Himeji, 4 July 2012

Dualities in superconductors with the local pairing

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Collaboration :

J. Ranninger (Grenoble) and M. Zapalska (Lublin)

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Outline

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- **Preliminaries**

/ pairing vs coherence /

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- **Dualities**

  / in strongly correlated systems /

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  / RG scaling equations /

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- ***Pre*-pairing above T_c**

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$\Rightarrow$ *Bogoliubov quasiparticles*

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- ***Pre*-pairing above T_c**

  - *Bogoliubov quasiparticles*
  - *residual Meissner effect*

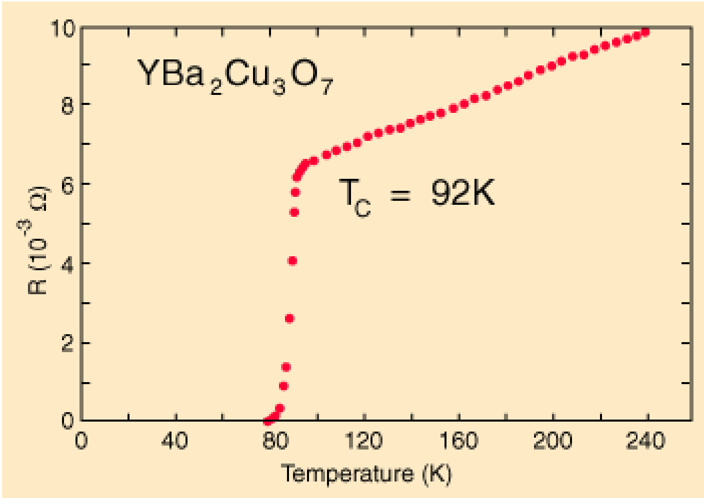
- **Summary**

1. Preliminaries

Superconducting state – properties

Superconducting state – properties

- ideal d.c. conductance



Superconducting state – properties

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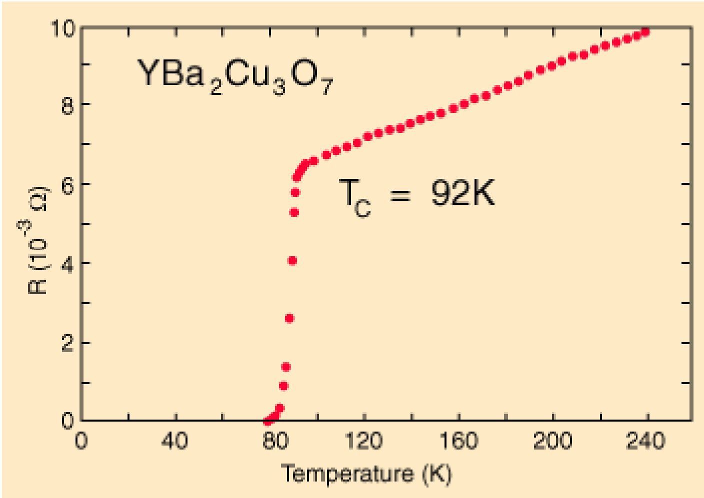


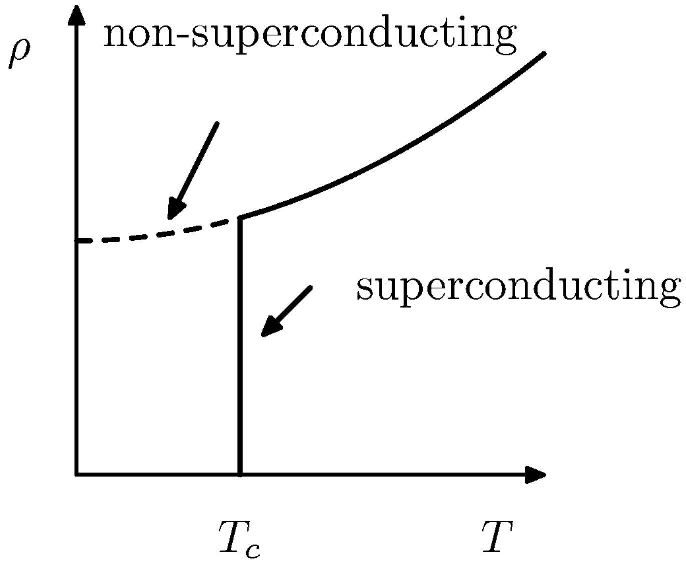
Normal conductors:

resistance $R = \rho \frac{l}{S}$

where $\rho \equiv 1/\sigma$

and $\sigma = \frac{ne^2\tau}{m}$

$\tau(T)$ – **relaxation time**



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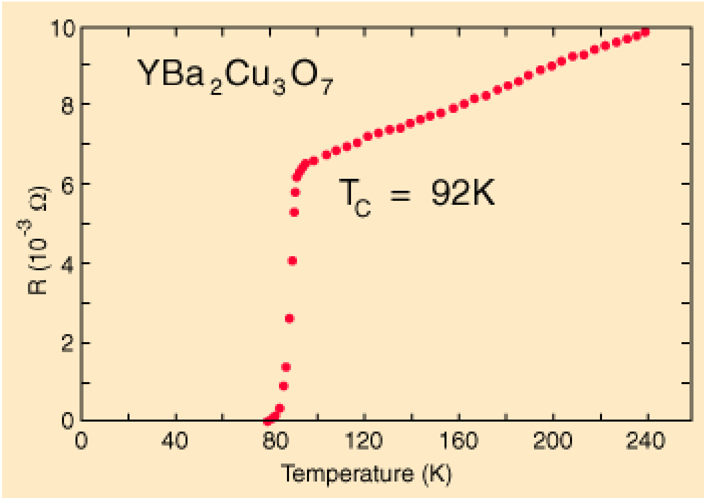


