Kraków, 4 marca 2013 r.

Procesy Andreeva w silnie skorelowanych układach fermionowych

T. Domański

Uniwersytet Marii Curie–Skłodowskiej

w Lublinie

http://kft.umcs.lublin.pl/doman/lectures

Outline

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1. **Introduction**

/ underlying idea /

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2. **Andreev transport via quantum dots**

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5. **Andreev scattering in ultracold gasses**

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1. Introduction

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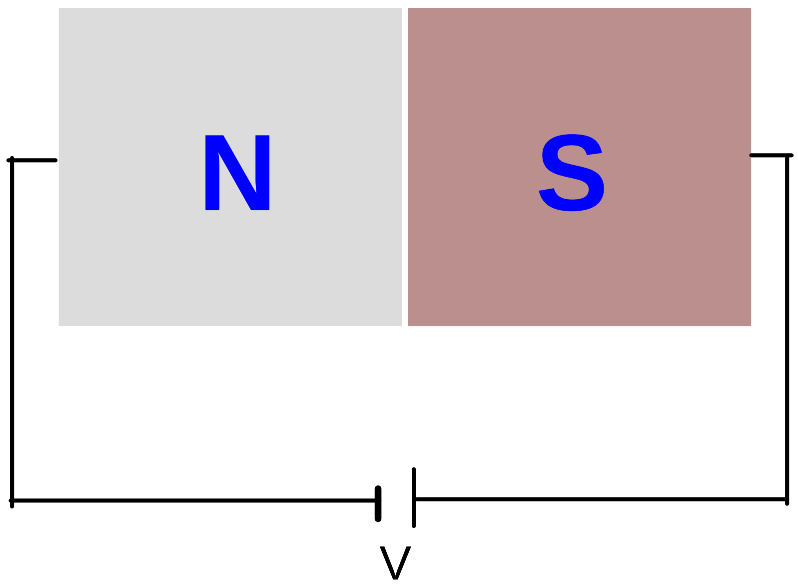

Let us restrict to the subgap regime $|eV| \ll \Delta$ of an applied bias V.

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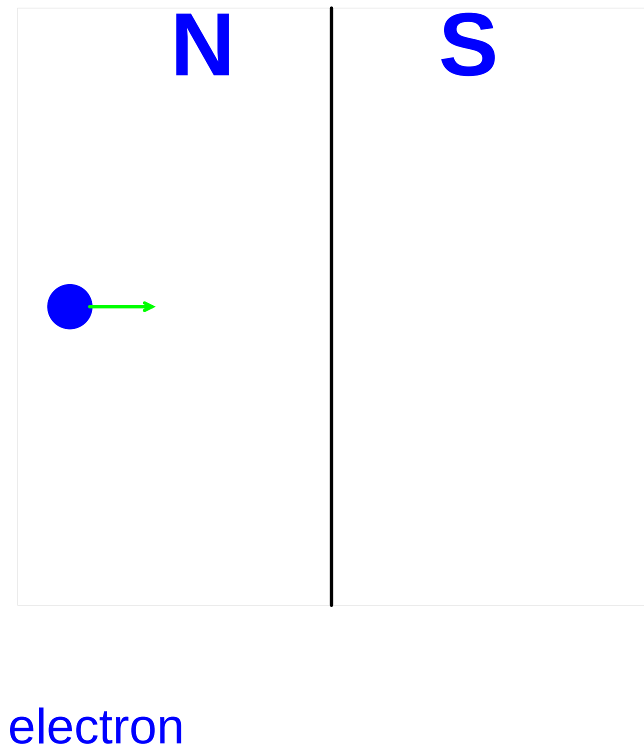

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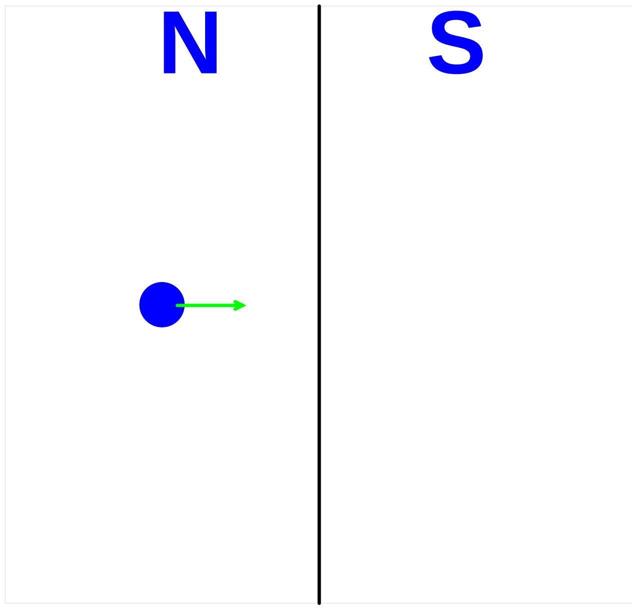

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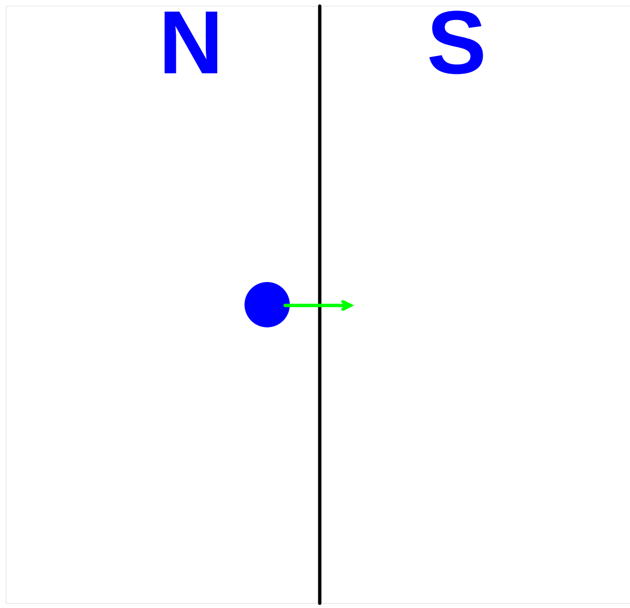

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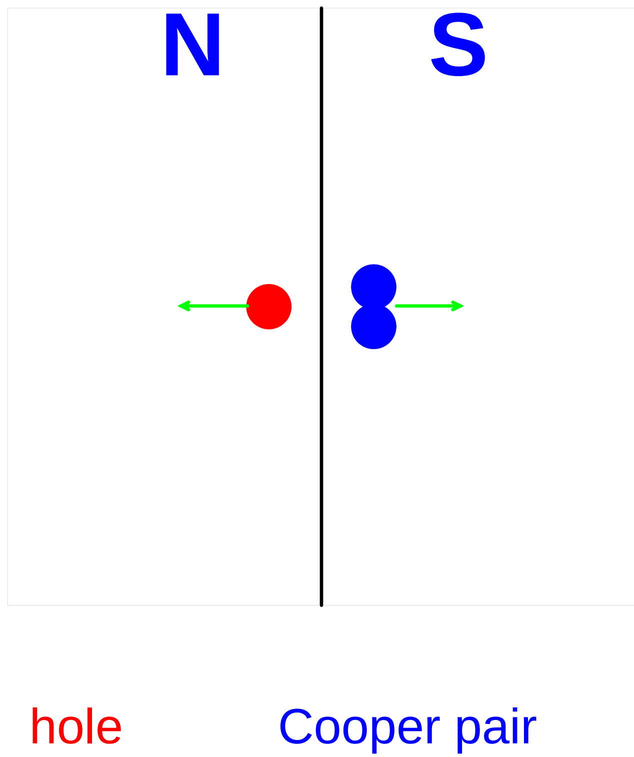

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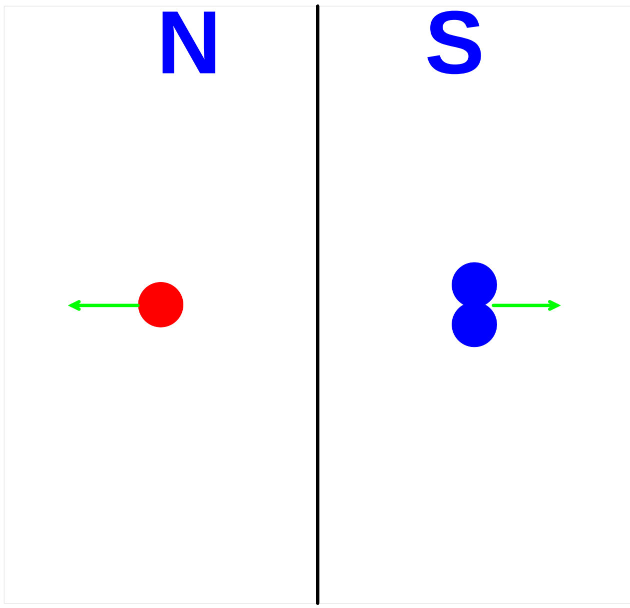

hole Cooper pair

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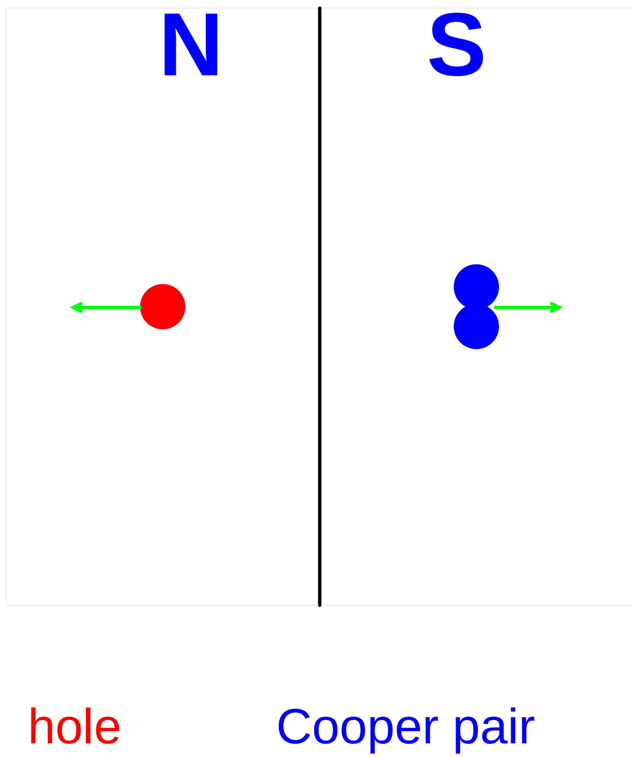

Such double-charge exchange is named the Andreev reflection (scattering).

Andreev reflections – historical remark

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A.F. Andreev

/ P. Kapitza Institute, Moscow (Russia) /

A.F. Andreev, Sov. Phys. JETP **19**, *1228 (1964).*

2. Andreev transport via quantum dot

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N-QD-S scheme

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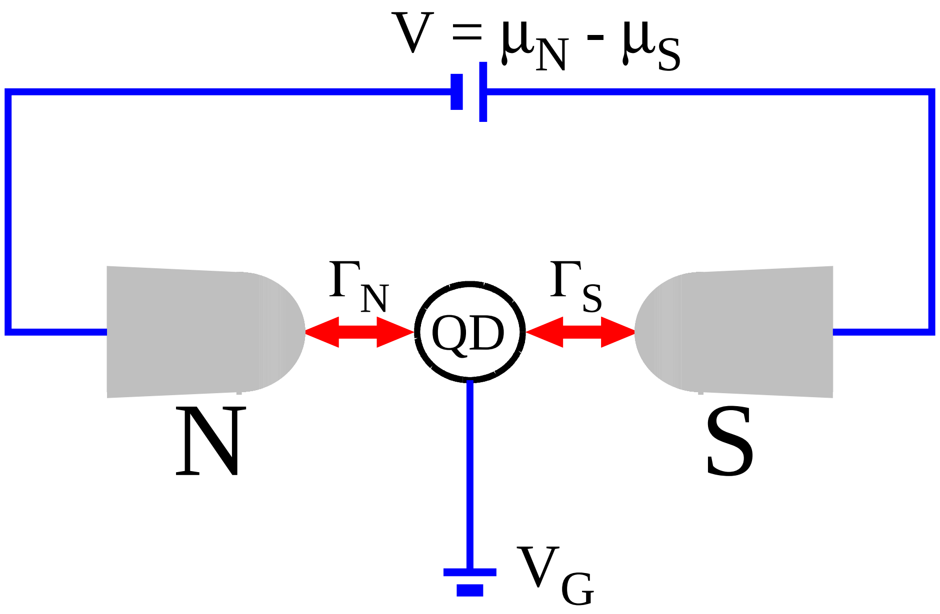

metallic lead **QD** **superconductor**

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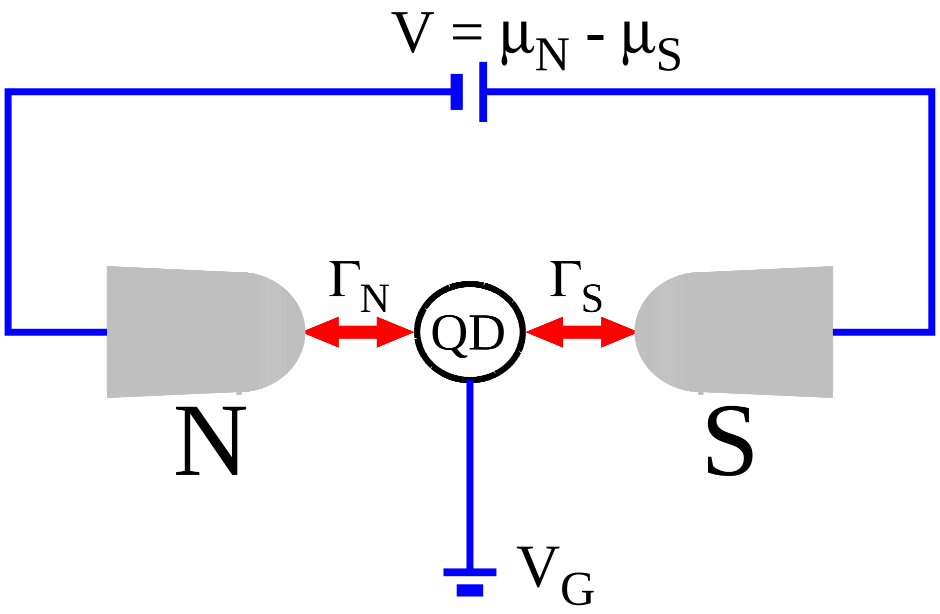

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This setup can be thought of as a particular version of the SET.

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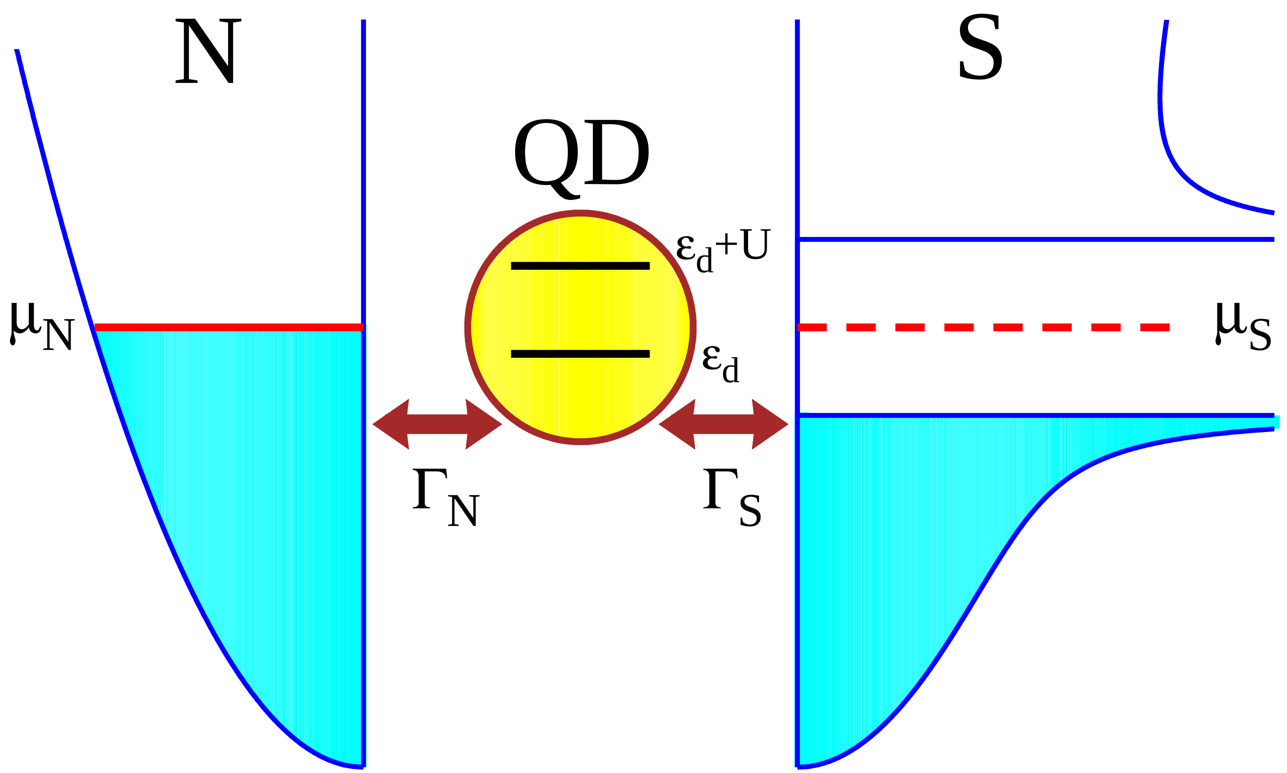

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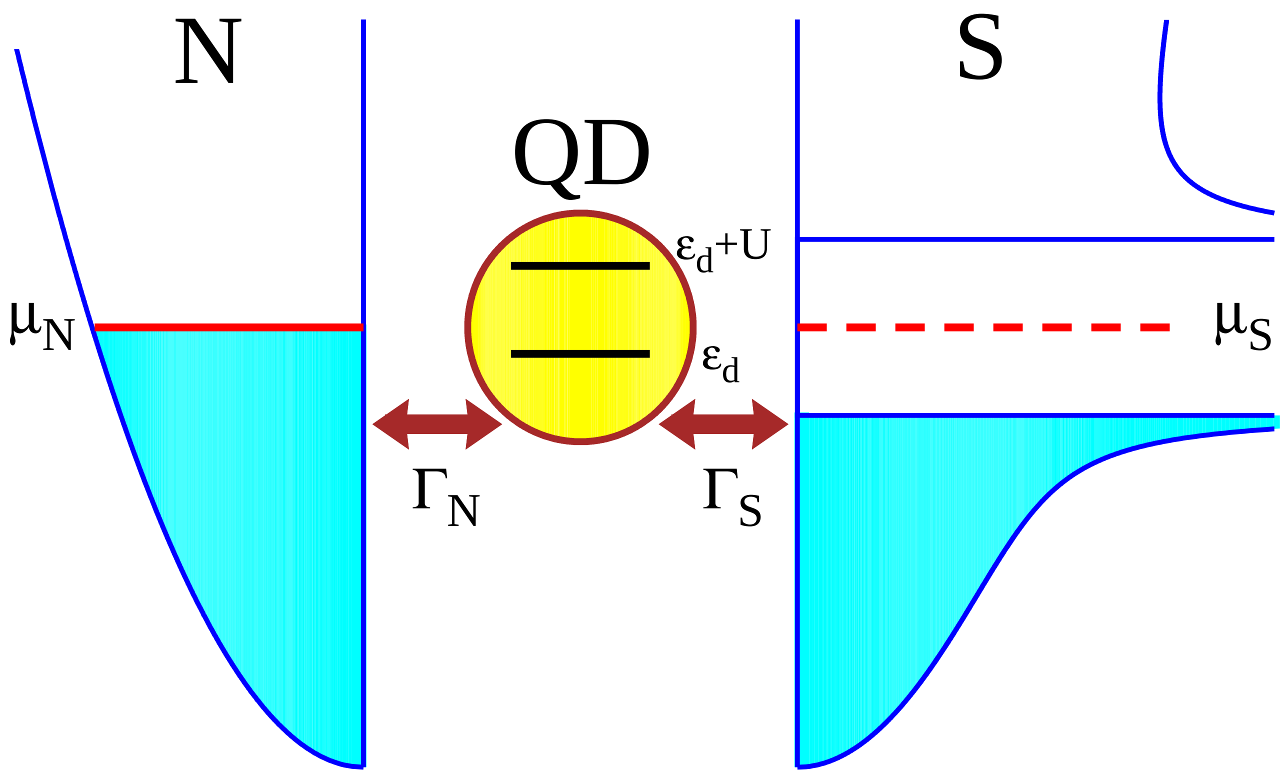

External bias $eV = \mu_N - \mu_S$ induces the current(s) through QD.

Microscopic model

The correlation effects

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$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U \, \hat{n}_{d\uparrow} \, \hat{n}_{d\downarrow}$$

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where

$$\hat{H}_N = \sum_{k,\sigma} \left(\varepsilon_{k,N} - \mu_N \right) \hat{c}^{\dagger}_{k\sigma N} \hat{c}_{k\sigma N}$$

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where

$$\hat{H}_S = \sum_{k,\sigma} \left(\varepsilon_{k,S} - \mu_S \right) \hat{c}^{\dagger}_{k\sigma S} \hat{c}_{k\sigma S} - \sum_{k} \left(\Delta \hat{c}^{\dagger}_{k\uparrow S} \hat{c}^{\dagger}_{k\downarrow S} + \text{h.c.} \right)$$

Relevant problems :

issue # 1

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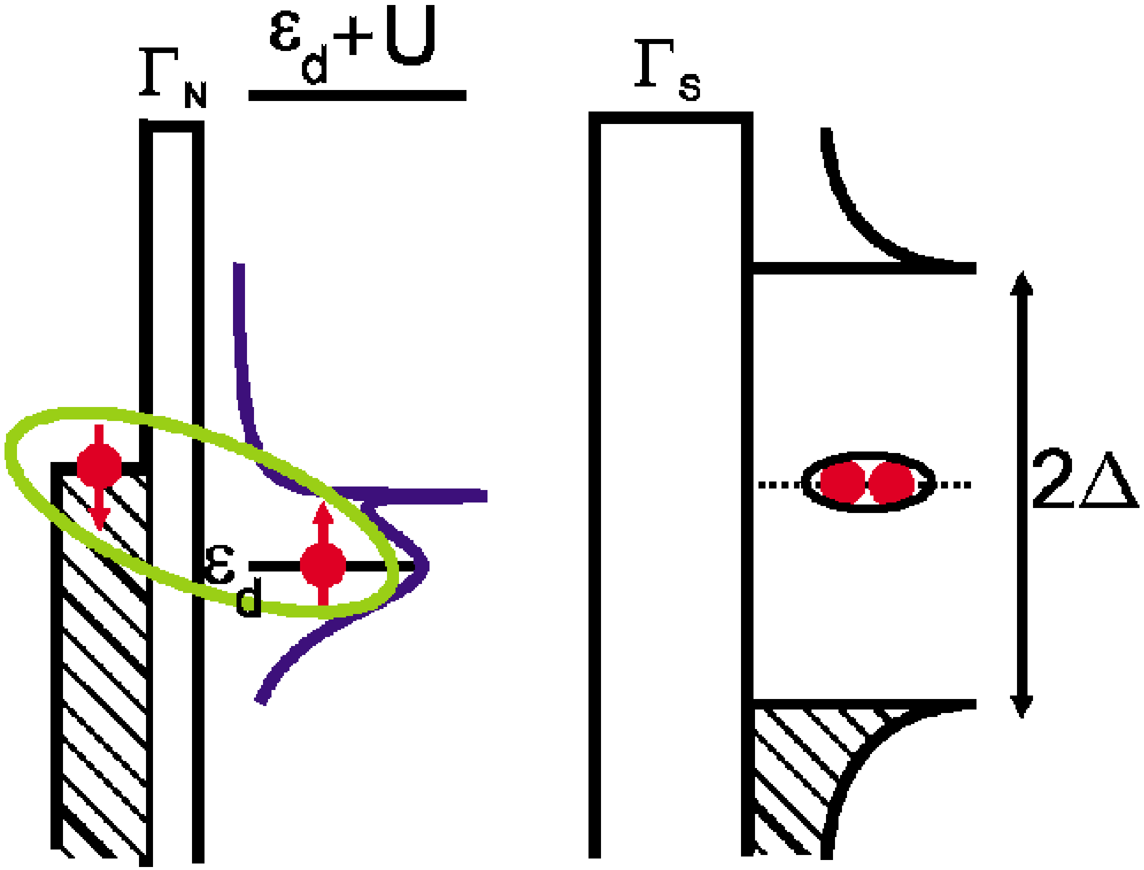

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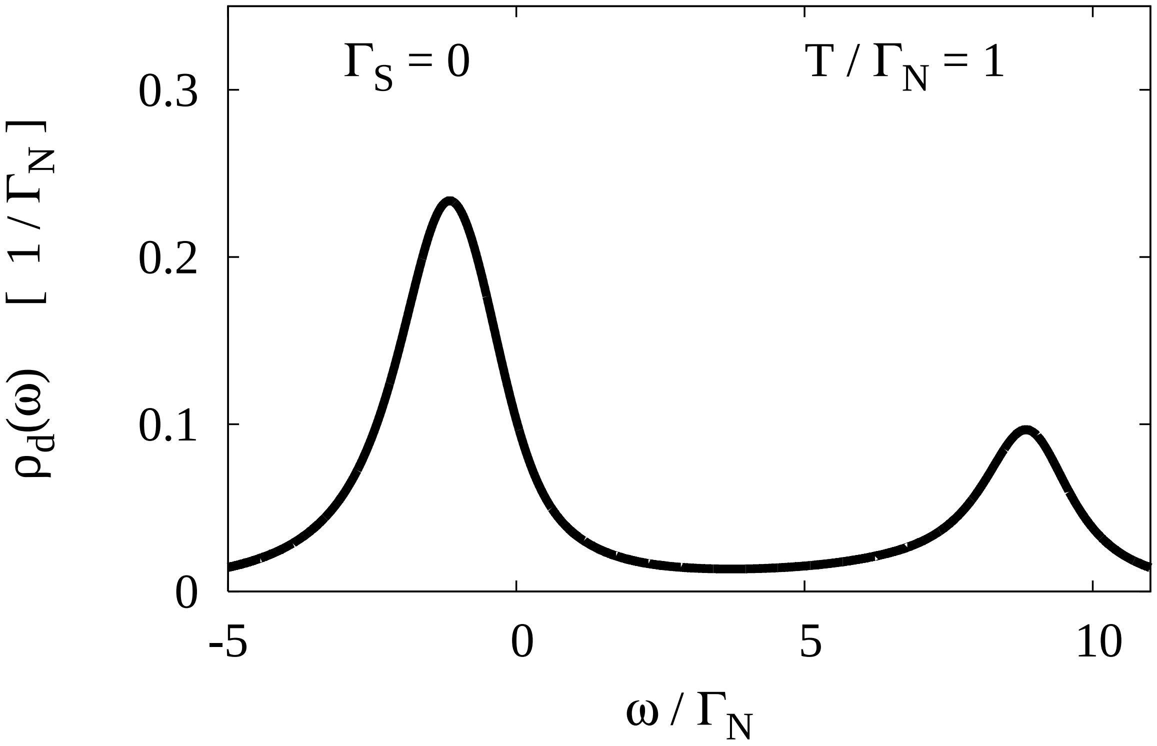

★ a broadening of QD levels

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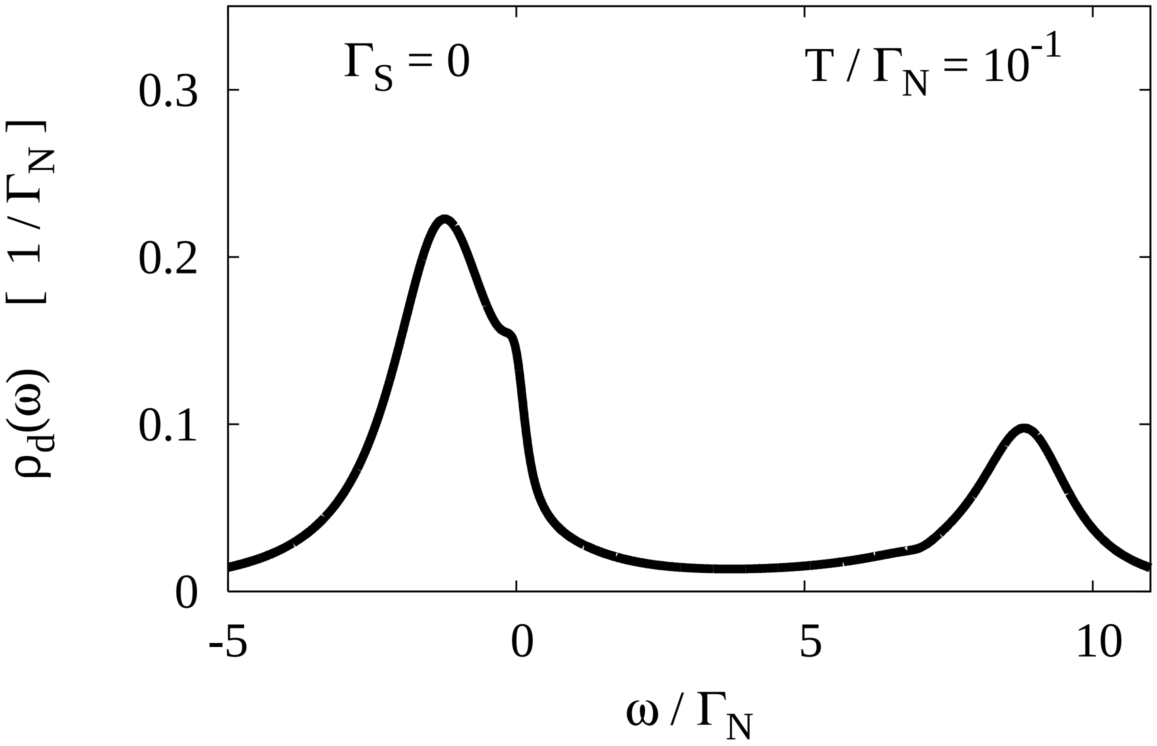

