Lublin, 21 lutego 2013 r.

Dualny charakter elektronów w nad-przewodnikach z parami lokalnymi

T. Domański

Instytut Fizyki UMCS

http://kft.umcs.lublin.pl/doman/lectures

Plan referatu

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* **Istota stanu nadprzewodzącego**

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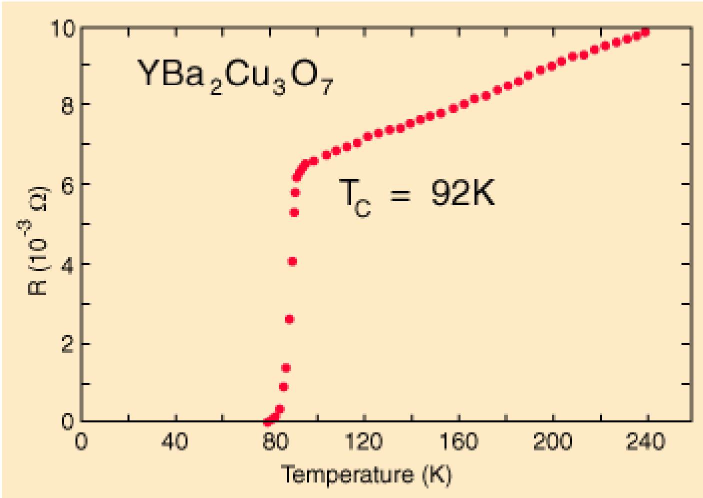
- **Podsumowanie**

1. Preliminaries

Superconducting state – properties

Superconducting state – properties

- ideal d.c. conductance



Superconducting state – properties

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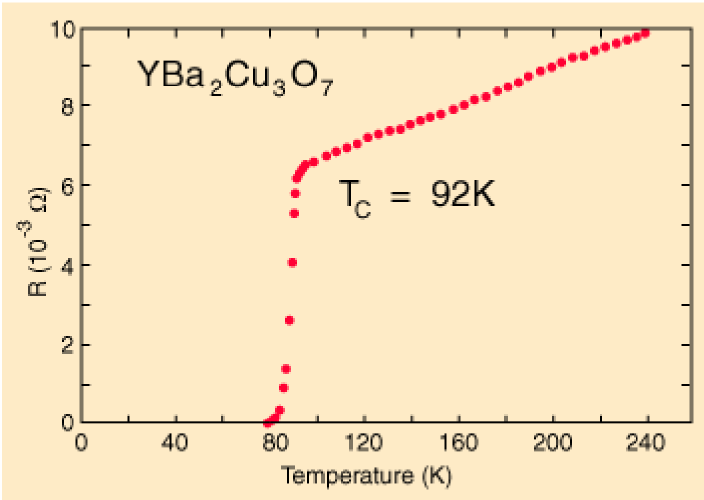


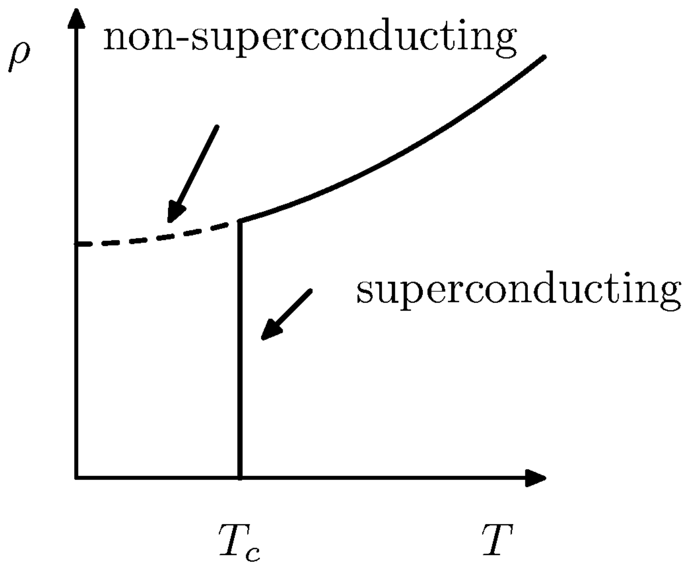
Normal conductors:

resistance $R = \rho \frac{l}{S}$

where $\rho \equiv 1/\sigma$

and $\sigma = \frac{ne^2\tau}{m}$

$\tau(T)$ – **relaxation time**



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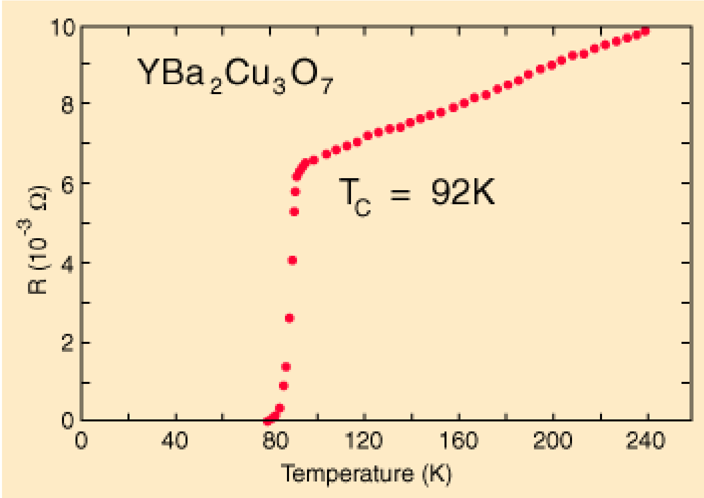


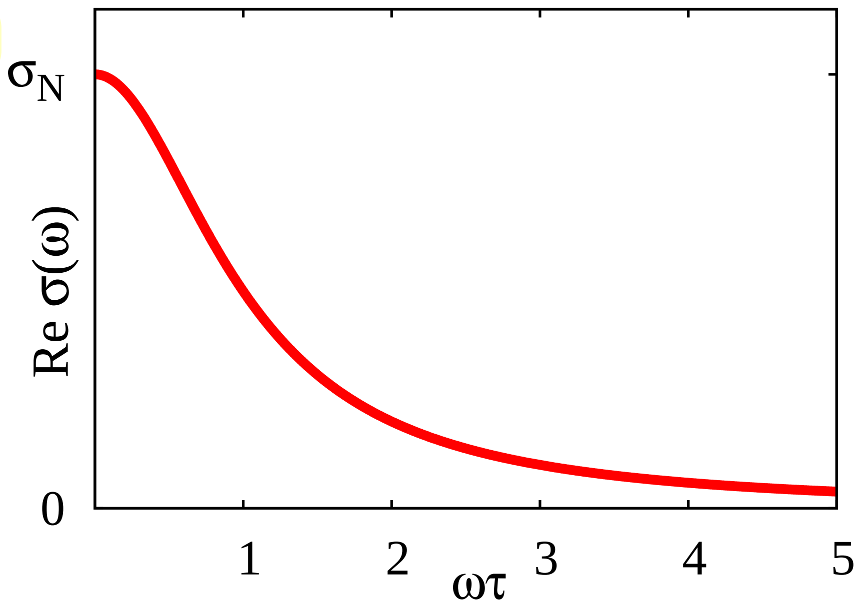
a.c. conductance (above T_c)

Drude conductance

$$\sigma(\omega) = \frac{ne^2\tau}{m}\,\frac{1}{1-i\omega\tau}$$

obeys the **f-sum rule**

$$\int_{-\infty}^{\infty} \mathrm{Re}\,\sigma(\omega) = \pi\,\frac{ne^2}{m}$$



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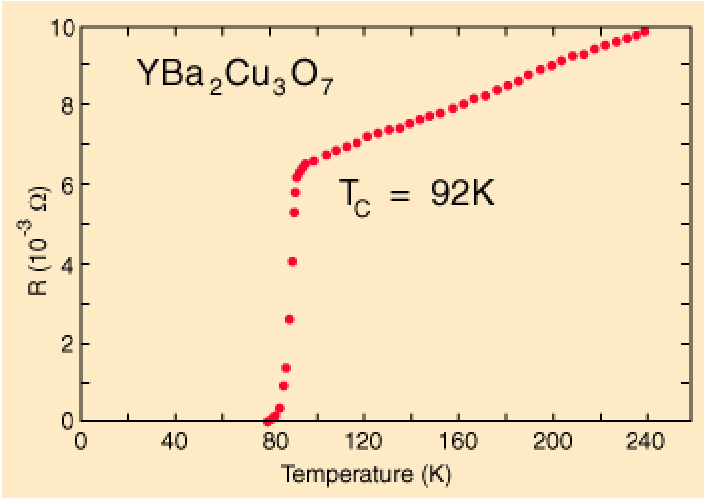


a.c. conductance (below T_c)

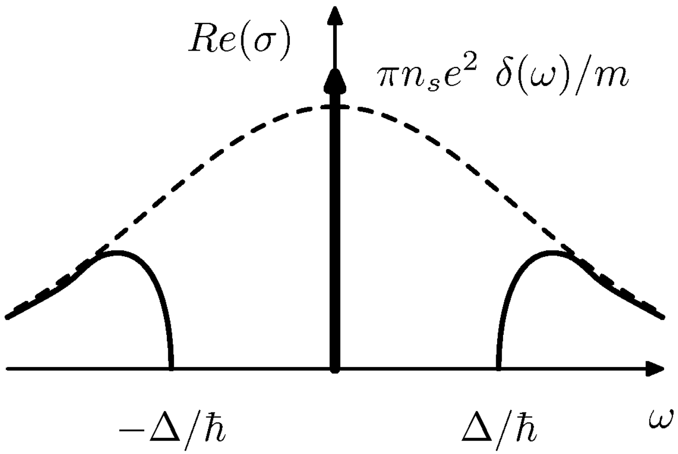
The f-sum rule

$$\int_{-\infty}^{\infty} \mathrm{Re}\, \boldsymbol{\sigma}(\boldsymbol{\omega}) = \pi \, \frac{ne^2}{m}$$

must be also obeyed

below T_c, however

$$n = n_n(T) + n_s(T)$$



$n_s(T)$ – **superfluid density**

Superconducting state – properties (continued)

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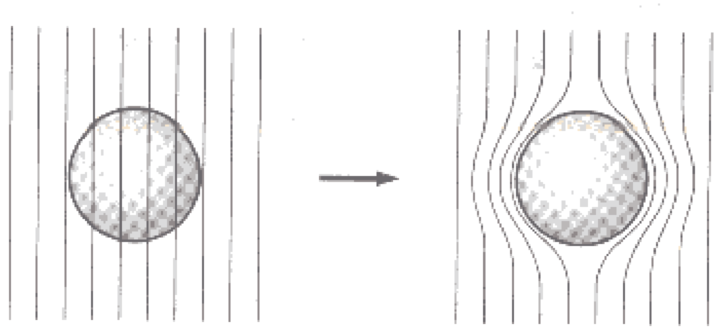
★ **ideal diamagnetism** /perfect screening of the d.c. magnetic field/



$T > T_c$ $\qquad$ $T < T_c$

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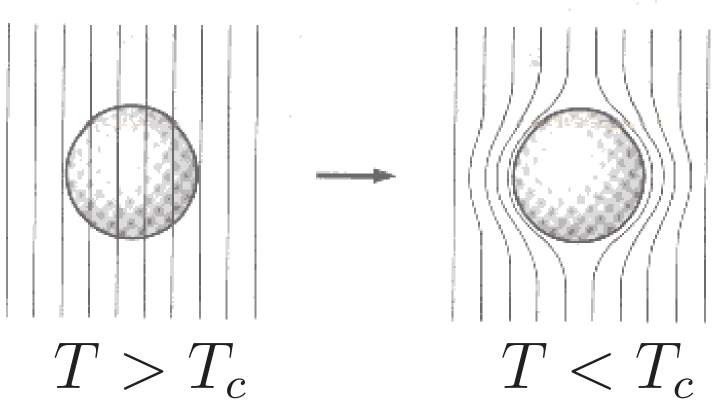


Meissner effect is described by the Londons' equation

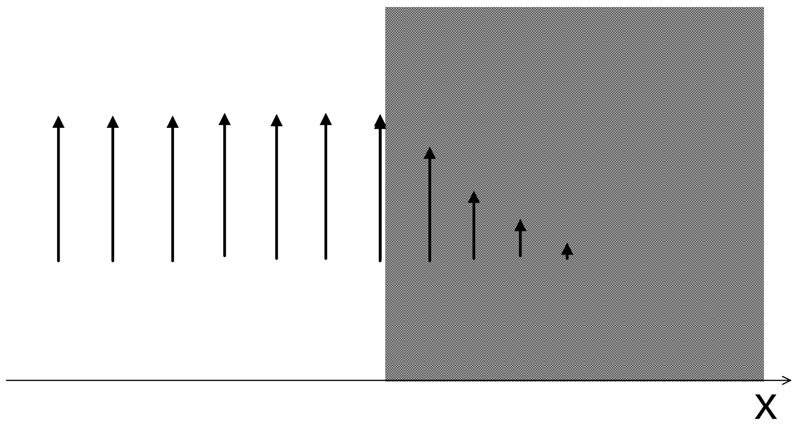
$$\vec{j} = -\frac{e^2 n_s(T)}{mc^2}\,\vec{A}$$

where the coefficient

$$\frac{e^2 n_s(T)}{mc^2} \equiv \rho_s(T) = \frac{1}{\lambda^2}$$

$\rho_s(T)$ – superfluid stiffness

$\lambda(T)$ – penetration depth



$$B(x) = B_0\, e^{-x/\lambda}$$

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Simultaneous appearance of:

$\Longrightarrow$ **the ideal (d.c.) conductance**

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is induced by the superfluid fraction

$$n_s(T)$$

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Onset of the fermion pairing often goes hand in hand with appearance of the superconductivity/superfluidity but it doesn't have to be a rule.

Formal issues – generalities

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The order parameter

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It has the following physical implications:

$|\chi| \neq 0 \longrightarrow$ **amplitude causes the energy gap**

$\nabla\theta \neq 0 \longrightarrow$ **phase slippage induces supercurrents**

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1. **closing the gap** . . . . **[conventional BCS superconductors]**

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2. **disordering the phase** . . . . **[HTSC compounds, URh_2Si_2 (?)]**

$$\lim_{T\to T_c} \; \langle\theta\rangle = 0$$

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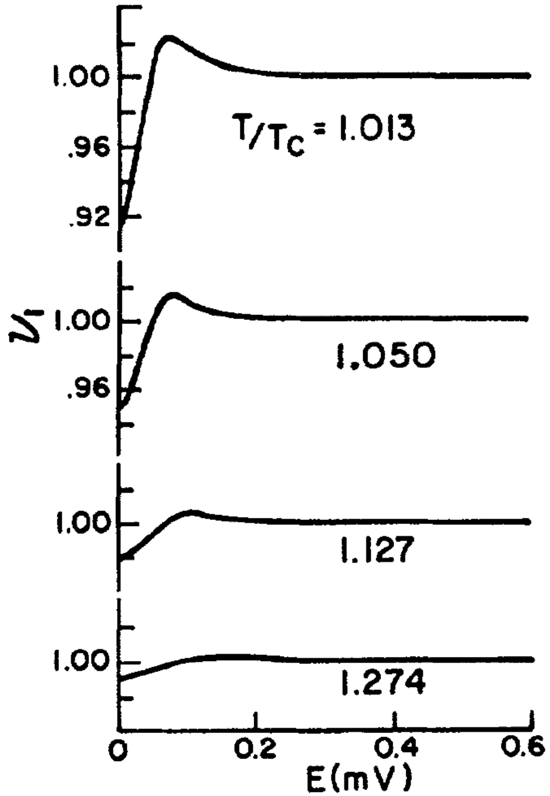
Tunneling conductance revealed a pseudogap surviving to $\sim 1.3T_c$.



