Planar Josephson junctions:
platform for topological superconductivity

Tadeusz DOMAŃSKI

UMCS, Lublin

ACMIN AGH (Kraków) **24 May 2023**

OUTLINE

1. **Josephson effect**

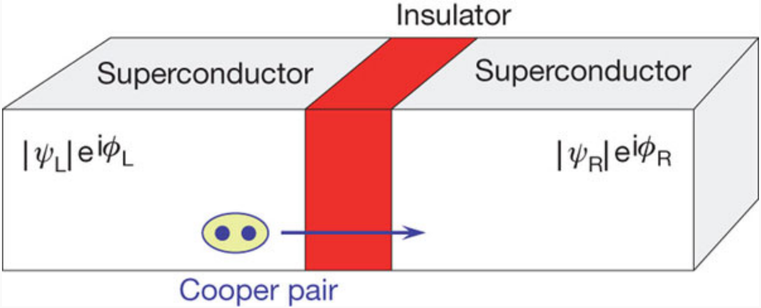
2. **Recent challenges (selected examples)**

3. **Topological superconductivity**

Part 1. Josephson effect

(prediction, discovery, advancements)

Two superconductors, differing in phase $\phi_L \neq \phi_R$, contacted through a narrow ($\leq \xi$) insulating region induce *super*flow of the Cooper pairs.



JOSEPHSON EFFECT

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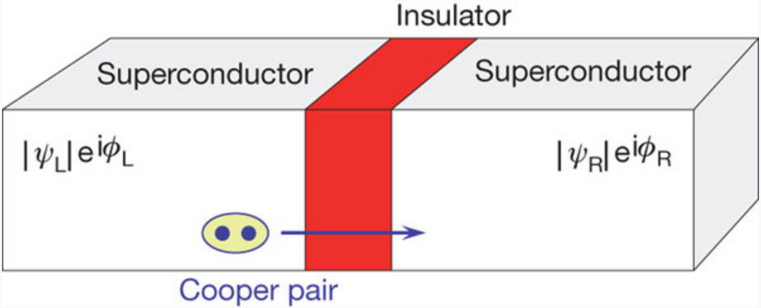


This effect has been predicted by B.D. Josephson in 1962.

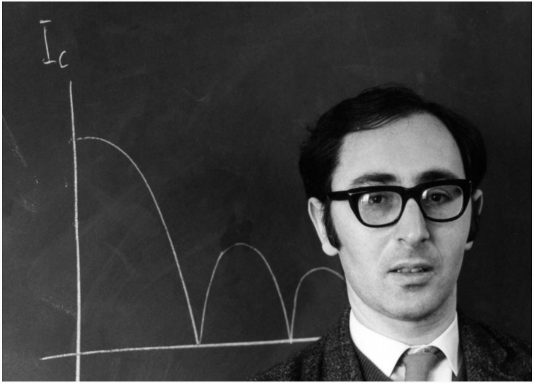
/ 22-year-old PhD student at Cambridge, England /

PREDICTION

B.D. Josephson, Physics Letters 1, 251 (1962).

$\Longrightarrow$ **finite current at zero bias (dc Josephson effect)**

$\Longrightarrow$ **current oscillating with frequency $2eV/\hbar$ in biased junction (ac Josephson effect).**



HYDRODYNAMIC REASONING

In quantum mechanics the probability current is defined by

$$\vec{j}(\vec{r},t) = -\frac{i\hbar}{2m}\left[\Psi^\star(\vec{r},t)\nabla\Psi(\vec{r},t) - \Psi(\vec{r},t)\nabla\Psi^\star(\vec{r},t)\right]$$

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Applying this formalism to the wave-function Φ_0 of Cooper pairs

$$\Phi_0(\vec{r},t) \equiv \underbrace{\left|\Phi_0(\vec{r},t)\right|}_{\sqrt{n(\vec{r},t)}}\; e^{i\phi(\vec{r},t)}$$

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we can express the Josephson current density

$$\vec{j}_J(\vec{r},t) = -\,q\,\frac{i\hbar}{2m}\left[\Phi_0^\star\nabla\Phi_0 - \Phi_0\nabla\Phi_0^\star\right]$$

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where the charge $q = 2e$ and $\vec{v}(\vec{r},t)$ is the Cooper pairs' velocity.

EXPERIMENTAL EVIDENCE

VOLUME 10, NUMBER 6 PHYSICAL REVIEW LETTERS 15 MARCH 1963

PROBABLE OBSERVATION OF THE JOSEPHSON SUPERCONDUCTING TUNNELING EFFECT

P. W. Anderson and J. M. Rowell
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 11 January 1963)

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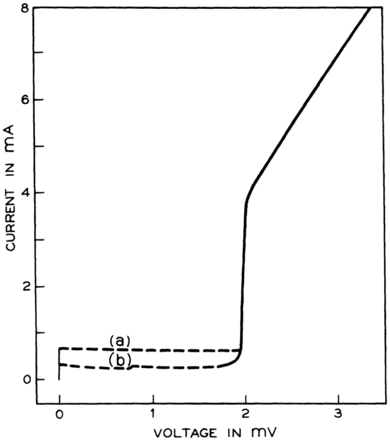


Authors reported on:

„dc tunneling current at or near zero voltage in very thin tin oxide barriers between superconducting Sn and Pb"

1973

B.D. Josephson (with L. Esaki & I. Giaver)

NOBEL PRIZE IN PHYSICS

1973

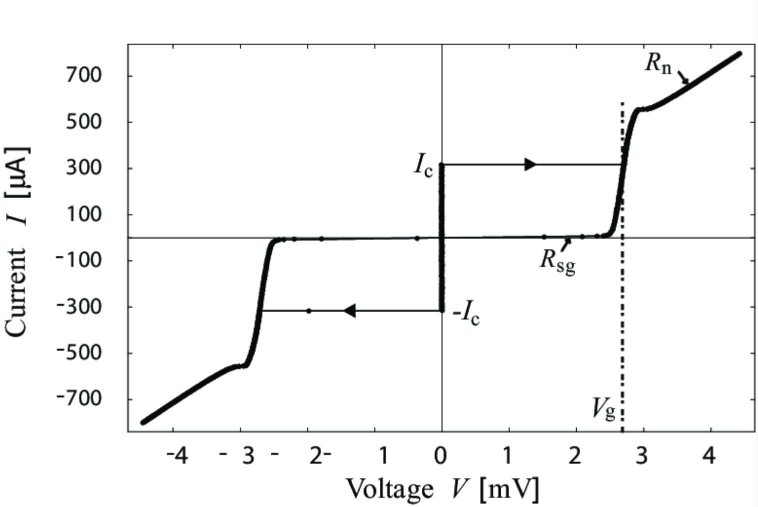
B.D. Josephson (with L. Esaki & I. Giaver)