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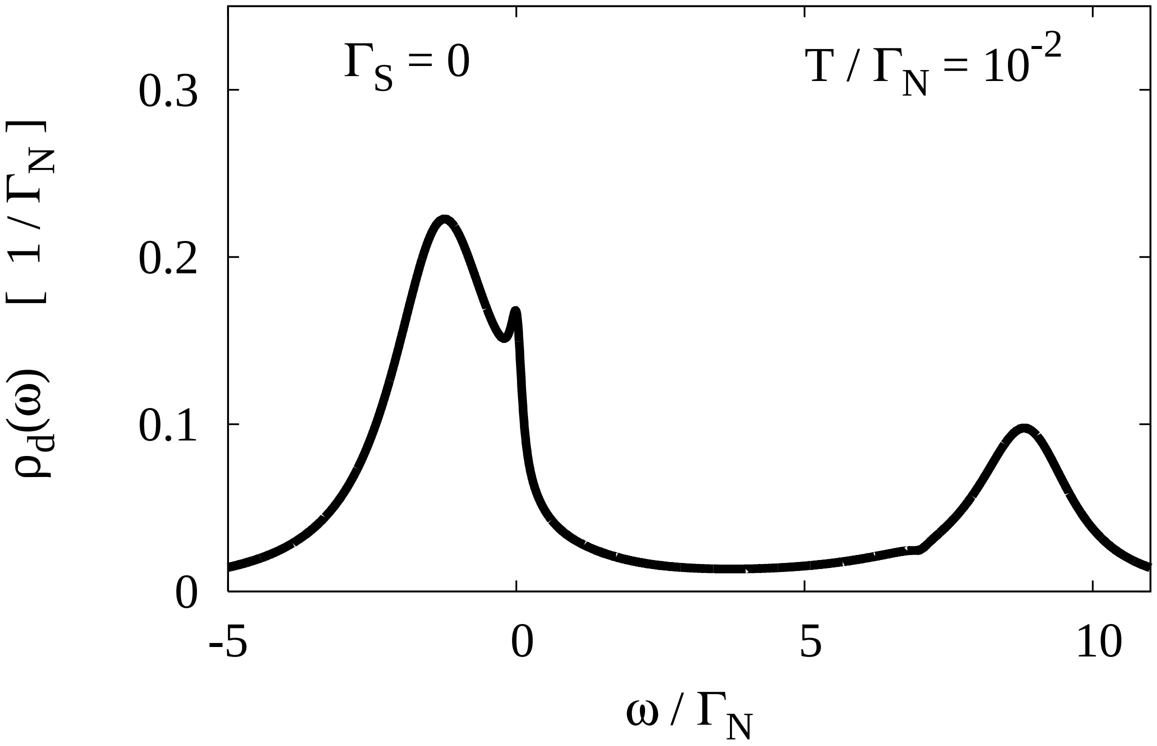

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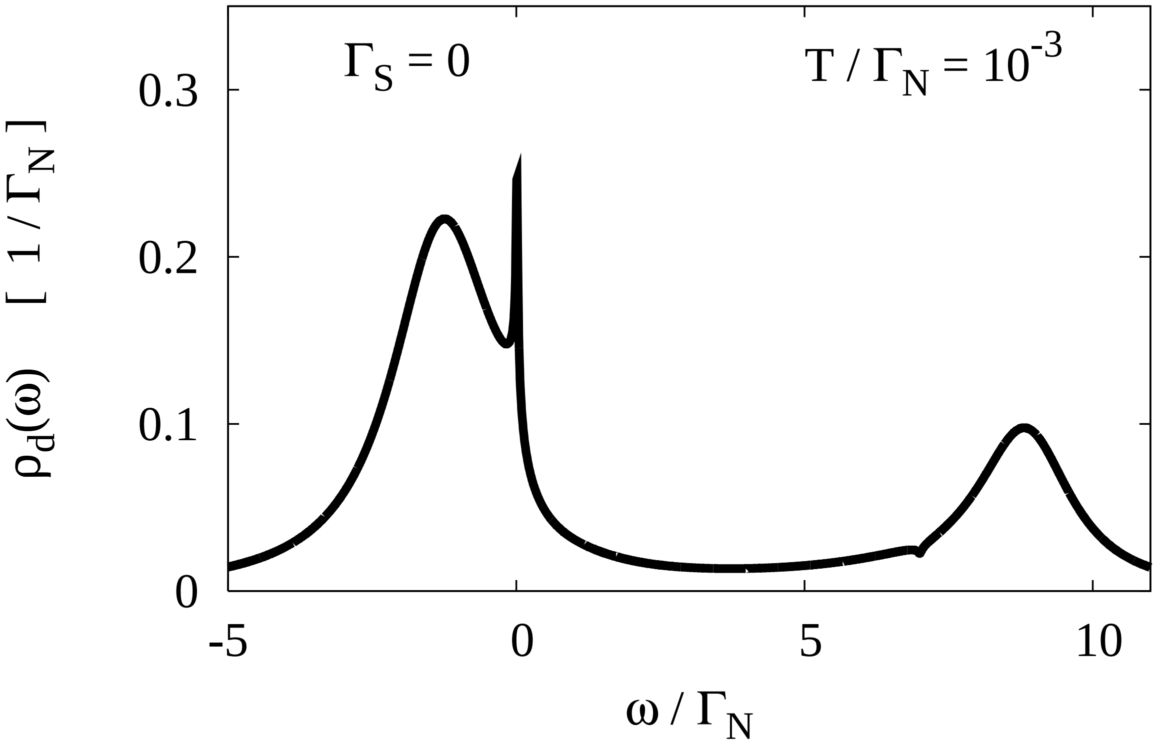

- a broadening of QD levels and
- appearance of the Kondo resonance below T_K.

Relevant problems : **issue $\#$ 2**

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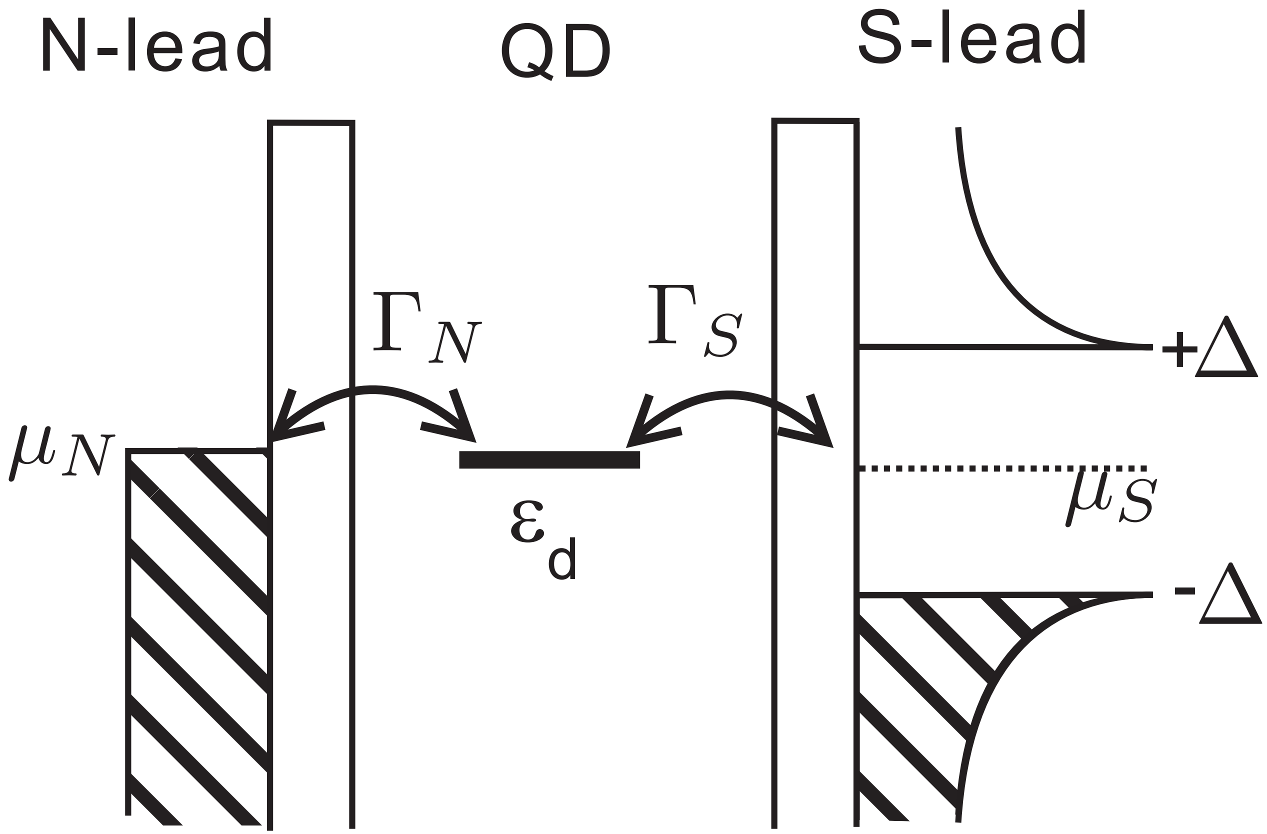
Hybridization of QD to the superconducting lead



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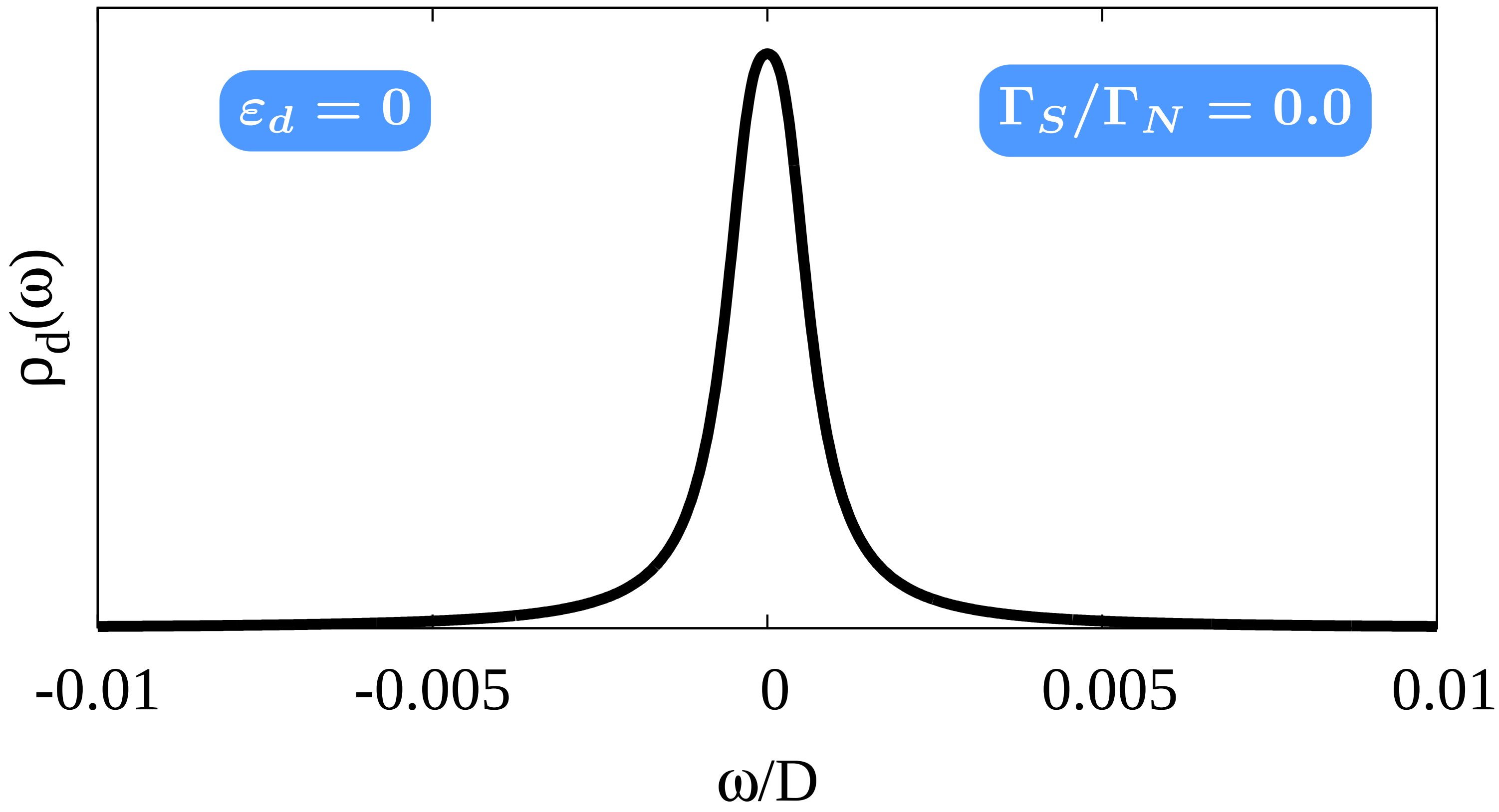

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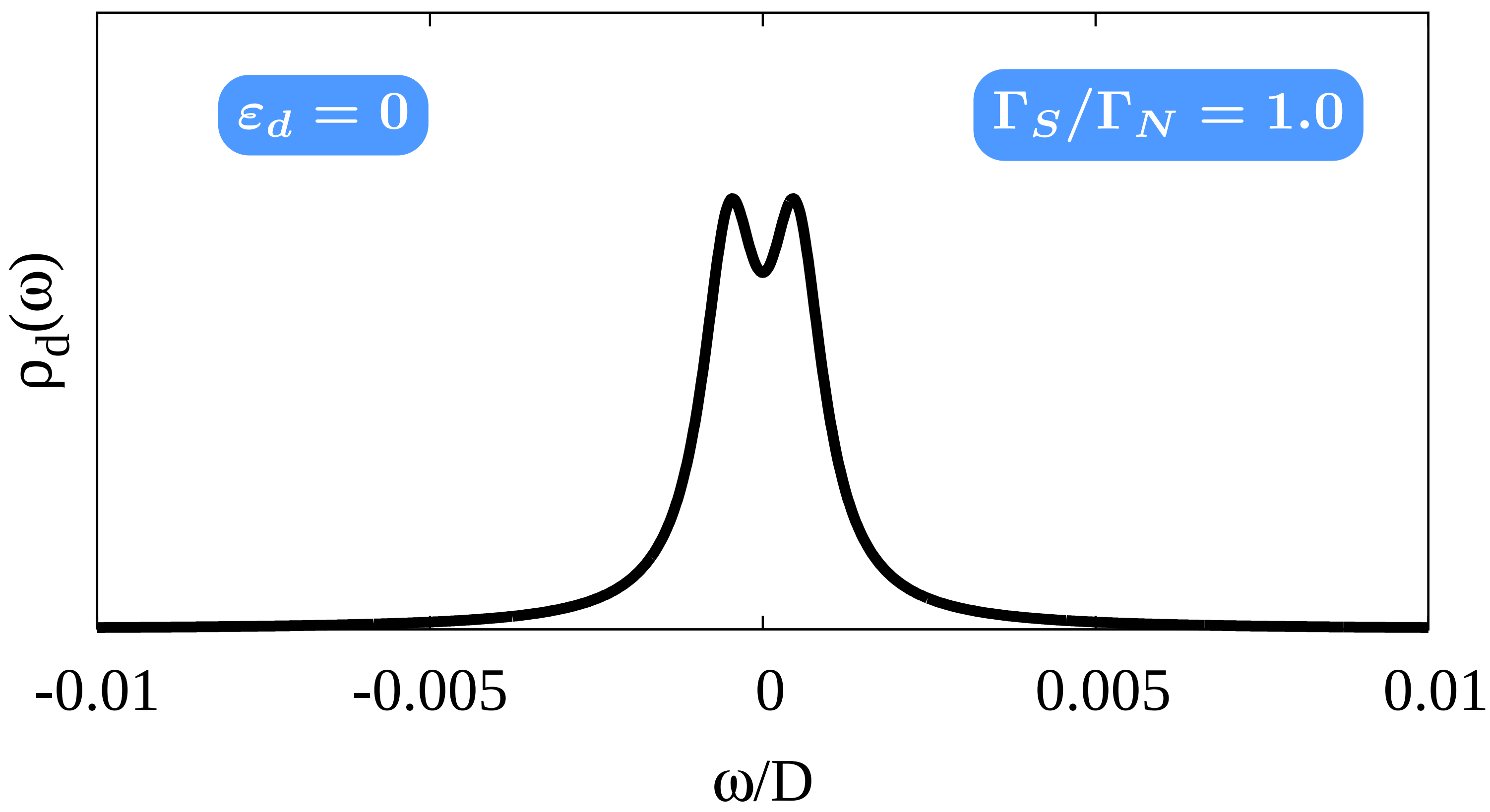

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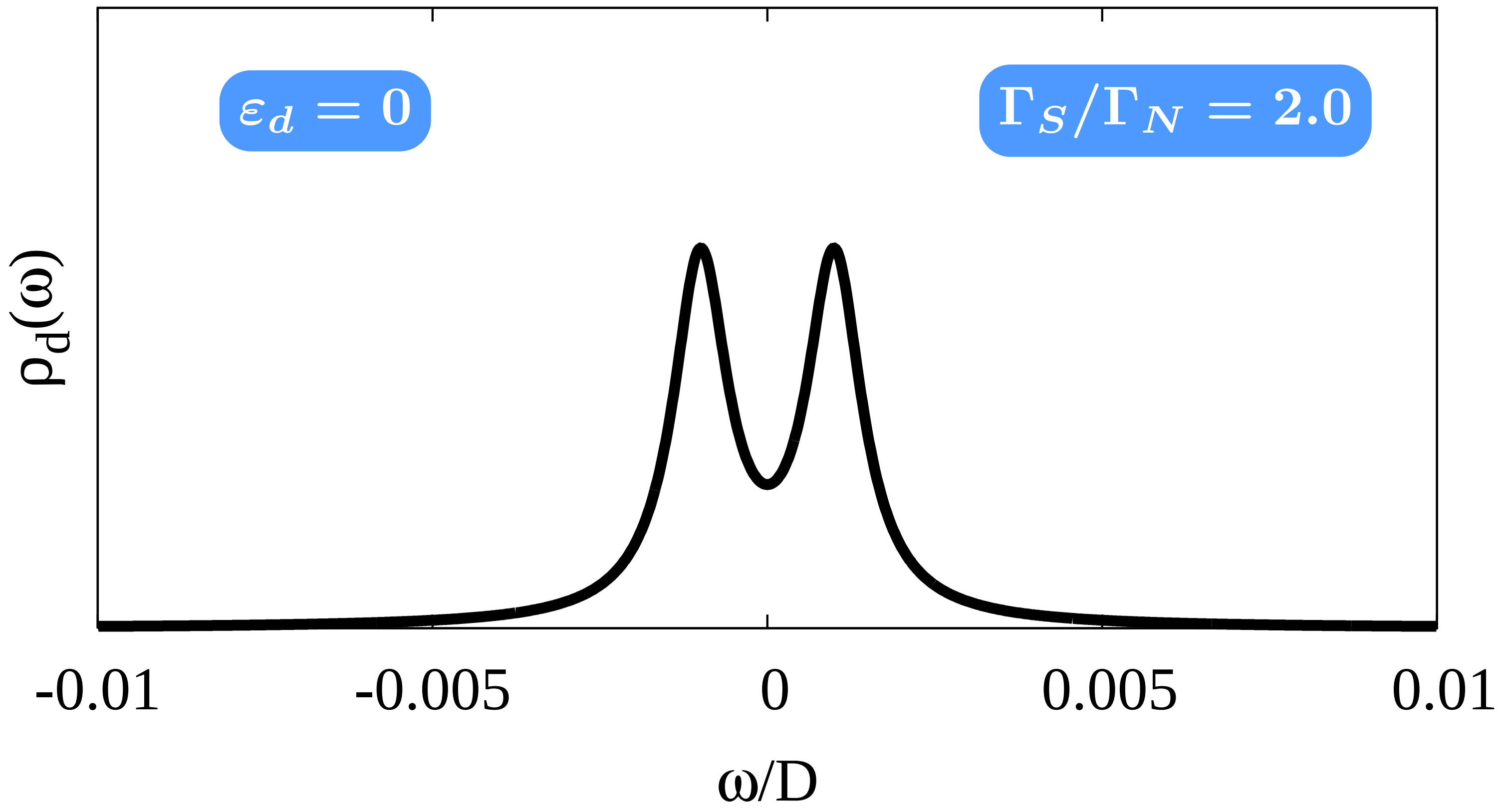

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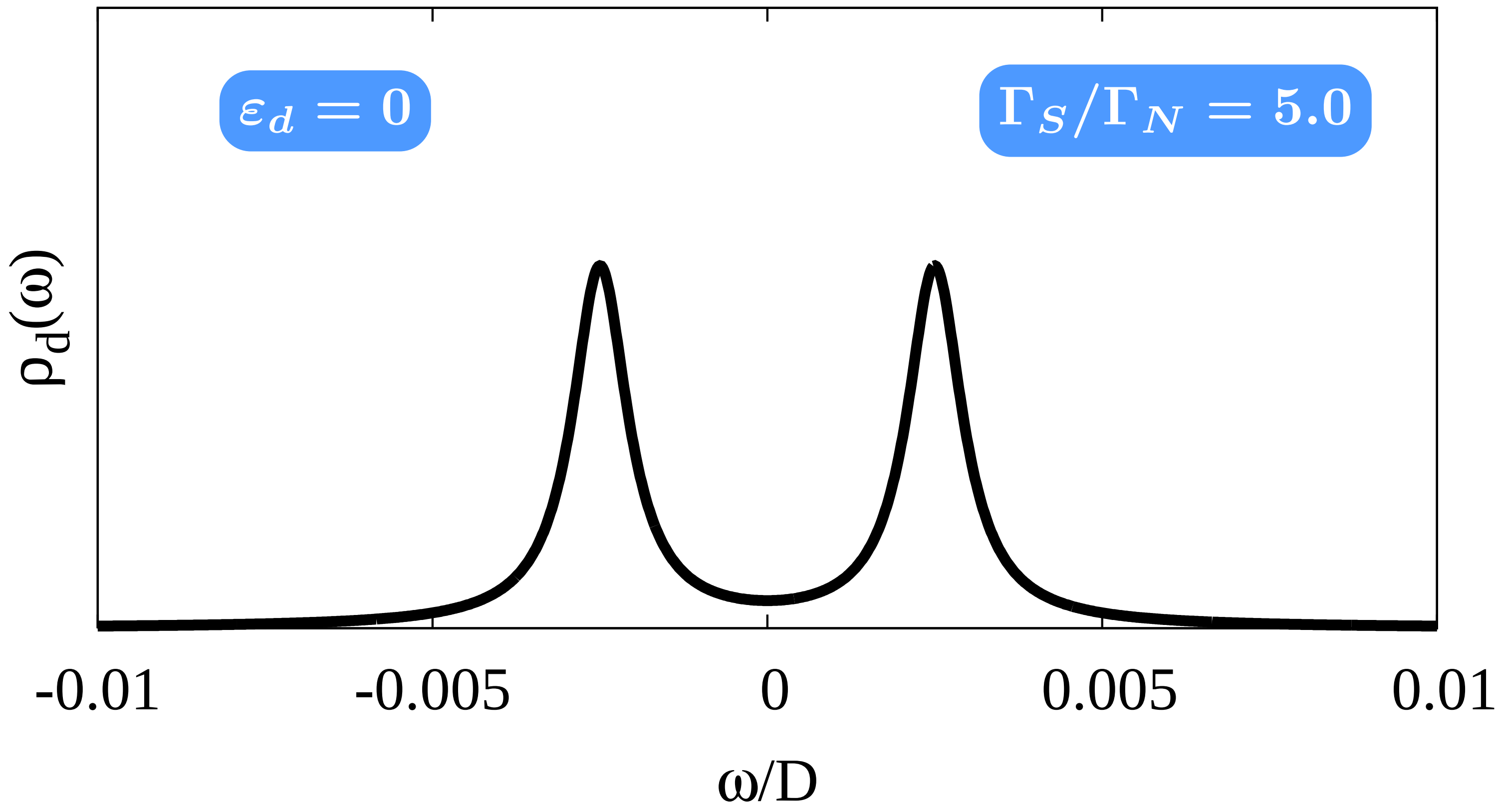

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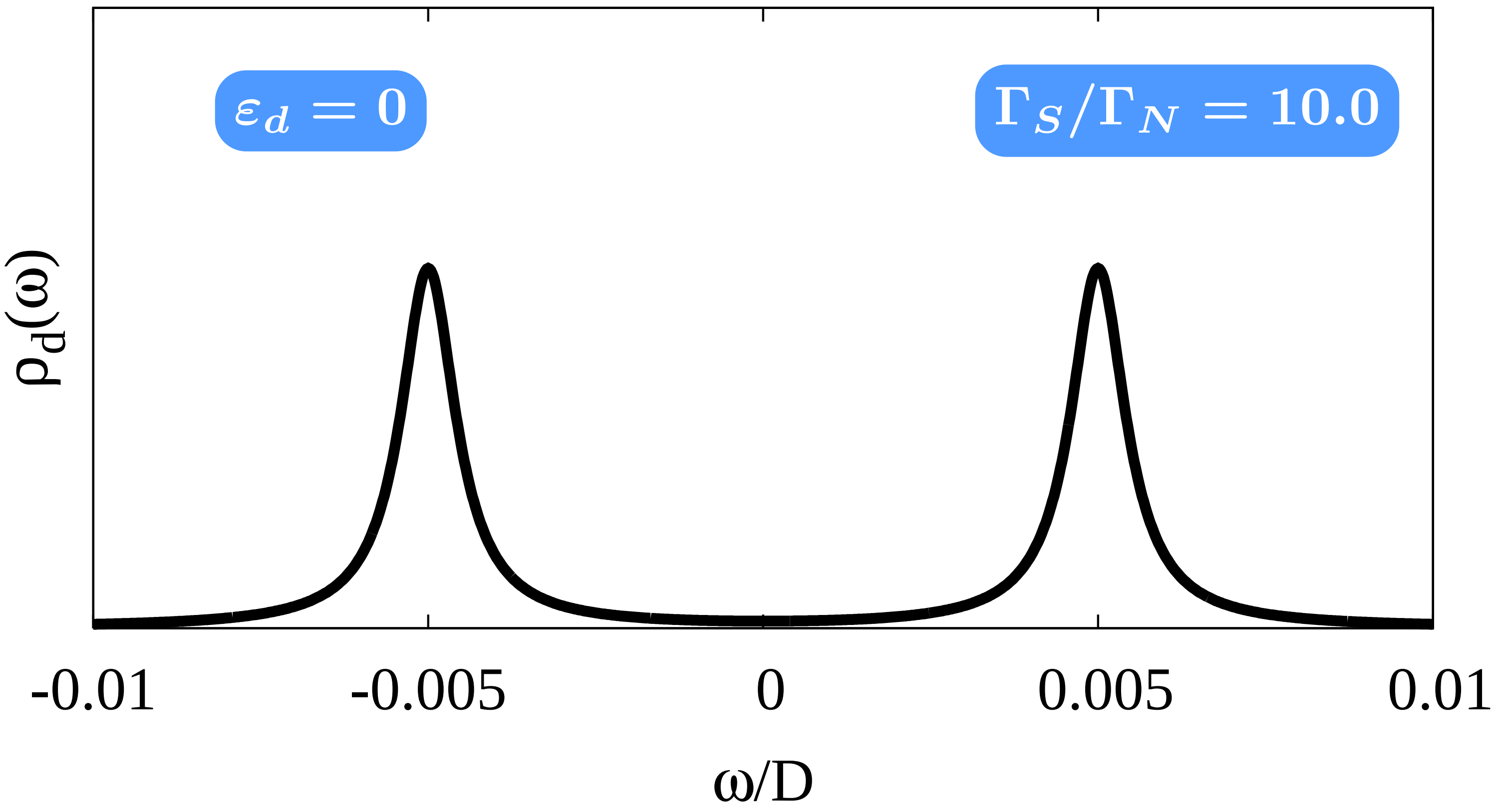

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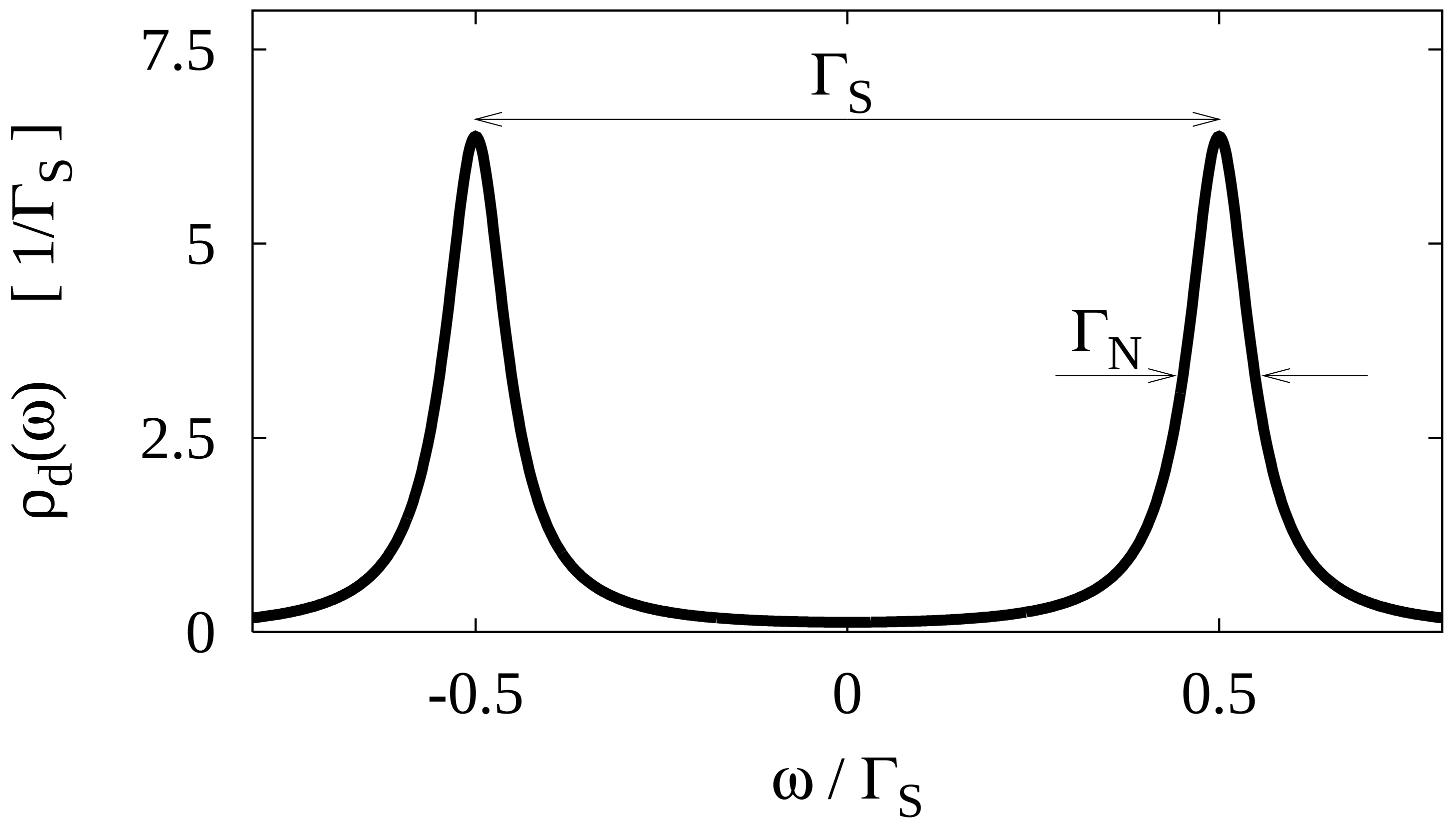