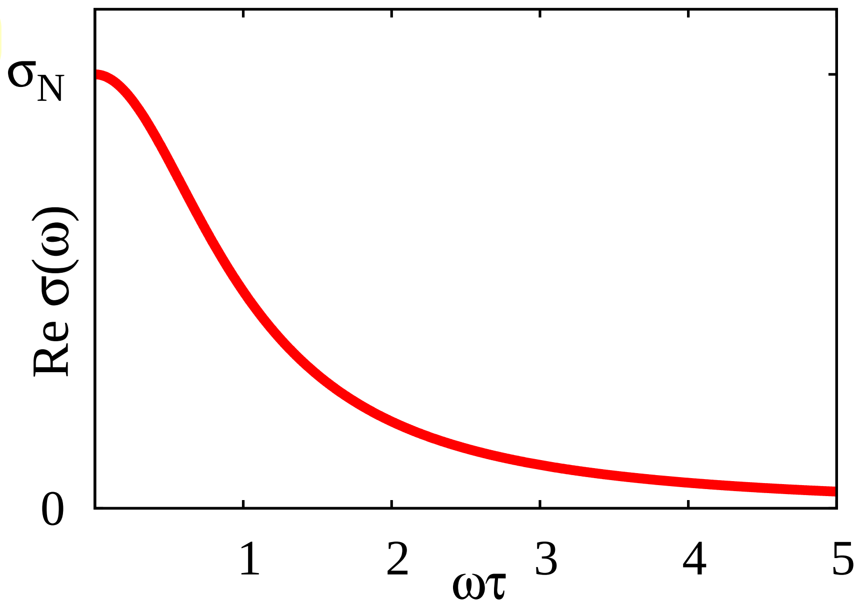
a.c. conductance (above T_c)

Drude conductance

$$\sigma(\omega) = \frac{ne^2\tau}{m} \frac{1}{1-i\omega\tau}$$

obeys the **f-sum rule**

$$\int_{-\infty}^{\infty} \mathrm{Re}\, \sigma(\omega) = \pi \frac{ne^2}{m}$$



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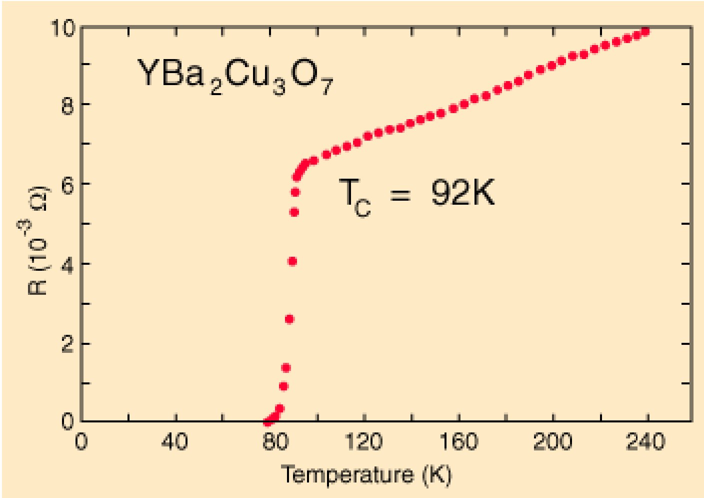


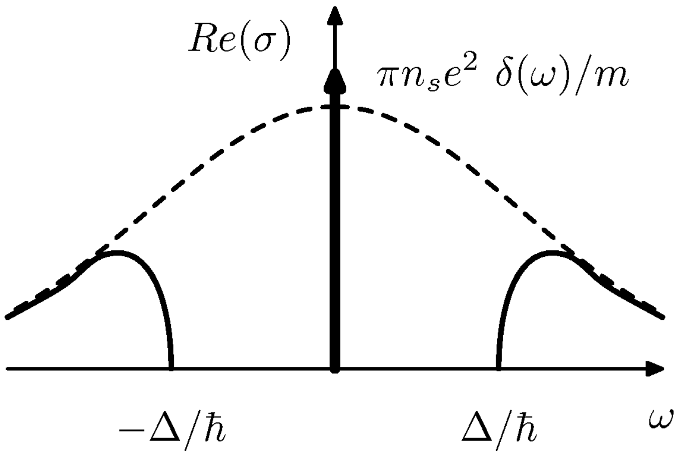
a.c. conductance (below T_c)

The f-sum rule

$$\int_{-\infty}^{\infty} \mathrm{Re}\, \boldsymbol{\sigma}(\omega) = \pi \, \frac{ne^2}{m}$$

must be also obeyed below T_c, however

$$n = n_n(T) + n_s(T)$$



$n_s(T)$ – **superfluid density**

Superconducting state – properties (continued)

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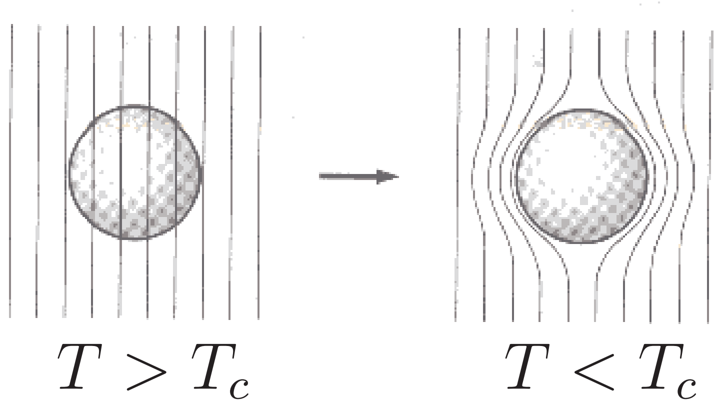
★ **ideal diamagnetism** /perfect screening of the d.c. magnetic field/



