

# Superconductivity of nanoscopic systems

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Zjazd Fizyków Polskich (Gdańsk)

7 IX 2023

# OUTLINE

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- ⇒ **electron pairing in nanostructures**
- ⇒ **in-gap bound states**
- ⇒ **interplay with correlations**
- ⇒ **static & dynamical phase transitions**
- ⇒ **topological phases**
- ⇒ **Majorana quasiparticles**

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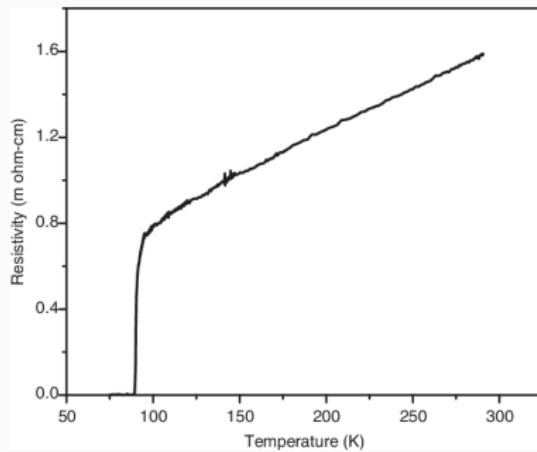
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- ⇒ Majorana quasiparticles

/ J. Spałek, D. Kaczorowski, I. Weymann /

# **Macroscopic superconductors**

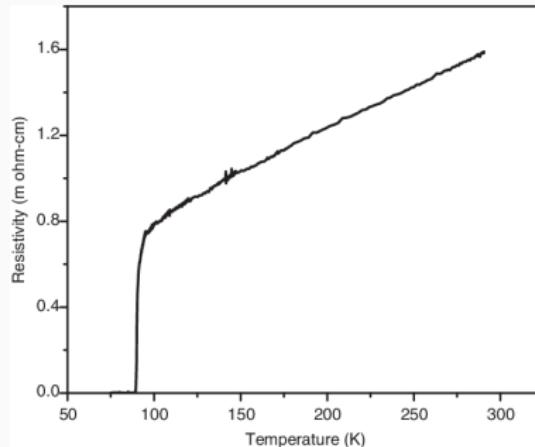
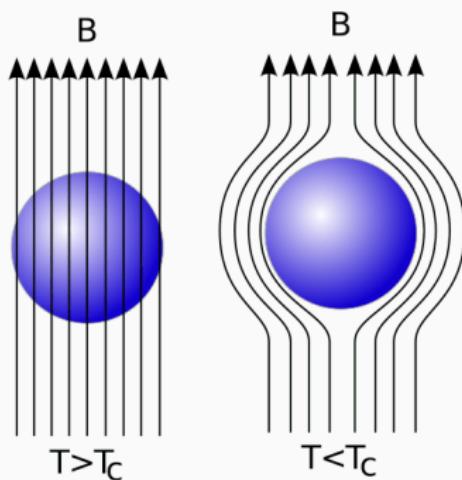
# SUPERCONDUCTOR: PROPERTIES

perfect conductor



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perfect conductor



perfect diamagnet

# ELECTRON PAIRING

BCS (non-Fermi liquid) ground state :

$$|\text{BCS}\rangle = \prod_k \left( \color{red}{u_k} + v_k \hat{c}_{k\uparrow}^\dagger \hat{c}_{-k\downarrow}^\dagger \right) |\text{vacuum}\rangle$$

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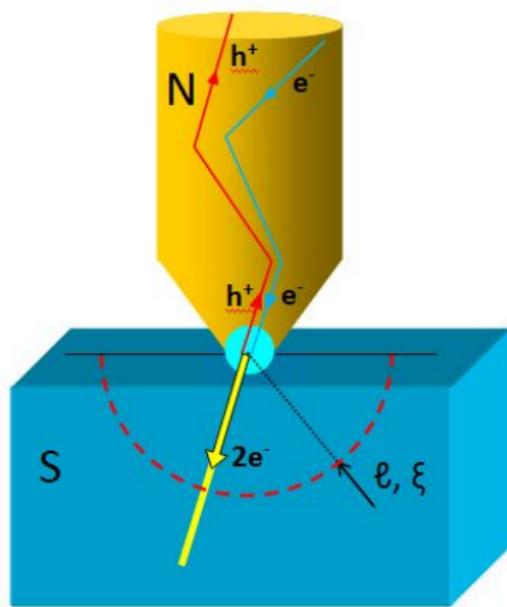
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Charge is conserved modulo-2e due to Bose-Einstein condensation of Cooper pairs

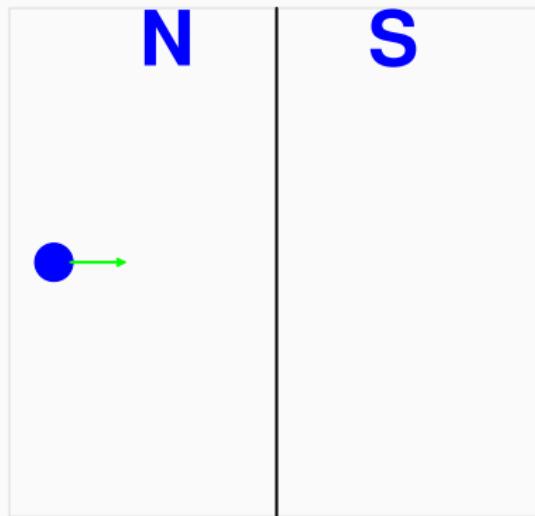
## PARTICLE VS HOLE

In superconductors the particle and hole degrees of freedom are mixed with one another (as particularly evident near  $E_F$ )



## PARTICLE VS HOLE

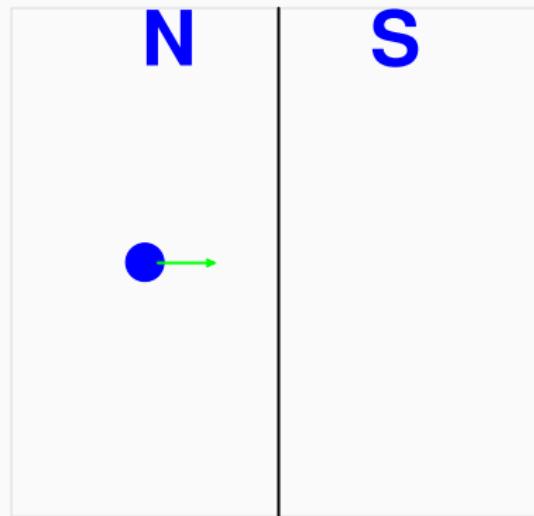
Let us consider the interface of metal **N** and superconductor **S**



where incident electron ...

# PARTICLE VS HOLE

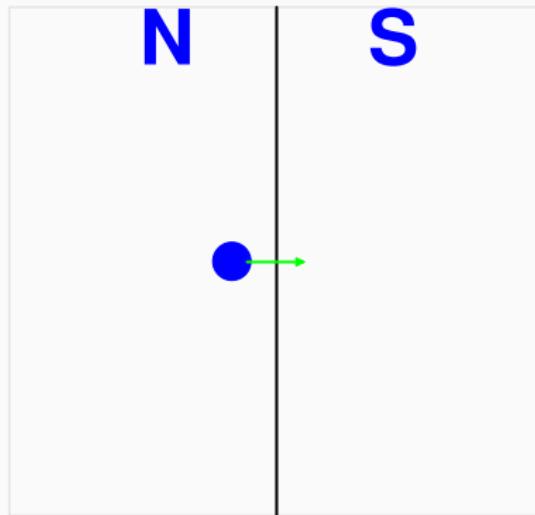
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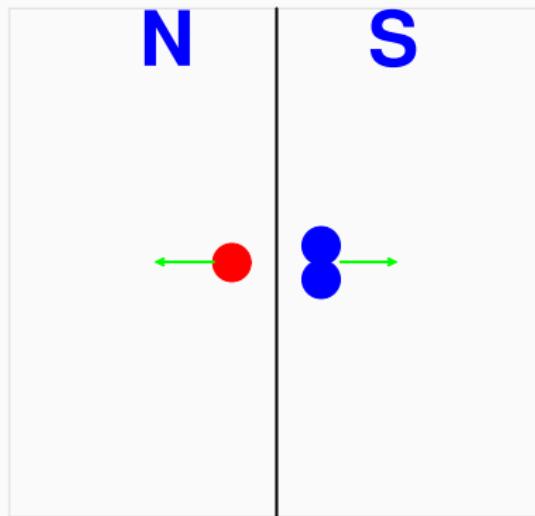
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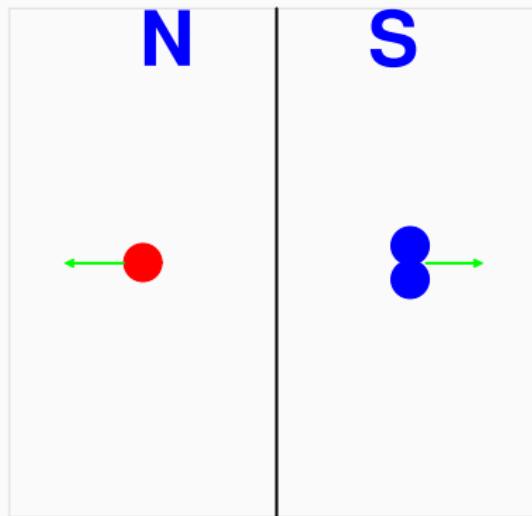
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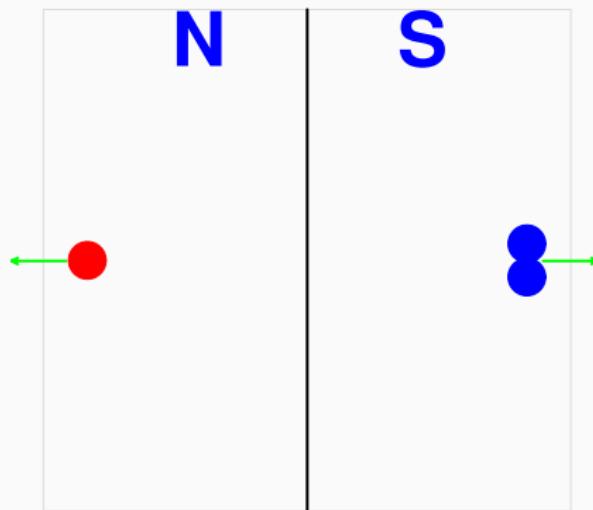
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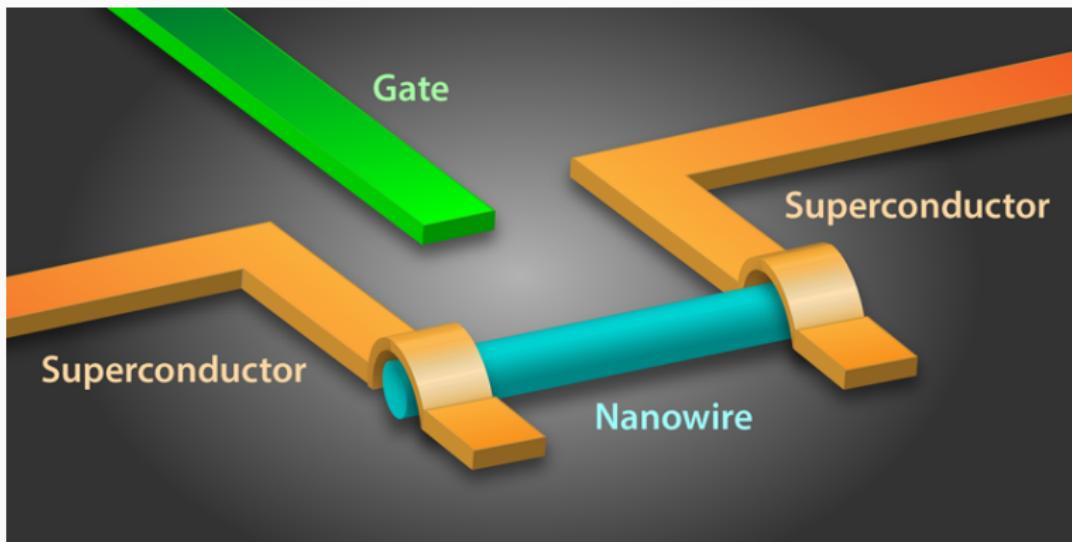
# Nanoscale superconductors

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a few examples ...

# 1. JOSEPHSON-TYPE NANOSTRUCTURE

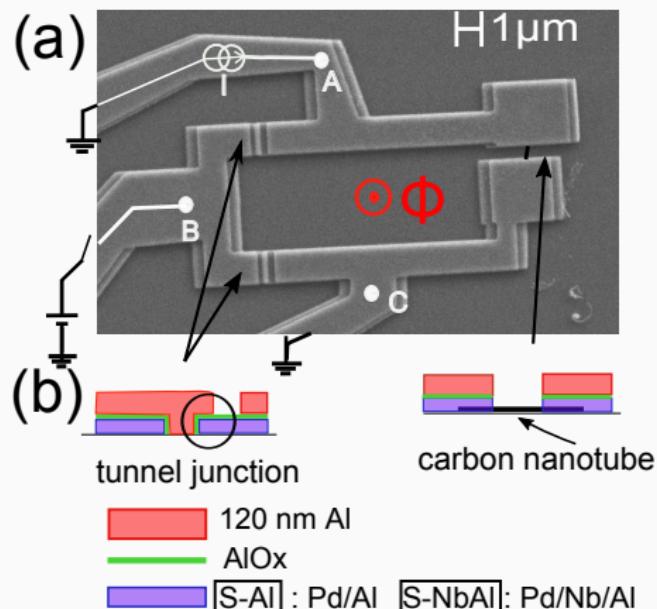
superconductor (S) - quantum dot (QD) - superconductor (S)



Cooper pairs can tunnel by imposing their phase-difference.

# 1. JOSEPHSON-TYPE NANOSTRUCTURE

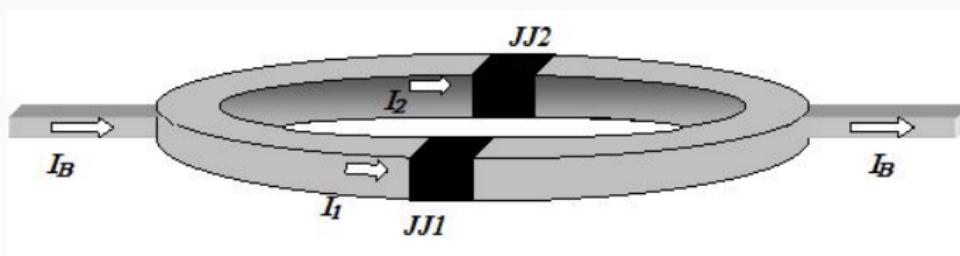
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R. Delagrange et al, Phys. Rev. B **93**, 195437 (2016).

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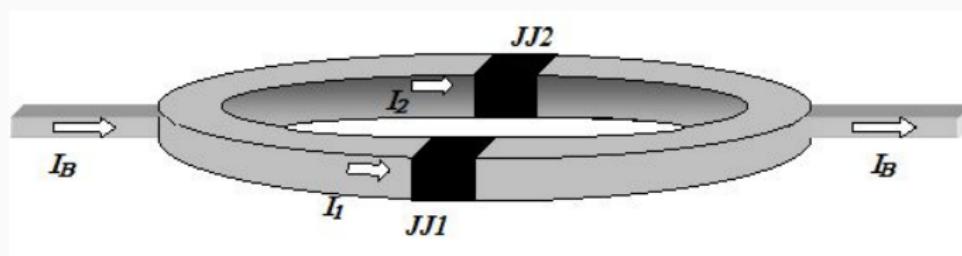
SQUID - superconducting quantum interferometer device



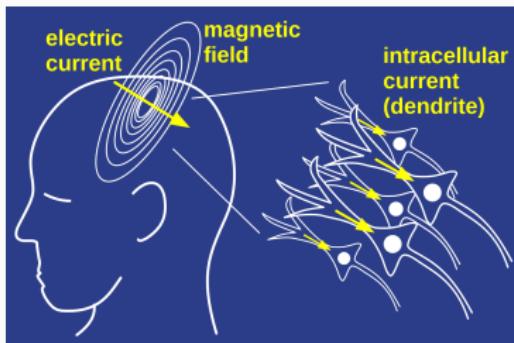
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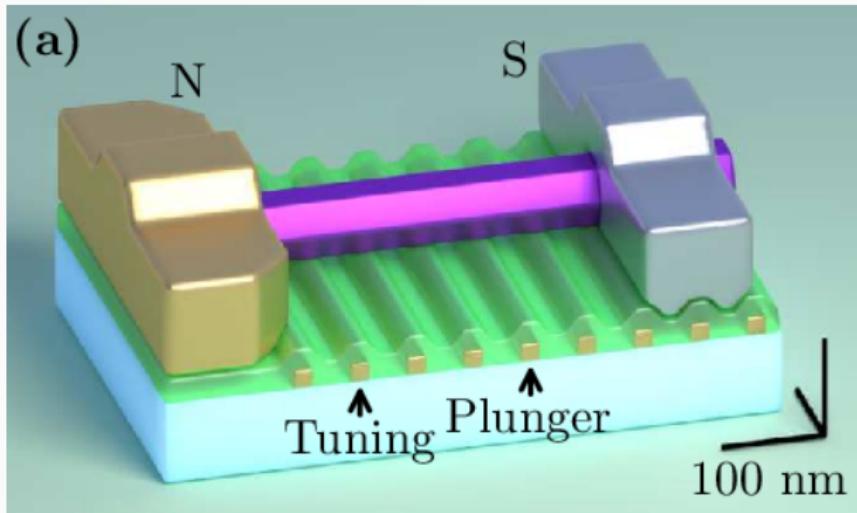
This device allows for extremely precise detection of magnetic fields



which is able to probe the neural currents in a human brain.