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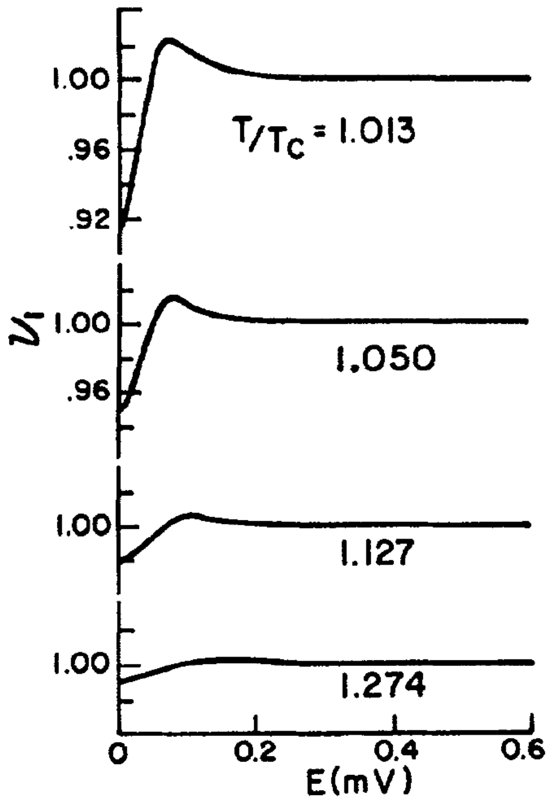

R.W. Cohen and B. Abels, Phys. Rev. **168**, *444 (1968).*

2. Dualities

/ in the strongly correlated systems /

Duality 1: **amplitude vs phase driven transition**

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Critical temperature T_c can be related to:

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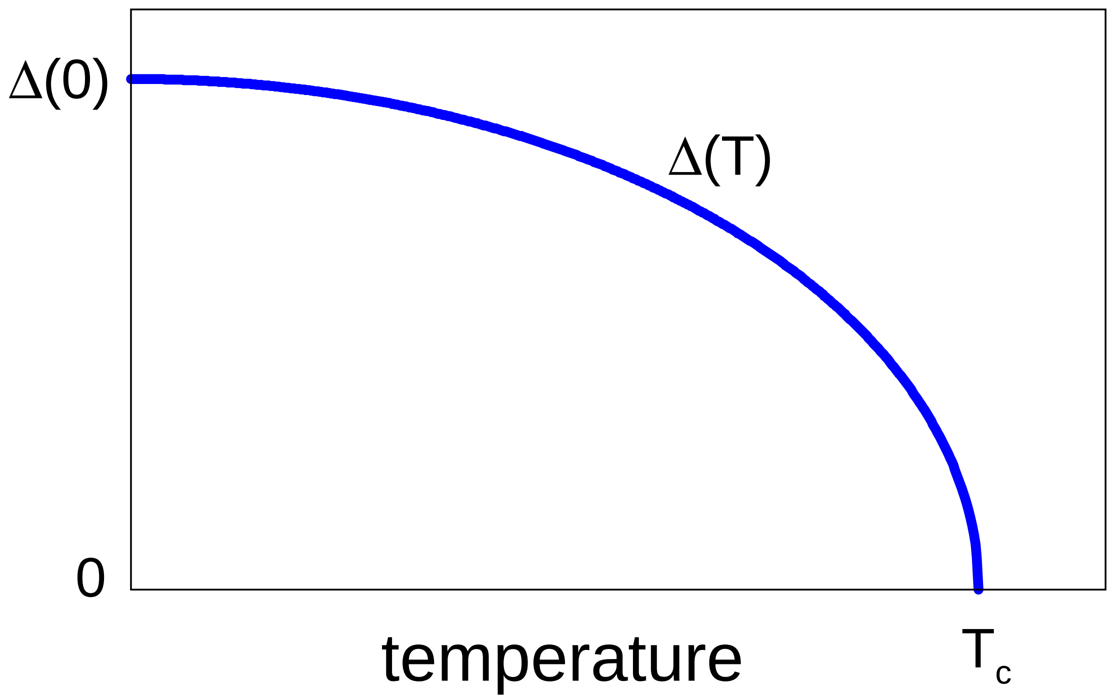

Pairing is responsible for the energy gap $\Delta(T)$ in a single particle spectrum

$\Delta(T_c) = 0$ **/ amplitude transition /**

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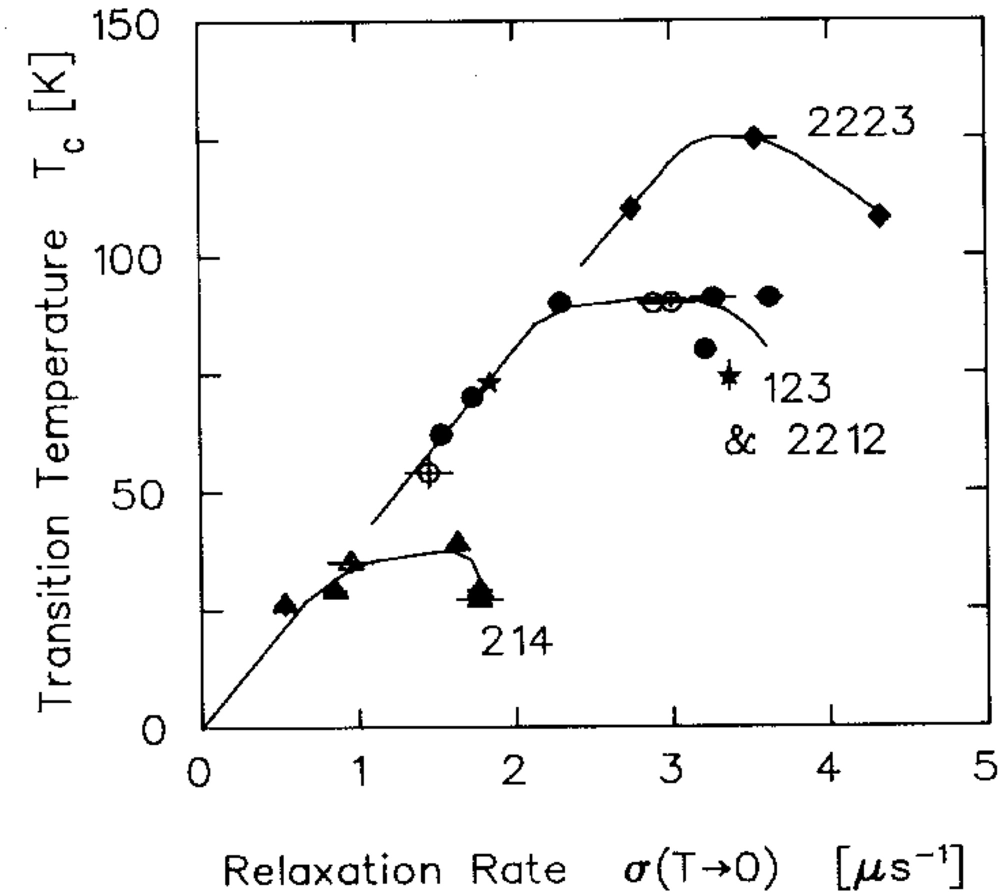

Y.J. Uemura et al, Phys. Rev. Lett. ***62****, 2317 (1989).*

Early experiments with the muon-spin relaxation have indicated that

$$T_c \propto \rho_s(0)$$

/ Uemura scaling /

The superfluid stiffness $\rho_s(T)$ is here defined by

$$\rho_s(T) \equiv \frac{1}{\lambda^2(T)} = \frac{4\pi e^2}{m^* c^2} n_s(T)$$

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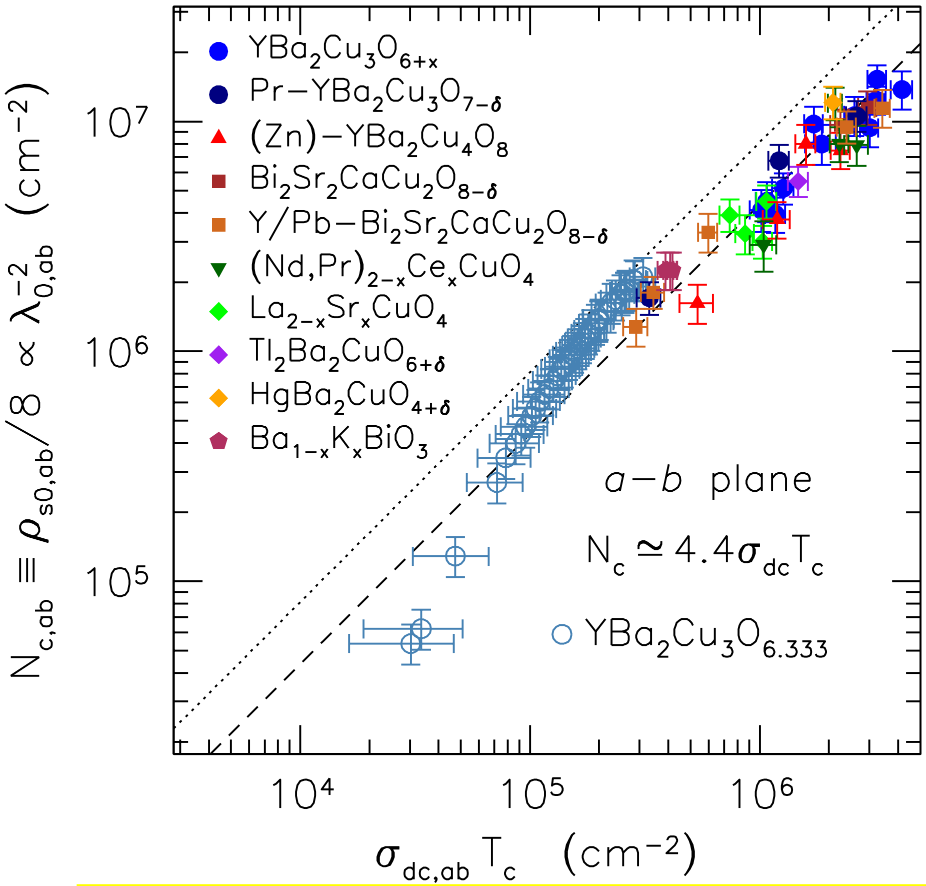

C.C. Homes, Phys. Rev. B **80**, *180509(R) (2009).*

Recently such scaling has been updated from transport measurements

$$\frac{1}{8}\rho_s = 4.4\sigma_{dc}T_c$$

/ Homes scaling /

This new relation is valid for all samples ranging from the underdoped to overdoped region.

/ ab - plane /

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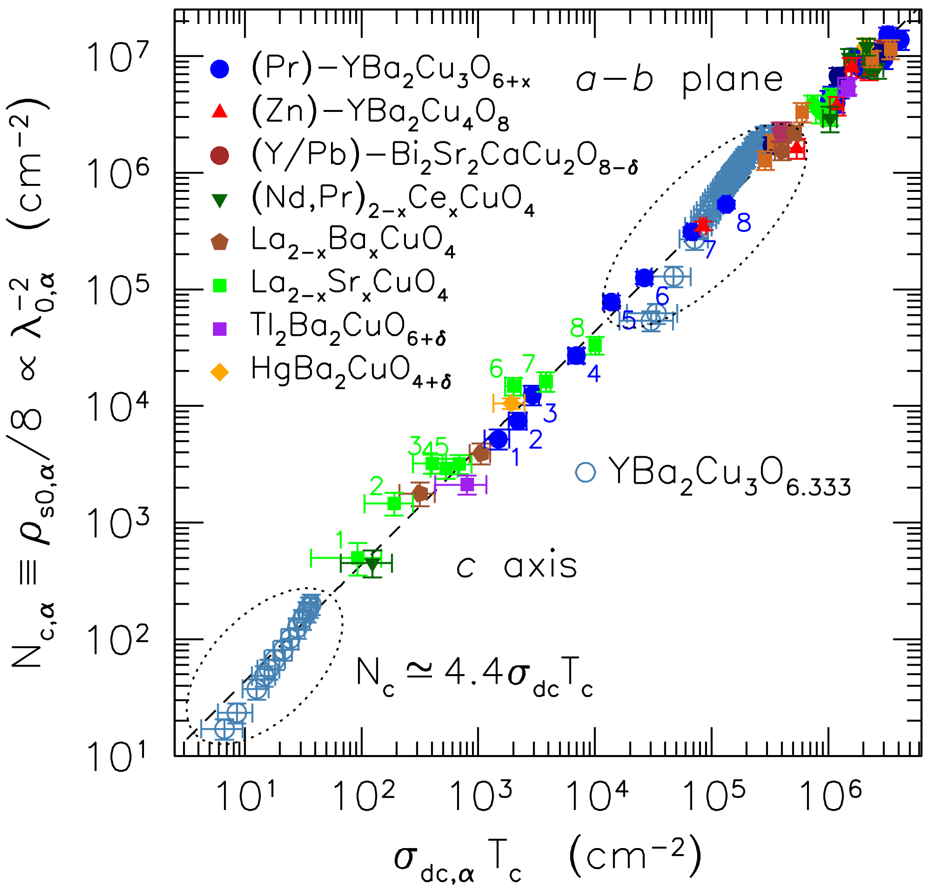

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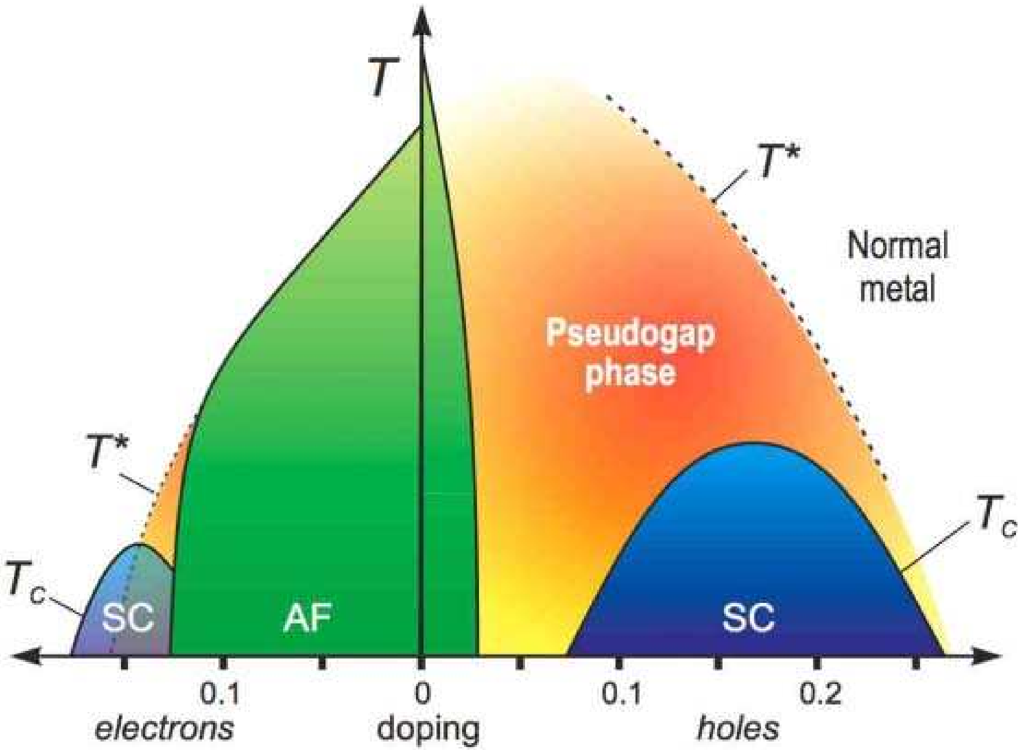

