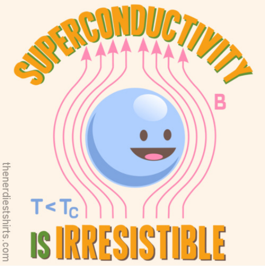
BOUND STATES IN CONVENTIONAL AND TOPOLOGICAL SUPERCONDUCTORS

Tadeusz DOMAŃSKI

M. Curie-Skłodowska University, Lublin



XIX KKN, Bronisławów **10 October 2019**

OUTLINE

- **Bogoliubov quasiparticles in superconductors**

$\Rightarrow$ **particle vs hole**

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N. Bogoliubov **J. Bardeen** **E. Majorana**

Seminal contributions to quantum field theory:

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I. On the theory of superfluidity

N.N. Bogoliubov, J. Phys.(USSR) 11, 23 (1947)

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II. **On a new method in the theory of superconductivity**

N.N. Bogoliubov, Nuovo Cim. 7, 794 (1958)

I. SUPERFLUIDITY

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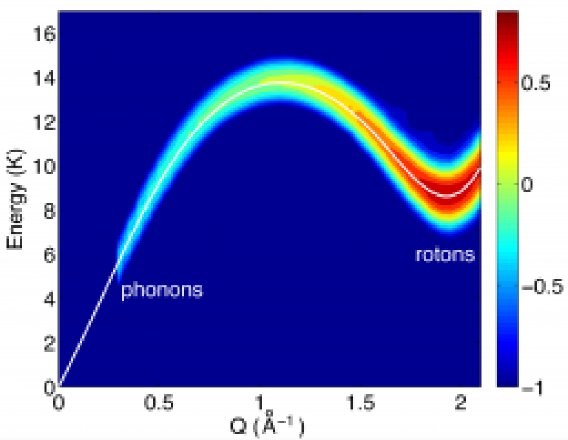

Effective spectrum of the superfluid ^{4}He

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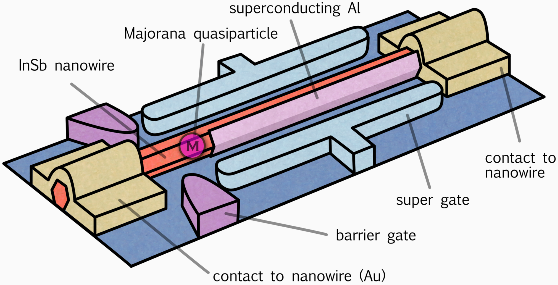

Bulk superconductors

HALLMARKS OF ELECTRON PAIRING

BCS ground state :

$$|\mathbf{BCS}\rangle = \prod_{k} \left(u_k + v_k \; \hat{c}^{\dagger}_{k\uparrow} \, \hat{c}^{\dagger}_{-k\downarrow} \right) \; |\mathbf{vacuum}\rangle$$

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Effective (Bogoliubov) quasiparticles

$$\begin{aligned} \hat{\gamma}_{k\uparrow} &= u_k \hat{c}_{k\uparrow} + v_k \hat{c}^\dagger_{-k\downarrow} \\ \hat{\gamma}^\dagger_{-k\downarrow} &= -v_k \hat{c}_{k\uparrow} + u_k \hat{c}^\dagger_{-k\downarrow} \end{aligned}$$

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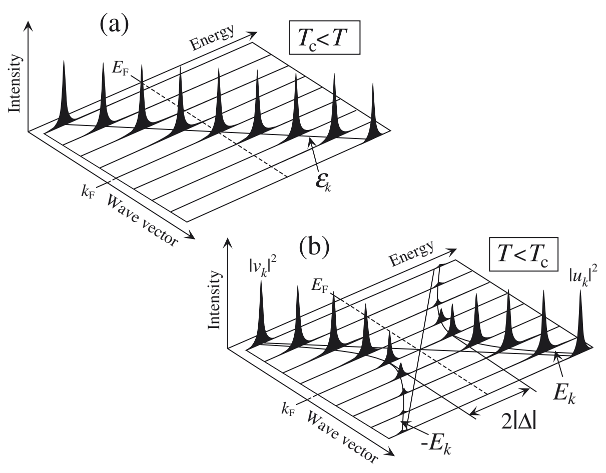
formally due to

$$\begin{aligned} \hat{\gamma}_{k\uparrow} &= u_k \hat{c}_{k\uparrow} + \tilde{v}_k\ \hat{b}_{q=0} \hat{c}^\dagger_{-k\downarrow} \\ \hat{\gamma}^\dagger_{-k\downarrow} &= -\tilde{v}_k\ \hat{b}^\dagger_{q=0}\ \hat{c}_{k\uparrow} + u_k \hat{c}^\dagger_{-k\downarrow} \end{aligned}$$

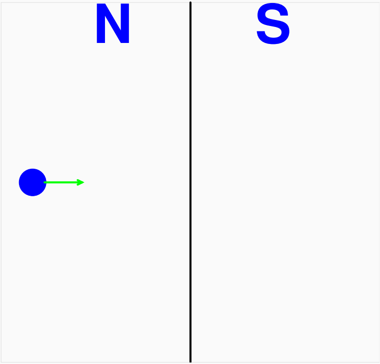
BOGOLIUBOV QUASIPARTICLES

Quasiparticle spectrum of conventional superconductors consists of the Bogoliubov (p/h) branches gaped around E_F



PARTICLE VS HOLE

Let us consider the interface of metal N and superconductor S



where incident electron ...

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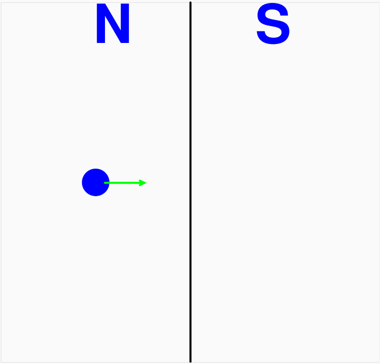

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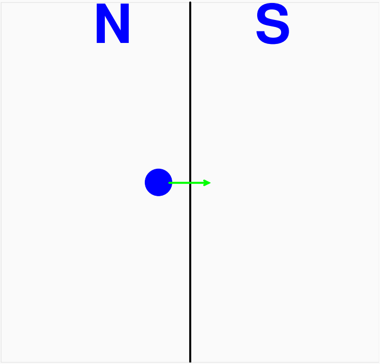

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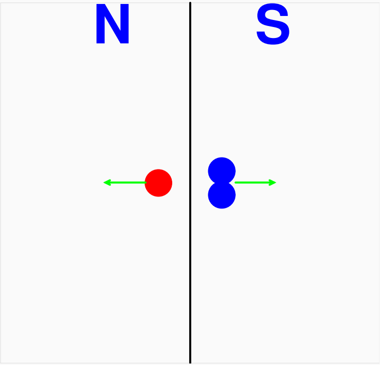

where incident electron is <u>converted</u> into: Cooper pair + hole.

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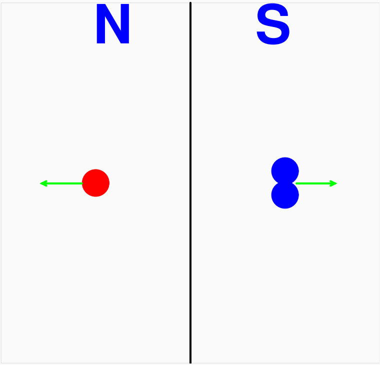

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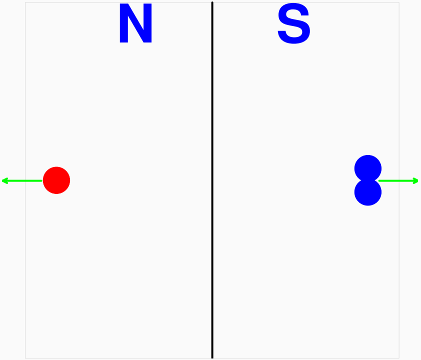

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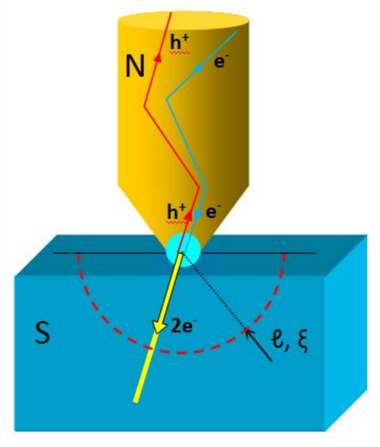
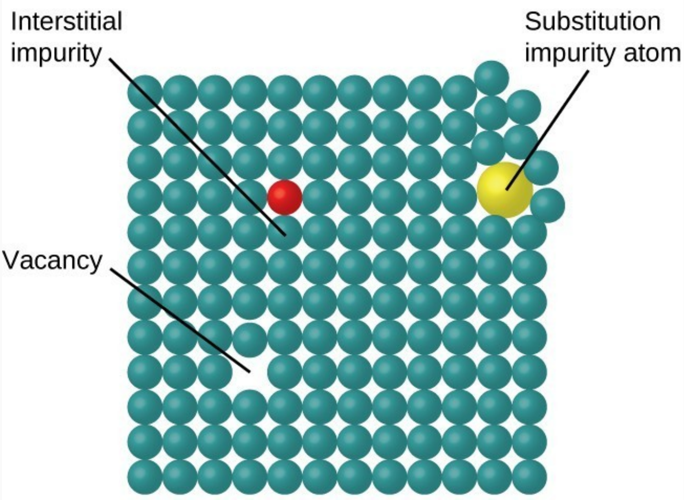
PARTICLE VS HOLE

In superconductors the particle and hole degrees of freedom are mixed via the electron pairing (efficient near the Fermi energy).



Superconductivity in nanosystems

IMPURITIES IN SOLIDS



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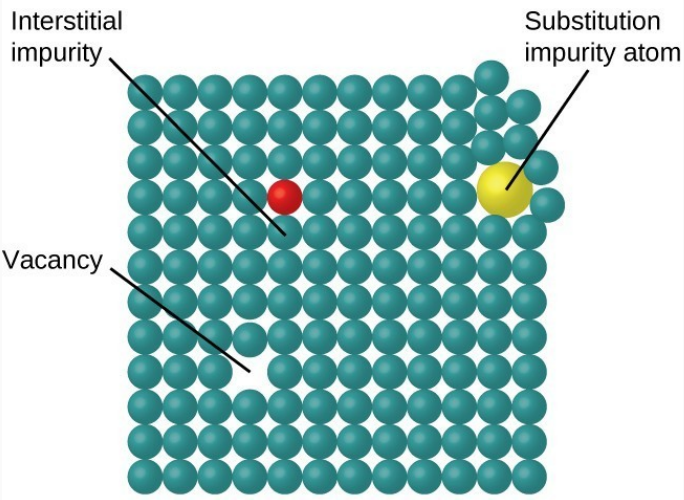
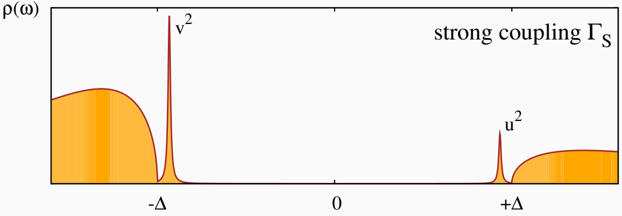


Are they foes or friends to a superconducting host ?

