

# Time-resolved Andreev states in superconducting hybrid structures

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**Tadeusz DOMAŃSKI**  
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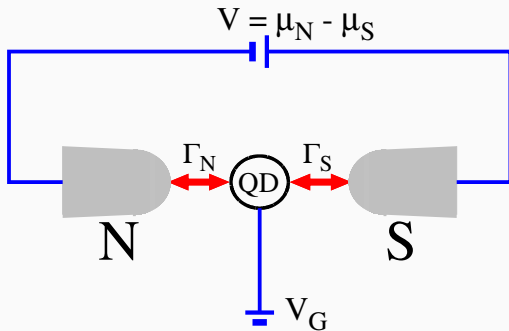
# **Superconducting nanostructures**

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**a few examples ...**

# 1. ANDREEV-TYPE JUNCTION

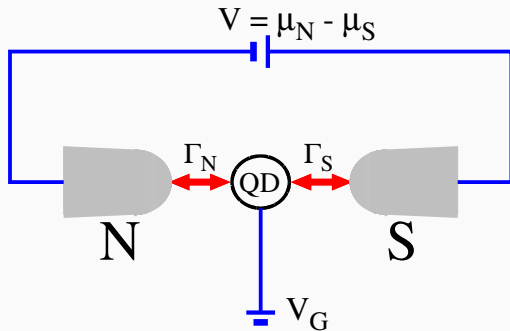
Normal metal (N) - Quantum Dot (QD) - Superconductor (S)





# 1. ANDREEV-TYPE JUNCTION

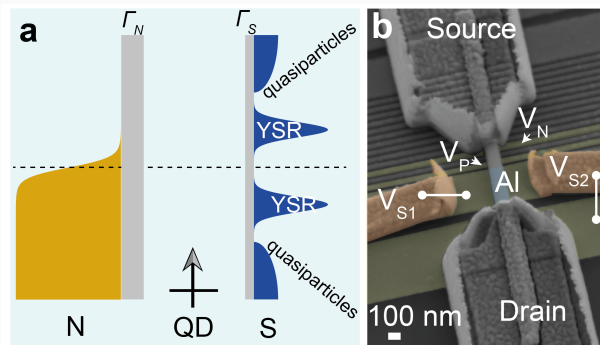
Normal metal (N) - Quantum Dot (QD) - Superconductor (S)



This is particular version of single-electron-transistor (SET) which operates on the Andreev (electron-to-hole) scattering mechanism.

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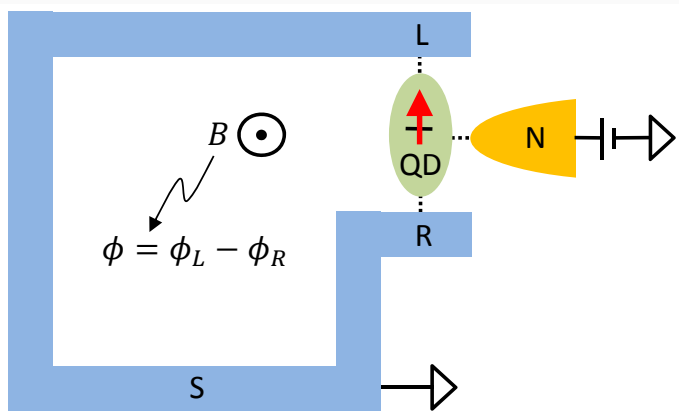
## Example of the recent empirical realization



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, *Commun. Phys.* **3**, 125 (2020).

## 2. JOSEPHSON JUNCTION

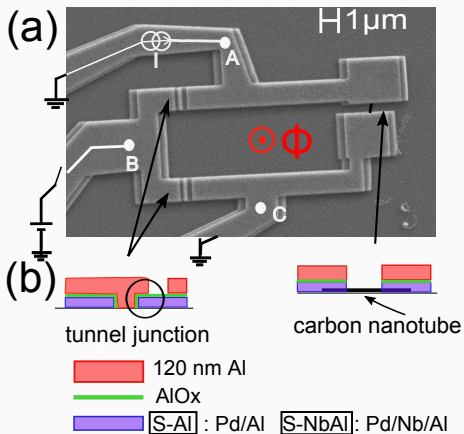
### Schematics of realistic $S_L$ -QD- $S_R$ (Josephson-type) junction



G. Kiršanskas, M. Goldstein, K. Flensberg, L.I. Glazman & J. Paaske,  
Phys. Rev. B 92, 235422 (2015)

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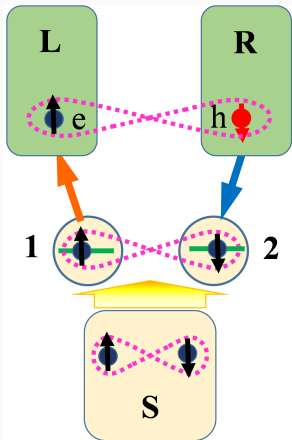
### Selected example of experimental realization



R. Delagrangé, R. Weil, A. Kasumov, M. Ferrier, H. Bouchiat, R. Deblock,  
*Phys. Rev. B* **93**, 195437 (2016).

### 3. COOPER PAIR SPLITTER

Three-terminal geometry of the Cooper pair splitter with two quantum dots (1,2) coupled to the normal ( $L, R$ ) electrodes and strongly coupled to the superconductor ( $S$ ) as a reservoir of Cooper pairs.



B.R. Buřka, Phys. Rev. B 104, 155410 (2021)

# SUPERCONDUCTING PROXIMITY EFFECT

- Quantum dot (QD) coupled to bulk superconductor (SC) induces:  
⇒ **on-dot pairing**

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# SUPERCONDUCTING PROXIMITY EFFECT

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- Such in-gap states originate from:

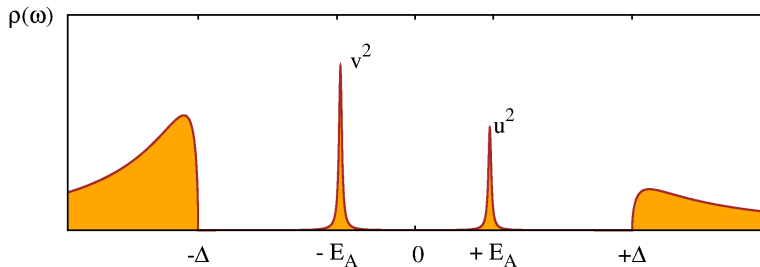
⇒ **leakage of Cooper pairs on QD** (Andreev)

⇒ **exchange int. of QD with SC** (Yu-Shiba-Rusinov)



# IN-GAP STATES

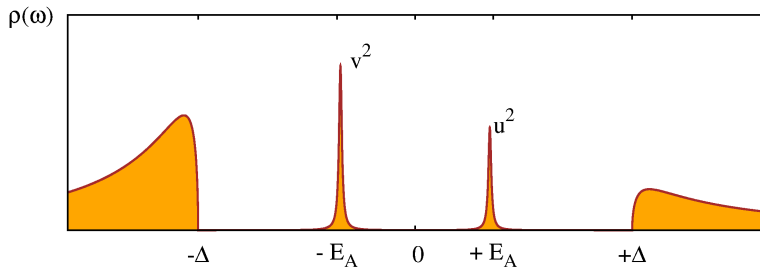
Spectrum of a single impurity coupled to bulk superconductor:



**Bound states appearing in the subgap region  $-\Delta < \omega < \Delta$**

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Spectrum of a single impurity coupled to bulk superconductor:



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**Yu-Shiba-Rusinov (Andreev) bound states**