In the high T_c cuprates

d-wave superconductivity

appears near the AF insulator. In both cases the order comes from the exchange coupling

$$J_{ij} = \frac{2t_{ij}^2}{U}$$

O. Fisher et al, *Rev. Mod. Phys.* **79**, *353 (2007).*

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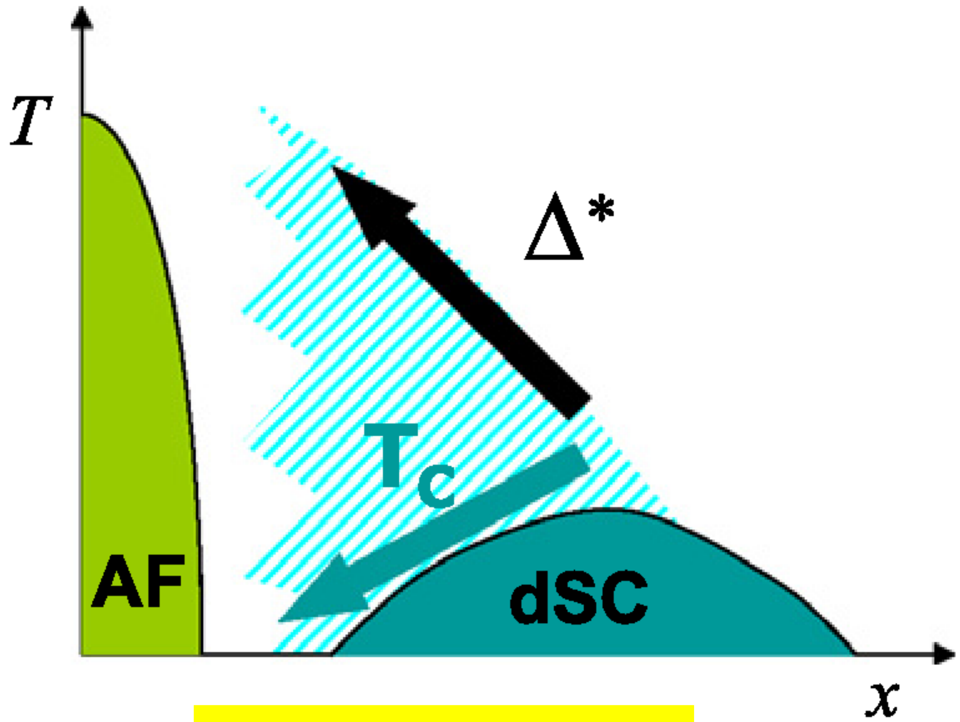

T. Valla, Physica C (2012).

1. What type of relationship does occur between

dSC and AF order ?

2. Are they related with the pseudogap ?

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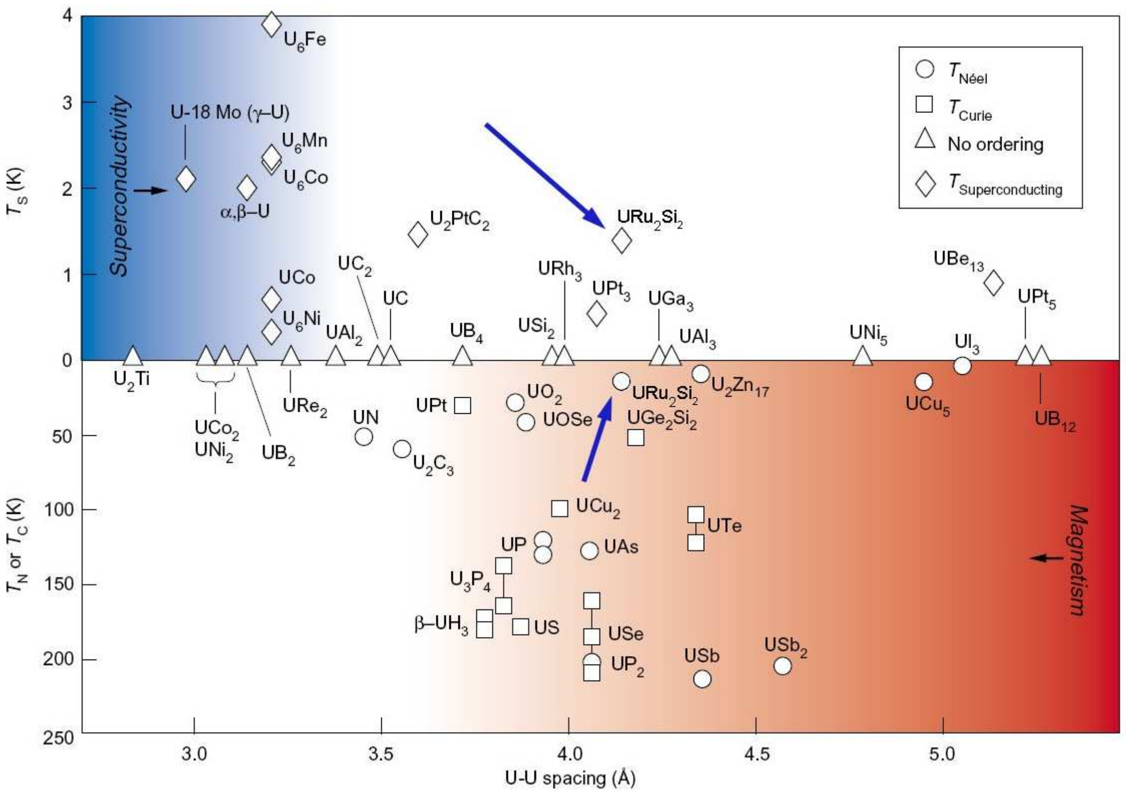

Magnetism plays also a role for superconductivity of

the uranium compounds

/ Hill plot /

J.A. Mydosh and P.M. Oppeneer, Rev. Mod. Phys. **83**, *1301 (2011).*

Duality 3: itinerant vs localized features

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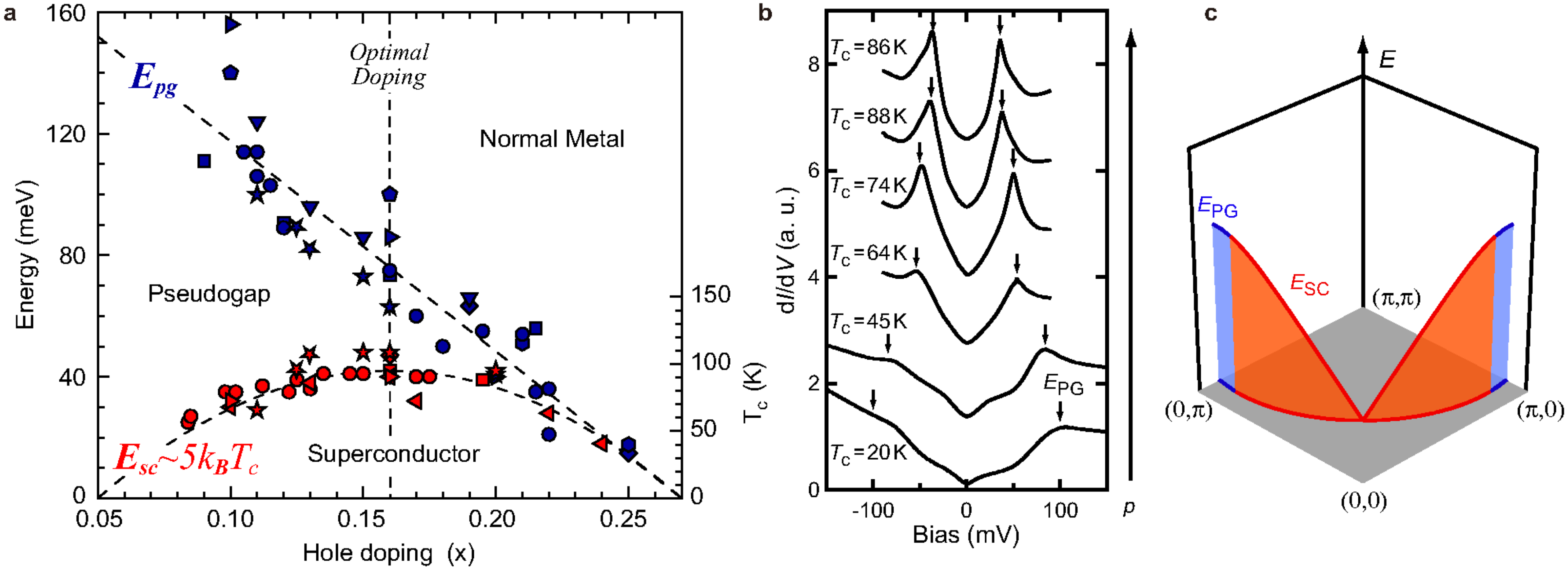

Y. Kohsaka, ... and J.C. Davis, Nature ***454****, 1072 (2008).*

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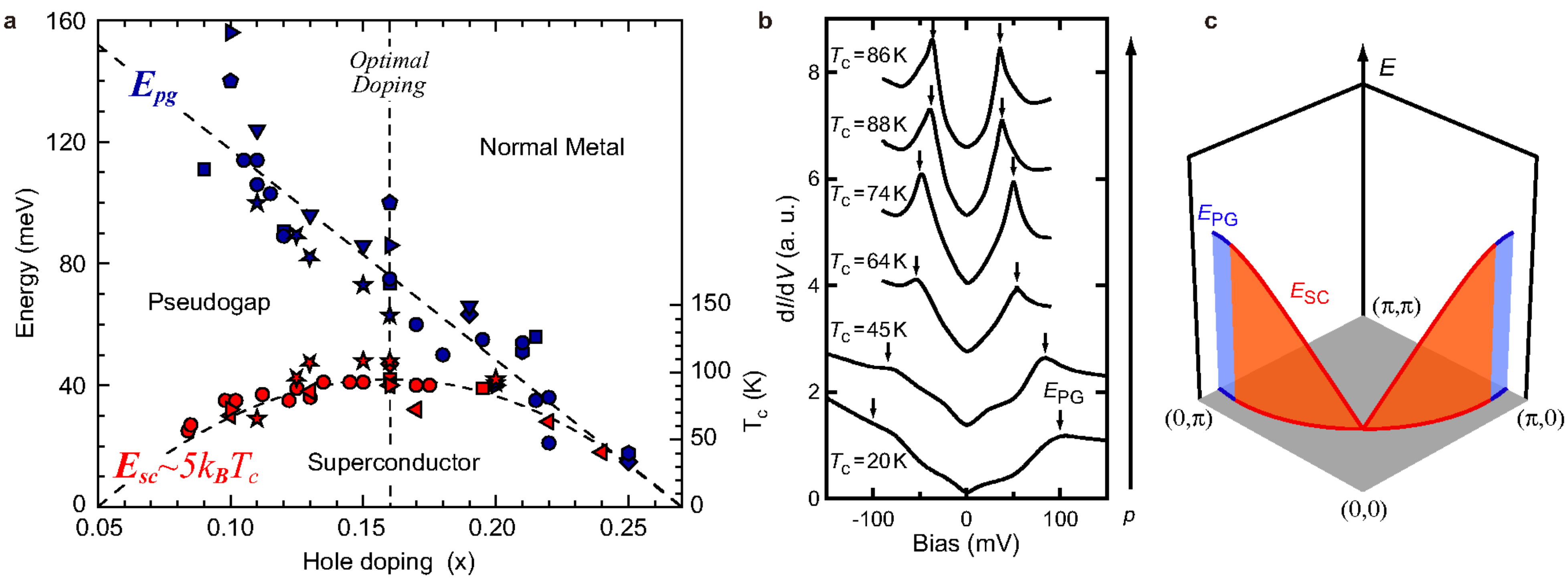

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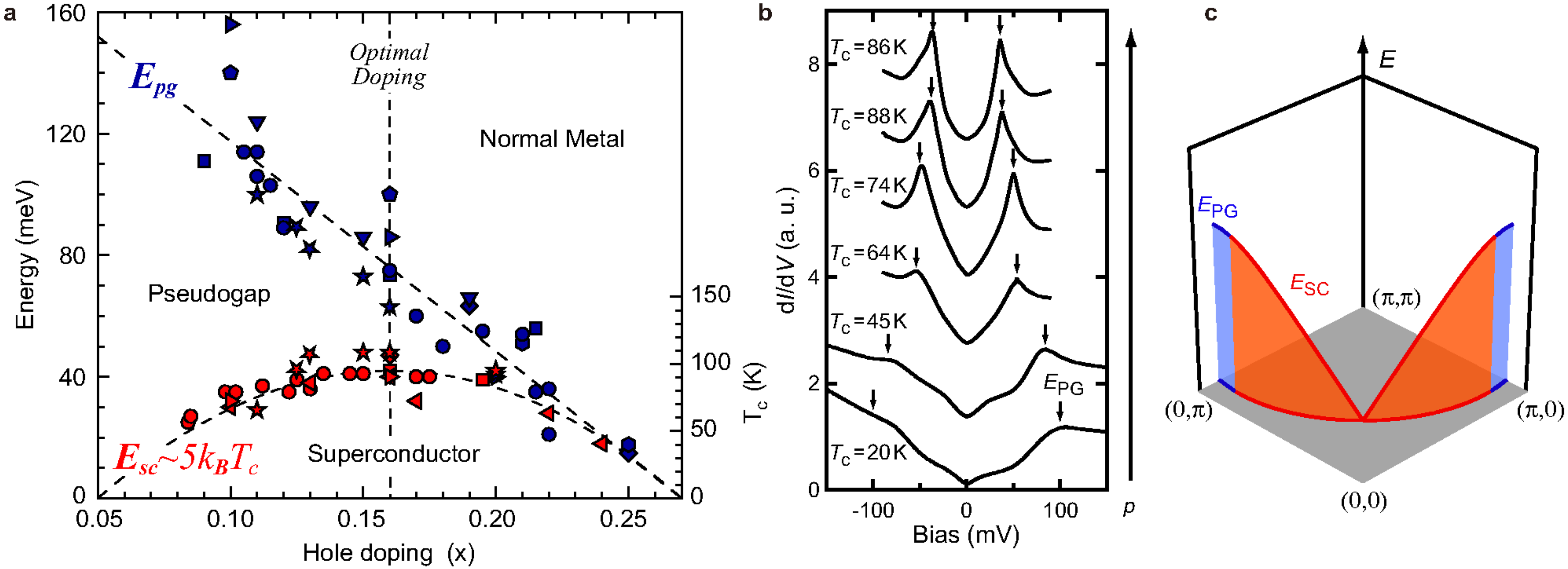

