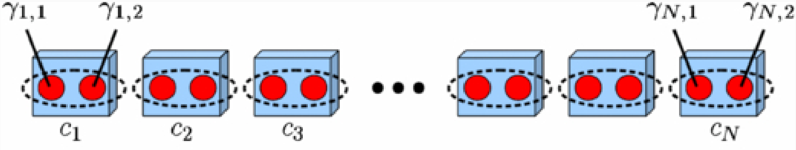
Itinerant 1D fermions with intersite (*p-wave*) pairing

$$\hat{H} = t\sum_{i=1}^{N-1}\left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.}\right) + \Delta\sum_{i=1}^{N-1}\left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \text{h.c.}\right) - \mu\sum_{i=1}^{N}\hat{c}_i^\dagger \hat{c}_i$$

Usual fermion operators can be recast in the Majorana basis

$$\hat{\gamma}_{j,1} \equiv \frac{1}{\sqrt{2}}\left(\hat{c}_j + \hat{c}_j^\dagger\right)$$

$$\hat{\gamma}_{j,2} \equiv \frac{1}{i\sqrt{2}}\left(\hat{c}_j - \hat{c}_j^\dagger\right)$$

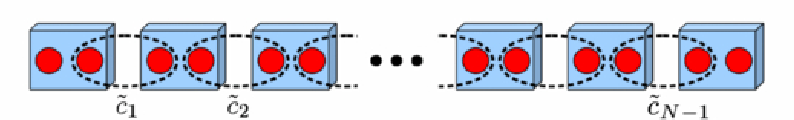


A.Y. Kitaev, Phys. Usp. 44, 131 (2001).

KITAEV CHAIN: PARADIGM FOR MAJORANA QPS

For $\Delta = t$ and $|\mu|$ being inside the band (for example $\mu = 0$)

the operators $\hat{\gamma}_{1,1}$ and $\hat{\gamma}_{2,N}$ *decouple* from all the rest



They correspond to zero-energy modes at the chain edges

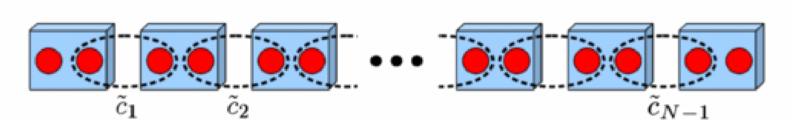
which can be regarded as *fractions* of non-local fermion

$$
\begin{aligned}
\hat{c}_{nonlocal} &\equiv \left(\hat{\gamma}_{1,1} + i\hat{\gamma}_{N,2}\right)/\sqrt{2} \\
\hat{c}^{\dagger}_{nonlocal} &\equiv \left(\hat{\gamma}_{1,1} - i\hat{\gamma}_{N,2}\right)/\sqrt{2}
\end{aligned}
$$

KITAEV CHAIN: PARADIGM FOR MAJORANA QPS

For $\Delta = t$ and $|\mu|$ being inside the band (for example $\mu = 0$)

the operators $\hat{\gamma}_{1,1}$ and $\hat{\gamma}_{2,N}$ *decouple* from all the rest



They correspond to zero-energy modes at the chain edges

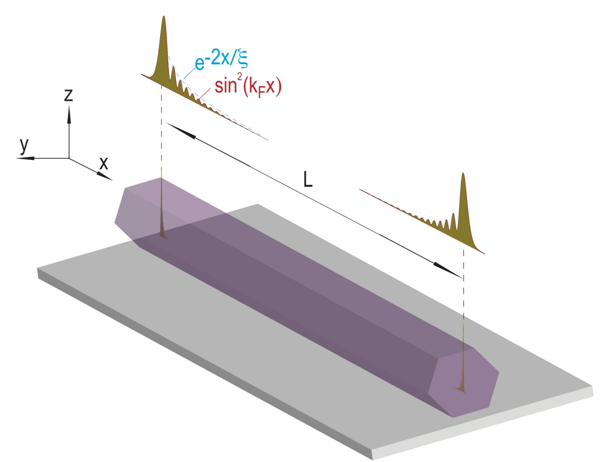
which can be regarded as *fractions* of non-local fermion

$$\hat{c}_{nonlocal} \equiv (\hat{\gamma}_{1,1} + i\hat{\gamma}_{N,2})/\sqrt{2}$$
$$\hat{c}^{\dagger}_{nonlocal} \equiv (\hat{\gamma}_{1,1} - i\hat{\gamma}_{N,2})/\sqrt{2}$$

Are these boundary modes distantly cross-correlated?