Relevant problems : $\#$ 1 + 2

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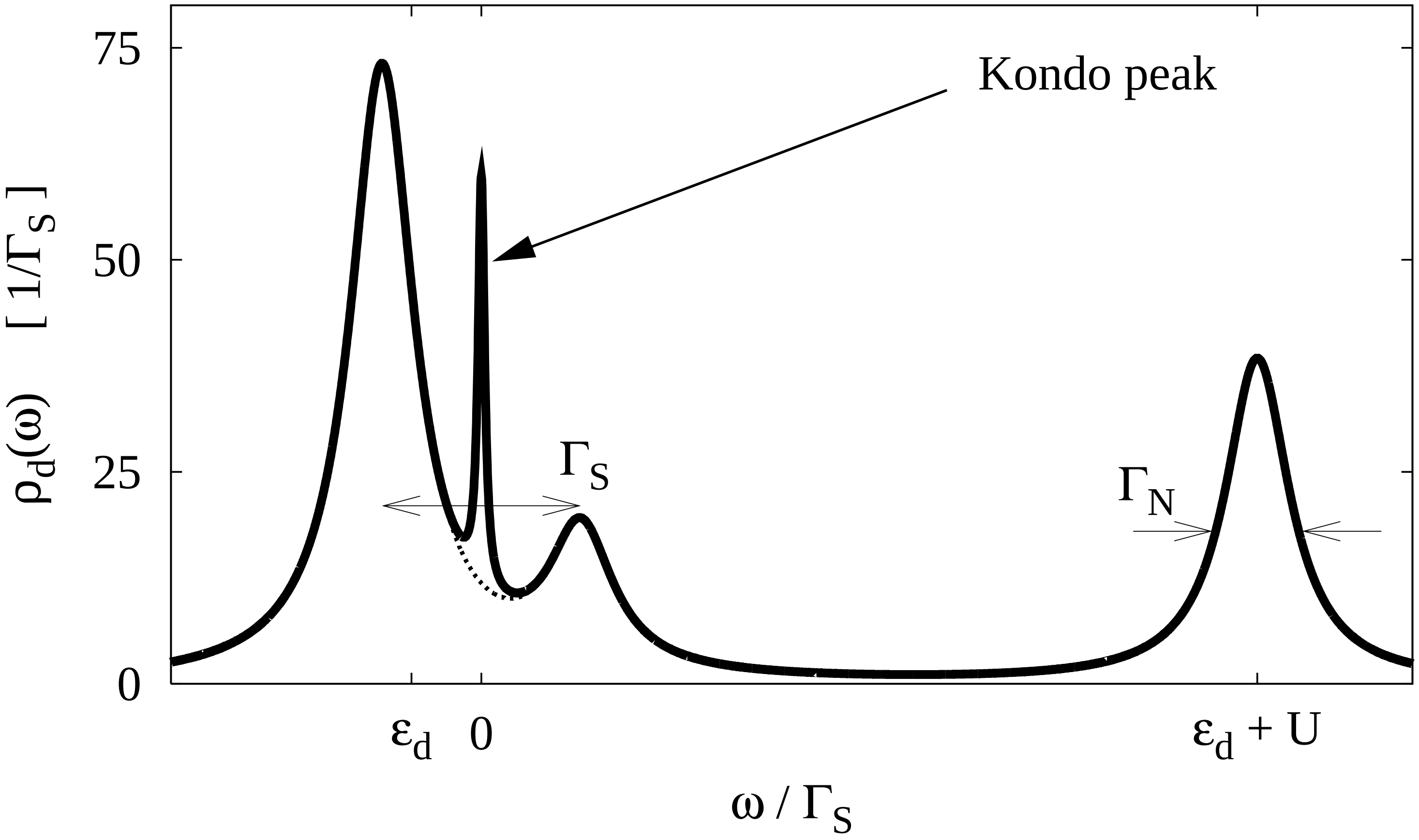

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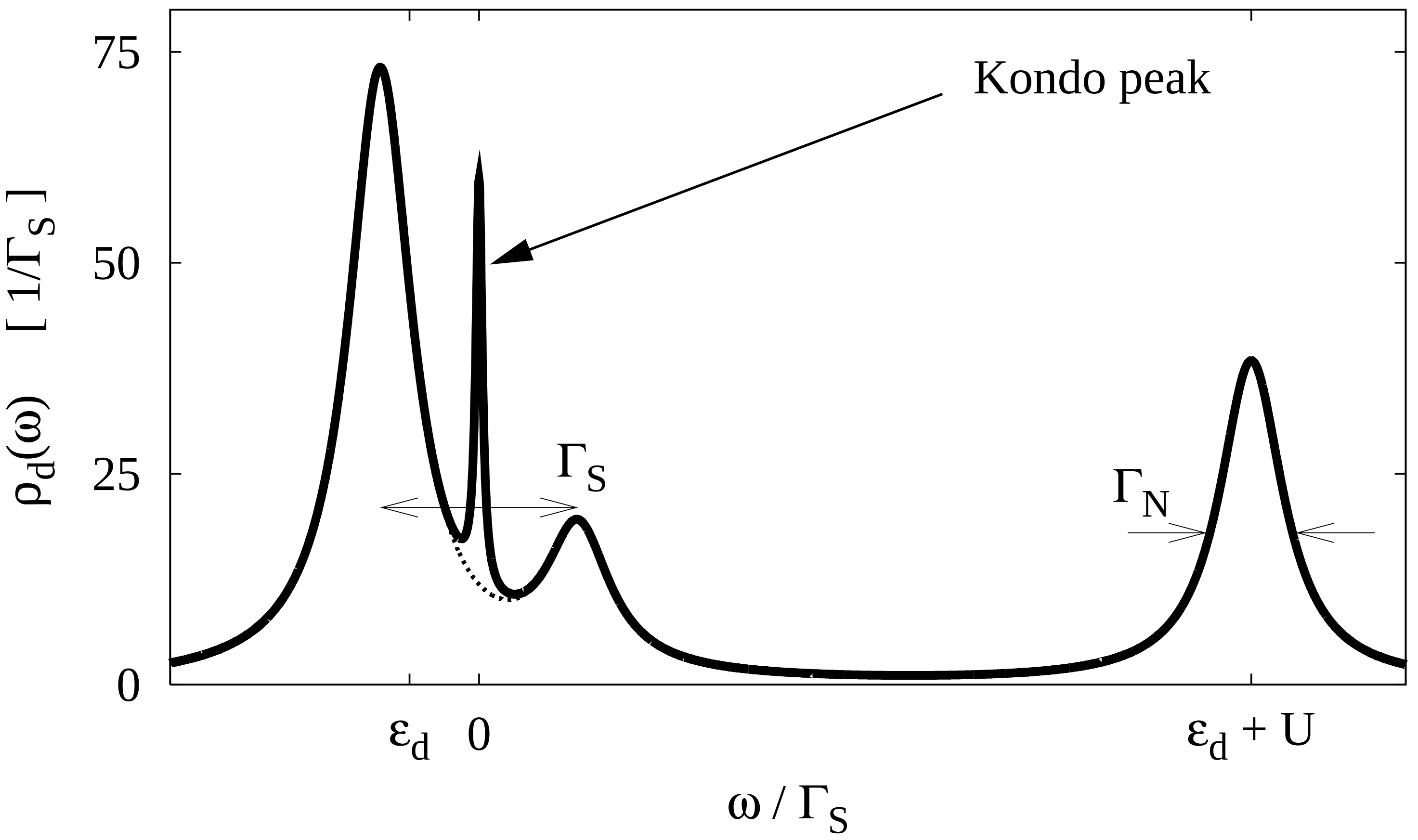

/ interplay between the Kondo effect and superconductivity /

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Are there any particular features ?

Formal aspects

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with

$\Sigma^U_d(\omega)$ **correction due to** $U \neq 0$.

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$$J_A(V) = \frac{2e}{h} \int d\omega \; T_A(\omega) \left[f(\omega + eV, T) - f(\omega - eV, T) \right]$$

with the transmittance

$$T_1(\omega) = \Gamma_N \Gamma_S \left(\left| G^r_{11}(\omega) \right|^2 + \left| G^r_{12}(\omega) \right|^2 - \frac{2\Delta}{|\omega|} \mathrm{Re} G^r_{11}(\omega) G^r_{12}(\omega) \right)$$

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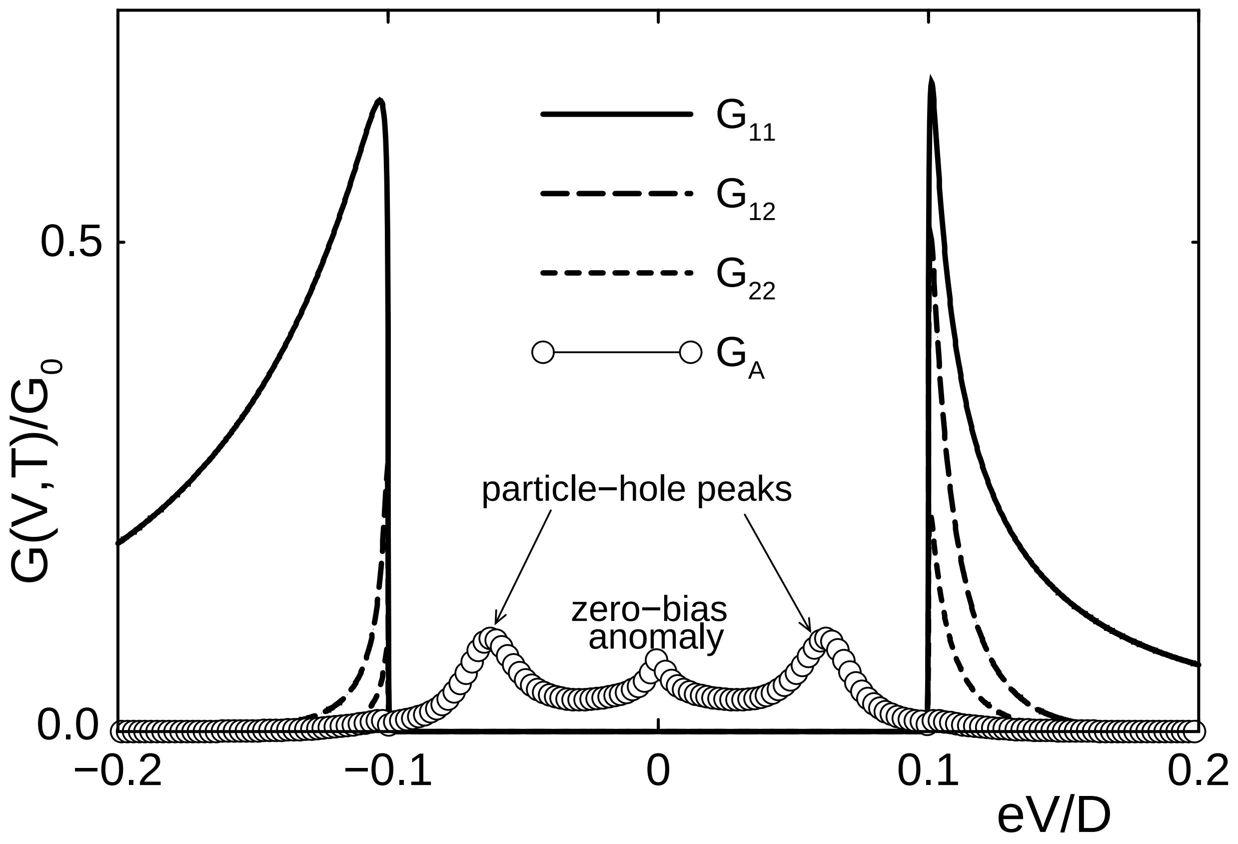

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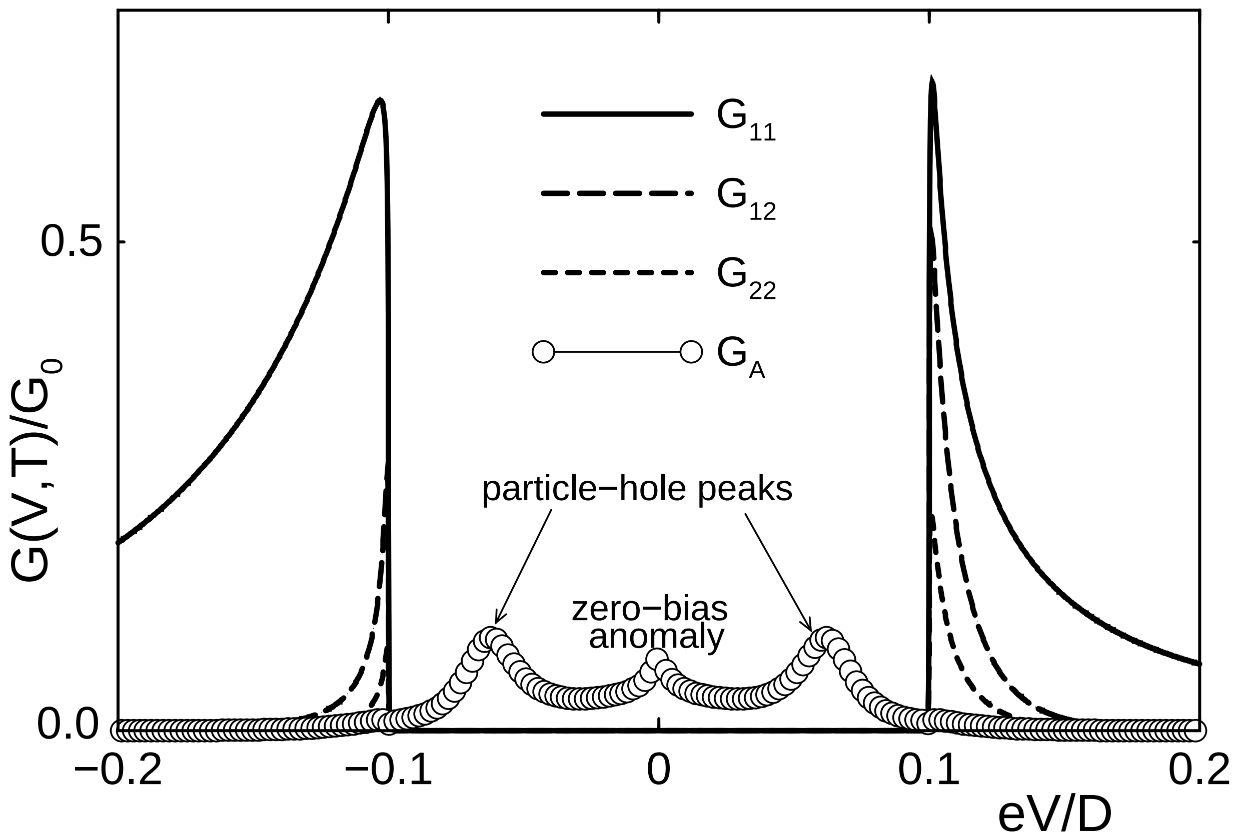

T. Domański, A. Donabidowicz, K.I. Wysokiński, PRB **76**, 104514 (2007).

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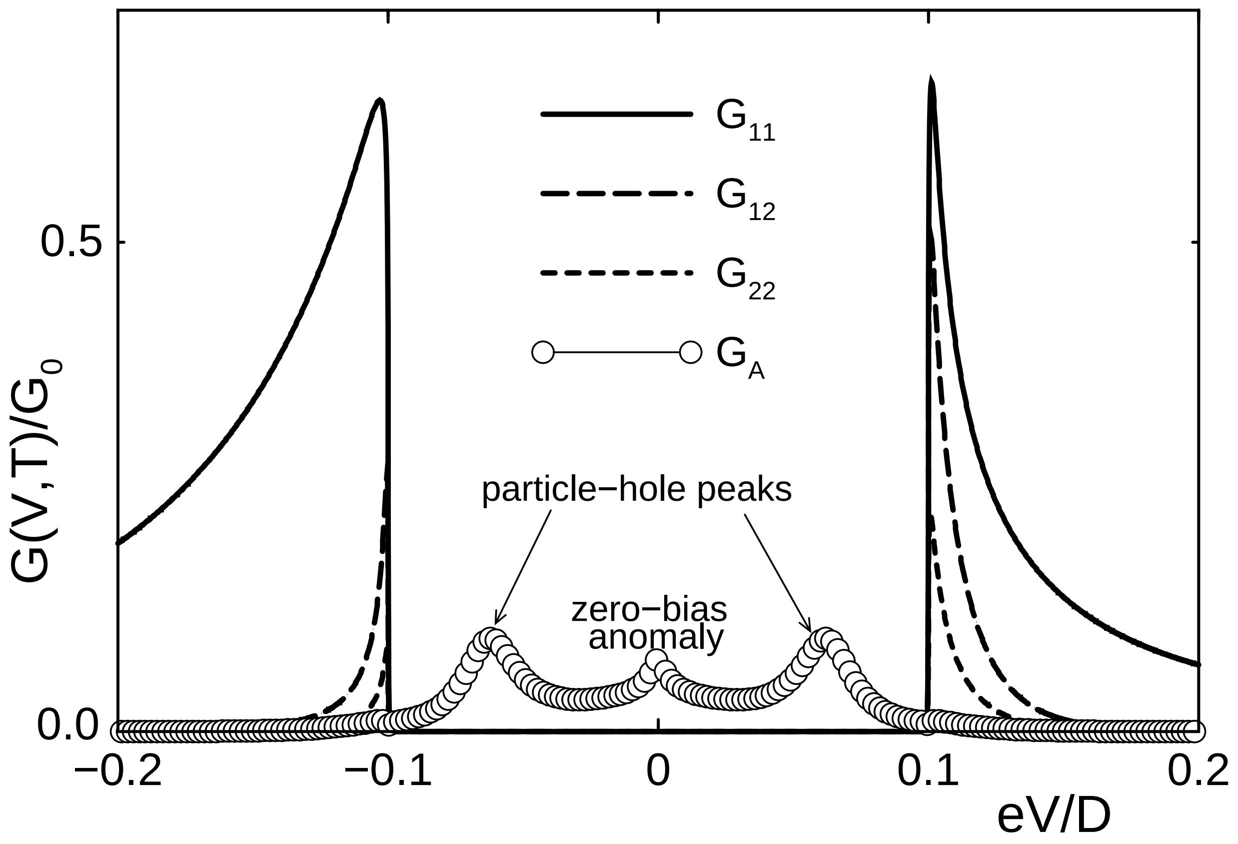

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We shall now focus on the subgap Andreev conductance.

Correlated QD – effect of the asymmetry Γ_S/Γ_N

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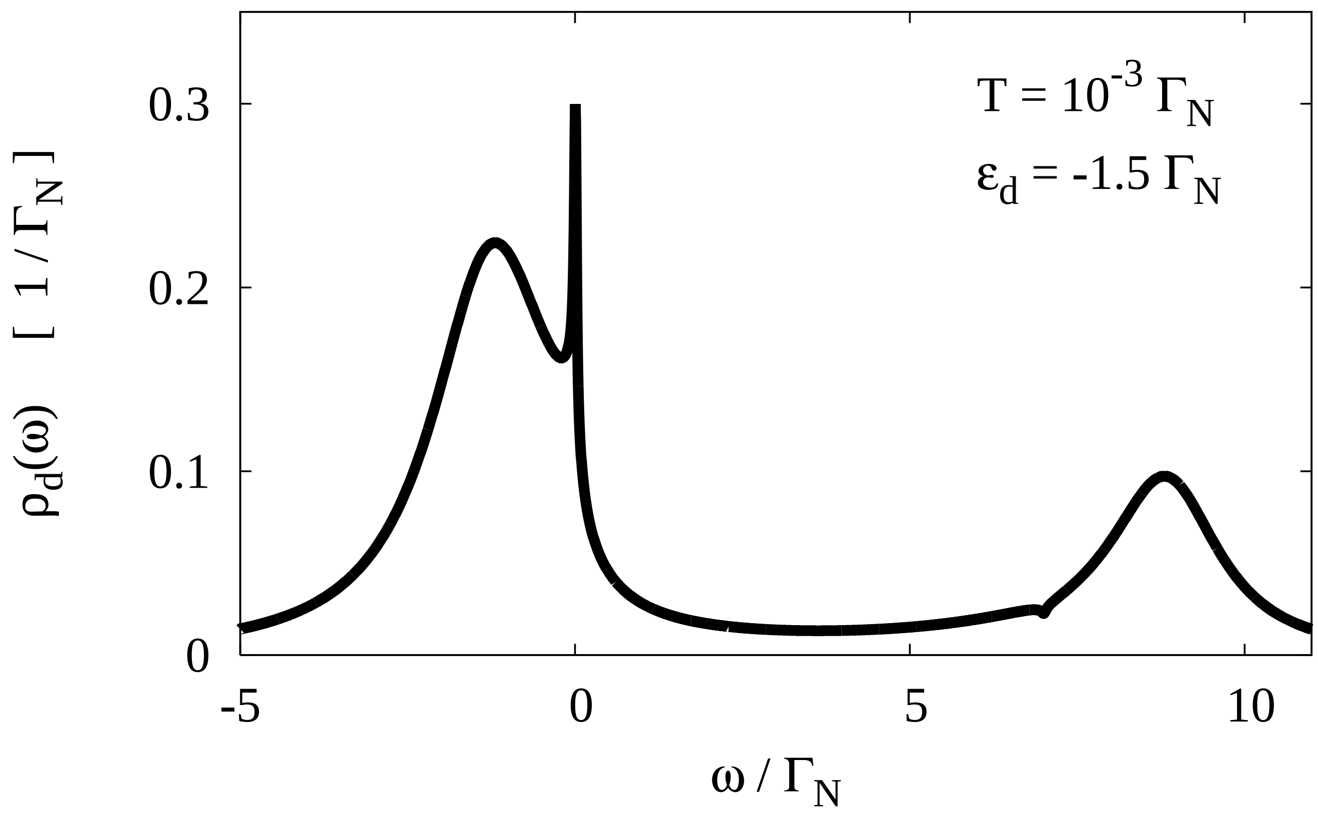
Spectral function obtained below T_K for $U = 10\Gamma_N$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

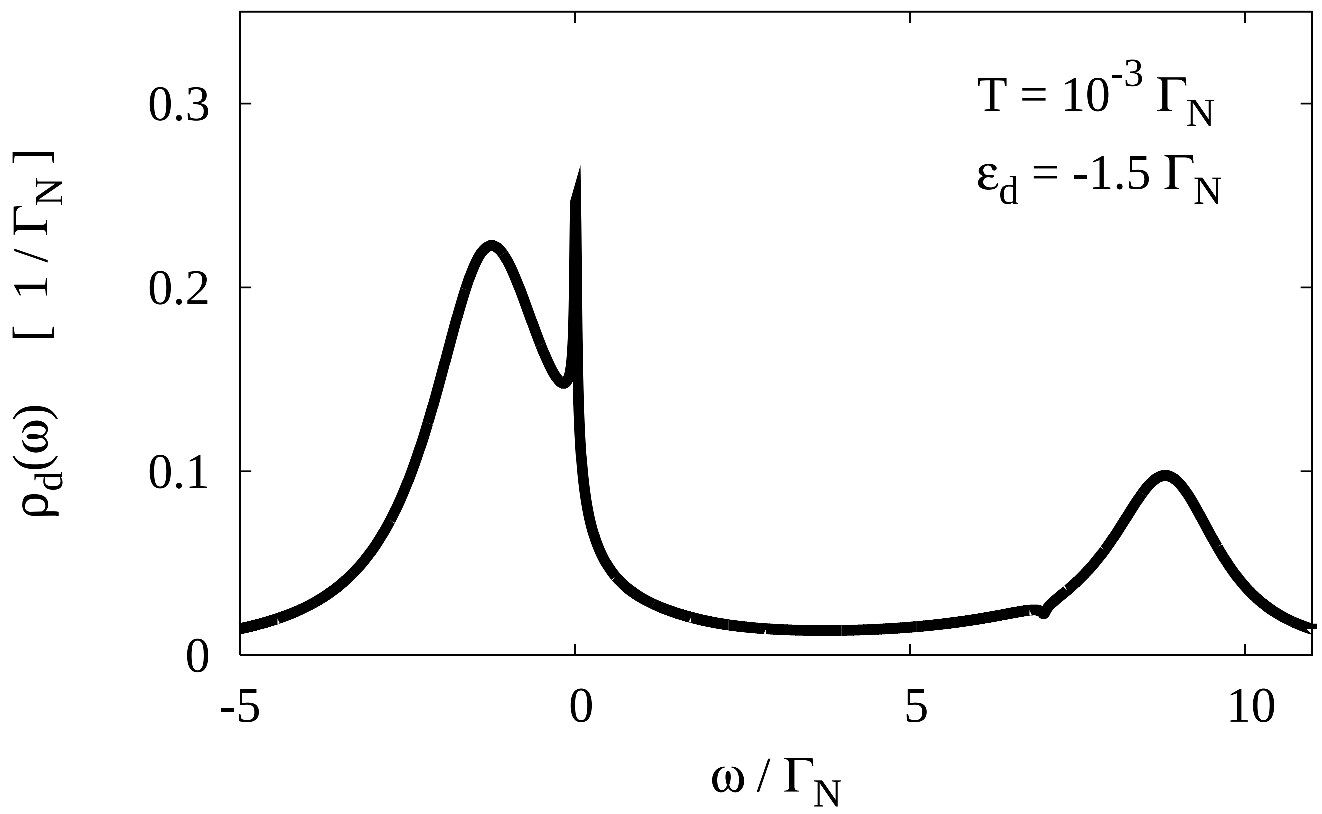
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 0$$

Correlated QD – **effect of the asymmetry Γ_S/Γ_N**

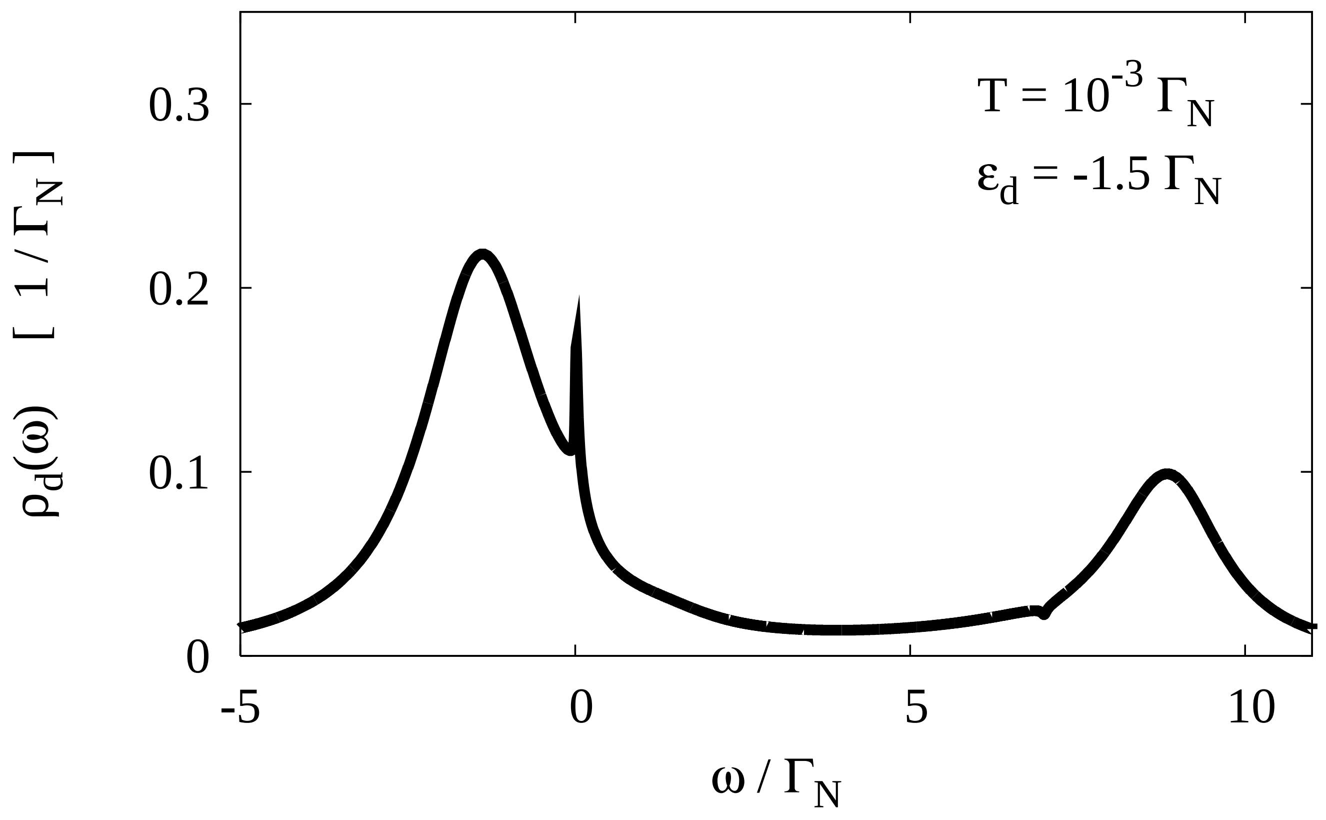
Spectral function obtained below T_K for $U=10\Gamma_N$



$$\Gamma_S/\Gamma_N = 1$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

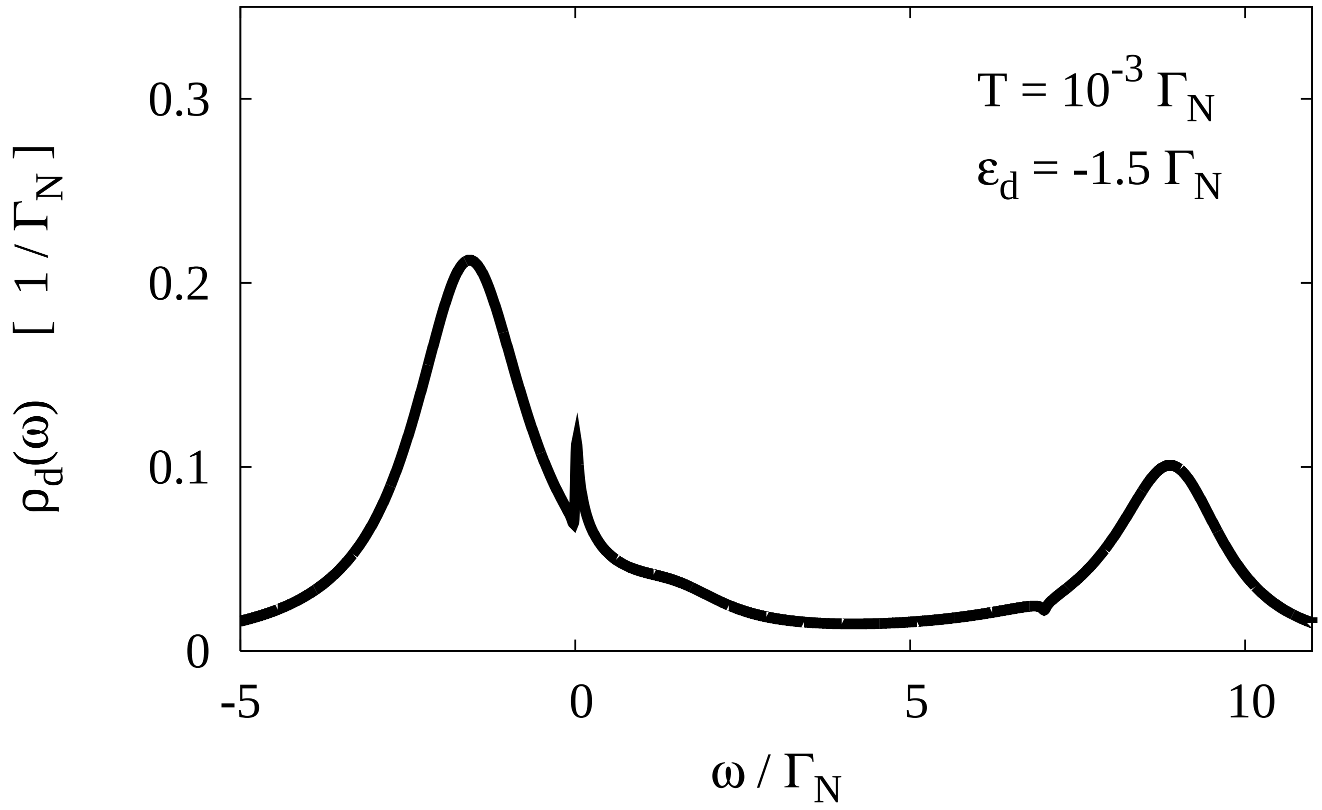
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 2$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