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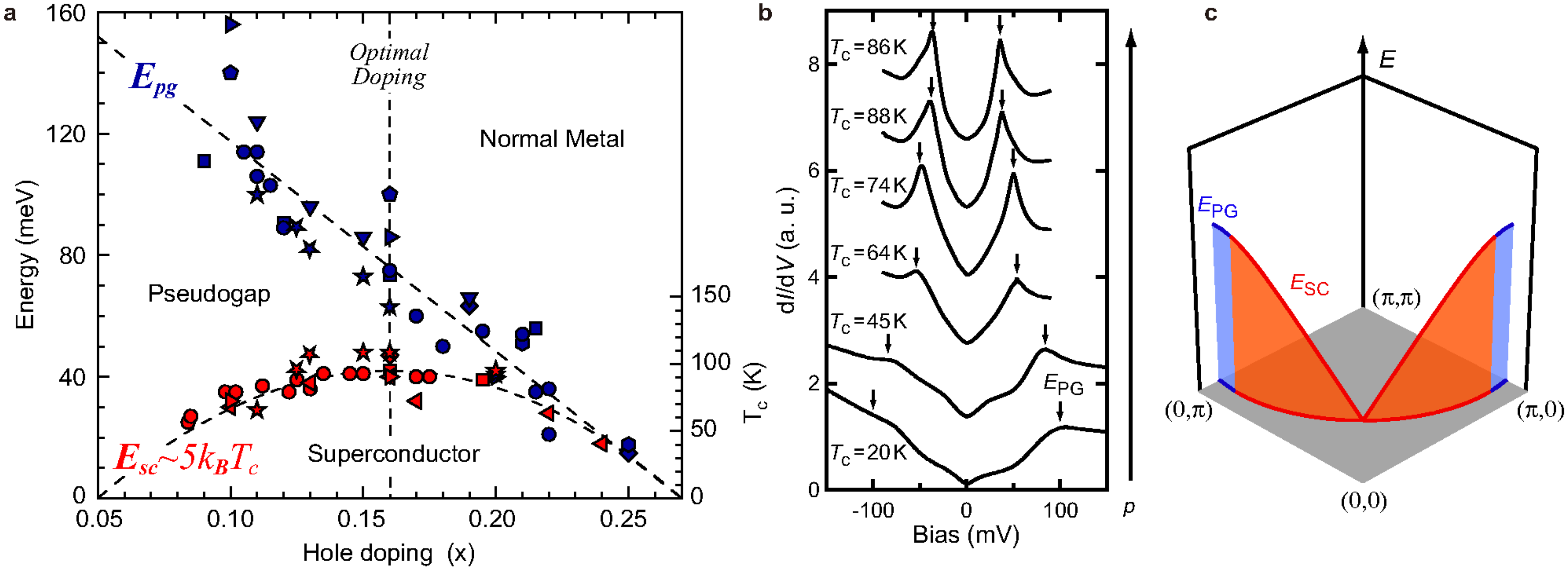

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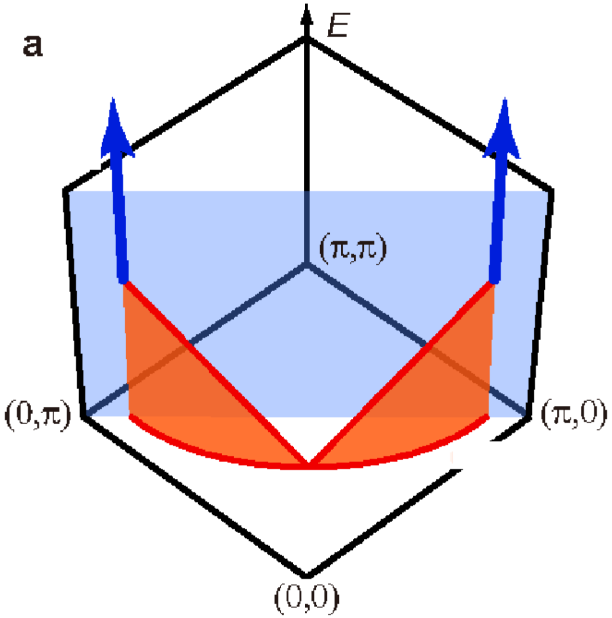

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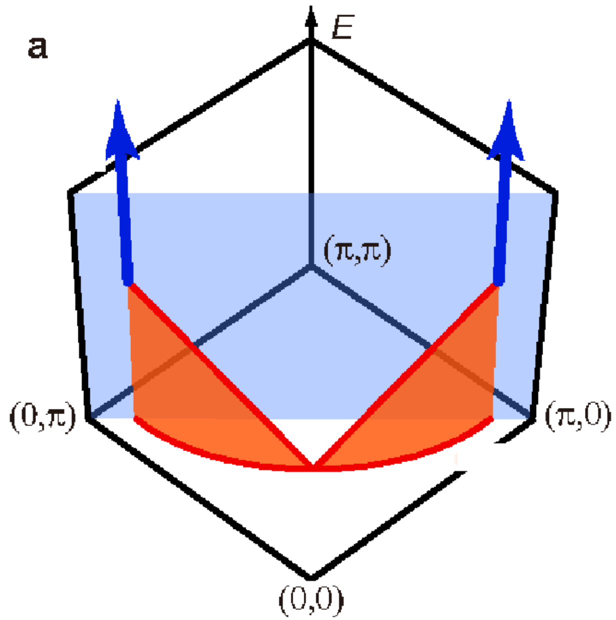

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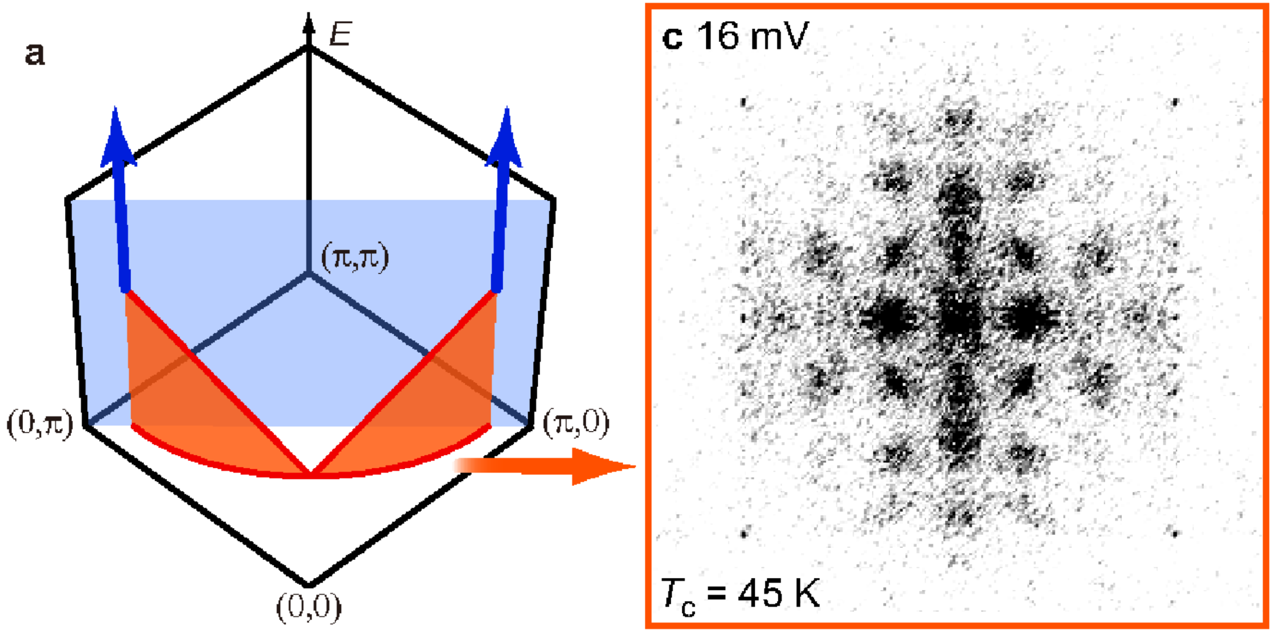

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⇒ **delocalized Cooper pairs** / *itinerant features at low energies* /

Cu-O-Cu centered patterns / *localized features in the $\vec{r}$-space* /

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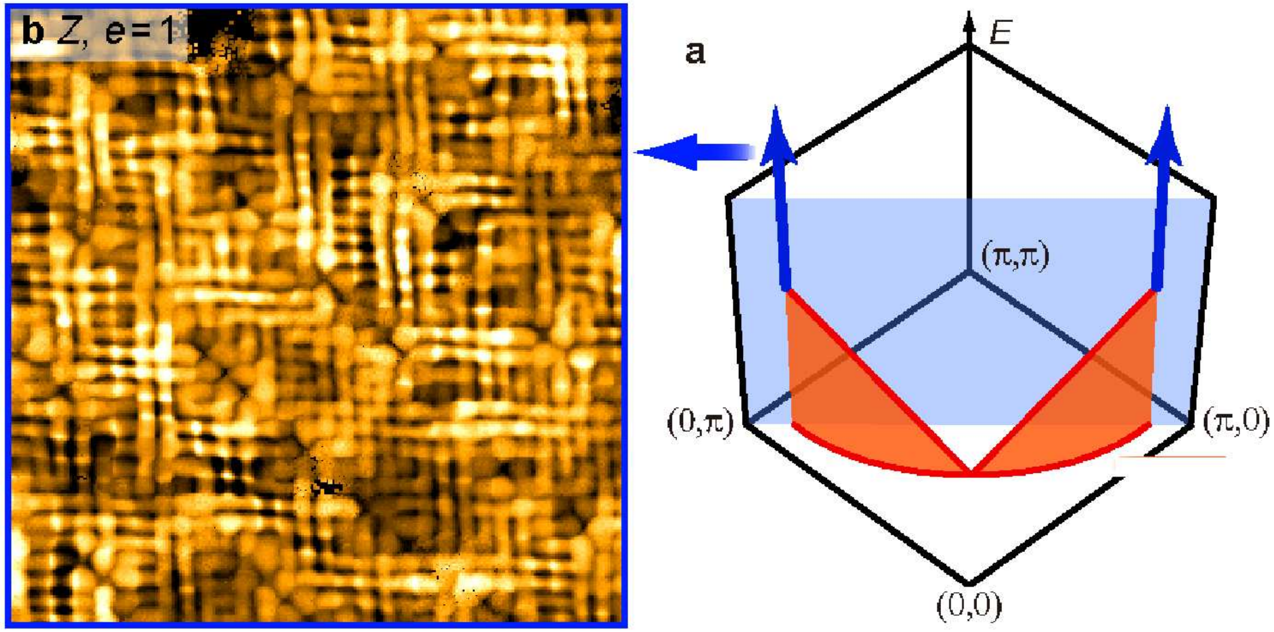

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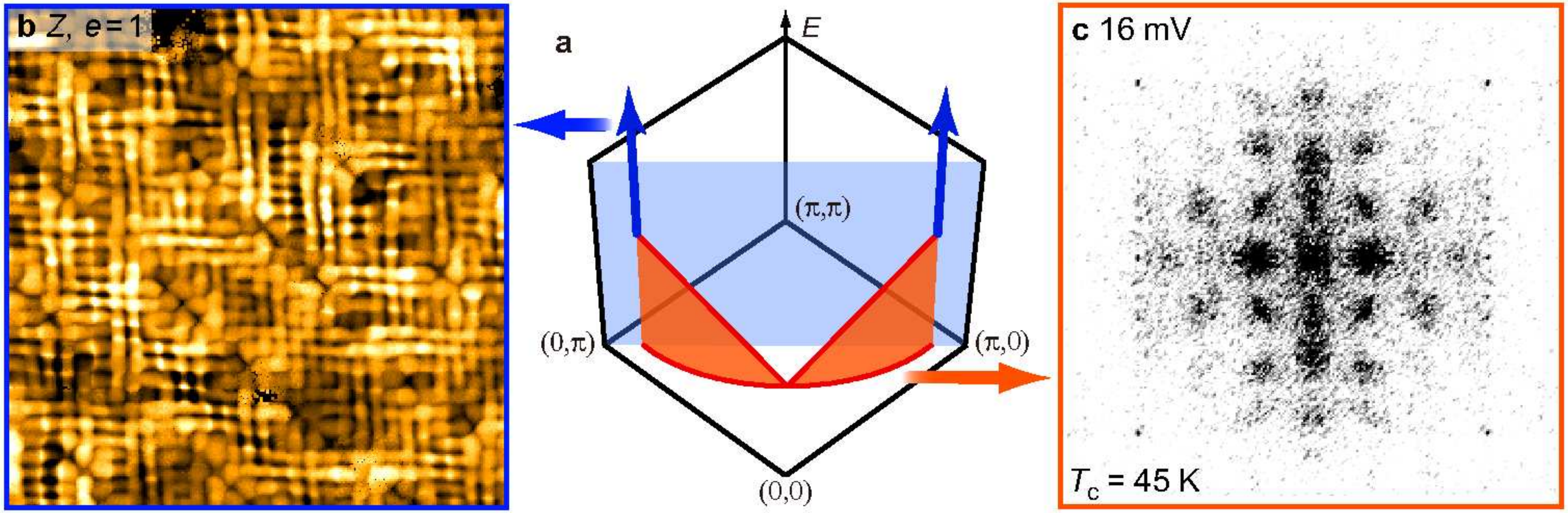

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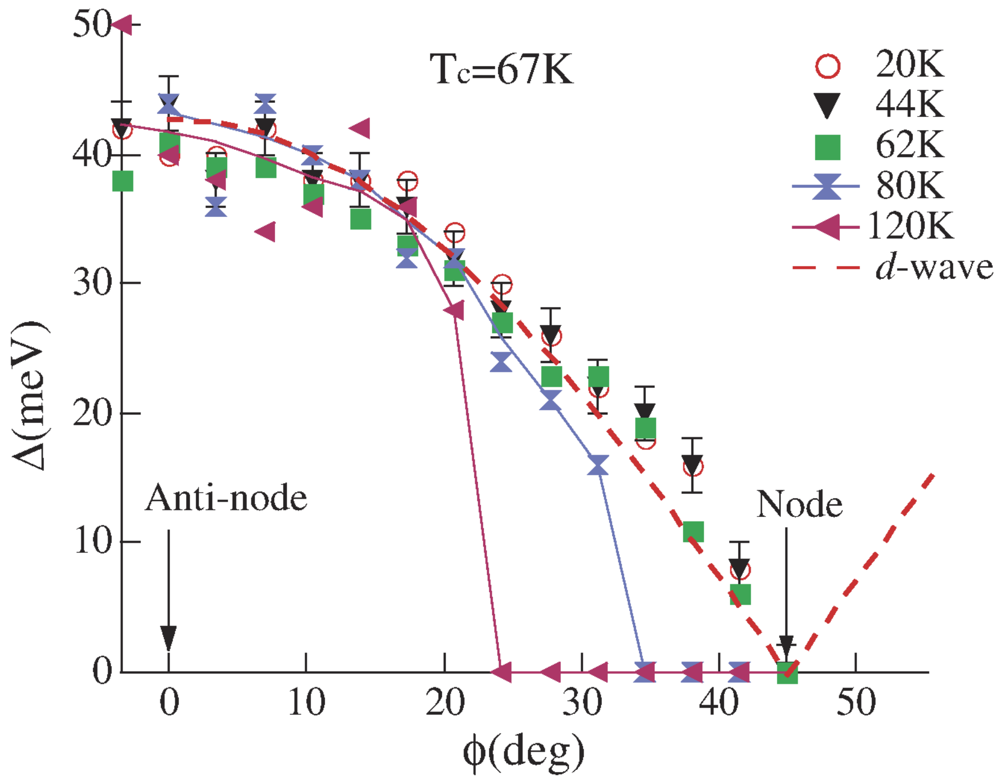

A. Kanigel et al, *Phys. Rev. Lett.* **99**, *157001 (2007).*

Duality 4: nodal antinodal dichotomy

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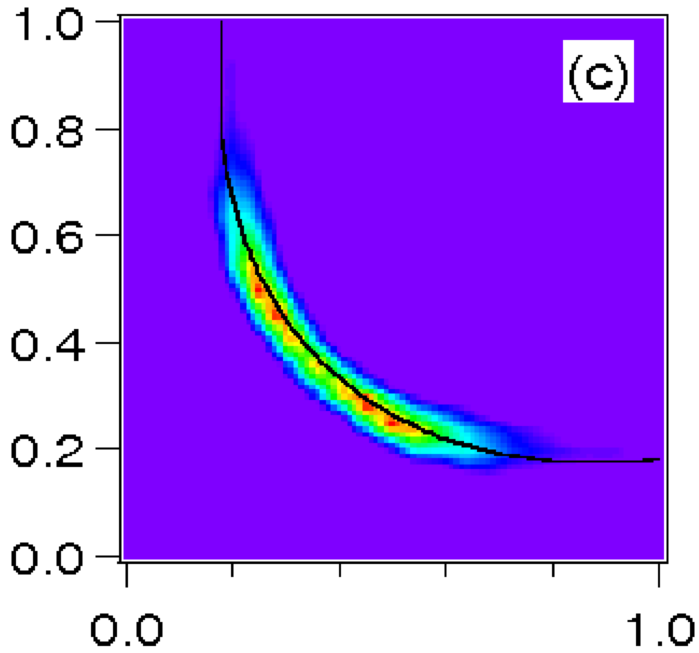

In antinodal parts the missing parts of Fermi surface are recovered at T^*.