1972

J. Bardeen, L.N. Cooper, J.R. Schrieffer

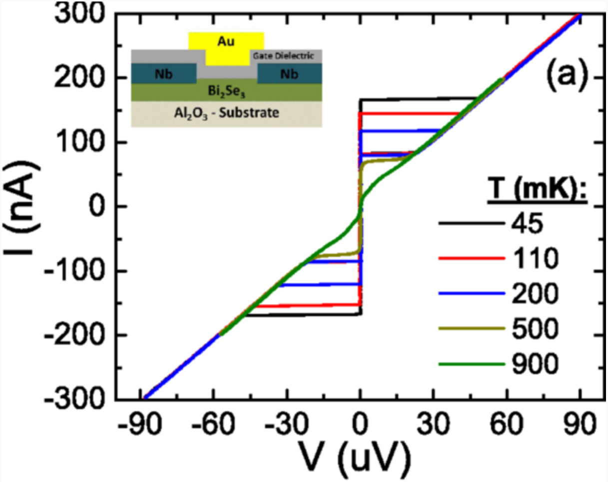
I(V) CHARACTERISTICS

Typical current-voltage plot, where $V_g = 2\Delta$



TEMPERATURE DEPENDENCE

The critical dc current I_c diminishes with increasing temperature.



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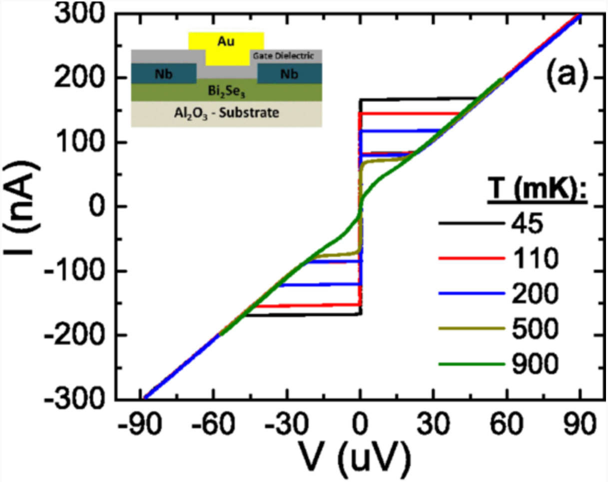
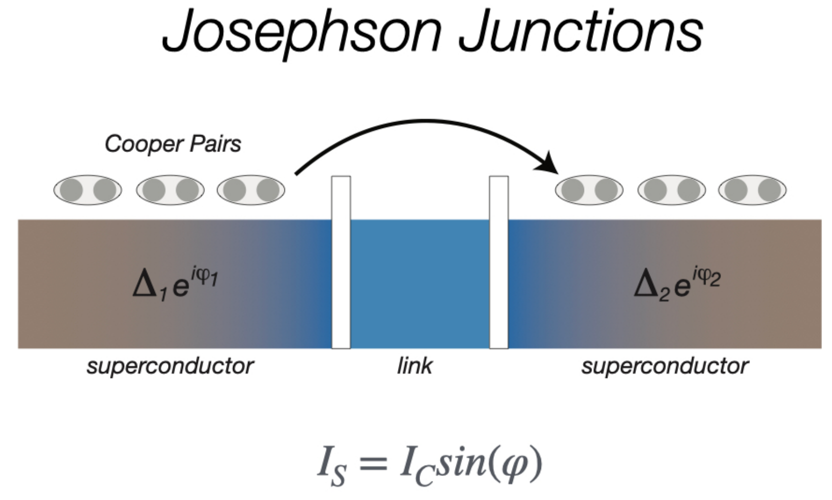


Switching from superflow to resistive current has stochastic character.

DC JOSEPHSON CURRENT

Periodicity: **Superflow of the Copper pairs depends on phase difference, therefore dc current is (usually) periodic with respect to** $\phi = \phi_L - \phi_R$

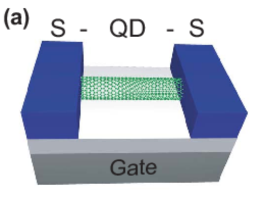


Part 2. Josephson junctions

(currently discussed issues)

1. JOSEPHSON CURRENT REVERSAL

Carbon nanotube interconnecting two superconductors, differing in phase.



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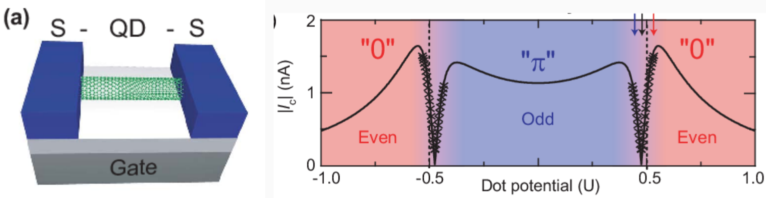
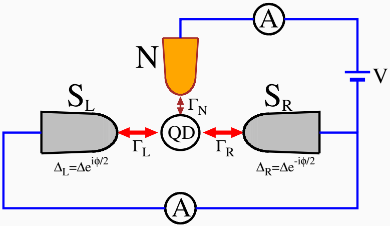


At certain gate potential the dc Josephson current abruptly changed its magnitude and direction (zero-pi transition).

H.I. Jorgensen, T. Novotný, K. Grove-Rasmussen, K. Flensberg, P.E. Lindelof, NanoLett. 7, 2441 (2007).

1. JOSEPHSON CURRENT REVERSAL

Three-terminal geometry



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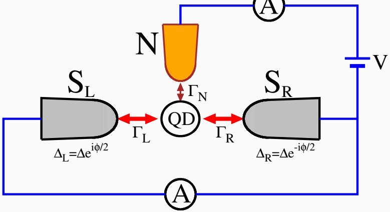
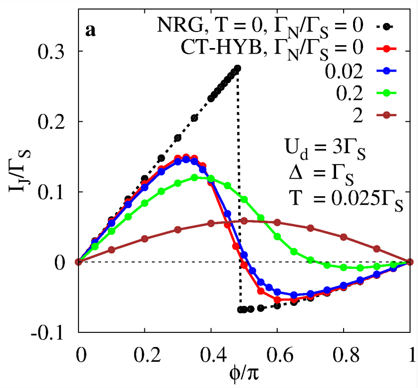




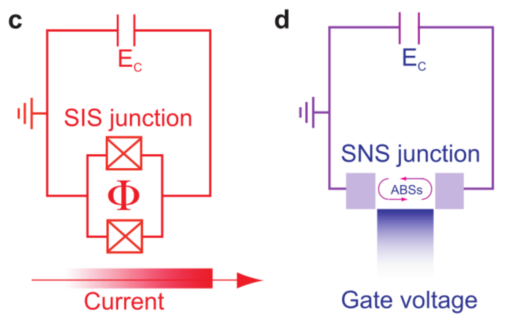
Reversal of dc Josephson current at certain phase difference ϕ is driven by a parity change of the Andreev bound states of QD.

T. Domański, M. Žonda, V. Pokorný, G. Górski, V. Janiš, T. Novotný

Phys. Rev. B 95, 045104 (2017).