## 2. ANDREEV-TYPE NANOSTRUCTURE

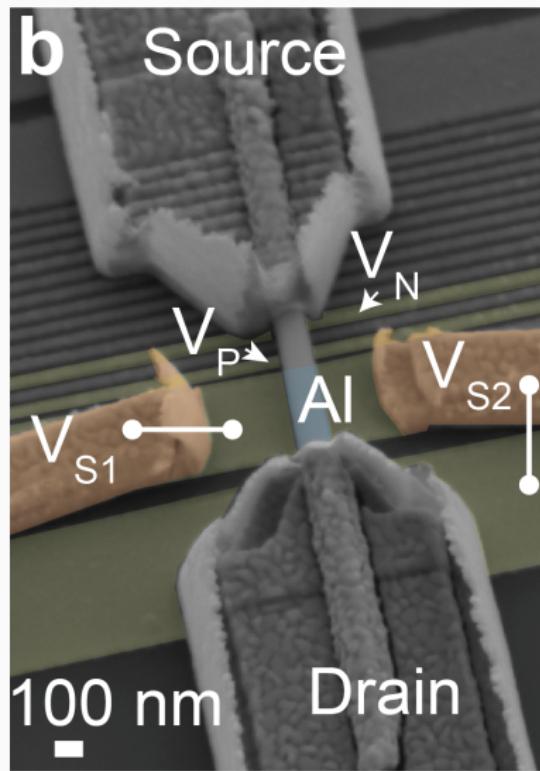
normal metal (N) - quantum dot (QD) - superconductor (S)



Charge tunneling via the electron-to-hole (Andreev) scattering.

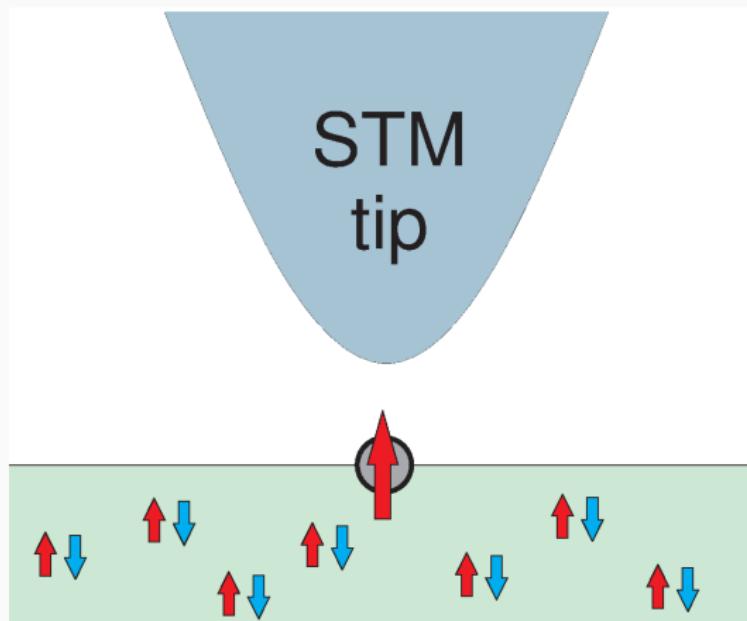
## 2. ANDREEV-TYPE NANOSTRUCTURE

normal metal (N) - quantum dot (QD) - superconductor (S)



### 3. SCANNING SPECTROSCOPY

Quantum impurity on a surface of superconductor + STM tip



Scanning spectroscopy can locally probe the electronic states of impurities deposited on surface of bulk superconductors.

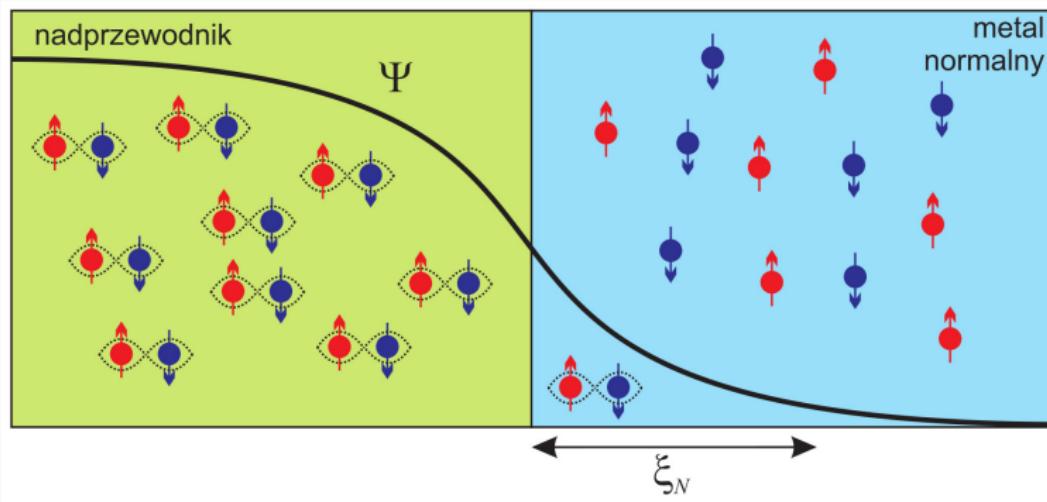
# **Cooper pairs in heterostructures**

# Cooper pairs in heterostructures

[ superconducting proximity effect ]

# LEAKAGE OF COOPER PAIRS

Any normal material contacted with a bulk superconductor  
absorbs the Cooper pairs



Cooper pairs leak into non-superconducting region up to spatial length  $\xi_N$ .

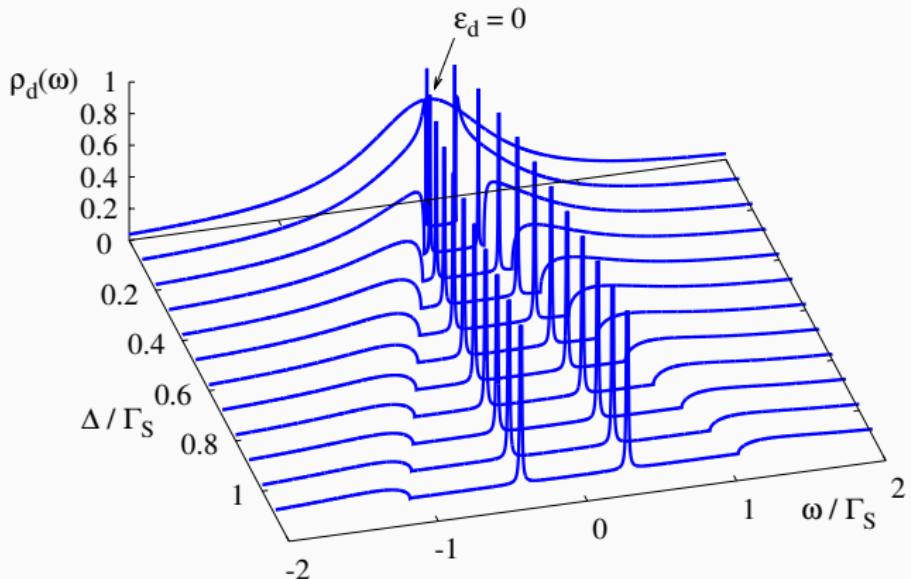
# Electron pairing in nanostructures

# Electron pairing in nanostructures

[ atoms, molecules, nanowires, etc ]

# QUANTUM DOT + SUPERCONDUTOR

Typical spectrum of the uncorrelated quantum dot (QD) coupled to the s-wave superconductor [exactly solvable case].



$\epsilon_d$  – energy level of QD,  $\Delta$  – gap of superconductor,  $\Gamma_S$  – hybridization

# SINGLE IMPURITY + BULK SUPERCONDUCTOR

## SINGLE IMPURITY + BULK SUPERCONDUCTOR

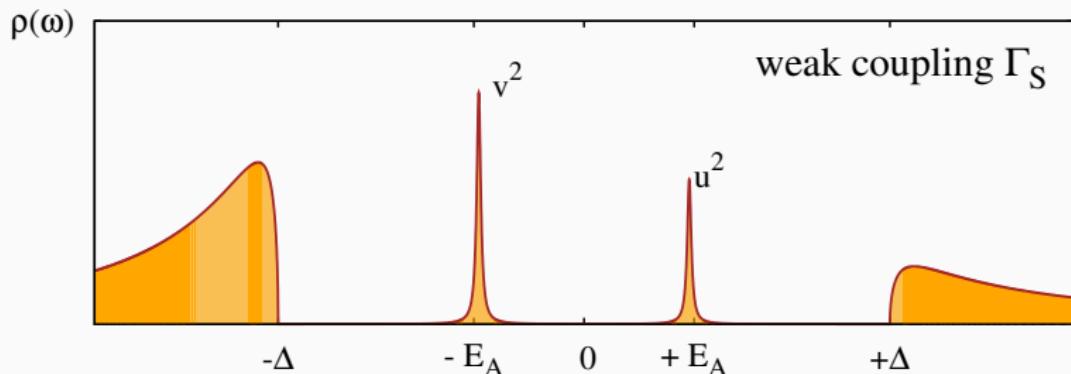
- Proximity-induced pairing is manifested by:  
⇒ in-gap bound states

# SINGLE IMPURITY + BULK SUPERCONDUCTOR

- Proximity-induced pairing is manifested by:
  - ⇒ in-gap bound states
- They originate from:
  - ⇒ leakage of Cooper pairs on QD (Andreev)
  - ⇒ exchange int. of QD with SC (Yu-Shiba-Rusinov)

# IN-GAP STATES

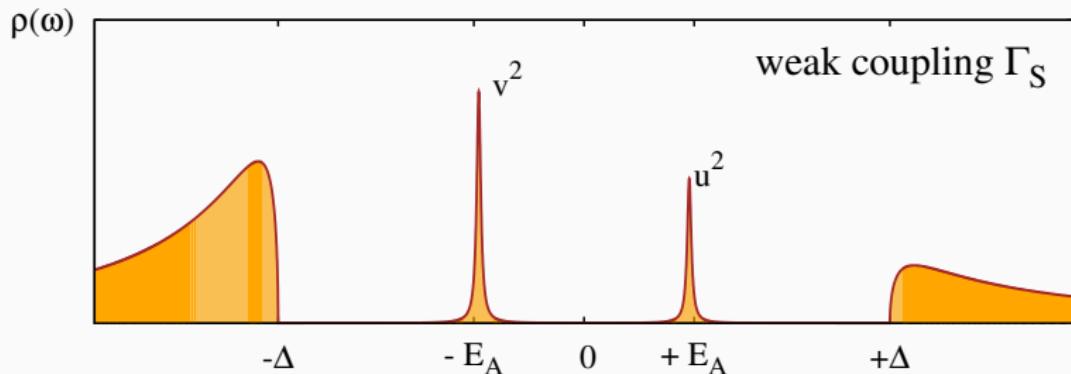
Spectrum of the quantum impurity coupled to superconductor



Bound states appear at  $\pm E_A$  in the subgap region  $E \in (-\Delta, \Delta)$

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Spectrum of the quantum impurity coupled to superconductor



Bound states appear at  $\pm E_A$  in the subgap region  $E \in (-\Delta, \Delta)$

Let us focus on these in-gap bound states ...

**Why are we interested in this issue ?**

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**selected headlines ...**

# SUPERCONDUCTING QUBITS

Applied Physics Letters

PERSPECTIVE

[scitation.org/journal/apl](https://scitation.org/journal/apl)

## A perspective on semiconductor-based superconducting qubits

Cite as: Appl. Phys. Lett. **117**, 240501 (2020); doi: [10.1063/5.0024124](https://doi.org/10.1063/5.0024124)

Submitted: 4 August 2020 · Accepted: 9 November 2020 ·

Published Online: 14 December 2020



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Ramón Aguado<sup>a)</sup> 

### AFFILIATIONS

Instituto de Ciencia de Materiales de Madrid (ICMM), Consejo Superior de Investigaciones Científicas (CSIC), Sor Juana Inés de la Cruz 3, 28049 Madrid, Spain

Quantum bits (qubits) can be constructed out of in-gap bound states, using either the Josephson junctions (transmons) or the semiconducting-superconducting hybrids (gatemons).

# SUPERCONDUCTING QUBITS

REPORT

QUANTUM DEVICES

## Coherent manipulation of an Andreev spin qubit

M. Hays<sup>1\*</sup>, V. Fatemi<sup>1\*</sup>, D. Bouman<sup>2,3</sup>, J. Cerrillo<sup>4,5</sup>, S. Diamond<sup>1</sup>, K. Serniak<sup>1†</sup>, T. Connolly<sup>1</sup>, P. Krogstrup<sup>6</sup>, J. Nygård<sup>6</sup>, A. Levy Yeyati<sup>5,7</sup>, A. Geresdi<sup>2,3,8</sup>, M. H. Devoret<sup>1\*</sup>

Two promising architectures for solid-state quantum information processing are based on electron spins electrostatically confined in semiconductor quantum dots and the collective electrodynamic modes of superconducting circuits. Superconducting electrodynamic qubits involve macroscopic numbers of electrons and offer the advantage of larger coupling, whereas semiconductor spin qubits involve individual electrons trapped in microscopic volumes but are more difficult to link. We combined beneficial aspects of both platforms in the Andreev spin qubit: the spin degree of freedom of an electronic quasiparticle trapped in the supercurrent-carrying Andreev levels of a Josephson semiconductor nanowire. We performed coherent spin manipulation by combining single-shot circuit–quantum-electrodynamics readout and spin-flipping Raman transitions and found a spin-flip time  $T_S = 17$  microseconds and a spin coherence time  $T_{2E} = 52$  nanoseconds. These results herald a regime of supercurrent-mediated coherent spin-photon coupling at the single-quantum level.

Hays *et al.*, *Science* **373**, 430–433 (2021) 23 July 2021

Recent evidence for experimental realization

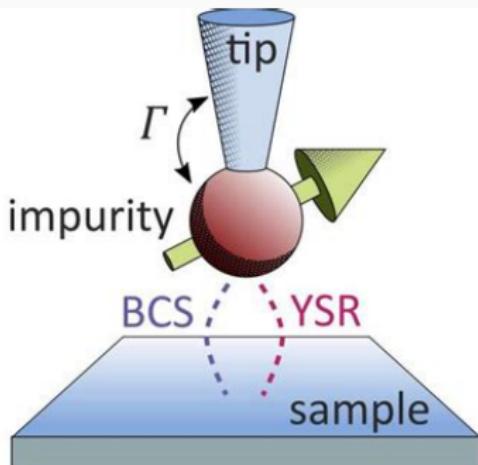
# **Conventional bound states**

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[ two scenarios ]

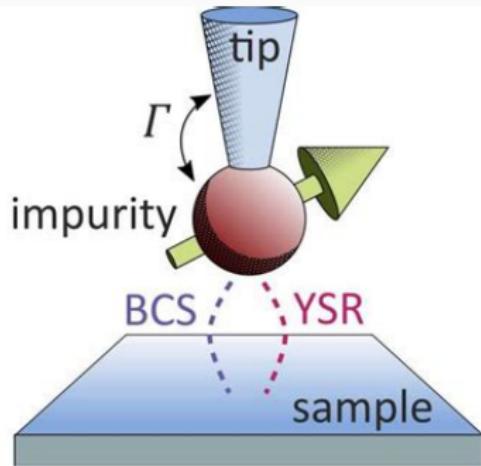
# 1. CLASSICAL IMPURITY

Classical magnetic impurity on surface of bulk superconductor

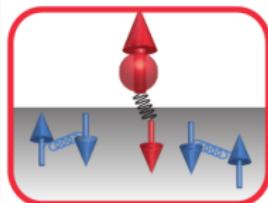


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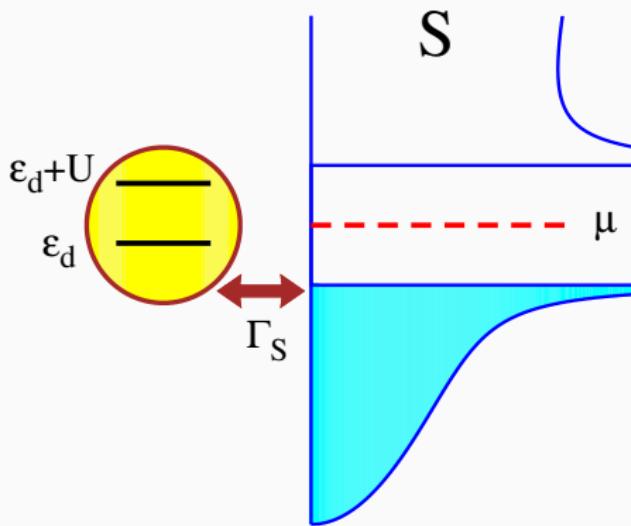
Impurity couples with unpaired electron of superconductor, forming a pair.