# ISSUES TO BE ADDRESSED

## **I. Dynamics of Andreev bound states:**

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⇒ transient phenomena

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⇒ periodic driving

## II. Phase transition in time-domain:

⇒ dynamical singlet-doublet transition

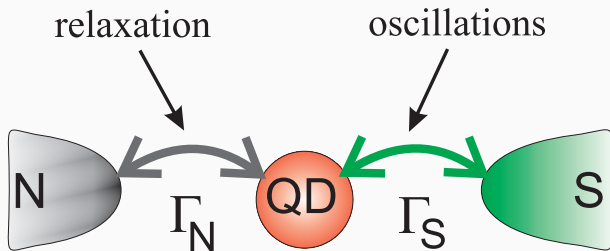
**Part I:**

**characteristic temporal scales of bound states**



# TRANSIENT EFFECTS OF IN-GAP STATES

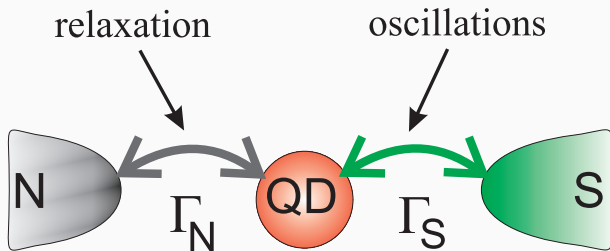
Consider a sudden coupling of QD to external leads



R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

# TRANSIENT EFFECTS OF IN-GAP STATES

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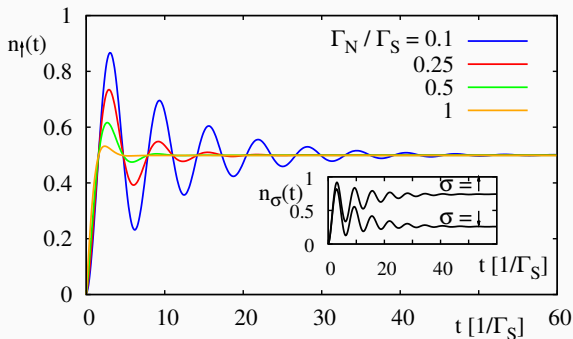


R. Taranko and T. Domański, Phys. Rev. B 98, 075420 (2018).

⇒ how much time is needed to create in-gap states ?

# RELAXATION VS QUANTUM OSCILLATIONS

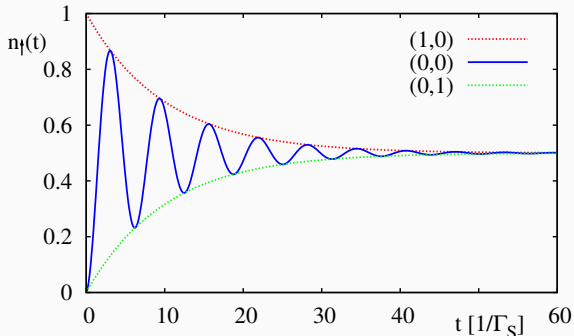
## Time-dependent charge of an initially empty QD



- relaxation time is proportional to  $1/\Gamma_N$
- oscillations depend on energies of in-gap states

# RELAXATION VS QUANTUM OSCILLATIONS

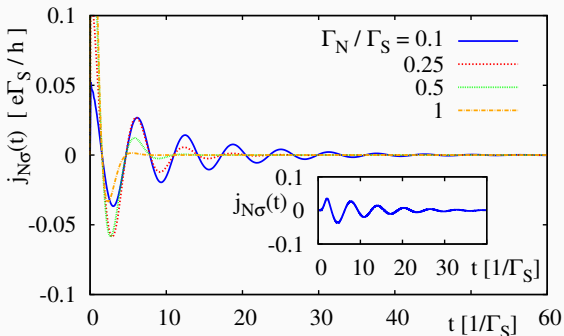
**t-dependent charge for various initial fillings ( $n_{\uparrow}, n_{\downarrow}$ )**



- **relaxation time is proportional to  $1/\Gamma_N$**
- **oscillations depend on energies of in-gap states**

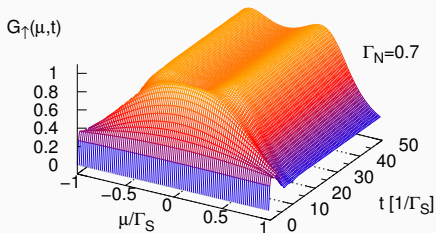
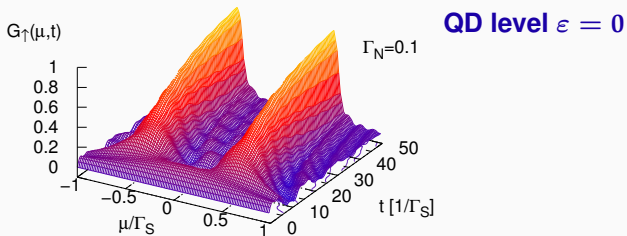
# RELAXATION VS QUANTUM OSCILLATIONS

## Time-dependent charge current of unbiased junction



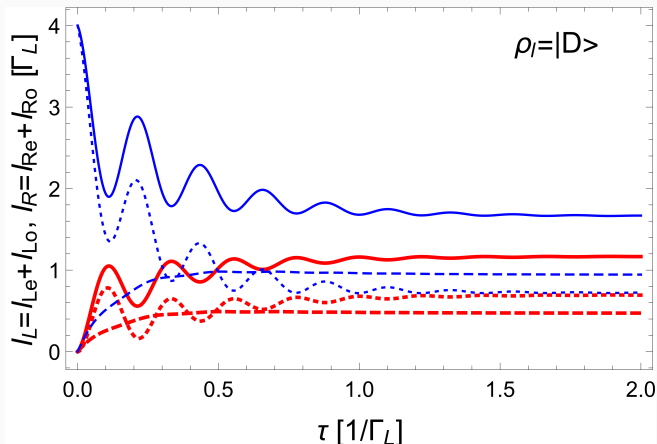
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# EXPERIMENTALLY ACCESSIBLE QUANTITIES



Subgap tunneling conductance  $G_{\sigma} = \frac{\partial I_{\sigma}}{\partial t}$  vs time ( $t$ ) and voltage ( $\mu$ )

# STATISTICS OF TUNNELING EVENTS



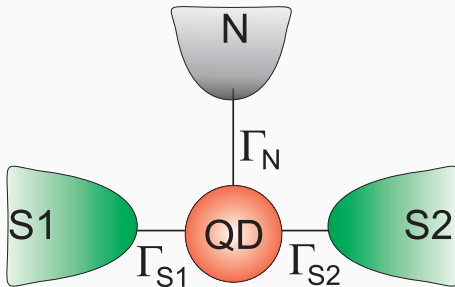
**Transient currents from 'Waiting Time Distribution' approach**

G. Michałek, B. Bułka, T. Domański & K.I. Wysokiński, Acta Phys. Polon. A 133, 391 (2018).

# Josephson-type structures



# PHASE-CONTROLLED TRANSIENT EFFECTS



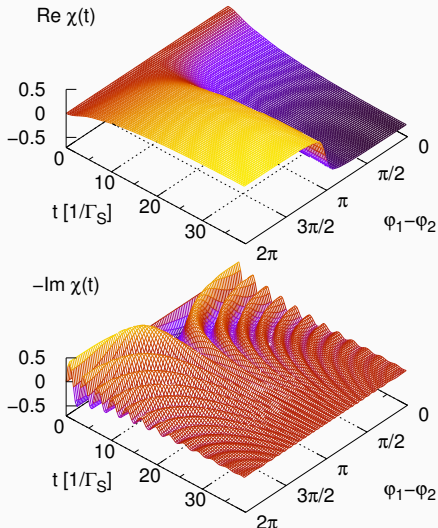
R. Taranko, T. Kwapiński & T. Domański, Phys. Rev. B 99, 165419 (2019).