"Death of a Fermi surface" *K. McElroy, Nature Physics* ***2****, 441 (2006)* .

Duality 4: nodal antinodal dichotomy

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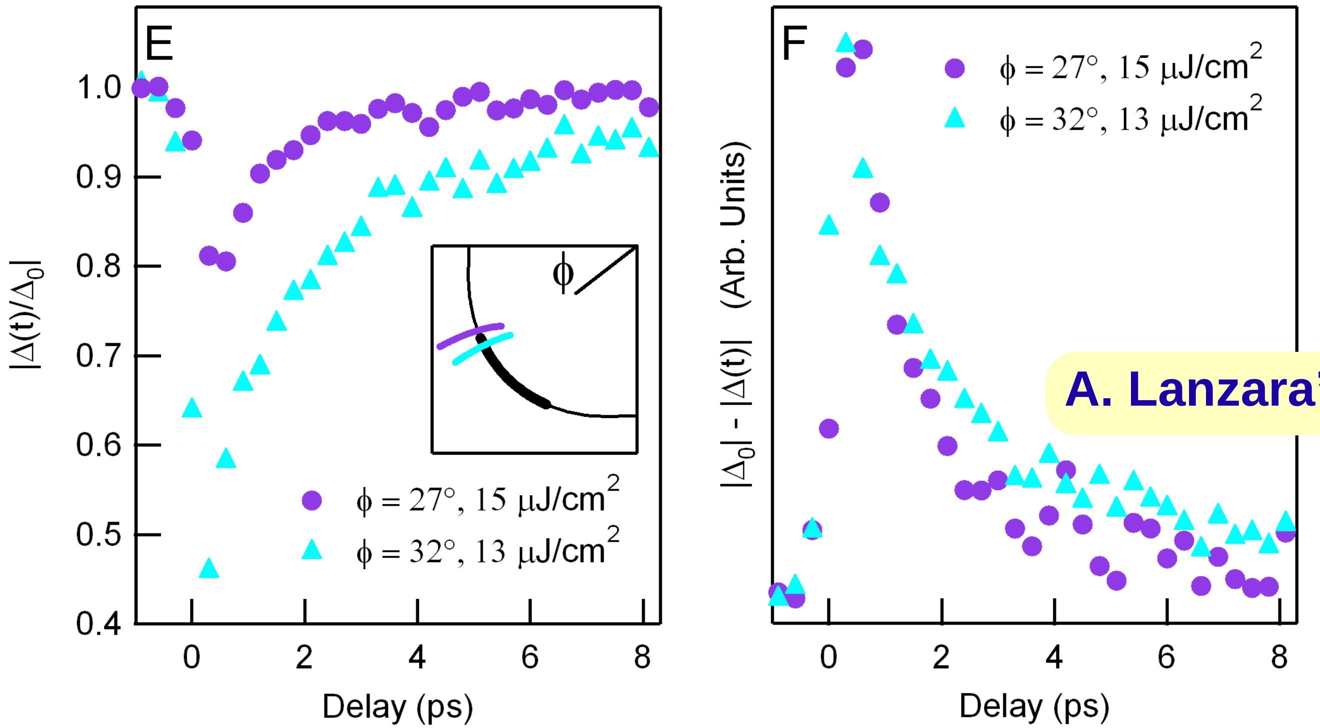

A. Lanzara's group

For $T < T_c$ the quasiparticle recovery time is $\sim$ 2 ps.

Ch.L. Smallwood et al, *Science* **336**, *1137 (2012).*

3. Methodology

Strongly correlated systems

/ Hubbard-Stratonovich transf. /

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We consider the strongly correlated fermion system

$$\hat{H} = \hat{T}_{kin} + U \int d\vec{r} \;\; \hat{c}^{\dagger}_{\uparrow}(\vec{r}) \; \hat{c}^{\dagger}_{\downarrow}(\vec{r}) \; \hat{c}_{\downarrow}(\vec{r}) \; \hat{c}_{\uparrow}(\vec{r})$$

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we can express the partition function by the path integral

$$Z = \int D \left[\bar{\psi}, \psi\right] e^{-S[\bar{\psi}, \psi]}$$

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where the imaginary-time fermionic action

$$S[\bar{\psi},\psi] = \int_0^{\beta} d\tau \int d\vec{r} \left[\sum_{\sigma} \bar{\psi}_{\sigma}(\vec{r},\tau) \left(\partial_{\tau} + \hat{\xi} \right) \psi_{\sigma}(\vec{r},\tau) \right.$$
$$\left. - \, g \; \bar{\psi}_{\uparrow}(\vec{r},\tau) \; \bar{\psi}_{\downarrow}(\vec{r},\tau) \; \psi_{\downarrow}(\vec{r},\tau) \psi_{\uparrow}(\vec{r},\tau) \right]$$

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and $\hat{\xi} \equiv -\hbar^2\nabla^2/2m - \mu, \qquad g = -U$.

Hubbard-Stratonovich – continued

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To eliminate the quartic term we introduce the auxiliary pairing fields

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The mean field (*saddle point*) solution usually relies on assumption of the static and uniform pairing field

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We try to go beyond this scheme treating the fermionic and bosonic degrees of freedom on an equal footing !

Boson-Fermion scenario

[in the lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\,\delta_{i,j})\,\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma} + \sum_{l}\left(E^{(B)}_{l} - 2\mu\right)\hat{b}^{\dagger}_{l}\hat{b}_{l}$$

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For some more specific derivation see for instance:

E. Altman and A. Auerbach, Phys. Rev. B **65***, 104508 (2002).*

or *Y. Yildirim and Wei Ku, Phys. Rev. X* **1***, 011011 (2011).*

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T. Domański and J. Ranninger, Phys. Rev. **B 63**, *134505 (2001).*

4. *Pre*-pairing above T_c

a) Bogoliubov quasiparticles

BCS excitation spectrum

/ in conventional superconductors /

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The effective (Bogoliubov) quasiparticles :

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$$A(\mathbf{k},\omega) = |u_{\mathbf{k}}|^2\, \delta(\omega - E_{\mathbf{k}}) + |v_{\mathbf{k}}|^2\, \delta(\omega + E_{\mathbf{k}})$$

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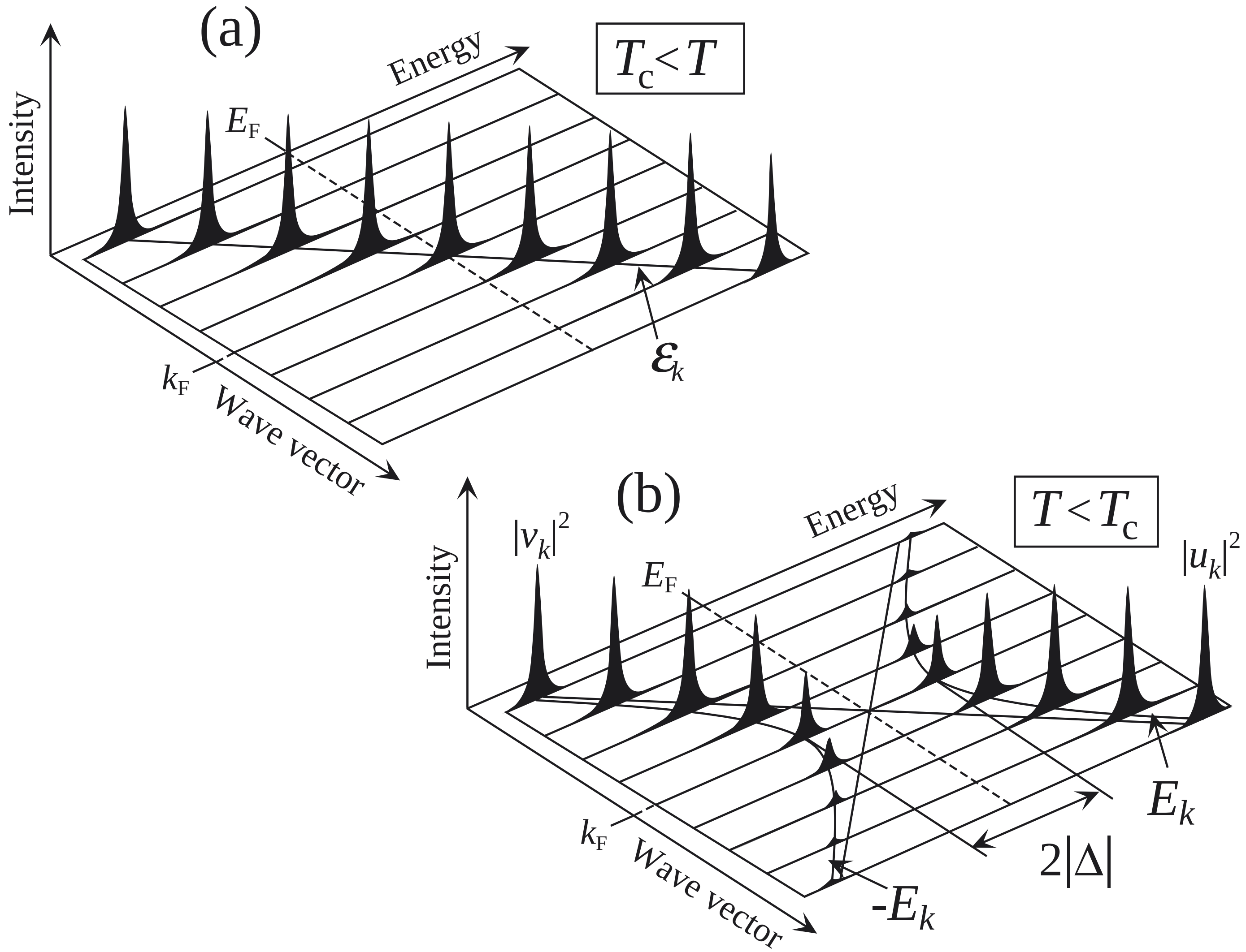
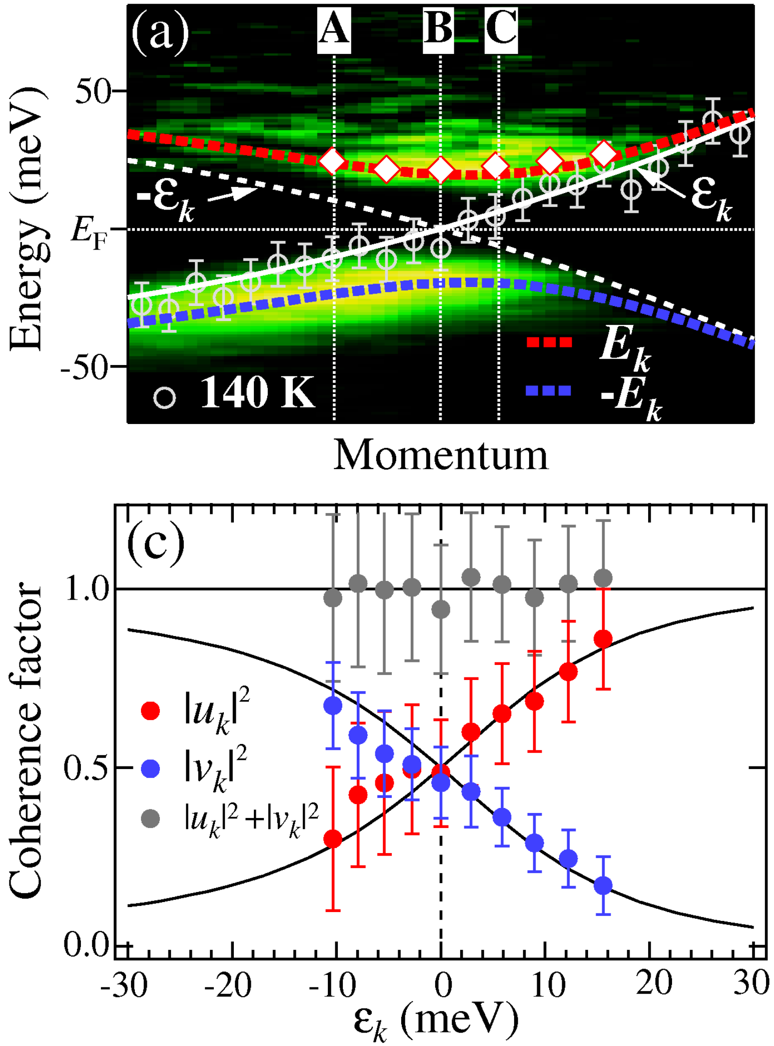
Occupancy of the momentum k is given by

$$n_k = |u_k|^2 \, f_{FD}(E_k) + |v_k|^2 \, \underbrace{f_{FD}(-E_k)}_{1-f_{FD}(E_k)}$$



The single particle spectrum (in conventional superconductors) consists of two Bogoliubov branches gaped around E_F.