SPATIAL PROFILE OF MAJORANA MODES

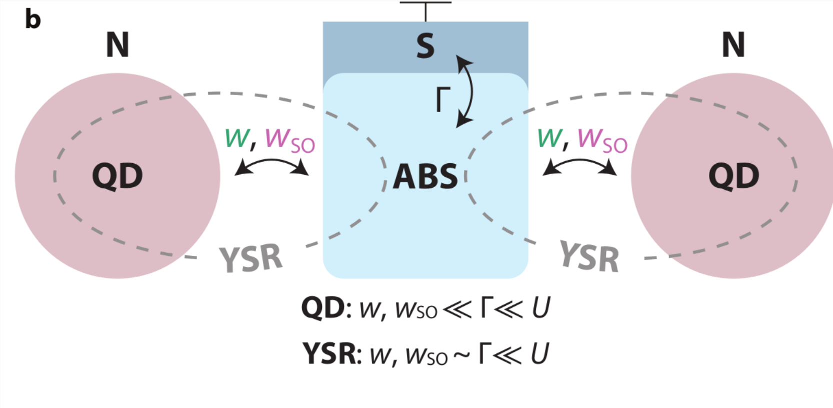
Majorana zero-energy modes are confined on edges (and/or defects)



R. Aguado, Riv. Nuovo Cim. 40, 523 (2017).

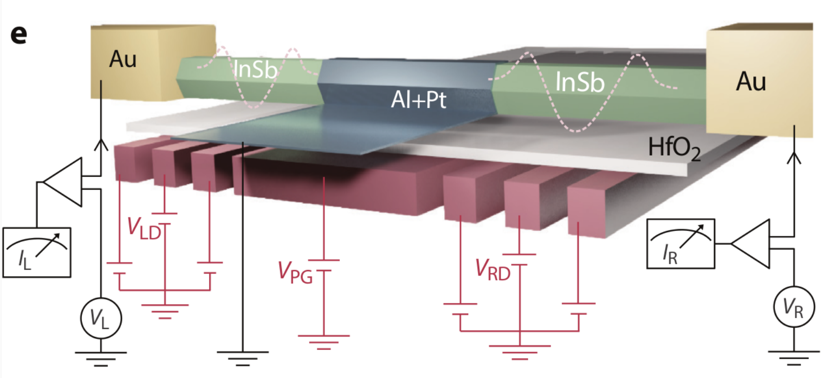
MINIMAL KITAEV CHAIN

Effective triplet pairing has been recently realized using two quantum dots interconnected by superconductor (Poor Man's Majorana states)



T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).

Two spin-polarized quantum dots in an InSb nanowire strongly coupled by elastic co-tunneling and crossed Andreev reflection



T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).

Part 2. Stationary phenomena

(in topological hybrid structures)

EARLY PROPOSALS

Hybrid structure: **quantum dot + topological superconductors**

EARLY PROPOSALS

Hybrid structure: quantum dot + topological superconductors

PHYSICAL REVIEW B **84**, 140501(R) (2011)

Scheme to measure Majorana fermion lifetimes using a quantum dot

Martin Leijnse and Karsten Flensberg
Nano-Science Center & Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark
(Received 30 August 2011; published 3 October 2011)

EARLY PROPOSALS

Hybrid structure: quantum dot + topological superconductors

PHYSICAL REVIEW B **84**, 140501(R) (2011)

Scheme to measure Majorana fermion lifetimes using a quantum dot

Martin Leijnse and Karsten Flensberg
Nano-Science Center & Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark
(Received 30 August 2011; published 3 October 2011)

PHYSICAL REVIEW B **84**, 201308(R) (2011)

Detecting a Majorana-fermion zero mode using a quantum dot

Dong E. Liu and Harold U. Baranger
Department of Physics, Duke University, Box 90305, Durham, North Carolina 27708-0305, USA
(Received 22 July 2011; revised manuscript received 13 September 2011; published 16 November 2011)

EARLY PROPOSALS

Hybrid structure: quantum dot + topological superconductors

PHYSICAL REVIEW B **84**, 140501(R) (2011)

Scheme to measure Majorana fermion lifetimes using a quantum dot

Martin Leijnse and Karsten Flensberg
Nano-Science Center & Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark
(Received 30 August 2011; published 3 October 2011)

PHYSICAL REVIEW B **84**, 201308(R) (2011)

Detecting a Majorana-fermion zero mode using a quantum dot

Dong E. Liu and Harold U. Baranger
Department of Physics, Duke University, Box 90305, Durham, North Carolina 27708-0305, USA
(Received 22 July 2011; revised manuscript received 13 September 2011; published 16 November 2011)

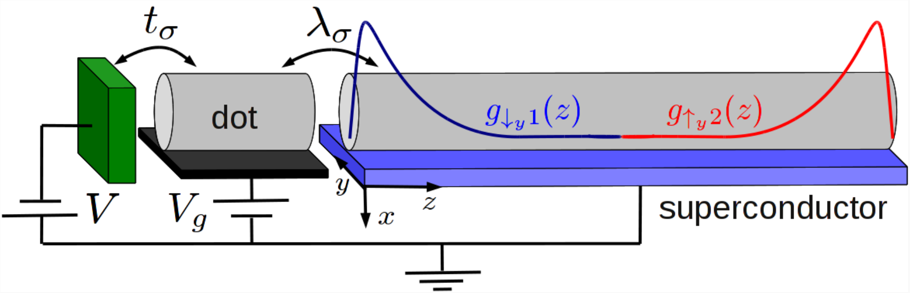
PHYSICAL REVIEW B **89**, 165314 (2014)