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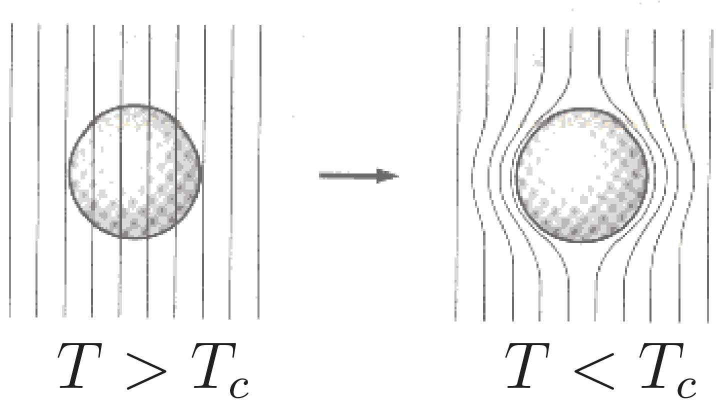


Meissner effect is described by the Londons' equation

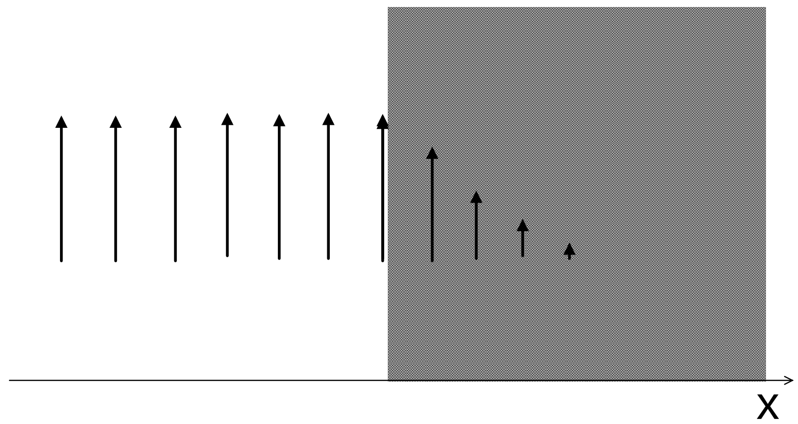
$$\vec{j} = -\frac{e^2 n_s(T)}{mc^2}\vec{A}$$

where the coefficient

$$\frac{e^2 n_s(T)}{mc^2} \equiv \rho_s(T) = \frac{1}{\lambda^2}$$

$\rho_s(T)$ **– superfluid stiffness**

$\lambda(T)$ – penetration depth



$$B(x) = B_0\, e^{-x/\lambda}$$

Superconducting state – basic concepts

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Simultaneous appearance of:

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⇒ **the ideal (d.c.) conductance**

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Superconducting state – basic concepts

Simultaneous appearance of:

$\Longrightarrow$ the ideal (d.c.) conductance

$\Longrightarrow$ and the Meissner effect

is induced by the superfluid fraction

$$n_s(T)$$

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Onset of the fermion pairing often goes hand in hand with appearance of the superconductivity/superfluidity but it doesn't have to be a rule.

Formal issues – generalities

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The order parameter

$$\chi(\vec{r}, t) \equiv \langle \hat{c}_{\downarrow}(\vec{r}) \hat{c}_{\uparrow}(\vec{r}) \rangle$$

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$|\chi| \neq 0 \longrightarrow$ **amplitude causes the energy gap**

$\nabla\theta \neq 0 \longrightarrow$ **phase slippage induces supercurrents**

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1. closing the gap . . . . [conventional BCS superconductors]

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2. disordering the phase . . . . [HTSC compounds, URh_2Si_2 (?)]

$$\lim_{T \to T_c} \langle \theta \rangle = 0$$

Historical remark

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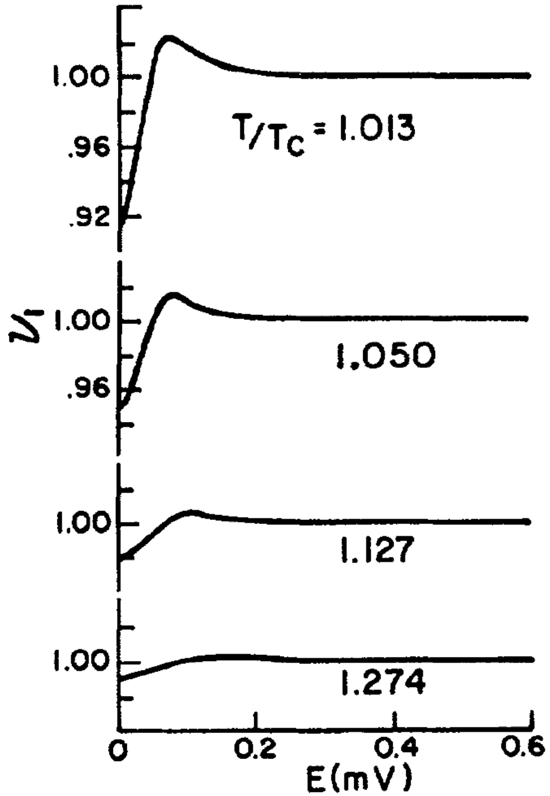
Tunneling conductance revealed a pseudogap surviving to $\sim 1.3T_c$.