Experimental data for cuprates – at $T < T_c$



H. Matsui, T. Sato, and T. Takahashi et al, *Phys. Rev. Lett.* **90**, *217002 (2003).*

Beyond the BCS approximation

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We have generalized the Bogoliubov ansatz, taking into account the non-condensed (preformed) pairs

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$$\begin{aligned}
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&\frac{1}{\sqrt{N}}\sum_{\mathbf{q}\neq 0}\left[u_{\mathbf{k},\mathbf{q}}(l)\,\hat{b}^{\dagger}_{\mathbf{q}}\hat{c}_{\mathbf{q}+\mathbf{k}\uparrow} + v_{\mathbf{k},\mathbf{q}}(l)\,\hat{b}_{\mathbf{q}}\hat{c}^{\dagger}_{\mathbf{q}-\mathbf{k}\downarrow}\right], \\
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with the boundary conditions

$u_{\mathbf{k}}(0) = 1$ **and** $v_{\mathbf{k}}(0) = v_{\mathbf{k},\mathbf{q}}(0) = u_{\mathbf{k},\mathbf{q}}(0) = 0.$

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The corresponding fixed point values $\lim_{l\rightarrow\infty} u_{\mathbf{k}}(l)$ (and other parameters) have been determined solving the set of coupled flow equations

$$\frac{\partial}{\partial l}u_{\mathbf{k}}(l)\,, \quad \frac{\partial}{\partial l}v_{\mathbf{k}}(l)\,, \quad \frac{\partial}{\partial l}u_{\mathbf{k},\mathbf{q}}(l)\,, \quad \frac{\partial}{\partial l}v_{\mathbf{k},\mathbf{q}}(l)\,.$$

Bogoliubov QPs above T_c

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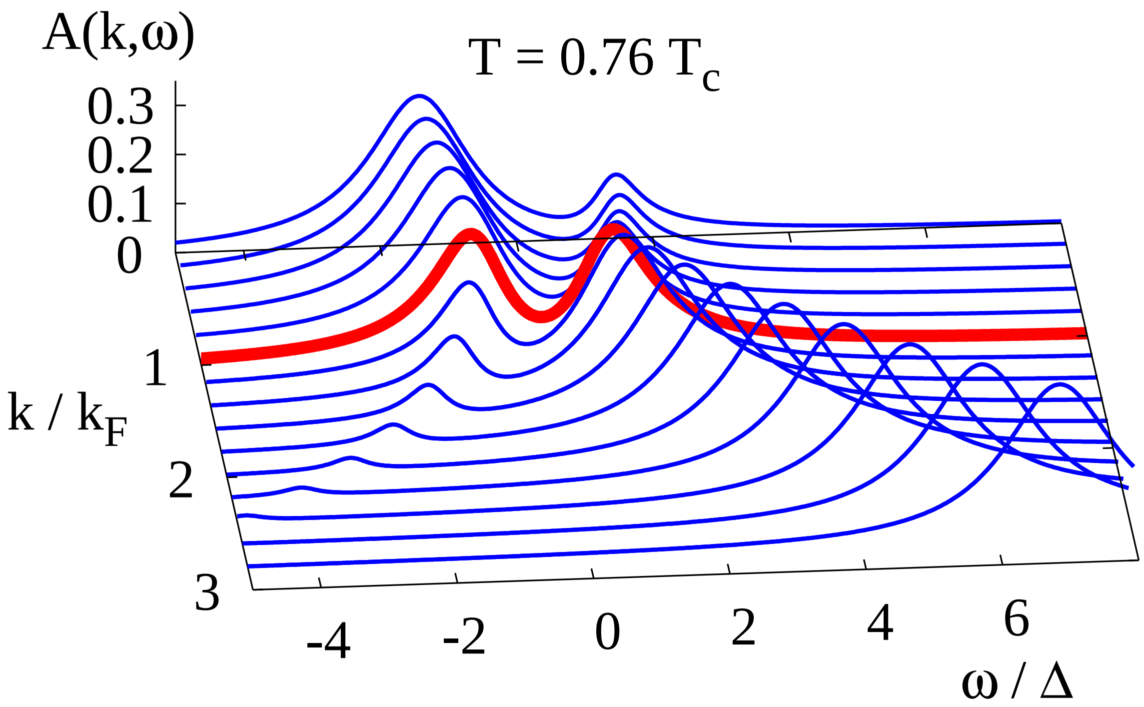

T. Domański and J. Ranninger, Phys. Rev. Lett. **91**, *255301 (2003).*

T. Domański, Phys. Rev. A **84**, *023634 (2011).*

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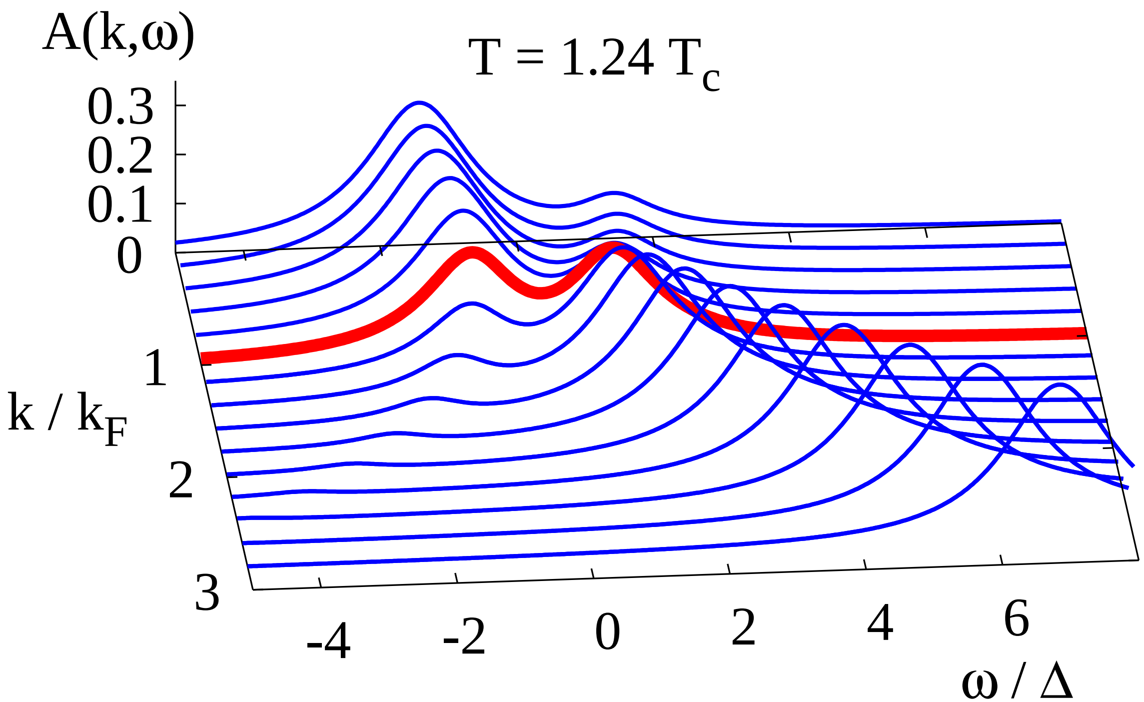

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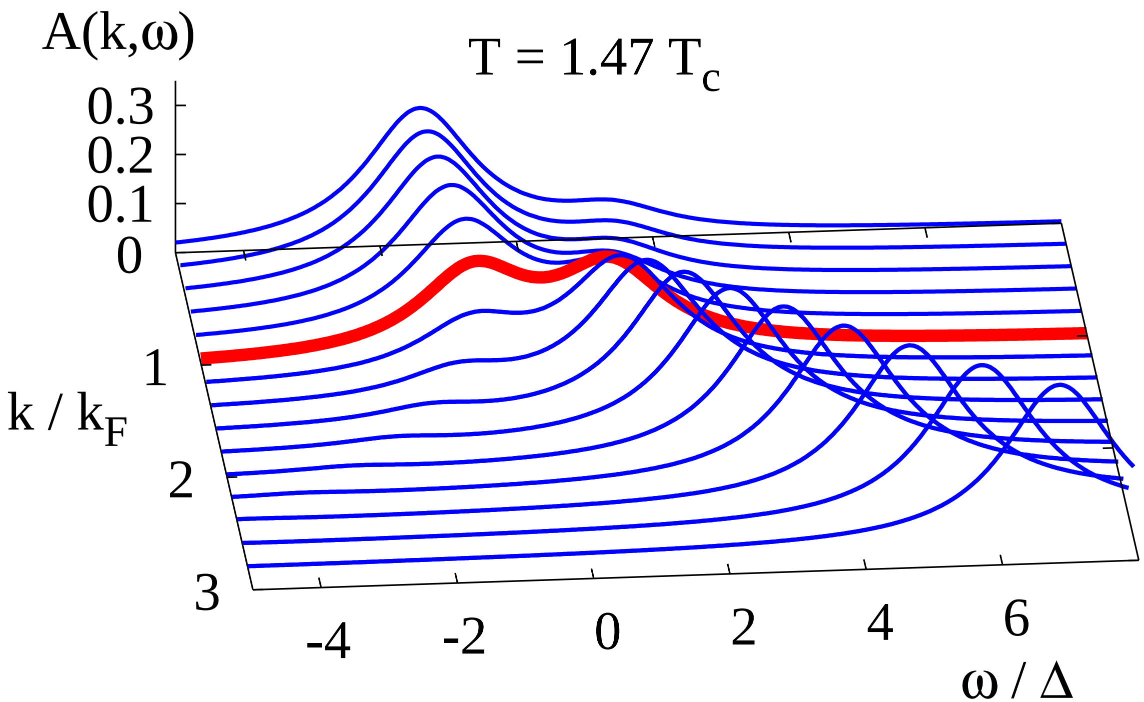

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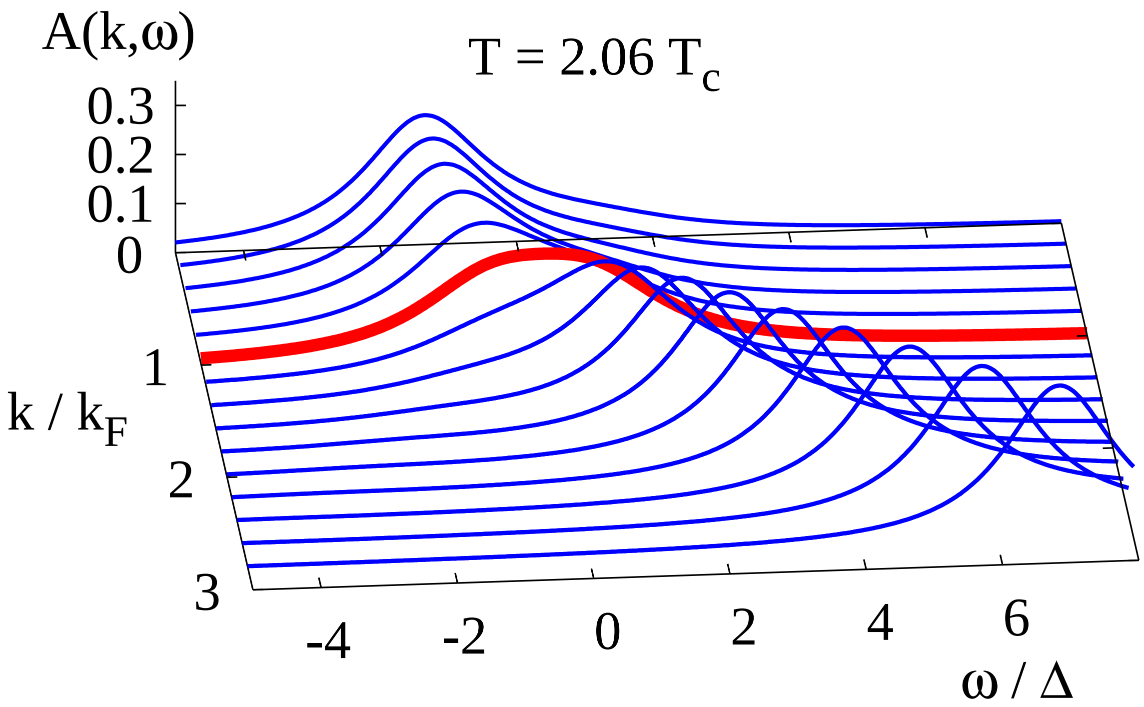

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Evidence for Bogoliubov QPs above T_c

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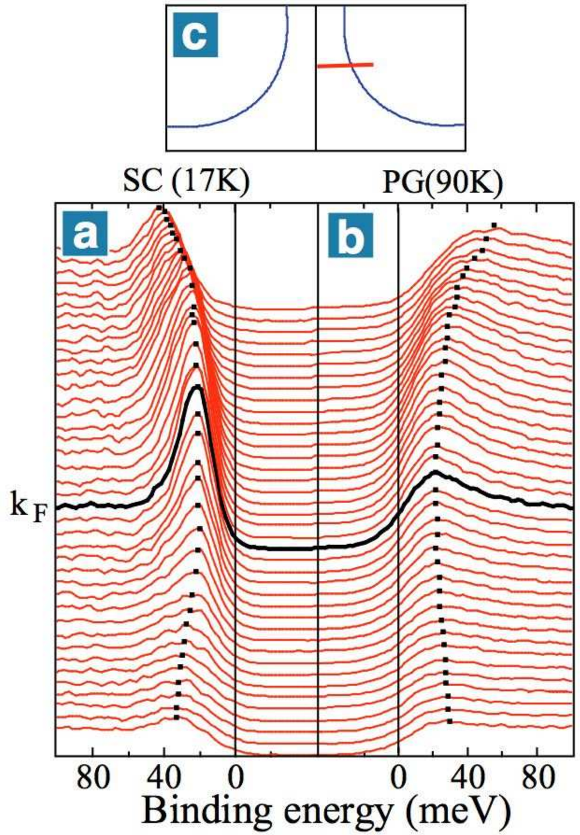
J. Campuzano group (Chicago, USA)

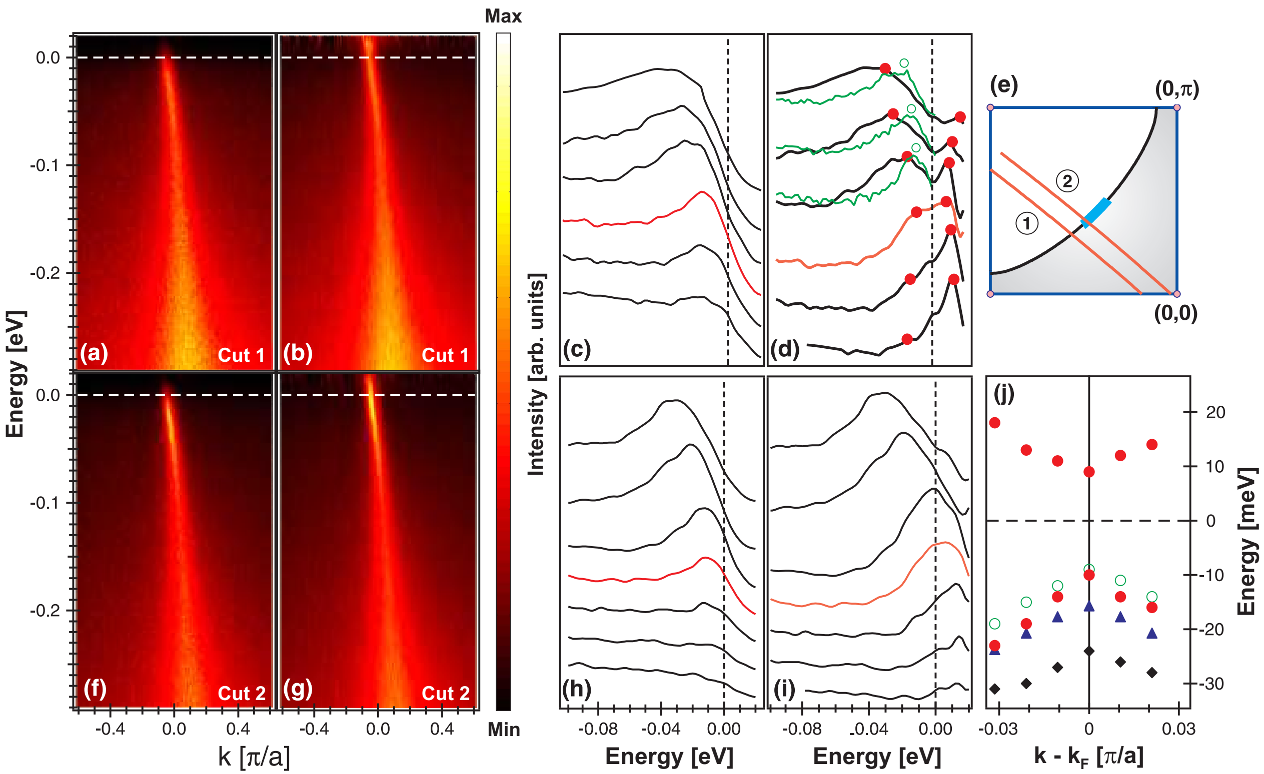


Results for: $Bi_2Sr_2CaCu_2O_8$

A. Kanigel et al, Phys. Rev. Lett. **101***, 137002 (2008).*

Evidence for Bogoliubov QPs above T_c

PSI group (Villigen, Switzerland)