Subtle leakage of a Majorana mode into a quantum dot

E. Vernek,[1,2] P. H. Penteado,[2] A. C. Seridonio,[3] and J. C. Egues[2]
[1]*Instituto de Física, Universidade Federal de Uberlândia, Uberlândia, Minas Gerais 38400-902, Brazil*
[2]*Instituto de Física de São Carlos, Universidade de São Paulo, São Carlos, São Paulo 13560-970, Brazil*
[3]*Departamento de Física e Química, Universidade Estadual Paulista, Ilha Solteira, São Paulo 15385-000, Brazil*
(Received 15 August 2013; revised manuscript received 10 April 2014; published 30 April 2014)

TRANSFER OF MAJORANA MODE ON QD

Hybrid structure: quantum dot + topological superconductor

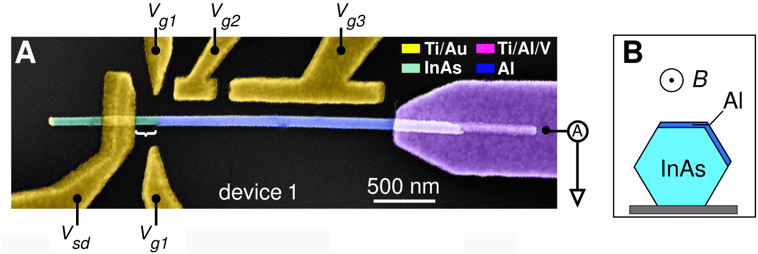


Issue: Majorana mode is partly transferred onto quantum dot where it can be detected by tunneling spectroscopy

M. Leijnse and K. Flensberg, Phys. Rev. B 84, 140501(R) (2011).

EXPERIMENTAL REALIZATION

Hybrid structure: **Epitaxial Al shell (blue) grown on two facets of the hexagonal InAs core (cyan), with a thickness of $\sim$ 10 nm. Magnetic field is applied parallelly to the nanowire.**

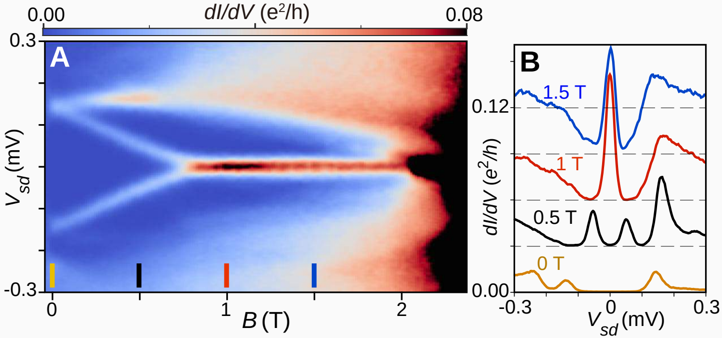


M.T. Deng et al, Science 354, 1557 (2016).

EXPERIMENTAL REALIZATION

Experimental data: (A) Tunneling spectrum for resonant dot-wire coupling obtained at $V_{bg} = -8.5$ V, $V_{g1} = 22$ V, and $V_{g2} = V_{g3} = -10$ V.

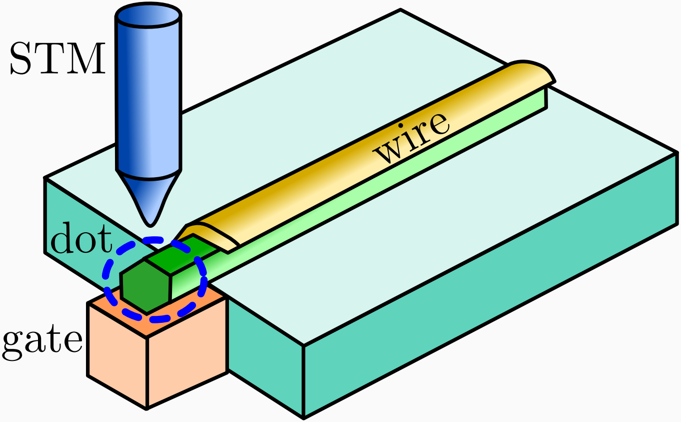
(B) Differential conductance at various values of the magnetic field.



M.T. Deng et al, Science 354, 1557 (2016).