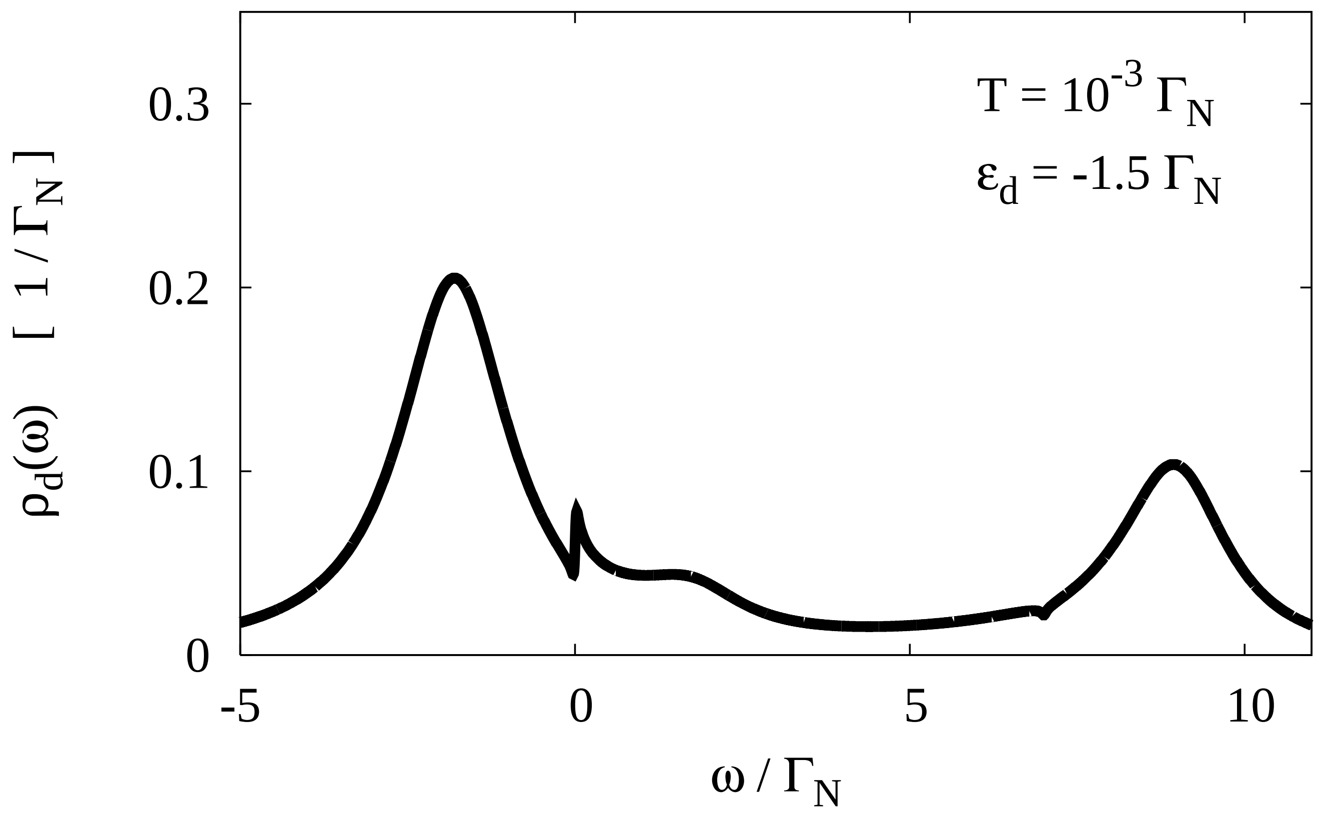
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 3$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

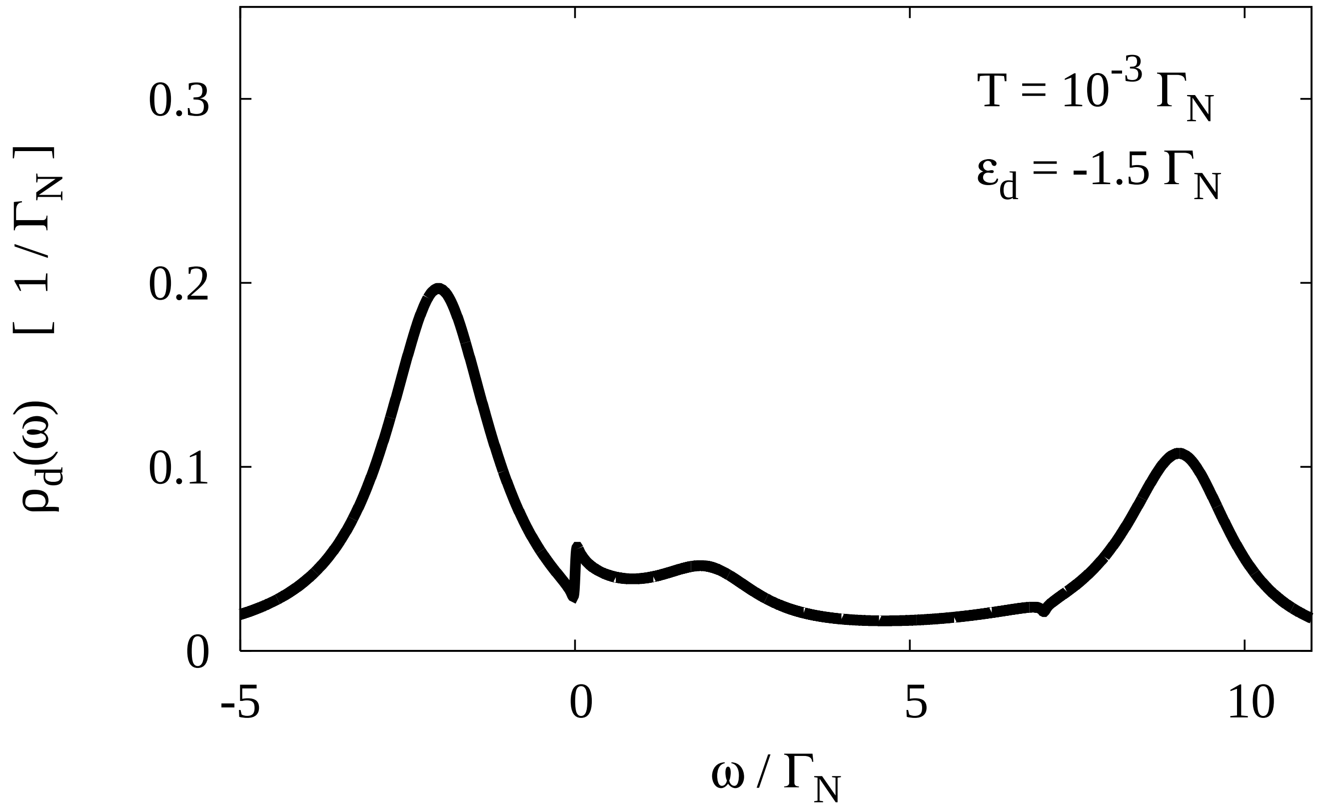
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 4$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

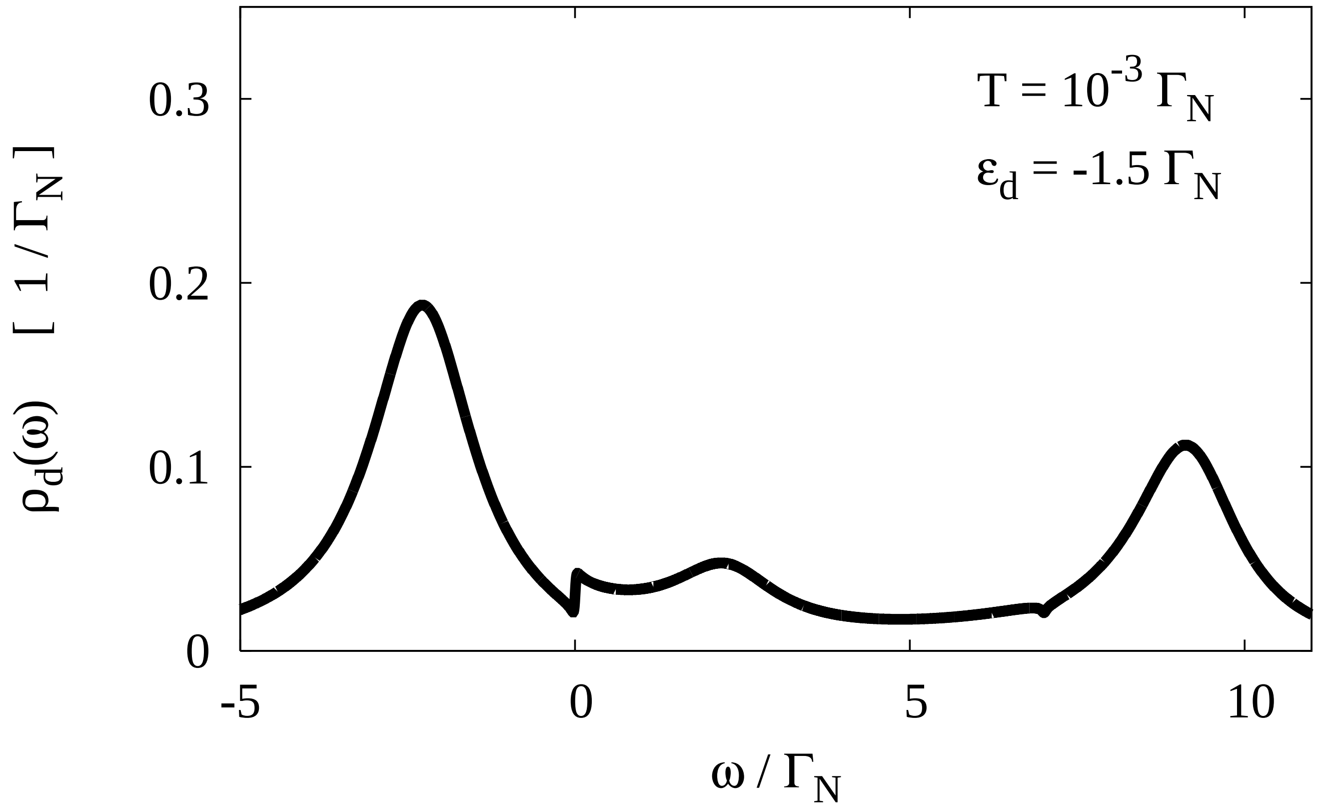
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 5$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

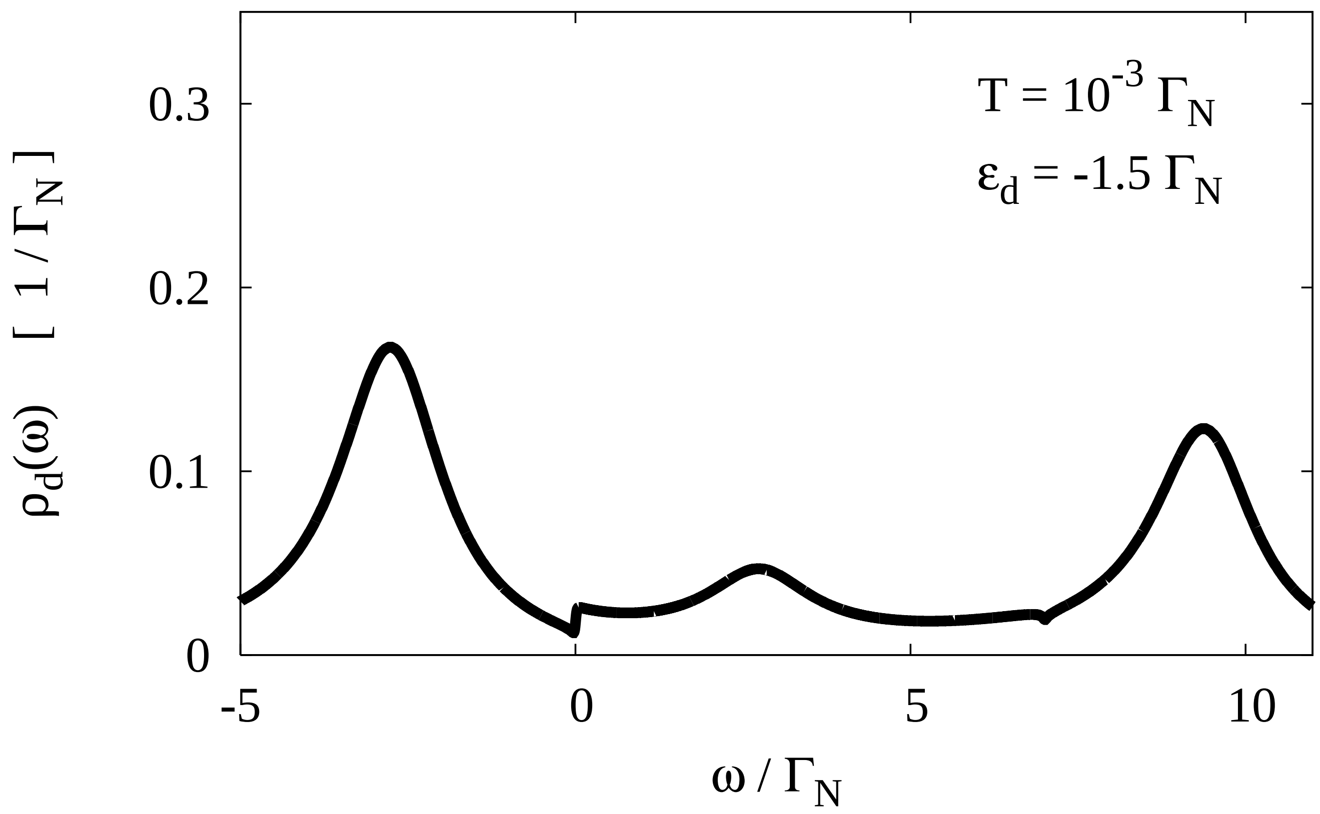
Spectral function obtained below T_K for $U = 10\Gamma_N$



$$\Gamma_S/\Gamma_N = 6$$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

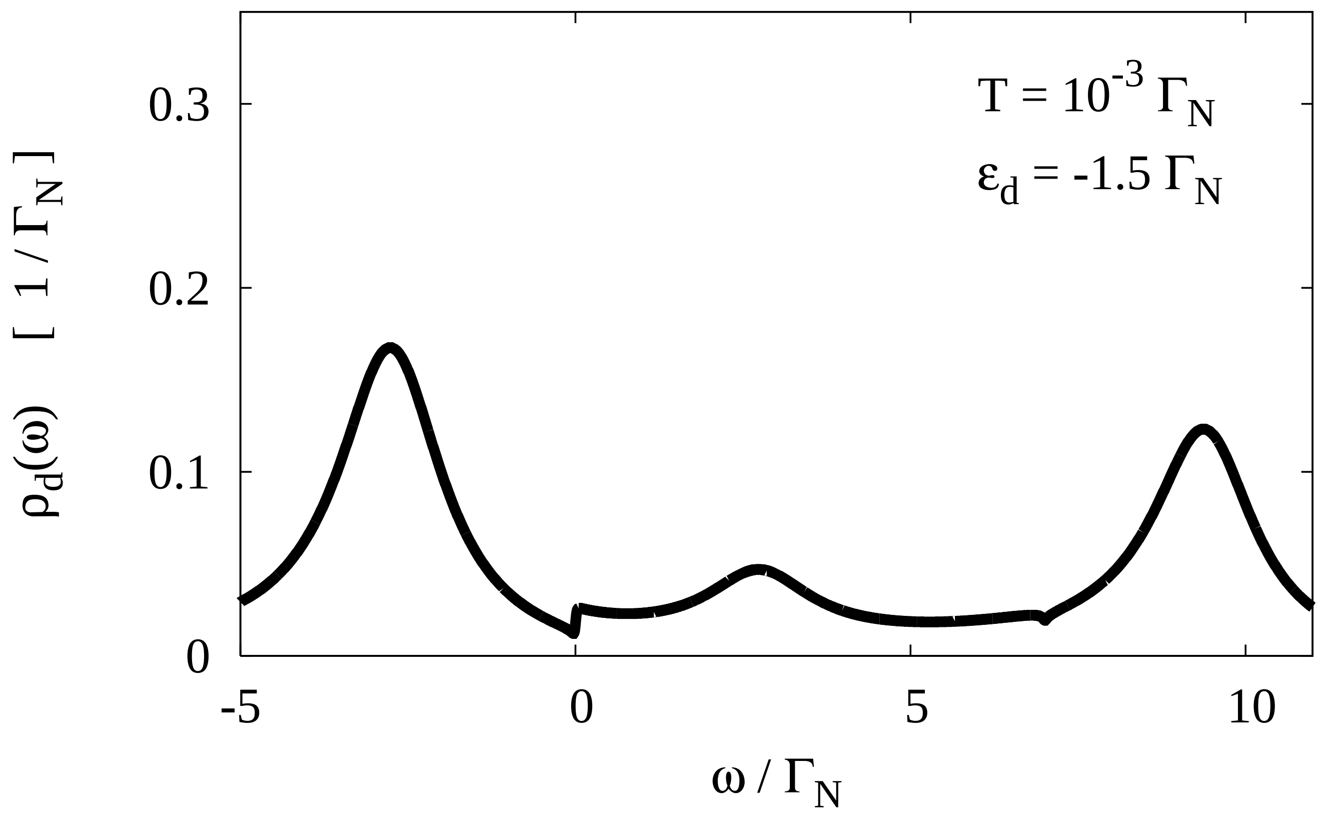
Spectral function obtained below T_K for $U = 10\Gamma_N$



$\Gamma_S/\Gamma_N = 8$

Correlated QD – effect of the asymmetry Γ_S/Γ_N

Spectral function obtained below T_K for $U=10\Gamma_N$



Superconductivity suppresses the Kondo resonance

Correlated QD – effect of the asymmetry Γ_S/Γ_N

Andreev conductance $G_A(V)$ for:

$U = 10\Gamma_N$

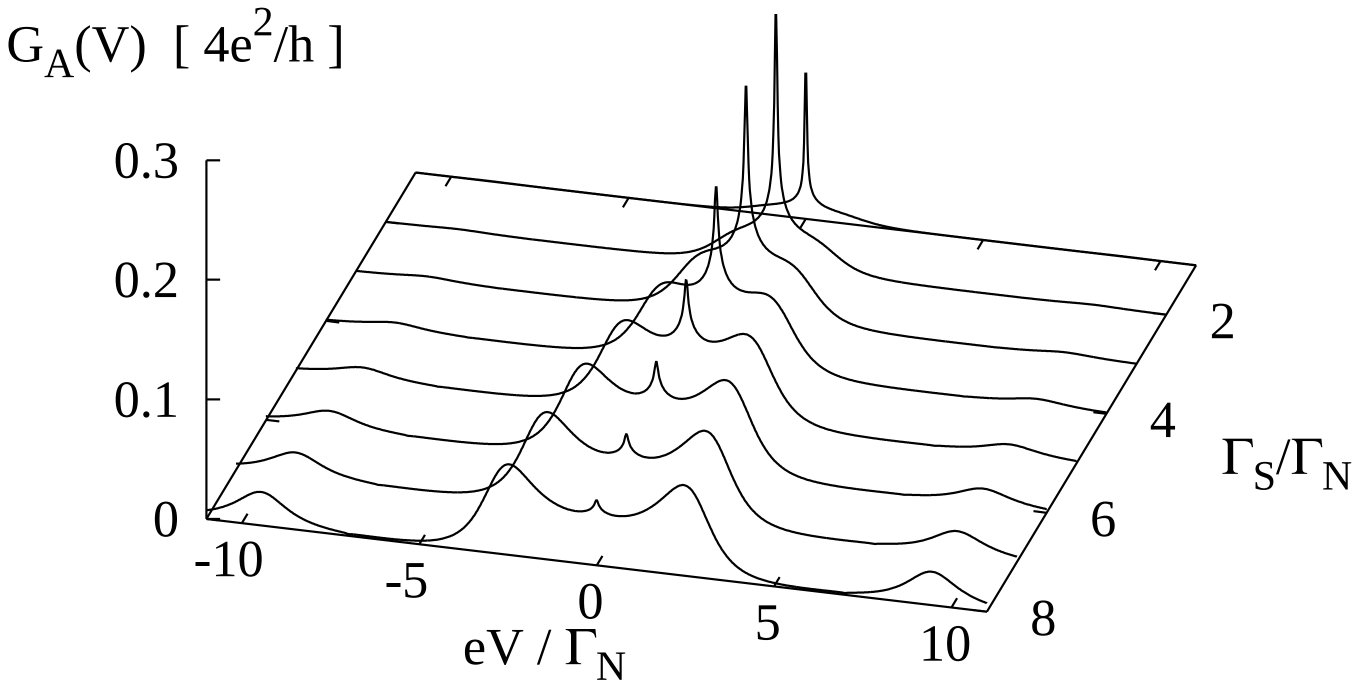
Correlated QD – **effect of the asymmetry Γ_S/Γ_N**

Andreev conductance $G_A(V)$ for: $U = 10\Gamma_N$

T. Domański and A. Donabidowicz, PRB **78**, 073105 (2008).

Correlated QD – effect of the asymmetry Γ_S/Γ_N

Andreev conductance $G_A(V)$ for: $U = 10\Gamma_N$



T. Domański and A. Donabidowicz, PRB **78**, 073105 (2008).

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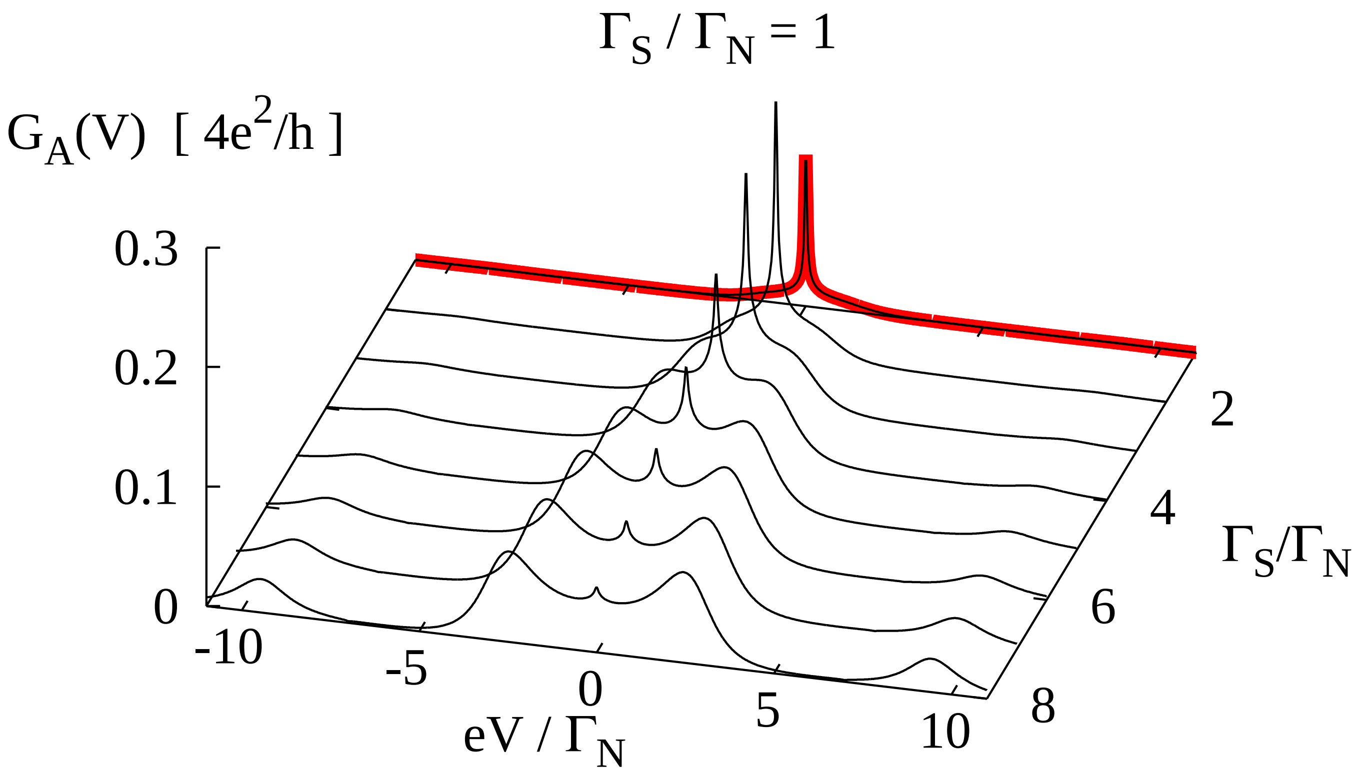

T. Domański and A. Donabidowicz, PRB **78**, 073105 (2008).

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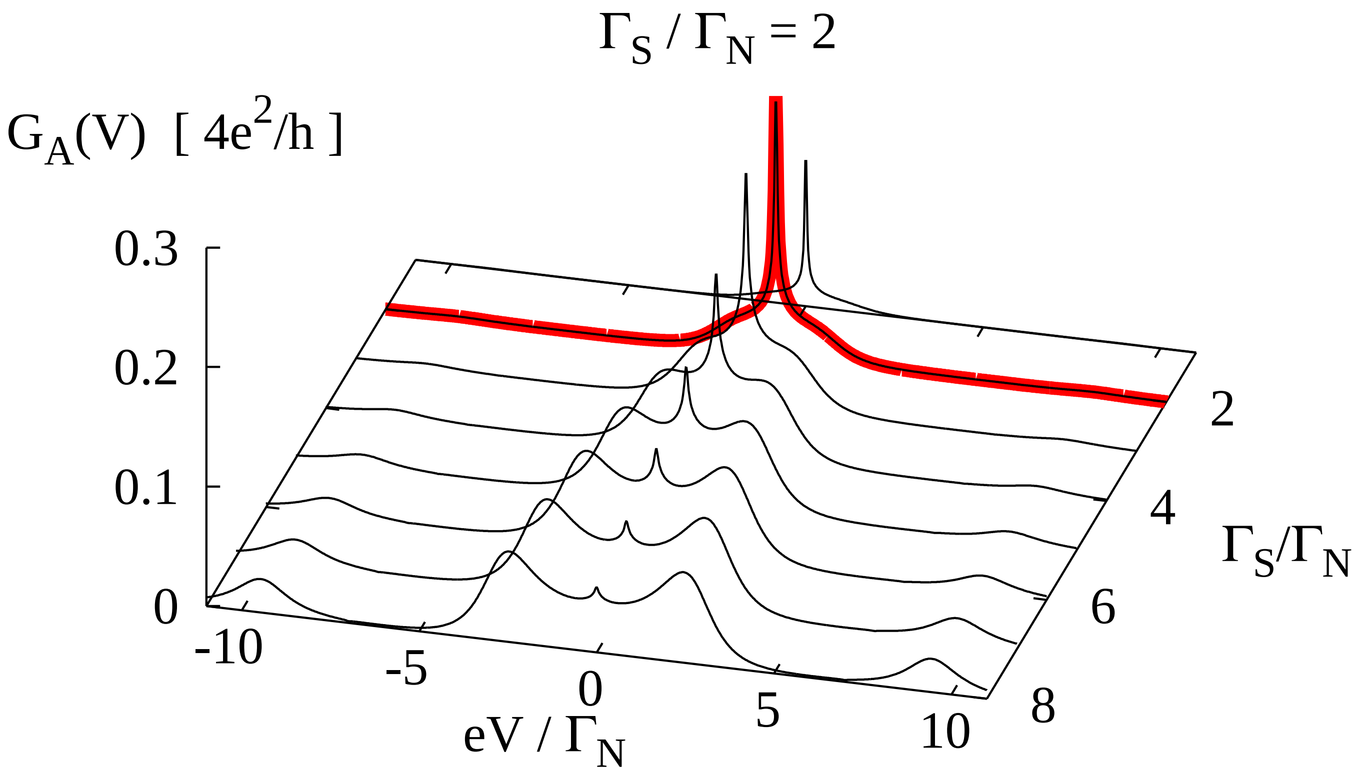