**Physical issue:**

**⇒ phase tunable evolution of in-gap states**

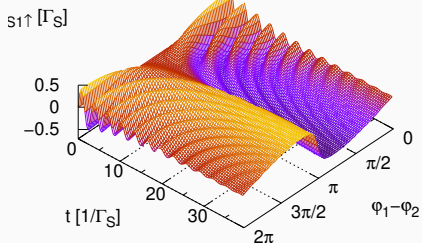
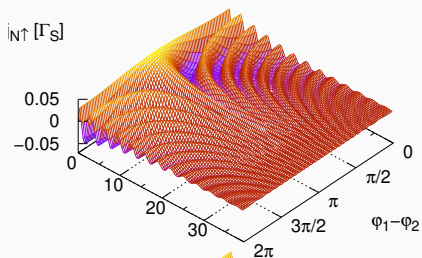
# DEVELOPMENT OF ON-DOT PAIRING

Complex order parameter  $\chi(t) = \langle \hat{d}_\downarrow \hat{d}_\uparrow \rangle$  induced by proximity effect



# TRANSIENT CURRENTS

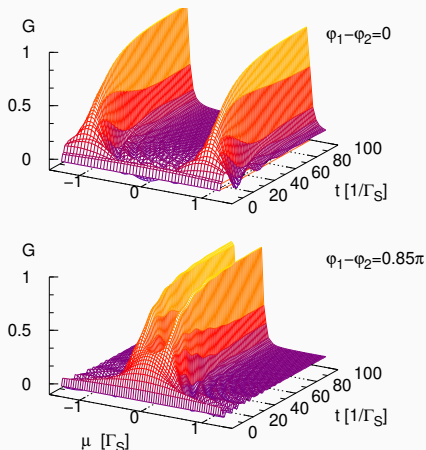
Time-dependent charge currents  $j_N(t)$  and  $j_{S1}(t)$ .



R. Taranko, T. Kwapiński & T. Domański, *Phys. Rev. B* 99, 165419 (2019).

# DYNAMICAL EFFECTS OF BIASED JUNCTION

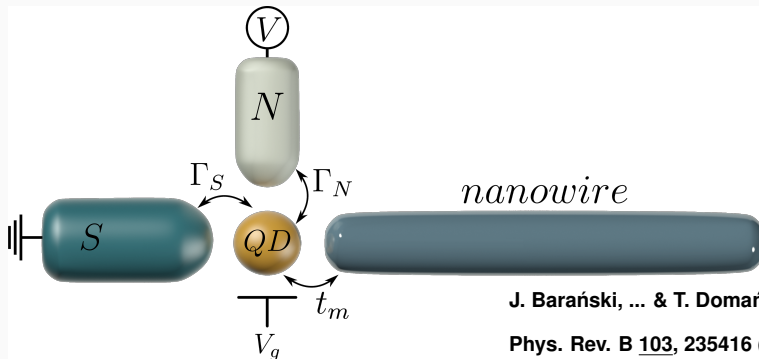
Conductance of the Andreev current  $j_N(t)$  versus voltage  $\mu$  applied between the normal lead and superconductors.



**Hybrid structure with topological superconductor**

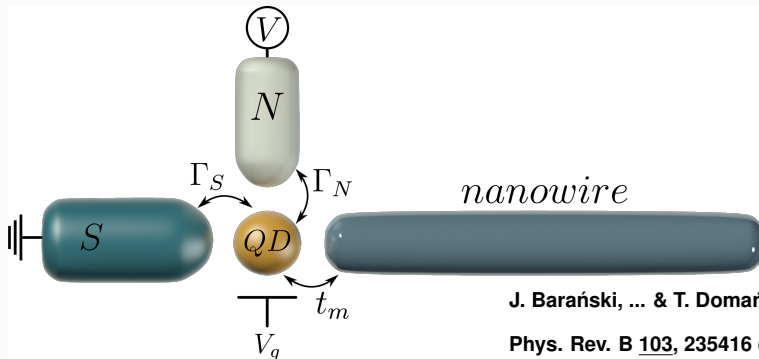
# MAJORANA LEAKAGE ON QUANTUM DOT

## N-QD-S circuit side-attached to topological superconductor



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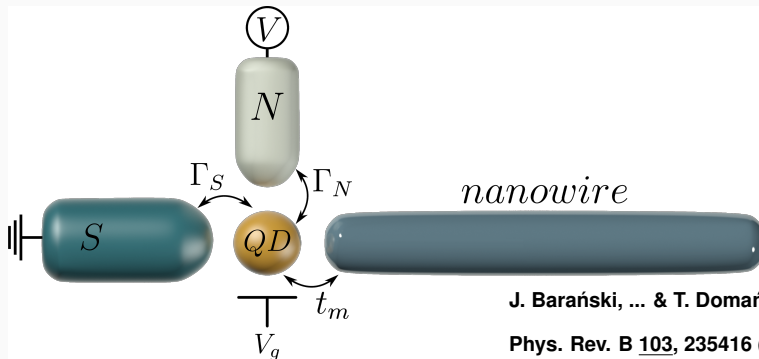
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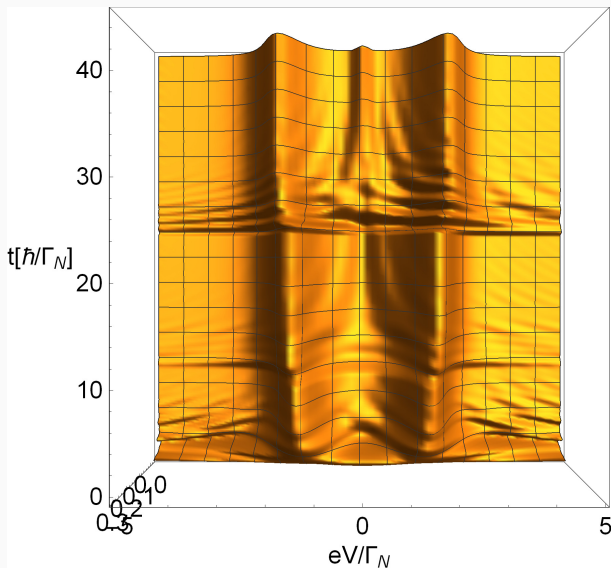