GATE-CONTROLLED BOUND STATES

Hybrid structure: **trivial + topological segments of nanowire**

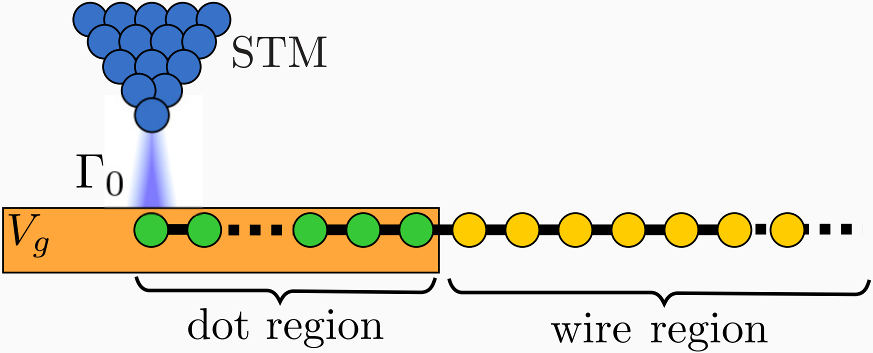


Issue: **bound states of trivial segment attached to topological sc**

A. Ptok, A. Kobiałka, T. Domański, Phys. Rev. B 96, 195430 (2017).

GATE-CONTROLLED BOUND STATES

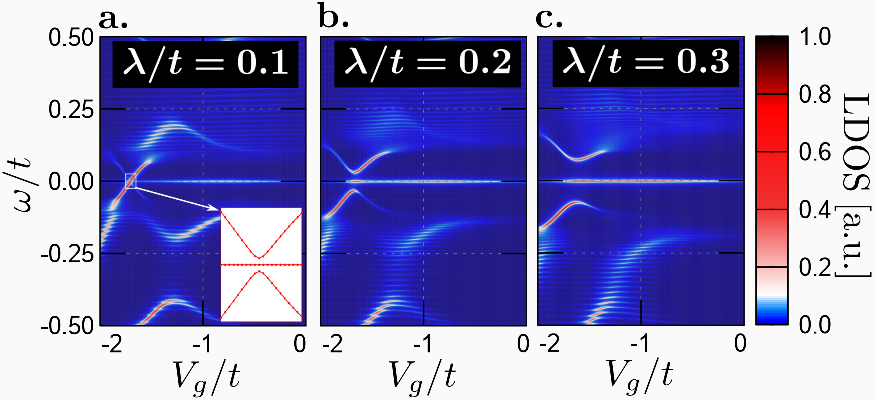
Hybrid structure: **trivial + topological segments of nanowire**



Remark: **bound states can be probed by STM measurements**

A. Ptok, A. Kobiałka, T. Domański, Phys. Rev. B <u>96</u>, 195430 (2017).

Hybrid structure: trivial + topological segments of nanowire



Variation the trivial (Andreev) & topological (Majorana) states vs the gate potential V_g for several spin-orbit couplings λ.

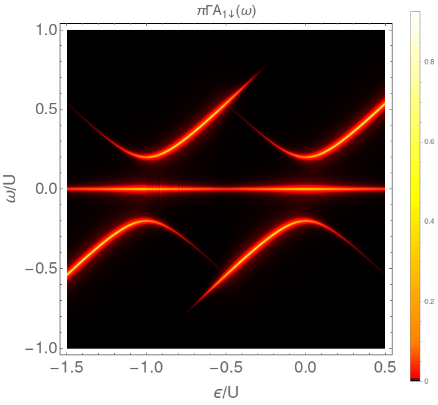
A. Ptok, A. Kobiałka, T. Domański, Phys. Rev. B 96, 195430 (2017).

What about correlations ?

(on-site Coulomb repulsion)

BOUND STATES OF STRONGLY CORRELATED DOT

Hybrid structure: Anderson impurity + topological nanowire

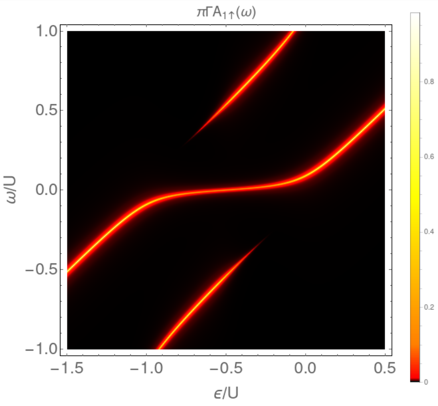


Spectrum of the Anderson impurity for ↓-spin electron which is side-coupled to the Majorana boundary mode of topological sc.

J. Barański, M. Barańska, T. Zienkiewicz, T. Domański, (2024) submitted.

BOUND STATES OF STRONGLY CORRELATED DOT

Hybrid structure: Anderson impurity + topological nanowire



Spectrum of the Anderson impurity for ↑-spin electron which is <u>not coupled</u> to the Majorana boundary mode of topological sc.

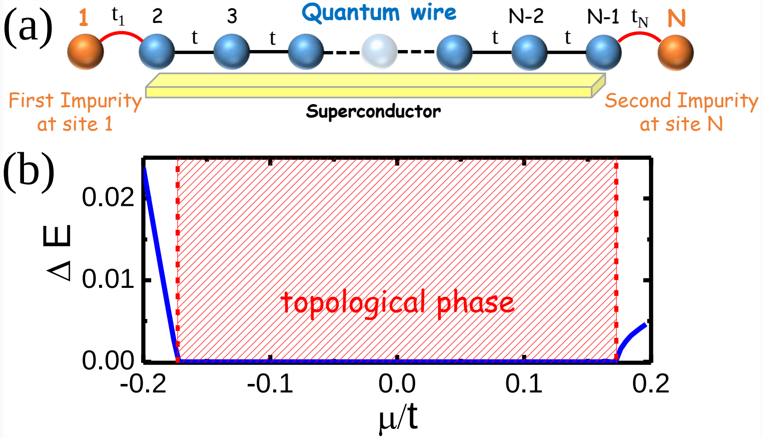
J. Barański, M. Barańska, T. Zienkiewicz, T. Domański, (2024) submitted.

Nonlocal cross-correlations ?