Yu - Shiba - Rusinov states

## 2. QUANTUM IMPURITY

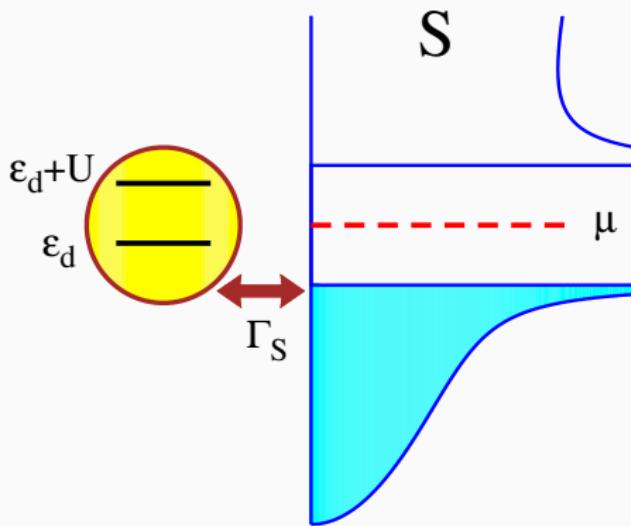
Correlated impurity coupled to the s-wave superconductor



$\epsilon_d$  – energy level,  $U$  – Coulomb potential,  $\Gamma_S$  – hybridization

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$\epsilon_d$  – energy level,  $U$  – Coulomb potential,  $\Gamma_S$  – hybridization

Coulomb repulsion competes with the Cooper pair leakage !

# **Pairing vs Coulomb repulsion**

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[ omnipresent antagonism ]

# SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

Hamiltonian of the quantum dot proximitized to superconductor

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \hat{d}_{\sigma}^{\dagger} \hat{d}_{\sigma} + U_d \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - (\Gamma_s \hat{d}_{\uparrow}^{\dagger} \hat{d}_{\downarrow}^{\dagger} + \text{h.c.})$$

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Eigen-states of this problem are represented by:

$$\begin{array}{lll} |\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle & \Leftarrow & \text{doublet states (spin } \frac{1}{2} \text{)} \\ u|0\rangle - v|\uparrow\downarrow\rangle \\ v|0\rangle + u|\uparrow\downarrow\rangle \end{array} \quad \left. \begin{array}{ll} \Leftarrow & \text{singlet states (spin 0)} \end{array} \right\}$$

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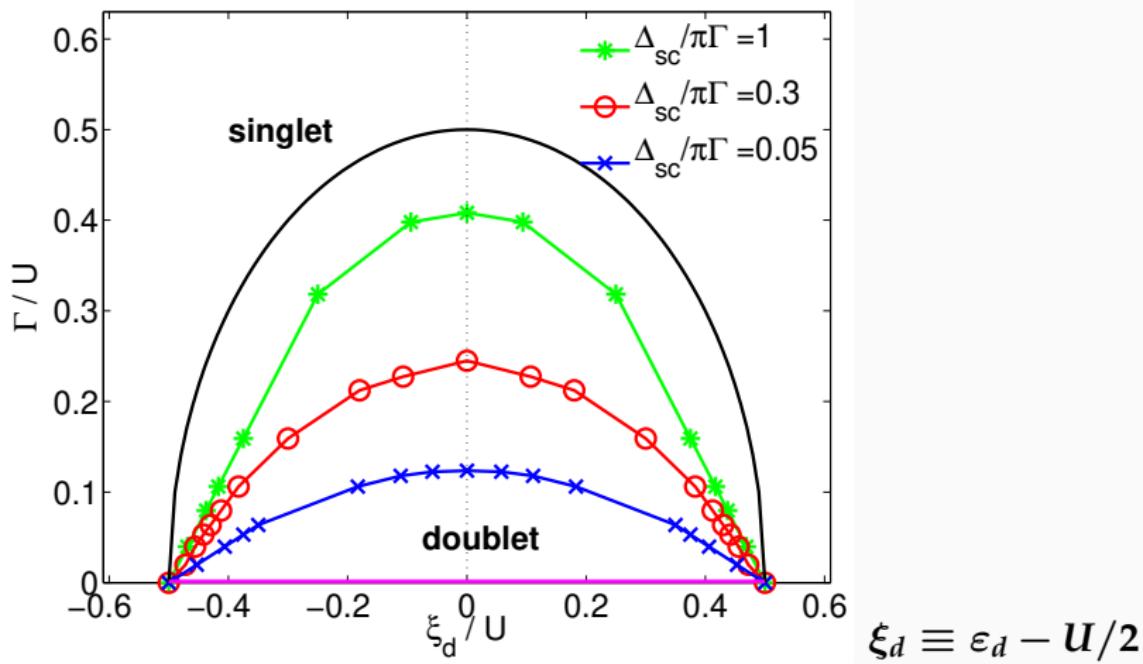
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Upon varying the ratio  $\epsilon_d/U_d$  or  $\Gamma_s/U_d$  the doublet-singlet transition can be induced between these ground states.

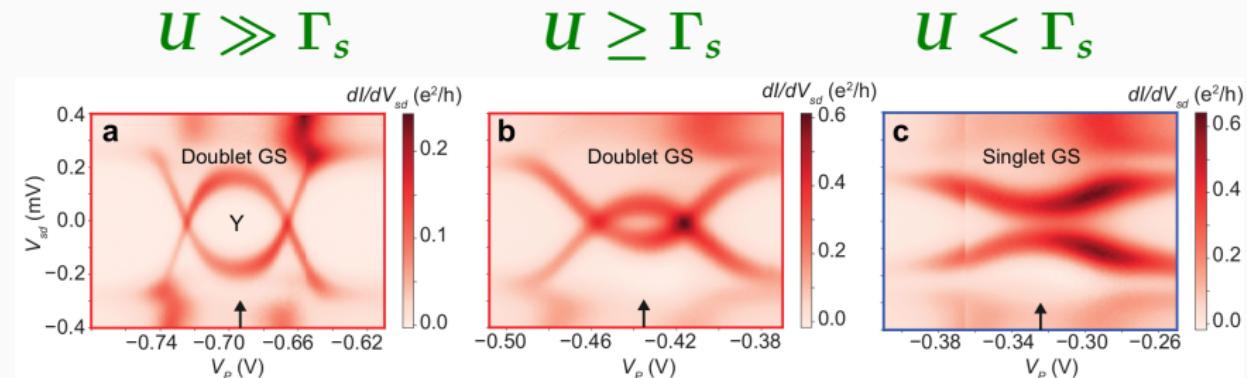
# QUANTUM PHASE TRANSITION

The singlet-doublet (quantum phase) transition): NRG results



# SINGLET VS DOUBLET: EXPERIMENT

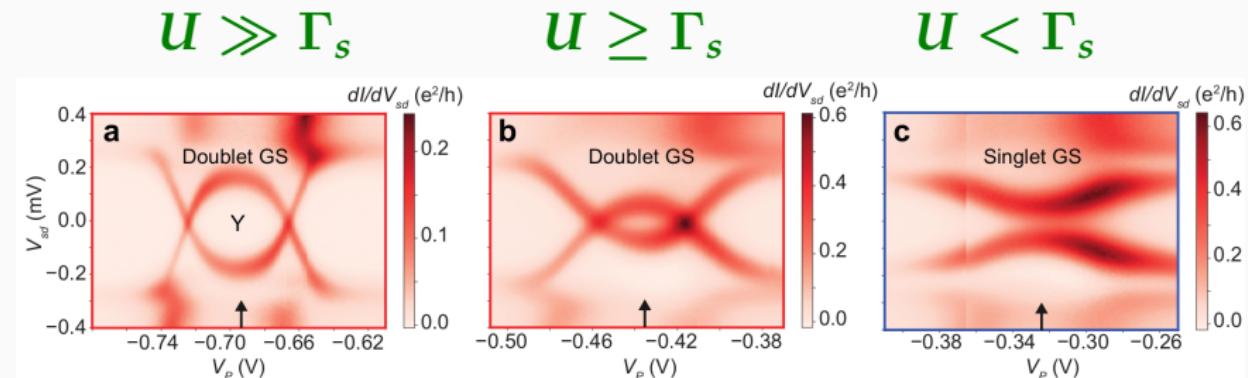
Differential conductance vs source-drain bias  $V_{sd}$  (vertical axis)  
and gate potential  $V_p$  (horizontal axis) measured for various  $\Gamma_s/U$



J. Estrada Saldaña et al, Commun. Phys. 3, 125 (2020).

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Crossings of in-gap states correspond to the singlet-doublet QPT.

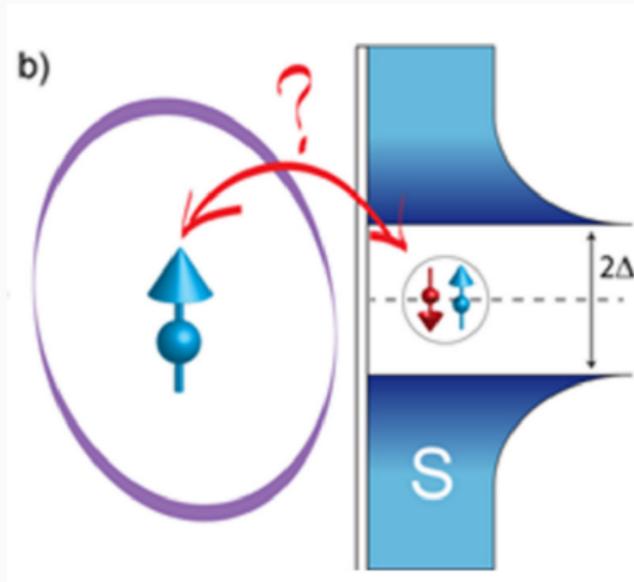
## **Localized vs itinerant electrons**

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[ Kondo effect ]

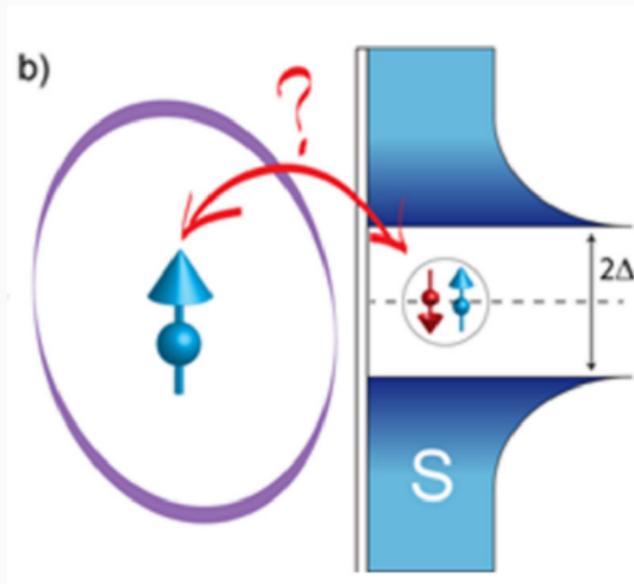
# RELEVANT QUESTION

Strongly correlated quantum dot coupled do superconductor



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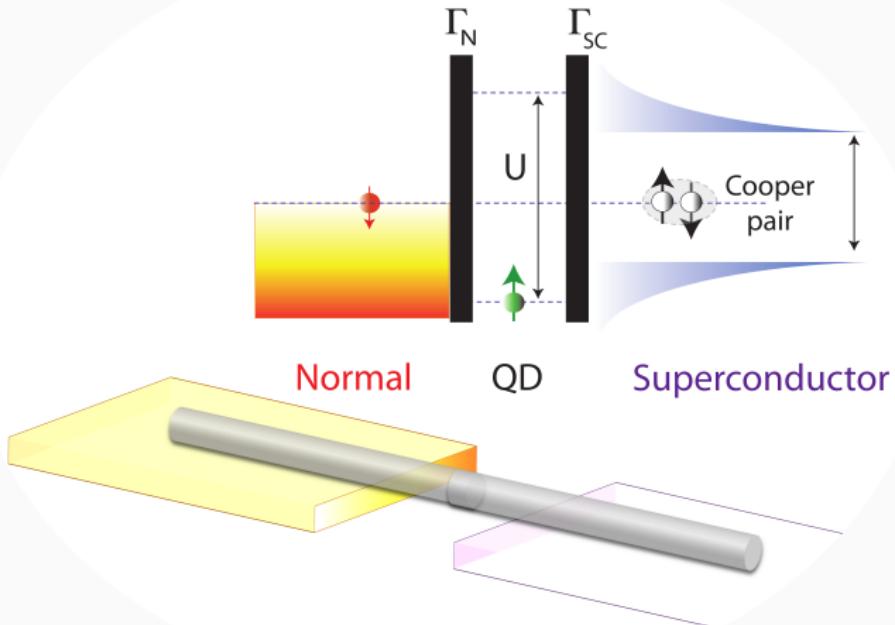
Strongly correlated quantum dot coupled do superconductor



To screen or not to screen ?

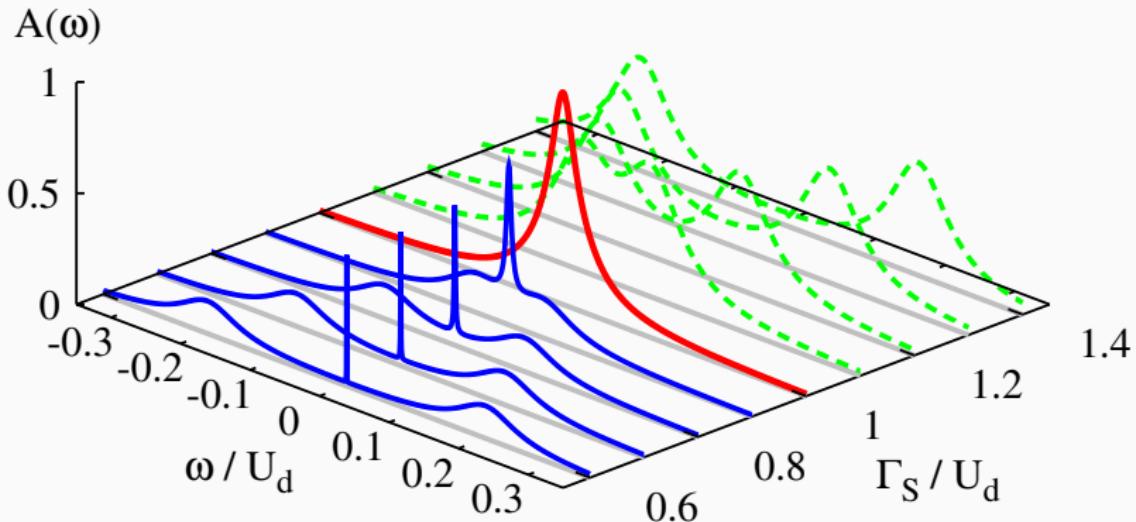
# ANDREEV-TYPE HETEROSTRUCTURE

Let us consider the correlated quantum dot (QD) placed between the normal (N) and superconducting (S) electrodes



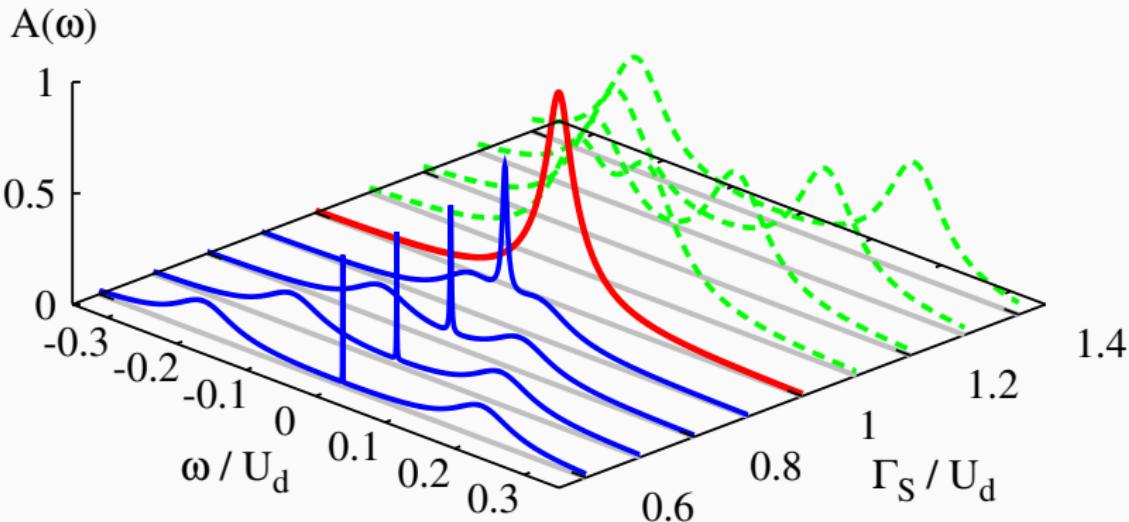
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Schrieffer-Wolff results obtained for half-filled QD ( $\varepsilon_d = -U/2$ )



# KONDO STATE VS PARITY CROSSING

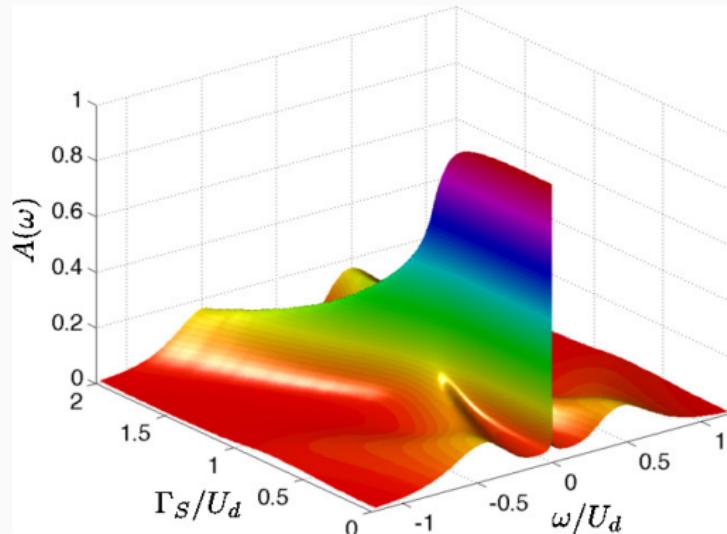
Schrieffer-Wolff results obtained for half-filled QD ( $\varepsilon_d = -U/2$ )



Kondo peak develops on the spinful (doublet) side and it is enhanced upon approaching the quantum phase transition.