⇒ time-resolved transfer of Majorana mode on QD

⇒ how does it show in the Andreev conductance

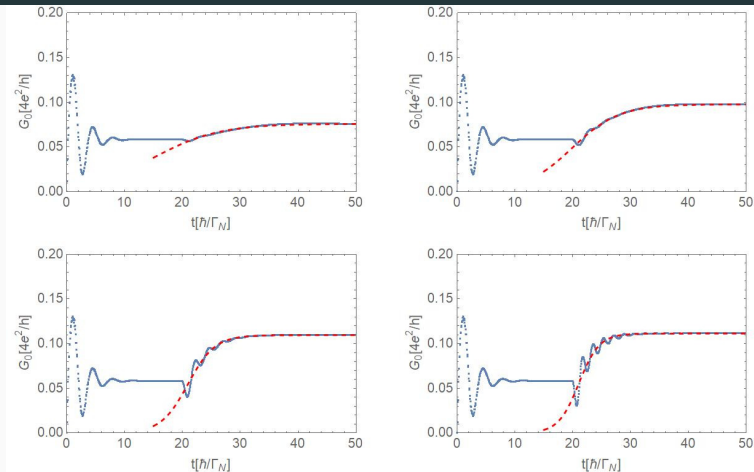


# TIME-RESOLVED MAJORANA LEAKAGE



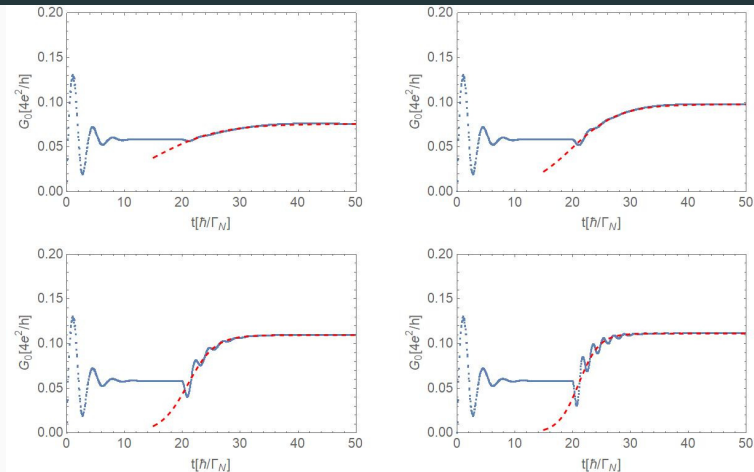
The differential Andreev conductance vs bias voltage  $V$  and time

# TIME-RESOLVED ZERO BIAS CONDUCTANCE



The zero-bias differential conductivity obtained for  $\Gamma_S = 3\Gamma_N$  and  $\epsilon_d = \Gamma_N$ , assuming:  $t_m = 0.25$  (upper left),  $0.5$  (upper right),  $1$  (lower left),  $1.5$  (lower right)  $\Gamma_N$ . QD is abruptly connected to Majorana mode at time  $t = 20\hbar/\Gamma_N$ .

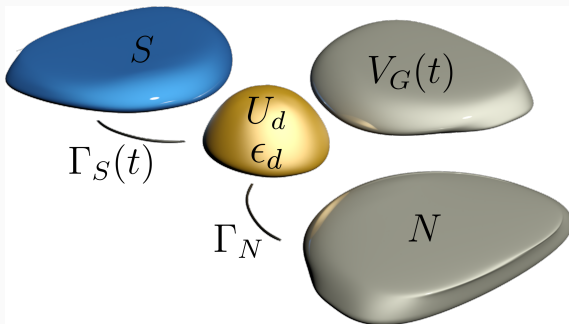
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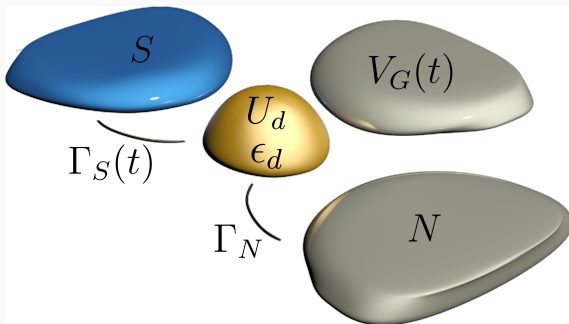
# Quench-induced dynamics

# QUENCH DRIVEN DYNAMICS



**Quantum quench protocols:**

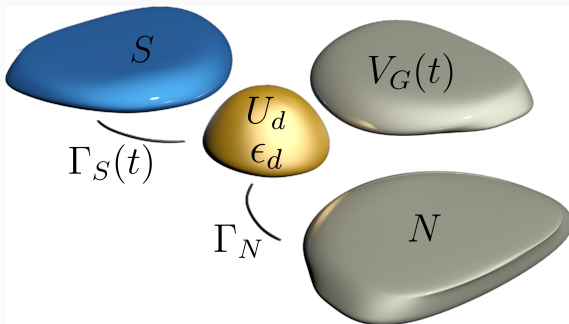
# QUENCH DRIVEN DYNAMICS



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**$\Rightarrow$  sudden change of the coupling  $\Gamma_S(t)$**

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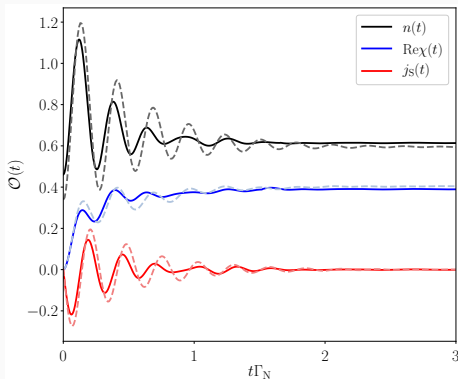
⇒ sudden change of the coupling  $\Gamma_S(t)$

⇒ abrupt application of gate potential  $V_G(t)$

# QUENCH OF COUPLING $\Gamma_S$

Time-dependent observables driven by the quantum quench

$\Gamma_S = 0 \rightarrow \Gamma_S = U$  obtained for  $\varepsilon_d = 0$ ,  $\Gamma_N = U/10$ .



**solid lines** - time dependent NRG

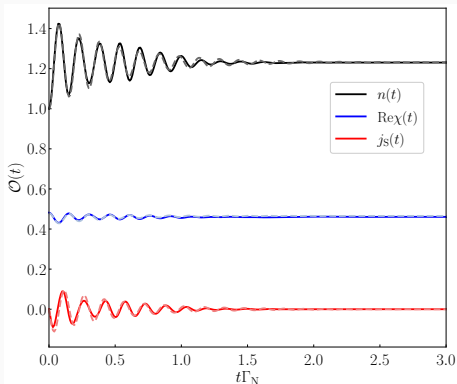
**dashed lines** - Hartree-Fock-Bogolubov



# QUENCH OF GATE POTENTIAL

Time-dependent observables driven by the quantum quench

$\epsilon_d = -U/2 \rightarrow \epsilon_d = -U$  obtained for  $\Gamma_S = 4U$ ,  $\Gamma_N = U/10$ .

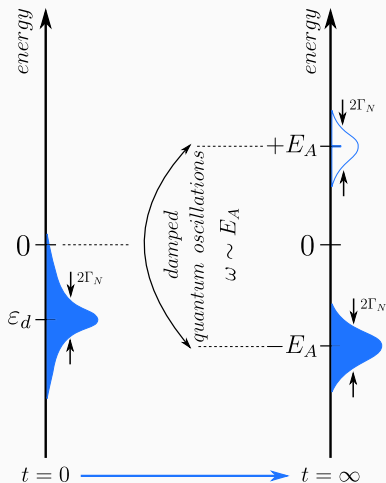


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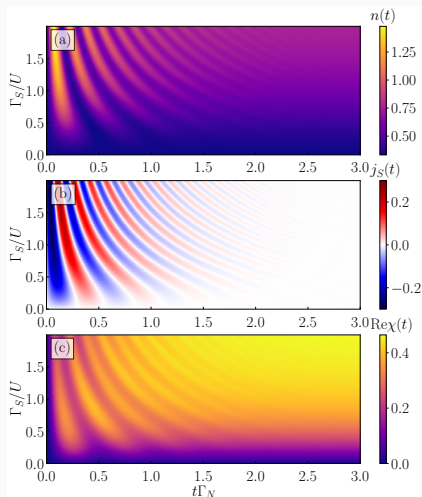
# UNIVERSAL TENDENCY

## Rabbi-type oscillations observable in development of the in-gap states



# POST-QUENCH QUANTUM OSCILLATIONS

Rabi-type oscillations induced by the quench from  $\Gamma_S = 0$  to  $\Gamma_S$  (as indicated).