(under stationary conditions)

TWO IMPURITIES + TOLOGOGICAL NANOWIRE

Hybrid structure: 2 quantum dots + topological nanowire

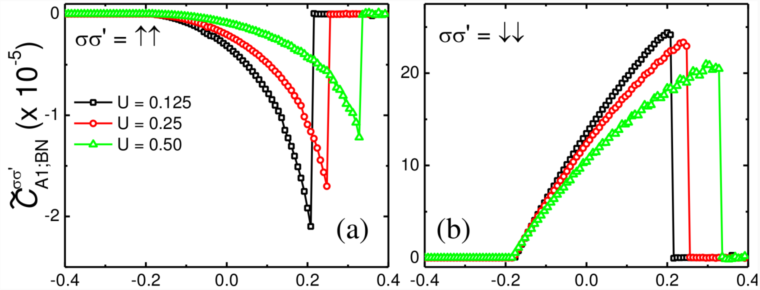


Issue: correlation effects ⟷ leakage of Majorana modes

G.S. Diniz and E. Vernek, Phys. Rev. B 107, 045121 (2023).

DMRG results:

(obtained in the Majorana operator representation)



Non-local spectral function vs μ for various Coulomb potentials

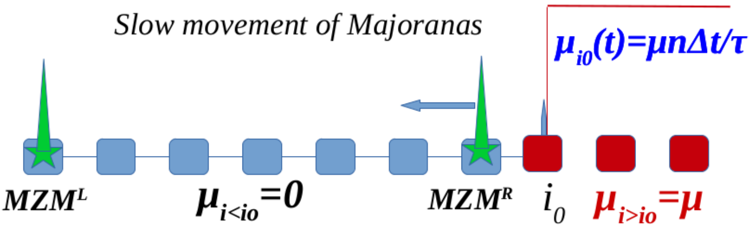
G.S. Diniz and E. Vernek, Phys. Rev. B 107, 045121 (2023).

Time - resolved phenomena

(imposed by slow/fast changes)

RELOCATION OF MAJORANAS

Hybrid structure: **switching on/off topological phase**

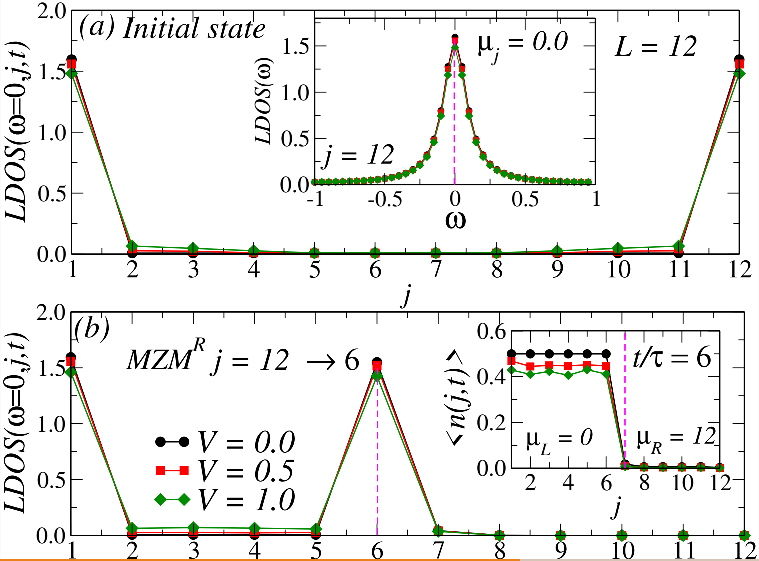


Issue: **gate-imposed relocation of the Majorana mode**

B. Pandey, L. Mohanta and E. Gagotto, Phys. Rev. B 107, L060304 (2023).

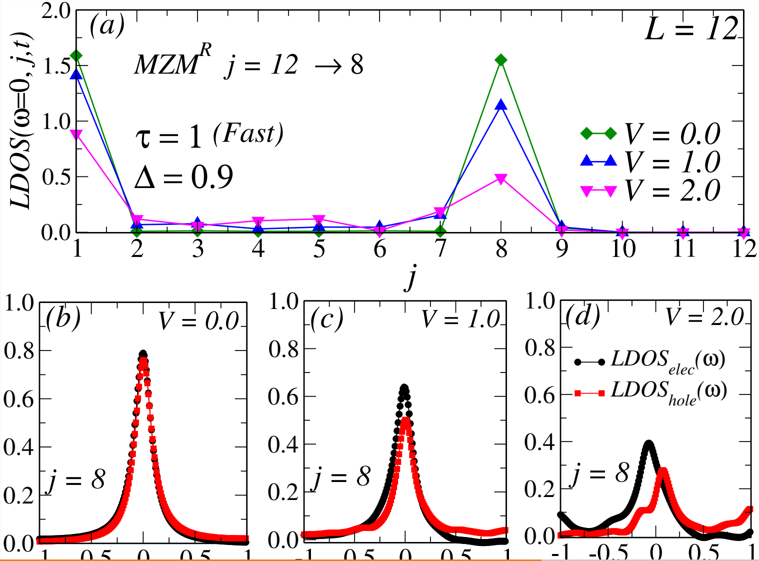
RELOCATION OF MAJORANAS

Hybrid structure: slow switching



RELOCATION OF MAJORANAS

Hybrid structure: fast switching

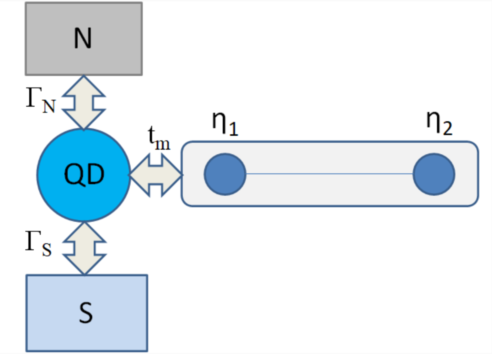


Part 3. Dynamical effects

(transmitted via Majorana modes)

TIME-RESOLVED LEAKAGE OF MAJORANA MODE

Hybrid structure: **quantum dot attached to topological nanowire**



Question:

How much time does it take to transfer the Majorana mode on QD ?