2. SUPERCONDUCTING QUBITS

Schematical view of the superconducting quantum bits in realization of: transmon (left) and gatemon (right h.s. panel).

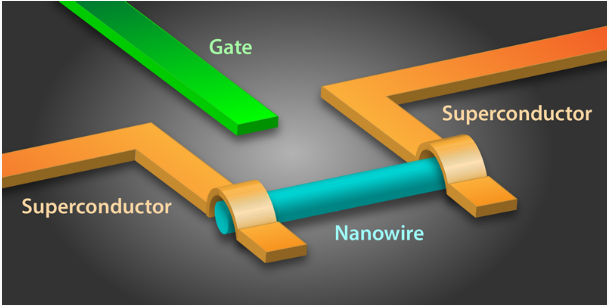


R. Aguado, Appl. Phys. Lett. 117, 240501 (2020).

Superconducting island circuit based on Josephson junction, which is capacitively shunted (E_C is the charging energy).

2. SUPERCONDUCTING QUBITS: GATEMON

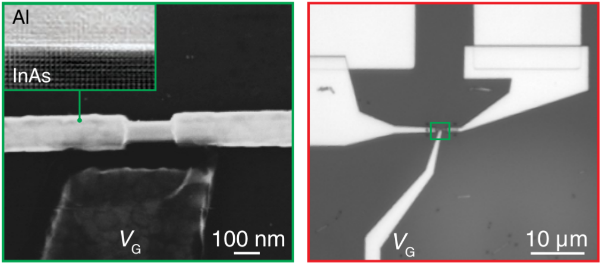
Idea: Electrical control over the Josephson supercurrent through semiconducting nanowire accomplished by a side-gate potential.



J.M. Nichol, Physics 8, 87 (2015).

2. SUPERCONDUCTING QUBITS: EXPERIMENT

Semiconducting (InAs) nanowire coupled to superconducting (Al) which is controlled by an electrostatic gate that depletes the carriers in a weak link region.



T.W. Larsen et al, Phys. Rev. Lett. 115, 127001 (2015).

Reported relaxation times $\sim$ 0.8 μs and dephasing times $\sim$ 1 μs exceeded the gate operation times by 2 orders.

3. JOSEPHSON DIODE

The field-free Josephson diode in a van der Waals heterostructure

https://doi.org/10.1038/s41586-022-04504-8

Received: 29 March 2021

Accepted: 2 February 2022

Published online: 27 April 2022

Check for updates

Heng Wu[1,2,3,6]✉, Yaojia Wang[1,3,6], Yuanfeng Xu[1,4], Pranava K. Sivakumar[1], Chris Pasco[5], Ulderico Filippozzi[3], Stuart S. P. Parkin[1], Yu-Jia Zeng[2], Tyrel McQueen[5] & Mazhar N. Ali[1,3]✉

The superconducting analogue to the semiconducting diode, the Josephson diode, has long been sought with multiple avenues to realization being proposed by theorists[1–3]. Showing magnetic-field-free, single-directional superconductivity with Josephson coupling, it would serve as the building block for next-generation superconducting circuit technology. Here we realized the Josephson diode by fabricating an inversion symmetry breaking van der Waals heterostructure of $NbSe_2$/Nb_3Br_8/$NbSe_2$. We demonstrate that even without a magnetic field, the junction can be superconducting with a positive current while being resistive with a negative current.

Nature | Vol 604 | 28 April 2022 | 653.

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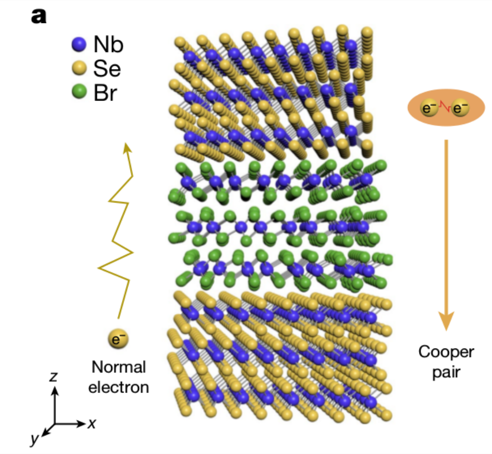
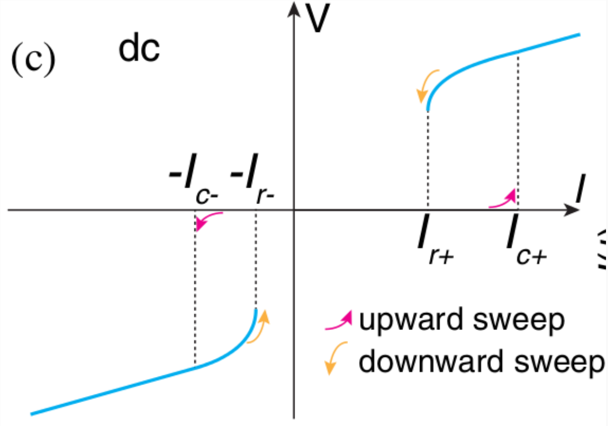
Discovery of the magnetic field - free superconducting diode in van der Waals heterostructure of $NbSe_2/Nb_3Br_8/NbSe_2$.

3. JOSEPHSON DIODE

Niobium bromide (just a few atoms thick) placed between layers of superconducting niobium diselenide does conduct electricity without resistance solely in one direction of the applied voltage.



Mechanism behind the Josephson diode effect in not fully understood.



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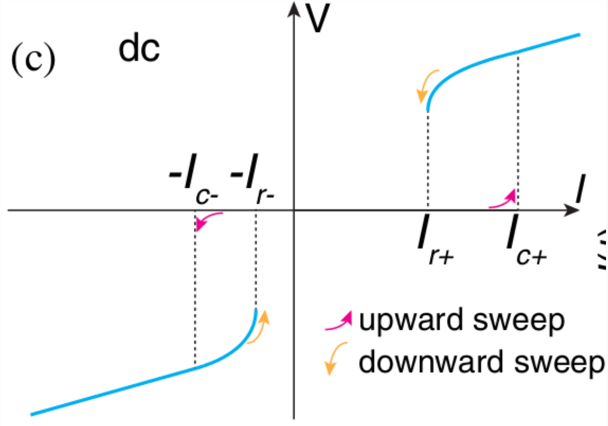


Speculations:

Major role plays the asymmetric sc proximity process inside the tunneling barrier due to: (a) inversion symmetry breaking, (b) time-reversal breaking.