## **Part II: dynamical singlet-doublet transition**

# SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

The proximitized quantum dot can be described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U_d \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left( \Delta_d \hat{d}_{\uparrow}^{\dagger} \hat{d}_{\downarrow}^{\dagger} + \text{h.c.} \right)$$

where  $\Delta_d = \Gamma_S/2$ .

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where  $\Delta_d = \Gamma_S/2$ . True eigen-states are represented by:

$|\uparrow\rangle$     **and**     $|\downarrow\rangle$      $\Leftarrow$     **doublet states (spin  $\frac{1}{2}$ )**

$u |0\rangle - v |\uparrow\downarrow\rangle$   
 $v |0\rangle + u |\uparrow\downarrow\rangle$      $\left. \vphantom{\begin{matrix} u |0\rangle - v |\uparrow\downarrow\rangle \\ v |0\rangle + u |\uparrow\downarrow\rangle \end{matrix}} \right\} \Leftarrow$     **singlet states (spin 0)**

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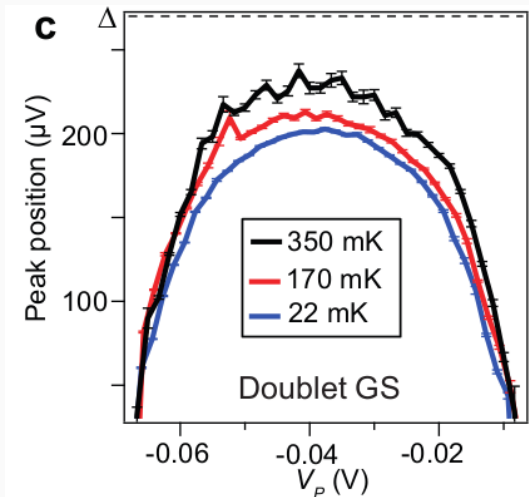
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Upon varying the parameters  $\epsilon_d$ ,  $U_d$  or  $\Gamma_S$  there can be induced **quantum phase transition** between these doublet/singlet states.

J. Bauer, A. Oguri & A.C. Hewson, J. Phys.: Condens. Matter 19, 486211 (2007).

# QUANTUM PHASE TRANSITION: EXPERIMENT



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, *Commun. Phys.* **3**, 125 (2020).



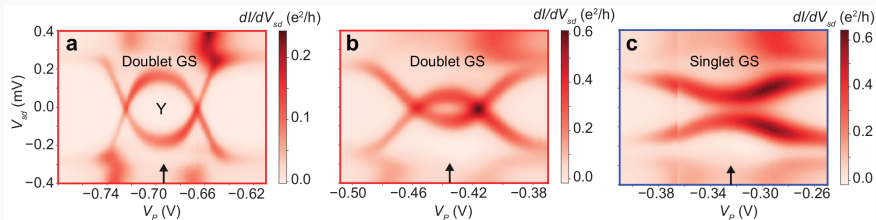
# SINGLET VS DOUBLET: EXPERIMENT

Differential conductance vs source-drain bias  $V_{sd}$  (vertical axis) and gate potential  $V_p$  (horizontal axis) measured for various  $\Gamma_s/U$

$$U \gg \Gamma_s$$

$$U \geq \Gamma_s$$

$$U < \Gamma_s$$



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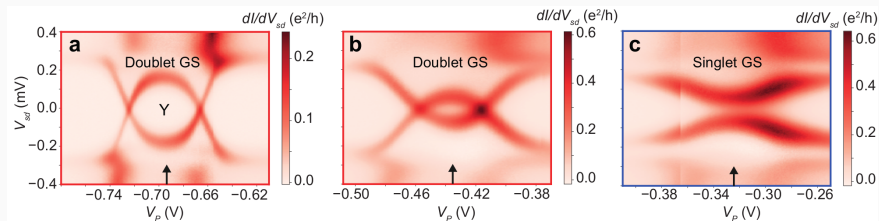
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**Crossings of in-gap states correspond to the singlet-doublet QPT.**

## **Outline of general concept**

## POST-QUENCH DYNAMICS

For  $t < 0$  we assume the system  $\hat{H}_0$  to be in its ground state:

$$\hat{H}_0 |\Psi_0\rangle = E_0 |\Psi_0\rangle$$

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**Loschmidt amplitude**



# STATIONARY VS DYNAMICAL PHASE TRANSITION

**Idea:** M. Heyl, A. Polkovnikov, S. Kehrein, *Phys. Rev. Lett.* 110, 135704 (2013).

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**Partition function**

$$\mathcal{Z} = \langle e^{-\beta \hat{H}} \rangle$$

**Loschmidt amplitude**

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where

$$\beta = \frac{1}{k_B T}$$

**Loschmidt amplitude**

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**Loschmidt echo**  $L(t)$

$$L(t) = \left| \langle \Psi_0 | e^{-it\hat{H}} | \Psi_0 \rangle \right|^2$$

# STATIONARY VS DYNAMICAL PHASE TRANSITION

**Idea:** M. Heyl, A. Polkovnikov, S. Kehrein, *Phys. Rev. Lett.* 110, 135704 (2013).

**Partition function**

$$\mathcal{Z} = \langle e^{-\beta \hat{H}} \rangle$$

where

$$\beta = \frac{1}{k_B T}$$

**Free energy**  $F(T)$

$$\mathcal{Z}(T) \equiv e^{-\beta F(T)}$$

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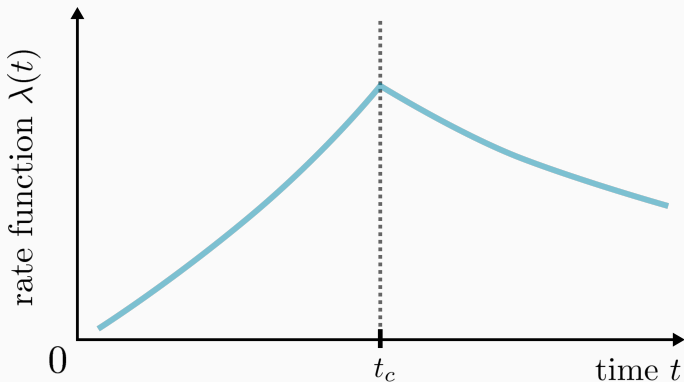
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# CRITICAL TIME

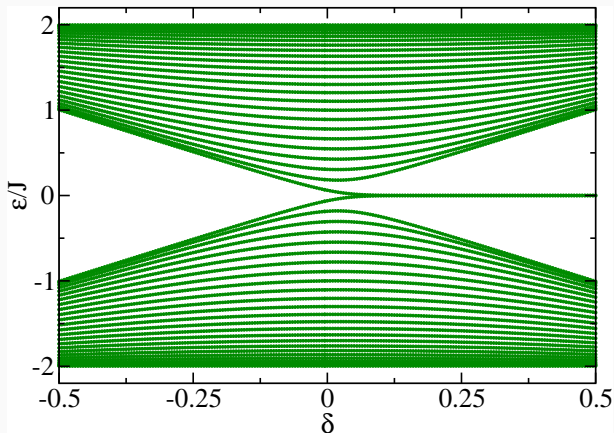


**At critical time  $t_c$  the rate function  $\lambda(t)$  of the Loschmidt echo  $L(t) \equiv e^{-\lambda(t)}$  exhibits a nonanalytic kink.**

**Some examples ...**

# SU-SCHRIEFFER-HEEGER MODEL

Quasiparticle spectrum of the SSH model under stationary conditions.