IN-GAP STATES

Spectrum of a single impurity hybridized with superconductor:



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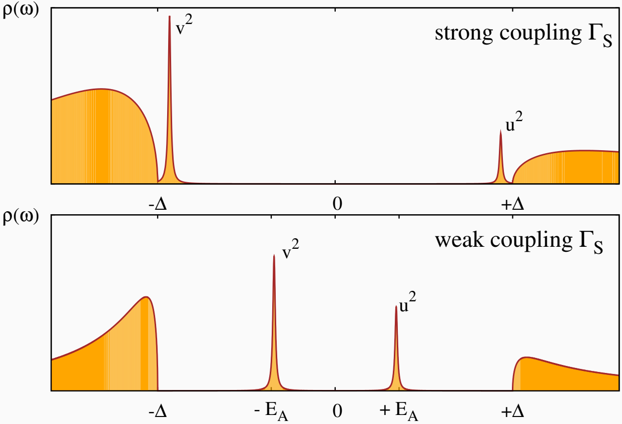

Bound states appearing in the subgap region $E \in \langle -\Delta, \Delta \rangle$

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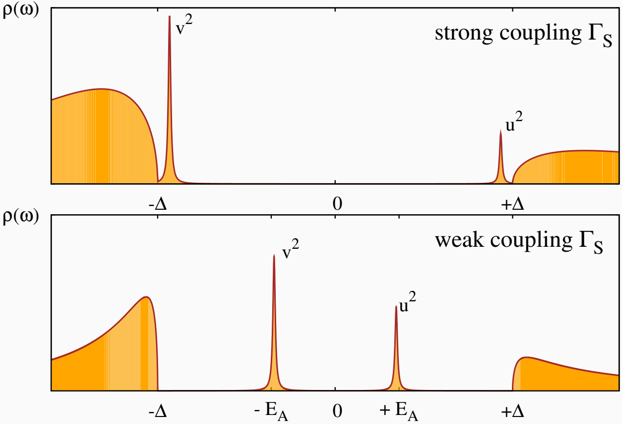
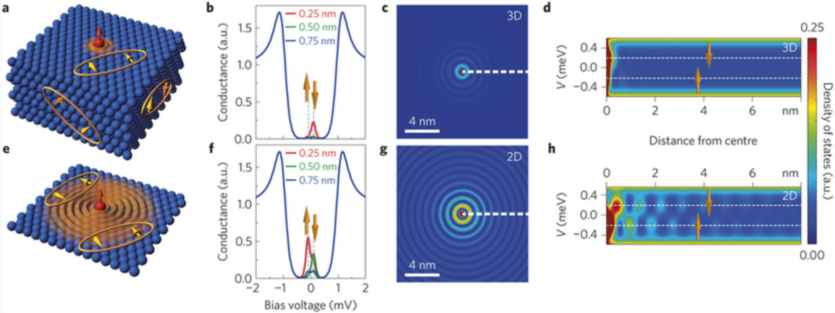


Bound states appearing in the subgap region $E \in \langle -\Delta, \Delta \rangle$

are dubbed Yu-Shiba-Rusinov (or Andreev) quasiparticles.

DIMENSIONALITY EFFECT

Empirical data obtained from STM measurements for $NbSe_2$



a) very small extent in dim=3

b) much longer extent in dim=2

G.C. Menard et al., Nature Phys. 11, 1013 (2015).

TOPOGRAPHY AND SPATIAL EXTENT

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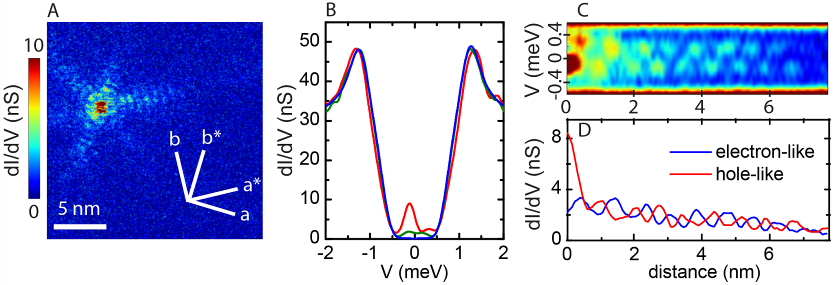

a) bound states extending to 10 nm

b) alternating particle-hole oscillations

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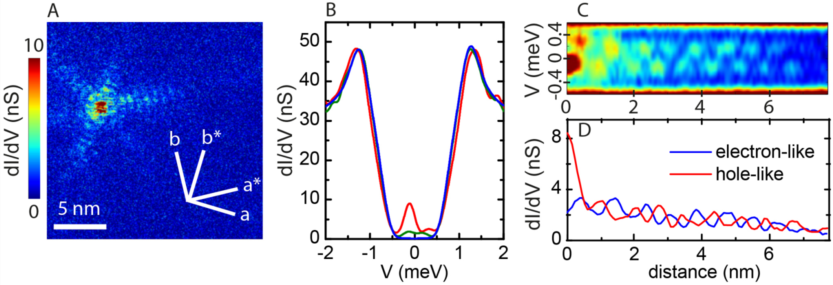

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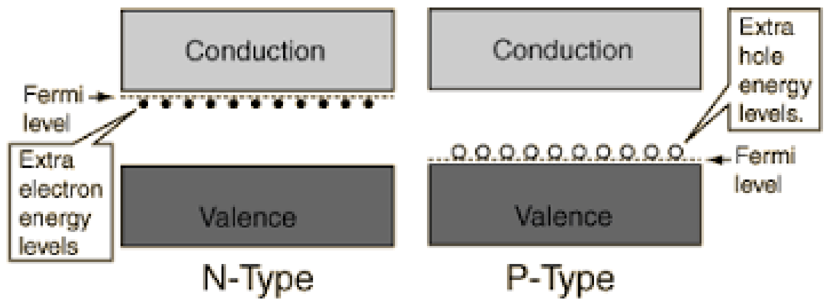
A. Ptok, Sz. Głodzik and T. Domański, Phys. Rev. B 96, 184425 (2017).

DOPED INSULATORS/SEMICONDUCTORS

Insulator doped by the in-gap donor/acceptor levels



benefits electrons/holes in its conduction/valence band.

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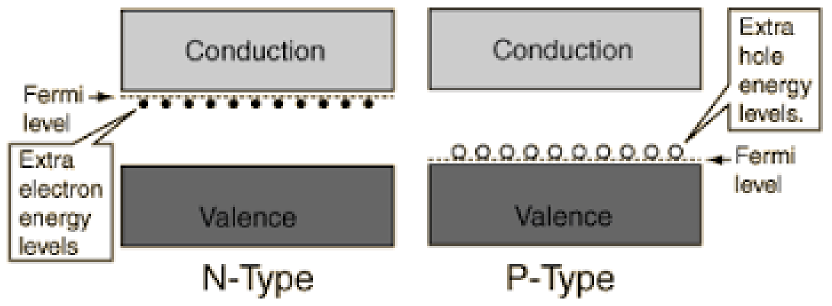

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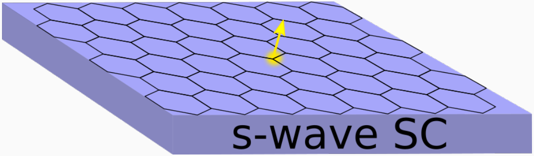
Any similarity to Yu-Shiba-Rusinov quasiparticles ?

Impurity in superconducting graphene sheet

IMPURITY IN SUPERCONDUCTING GRAPHENE

Magnetic impurity in graphene proximitized to s-wave superconductor



Sz. Głodzik and T. Domański, arXiv:1811.09295 (2018).

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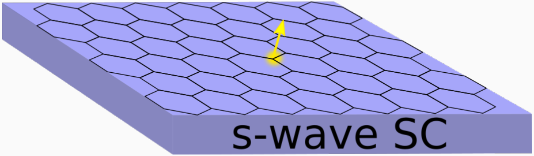

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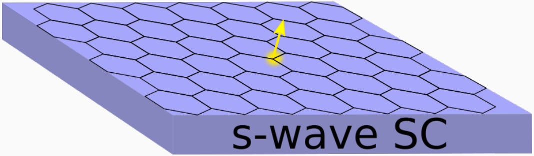

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Possible realizations:

$\Longrightarrow$ **graphene grown on rhenium**

C. Tonnoir et al., PRL 111, 246805 (2013) ...... CEA (Grenoble, France)

$\Longrightarrow$ **graphene epitaxially deposited on aluminum**

F.D. Natterer et al., PRB 93, 045406 (2016) ....NIST (Maryland, USA)

$\Longrightarrow$ **Al–graphene–Al on hexagonal boron-nitride (HBN)**

L. Bretheau et al., Nature Phys. 13, 756 (2018) .. MIT (Cambridge, USA)

MICROSCOPIC SCENARIO

We use the Kane-Mele model [PRL 95, 226801 (2005)] for graphene:

$$\hat{H}_{K-M} = \sum_{\langle ij\rangle\sigma} \left(t_{ij} - \mu\delta_{ij}\right) \hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma} + i\lambda_{SO} \sum_{\langle\langle ij\rangle\rangle\sigma} \nu_{ij}\hat{c}^{\dagger}_{i\sigma}\mathrm{s}_z^{\sigma\sigma'}\hat{c}_{j\sigma'}$$

with $\nu_{ij} = +1$ for the clockwise and -1 for anticlockwise electron hopping between the next-nearest-neighbor sites.

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We next consider a single magnetic imuprity embedded into the proximitized graphene

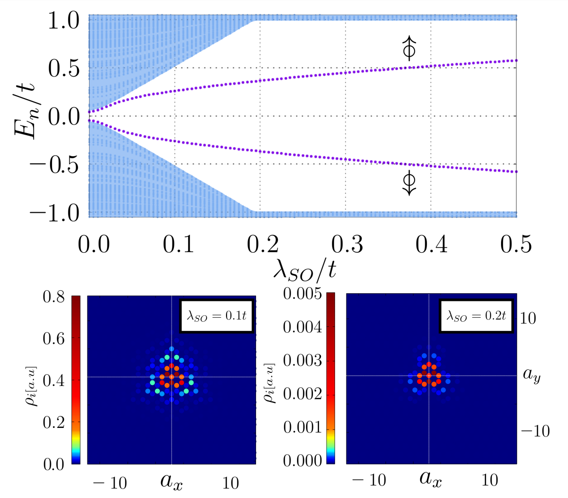
$$\hat{H} = \hat{H}_{imp} + \hat{H}_{K-M} + \hat{H}_{Rashba} + \hat{H}_{prox}$$

where electron pairing is described by the BCS term

$$\hat{H}_{prox} = \sum_{\mathrm{i}} \left(\Delta c^{\dagger}_{\mathrm{i}\uparrow} \; c^{\dagger}_{\mathrm{i}\downarrow} + \mathrm{h.c.} \right)$$

IN-GAP STATES OF INSULATING PHASE

Quasiparticle energies of the quantum spin Hall insulator + impurity

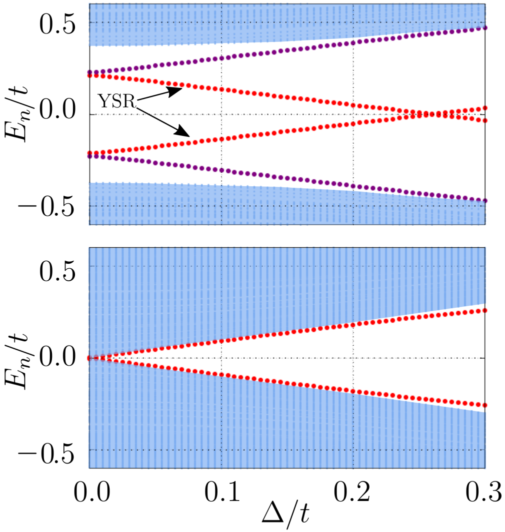


$\Delta = 0$

Sz. Głodzik and T. Domański, arXiv:1811.09295 (2018).