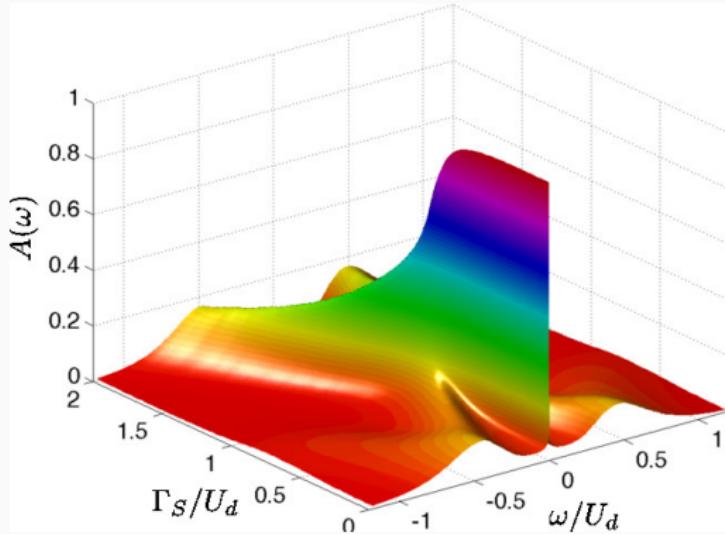
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Results obtained by NRG (Budapest code) for the half-filled QD.



# KONDO STATE VS PARITY CROSSING

Results obtained by NRG (Budapest code) for the half-filled QD.



The zero-energy (Kondo) peak exist only in the doublet phase  
and it disappears upon traversing to the BCS-type phase.

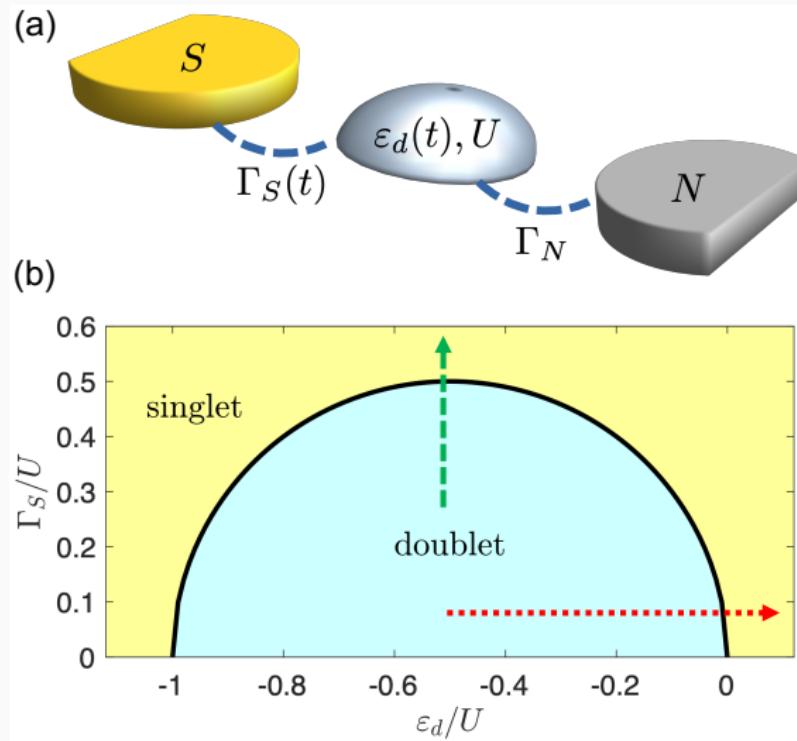
T. Domański, I. Weymann, M. Barańska & G. Górski, Sci. Rep. 6, 23336 (2016).

# Dynamical singlet-doublet transition

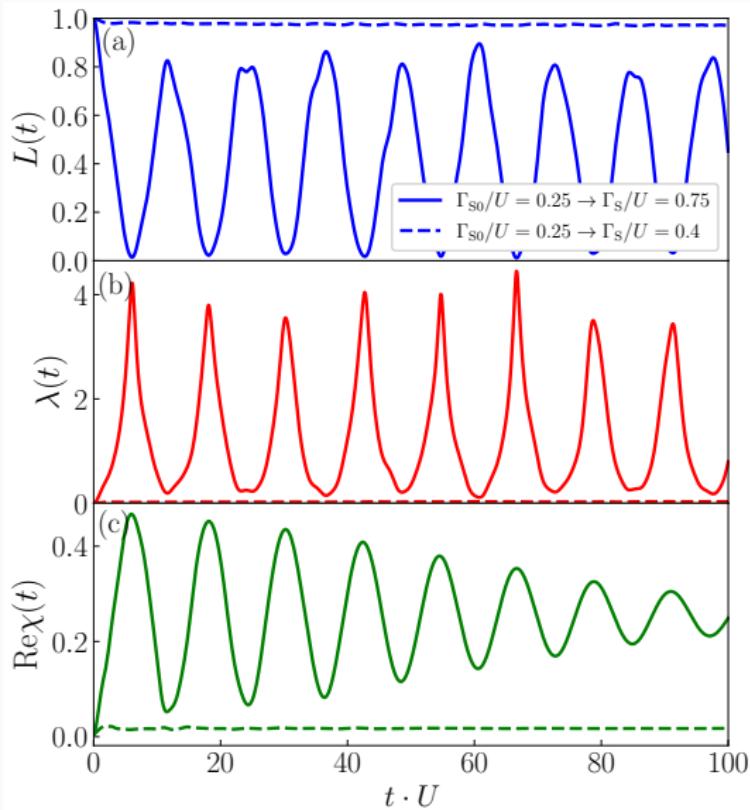
# Dynamical singlet-doublet transition

[ transition in time-domain ]

# QUENCH ACROSS QPT BOUNDARY



# $t$ -NRG RESULTS



**Loschmidt echo**

$$L(t) \equiv | \langle \Psi(t) | \Psi(0) \rangle |^2$$

**Return rate**

$$\lambda(t) = -(1/N) \log L(t)$$

**On-dot pairing**

$$\text{Re} \langle \hat{d}_{\downarrow}(t) \hat{d}_{\uparrow}(t) \rangle$$

## EMPIRICAL DETECTION

Dynamical singlet-doublet transition(s) can be detected:

- by measuring the time-resolved Andreev current

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Dynamical singlet-doublet transition(s) can be detected:

- by measuring the time-resolved Andreev current
- by detecting the time-dependent magnetic moment

# **Topological nano-superconductors**

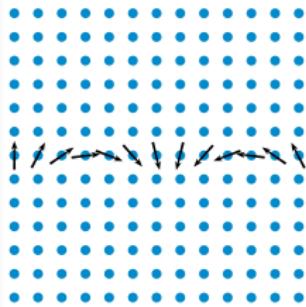
# **Topological nano-superconductors**

**[ 1- and 2-dimensional platforms ]**

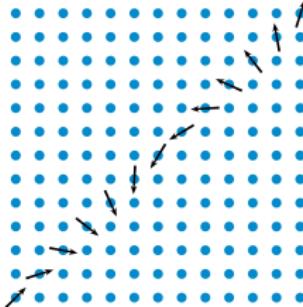
# MAGNETIC OBJECTS IN SUPERCONDUCTORS

## Magnetic chains and/or islands embedded in superconductors

Chain along (10)



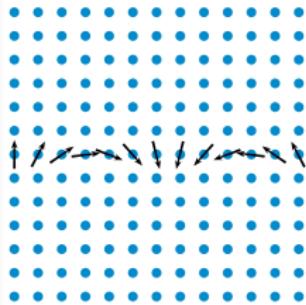
Chain along (11)



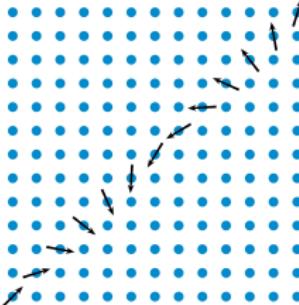
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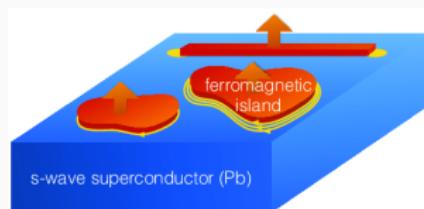
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Chain along (11)



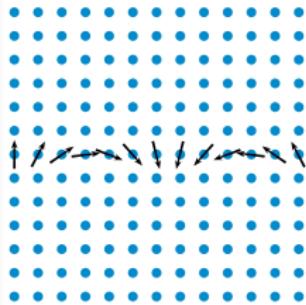
or magnetic islands



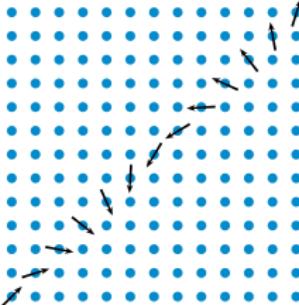
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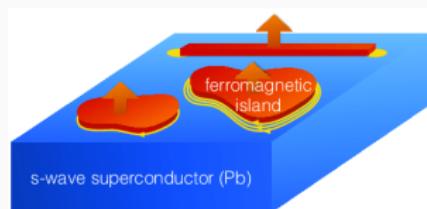
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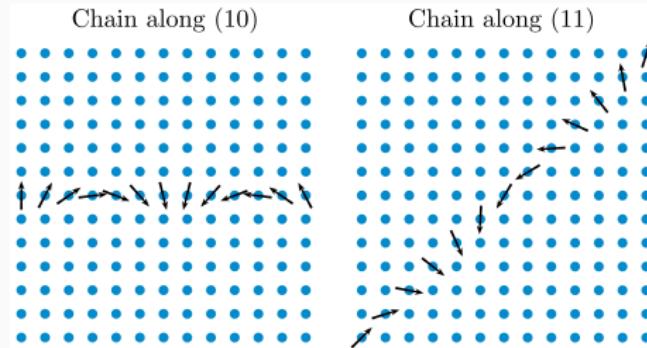
**or magnetic islands**



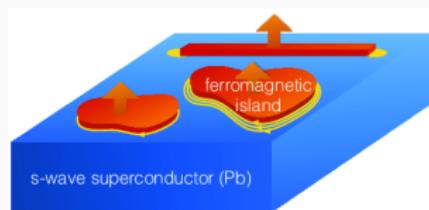
arrange their in-gap bound states into Shiba-bands.

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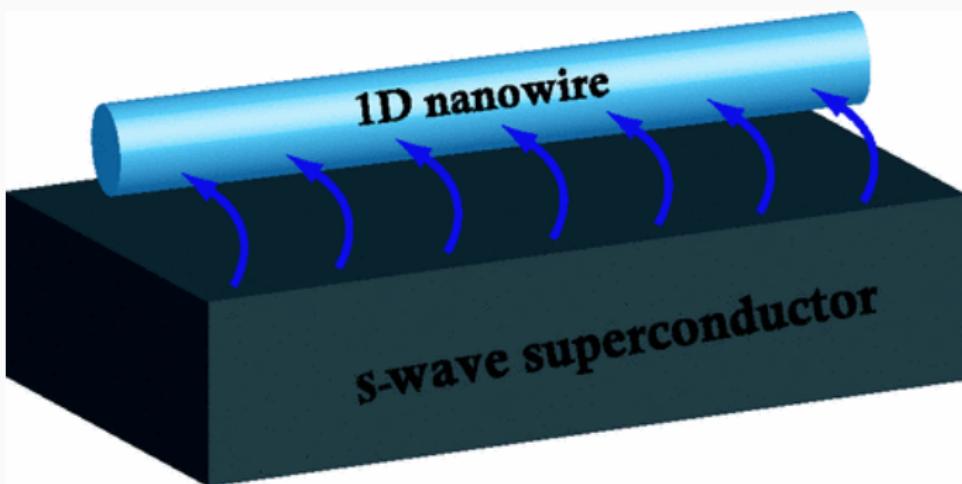
arrange their in-gap bound states into Shiba-bands.

In particular, the proper magnetic textures in chains and islands can guarantee their topologically non-trivial character, hosting the exotic Majorana-type boundary modes !

# **1. Rashba nanowires**

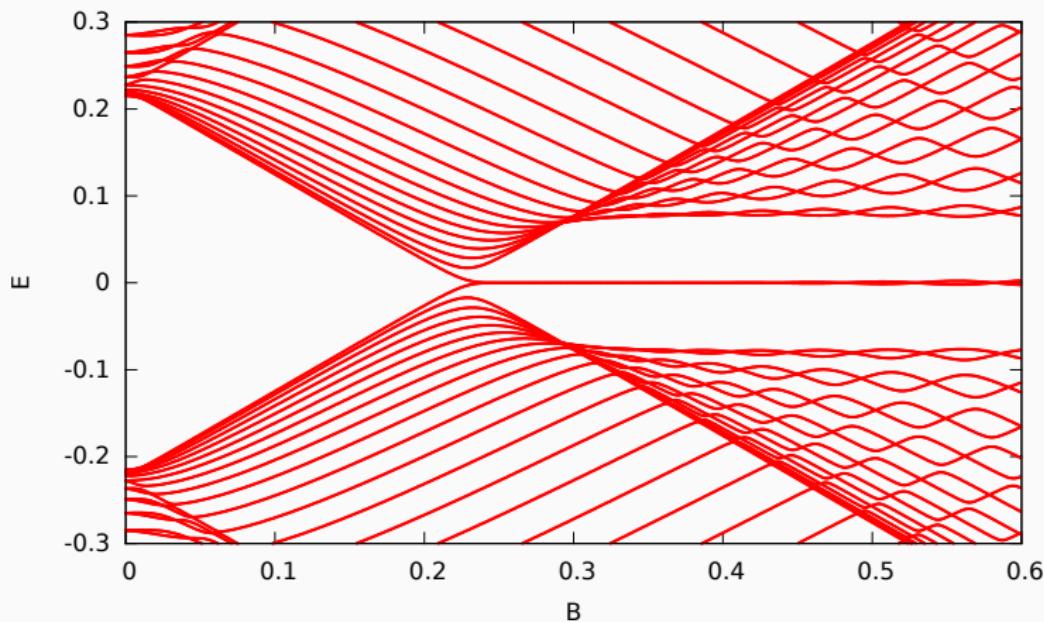
# SHIBA / MAJORANA: A STORY OF MUTATION

Leakage of Cooper pairs + spin-orbit coupling + Zeeman field induce effectively the *p*-wave electron pairs (Kitaev scenario).