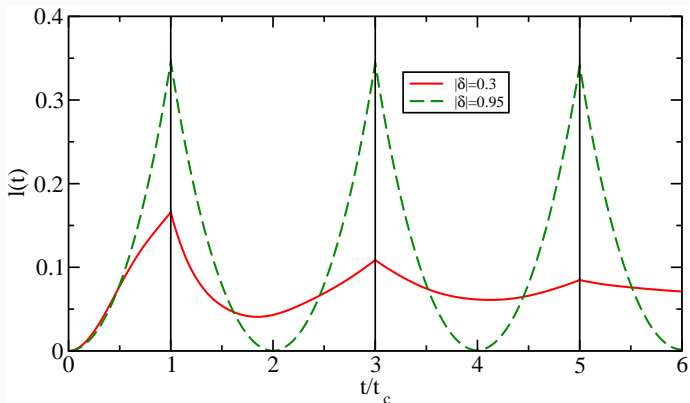


$$\hat{H} = -J \sum_j \left[ (1 + \delta e^{i\pi j}) \hat{c}_j^\dagger \hat{c}_{j+1} + \text{h.c.} \right]$$

[ N. Sedlmayr, Acta Phys. Polon. A 135, 1191 (2019) ]



# QUENCH DRIVEN TRANSITION



$$\hat{H} = -J \sum_j \left[ (1 + \delta e^{i\pi j}) \hat{c}_j^\dagger \hat{c}_{j+1} + \text{h.c.} \right]$$

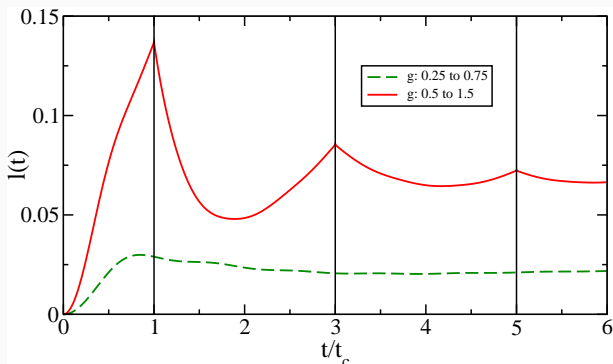
**solid red line:**  $\delta = -0.3 \longrightarrow \delta = +0.3$

**dashed green line:**  $\delta = 0.95 \longrightarrow \delta = -0.95$

[ N. Sedlmayr, Acta Phys. Polon. A 135, 1191 (2019) ]

# QUENCH OF TRANSVERSE FIELD $h$

Post-quench return rate of the Ising model ( $g \equiv h/J$ )



$$\hat{H} = -\frac{J}{2} \sum_{j=1}^{N-1} \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z + \frac{h}{2} \sum_{j=1}^N \hat{\sigma}_j^x$$

**solid red line** - across a phase transition ( $g_c = 1$ )

**dashed green line** - inside the same phase

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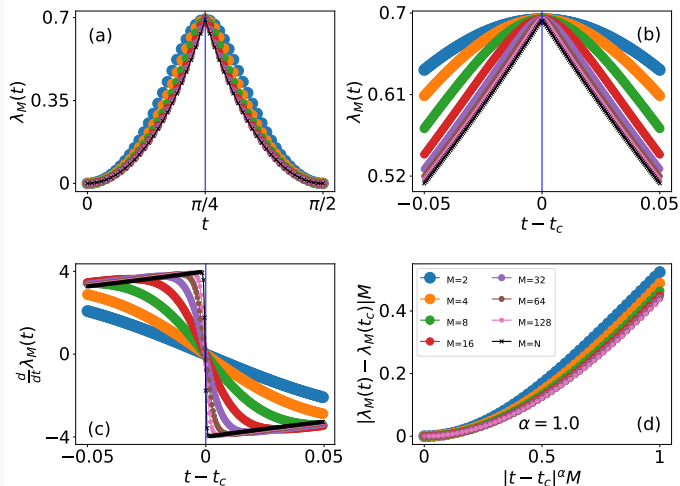
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(however there are exceptions from this rule)
- ⇒ at equidistant critical times**  
(in most cases, though not always)
- ⇒ at finite temperatures**  
(but they are no longer sharp)

# FINITE-SIZE EFFECTS



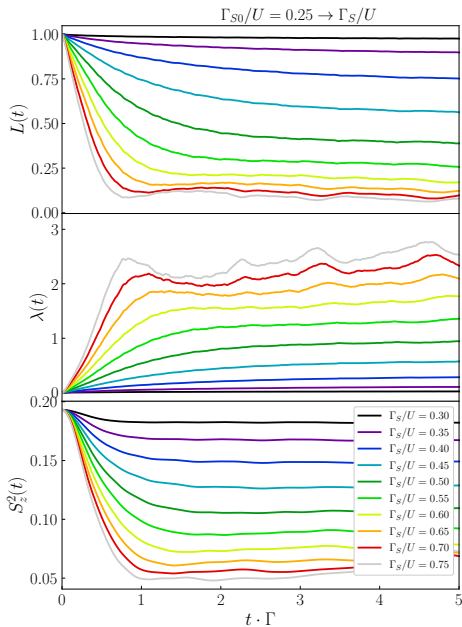
**"Local measures of dynamical quantum phase transitions"**

J.C. Halimeh, D. Trapin, M. Damme & M. Heyl, Phys. Rev. B 104, 075130 (2021).

# **Dynamical singlet-doublet transition**



# $t$ NRG RESULTS: ABRUPT CHANGE OF $\Gamma_S$



**Loschmidt echo**

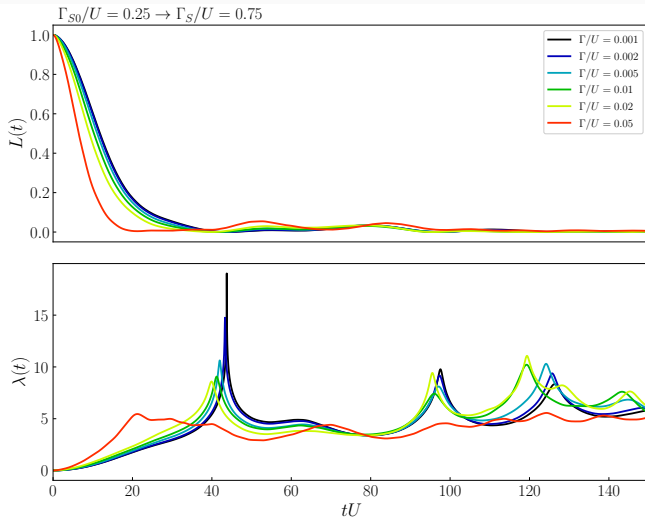
$$L(t) \equiv |\langle \Psi(0) | \Psi(t) \rangle|^2$$

**Return rate**

$$\lambda(t) \equiv -\ln \{L(t)\}$$

**The squared magnetic moment  $\langle S_z^2(t) \rangle$**

**$t$ NRG RESULTS:**  $\Gamma_S = U/4 \longrightarrow \Gamma_S = 3U/4$



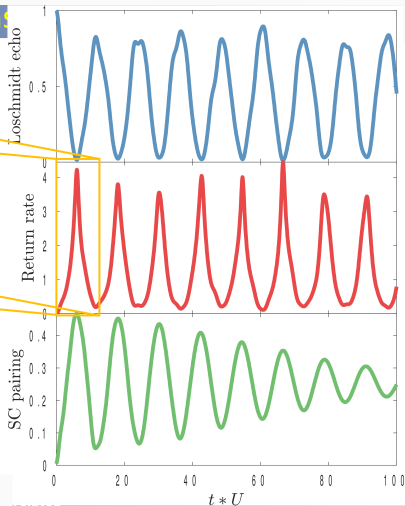
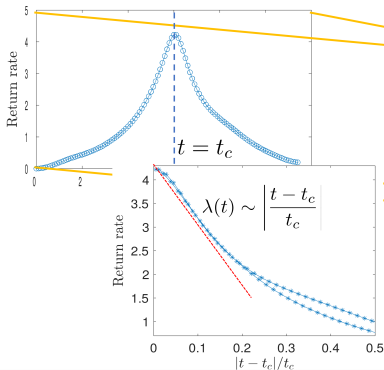
**Loschmidt echo  $L(t)$  and return rate  $\lambda(t)$  obtained for various  $\Gamma_N \equiv \Gamma$**