SHIBA QUASIPARTICLES

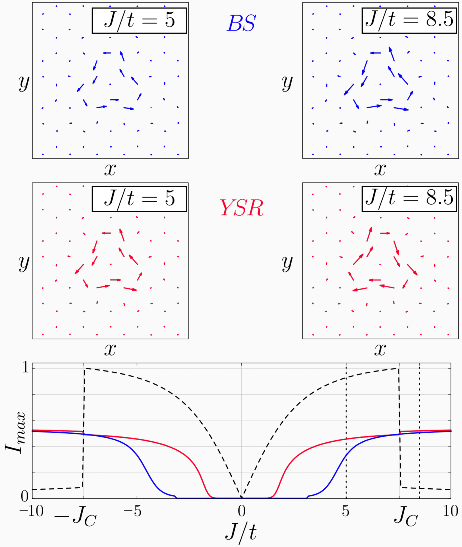
Shiba quasiparticles emerge from in-gap states of the QSH insulator



$\lambda_{SO} \neq 0$

$\lambda_{SO} = 0$

REVERSAL OF PERSISTENT CURRENTS



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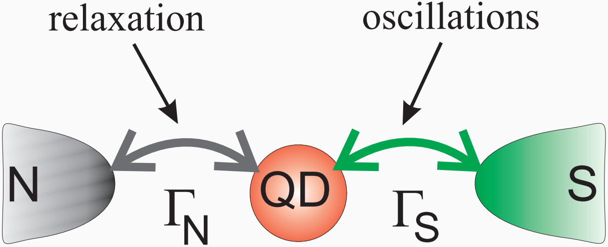
TRANSIENT EFFECTS FOR IN-GAP STATES

Let's consider abrupt coupling of QD to external leads



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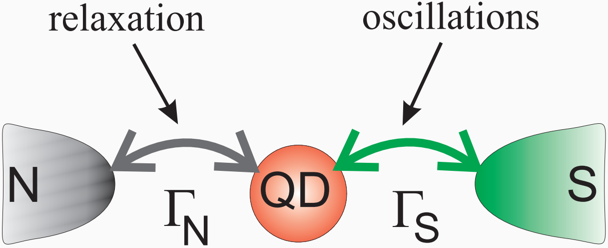

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Relevant questions:

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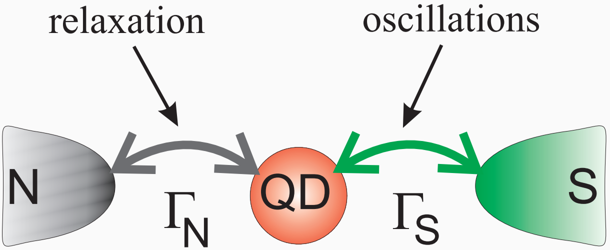

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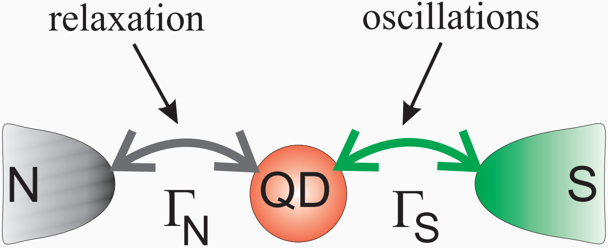

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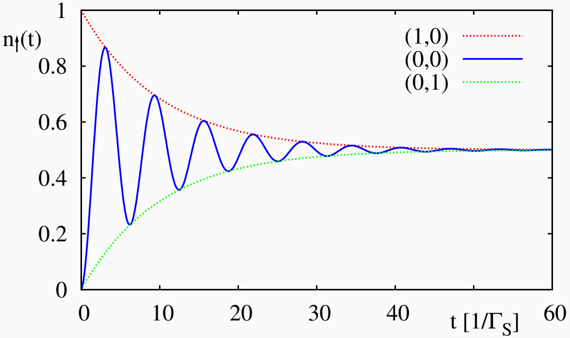
Relevant questions:

- **how much time is needed to form the in-gap states?**
- **any other characteristic time-scales?**

RELAXATION VS QUANTUM OSCILLATIONS

Time-dependent charge of the quantum dot

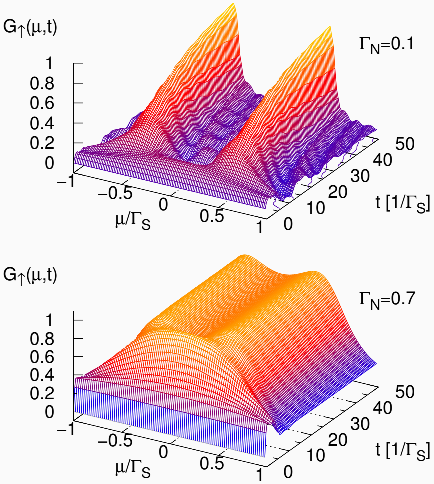


- **relaxation time is proportional to $1/\Gamma_N$**
- **oscillations depend on energies of in-gap states**

R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

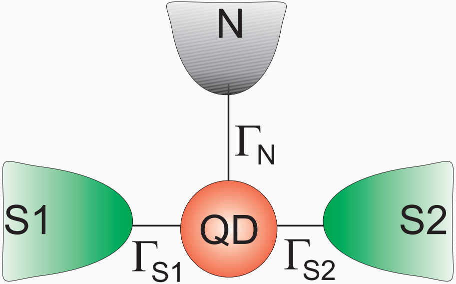
TIME-DEPENDENT CONDUCTANCE

QD level $\varepsilon = 0$



Subgap tunneling conductance $G_\sigma = \frac{\partial I_\sigma}{\partial t}$ vs time (t) and voltage (μ)

PHASE-CONTROLLED TRANSIENT EFFECTS



R. Taranko, T. Kwapiński and T. Domański Phys. Rev. B 99, 165419 (2019).

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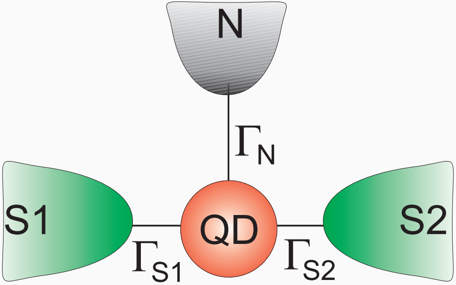

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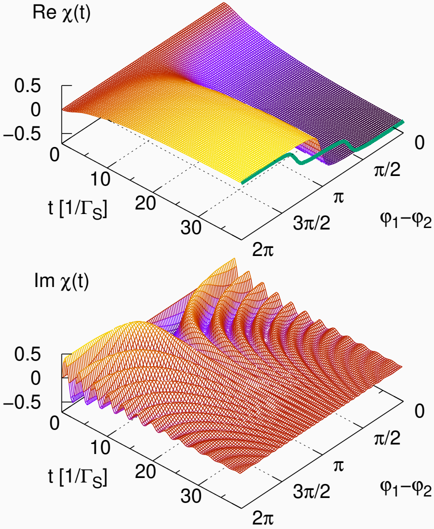
Physical issues:

- **phase-controlled emergence of in-gap states,**
- **dynamics of $0-\pi$ transition.**

PHASAL TRANSIENT EFFECTS

Phase & time dependence of the order parameter

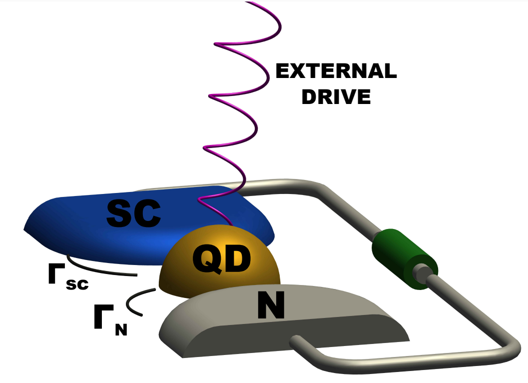


R. Taranko, T. Kwapiński and T. Domański Phys. Rev. B 99, 165419 (2019).

Floquet description of bound states

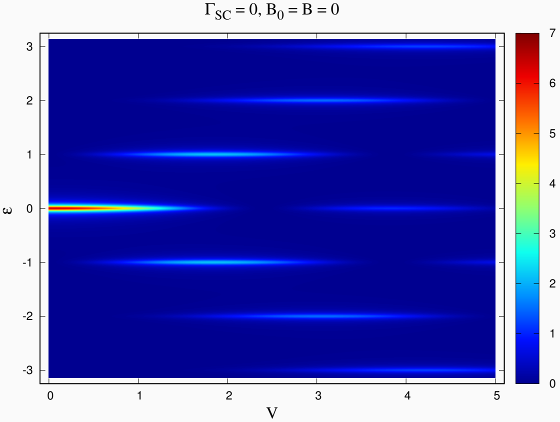
SHIBA STATES OF A DRIVEN QUANTUM IMPURITY

Quantum impurity with periodically oscillating energy level



$$\varepsilon(t) = \varepsilon_0 + V \times \cos(\omega t)$$

Floquet spectrum averaged over a period $T = 2\pi/\omega$

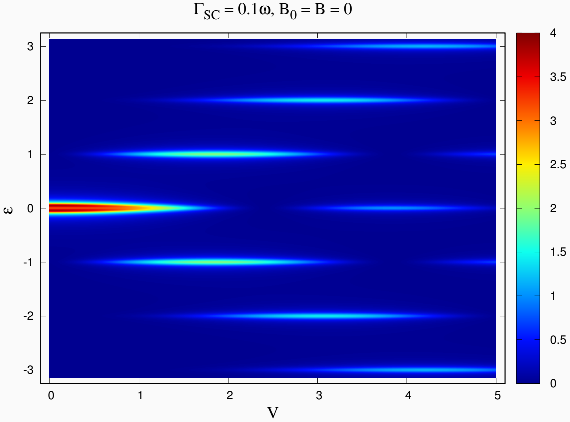


$\Gamma_S = 0.0$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

BOUND STATES OF A DRIVEN QUANTUM IMPURITY

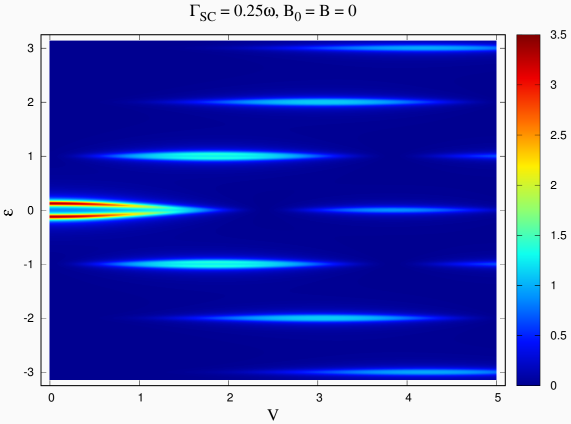
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.1\omega$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

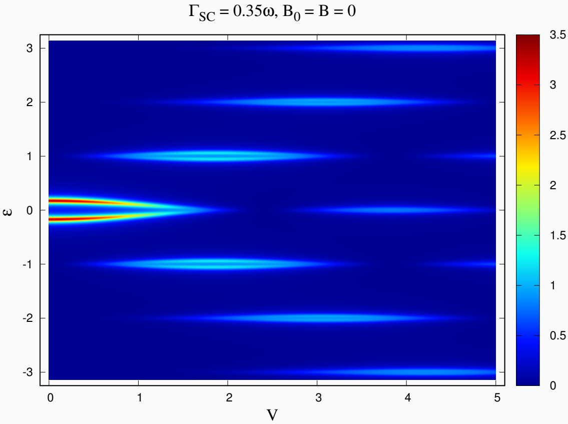
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.25\omega$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