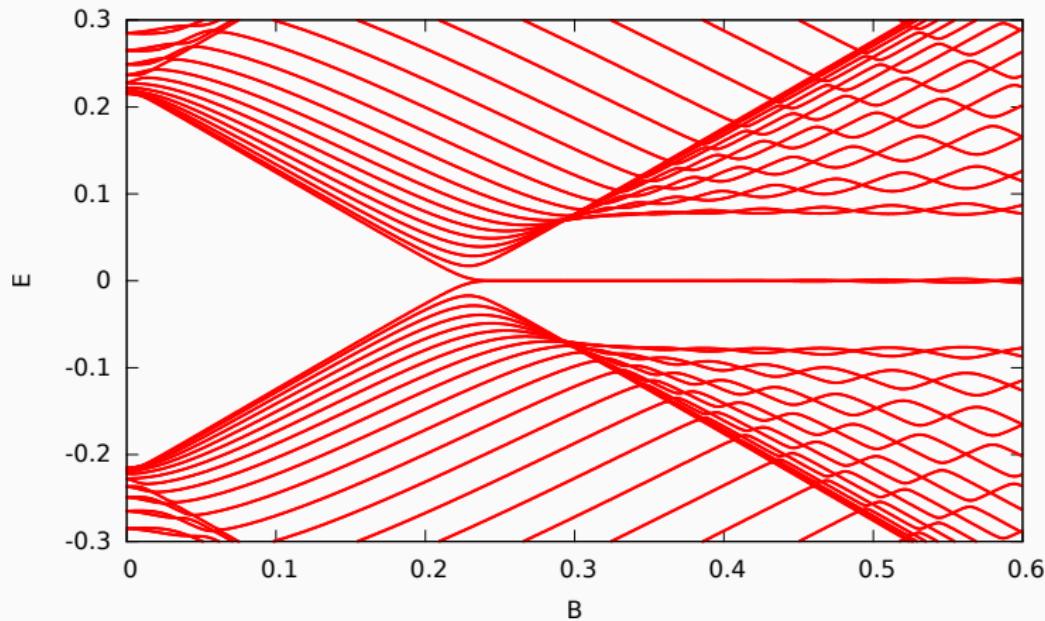
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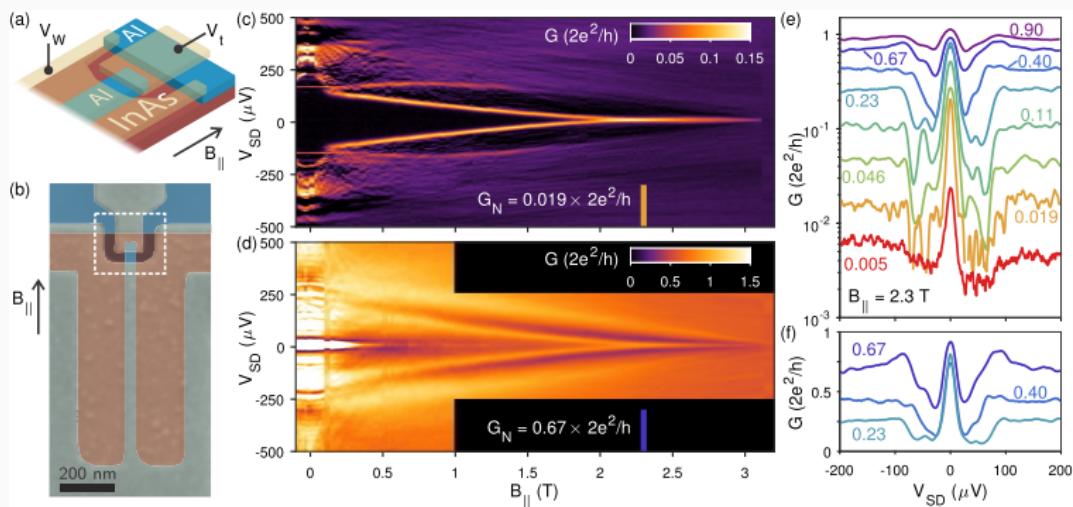


closing/reopening of a gap  $\Leftrightarrow$  band-inversion of topological insulators

M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

# EXAMPLE OF EMPIRICAL REALIZATION

## Lithographically fabricated Al nanowire contacted to InAs

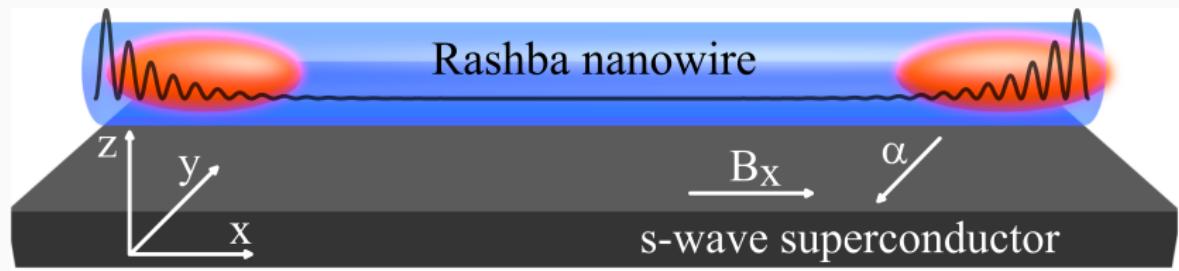


F. Nichele, ..., and Ch. Marcus, Phys. Rev. Lett. **119**, 136803 (2017).

/ Niels Bohr Institute, Copenhagen, Denmark /

# SHIBA / MAJORANA: A STORY OF MUTATION

Pairing of identical spin electrons is driven by the spin-orbit (Rashba) interaction in presence of magnetic field, using the semiconducting nanowires proximitized to conventional (*s*-wave) superconductor.



## PROPERTIES OF MAJORANA QPS

- **particle = antiparticle**

$$\hat{\gamma}_{i,n}^\dagger = \hat{\gamma}_{i,n}$$

- ⇒ **neutral in charge**
- ⇒ **of zero energy**

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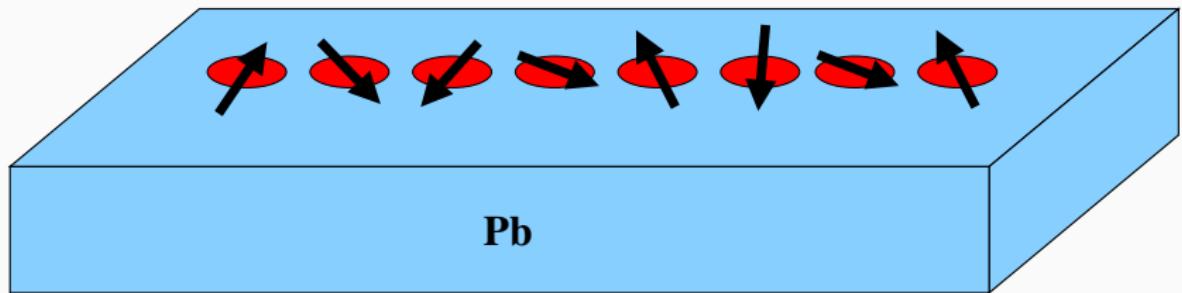
- **topologically protected**

- ⇒ immune to dephasing/decoherence

## **2. Selforganised magnetic chains**

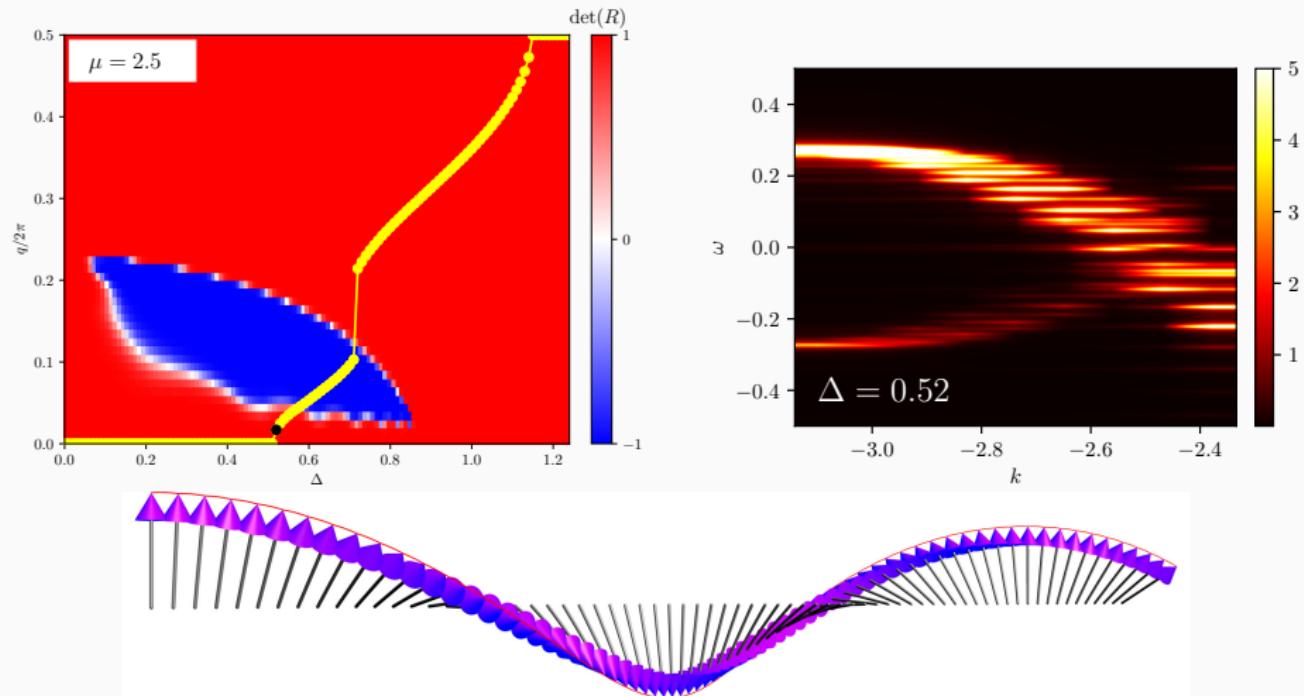
# MAGNETIC CHAINS ON SUPERCONDUCTORS

Magnetic atoms (like Fe) on a surface of s-wave superconductor (for example Pb) arrange themselves into such spiral order, where topological superconducting phase is selfsustained



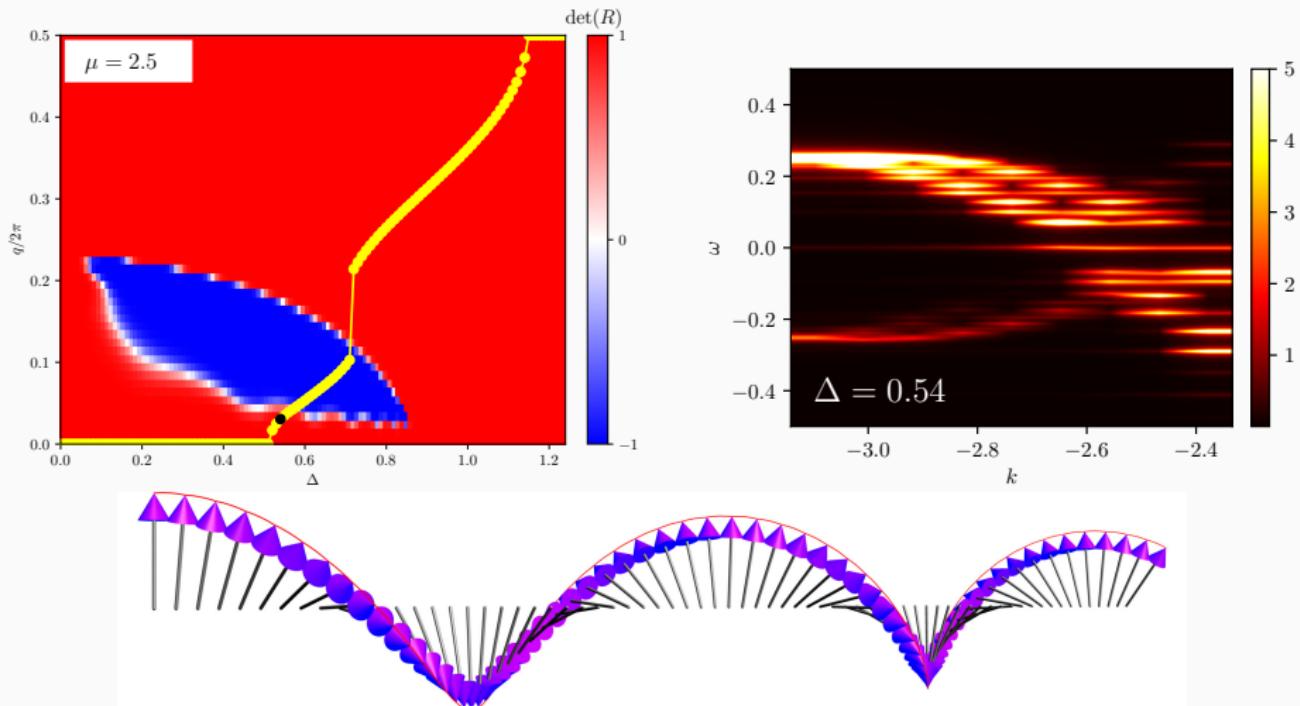
# HELICAL SELFORGANISATION (TOPOFILIA)