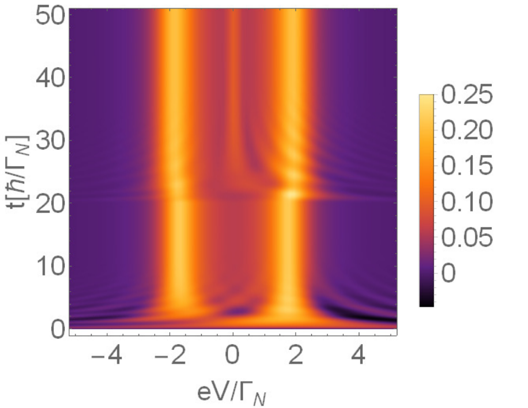
J. Barański, M. Barańska, T. Zienkiewicz, R. Taranko, T.Domański, PRB 103, 235416 (2021).

TIME-RESOLVED LEAKAGE OF MAJORANA MODE

Transient effects:

$\Rightarrow$ **at $t = 0$ QD is coupled to the external N and S electrodes,**

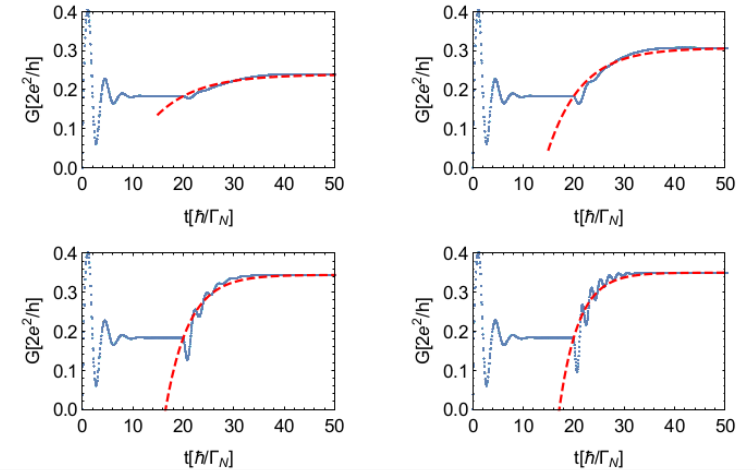
$\Rightarrow$ **at $t = 10$ topological nanowire is attached to N-QD-S setup.**



Gradual development of the trivial (Andreev) and topological (Majorana) states manifested in the differential conductance.

TIME-RESOLVED LEAKAGE OF MAJORANA MODE

Time-dependent zero-bias conductance



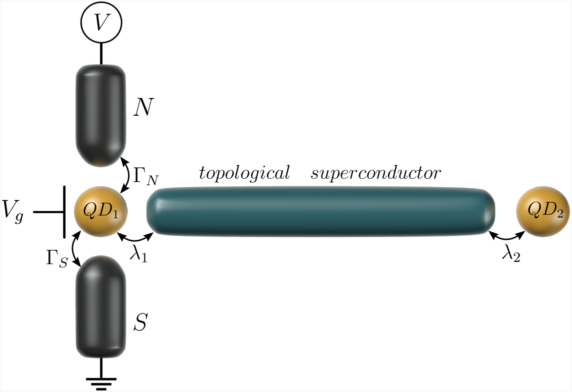
Majorana zero-bias feature establishes in about <u>nanoseconds</u>.

J. Barański, M. Barańska, T. Zienkiewicz, R. Taranko, T.Domański, PRB <u>103</u>, 235416 (2021).

What about distant dynamical phenomena ?

DYNAMICAL CROSS-CORRELATIONS

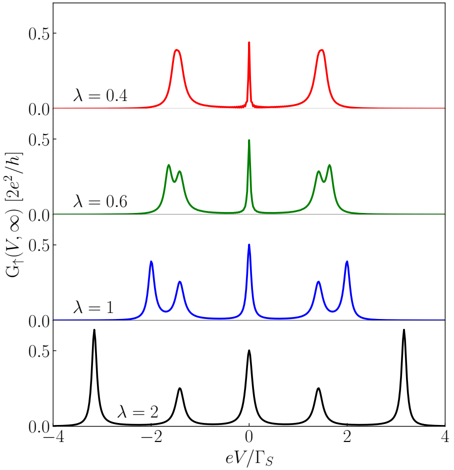
Two quantum dots interconnected via topological superconductor



Question: Is any nonlocal communication transmitted between QD_1 and QD_2 through the Majorana boundary modes ?