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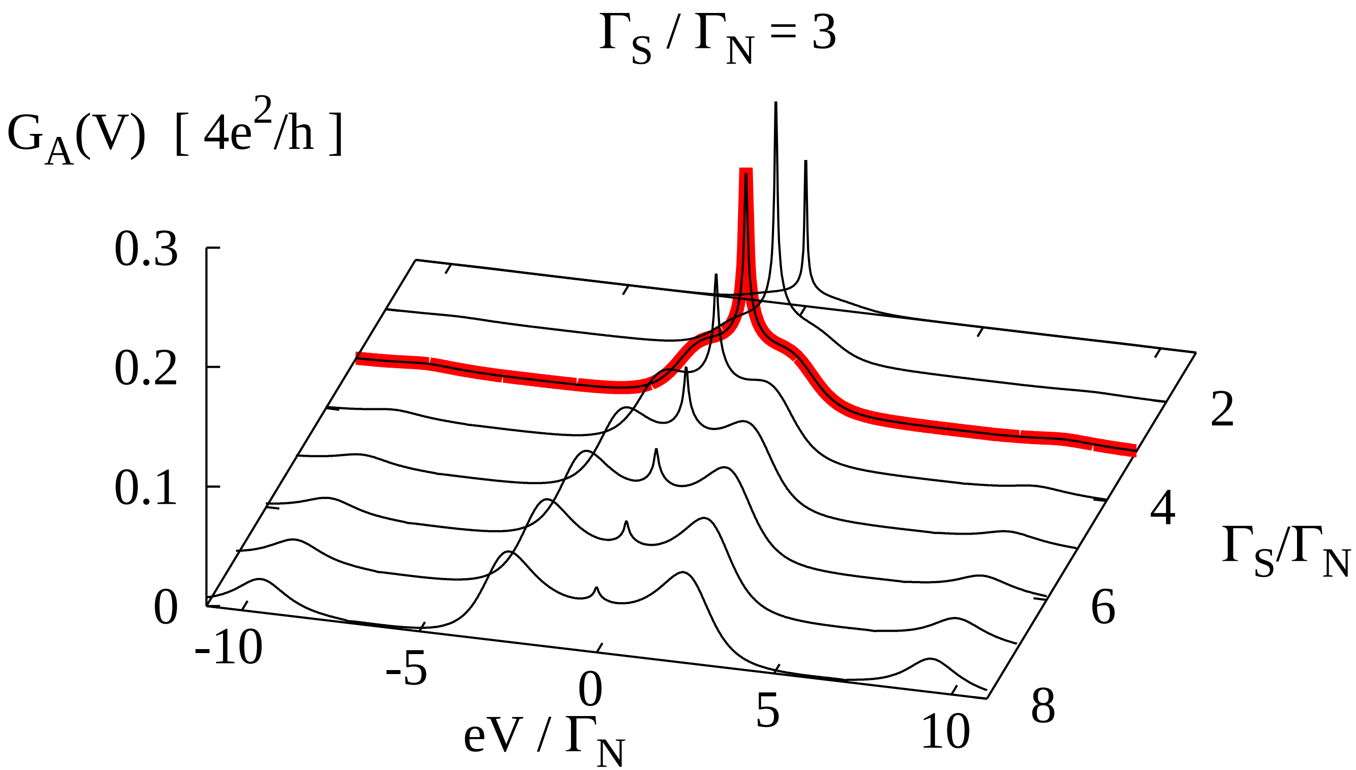

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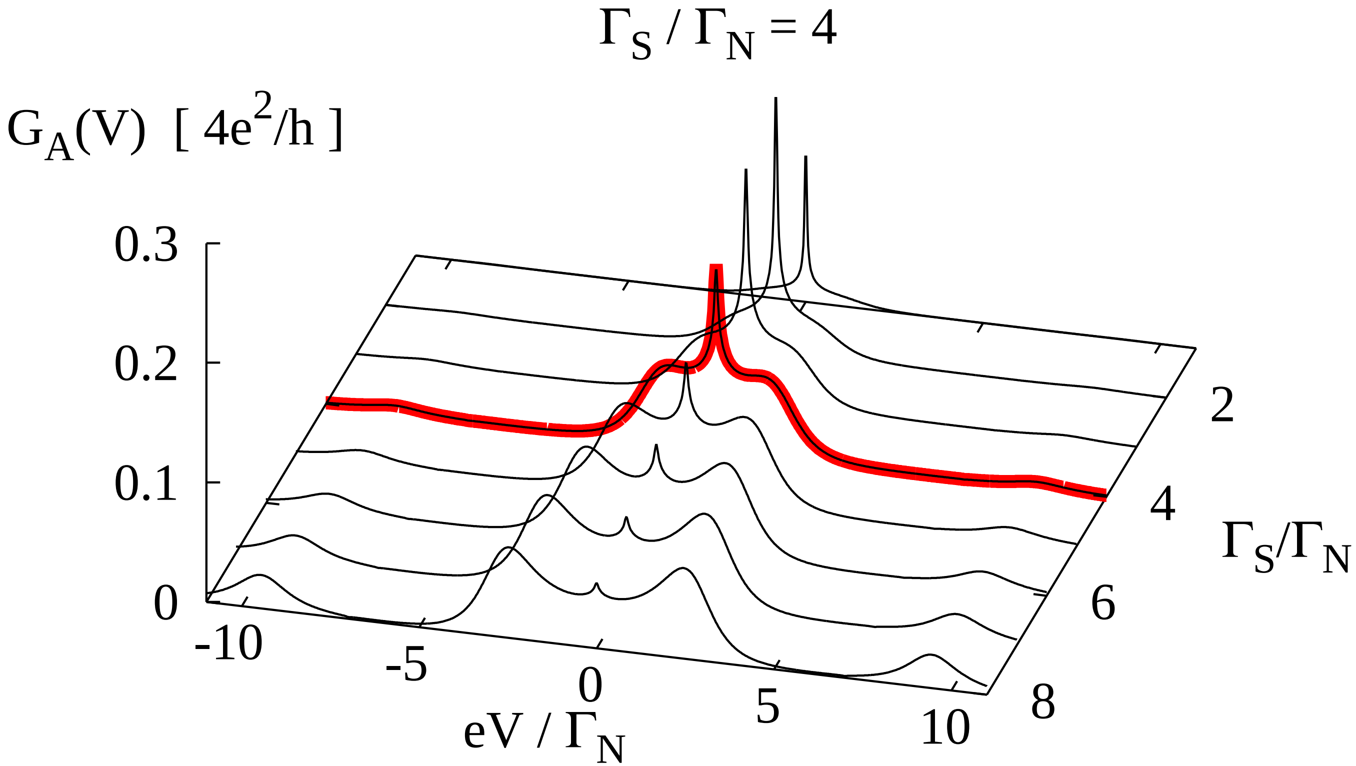

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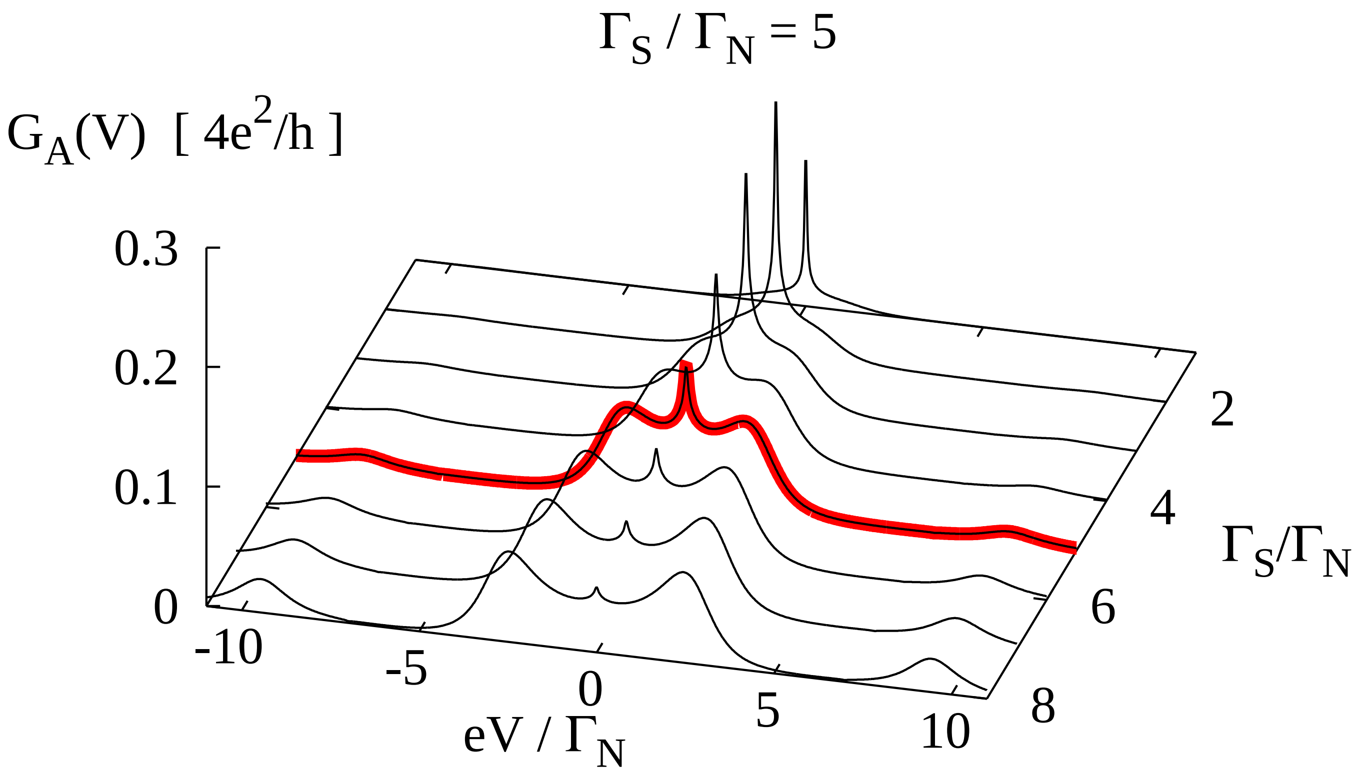

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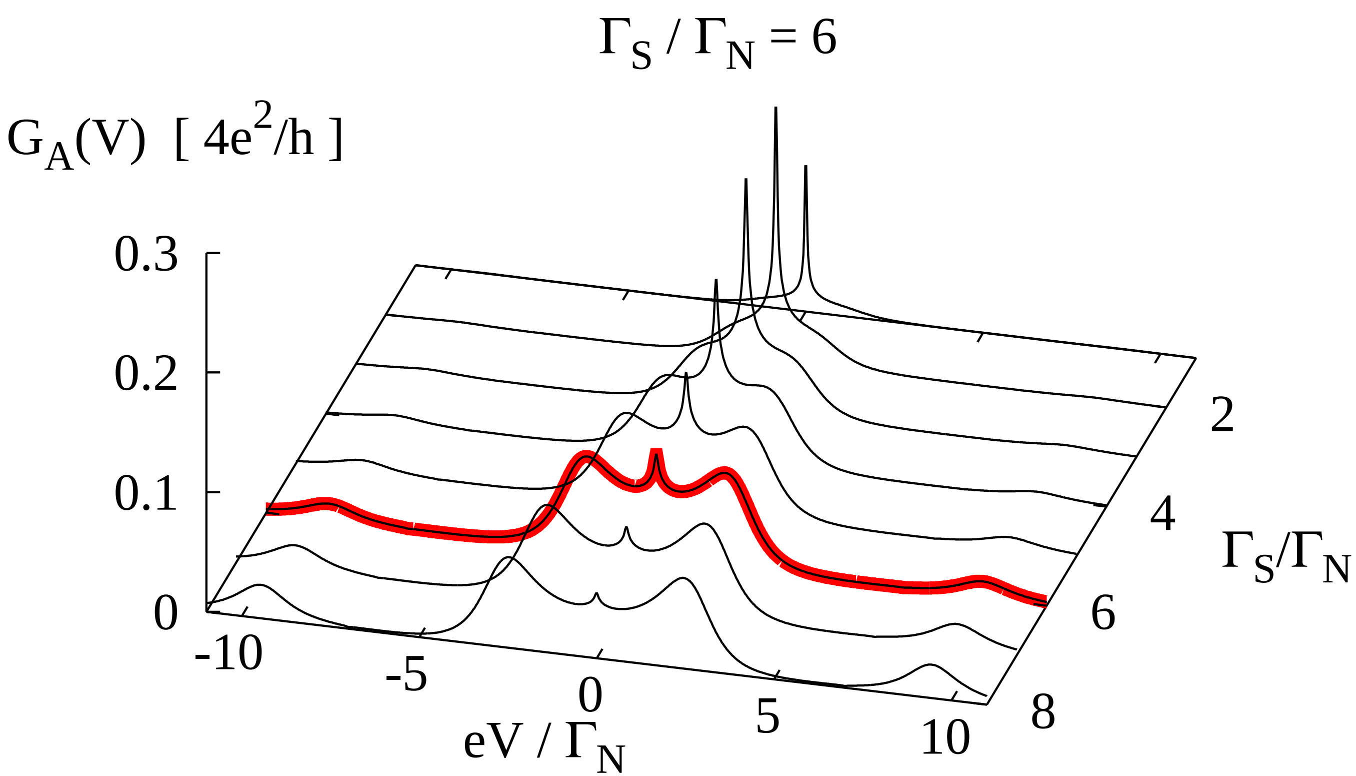

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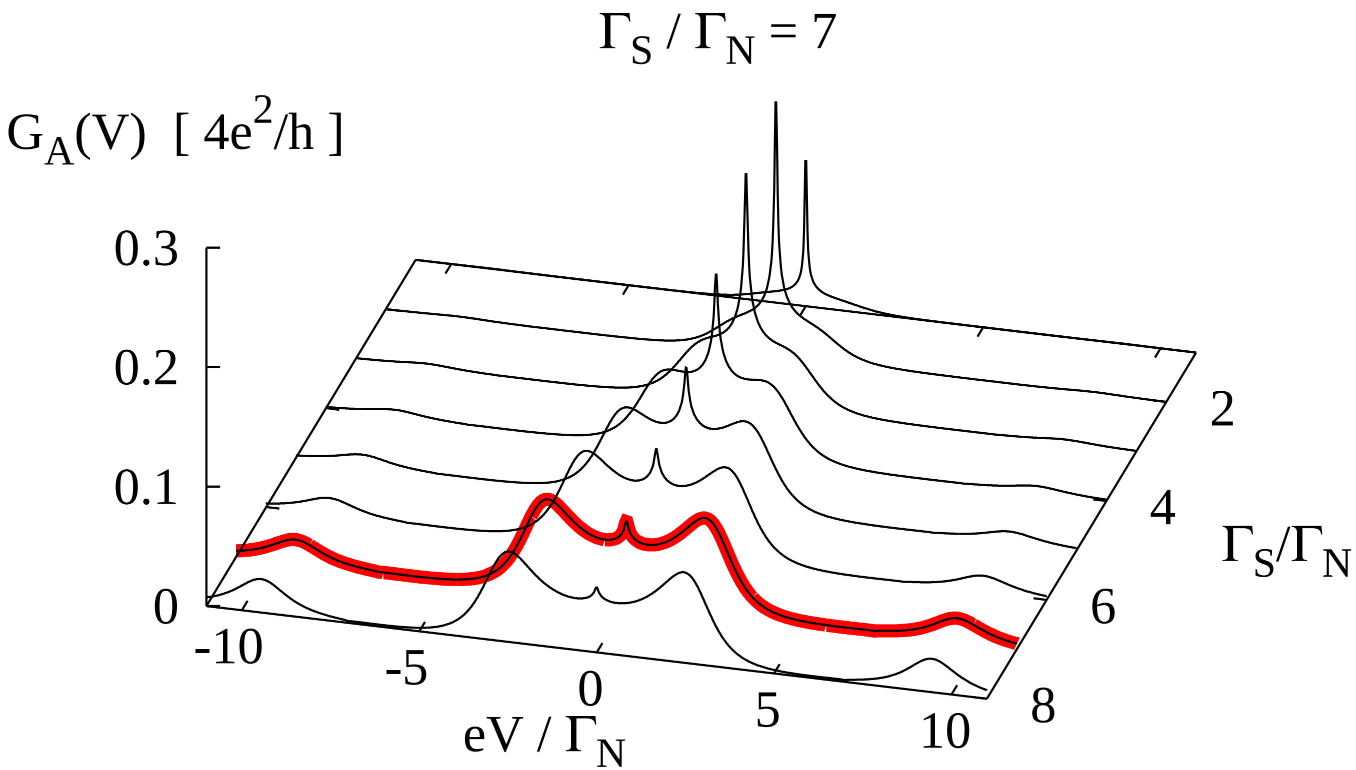

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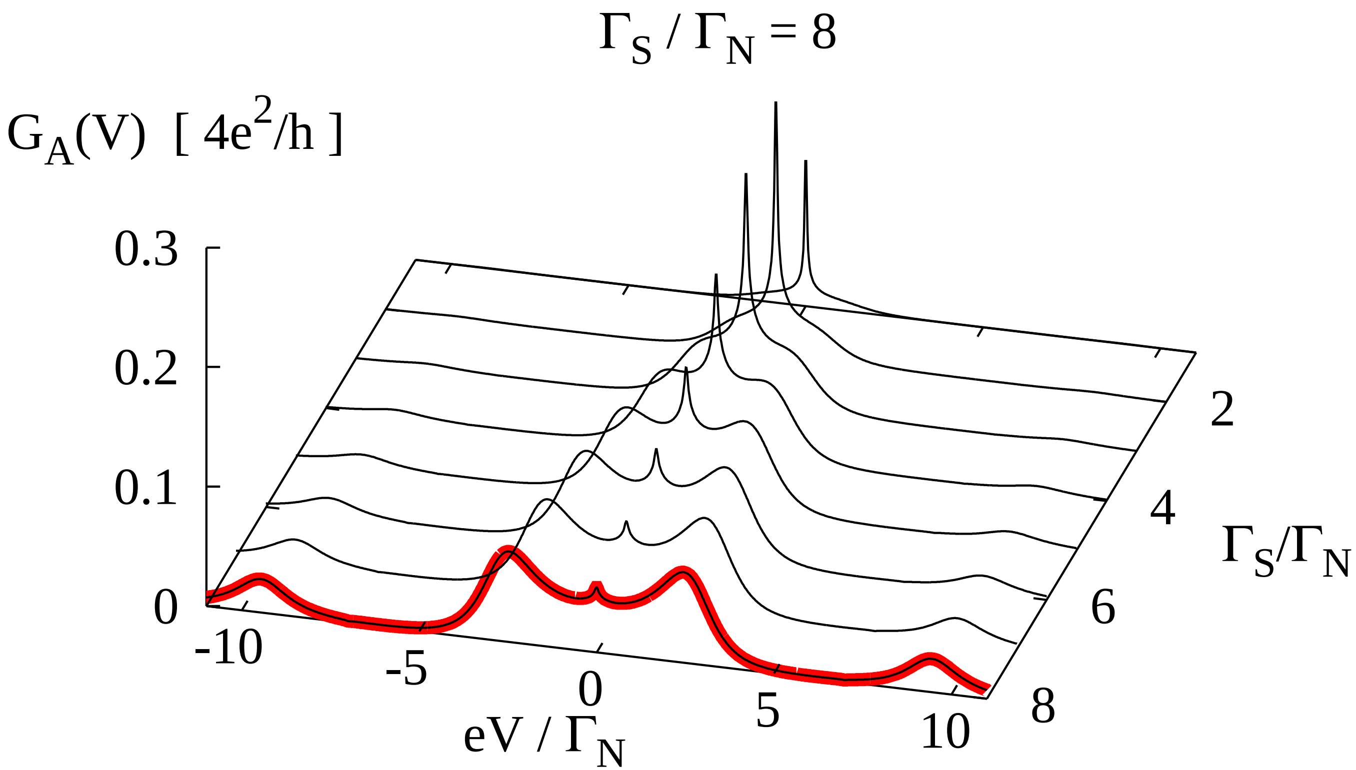

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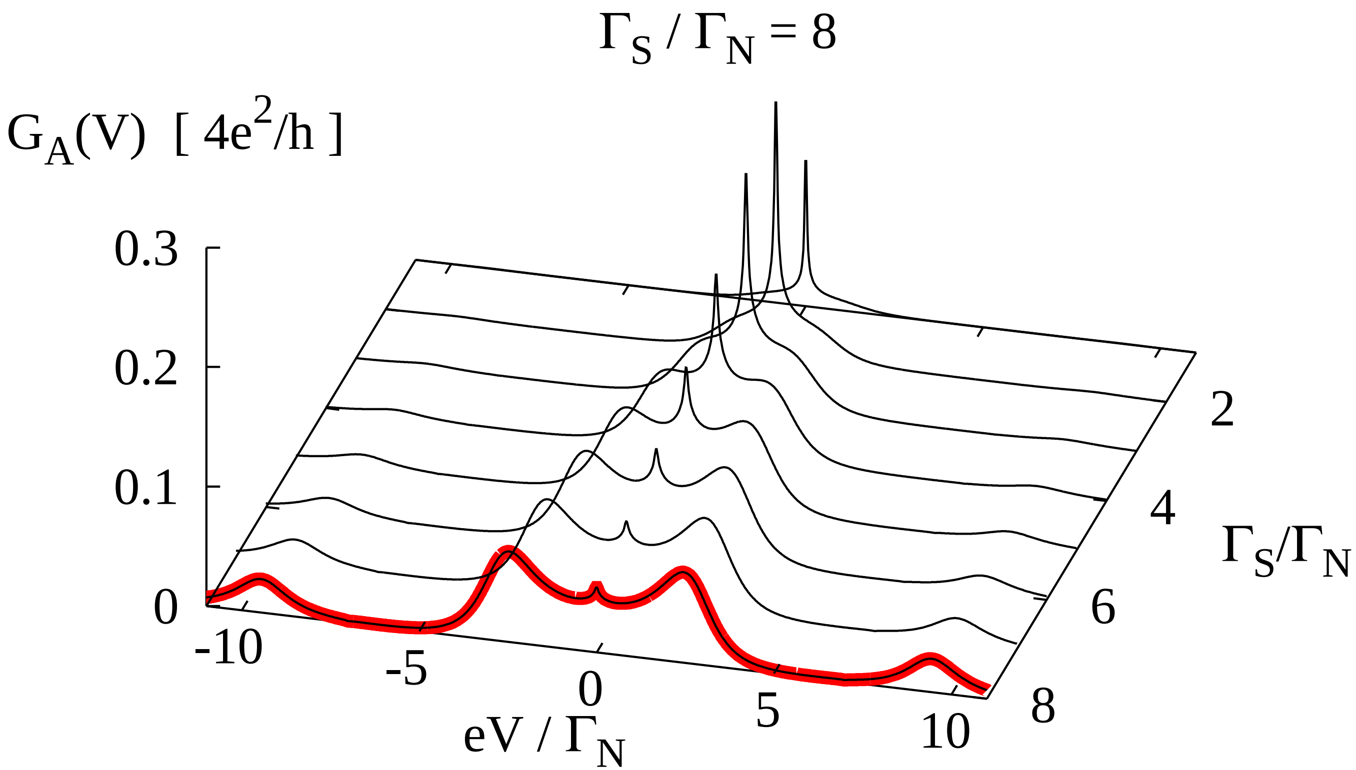


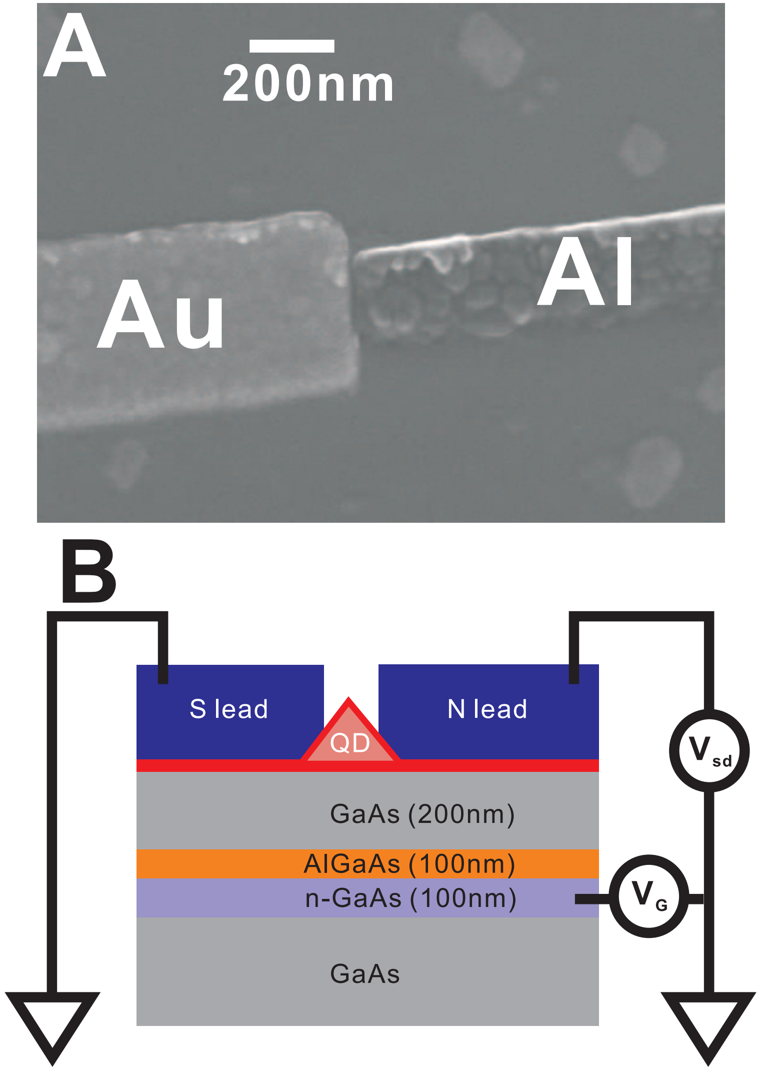
T. Domański and A. Donabidowicz, PRB **78**, 073105 (2008).

Kondo resonance slightly enhances the zero-bias Andreev conductance, especially for $\Gamma_S \sim \Gamma_N$!

Experimental setup

Experimental setup

/ University of Tokyo /



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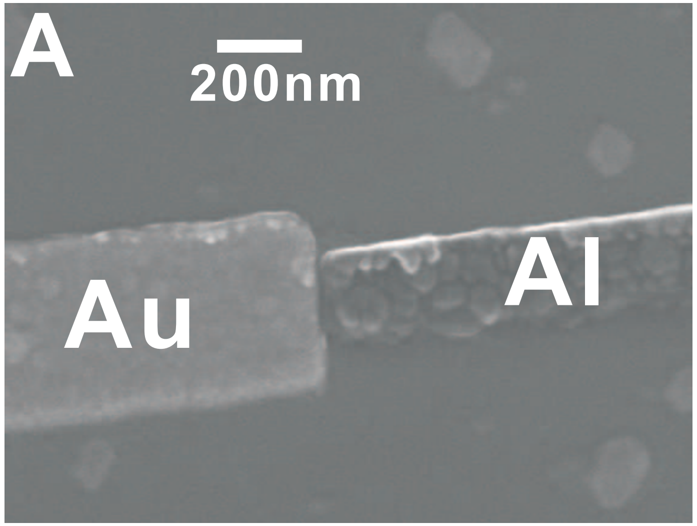
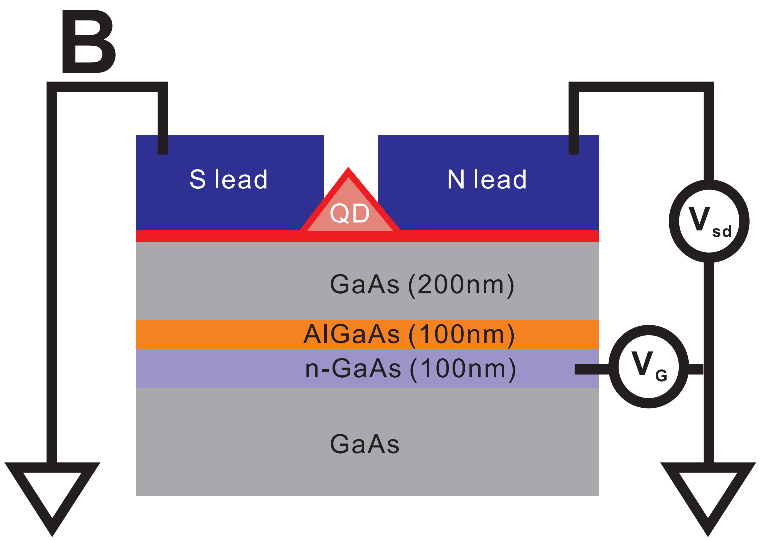
/ University of Tokyo /





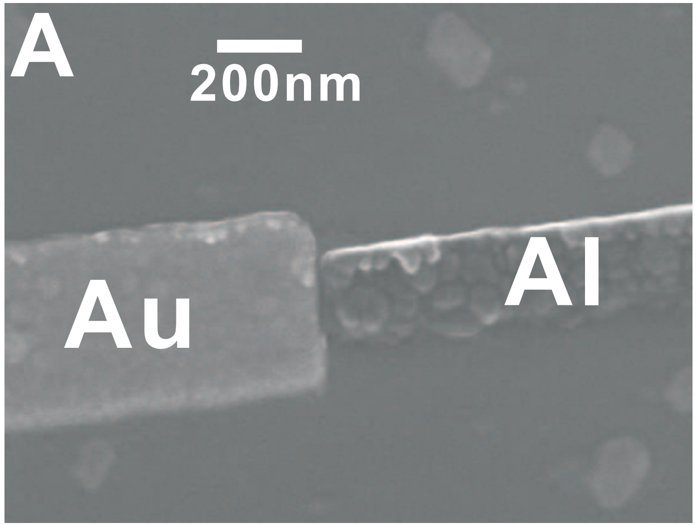
QD : self-assembled InAs

diameter ～ 100 nm

backgate : Si-doped GaAs

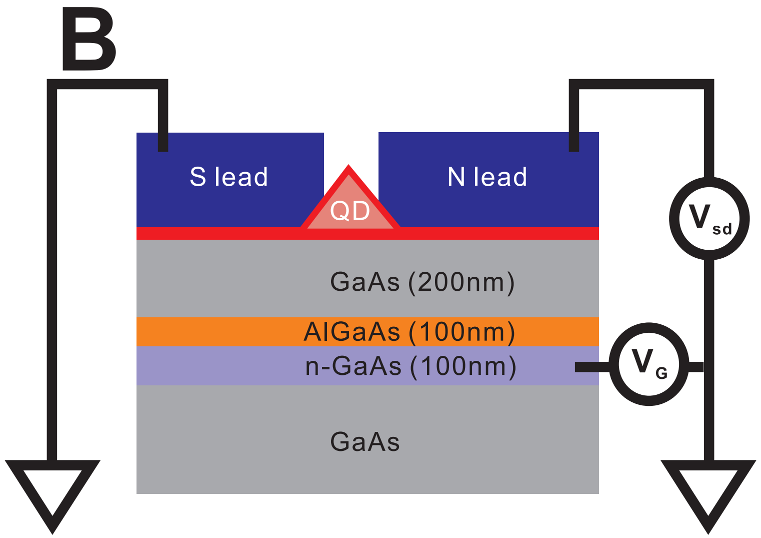
Experimental setup

/ University of Tokyo /



$T_c \simeq 1\text{K}$

$\Delta \simeq 152\mu\text{eV}$



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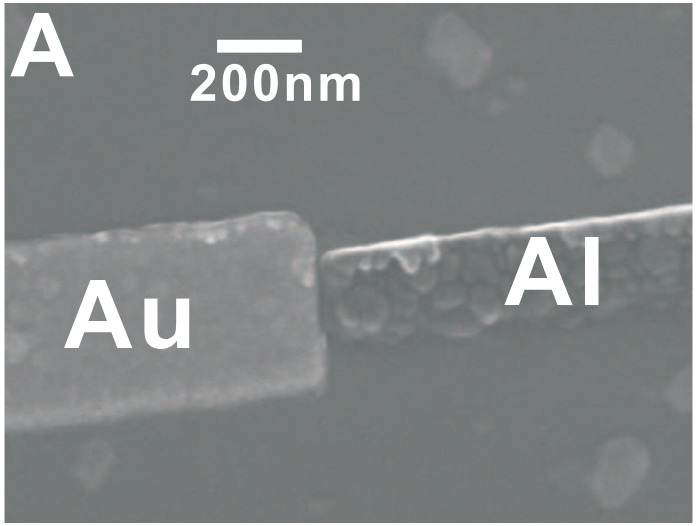
Experimental setup