Floquet spectrum averaged over a period $T = 2\pi/\omega$

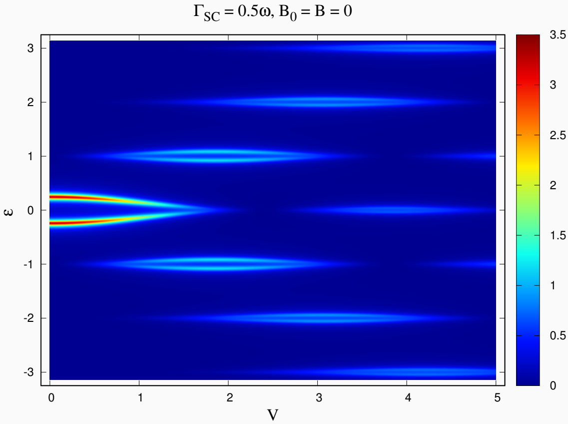


$\Gamma_S = 0.35\omega$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

BOUND STATES OF A DRIVEN QUANTUM IMPURITY

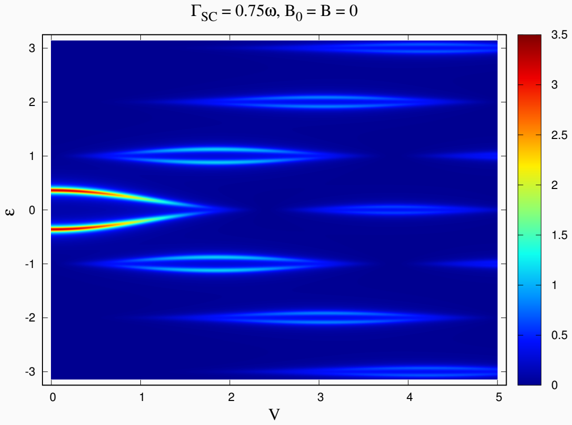
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.5\omega$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

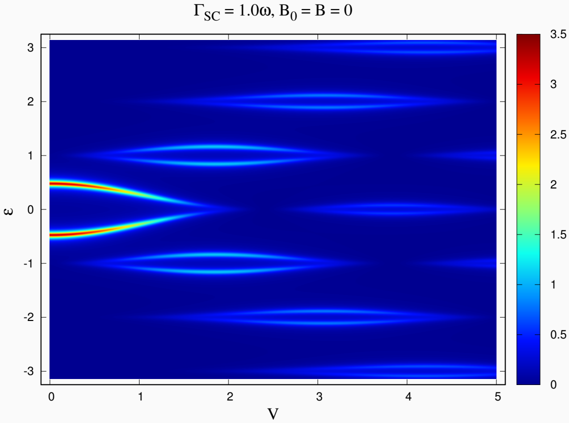
Floquet spectrum averaged over a period $T = 2\pi/\omega$



$\Gamma_S = 0.75\omega$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

Floquet spectrum averaged over a period $T = 2\pi/\omega$

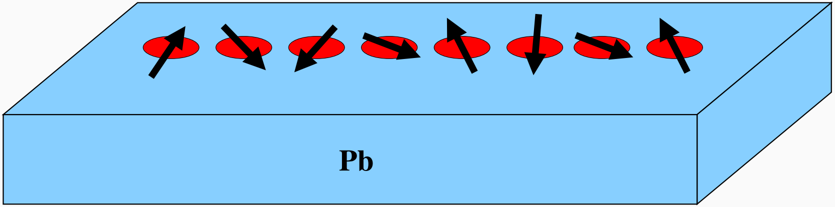


$\Gamma_S = 1.0\omega$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

MAGNETIC CHAINS IN SUPERCONDUCTORS

Nanochain of magnetic impurities embedded in superconductor:

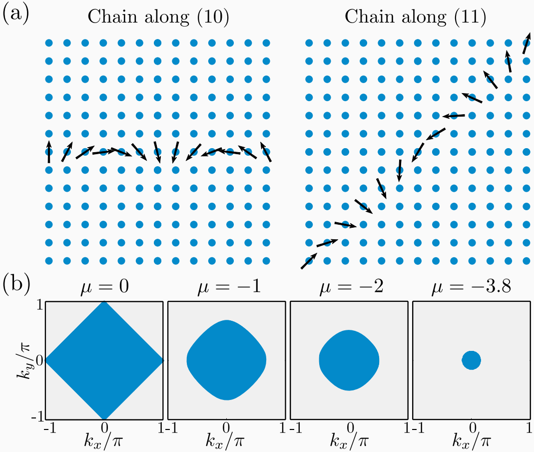


T.-P. Choy, J.M. Edge, A.R. Akhmerov, and C.W.J. Beenakker,

Phys. Rev. B 84, 195442 (2011).

MAGNETIC CHAINS IN SUPERCONDUCTORS

A chain of magnetic impurities embedded in superconductor:



MAGNETIC CHAINS IN SUPERCONDUCTORS

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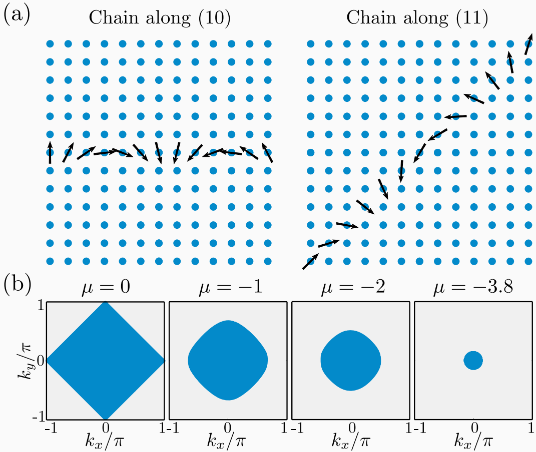


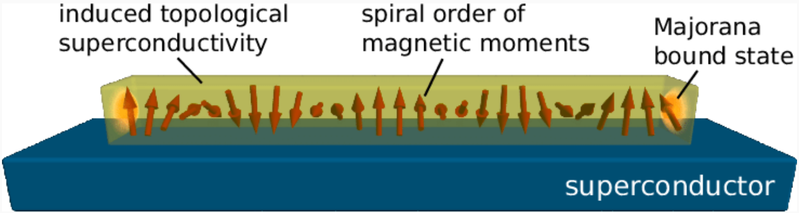
arranges its in-gap bound states into Shiba-band(s).

M.H. Christensen ... J. Paaske, Phys. Rev. B 94, 144509 (2016).

Two scenarios for topological sc phase

SCENARIO 1: RASHBA + ZEEMAN + PAIRING

Topological superconductivity can be driven e.g. by the spin-orbit Rashba interaction combined with the external magnetic field.

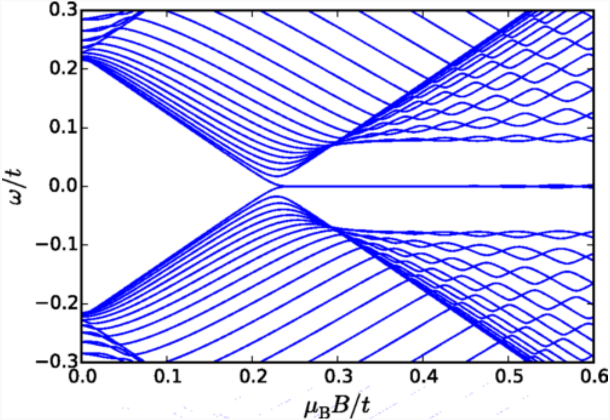


R. Lutchyn, J. Sau, S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010).

Y. Oreg, G. Refael, F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010).

TRANSITION FROM TRIVIAL TO TOPOLOGICAL PHASE

A pair of the Shiba (Andreev) states evolve into the Majorana qps

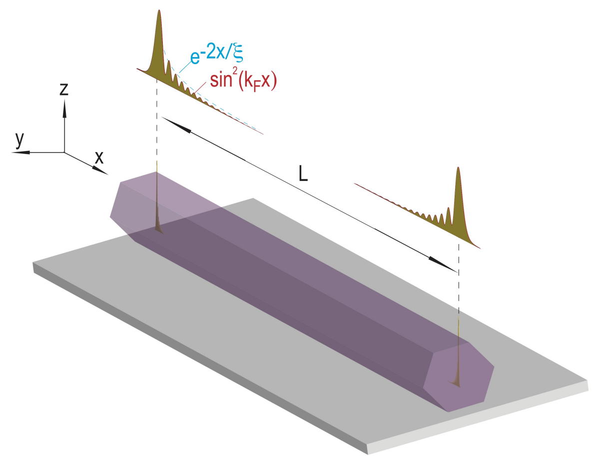


Mutation of the trivial bound states into the nontrivial Majorana modes

M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

SPATIAL PROFILE OF MAJORANA QPS

Majorana qps are exponentially localized at the edges



R. Aguado, Riv. Nuovo Cim. 40, 523 (2017).

PROPERTIES OF MAJORANA QPS

- **particle = antiparticle** $\hat{\gamma}_{i,n}^{\dagger} = \hat{\gamma}_{i,n}$

$\Rightarrow$ **neutral in charge**

$\Rightarrow$ **at zero energy**

PROPERTIES OF MAJORANA QPS

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- **fractional character** $\hat{\gamma}_{i,n}^{\dagger}\, \hat{\gamma}_{i,n} = \frac{1}{2}$

⇒ **half occupied/empty** $\hat{\gamma}_{i,n}\, \hat{\gamma}_{i,n}^{\dagger} = \frac{1}{2}$