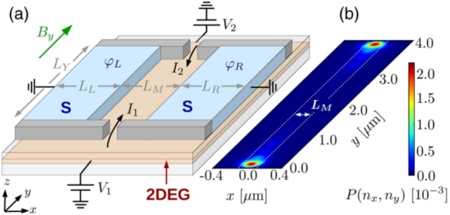
Part 3. Topological superconductivity

(in Josephson junctions)

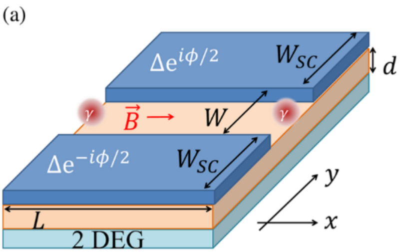
Theoretical concept (2017)

PLANAR JOSEPHSON JUNCTIONS

Idea: Narrow metallic region with the strong spin-orbit interaction and in presence of magnetic field embedded between external superconductors.



Michael Hell et al., PRL 118, 107701 (2017)



F. Pientka et al., Phys. Rev. X 7,021032 (2017)

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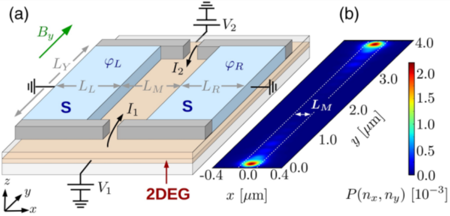
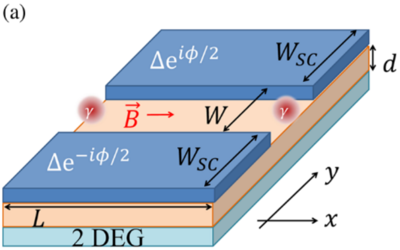


Michael Hell et al., PRL 118, 107701 (2017)



F. Pientka et al., Phys. Rev. X 7,021032 (2017)

Benefit:

Phase-tunable topological superconductivity induced in the metallic stripe.

PLANAR JOSEPHSON JUNCTIONS

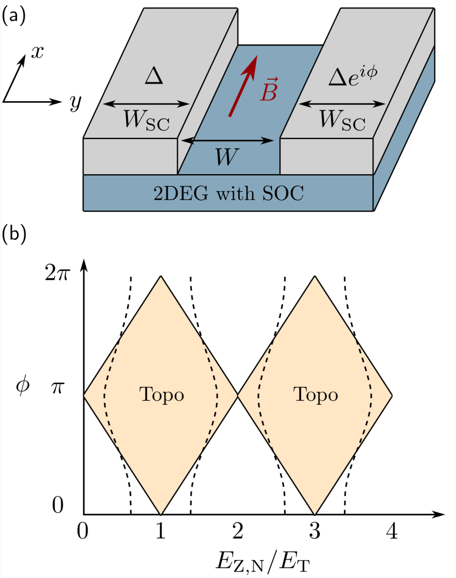
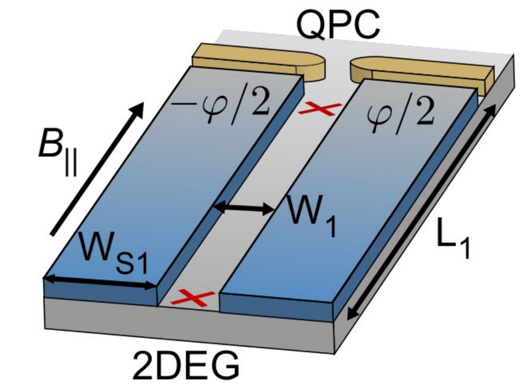


Diagram of topological superconducting state vs
– phase difference ϕ,
– magnetic field E_z.

Experimental realization (2019)

PLANAR JOSEPHSON JUNCTIONS

Two-dimensional electron gas of InAs epitaxially covered by a thin Al layer



Width:

W_1 = 80 nm

Length:

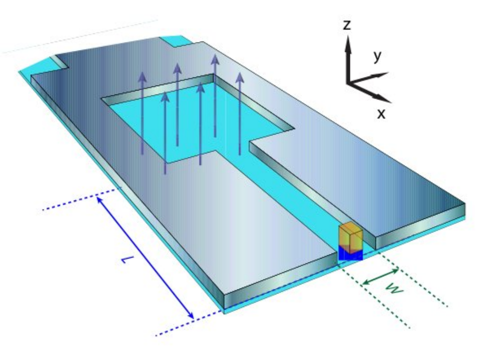
L_1 = 1.6 μm

A. Fornieri, ..., Ch. Marcus and F. Nichele, Nature 569, 89 (2019).

Niels Bohr Institute (Copenhagen, Denmark)

PLANAR JOSEPHSON JUNCTIONS

Two-dimensional HgTe quantum well coupled to 15 nm thick Al film



Width:

W = **600 nm**

Length:

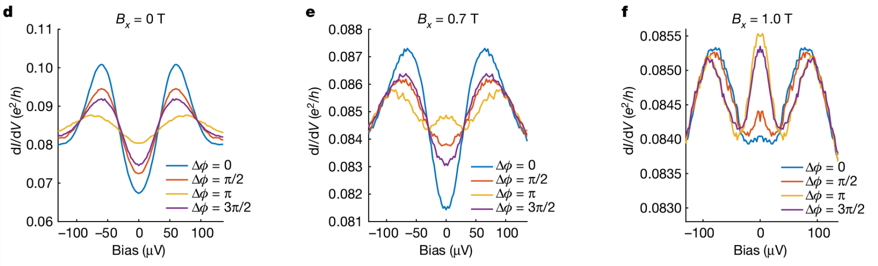
L = **1.0 μm**

H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).

Würzburg Univ. (Germany) + Harvard Univ. (USA)

PLANAR JOSEPHSON JUNCTION: EXPERIMENT

H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).



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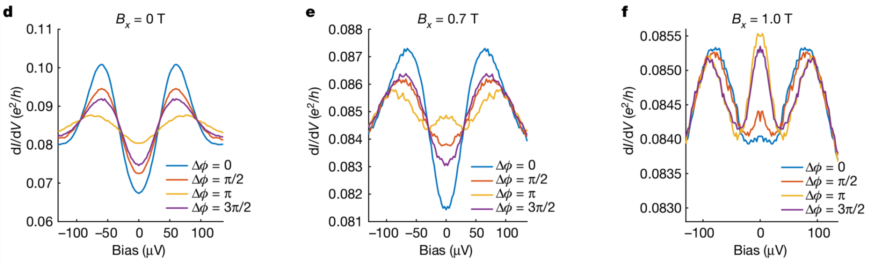
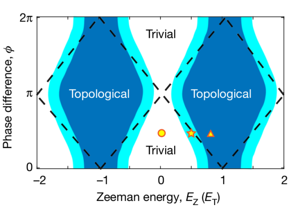


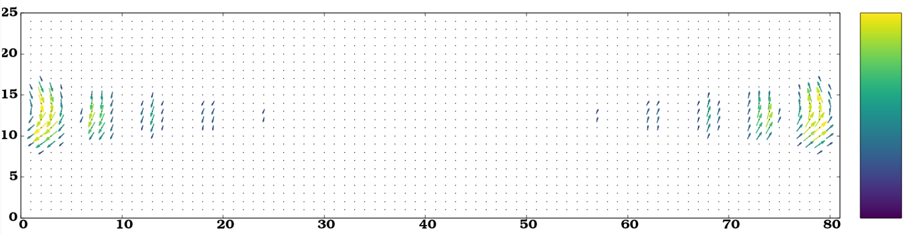
Experimental data obtained for three different magnetic fields indicated by the symbols in phase diagram ⇒.