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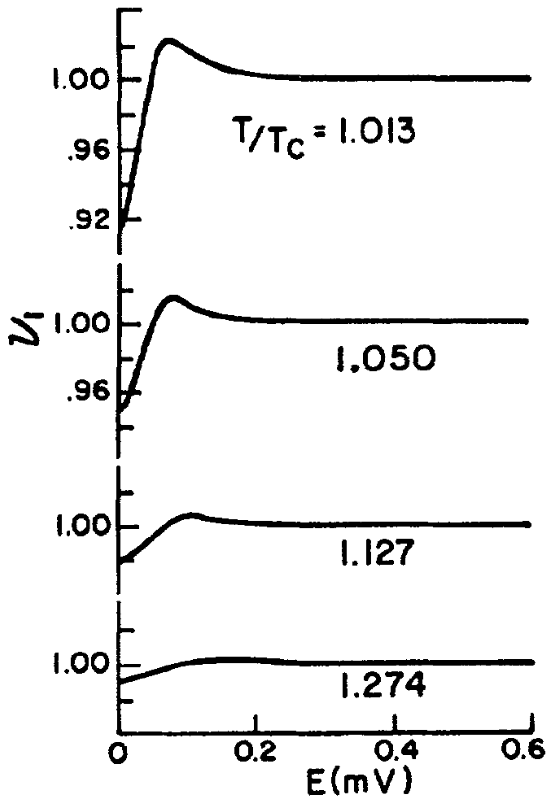

R.W. Cohen and B. Abels, *Phys. Rev.* **168**, 444 (1968).

2. Dualities

/ in the strongly correlated systems /

Duality 1: **amplitude vs phase driven transition**

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Critical temperature T_c can be related to:

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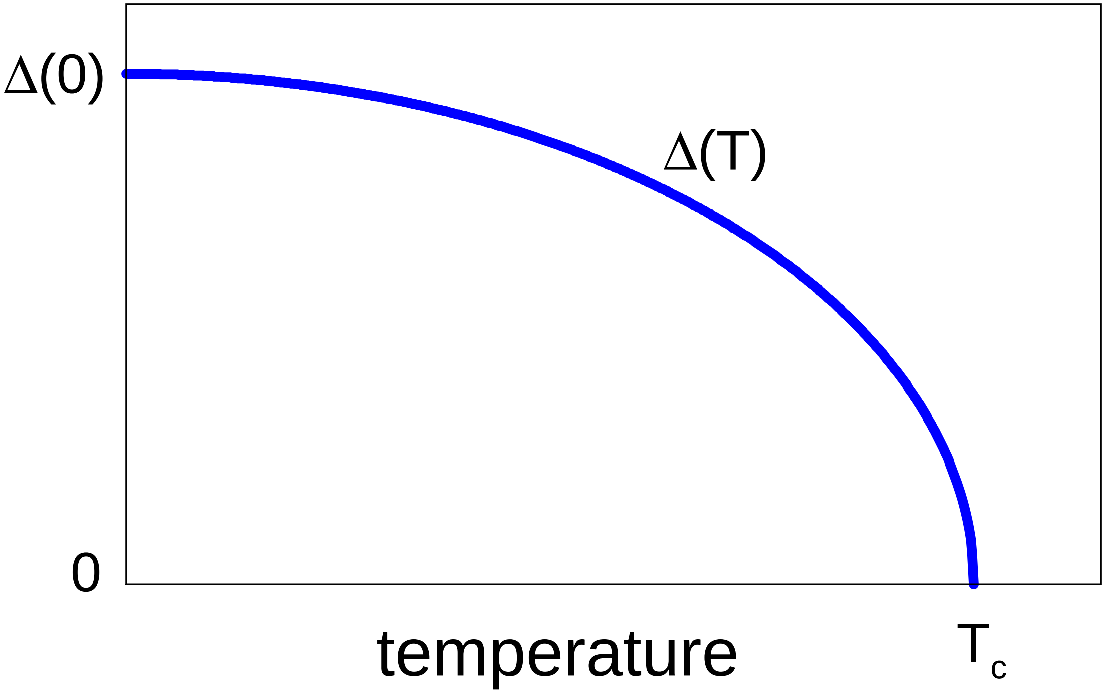

Pairing is responsible for the energy gap $\Delta(T)$ in a single particle spectrum

$\Delta(T_c) = 0$ **/ amplitude transition /**

Duality 1: amplitude vs phase driven transition

Critical temperature T_c can be related to:

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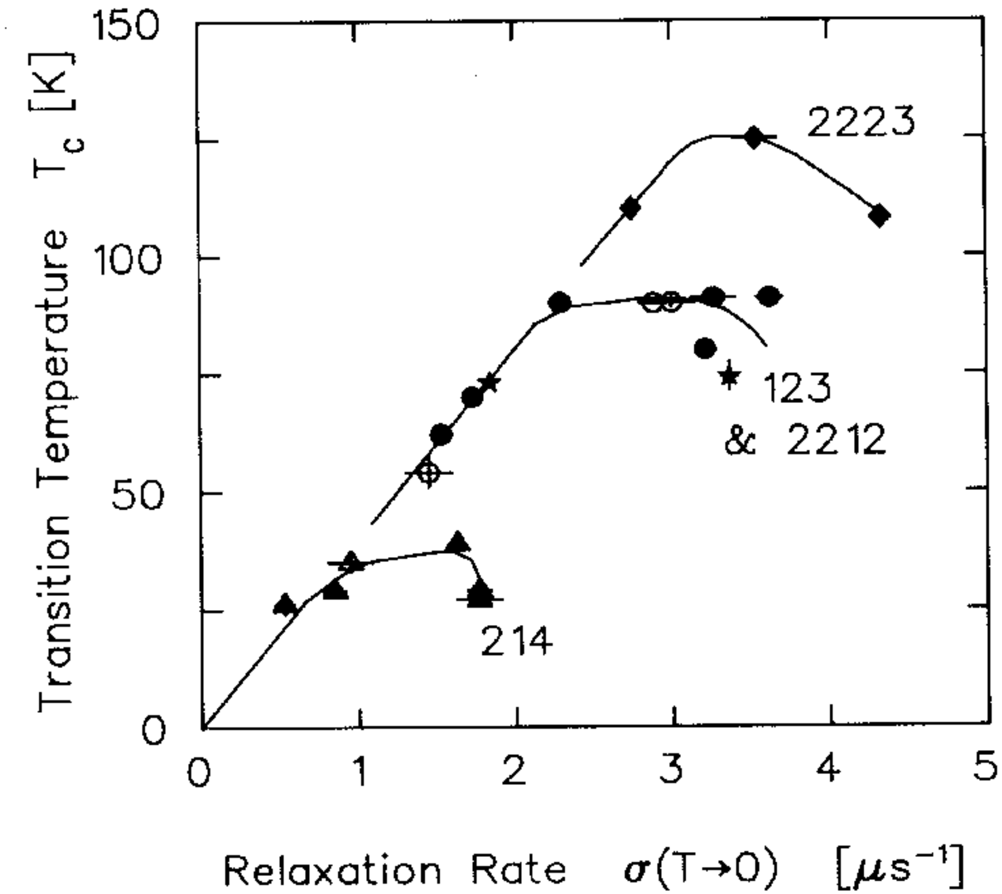

Y.J. Uemura et al, Phys. Rev. Lett. ***62****, 2317 (1989).*

Early experiments with the muon-spin relaxation have indicated that

$$T_c \propto \rho_s(0)$$

/ Uemura scaling /

The superfluid stiffness $\rho_s(T)$ is here defined by

$$\rho_s(T) \equiv \frac{1}{\lambda^2(T)} = \frac{4\pi e^2}{m^* c^2} n_s(T)$$

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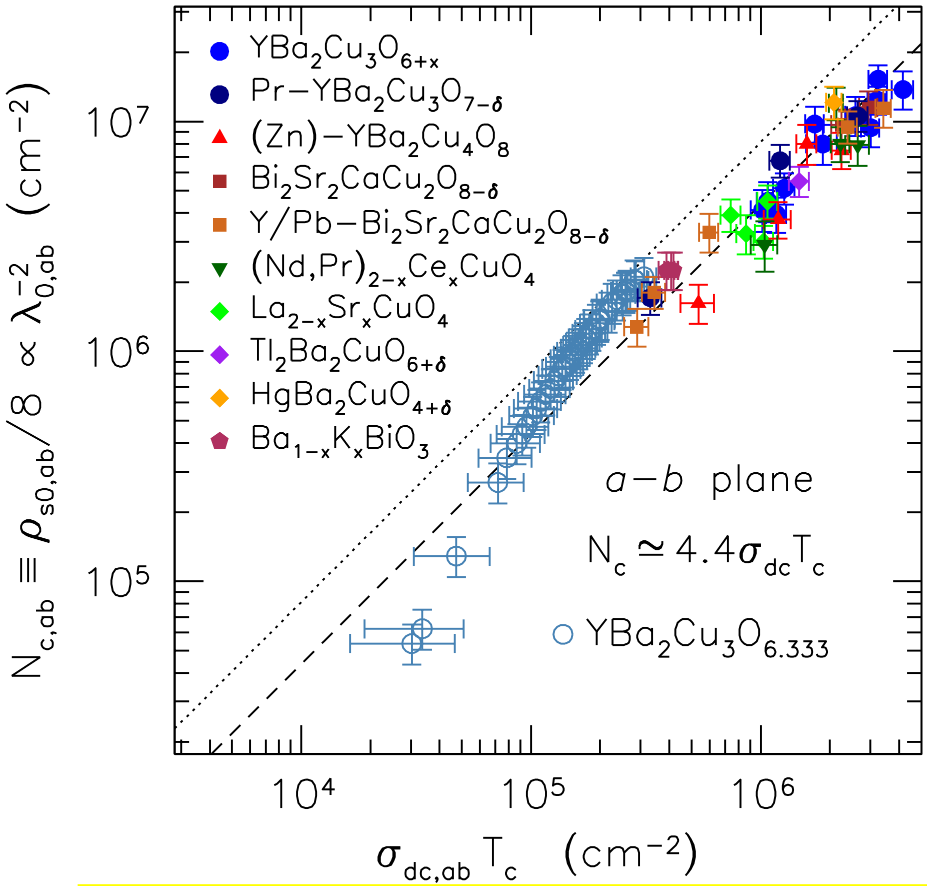

C.C. Homes, Phys. Rev. B **80***, 180509(R) (2009).*

Recently such scaling has been updated from transport measurements

$$\frac{1}{8}\rho_s = 4.4\sigma_{dc} T_c$$

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This new relation is valid for all samples ranging from the underdoped to overdoped region.