**$t$ NRG RESULTS:**  $\Gamma_S = U/4 \longrightarrow \Gamma_S = 3U/4$

## Dynamical quantum phase transition

from the doublet to the singlet phase

$$\Gamma_S = U/4 \rightarrow \Gamma_S = 3U/4$$



**K. Wrzeńniewski, N. Sedlmayr, T. Domański & I. Weymann, (2021)**

# CONCLUSIONS

**Evolution of the Andreev in-gap states:**

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- **activates Rabi-type oscillations** /due to particle-hole mixing/
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- **activates Rabi-type oscillations** /due to particle-hole mixing/
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- **may exhibit dynamical transition(s)** /upon varying ground states/

**These phenomena are detectable in transport properties !**

# ACKNOWLEDGEMENTS

- **transient phenomena, Floquet formalism & machine-learning**

⇒ R. Taranko (Lublin), B. Baran (Lublin),

- **statistical approach to Andreev transport**

⇒ B.R. Bułka & G. Michałek (Poznań), K.I. Wysokiński (Lublin),

- **time-resolved leakage of Majorana mode**

⇒ J. Barański (Dęblin),

- **dynamical singlet-doublet transition**

⇒ I. Weymann & K. Wrześniewski (Poznań),

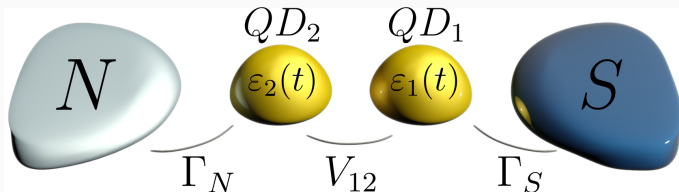
N. Sedlmayr (Lublin)

**Andreev (triplet) blockade**



# DYNAMICAL EFFECTS IN DOUBLE QUANTUM DOT

**Setup:** let's consider two quantum dots ( $QD_{1,2}$ ) placed between normal metal (N) and superconducting (S) electrodes

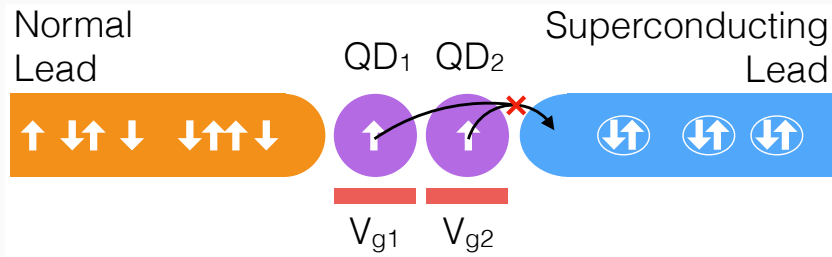


<sup>(1)</sup> R. Taranko, K. Wrześniewski, B. Baran, I. Weymann & T. Domański, *Phys. Rev. B* **103**, 165430 (2021).

<sup>(2)</sup> B. Baran, R. Taranko & T. Domański, *Sci. Rep.* **11**, 11148 (2021).

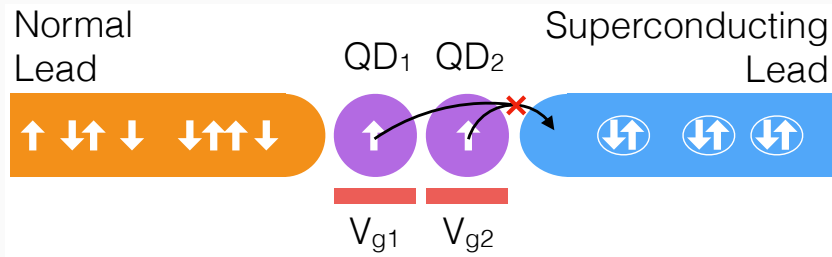
# ANDREEV BLOCKADE

D. Pekker, P. Zhang & S.M. Frolov, *SciPost Phys.* 11, 081 (2021).



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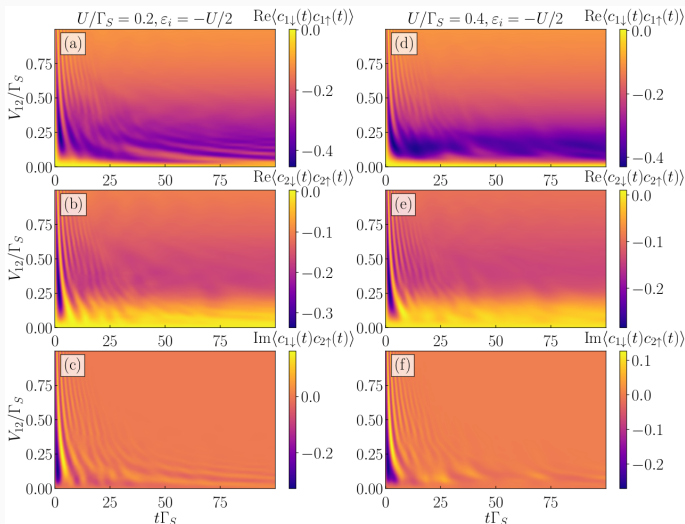
D. Pekker, P. Zhang & S.M. Frolov, SciPost Phys. 11, 081 (2021).



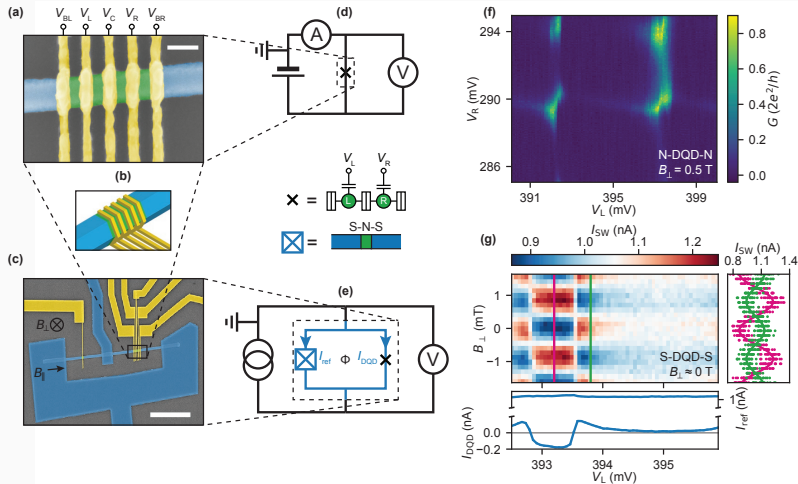
**Problem:** initial triplet configuration does not allow for development of the superconducting proximity effect