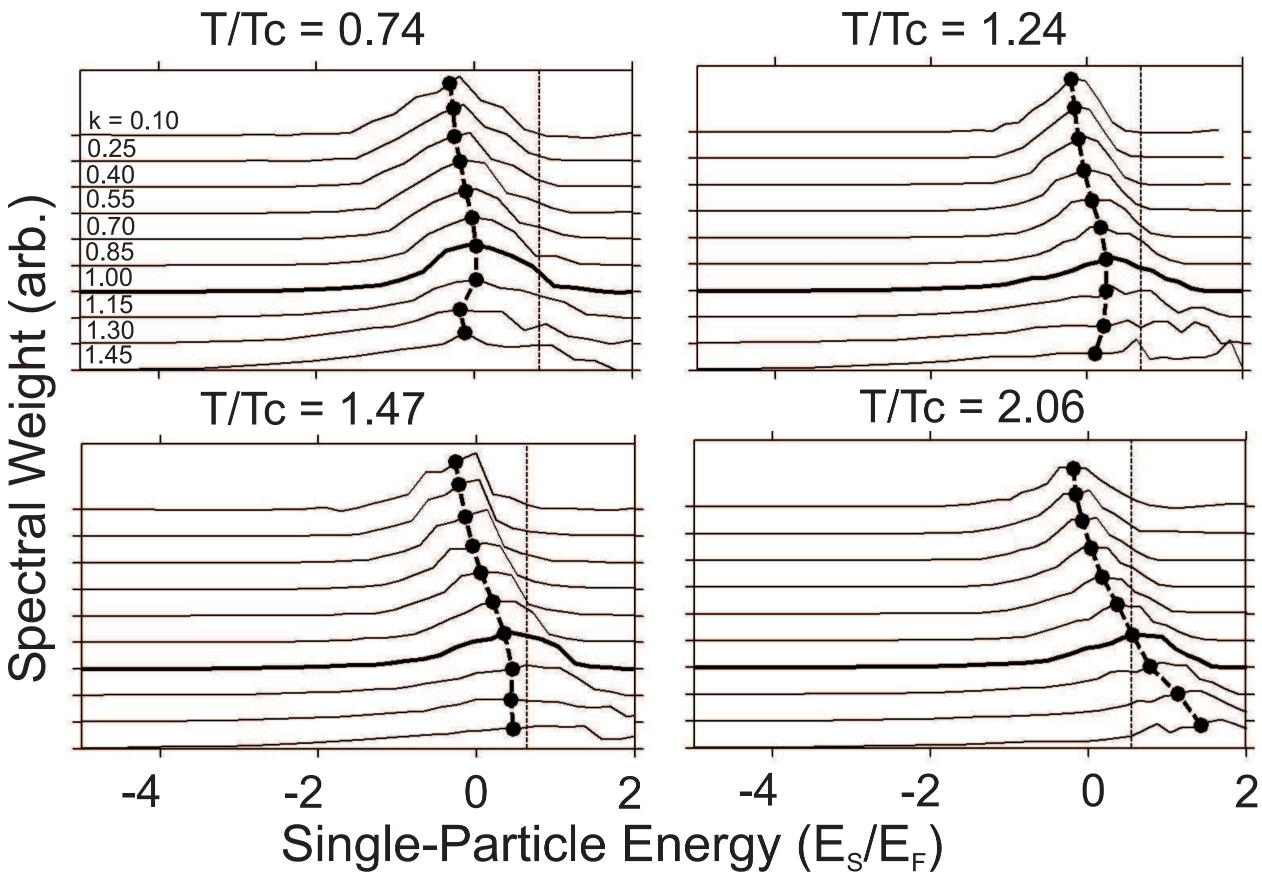


Results for: $La_{1.895}Sr_{0.105}CuO_4$

M. Shi et al, Eur. Phys. Lett. **88**, *27008 (2009).*

Evidence for Bogoliubov QPs above T_c

D. Jin group (Boulder, USA)



Results for: ultracold ^{40}K atoms

J.P. Gaebler et al, Nature Phys. **6**, *569 (2010).*

Local spectrum

experimental STM data

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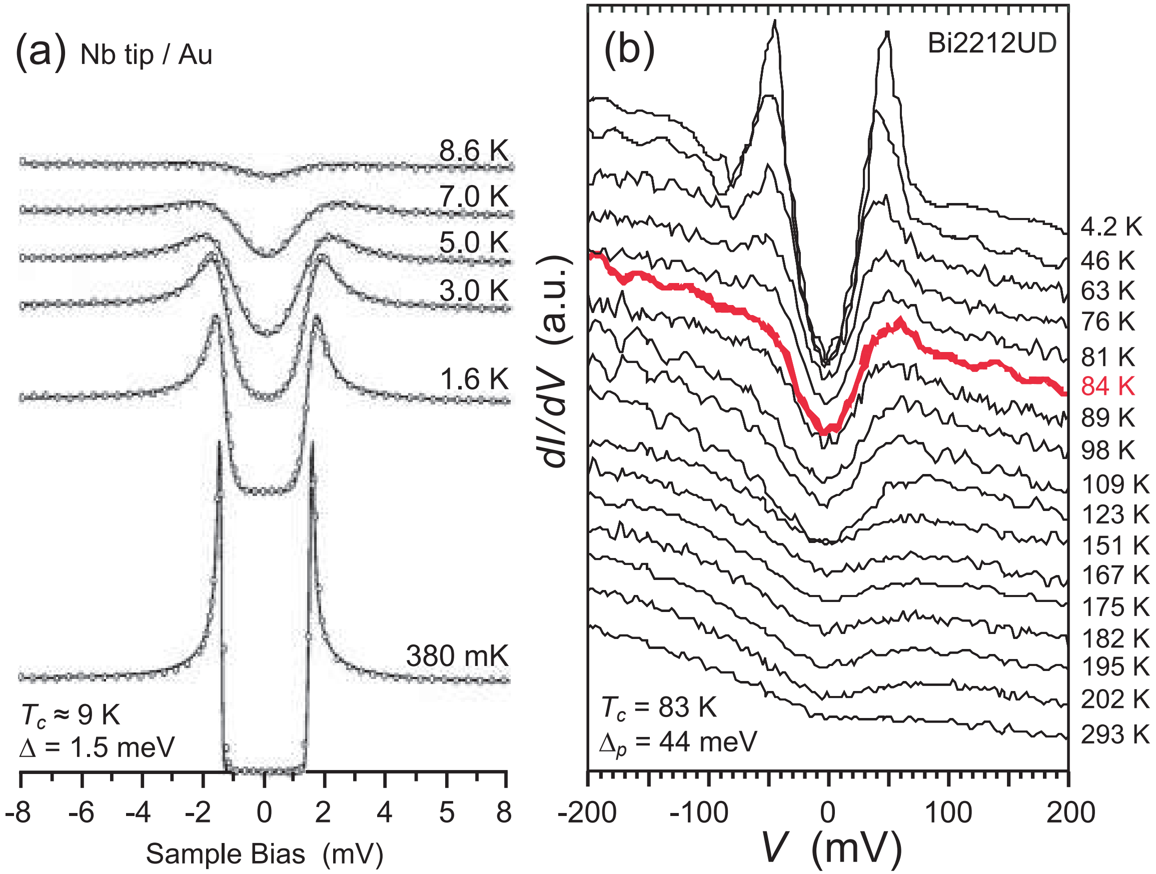
conv. sc. | the high T_c cuprates



Ch. Renner et al, *Phys. Rev. Lett.* **80**, *149 (1998).*

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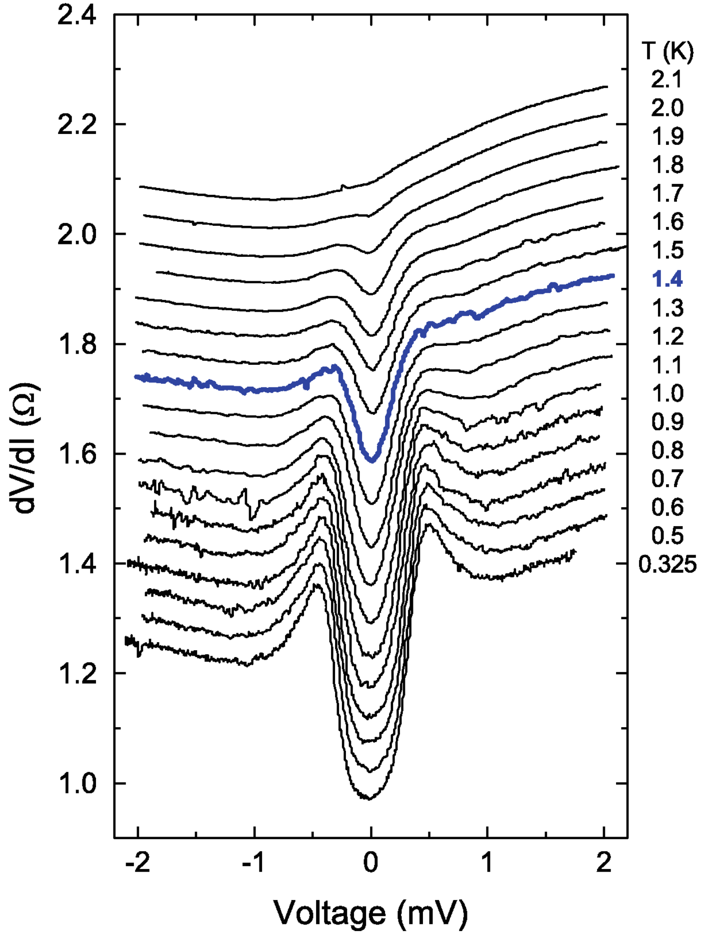
URu_2Si_2



F. Morales and R. Escudero, J. Low Temp. Phys. **154**, *68 (2009).*

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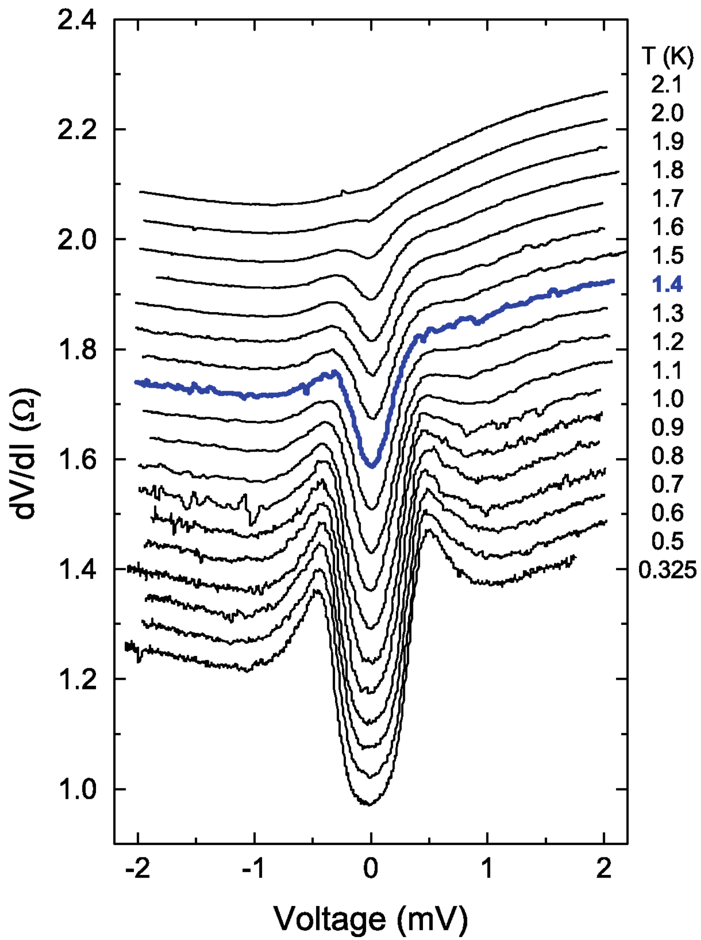
The superconducting pseudogap seems to persist up to $\sim 1.5T_c$



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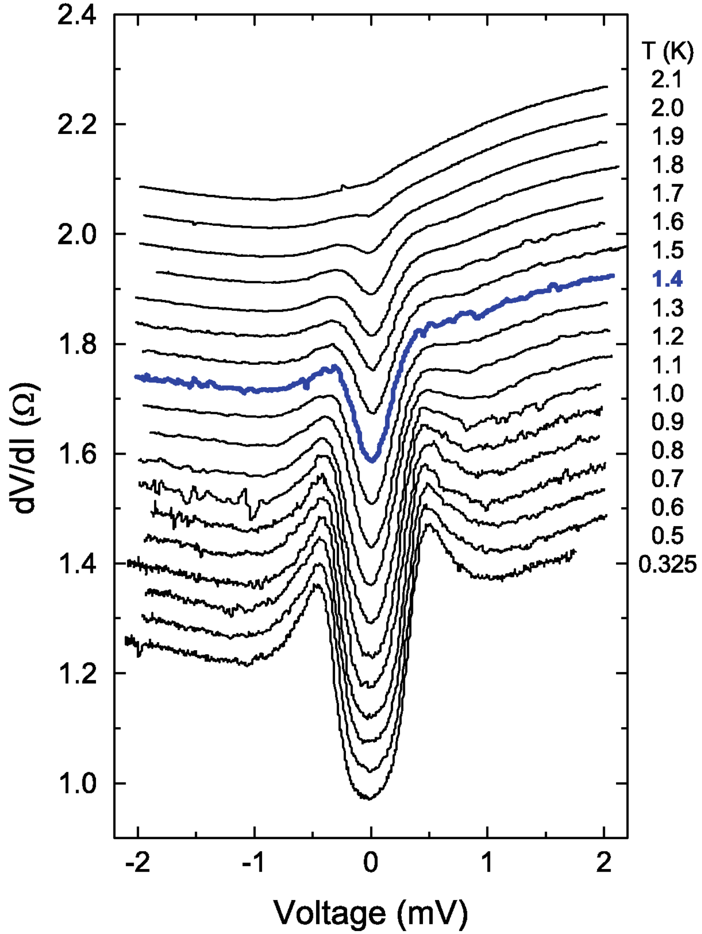
The Bogoliubov quasiparticle branches should be observable too !!!