/ ab - plane /

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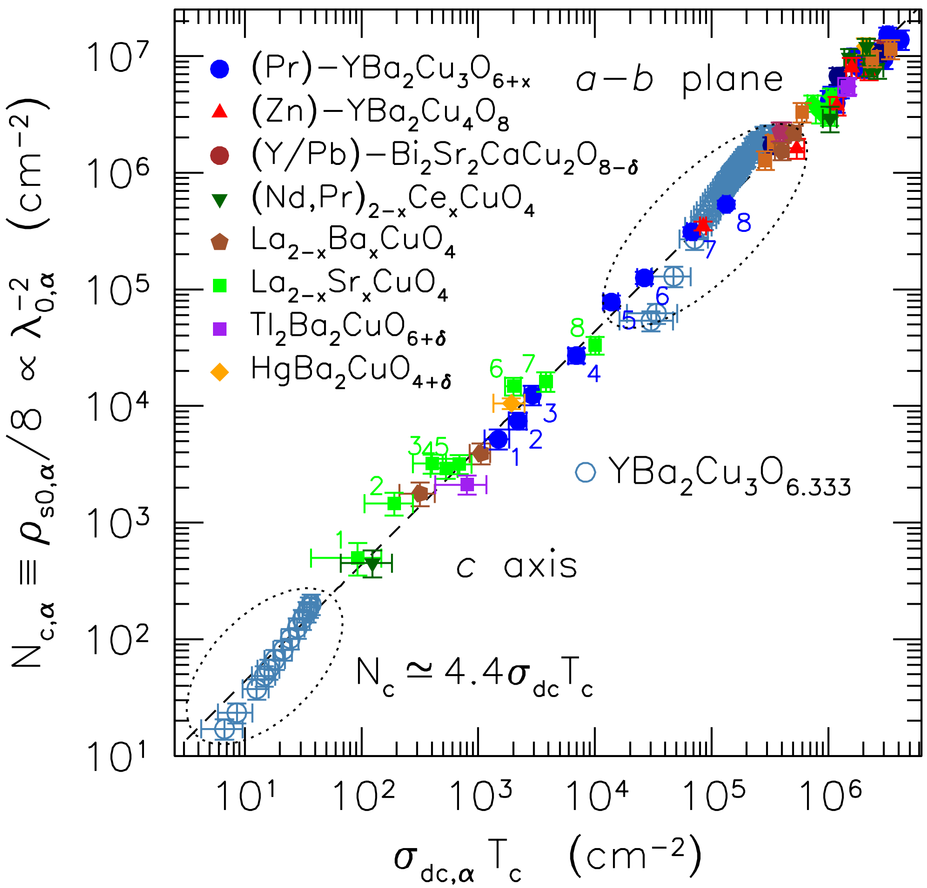

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/ c - axis /

Duality 2: superconducting vs magnetic order

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Conventional superconductors usually compete with the magnetic order. This relation is, however, no longer obvious for the local pair superconductors, where the pairing and magnetism might have a common origin.

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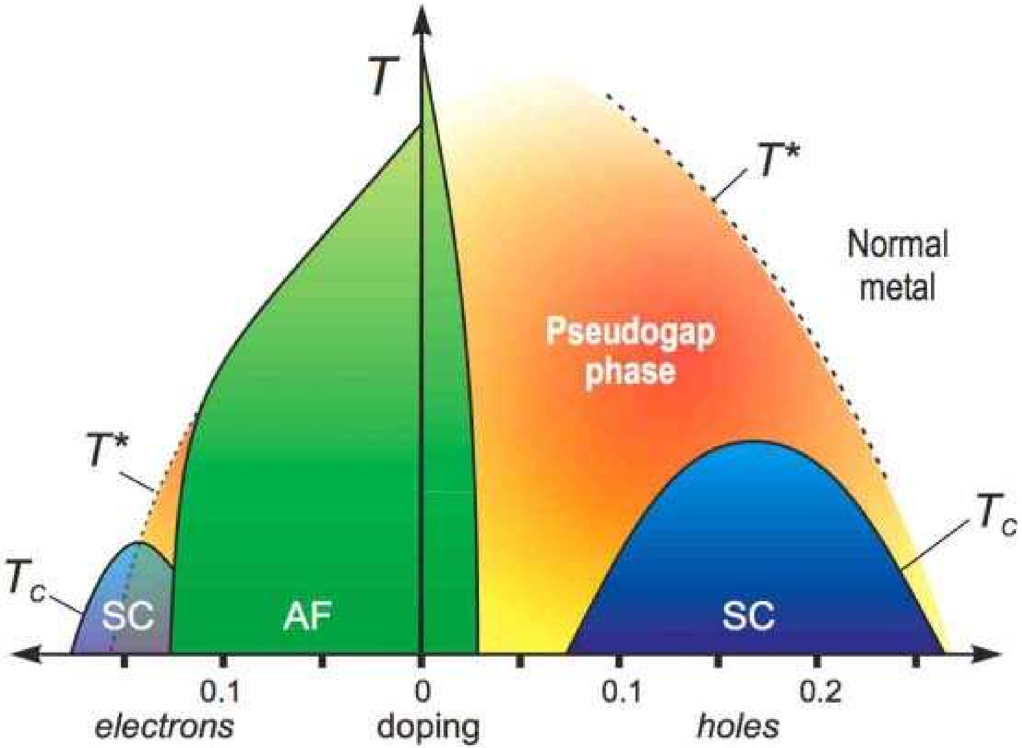