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- spatially nonlocal

⇒ **exist always in pairs at boundaries/defects**

PROPERTIES OF MAJORANA QPS

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- **spatially nonlocal**

$\Rightarrow$ **exist always in pairs at boundaries/defects**

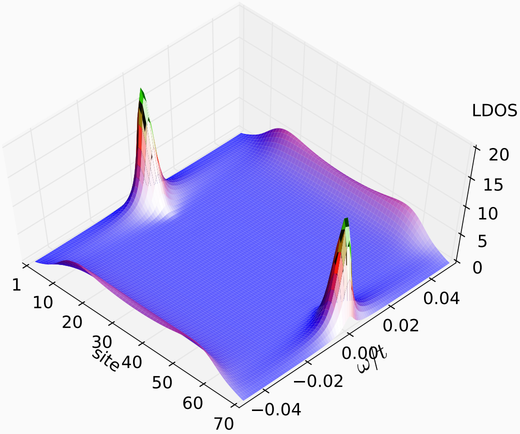
- **topologically protected**

$\Rightarrow$ **immune to dephasing/decoherence**

TOPOLOGICAL PROTECTION

Low energy quasiparticles of the Rashba nanowire

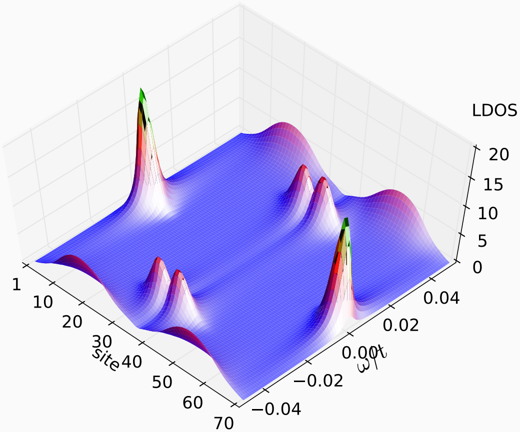
$t_{35}/t = 1.0$



M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

Low energy quasiparticles of the Rashba nanowire

$t_{35}/t = 0.8$

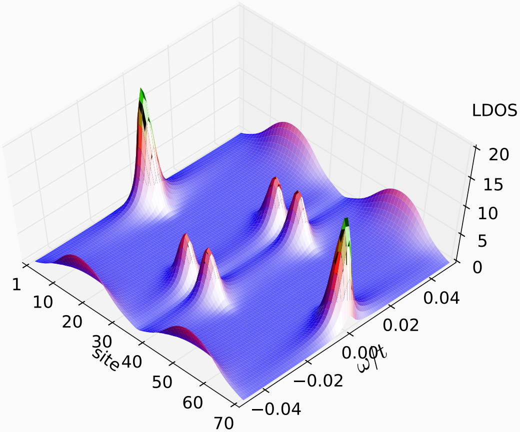


M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

TOPOLOGICAL PROTECTION

Low energy quasiparticles of the Rashba nanowire

$t_{35}/t = 0.6$

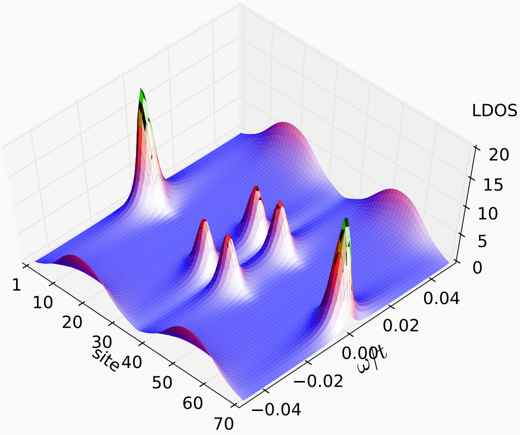


M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

TOPOLOGICAL PROTECTION

Low energy quasiparticles of the Rashba nanowire

$t_{35}/t = 0.4$

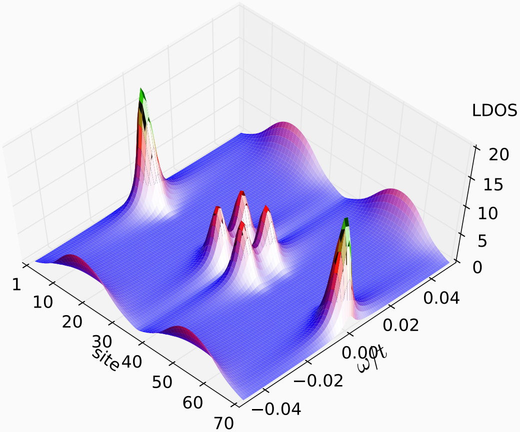


M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

TOPOLOGICAL PROTECTION

Low energy quasiparticles of the Rashba nanowire

$t_{35}/t = 0.2$

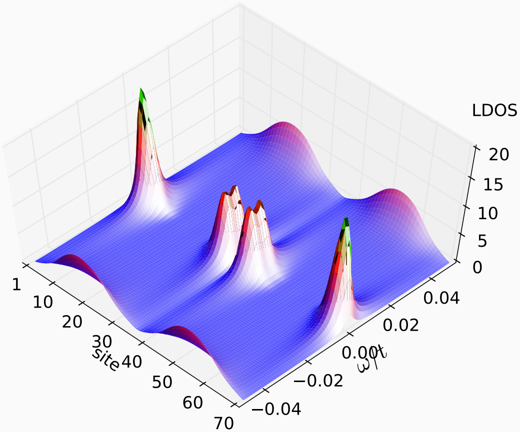


M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

TOPOLOGICAL PROTECTION

Low energy quasiparticles of the Rashba nanowire

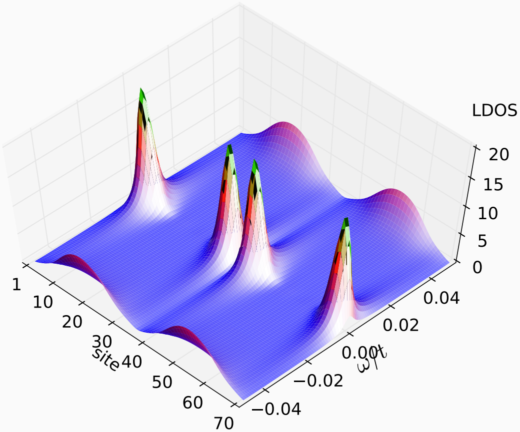
$t_{35}/t = 0.1$



M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

Low energy quasiparticles of the Rashba nanowire

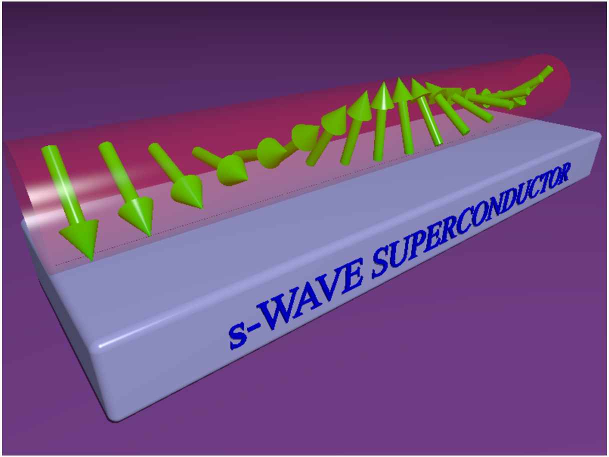
$t_{35}/t = 0.0$



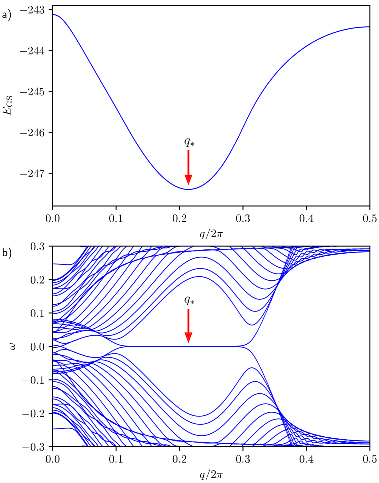
M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

SCENARIO 2: HELICAL ORDER + PAIRING

Topological superconductivity can be driven by helically ordered magnetic moments coupled to the itinerant electrons + pairing.



This nanochain self-tunes to its *topological phase* (topofilia)



Ground state energy

vs the pitch vector q

In-gap Shiba states

A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).

TOPOLOFILIA

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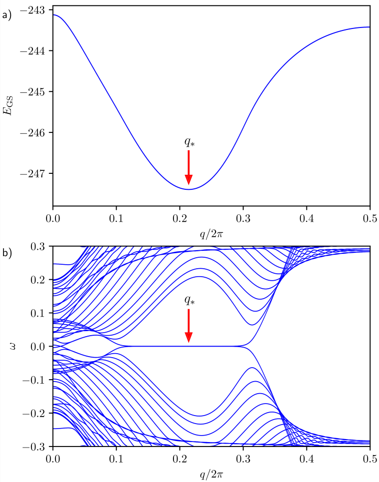

Ground state energy

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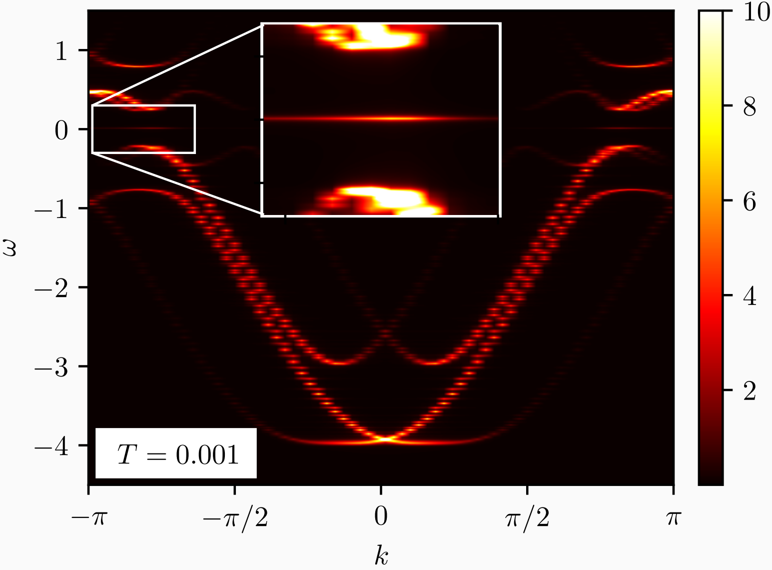
A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).

More details will provided in the lecture by Maciek Maśka.

THERMAL EFFECTS IN HELICAL SCENARIO



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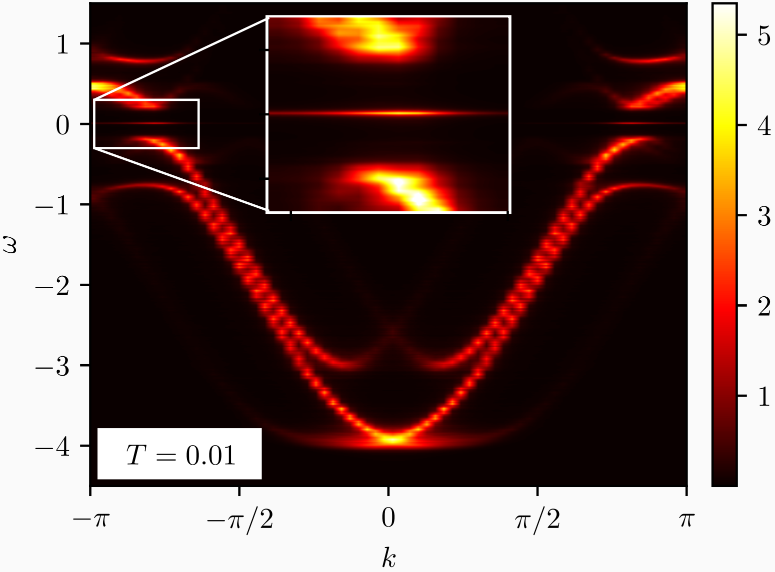

A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).

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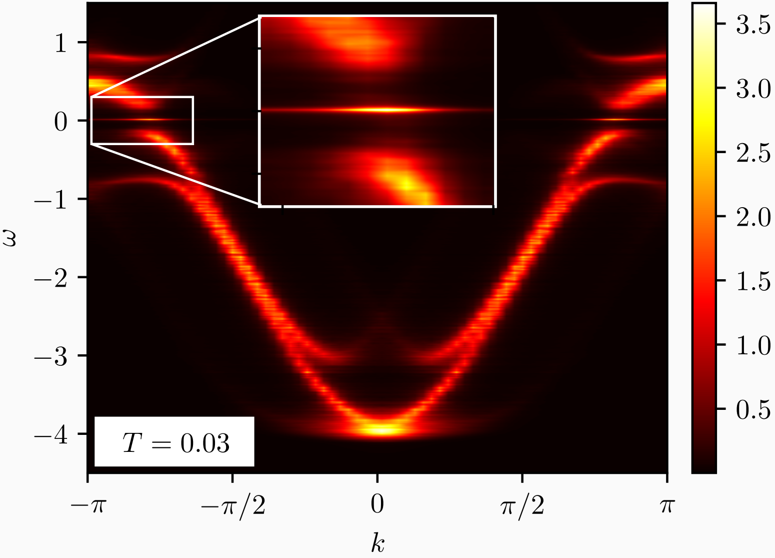
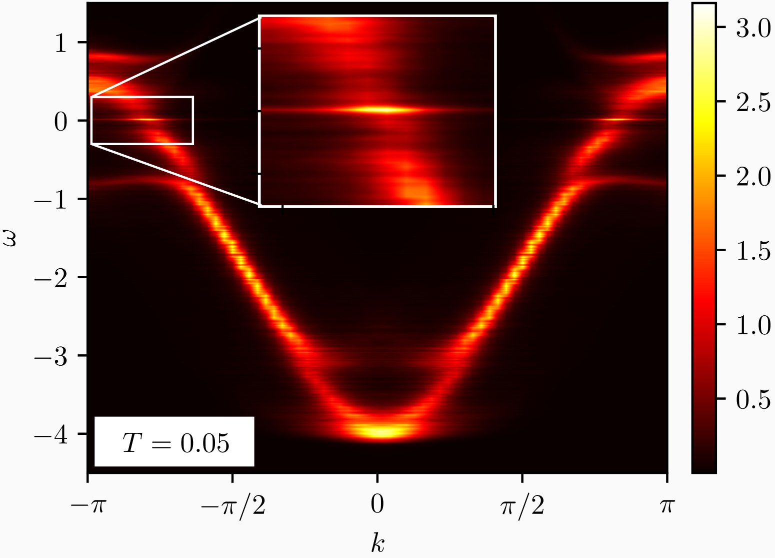


A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).