/ University of Tokyo /



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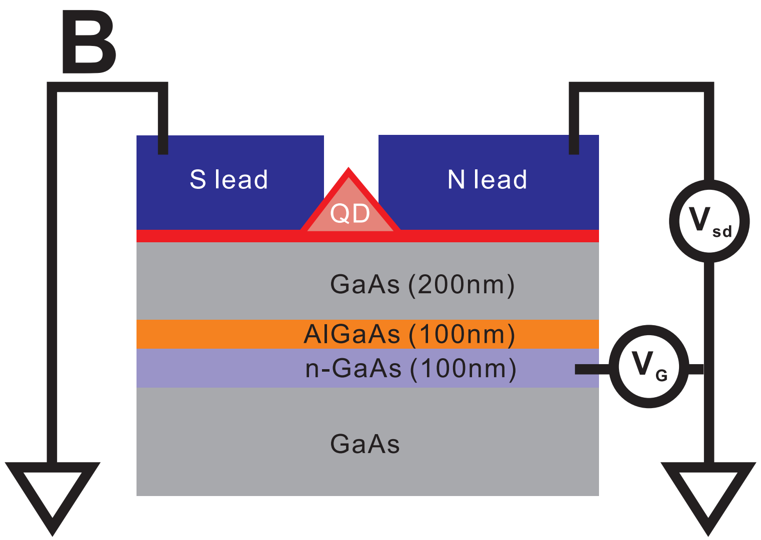

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R.S. Deacon et al, *Phys. Rev. Lett.* **104**, *076805 (2010).*

Interplay with the Kondo effect

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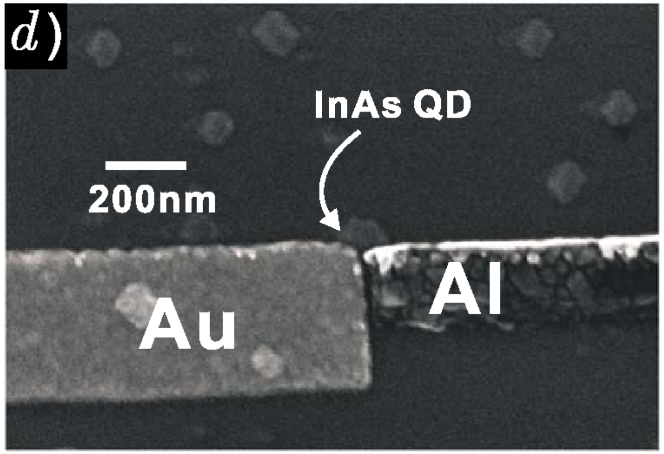

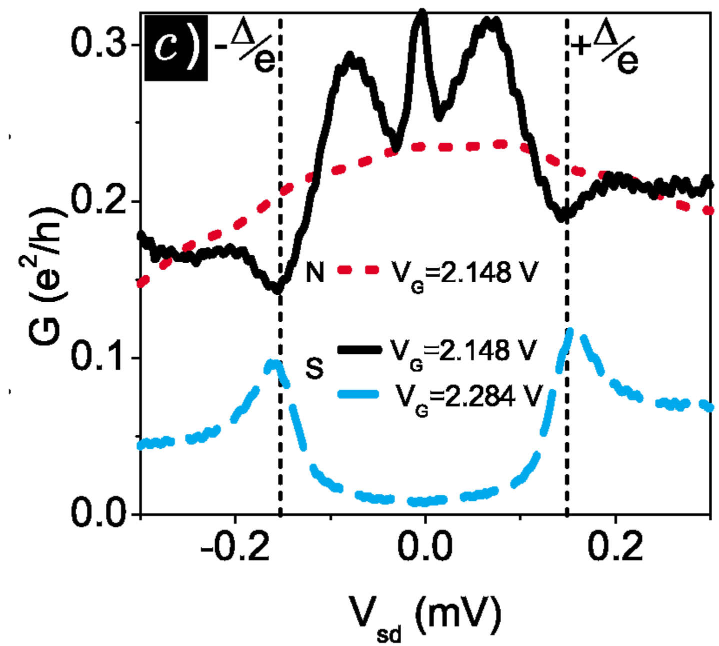

R.S. Deacon et al, *Phys. Rev. B* **81**, *121308(R) (2010).*

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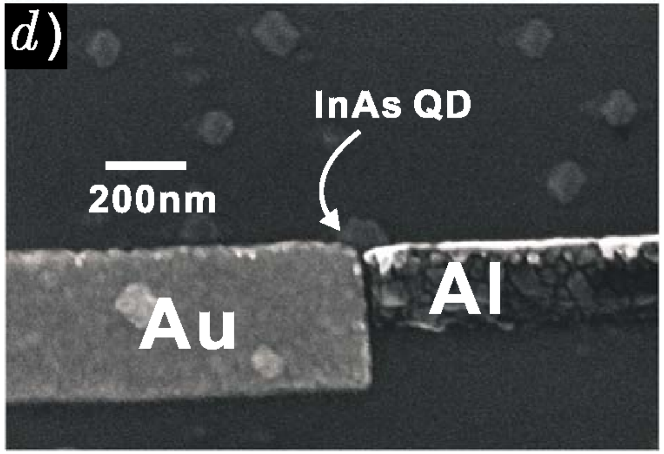
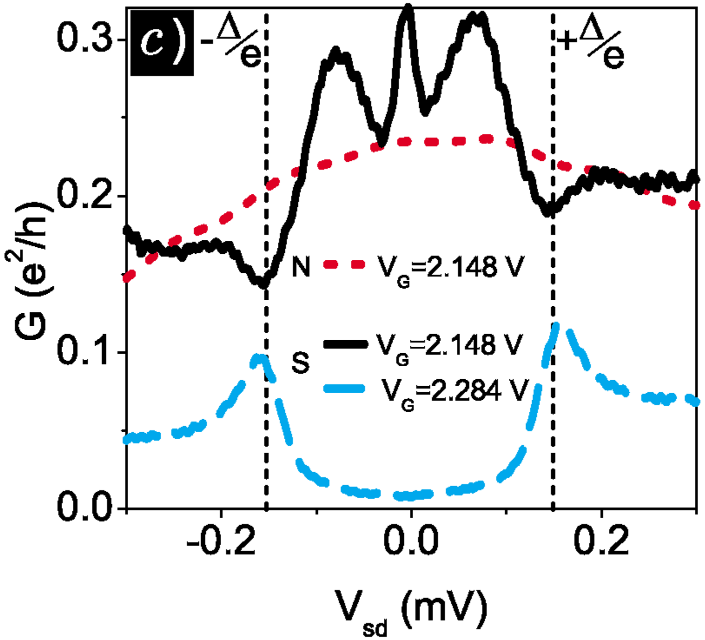


"The zero-bias conductance peak is consistent with Andreev transport enhanced by the Kondo singlet state"



R.S. Deacon et al, *Phys. Rev. B* **81**, *121308(R) (2010).*

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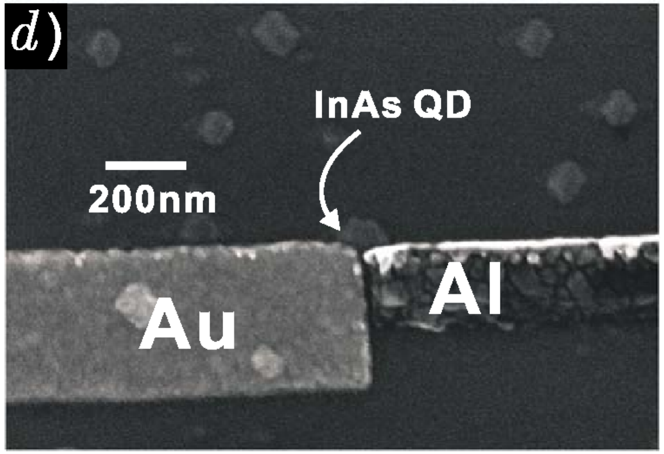

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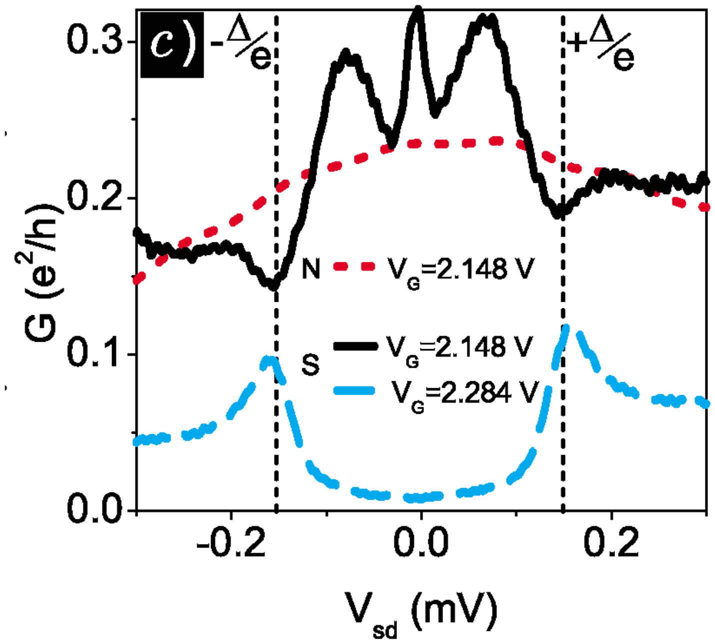

"We note that the feature exhibits excellent qualitative agreement with a recent theoretical treatment by Domanski et al"

R.S. Deacon et al, *Phys. Rev. B* **81**, *121308(R) (2010).*

Summary

/ for part 2 /

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QD coupled between N and S electrodes:

Summary

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$\Rightarrow$ **absorbs the superconducting order** / proximity effect /

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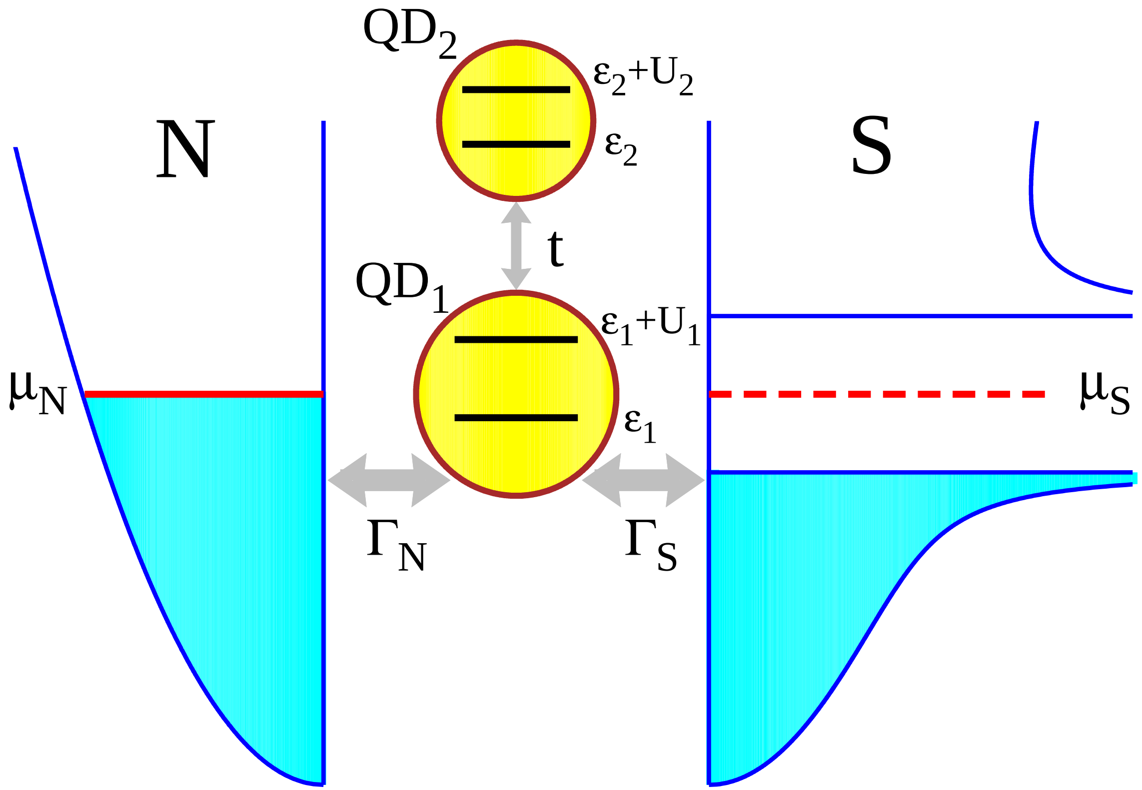
$\Rightarrow$ zero-bias enhancement / below T_K /

3. Further extensions

Double QD – **between a metal and superconductor**



(T-shape configuration)

Relevant issues:

⇒ **induced on-dot pairing** . . . . (due to $\boldsymbol{\Gamma_S}$)

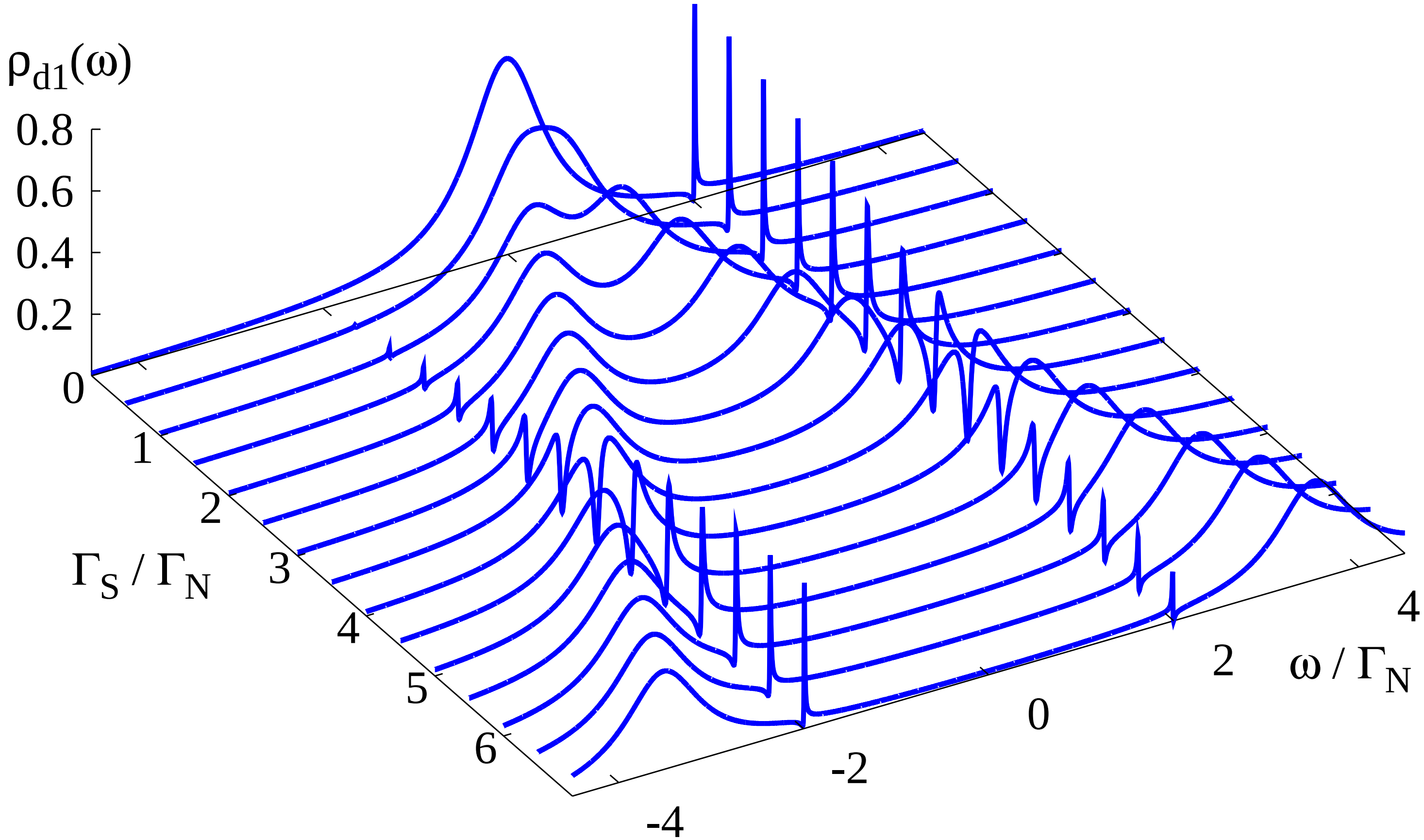
⇒ **Coulomb blockade & Kondo effect** . . . . (via $\boldsymbol{U_1}$ and $\boldsymbol{\Gamma_N}$)

⇒ **quantum interference** . . . . (because of $\boldsymbol{t}$)

Quantum interference – in the particle and hole channels

Quantum interference – in the particle and hole channels

Fano-type lineshapes appear simultaneously at $\pm\varepsilon_2$

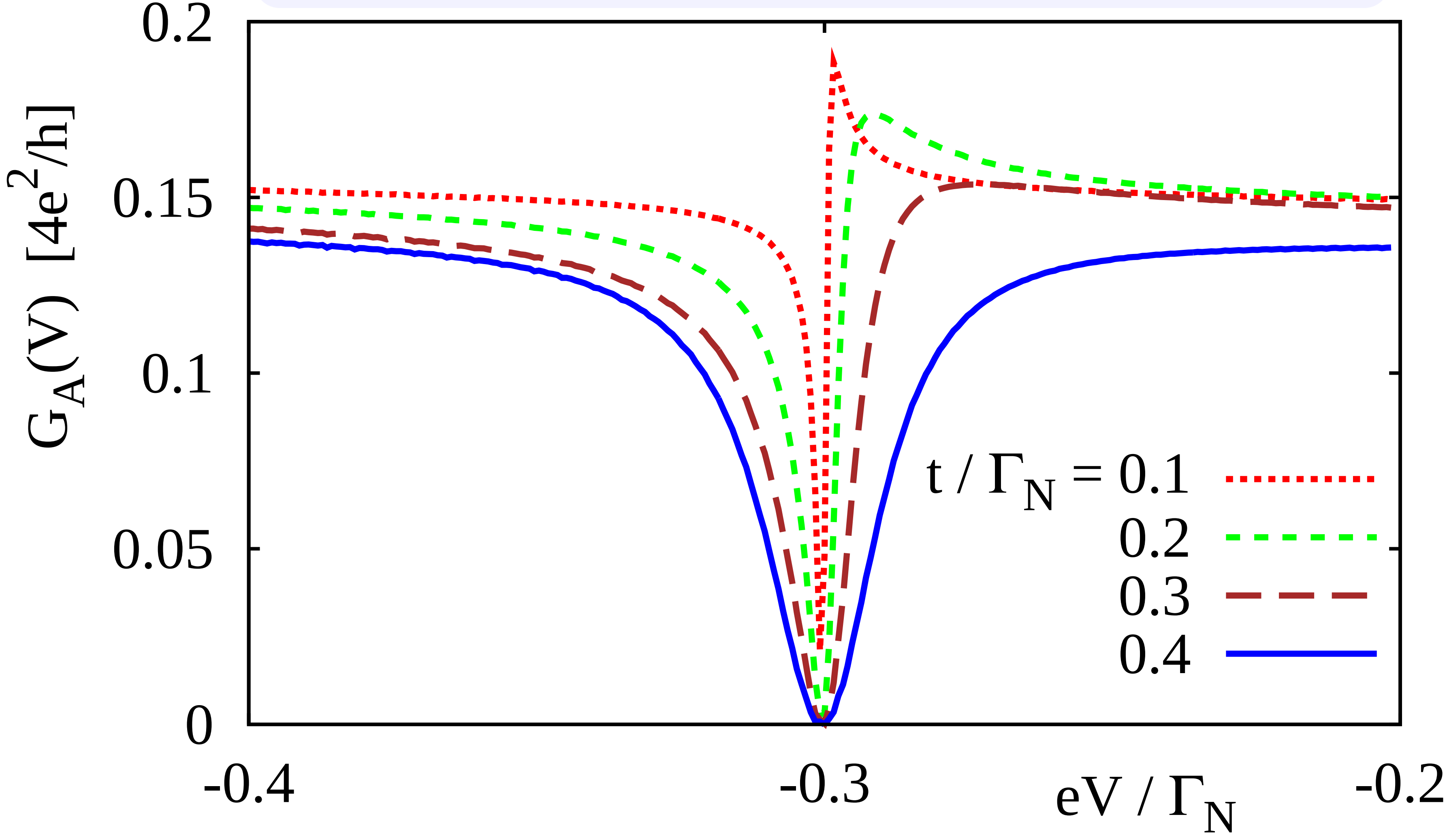


/ The case: $U_1 = 0$ and $U_2 = 0$ /

J. Barański and T. Domański, Phys. Rev. B (2012).

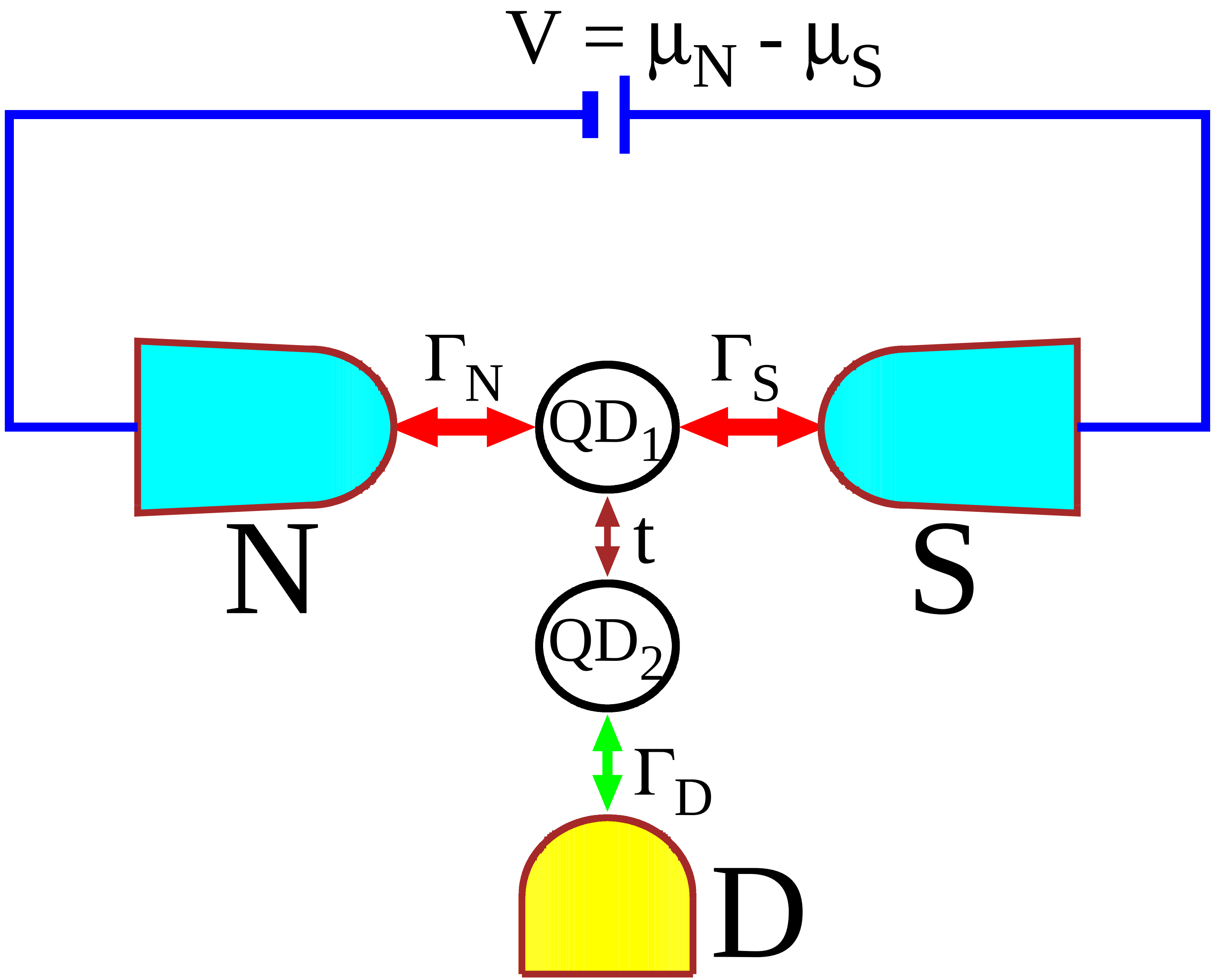
Quantum interference – in the particle and hole channels

Differential Andreev conductance $G_A = \frac{\partial J_A(V)}{\partial V}$



J. Barański and T. Domański, Phys. Rev. B **84**, 195424 (2011).

Double QD – **decoherence effects**

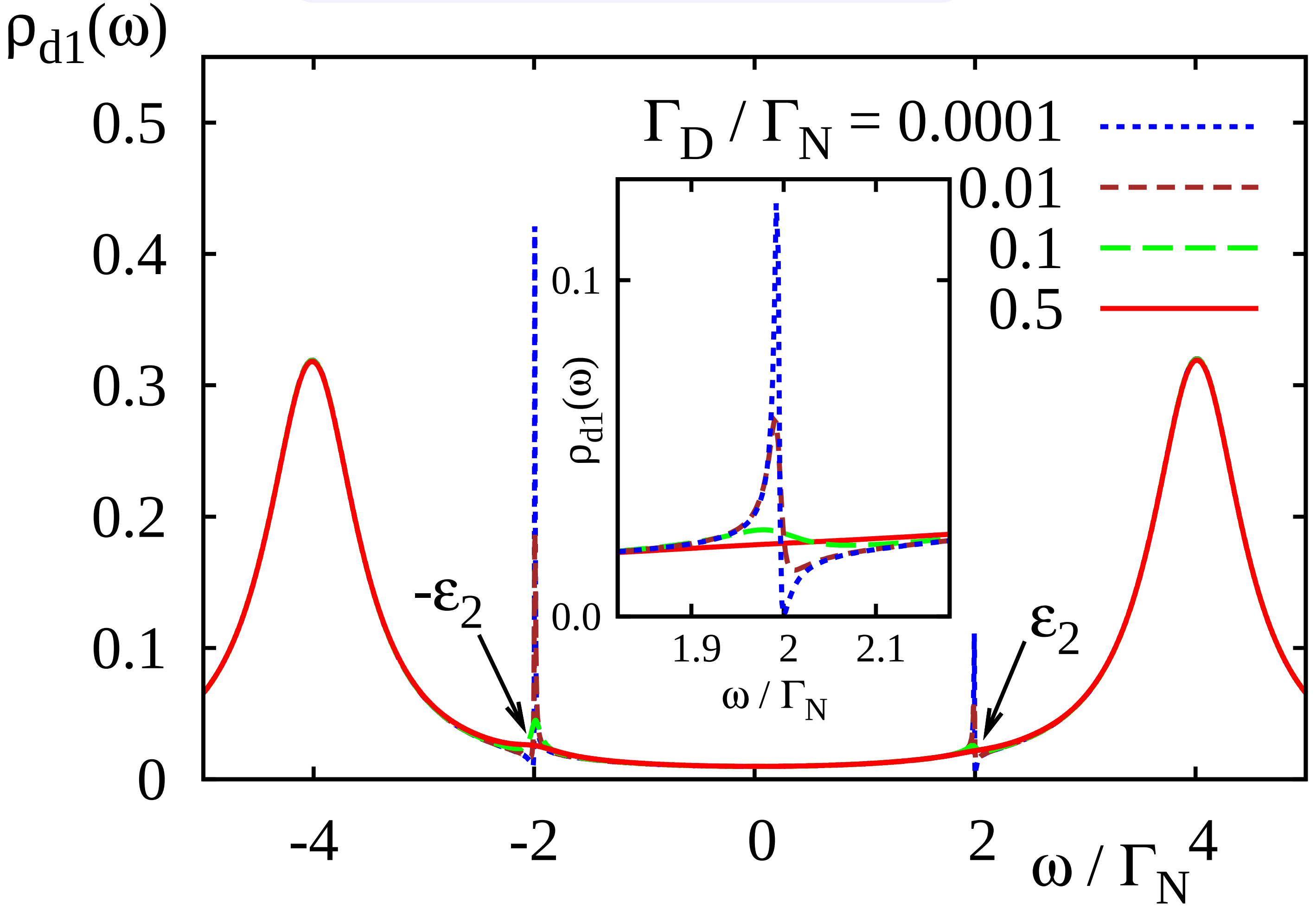


In this setup the floating lead (D) is responsible for a dephasing.