Topography of Majorana modes

TOPOGRAPHY OF MAJORANA MODES

Spatial profile of the zero-energy quasiparticles of a homogeneous metallic strip embedded into the Josephson junction for the phase difference $\phi = \pi$ (which is optimal for topological state).



"Majorana polarization" $u_{\uparrow,n}v_{\uparrow,n} - u_{\downarrow,n}v_{\downarrow,n}$ obtained for eigenvalue $E_n = 0$.

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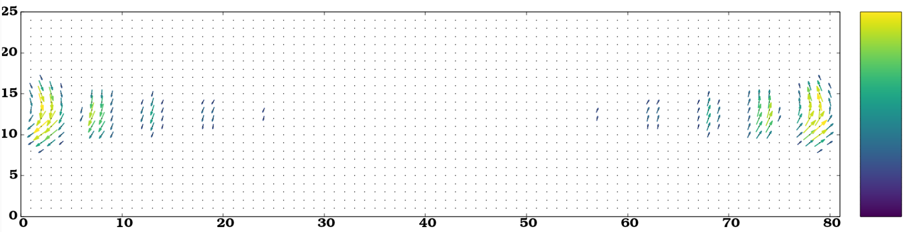
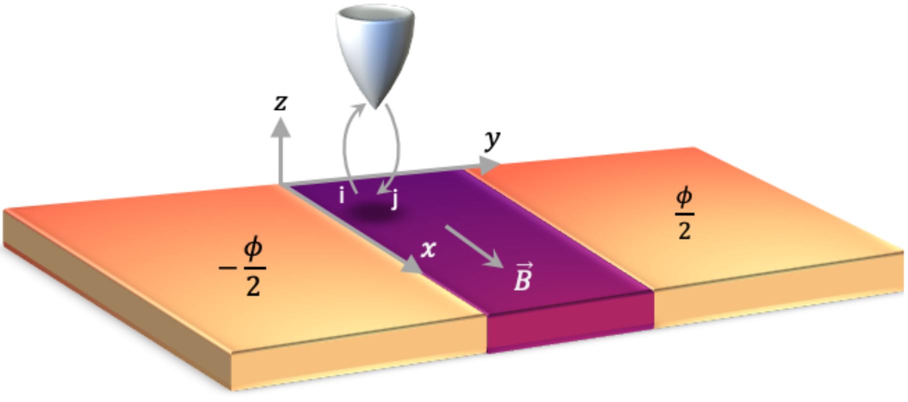


"Majorana polarization" $u_{\uparrow,n}v_{\uparrow,n} - u_{\downarrow,n}v_{\downarrow,n}$ obtained for eigenvalue $E_n = 0$. Magnitude of this quantity is measurable by the conductance of SESAR spectroscopy. For details see:

Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

Selective Equal Spin Andreev Reflection (SESAR) spectroscopy:

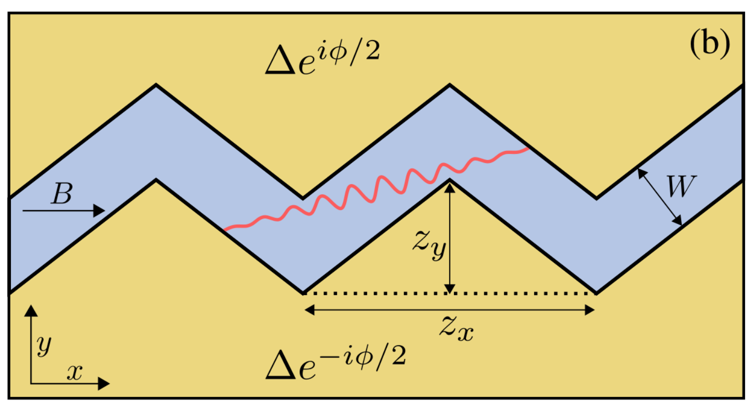


Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

Means to localize Majoranas

I. DESHAPED JOSEPHSON JUNCTION

To reduce spatial extent of the Majorana modes and increase the topological gap one can use zigzag-shape metallic stripe.



T. Laeven, B. Nijholt, M. Wimmer & A.R. Akhmerov, PRL 102, 086802 (2020).

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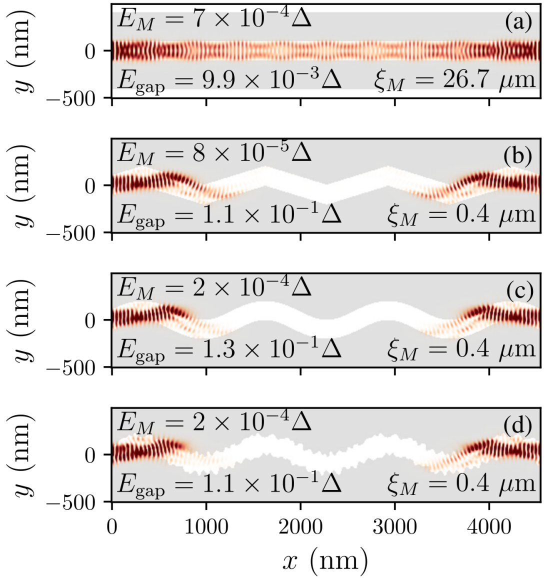
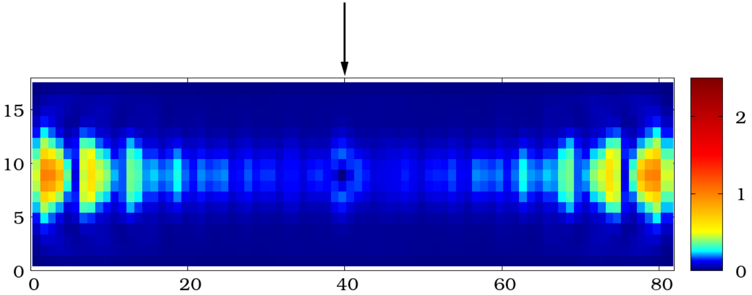


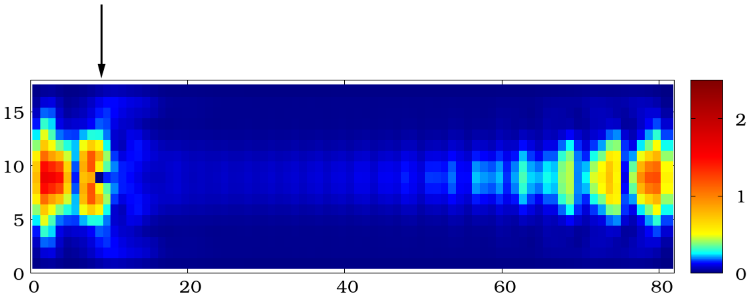
T. Laeven, B. Nijholt, M. Wimmer & A.R. Akhmerov, PRL 102, 086802 (2020).

Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed in the center.



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed near the edge.



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

III. RANDOM DISORDER

"Benefits of Weak Disorder in dim=1 Topological Superconductors"

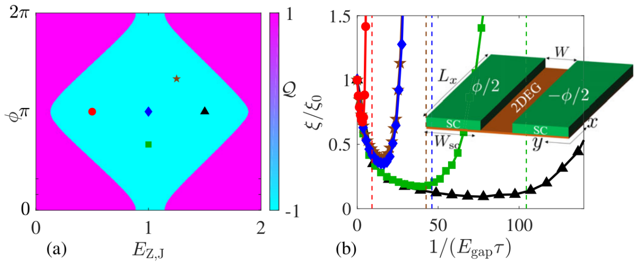
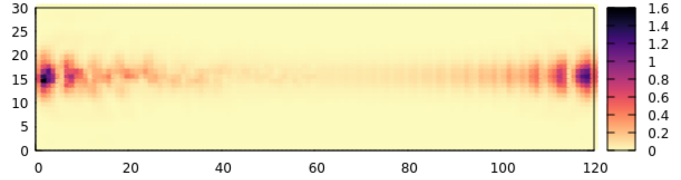


FIG. 1. (a) Phase diagram of the planar Josephson junction Eq. (1) in the clean limit. In the topological phase ($\mathcal{Q} = -1$), the system supports zero-energy MBSs at each end of the junction. (b) The Majorana localization length ξ versus the disorder-induced inverse mean free time τ^{-1} for different points inside the topological phase [see markers in (a)].

A. Haim & A. Stern, Phys. Rev. Lett. 122, 126801 (2019).

III. RANDOM DISORDER

The left-hand-side part of the metallic stripe is randomly disordered



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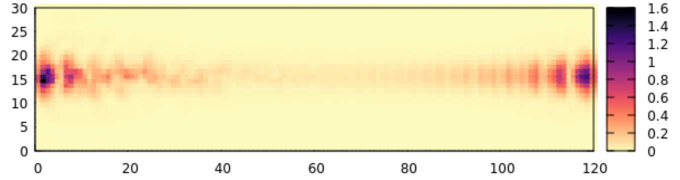
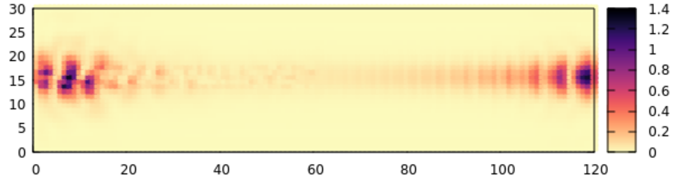


weak disorder



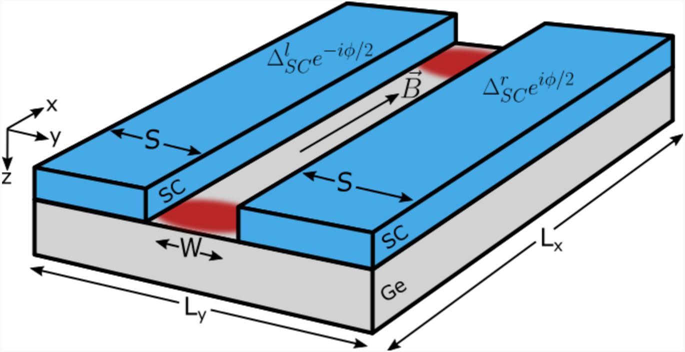
moderate disorder

Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

New proposals

1. GERMANIUM BASED PLANAR JJ

2D hole gas of germanium (Ge) exhibits strong and tunable spin-orbit interaction (cubic in momentum). Such Ge structures can be compatible with the existing metal-oxide-semiconductor (CMOS) technology.



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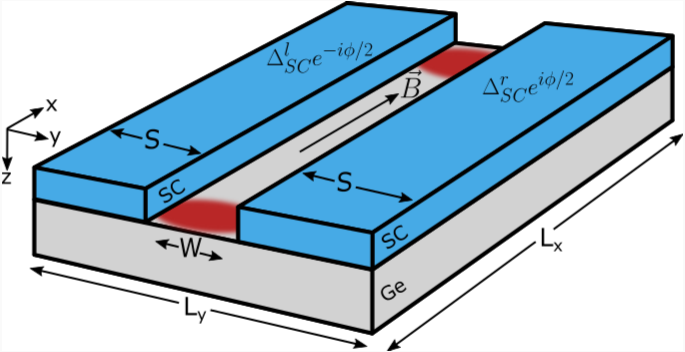

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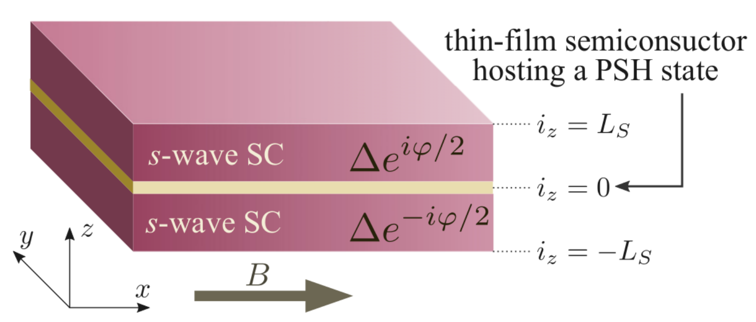
$\Longrightarrow$ **topological phase is asymmetric on phase reversal $\phi \longrightarrow -\phi$**

2. VERTICAL JOSEPHSON JUNCTION

Josephson junction, comprising a thin semiconducting film sandwiched between conventional s-wave superconductors.



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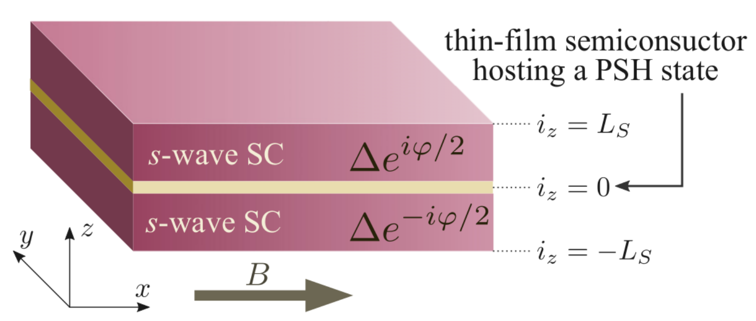