THERMAL EFFECTS IN HELICAL SCENARIO



A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).

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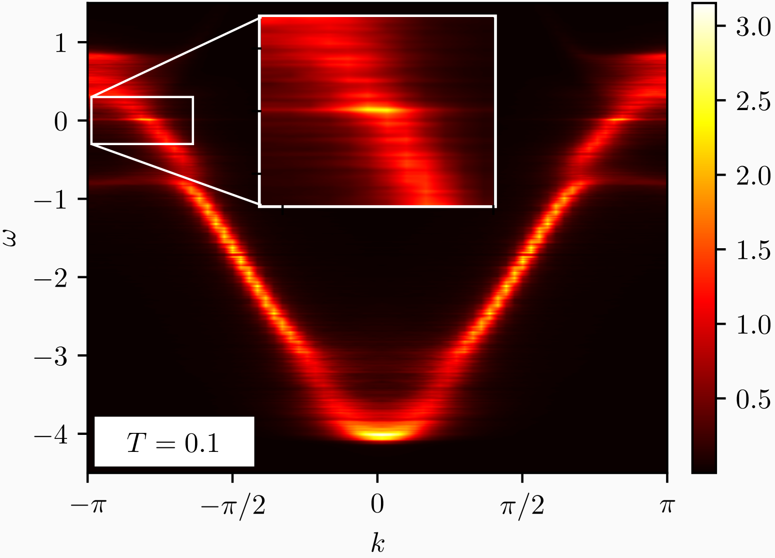

A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).

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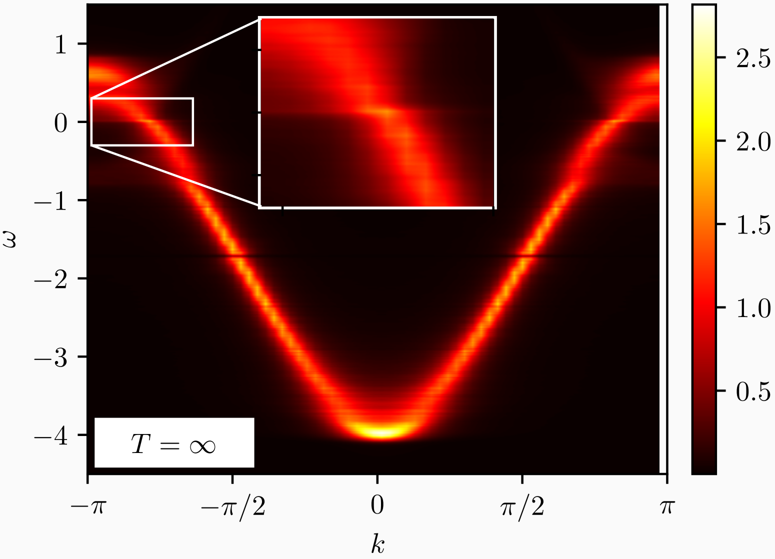
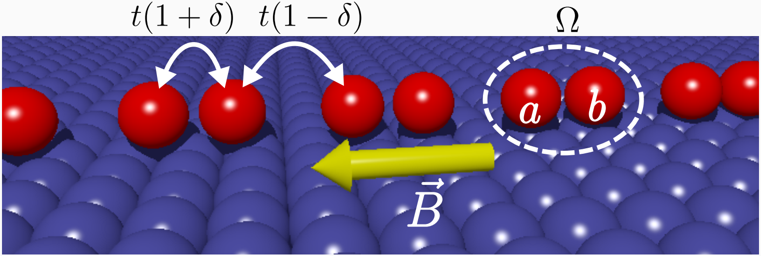


A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).

FURTHER EFFECTS

Role of the dimerization shall be discussed on Friday by Aksel Kobiałka.

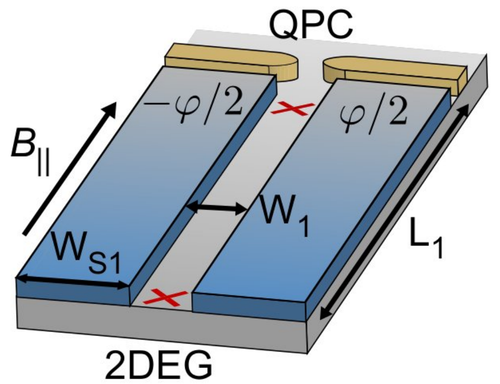


A. Kobiałka, N. Sedlmayr, M.M. Maśka, and T. Domański, arXiv:1909.11550 (2019).

Localized Majorana modes in dim=2

PLANAR JOSEPHSON JUNCTIONS

Two-dimensional electron gas of InAs epitaxially covered by a thin Al layer



Width:

W_1 = 80 nm

Length:

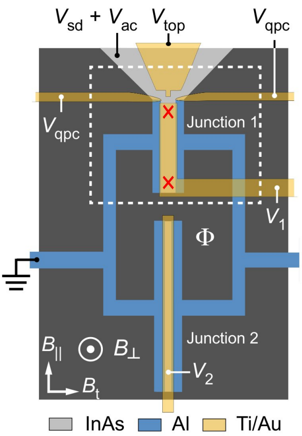
L_1 = 1.6 μm

A. Fornieri, ..., Ch. Marcus and F. Nichele, Nature 569, 89 (2019).

Niels Bohr Institute (Copenhagen, Denmark)

PLANAR JOSEPHSON JUNCTIONS

Majorana qps at the ends of 2DEG depend on the phase-difference Φ

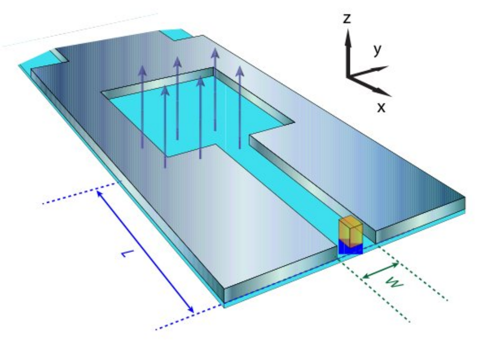


A. Fornieri, ..., Ch. Marcus and F. Nichele, Nature 569, 89 (2019).

Niels Bohr Institute (Copenhagen, Denmark)

PLANAR JOSEPHSON JUNCTIONS

Two-dimensional HgTe quantum well coupled to thin Al film

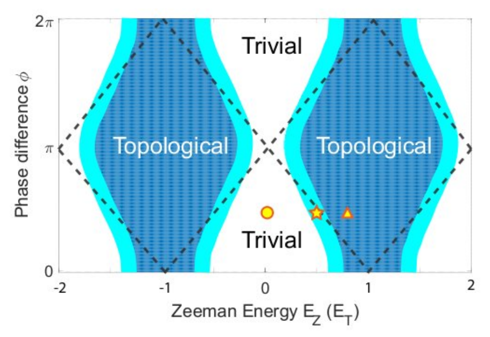


H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).

Würzburg Univ. (Germany) + Harvard Univ. (USA)

PLANAR JOSEPHSON JUNCTIONS

Tuning between the trivial and topological superconducting state by phase difference ϕ and in-plane magnetic field

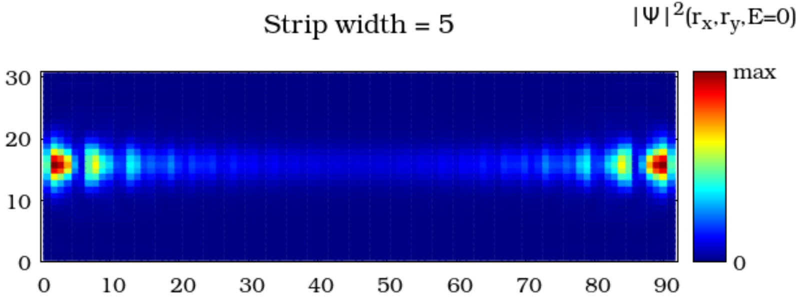


H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).

Würzburg Univ. (Germany) + Harvard Univ. (USA)

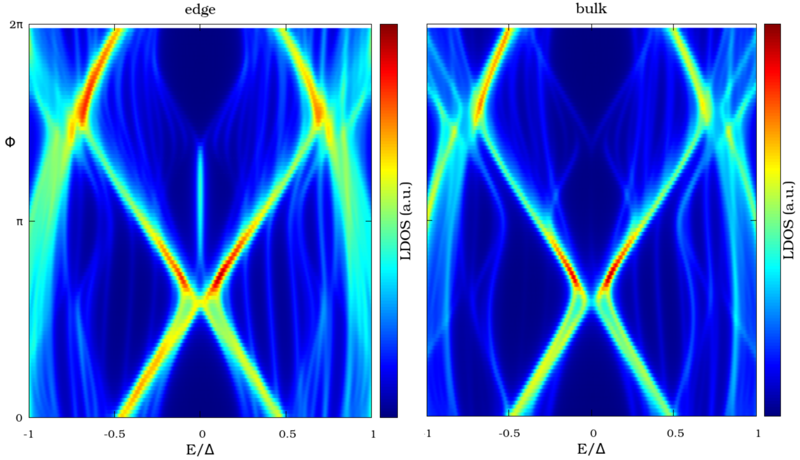
PLANAR JOSEPHSON JUNCTIONS

Majorana modes are localized near edges of the metallic nanostrip



Results obtained by Sz. Głodzik (2019).

PLANAR JOSEPHSON JUNCTIONS

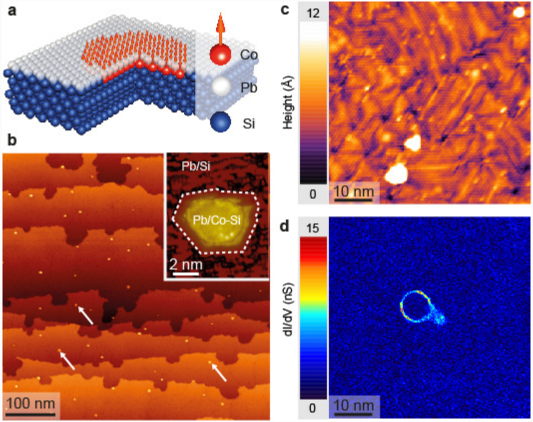


Results obtained by Sz. Głodzik (2019).