Quantum interference – influence of the decoherence

Quantum interference – **influence of the decoherence**

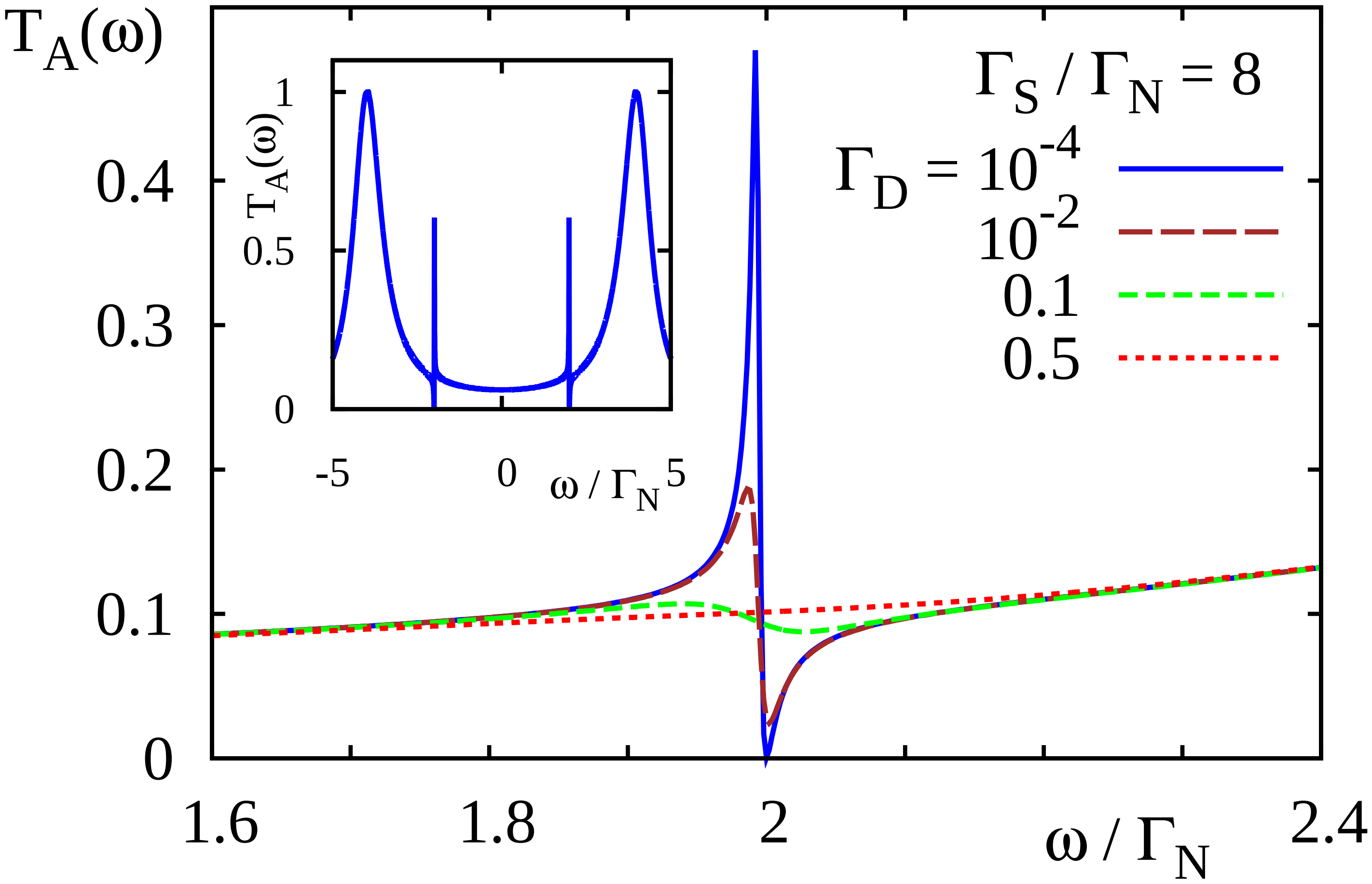
Density of states $\rho_{d1}(\omega)$



J. Barański and T. Domański, Phys. Rev. B (2012).

Quantum interference – influence of the decoherence

Andreev transmittance $T_A(\omega)$



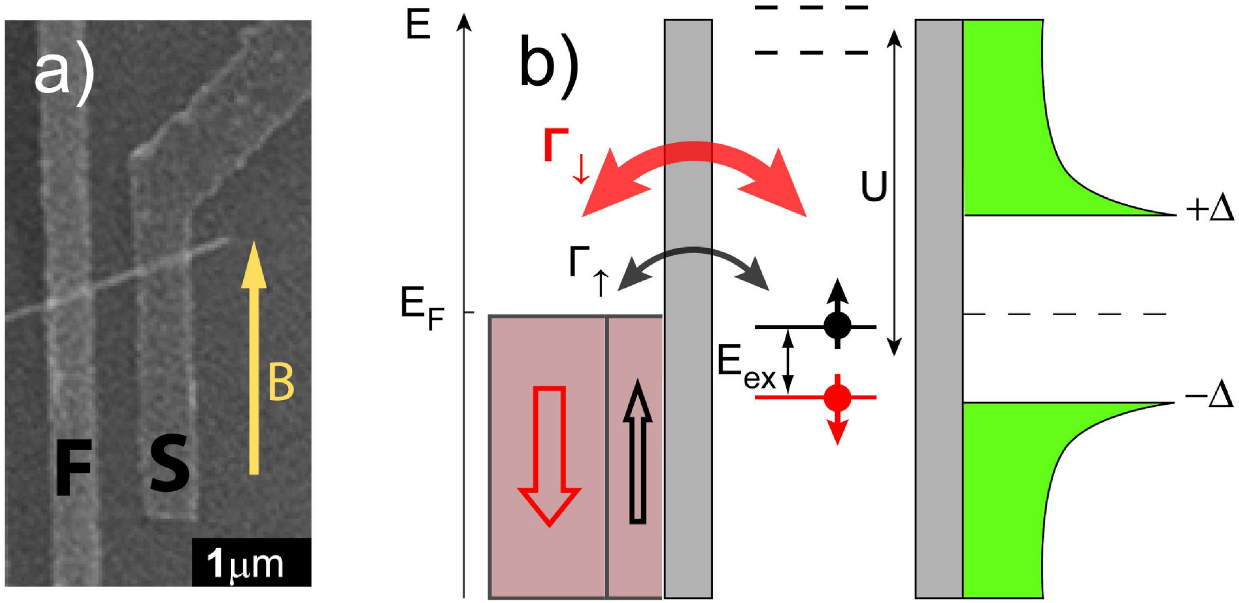
J. Barański and T. Domański, Phys. Rev. B (2012).

Interplay with ferromagnetism

– Univ. of Basel group

Interplay with ferromagnetism

– **Univ. of Basel group**



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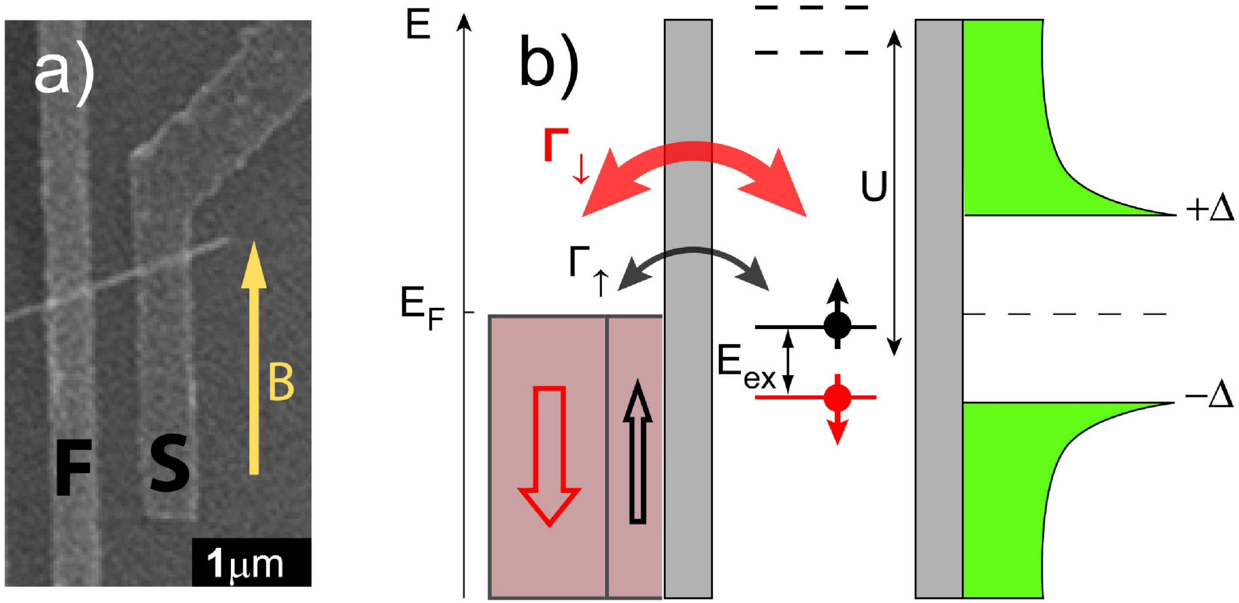
L. Hofstetter et al, *Phys. Rev. Lett.* **104**, *246804 (2010).*



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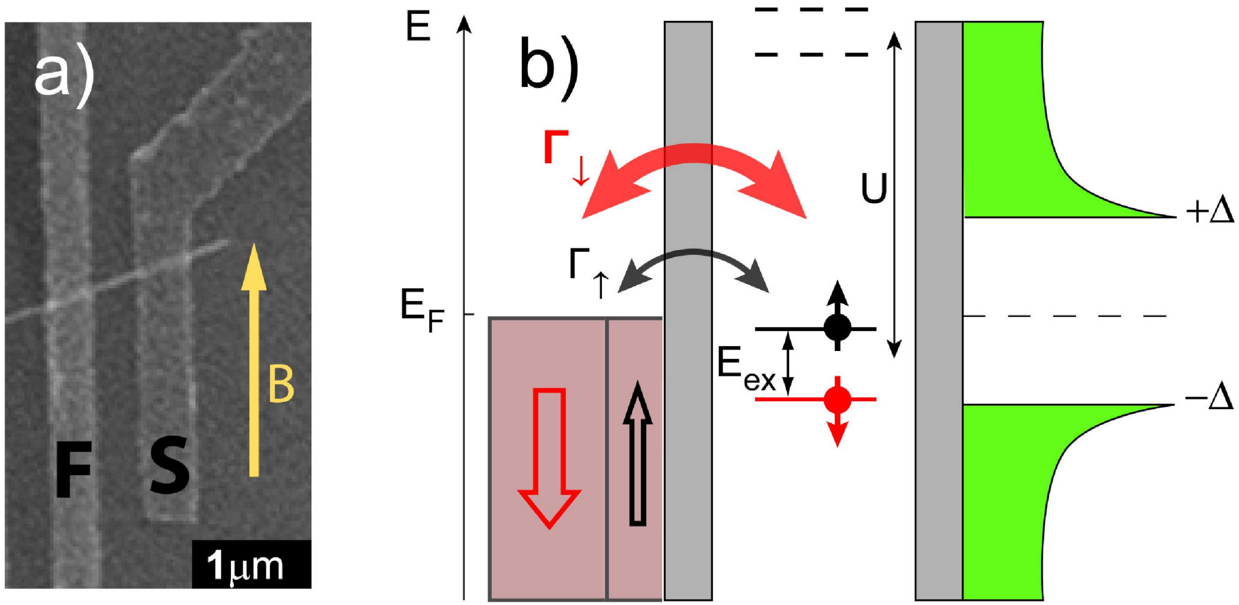
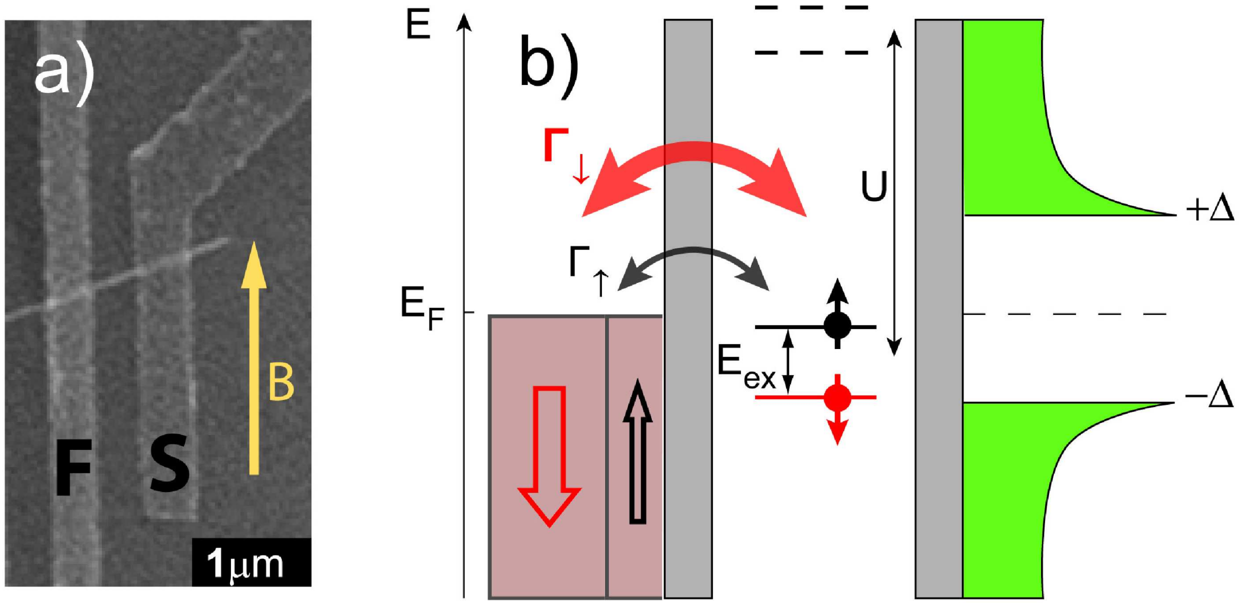


Effects of ferromagnetism and superconductivity

Interplay with ferromagnetism – Univ. of Basel group

L. Hofstetter et al, *Phys. Rev. Lett.* **104**, *246804 (2010).*



Effects of ferromagnetism and superconductivity

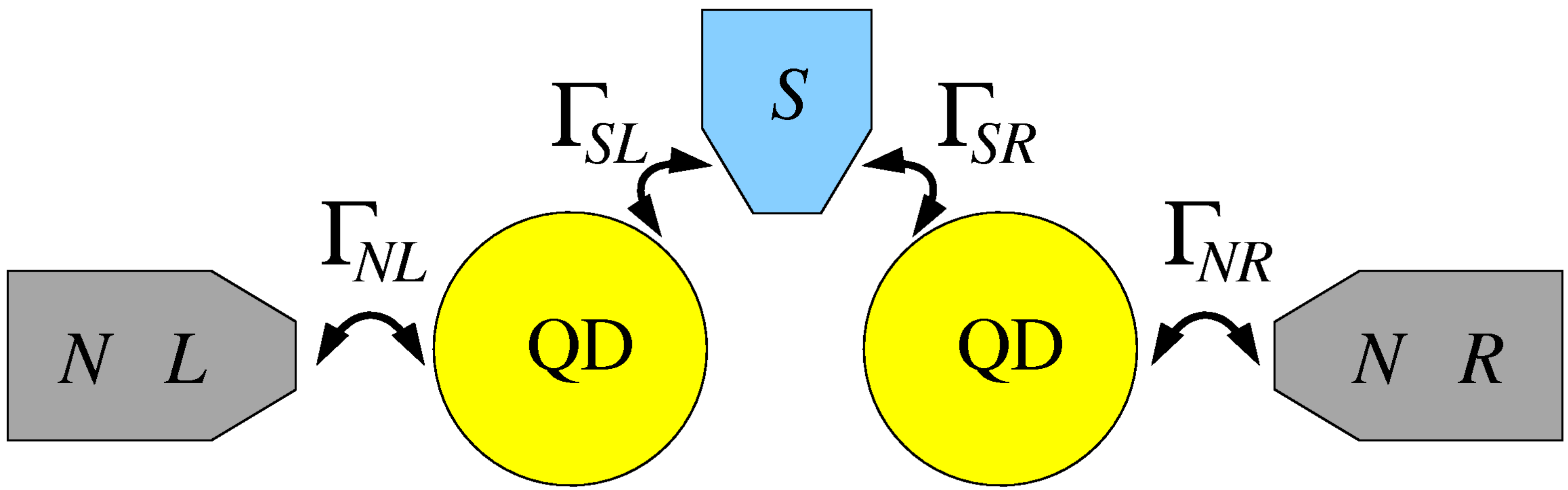
left – Ni/Co/Pd trilayer ferromagnet

QD – InAs nanowire

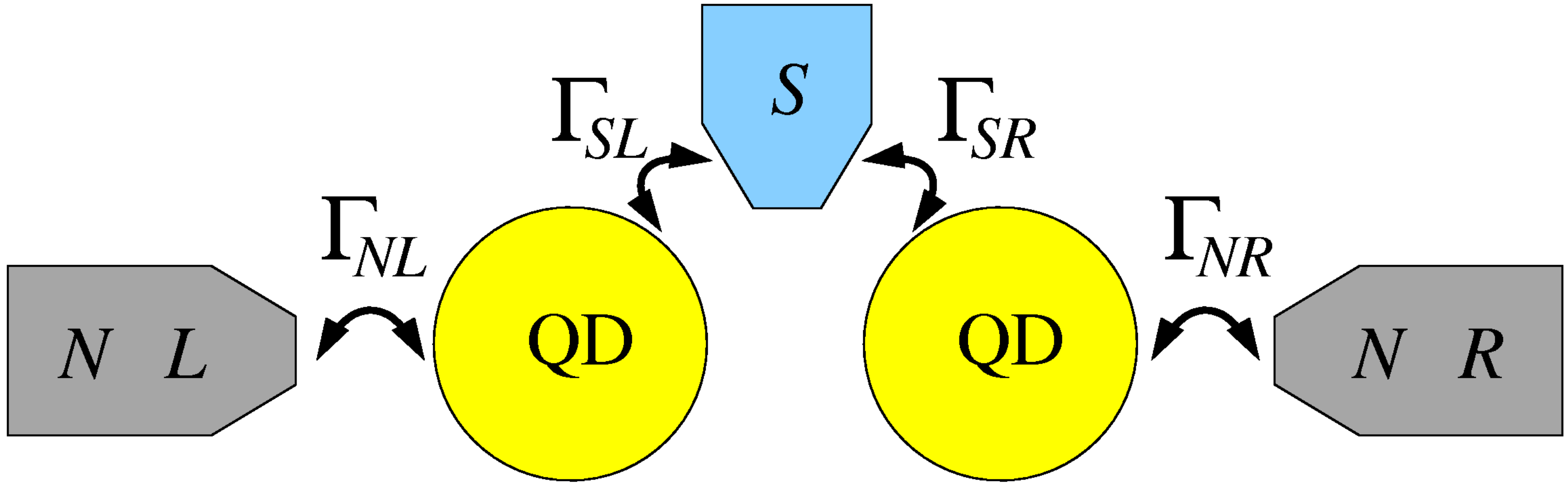
right – Ti/Al bilayer superconductor

Three terminal junctions

Three terminal junctions

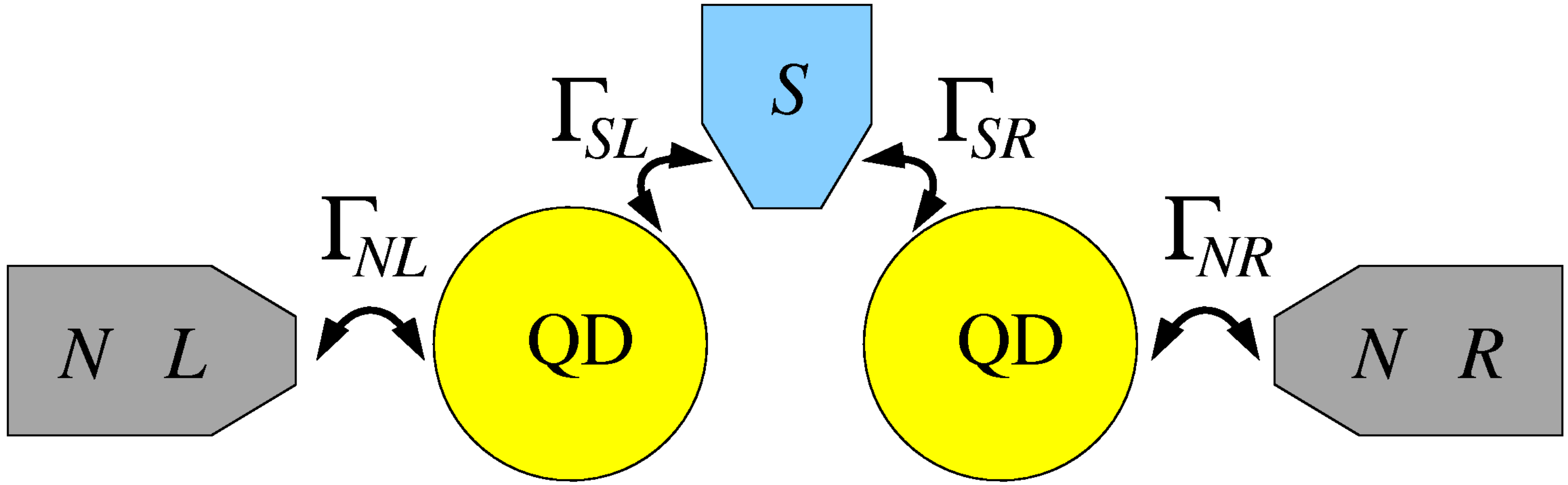


Three terminal junctions



Crossed Andreev reflections tunable via gate voltages

Three terminal junctions



Crossed Andreev reflections tunable via gate voltages

J. Eldridge, M.G. Pala, M. Governale, J. König, Phys. Rev. B **82***, 184507 (2010)*

Cooper pair splitter

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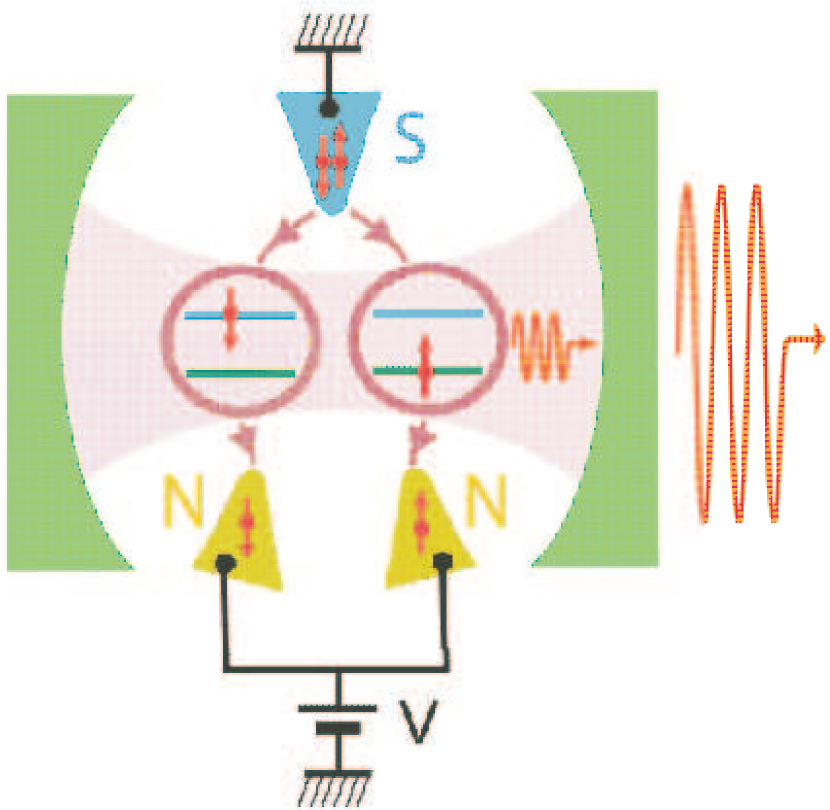

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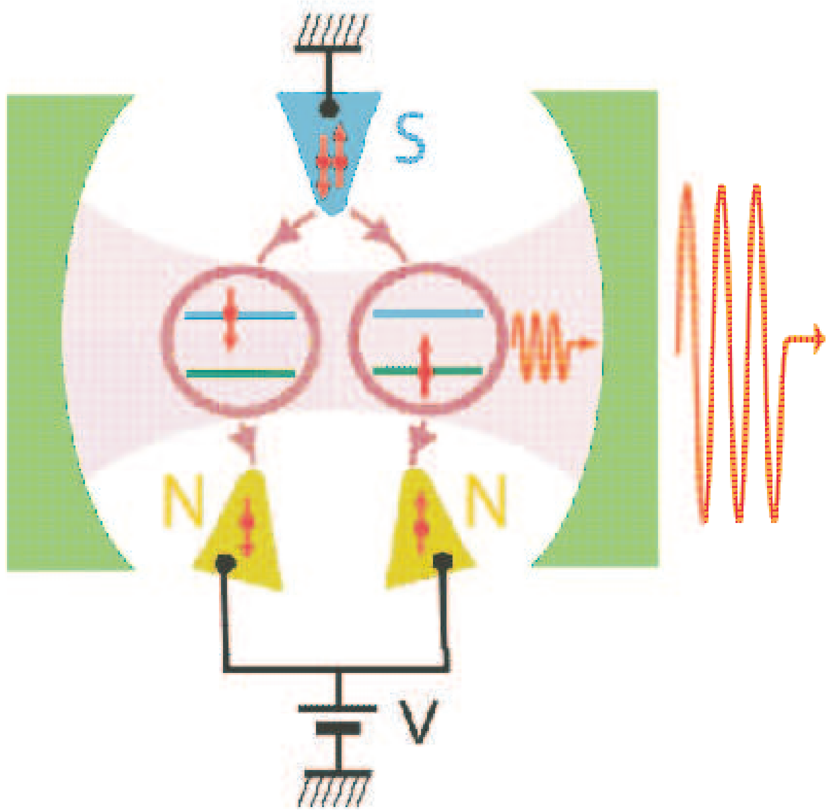
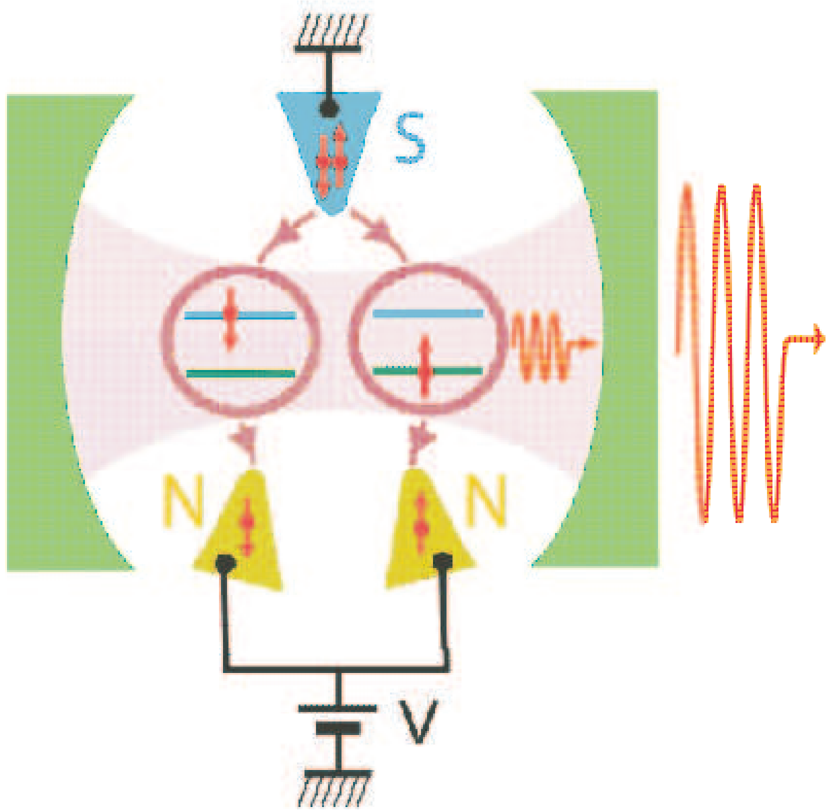


Realization of the Cooper pair splitting in a microwave cavity

Cooper pair splitter



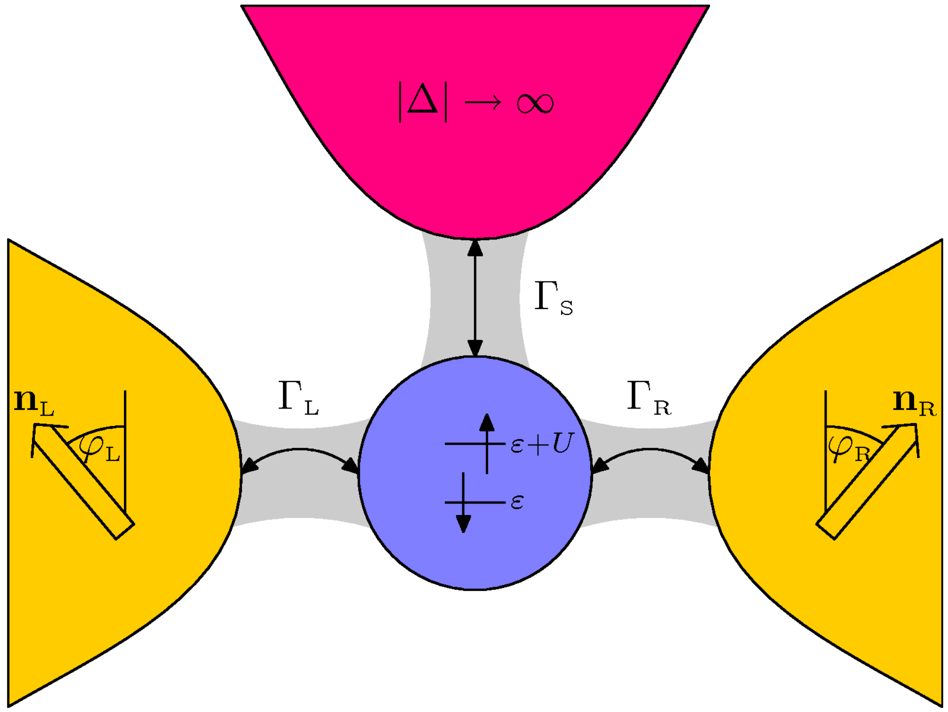
Realization of the Cooper pair splitting in a microwave cavity