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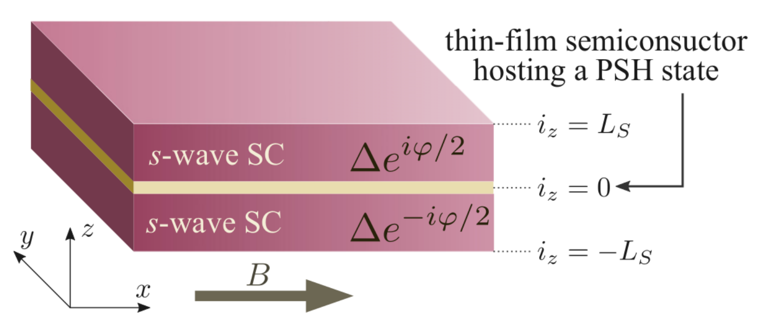

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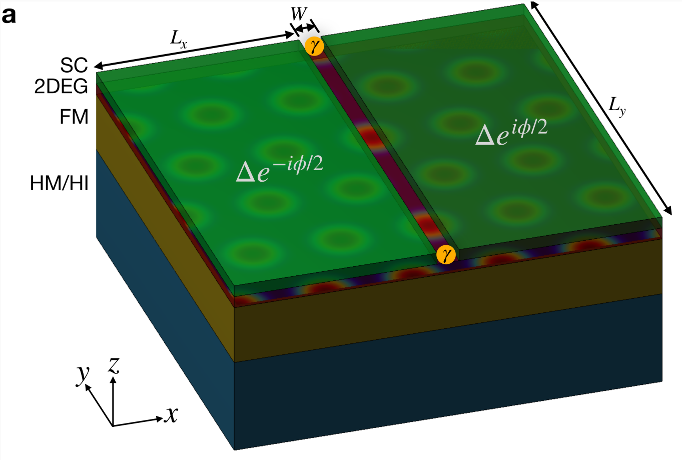
$\Longrightarrow$ **semiconducting region hosts a Persistent Spin-Helix state**

$\Longrightarrow$ **a weak Zeeman field can induce p_x-wave superconductivity**

3. SKYRMIONS UNDER JOSEPHSON JUNCTION

Josephson junction deposited on top of a skyrmion crystal.



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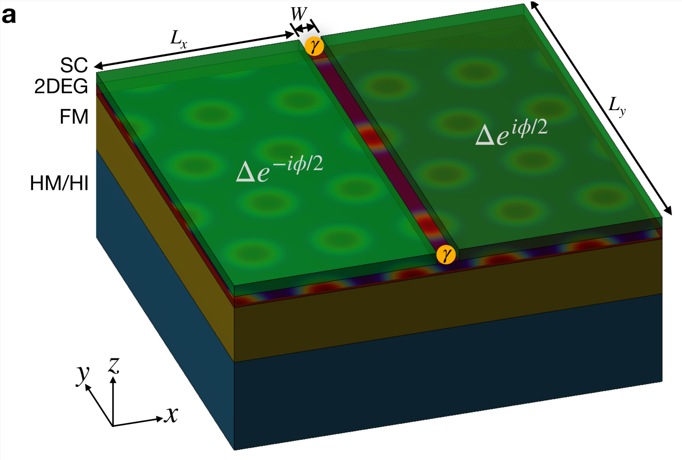

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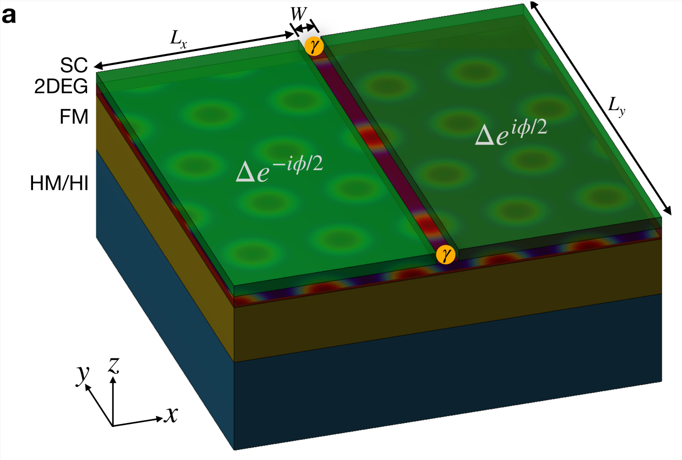

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FM - ferromagnetic layer **HM/HI - heavy metal or heavy insulator**

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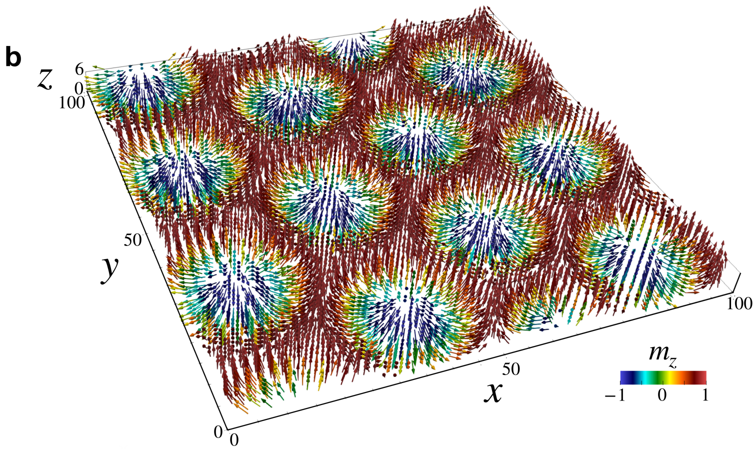

Skyrmions are driven in FM region by:

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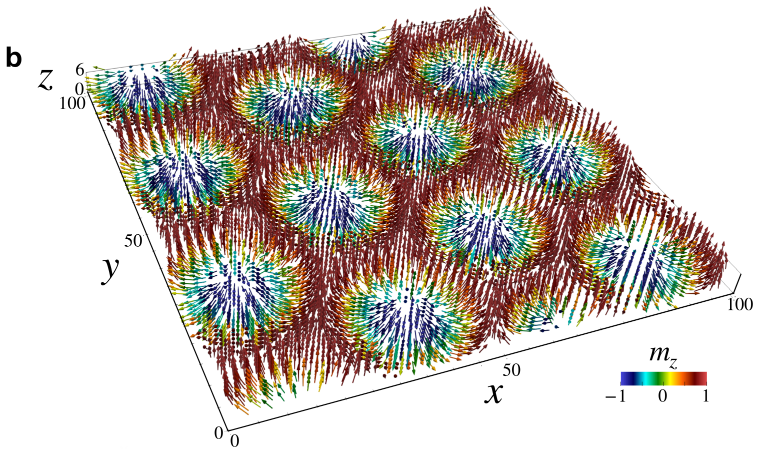