/ by ARPES or SI-STM /



F. Morales and R. Escudero, J. Low Temp. Phys. **154**, *68 (2009).*

4. *Pre*-pairing above T_c

b) residual Meissner effect

Correlation functions

For studying the diamagnetic response (in the Kubo formalism)

we have to determine the current-current correlation function

$$- \hat{T}_\tau \langle \hat{j}_{\mathrm{q}}(\tau) \; \hat{j}_{-\mathrm{q}}(0) \rangle$$

with statistical averaging defined as

$$\langle ... \rangle = \mathrm{Tr} \left\{ e^{-\beta \hat{H}} ... \right\} / \mathrm{Tr} \left\{ e^{-\beta \hat{H}} \right\}$$

and $\beta^{-1} = k_B T$.

This can be achieved using the following invariance

$$\begin{aligned}
\mathrm{Tr} \left\{ e^{-\beta \hat{H}} \hat{O} \right\} &= \mathrm{Tr} \left\{ e^{\hat{S}(l)} e^{-\beta \hat{H}} \hat{O} e^{-\hat{S}(l)} \right\} \\
&= \mathrm{Tr} \left\{ e^{\hat{S}(l)} e^{-\beta \hat{H}} e^{-\hat{S}(l)} e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)} \right\} \\
&= \mathrm{Tr} \left\{ e^{-\beta \hat{H}(l)} \hat{O}(l) \right\}
\end{aligned}$$

where

$$\hat{H}(l) = e^{\hat{S}(l)} \hat{H} e^{-\hat{S}(l)} \qquad \hat{O}(l) = e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)}$$

Technicalities

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The initial current operator

$$\hat{j}_{\mathbf{q},\sigma} = \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}+\frac{\mathbf{q}}{2}} \hat{c}^{\dagger}_{\mathbf{k},\sigma} \hat{c}_{\mathbf{k}+\mathbf{q},\sigma}$$

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is constrained (from the flow equation) in the form

$$\hat{\mathbf{j}}_{\mathbf{q},\uparrow}(l) = \sum_{\mathbf{k}} \mathrm{v}_{\mathbf{k}+\frac{\mathbf{q}}{2}} \Big(\mathcal{A}_{\mathbf{k},\mathbf{q}}(l) \hat{c}^{\dagger}_{\mathbf{k},\uparrow} \hat{c}_{\mathbf{k}+\mathbf{q},\uparrow} + \mathcal{B}_{\mathbf{k},\mathbf{q}}(l) \hat{c}_{-\mathbf{k},\downarrow} \hat{c}^{\dagger}_{-(\mathbf{k}+\mathbf{q}),\downarrow}$$
$$+ \sum_{\mathbf{p}} \Big(\mathcal{D}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) \hat{b}_{\mathbf{k}+\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{k},\uparrow} \hat{c}^{\dagger}_{\mathbf{p}-\mathbf{q},\downarrow} + \mathcal{F}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) \hat{b}^{\dagger}_{\mathbf{k}+\mathbf{p}} \hat{c}_{\mathbf{p},\downarrow} \hat{c}_{\mathbf{k}+\mathbf{q},\uparrow} \Big) \Big)$$

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with the boundary conditions

$\mathcal{A}_{\mathbf{k},\mathbf{q}}(0) = 1$ **and** $\mathcal{B}_{\mathbf{k},\mathbf{q}}(0) = \mathcal{D}_{\mathbf{k},\mathbf{p},\mathbf{q}}(0) = \mathcal{F}_{\mathbf{k},\mathbf{p},\mathbf{q}}(0) = 0$

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We next determine all fixed point values $\lim_{l\to\infty} \mathcal{A}_{\mathbf{k},\mathbf{q}}(l) \equiv \tilde{\mathcal{A}}_{\mathbf{k},\mathbf{q}}$ **etc**

from the set of flow equations

$$\frac{\partial}{\partial l}\mathcal{A}_{\mathbf{k},\mathbf{q}}(l) \;,\quad \frac{\partial}{\partial l}\mathcal{B}_{\mathbf{k},\mathbf{q}}(l) \;,\quad \frac{\partial}{\partial l}\mathcal{D}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) \;,\quad \frac{\partial}{\partial l}\mathcal{F}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) .$$

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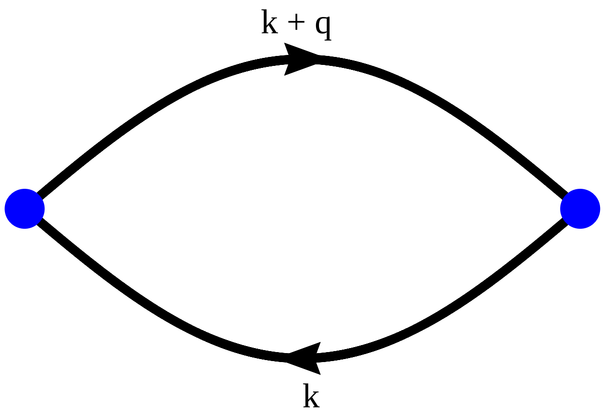

$\Longleftarrow$ **the usual bubble diagram**

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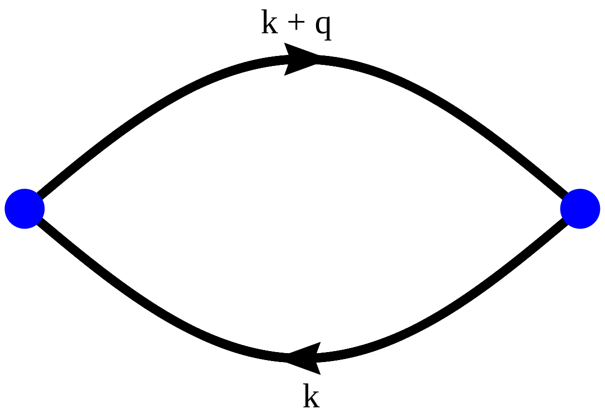

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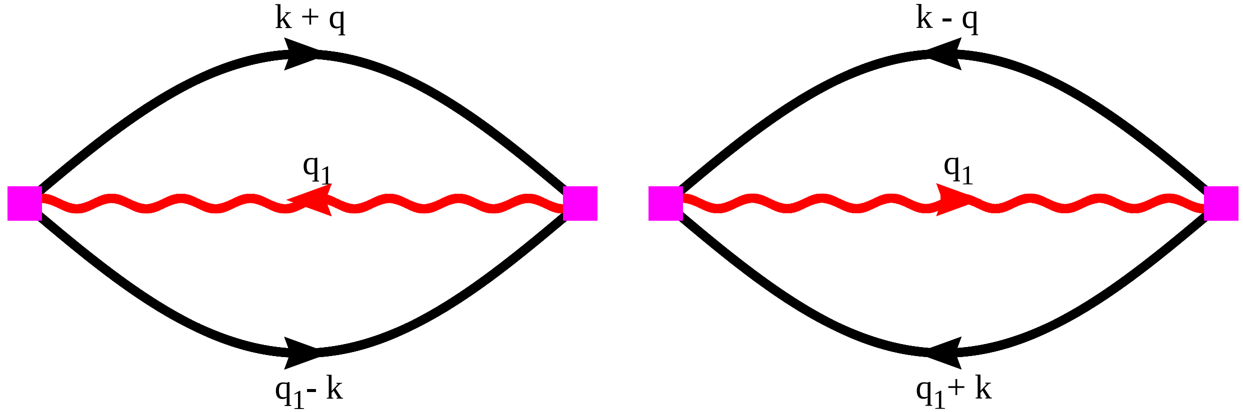

anomalous diagrams

Vertex functions have to be determined from the flow equations.

M. Zapalska, T. Domański, Phys. Rev. B ***84****, 174520 (2011).*

Onset of diamagnetism above T_c

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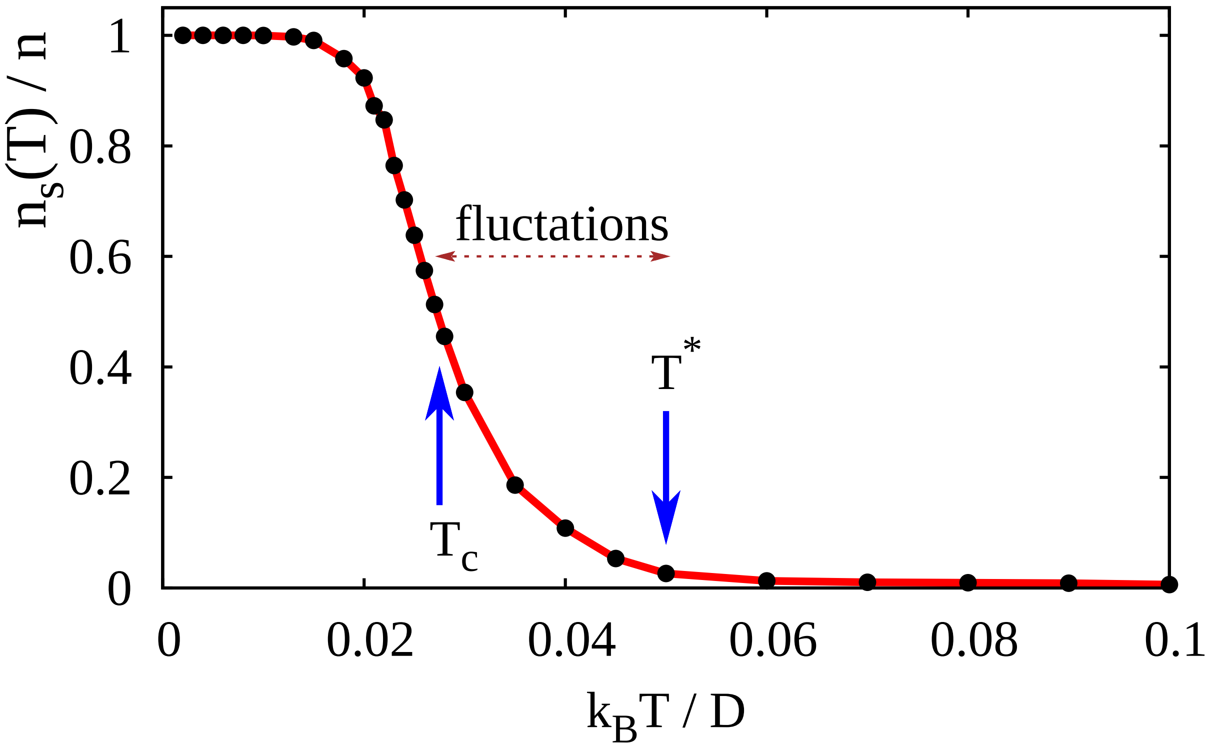
Residual diamagnetism emerges simultaneously with the collective features.

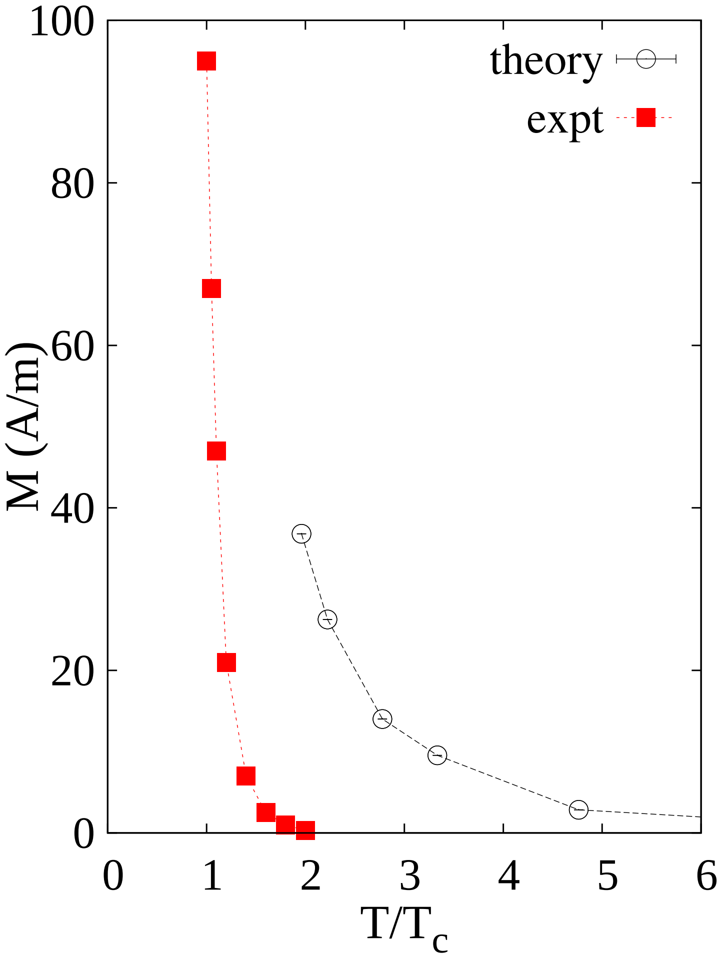


$$\vec{j} = -\frac{e^2 n_s(T)}{m}\,\vec{A}$$

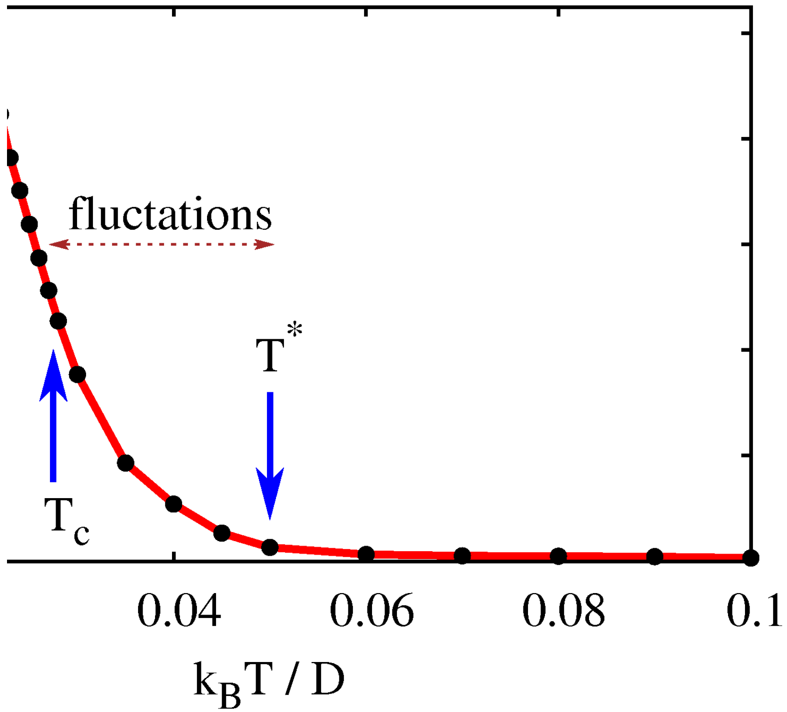
M. Zapalska, T. Domański, Phys. Rev. B **84**, *174520 (2011).*

Onset of diamagnetism above T_c

Comparison to the Quantum Monte Carlo simulations & experimental data



M. Zapalska, T.D., Phys. Rev. B ***84****, 174520 (2011).*



QMC: *K.-Y. Yang, ... and M. Troyer, Phys. Rev. B* ***83****, 214516 (2011).* **/ ETH Zürich, Switzerland /**

expt: *L. Li, ... and N.P. Ong, Phys. Rev. B* ***81****, 054510 (2010).* **/ Princeton, USA /**

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