A. Cottet, T. Kontos, and A. Levy Yeyati, Phys. Rev. Lett. **108**, *166803 (2012)*

QD spin-valve

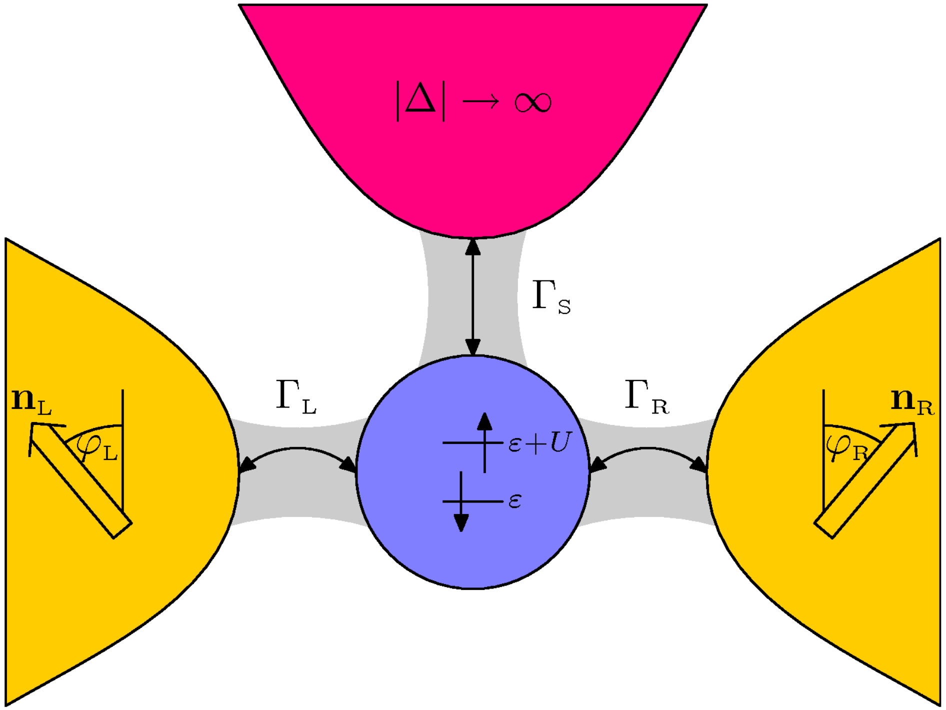
– in three terminal junctions

QD spin-valve – in three terminal junctions



QD spin-valve

– in three terminal junctions



Idea of the spin valves using the Andreev reflections

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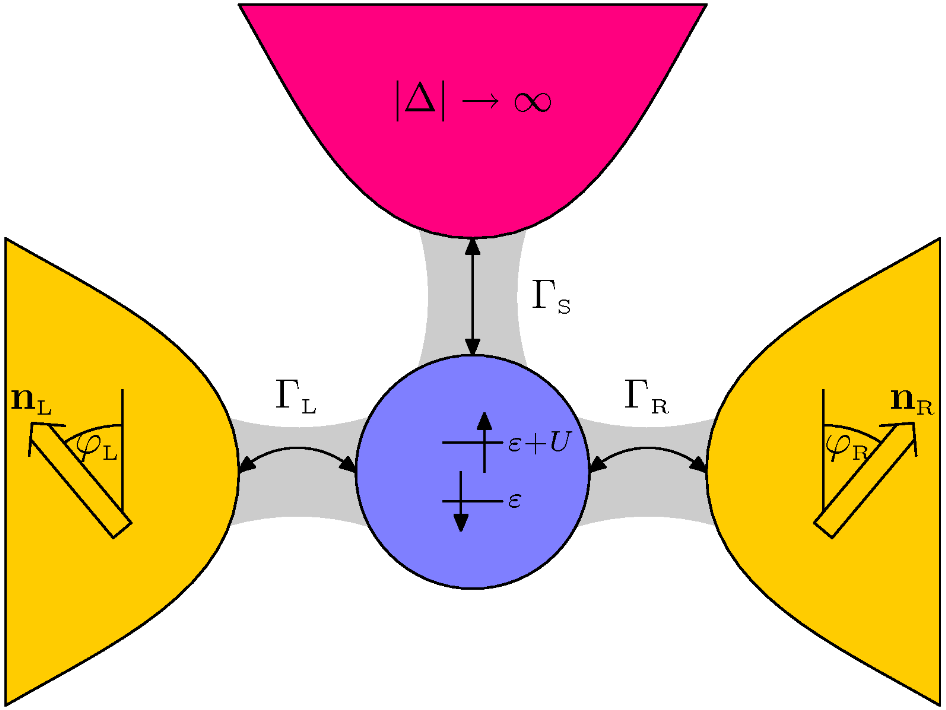

Idea of the spin valves using the Andreev reflections

B. Sothmann, D. Futterer, M. Governale, J. König, Phys. Rev. B **82***, 094514 (2010).*

4. Bulk superconductors

Andreev spectroscopy – for bulk superconductors

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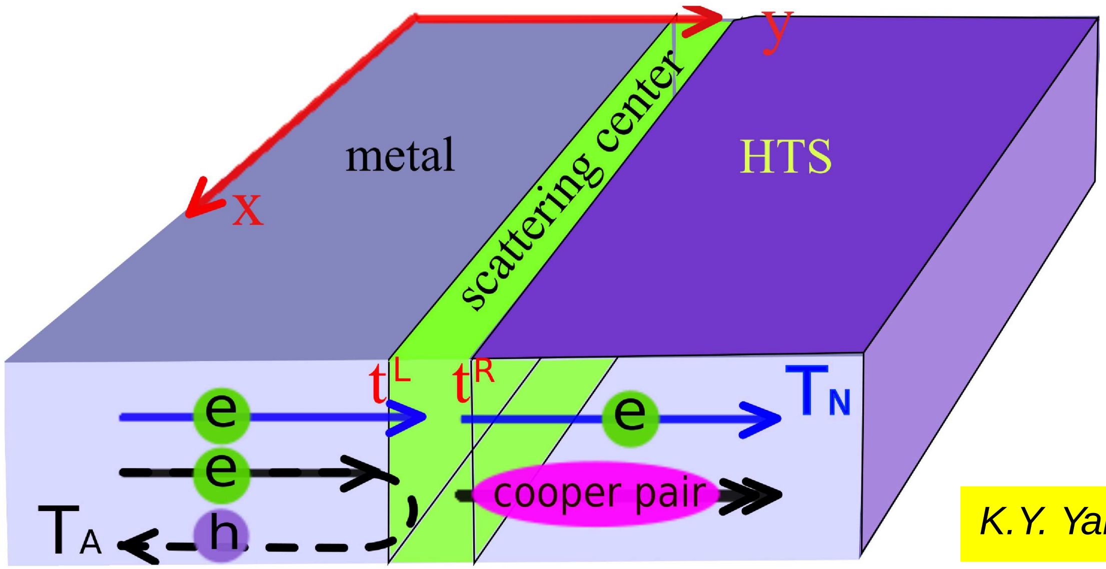
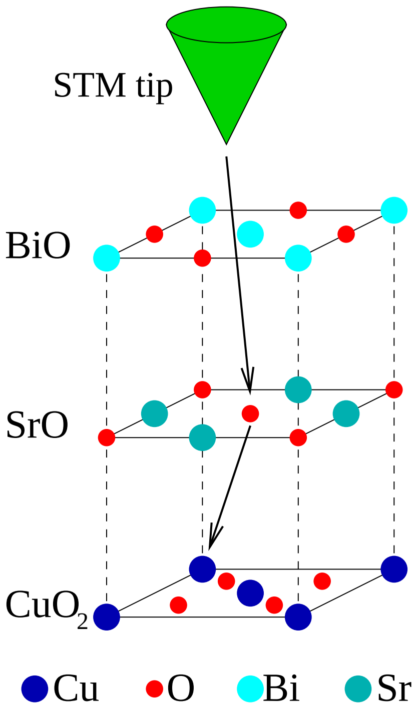


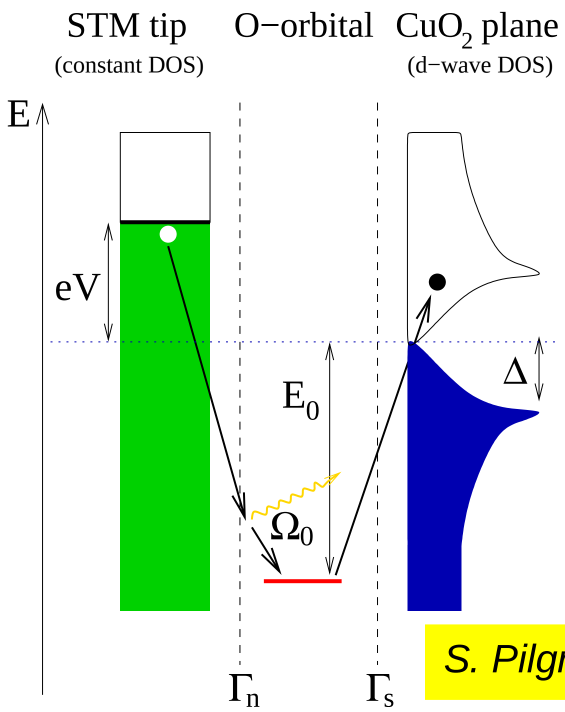
K.Y. Yang et al, *Phys. Rev. Lett.* **105**, *167004 (2010).*

For practical experimental realizations one can e.g. use an insulating barrier sandwiched between the conducting (N) and the probed superconductor (S).

Andreev spectroscopy – for bulk superconductors

The subgap Andreev spectroscopy is also a valuable tool for studying various superconducting compounds.





S. Pilgram et al, *Phys. Rev. Lett.* **97**, *117003 (2006).*

Other experimental realizations are also possible in the STM configuration, where the apex oxygen atoms play a role similar to QD in the N-QD-S setup.

Andreev spectroscopy – **for bulk superconductors**

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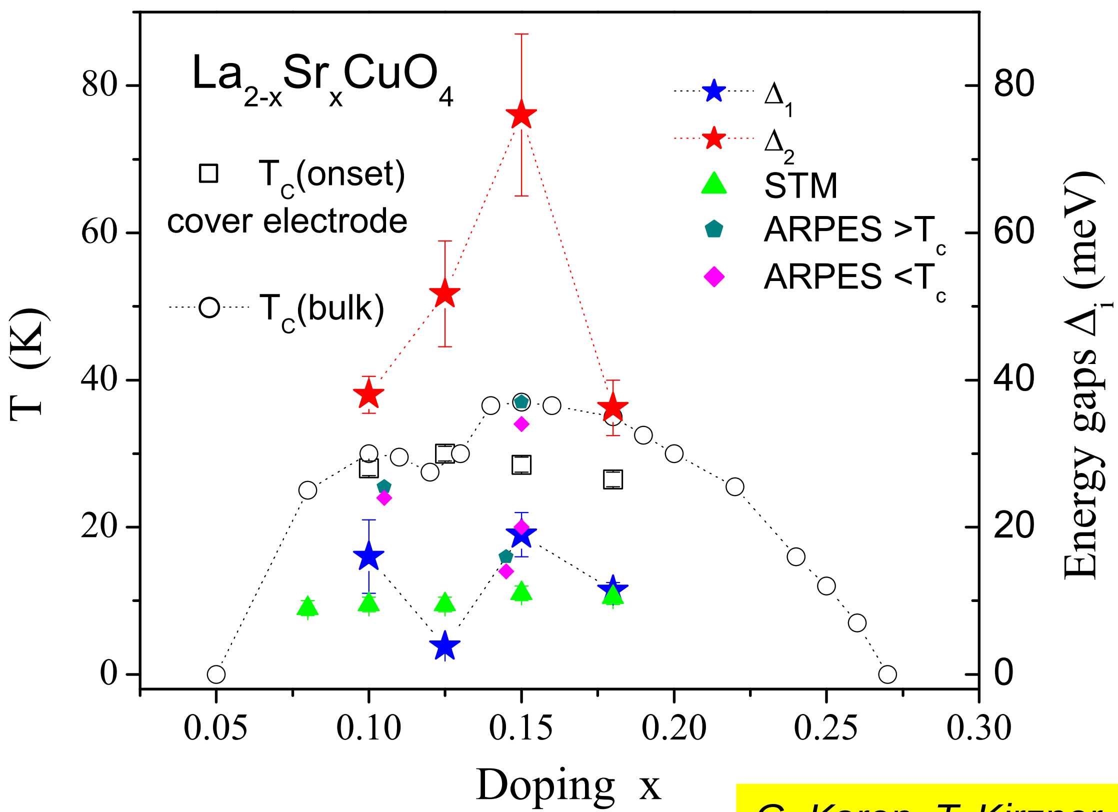

G. Koren, T. Kirzner, Phys. Rev. Lett. **106**, *017002 (2011).*

Such Andreev spectroscopy has revealed the intriguing two-gap feature.

5. Ultracold gasses

Andreev spectroscopy – for ultracold atoms

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Proposal for the Andreev-type spectroscopy has been discussed also in a context of the superfluid ultracold fermion atom systems.

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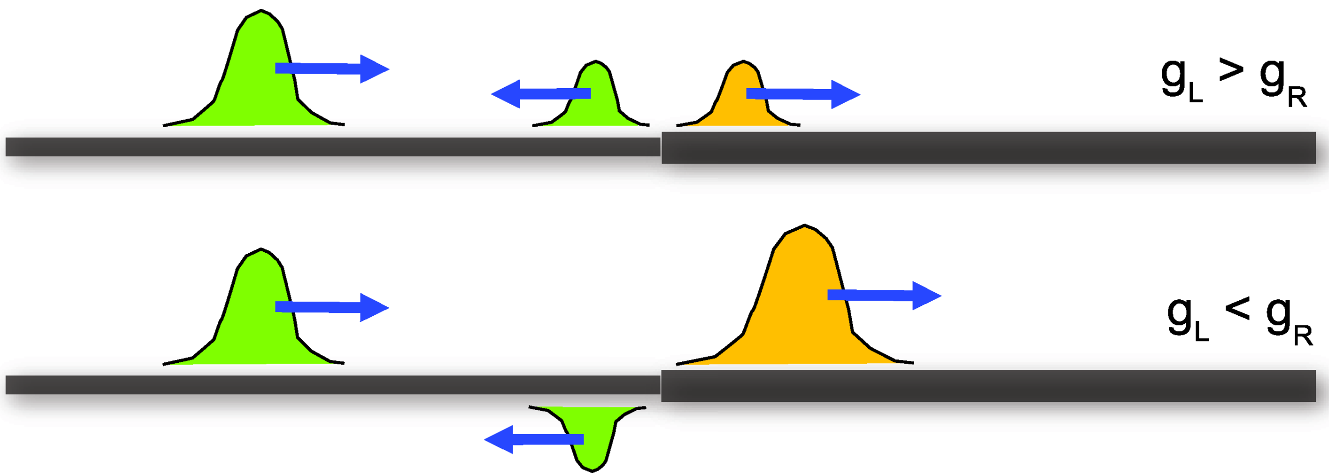

A.J. Daley, P. Zoller, and B. Trauzettel, Phys. Rev. Lett. **100**, *110404 (2008).*

The wave packet propagating along the 1-dimensional optical lattice can be scattered at an interaction boundary in the Andreev-type fashion.

Feshbach resonance

[local problem]

$$\hat{H}_{loc}(\mathbf{r}) = \sum_{\sigma} \varepsilon(\mathbf{r})\ \hat{c}^{\dagger}_{\sigma}(\mathbf{r})\hat{c}_{\sigma}(\mathbf{r}) + E(\mathbf{r})\ \hat{b}^{\dagger}(\mathbf{r})\hat{b}(\mathbf{r})$$
$$+g\left(\hat{b}^{\dagger}(\mathbf{r})\hat{c}_{\downarrow}(\mathbf{r})\hat{c}_{\uparrow}(\mathbf{r}) + \hat{c}^{\dagger}_{\uparrow}(\mathbf{r})\hat{c}^{\dagger}_{\downarrow}(\mathbf{r})\hat{b}(\mathbf{r})\right)$$

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M.L. Chiofalo, S.J.J.M.F. Kokkelmans, J.N. Milstein, and M.J. Holland, Phys. Rev. Lett. **88**, *090402 (2002).*

Local solution

[exact]

$$\mathcal{G}_{loc}(i\omega_n) = [1 - Z(T)] \left(\frac{u^2}{i\omega_n - \varepsilon_+} + \frac{v^2}{i\omega_n - \varepsilon_-} \right) + \frac{Z(T)}{i\omega_n - \varepsilon}$$

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T. Domański, Eur. Phys. J. B **33**, *41 (2003);* *T. Domański* et al, *Sol. State Commun.* **105**, *473 (1998).*

Approximate general solution

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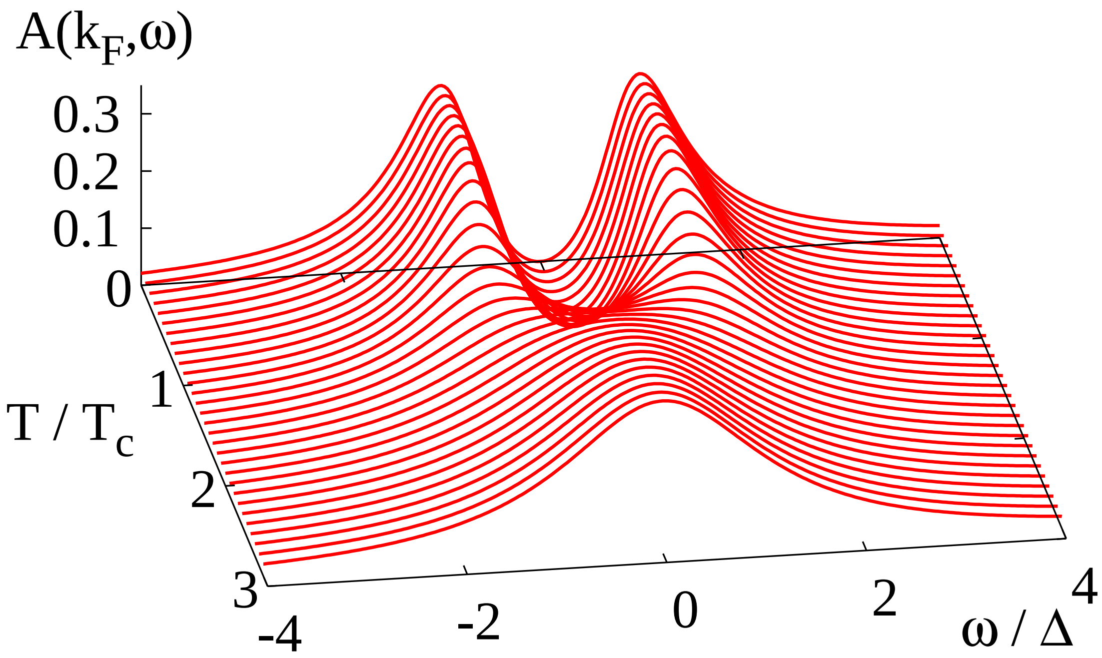

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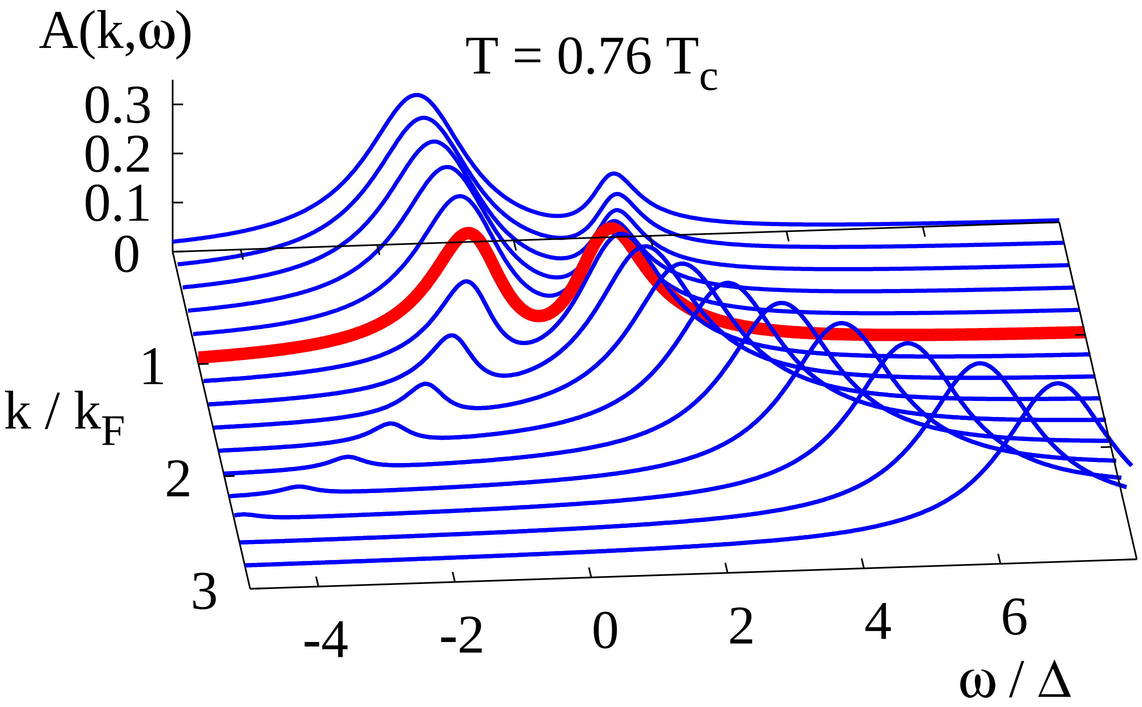

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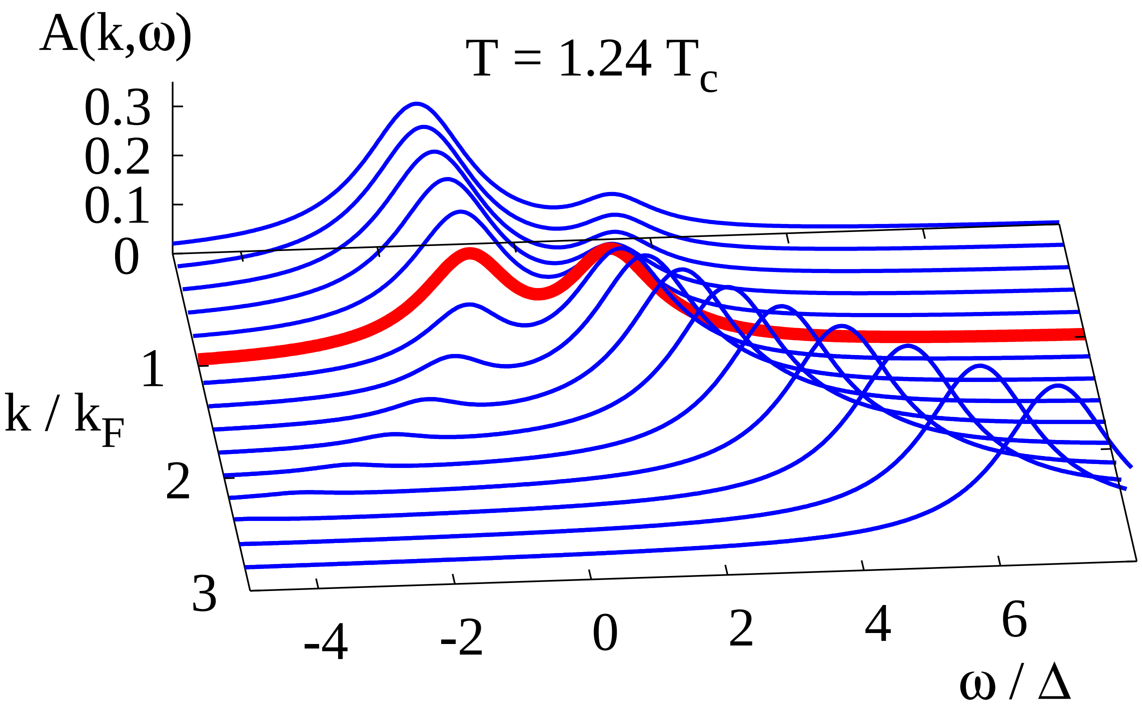

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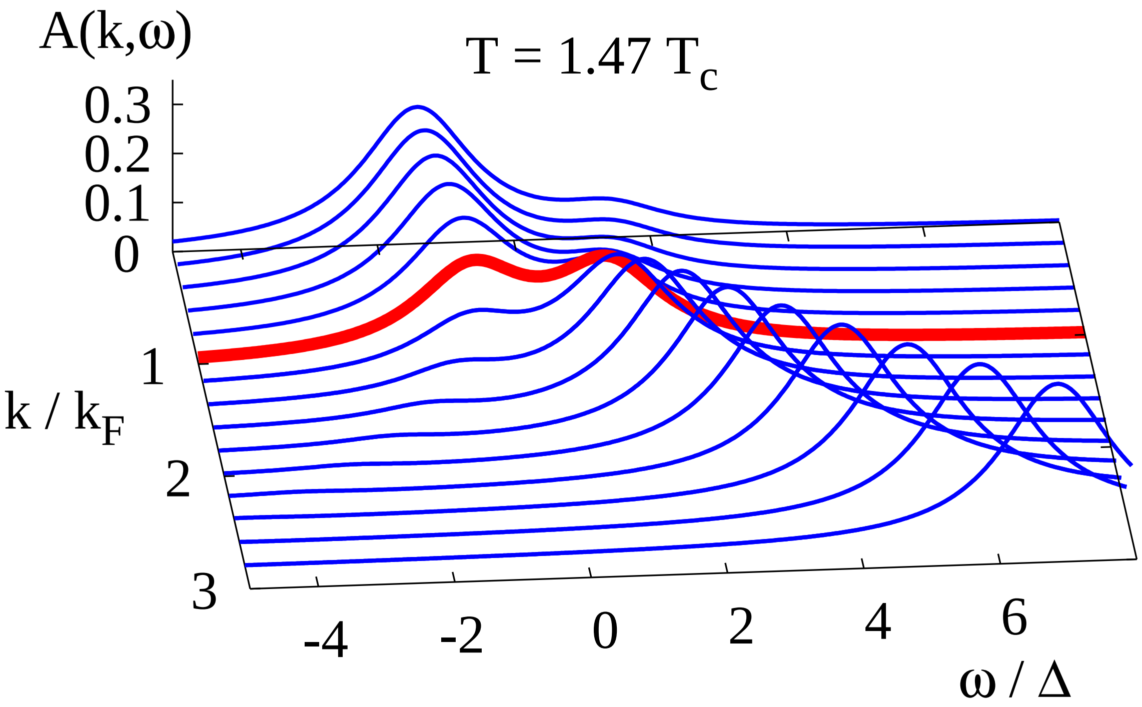

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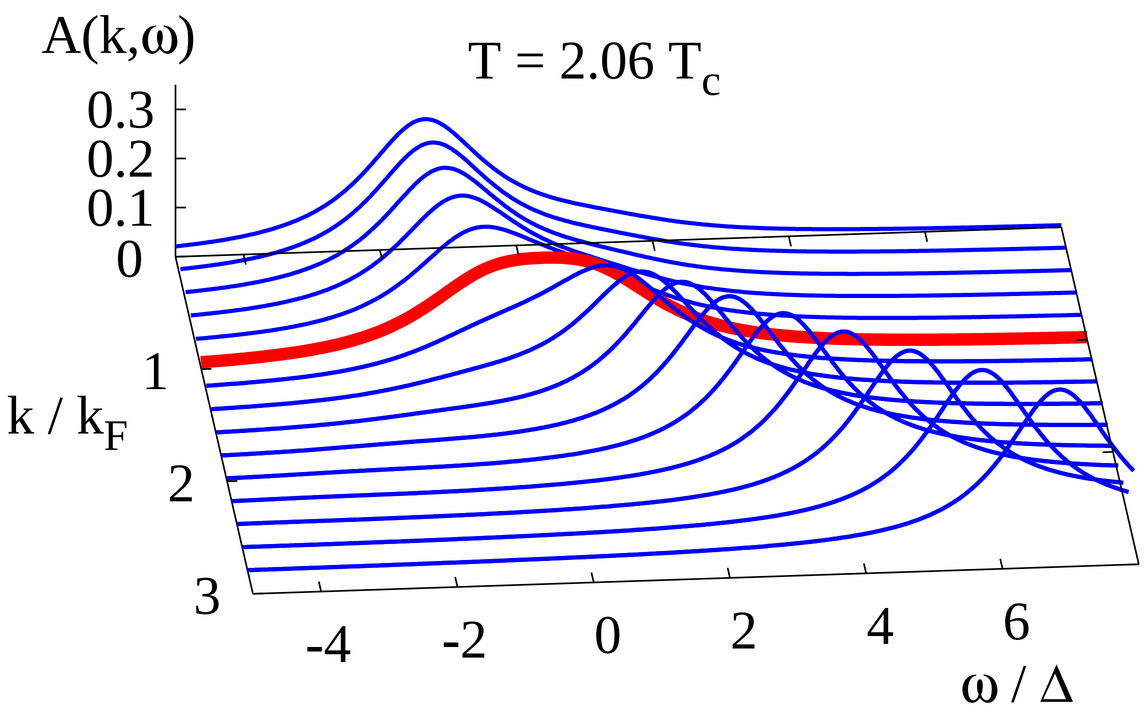

T. Domański, Phys. Rev. A **84**, *023634 (2011).*

Evidence for Bogoliubov QPs above T_c

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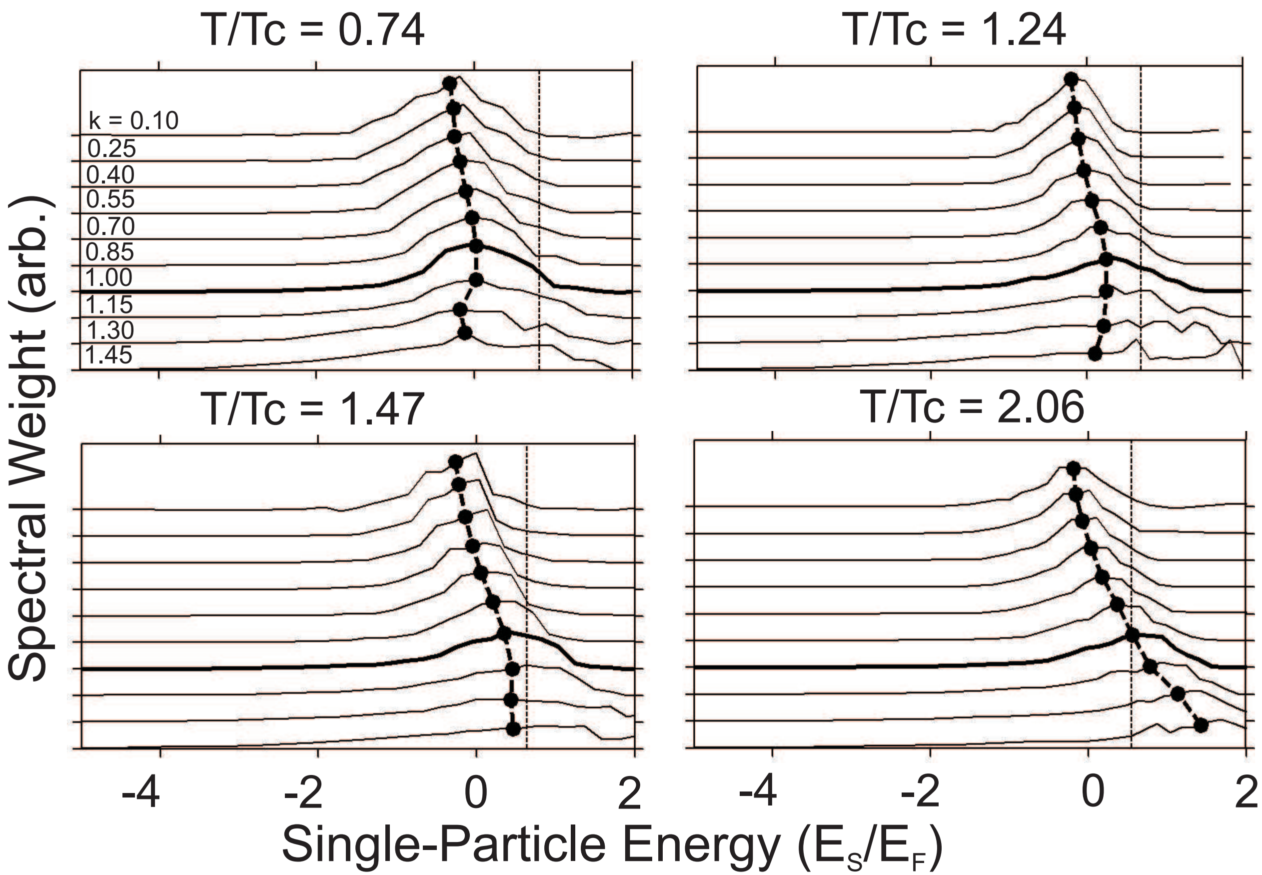
D. Jin group (Boulder, USA)



Results for the ultracold ^{40}K atoms

J.P. Gaebler et al, Nature Phys. **6**, *569 (2010).*

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- **bulk materials**

  / superconductors, atomic superfluids, black holes (?) /