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

STEADY-LIMIT CONDUCTANCE

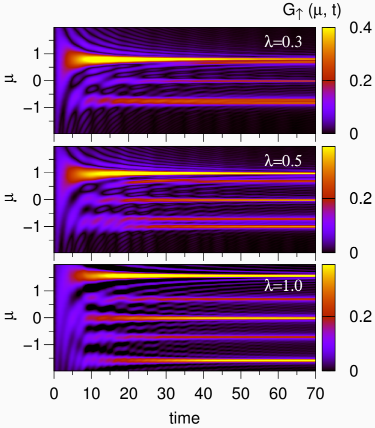


Differential conductance $G(V, t \to \infty)$ versus bias V for several couplings λ between QD$_{1,2}$ and topological superconductor.

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

TIME-RESOLVED CONDUCTANCE

Time-dependent conductance of the biased N-QD$_1$-S junction

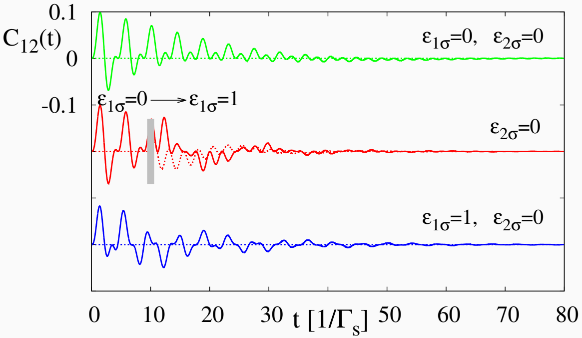


Signatures of the (trivial) molecular bound states and (topological) Majorana mode obtained for $\varepsilon_1 = 0$, $\varepsilon_2 = 2$.

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

NONLOCAL CROSS-CORRELATIONS

Evolution of the interdot electron pairing $C_{12}(t) = \left\langle \hat{d}_{1\downarrow}\hat{d}_{2\uparrow} \right\rangle$



The nonlocal electron pairing persists only over a short transient time-scale. It could be detected by crossed Andreev refelections.

R. Taranko, K. Wrześniewski, I. Weymann, T. Domański, Phys. Rev. B 110, 035413 (2024).

CONCLUSIONS: PART 1

Two quantum dots interconnected through (non-overlaping) Majorana modes

CONCLUSIONS: PART 1

Two quantum dots interconnected through (non-overlaping) Majorana modes

$\Longrightarrow$ **are distantly cross-correlated only briefly after attaching them to topological superconductor,**

CONCLUSIONS: PART 1

Two quantum dots interconnected through (non-overlaping) Majorana modes

$\Rightarrow$ **are distantly cross-correlated only briefly after attaching them to topological superconductor,**

$\Rightarrow$ **beyond this transient region they do not show any mutual interdependence.**

CONCLUSIONS: PART 1

Two quantum dots interconnected through (non-overlaping) Majorana modes

$\Longrightarrow$ **are distantly cross-correlated only briefly after attaching them to topological superconductor,**

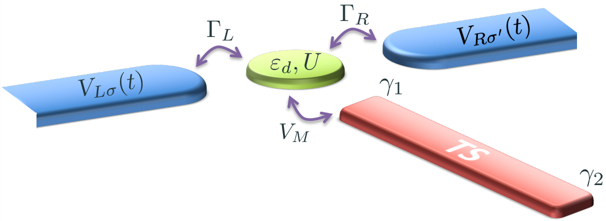
$\Longrightarrow$ **beyond this transient region they do not show any mutual interdependence.**

$\Longrightarrow$ **It implies absence of charge teleportation and/or other nonlocal phenomena outside transient region.**

Other related phenomena

MAJORANA SIGNATURES IN AC-CONDUCTANCE

Quantum dot coupled to the topological nanowire under ac-voltage



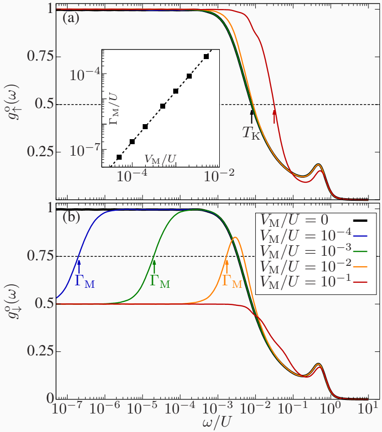
Question:

Can we resolve Majorana and Kondo states in ac-response ?

K.P. Wójcik, T. Domański, I. Weymann, Phys. Rev. B 109, 075432 (2024).