# DYNAMICS OF ANDREEV BLOCKADE

## Disappearance of Andreev blockade obtained for half-filled DQD.



# TRIPLET BLOCKADE IN JOSEPHSON JUNCTION



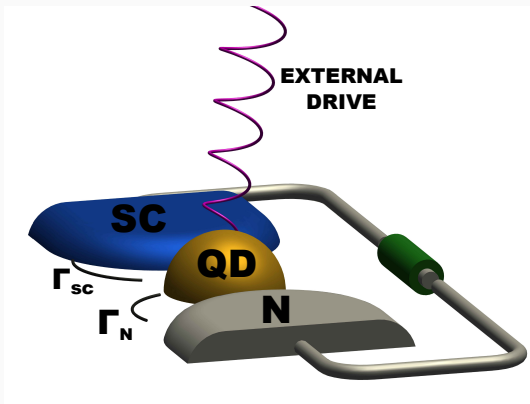
„Triplet-blockaded Josephson supercurrent in double quantum dots”

D. Bouman et al, Phys. Rev. B 102, 220505(R) (2020).

## Periodically driven QD

# BOUND STATES OF A DRIVEN QUANTUM IMPURITY

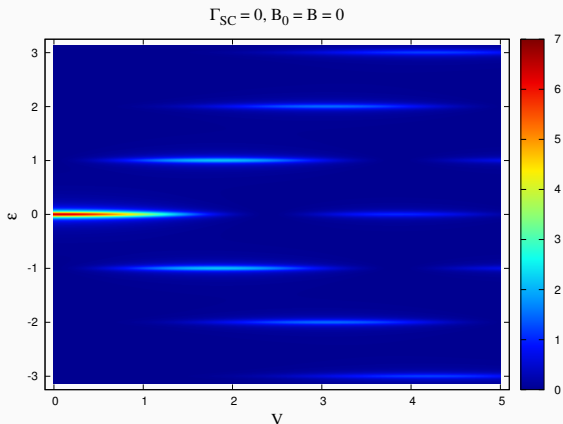
Quantum impurity with periodically oscillating energy level



$$\epsilon(t) = \epsilon_0 + V \times \cos(\omega t)$$

# BOUND STATES OF A DRIVEN QUANTUM IMPURITY

Floquet spectrum averaged over a period  $T = 2\pi/\omega$



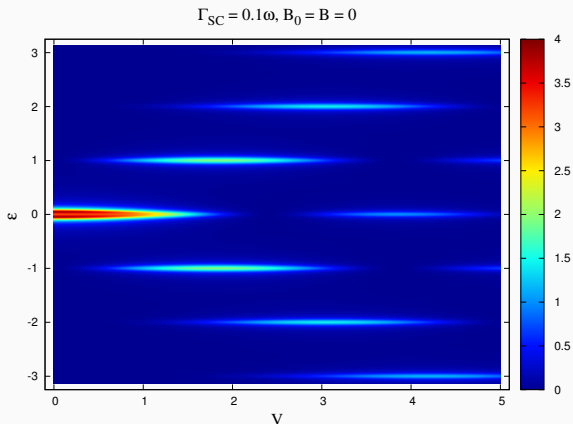
$\Gamma_S = 0.0$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).



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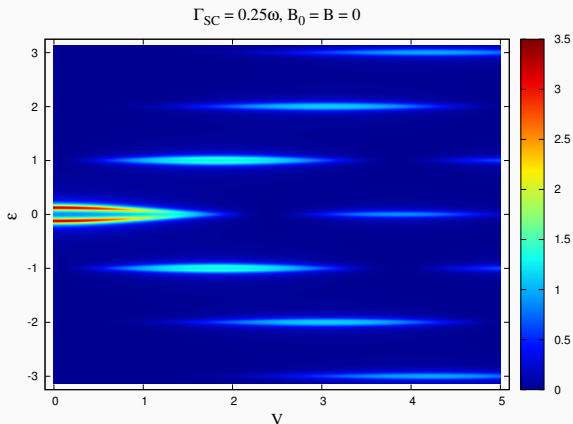


$$\Gamma_S = 0.1\omega$$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

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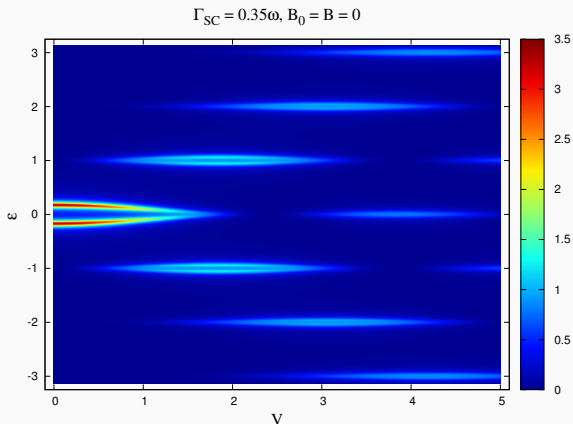


$$\Gamma_S = 0.25\omega$$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

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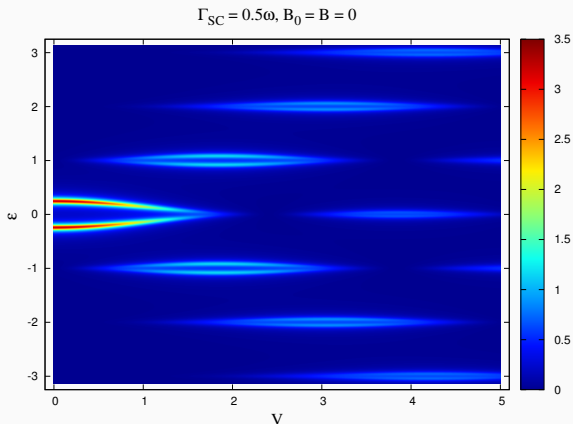


$$\Gamma_S = 0.35\omega$$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

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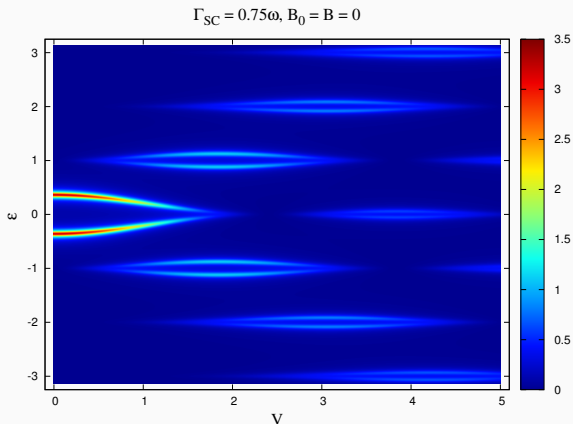


$$\Gamma_S = 0.5\omega$$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

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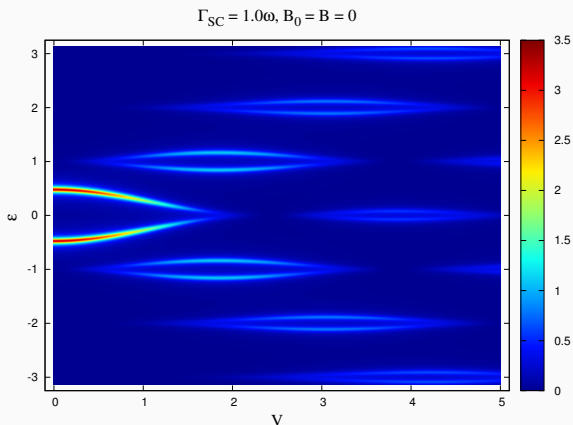


$$\Gamma_S = 0.75\omega$$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).

# BOUND STATES OF A DRIVEN QUANTUM IMPURITY

Floquet spectrum averaged over a period  $T = 2\pi/\omega$



$$\Gamma_S = 1.0\omega$$

B. Baran and T. Domański, Phys. Rev. B 100, 085414 (2019).