Edge modes in dim=2 systems

TWO-DIMENSIONAL MAGNETIC STRUCTURES

Magnetic island of Co atoms deposited on the superconducting Pb surface



Diameter of island:

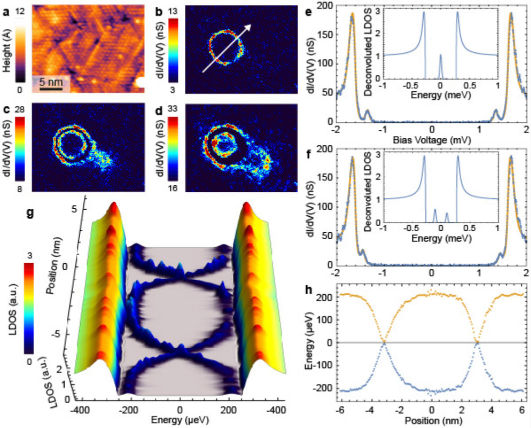
5 – 10 nm

G. Ménard, ..., and P. Simon, Nature Commun. 8, 2040 (2017).

Pierre & Marie Curie University (Paris, France)

EVIDENCE FOR DELOCALIZED MAJORANA MODES

Majorana modes propagating along magnetic islands

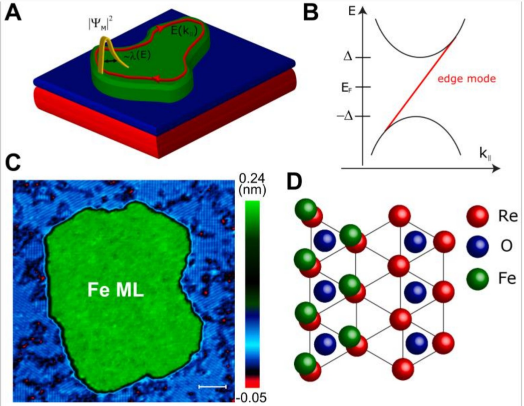


G. Ménard, ..., and P. Simon, Nature Commun. 8, 2040 (2017).

Pierre & Marie Curie University (Paris, France)

PROPAGATING MAJORANA EDGE MODES

Magnetic island of Fe atoms deposited on the superconducting Re surface



Chern number:

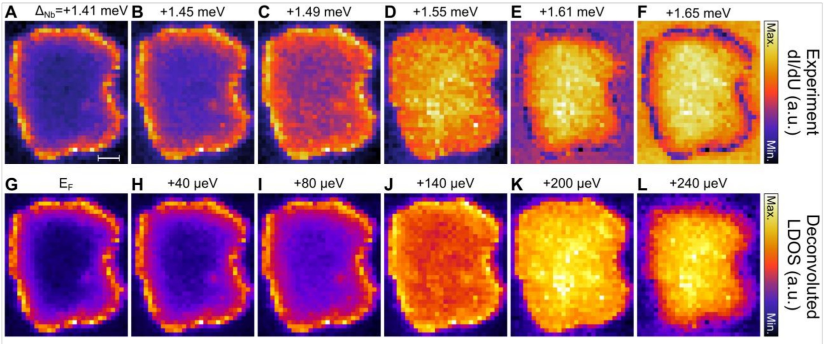
C = 20

A. Palacio-Morales, ... & R. Wiesendanger, Science Adv. 5, eaav6600 (2019).

University of Hamburg (Germany)

PROPAGATING MAJORANA EDGE MODES

Real space maps of the tunneling conductance (top panel) and deconvoluted DOS (bottom panel) obtained for various energies (as indicated) in the subgap regime ($\Delta = 240\mu eV$).



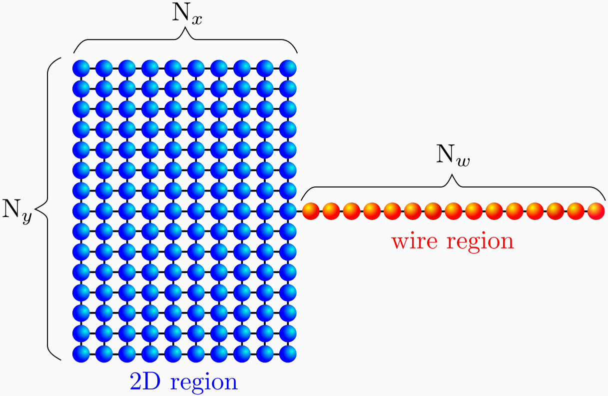
A. Palacio-Morales, ... & R. Wiesendanger, Science Adv. 5, eaav6600 (2019).

University of Hamburg (Germany)

Mixed – dimensionality structures

CAN MAJORANA QPS BE DECONFINED ?

Main idea: **Majorana qps in 1D–2D hybrid structure**



A. Kobiałka, T. Domański & A. Ptok, Scientific Reports 9, 12933 (2019).

TOPOLOGICAL INVARIANTS

Constituents of this hybrid-system belong to different homotopy groups:

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which can be characterized by the Chern number, that is equivalent to the Thouless–Kohmoto–Nightingale–den Nijs number.

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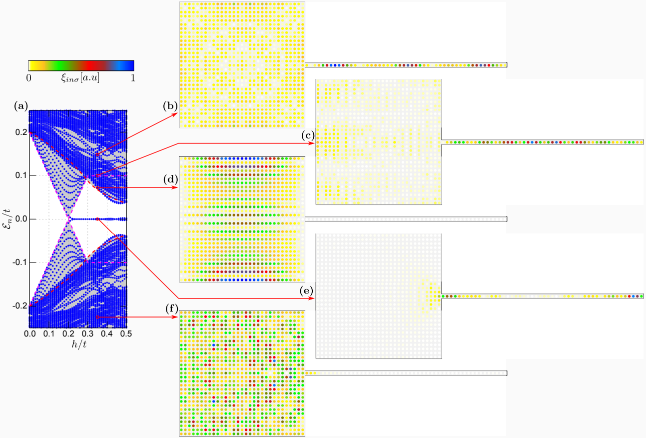
For details, concerning the topological criteria see e.g.

- **A. Kitaev, AIP Conf. Proc. 1134, 22 (2009);**
- **M.Z. Hasan & C.L. Kane, Rev. Mod. Phys. 82, 3045 (2010);**
- **X.-L. Qi & S.-C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).**

TRIVIAL VS MAJORANA MODES

Majorana/Andreev quasiparticles of a wire-plaquette hybrid

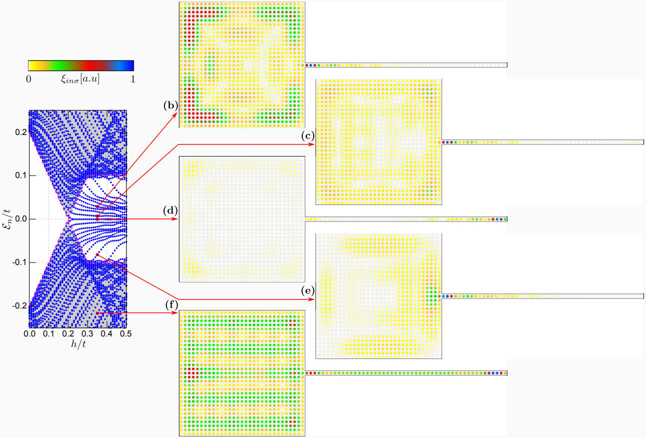


plaquette: nontopological ***nanowire: topological***

A. Kobiałka, T. Domański & A. Ptok, Scientific Reports 9, 12933 (2019).

TRIVIAL VS MAJORANA MODES

Majorana/Andreev quasiparticles of a wire-plaquette hybrid

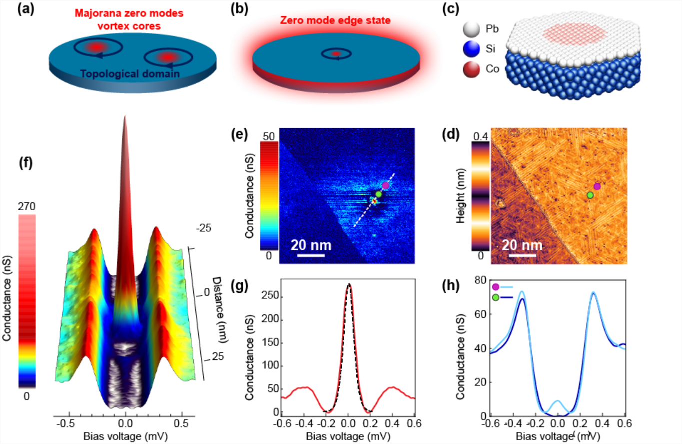


Both regions are assumed to be in topological sc phase.

A. Kobiałka, T. Domański & A. Ptok, Scientific Reports 9, 12933 (2019).

SIMILAR IDEAS: DEFECTS IN MAGNETIC ISLAND

Localized Majorana at point-like defect, coexisting with itinerant Majorana edge mode (observed in Co-Si island on disordered Pb)

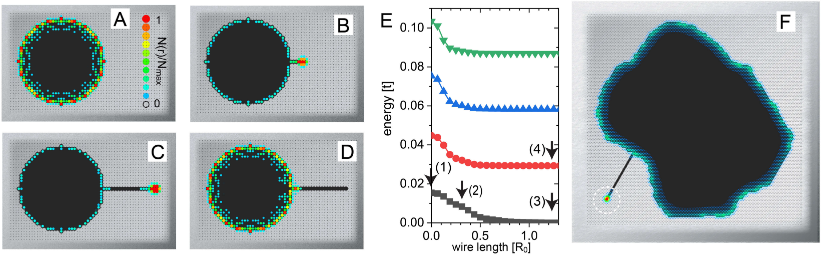


G.C. Ménard, ..., P. Simon and T. Cren, Nature Comm. 10, 2587 (2019).

Paris (France)

SIMILAR IDEAS: ISLAND + NONOWIRE

Itinerant Majorana mode leaking into side-attached nanowire.

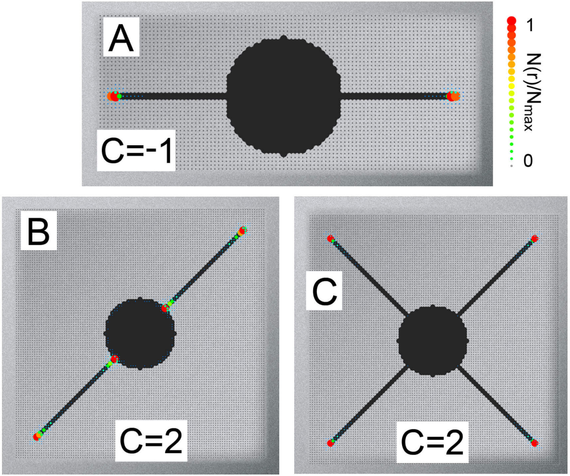


E. Mascot, S. Cocklin, S. Rachel, and D.K. Morr, arXiv:1909.06360 (2019).

Univ. of Illinois at Chicago (USA)

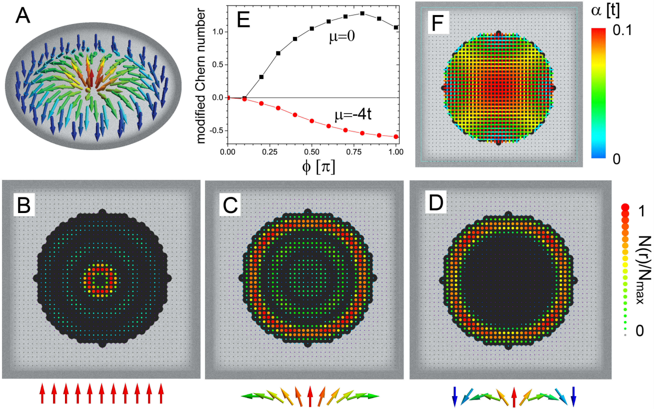
SIMILAR IDEAS: ISLAND + NONOWIRE

Majorana modes leaking to the side-attached nanowires.