A. Gorczyca-Goraj, T. Domański & M.M. Maśka, Phys. Rev. B 99, 235430 (2019).



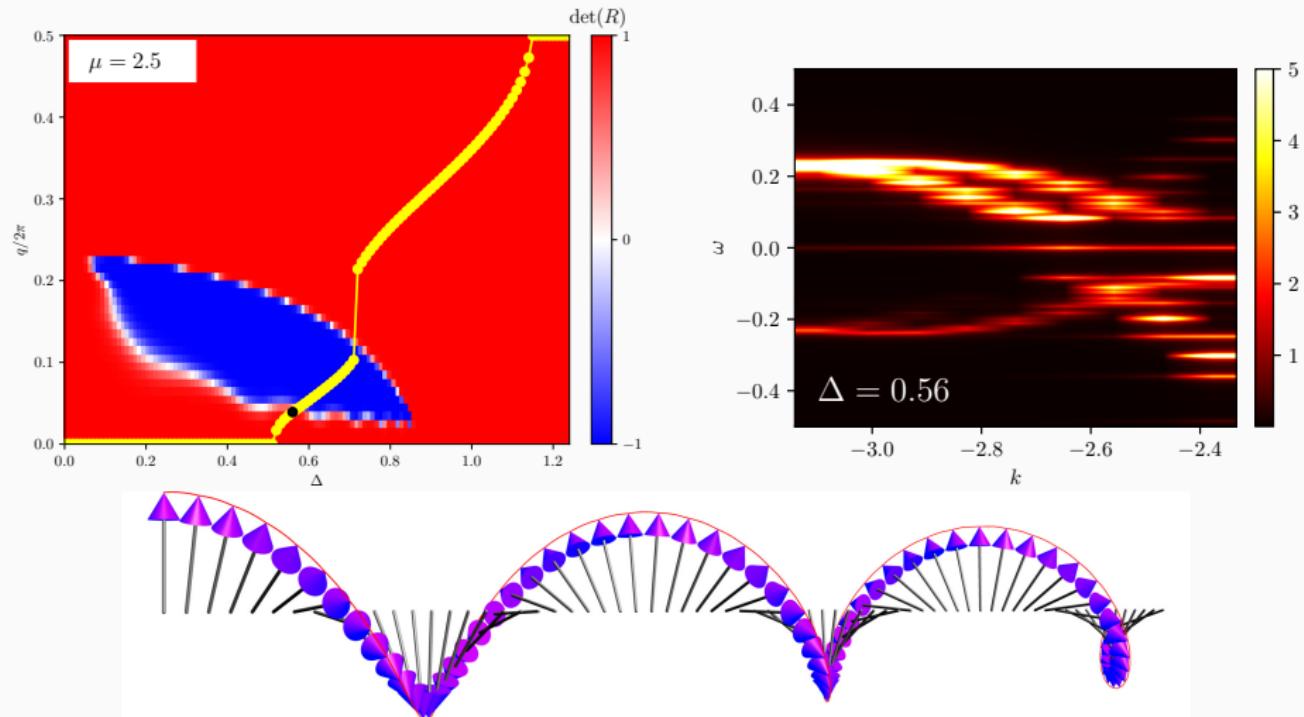
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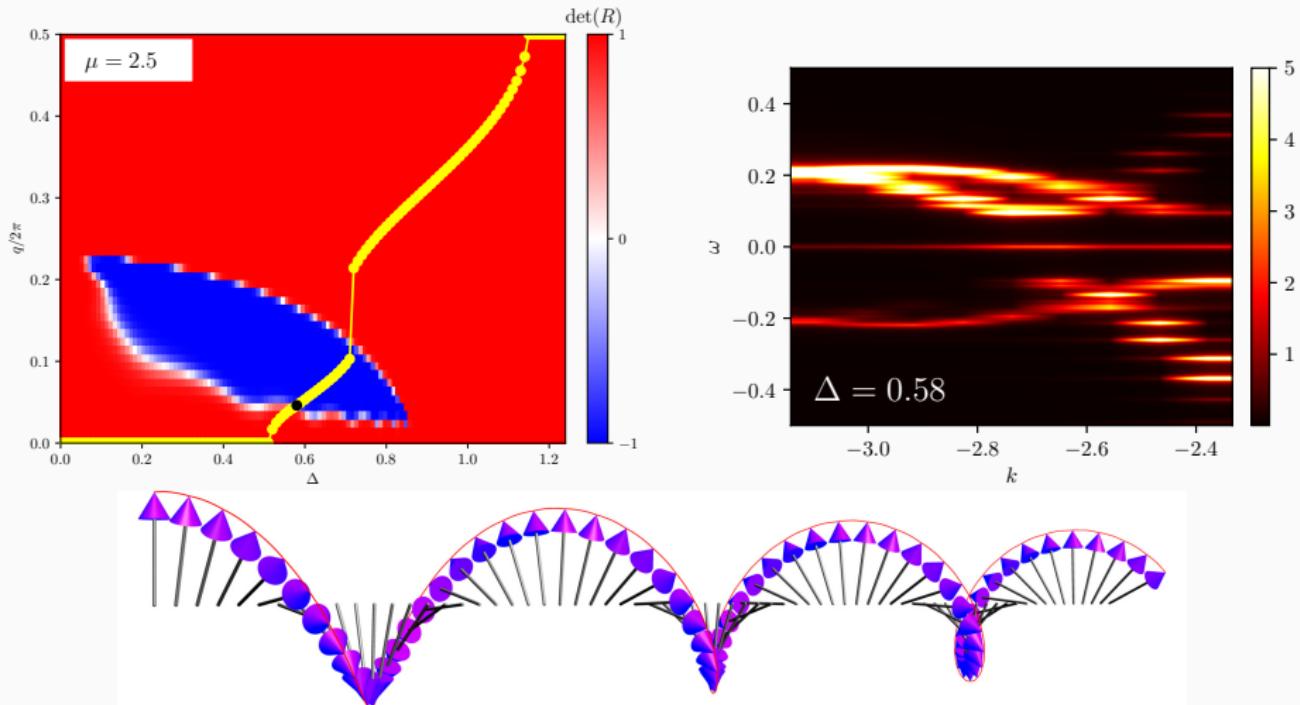
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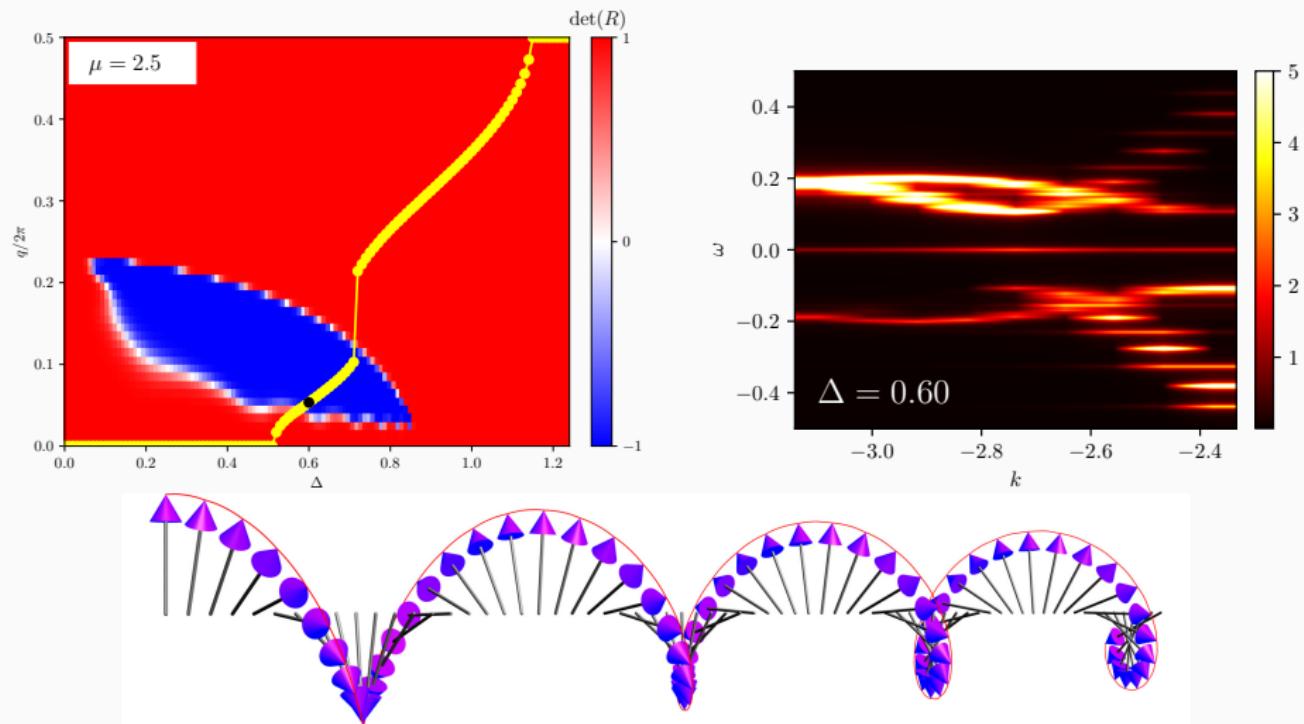
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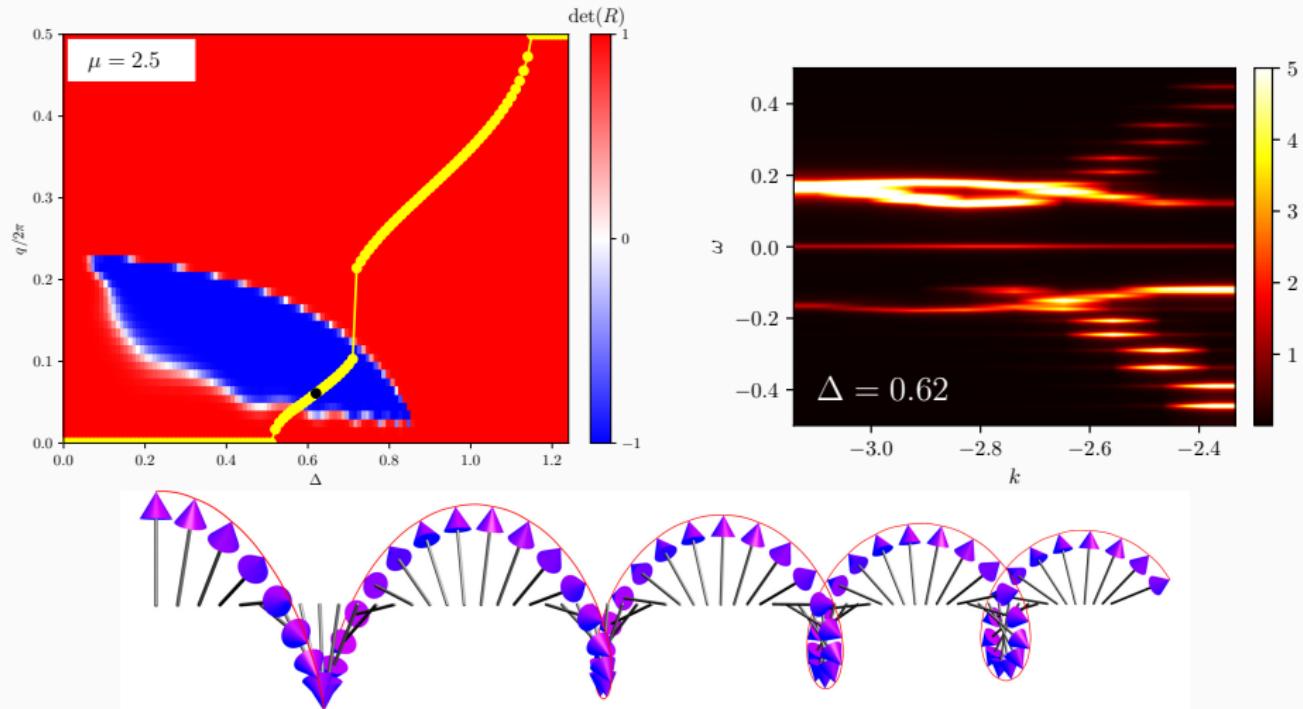
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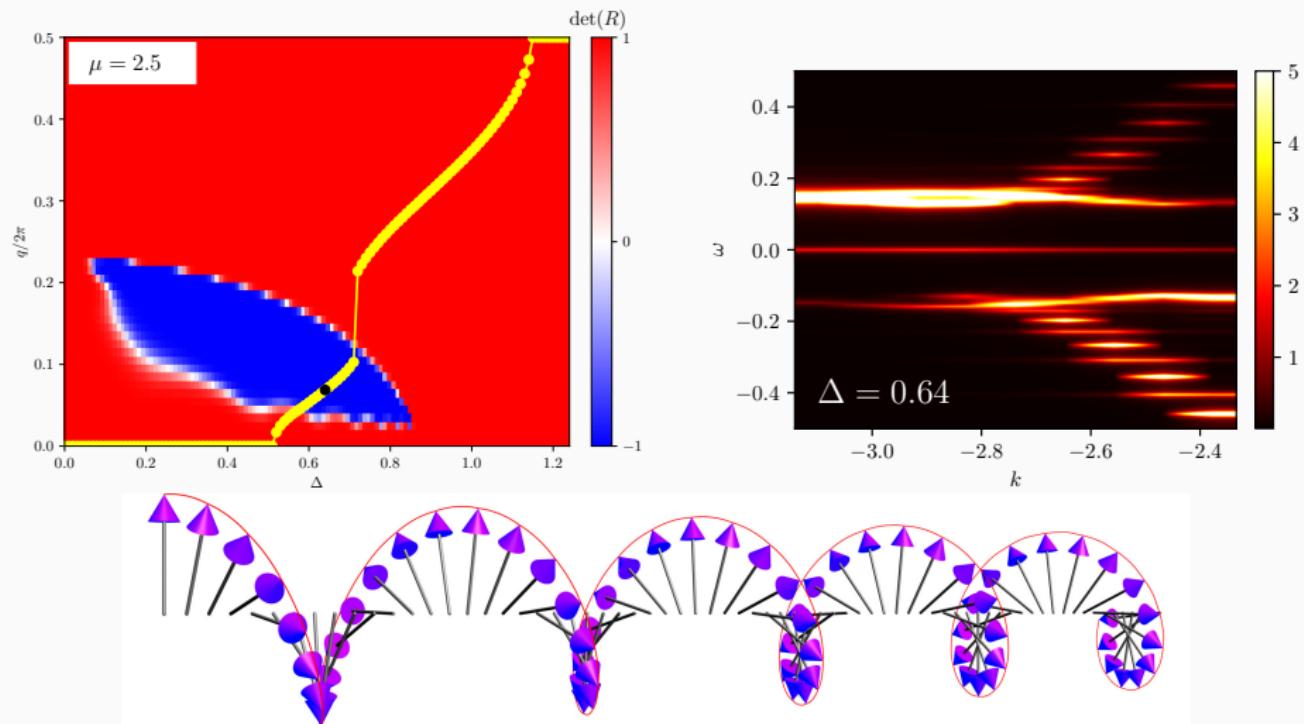
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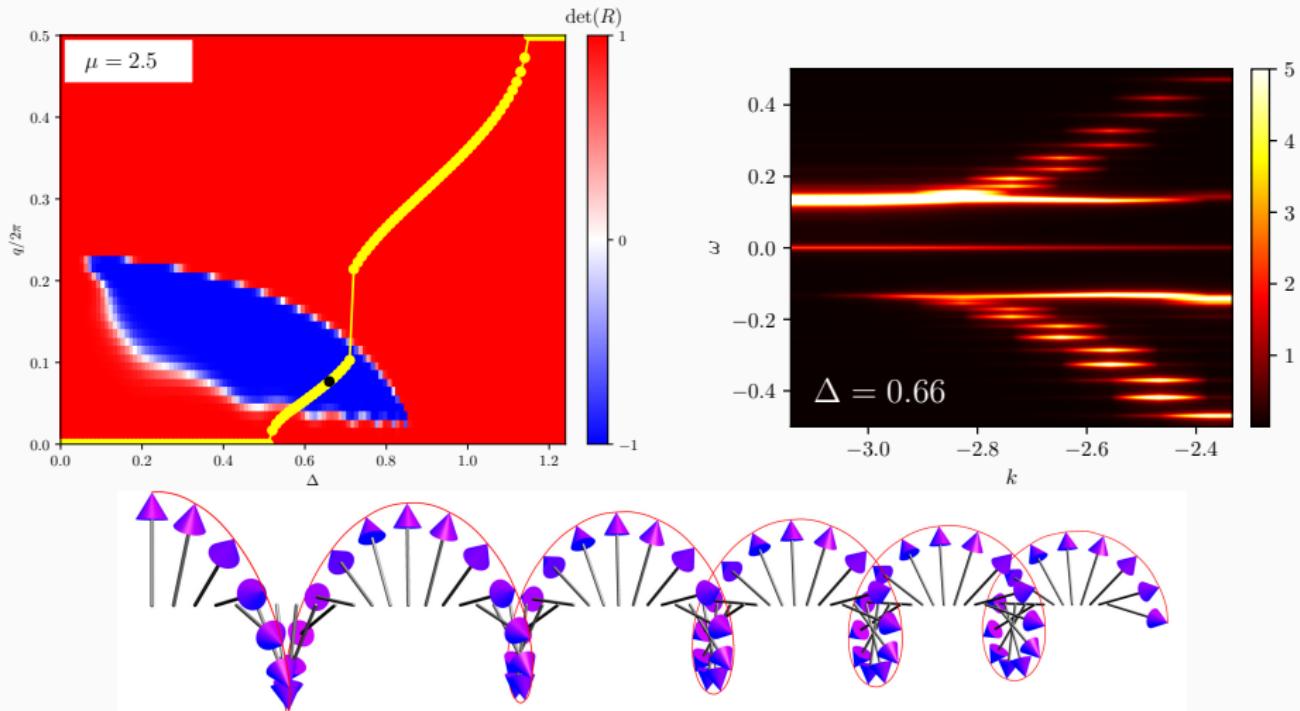
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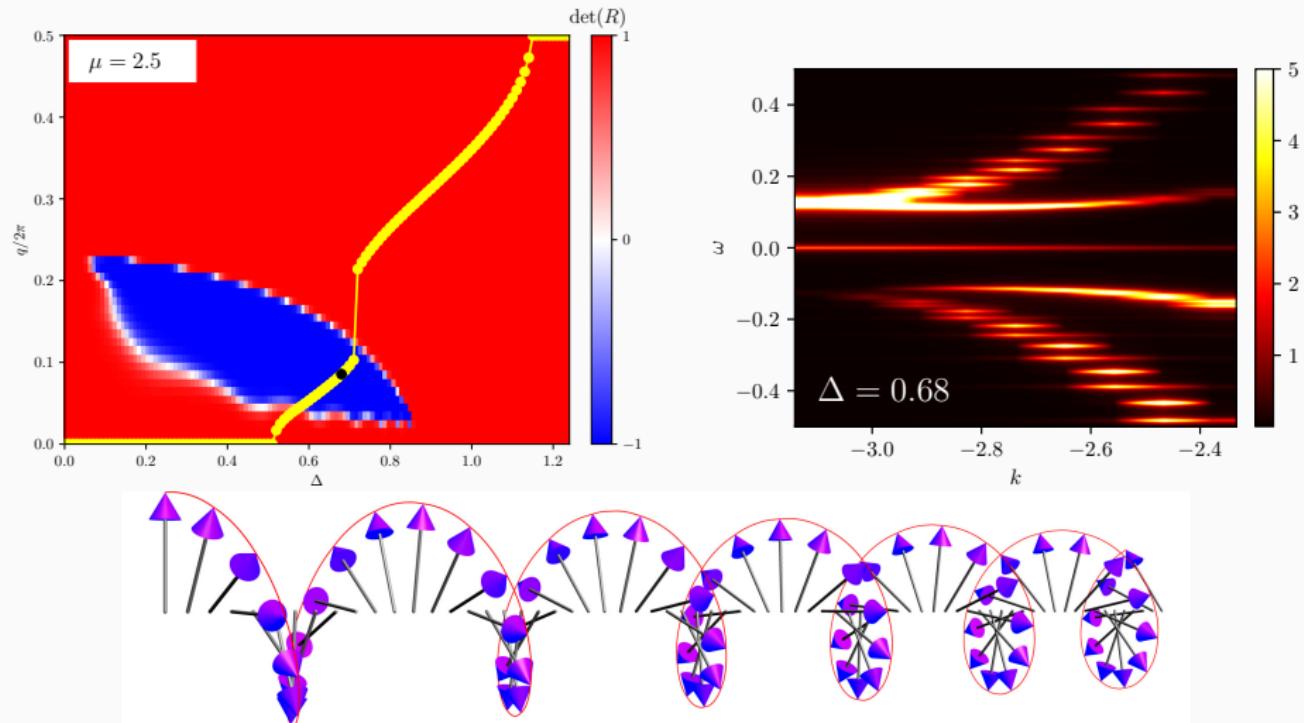
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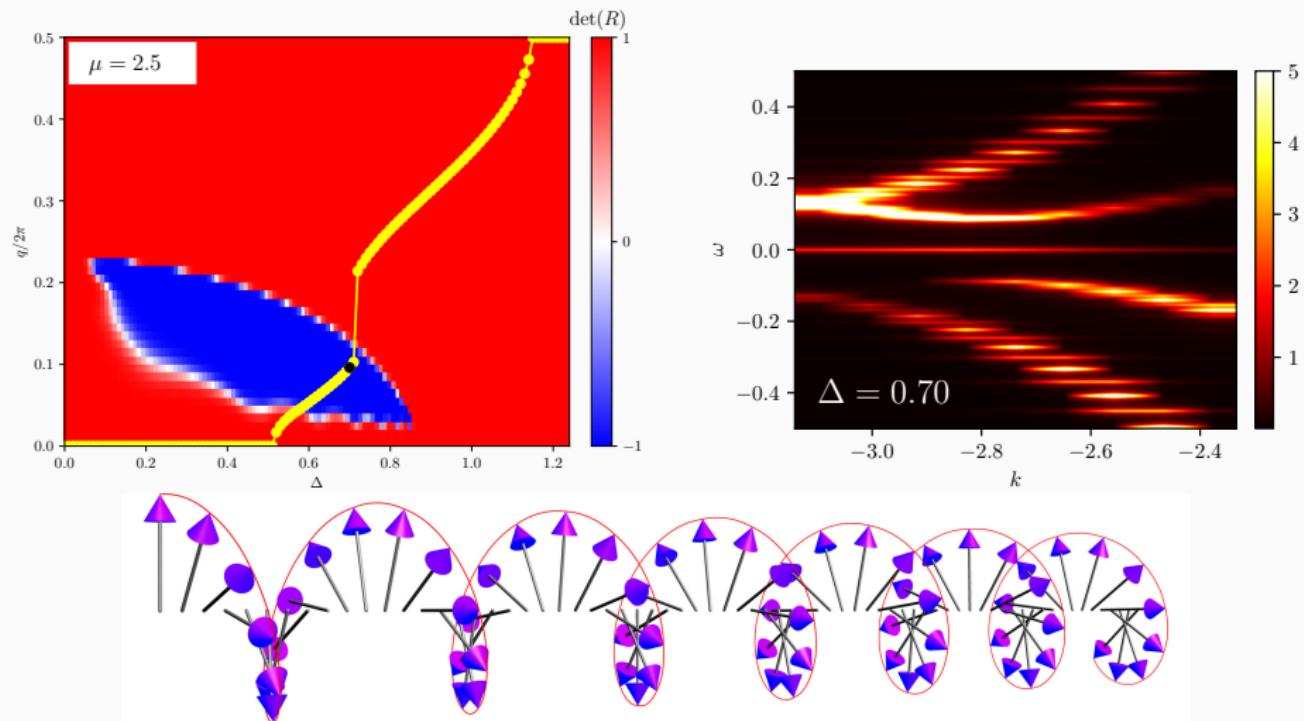
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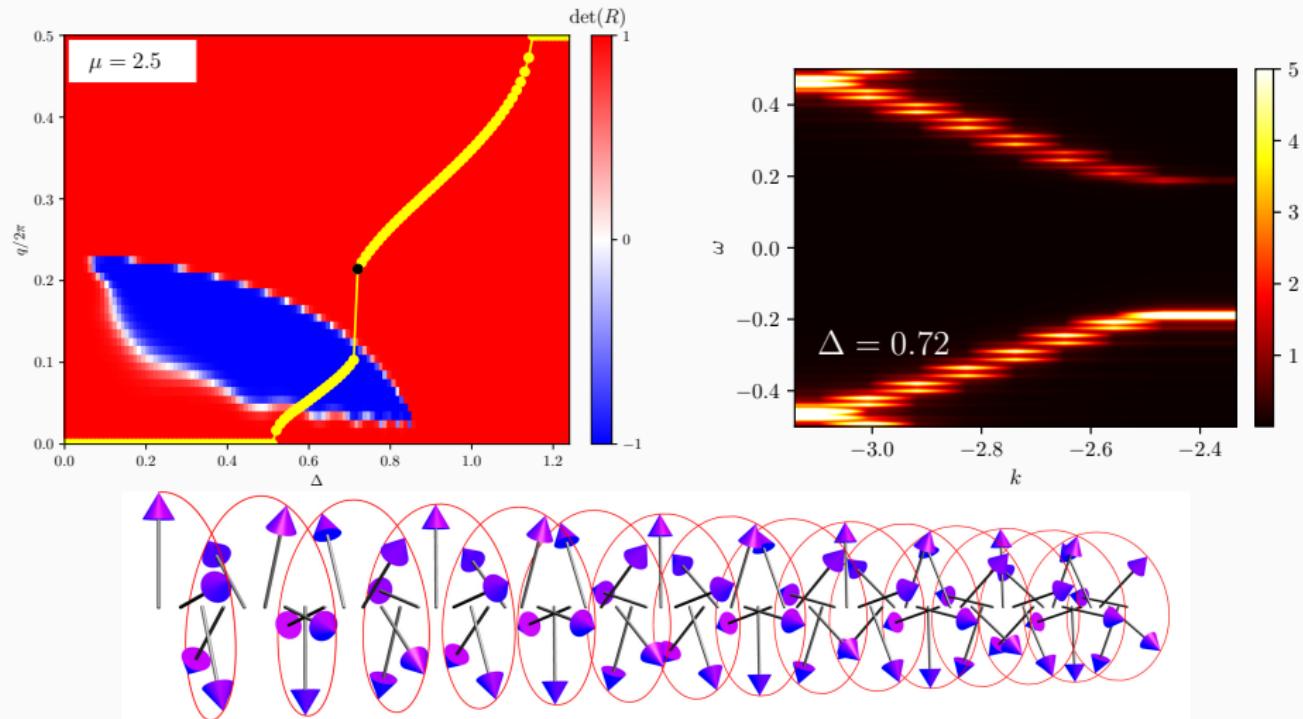
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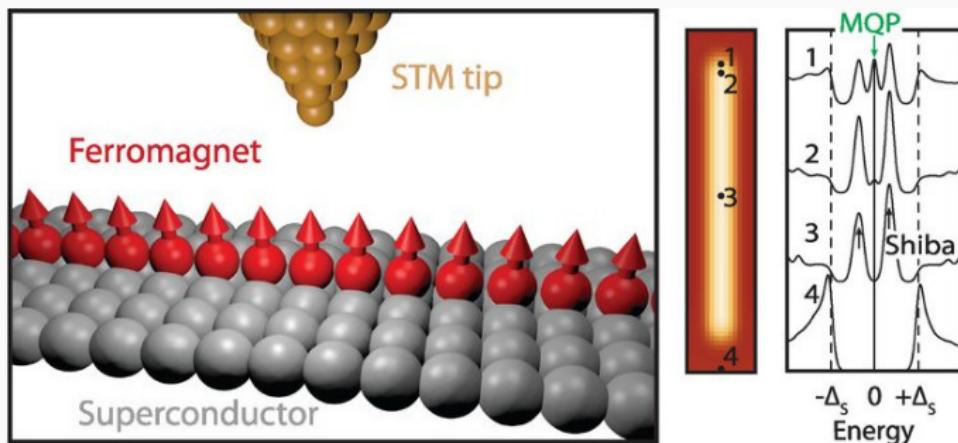
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# EXAMPLE OF EMPIRICAL REALIZATION