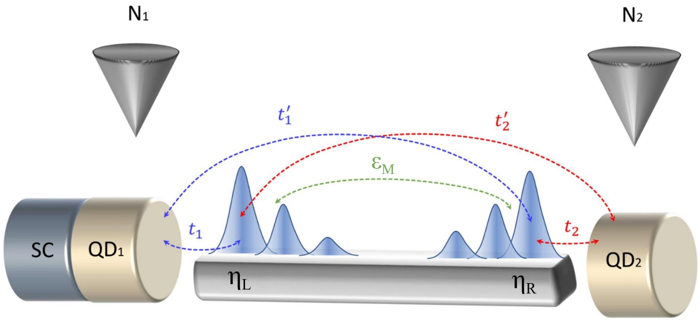
DYNAMICAL FEATURES

The frequency dependent conductance of the *ac*-driven junction



Spin-resolved conductances: **Signatures of the Coulomb peak and the Kondo effect can be clearly distinguished at finite-frequencies.**

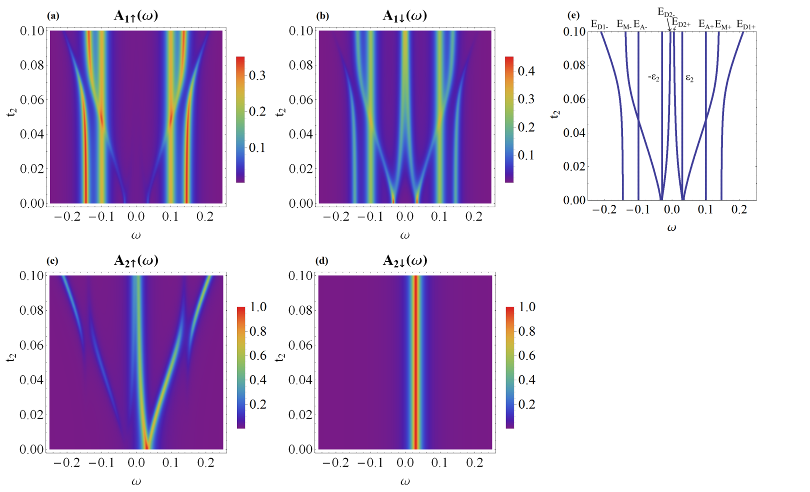
QUASIPARTICLE SPECTRUM OF QUANTUM DOTS



Issue: Molecular spectrum of the quantum dots connected via the overlapping Majorana modes

G. Górski, K.P. Wójcik, J. Barański, I. Weymann & T. Domański, Sci. Rep. 14, 13848 (2024).

QUASIPARTICLE SPECTRUM OF QUANTUM DOTS



The same quasiparticle states are present in both quantum dots , however, with very different spectral weights.

G. Górski, K.P. Wójcik, J. Barański, I. Weymann & T. Domański, Sci. Rep. 14, 13848 (2024).

QUANTUM ENTAGLEMENT OF DOUBLE DOTS

Setup: Quantum dots interconnected via short topological nanowire

Scientific issue:

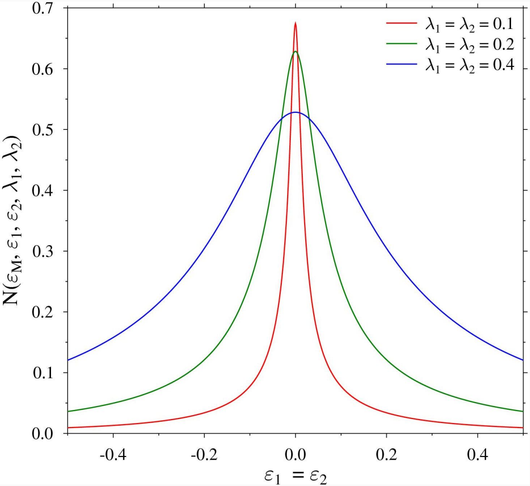
Entanglement of QD's quantified by their fermionic negativity

C. Jasiukiewicz, A. Sinner, I. Weymann, T. Domański & L. Chotorlishvili, (2024)

/to be submitted/.

QUANTUM ENTAGLEMENT OF DOUBLE DOTS

Setup: Quantum dots interconnected via short topological nanowire



Logarythmic negativity versus the energy levels QD's obtained for $\varepsilon_M \neq 0$.

CONCLUSIONS: PART 2

Nanoscopic superconducting structures are useful platform for realization of the bound states:

CONCLUSIONS: PART 2

Nanoscopic superconducting structures are useful platform for realization of the bound states:

$\Longrightarrow$ **conventional (Andreev-type),**

CONCLUSIONS: PART 2

Nanoscopic superconducting structures are useful platform for realization of the bound states:

$\Longrightarrow$ **conventional (Andreev-type),**

$\Longrightarrow$ **topological (Majorana-type).**

CONCLUSIONS: PART 2

Nanoscopic superconducting structures are useful platform for realization of the bound states:

$\Longrightarrow$ **conventional (Andreev-type),**

$\Longrightarrow$ **topological (Majorana-type).**

Both types are promising candidates for stable qubits and/or quantum computations.

CONCLUSIONS: PART 2

Nanoscopic superconducting structures are useful platform for realization of the bound states:

$\Rightarrow$ **conventional (Andreev-type),**

$\Rightarrow$ **topological (Majorana-type).**

Both types are promising candidates for stable qubits and/or quantum computations.

http://kft.umcs.lublin.pl/doman/lectures

ACKNOWLEDGEMENTS

⇒ **M.M. Maśka, M. Dziurawiec**

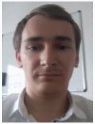
(Technical University, Wrocław)

⇒ **I. Weymann, K. Wrześniewski**

(A. Mickiewicz University, Poznań)

⇒ **K. Wójcik**

(Institute of Molecular Physics PAS, Poznań)



⇒ **Sz. Głodzik, R. Taranko, N. Sedlmayr**

(M. Curie-Skłodowska Univ., Lublin)