PERSPECTIVES: SKYRMIONS IN SUPERCONDUCTORS

Creation of topological phase through skyrmions.



E. Mascot, S. Cocklin, S. Rachel, and D.K. Morr, arXiv:1811.06664

Univ. of Illinois at Chicago (USA)

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- Majorana quasiparticles

⟹ **M. Maśka & A. Gorczyca-Goraj (Katowice),**
A. Kobiałka (Lublin), A. Ptok (Kraków),
J. Tworzydło (Warsaw), N. Sedlmayr (Lublin).

- Shiba states/bands in topological phases

⟹ **Sz. Głodzik (Lublin)**

- Dynamics of Shiba states

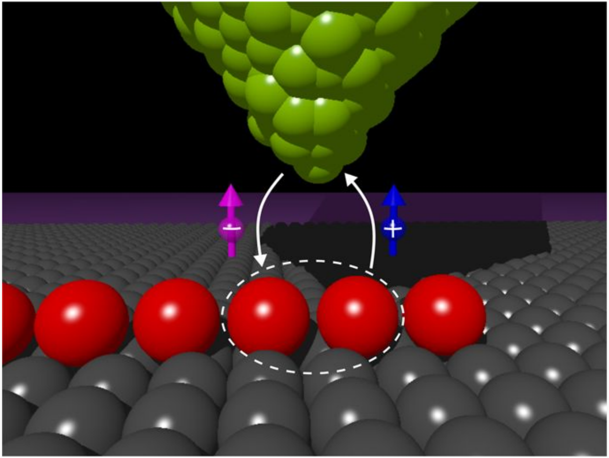
⟹ **B. Baran & R. Taranko (Lublin)**
G. Michałek & B.R. Bułka (Poznań).

- Majorana vs Kondo

⟹ **I. Weymann (Poznań), G. Górski (Rzeszów),**
T. Novotný, M. Žonda & V. Janiš (Prague).

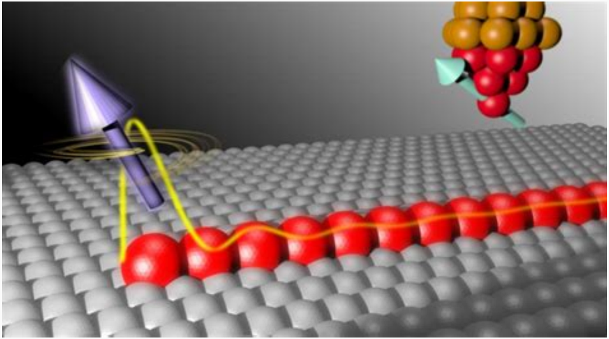
SELECTIVE EQUAL SPIN ANDREEV REFLECTIONS

Microscopic idea of the SESAR mechanism



M. Maśka and T. Domański, Scientific Reports **7**, 16193 (2017).

STM-type measurements for probing the Majorana qps



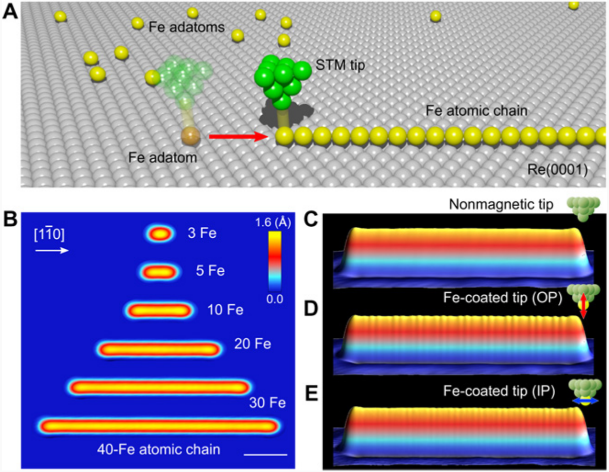
S. Jeon, ... and A. Yazdani, Science **358**, 772 (2017).

/ Princeton University, USA /

Kondo vs Majorana

POSSIBLE EXPERIMENTAL REALISATION

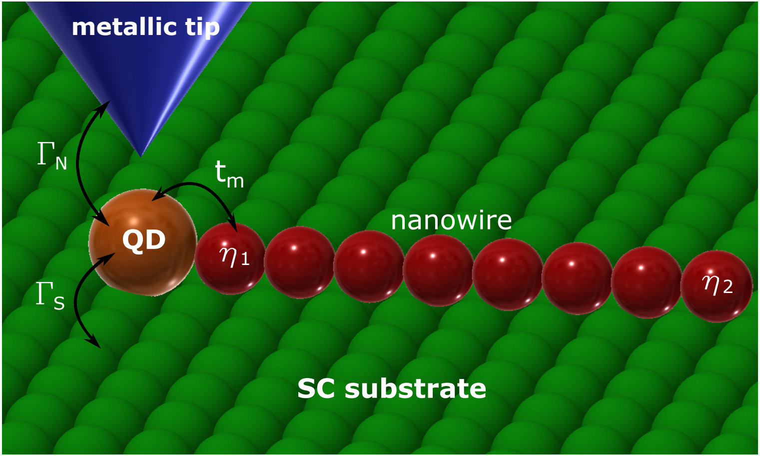
Deposition of individual atoms on superconducting surface



H. Kim, ..., and R. Wiesendanger, Science Adv. 4, eaar5251 (2018).

KONDO AND MAJORANA PHYSICS

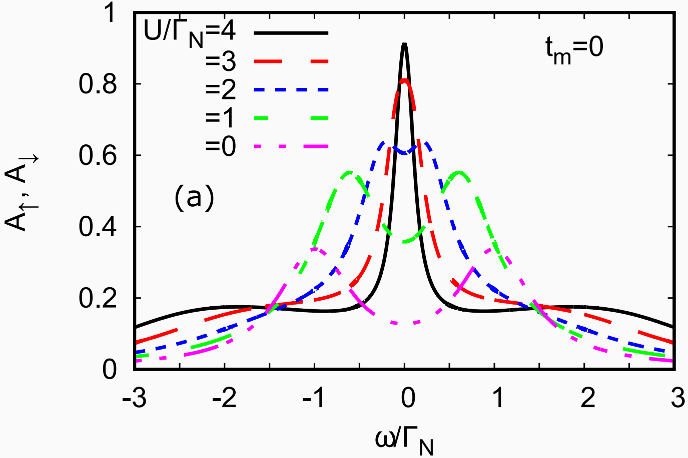
STM-type setup for probing the Kondo – Majorana – pairing effects.



G. Górski, ... and T. Domański, Scientific Reports 8, 15717 (2018).

SUBGAP KONDO PHYSICS

Spectrum of a quantum dot in absence of the Majoranas.

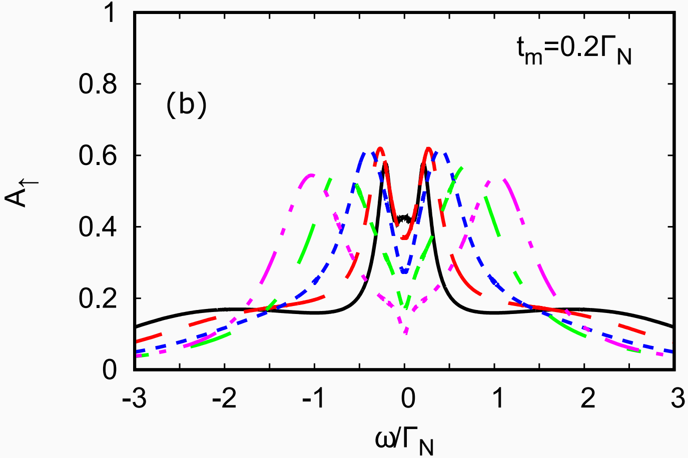


Results obtained for $t_m = 0$

G. Górski, ... and T. Domański, Scientific Reports 8, 15717 (2018).

KONDO VS MAJORANA

Spectrum of a quantum dot in its Kondo regime.

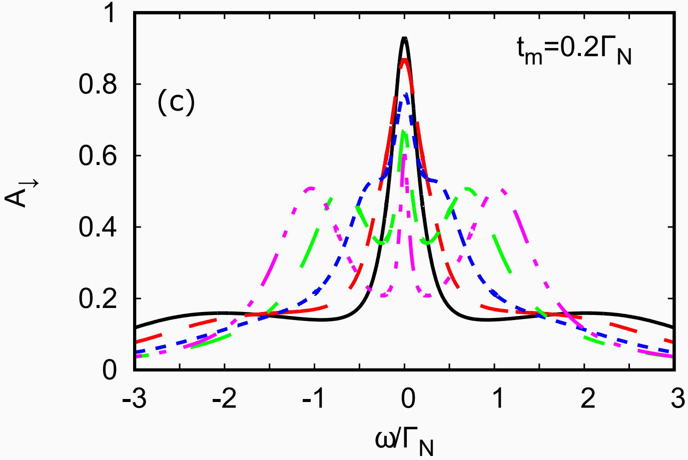


Results obtained for ↑ spin, assuming $\underline{t_m = 0.2\Gamma_N}$

G. Górski, ... and T. Domański, Scientific Reports 8, 15717 (2018).

KONDO VS MAJORANA

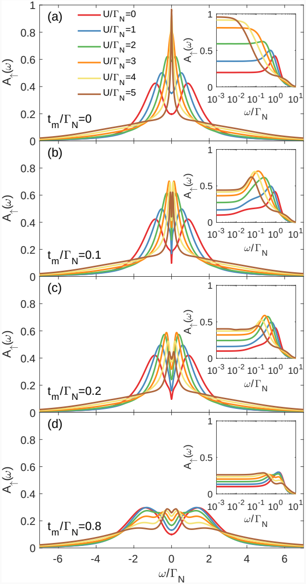
Spectrum of the correlated QD in its Kondo regime.



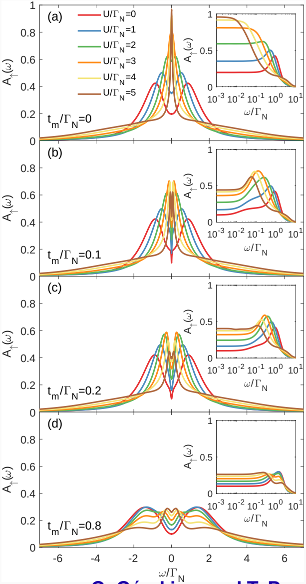
Results obtained for ↓ spin, assuming $\underline{t_m = 0.2\Gamma_N}$

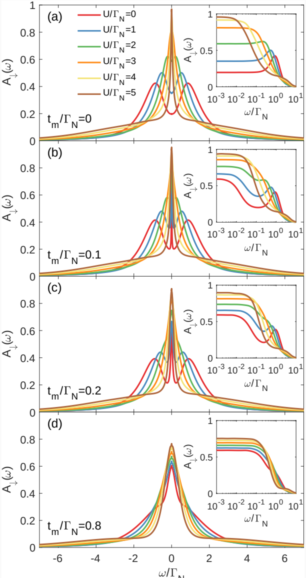
G. Górski, ... and T. Domański, Scientific Reports 8, 15717 (2018).

SPIN-RESOLVED NRG DATA



SPIN-RESOLVED NRG DATA





G. Górski, ... and T. Domański, Scientific Reports 8, 15717 (2018).

KONDO VS MAJORANA: CONCLUSIONS

- **influence of the Majorana on Kondo states:**

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$\Longrightarrow$ **destructive for ↑ electrons**

KONDO VS MAJORANA: CONCLUSIONS

- **influence of the Majorana on Kondo states:**

⇒ **constructive for ↓ electrons**

⇒ **destructive for ↑ electrons**

- **empirical observability via:**

⇒ **selective equal spin Andreev reflections (SESAR)**