Skyrmions are driven in FM region by: ★ **ferromagnetic exchange**

★ **Dzialoshinskii-Moriya interaction** ★ **magnetic field**

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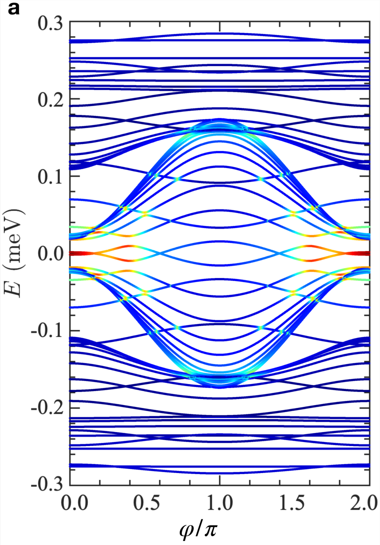
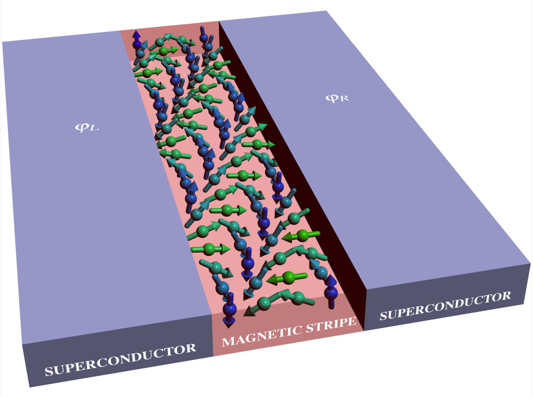


Phase difference has detrimental influence on the Majorana modes.

4. JJ WITH SELFORGANIZED MAGNETIC STRIPE

Narrow metallic stripe with the classical magnetic moments placed between two s-wave superconductors, differing in phase $\phi_L \neq \phi_R$.



M.M. Maśka, M. Dziurawiec, M. Strzałka & T.D. **– work in progress**

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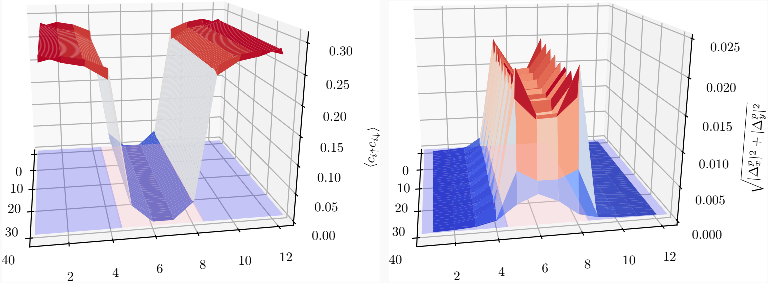
4. JJ WITH SELFORGANIZED MAGNETIC STRIPE

The effective s-wave (left) and induced p-wave (right) pairings.



Width: left superconductor (sites 1-4),
metallic stripe (sites 5-8),
right superconductor (sites 9-12),

Length: 40 sites.

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Classical moments of the metallic region develop such magnetic textures, which support the topological superconducting state

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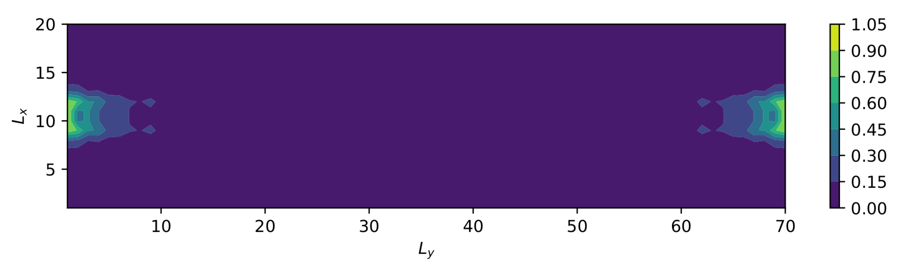

Spatial profile of the Majorana modes.

M.M. Maśka, M. Dziurawiec, M. Strzałka & T.D. – work in progress

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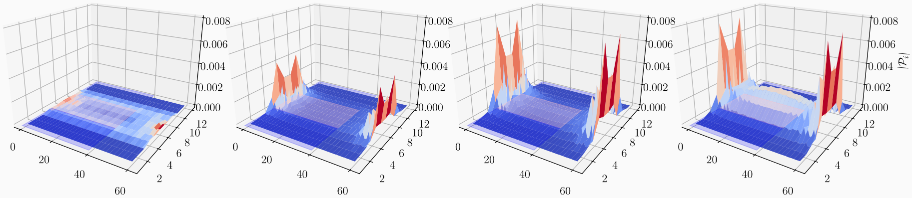
Spatial profiles of the (zero-energy) Majorana quasiparticles for selected values of the phase difference $\phi_R - \phi_L$.



$\phi_R - \phi_L = 0.6\pi$ $\quad$ $\phi_R - \phi_L = 0.4\pi$ $\quad$ $\phi_R - \phi_L = 0.2\pi$ $\quad$ $\phi_R - \phi_L = 0.0$

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4. JJ WITH SELFORGANIZED MAGNETIC STRIPE

In analogy to the skyrmion based Josephson junction, we observe that the optimal condition for the topological phase occurs at $\phi_L - \phi_R = 0$.

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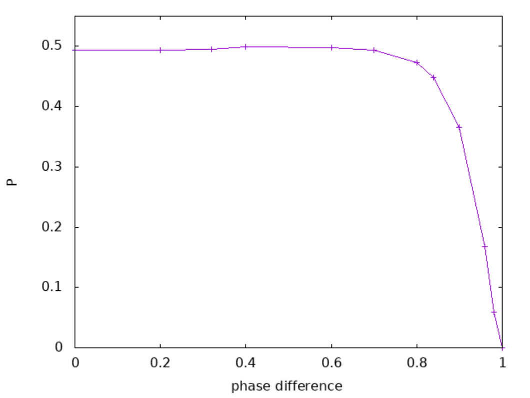

The integrated Majorana spectral weight versus the phase difference.

M.M. Maśka, M. Dziurawiec, M. Strzałka & T.D. – work in progress

Magnetism in Josephson-type geometries:

PART 3: SUMMARY

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$\Rightarrow$ **cooperates with superconductivity**

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http://kft.umcs.lublin.pl/doman/lectures

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⇒ **Maciek Maśka & coworkers**

(Technical University, Wrocław)

⇒ **Nick Sedlmayr**

(M. Curie-Skłodowska University, Lublin)

⇒ **Aksel Kobiałka**

(University of Basel, Switzerland)

⇒ **Szczepan Głodzik**

(University of Ljubljana, Slovenia)

SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

The proximitized quantum dot can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^\dagger_\sigma \, \hat{d}_\sigma \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \, \hat{d}^\dagger_\uparrow \hat{d}^\dagger_\downarrow + \text{h.c.} \right)$$

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Eigen-states of this problem are represented by:

$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left. \begin{array}{l} u \, |0\rangle - v \, |\uparrow\downarrow\rangle \\ v \, |0\rangle + u \, |\uparrow\downarrow\rangle \end{array} \right\} \Leftarrow$$ **singlet states (spin 0)**

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Upon varrying the parameters ε_d, U_d or Γ_S there can be induced quantum phase transition between these doublet/singlet states.