**STM measurements for the nanochain of Fe atoms self-organized on a surface of superconducting Pb.**



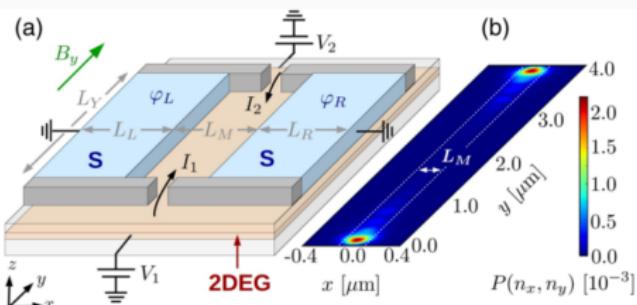
S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, USA /

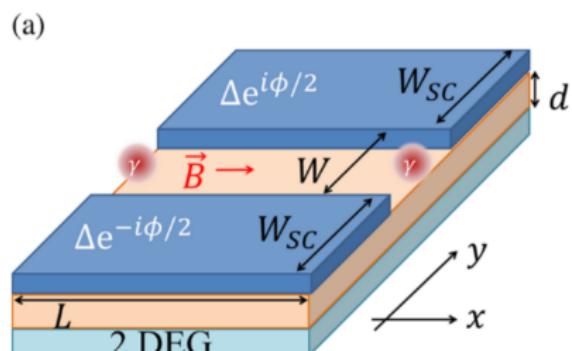
# **Majorana modes in Josephson junctions**

# PLANAR JOSEPHSON JUNCTIONS

Idea: Narrow metallic region with the strong spin-orbit interaction and in presence of magnetic field embedded between external superconductors.



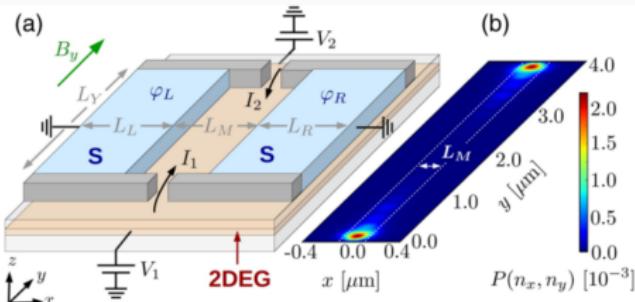
Michael Hell et al., PRL 118, 107701 (2017)



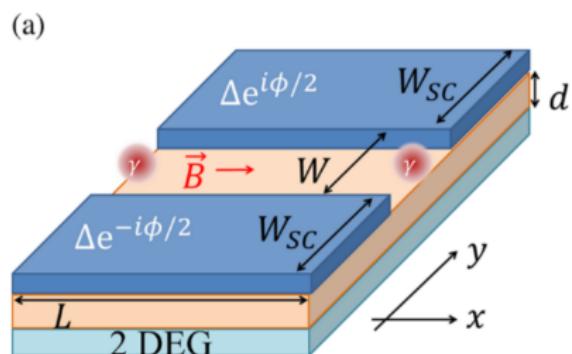
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Benefit:

Phase-tunable topological superconductivity induced in the metallic stripe.

# PLANAR JOSEPHSON JUNCTIONS

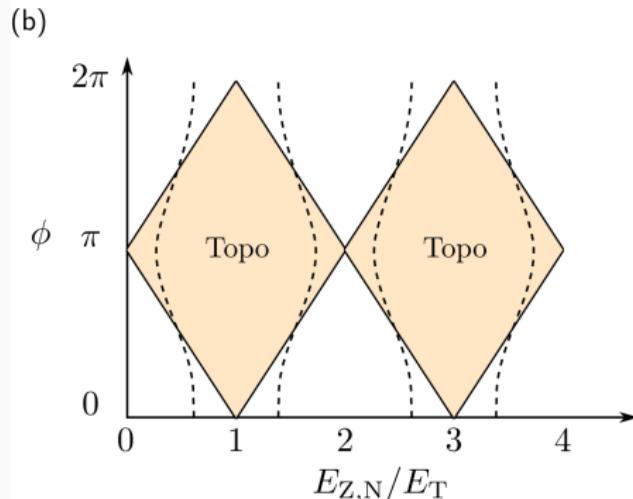
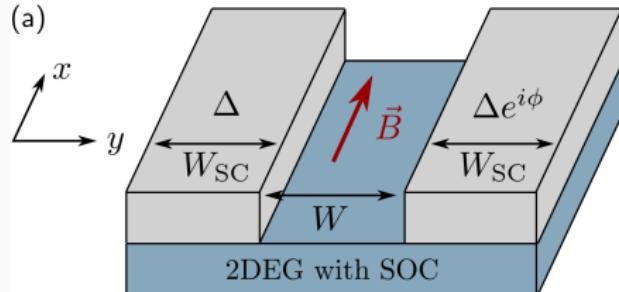
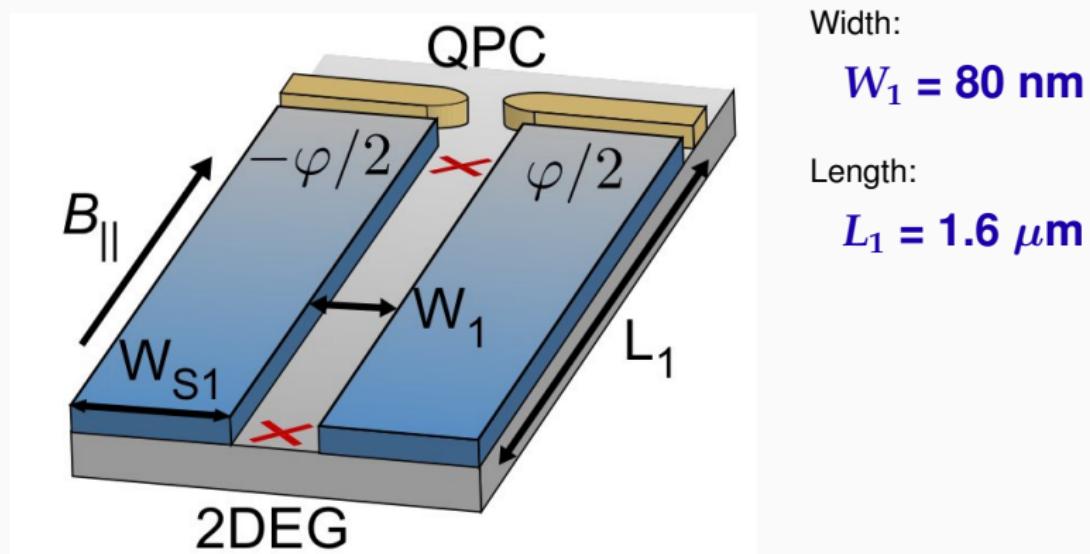


Diagram of topological superconducting state vs  
– phase difference  $\phi$ ,  
– magnetic field  $E_z$ .

# PLANAR JOSEPHSON JUNCTIONS: EXPERIMENT

Two-dimensional electron gas of **InAs** epitaxially covered by a thin **Al** layer

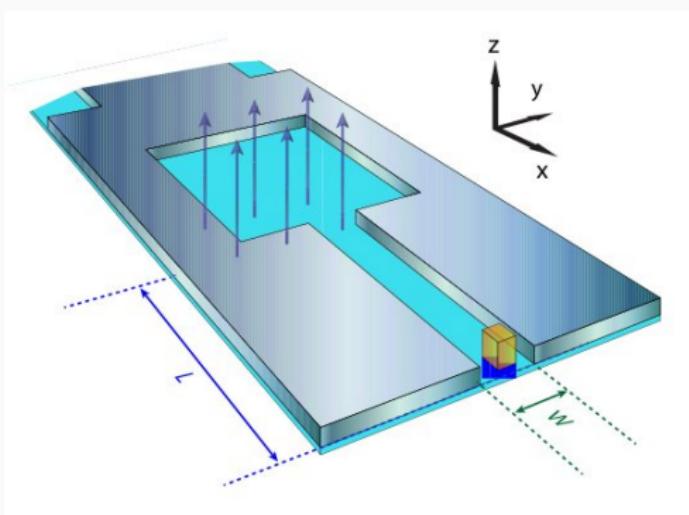


A. Fornieri, ..., Ch. Marcus and F. Nicelle, Nature **569**, 89 (2019).

Niels Bohr Institute (Copenhagen, Denmark)

# PLANAR JOSEPHSON JUNCTIONS: EXPERIMENT

Two-dimensional HgTe quantum well coupled to 15 nm thick Al film



Width:

$$W = 600 \text{ nm}$$

Length:

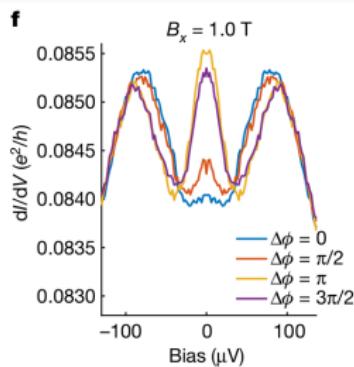
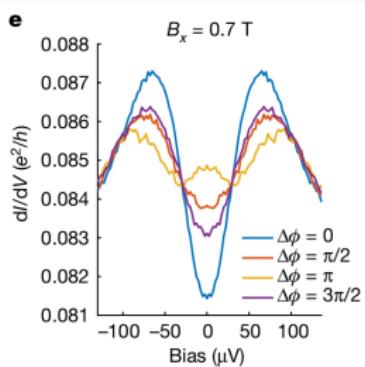
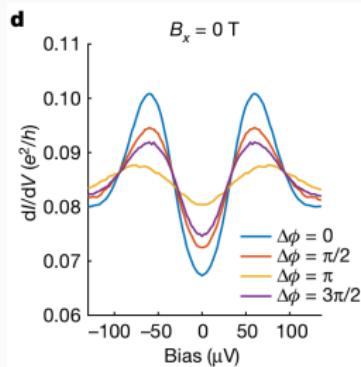
$$L = 1.0 \mu\text{m}$$

H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).