O. Fisher et al, *Rev. Mod. Phys.* **79**, *353 (2007).*

In the high T_c cuprates

d-wave superconductivity

appears near the AF insulator. In both cases the order comes from the exchange coupling

$$J_{ij} = \frac{2t_{ij}^2}{U}$$

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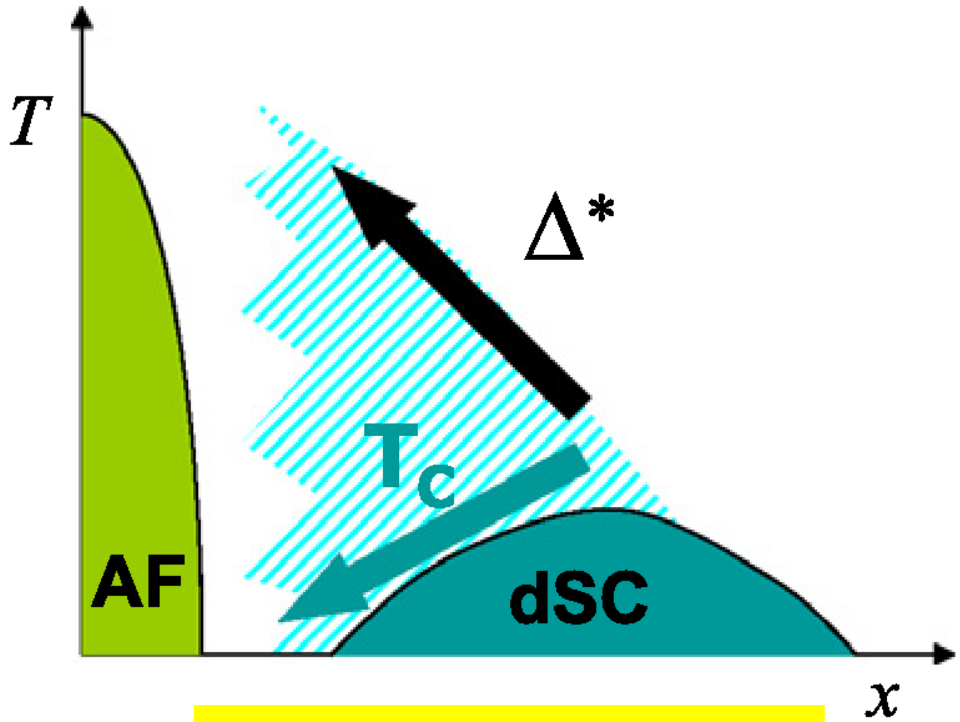

T. Valla, Physica C (2007) in print.

1. What type of relationship does occur between

dSC and AF order ?

2. Are they related with the pseudogap ?

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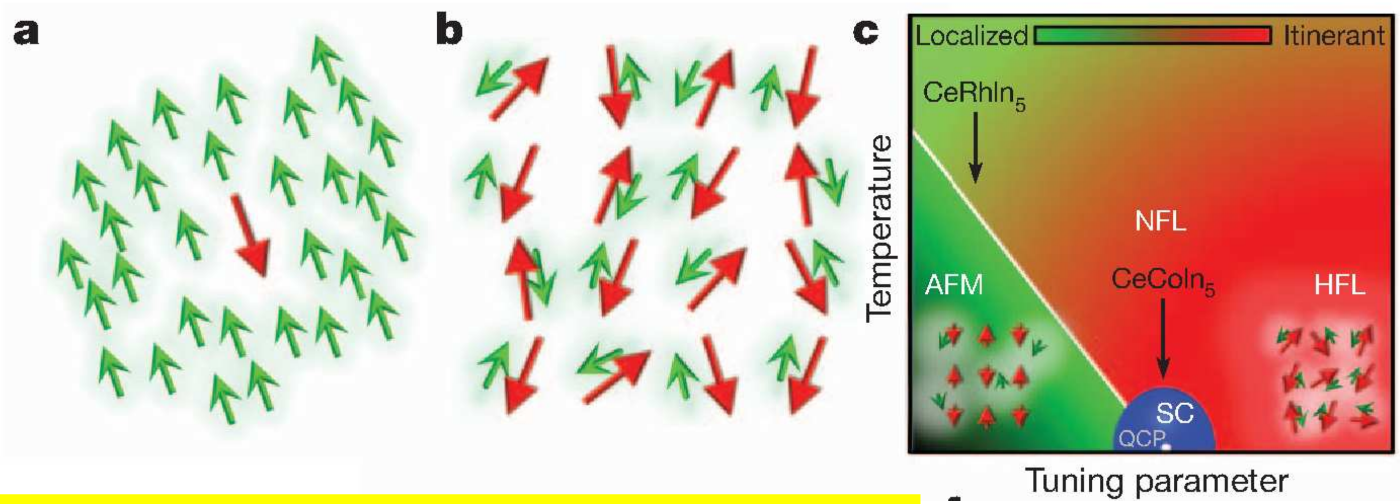

P. Aynajian, ... and A. Yazdani, Nature **486**, *201 (2012).*

Nontrivial intrelation between magnetism and superconductivity occurs in

the heavy fermion compounds

Their properties depend qualitavely on an interplay between

localized and itinerant electrons.