Würzburg Univ. (Germany) + Harvard Univ. (USA)

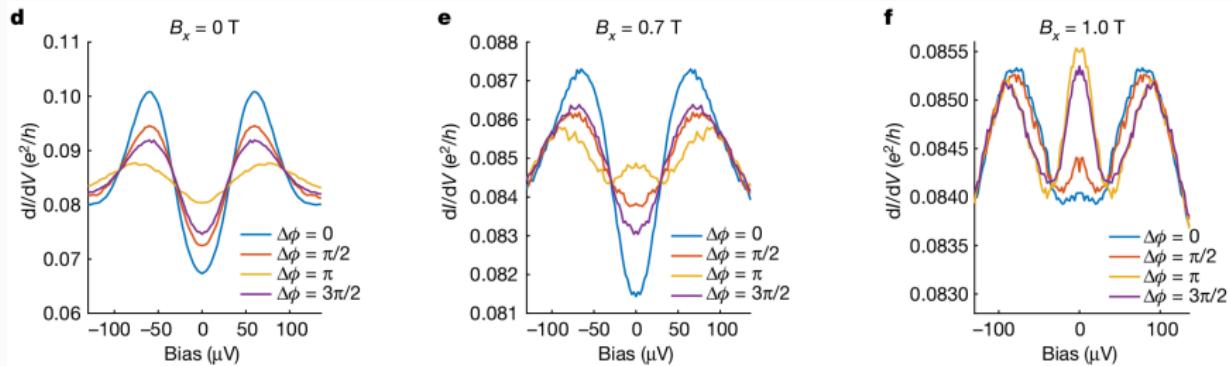
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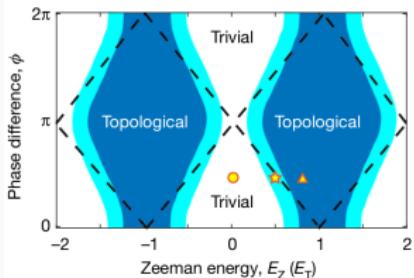


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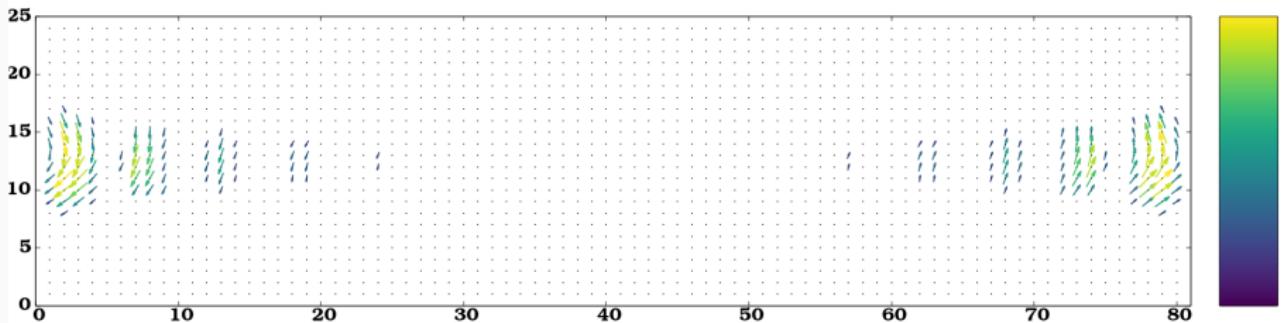
Experimental data obtained for three different magnetic fields indicated by the symbols in phase diagram  $\Rightarrow$ .



# **Topography of Majorana modes**

# TOPOGRAPHY OF MAJORANA MODES

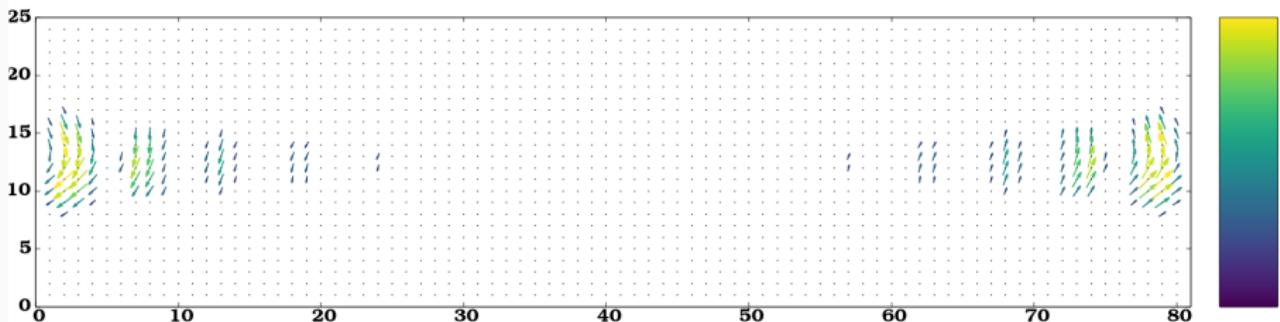
Spatial profile of the zero-energy quasiparticles of a homogeneous metallic strip embedded into the Josephson junction for the phase difference  $\phi = \pi$  (which is optimal for topological state).



“Majorana polarization”  $u_{\uparrow,n}v_{\uparrow,n} - u_{\downarrow,n}v_{\downarrow,n}$  obtained for eigenvalue  $E_n = 0$ .

# TOPOGRAPHY OF MAJORANA MODES

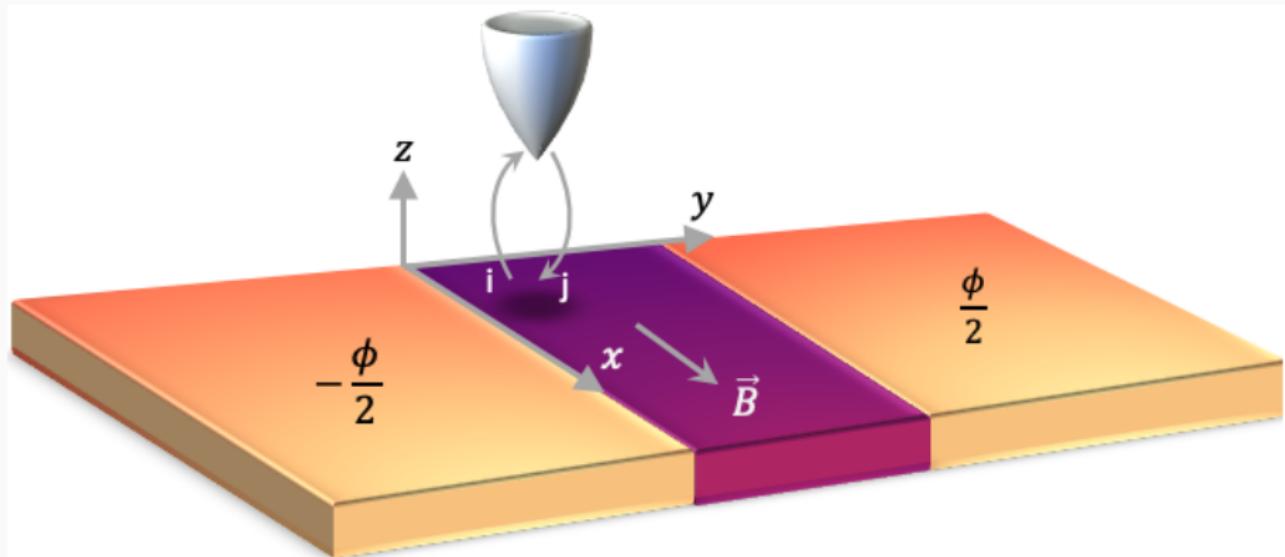
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“Majorana polarization”  $u_{\uparrow,n}v_{\uparrow,n} - u_{\downarrow,n}v_{\downarrow,n}$  obtained for eigenvalue  $E_n = 0$ . Magnitude of this quantity is measurable by the conductance of SESAR spectroscopy. For details see:

# TOPOGRAPHY OF MAJORANA MODES

Selective Equal Spin Andreev Reflection (SESTAR) spectroscopy:



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

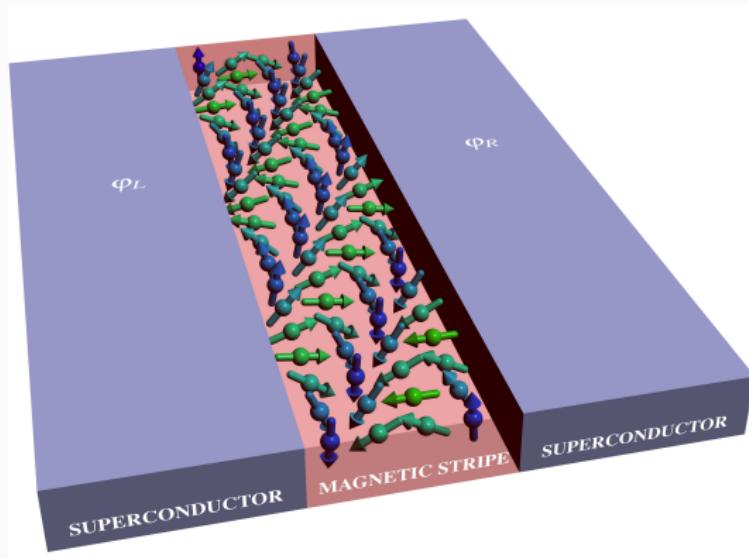
# **Topological Josephson junctions**

# **Topological Josephson junctions**

**with self-organized magnetic stripe**

# JJ WITH SELFORGANIZED MAGNETIC STRIPE

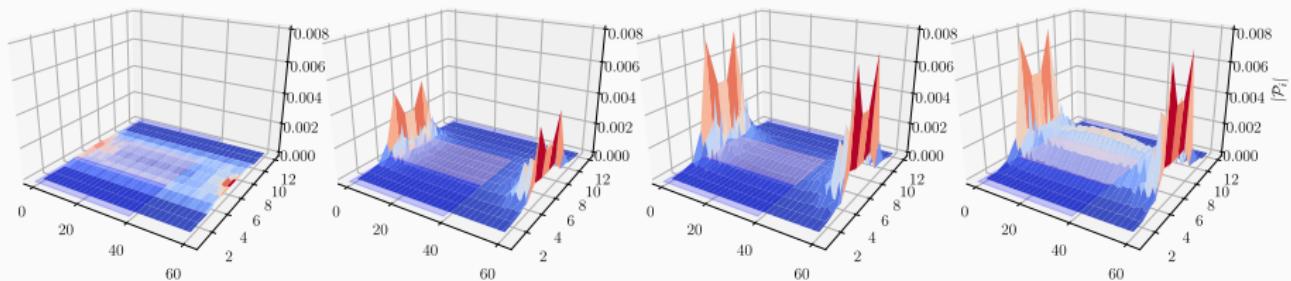
Narrow metallic stripe with the classical magnetic moments placed between two s-wave superconductors, differing in phase  $\phi_L \neq \phi_R$ .



M.M. Maśka, M. Dziurawiec, M. Strzałka & T.D. – work in progress  
/ Technical University (Wrocław) & UMCS (Lublin) /

# JJ WITH SELFORGANIZED MAGNETIC STRIPE

Spatial profiles of the (zero-energy) Majorana quasiparticles for selected values of the phase difference  $\phi_R - \phi_L$ .



$$\phi_R - \phi_L = 0.6\pi$$

$$\phi_R - \phi_L = 0.4\pi$$

$$\phi_R - \phi_L = 0.2\pi$$

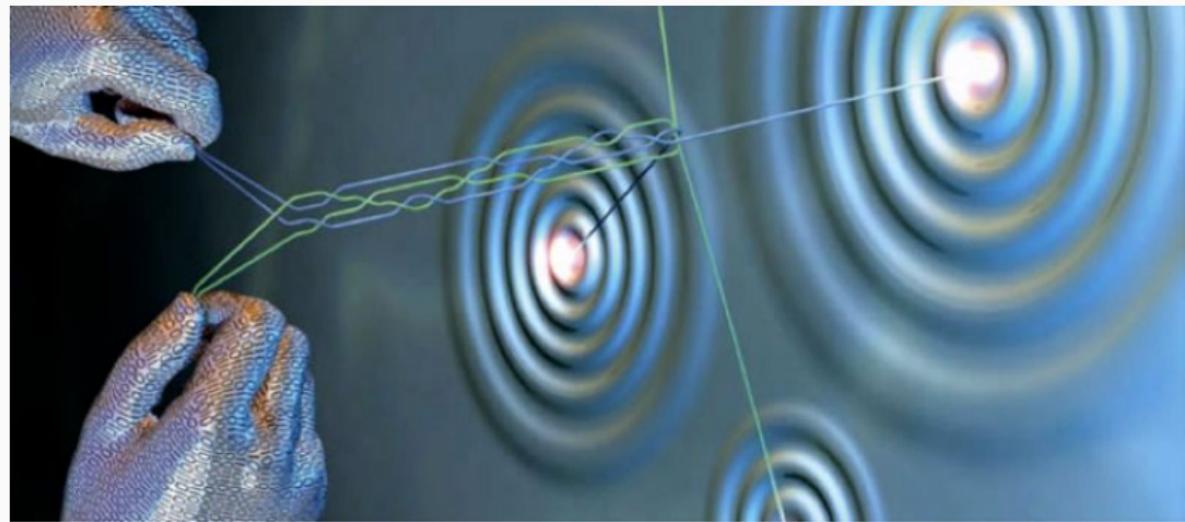
$$\phi_R - \phi_L = 0.0$$

M.M. Maśka, M. Dziurawiec, M. Strzałka & T.D. – work in progress

/ Technical University (Wrocław) & UMCS (Lublin) /

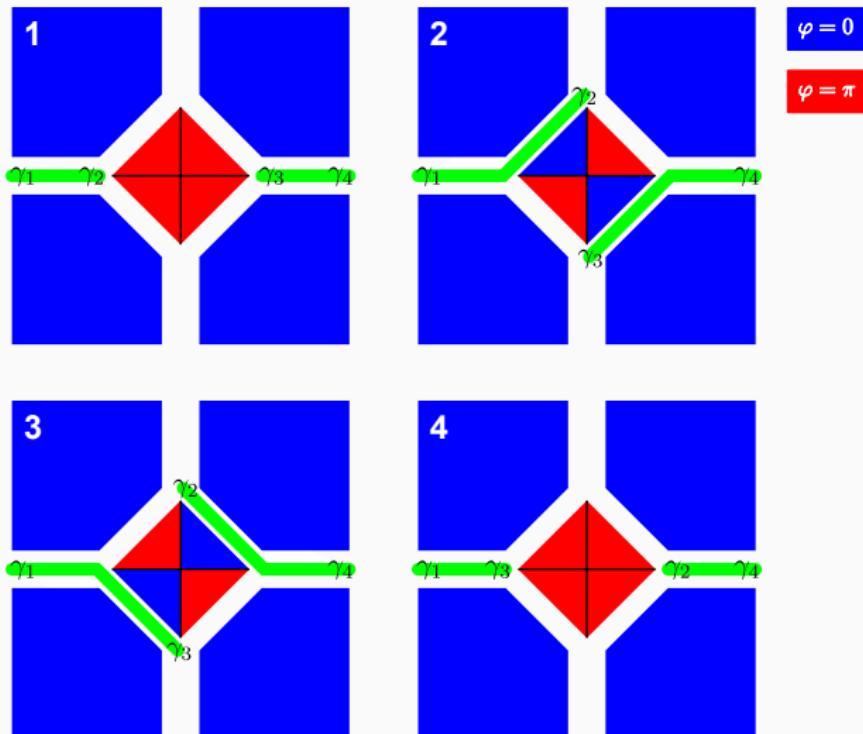
# CHALLENGING ISSUE

**Can one braid the Majorana modes ?**



# BRAIDING PROTOCOL

## Braiding of the Majorana pairs in Josephson platform



## TAKE-HOME MESSAGE

**Magnetism and superconductivity in nanoscopic systems:**

- ⇒ can cooperate between themselves
- ⇒ inducing novel (topological) states of matter
- ⇒ hosting in-gap (Andreev/Yu-Shiba-Rusinov/Majorana) states
- ⇒ useful for quantum bits / quantum computing

**<http://kft.umcs.lublin.pl/doman/lectures>**

## ACKNOWLEDGEMENTS

- K.I. Wysokiński, N. Sedlmayr, R. Taranko, (UMCS Lublin)  
Sz. Głodzik, A. Kobiałka, B. Baran
- I. Weymann, K. Wrześniewski, K. Wójcik (UAM Poznań)
- M. Maśka et al (Wrocław/Katowice)
- K. Wójcik, B.R. Bułka, G. Michałek (IFM PAN Poznań)
- J. Barański, M. Barańska (Dęblin)
- G. Górski (Rzeszów)
- T. Novotny, V. Janis, M. Zonda, V. Pokorný (Prague)