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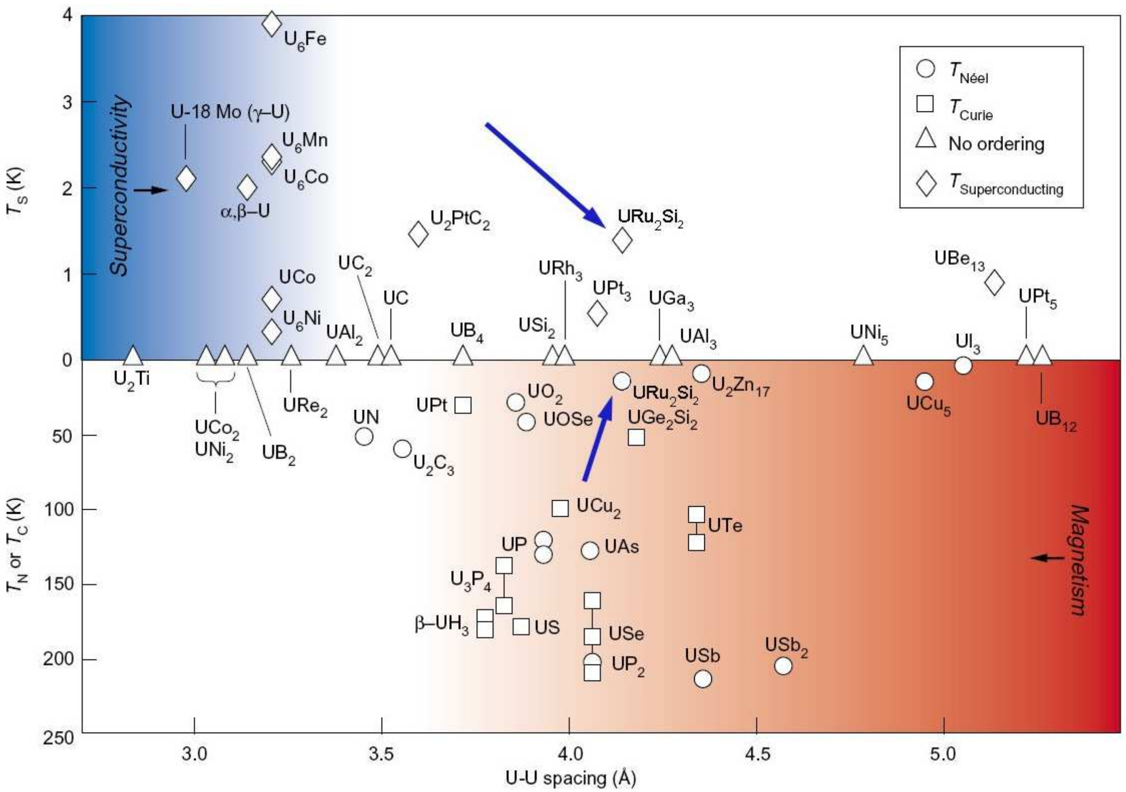

Magnetism plays also a major role for superconductivity in **the uranium compounds**

/ Hill plot /

J.A. Mydosh and P.M. Oppeneer, Rev. Mod. Phys. **83***, 1301 (2011).*

Duality 3: itinerant vs localized features

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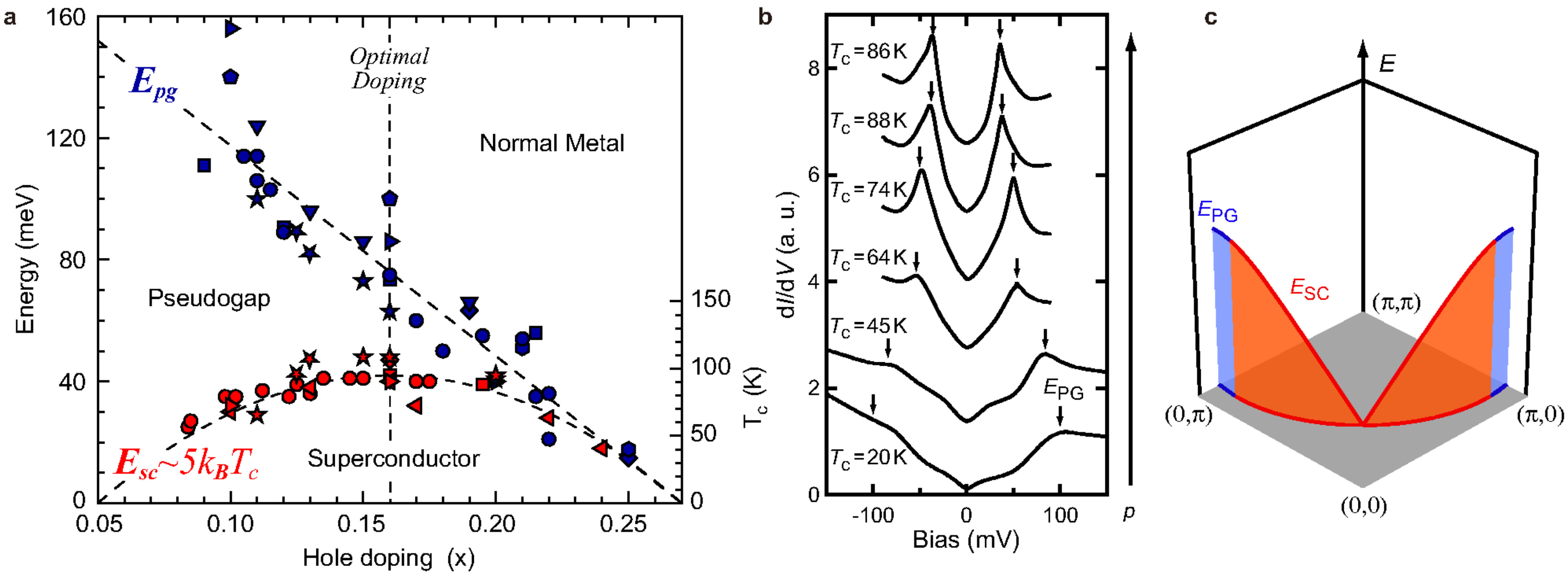

Y. Kohsaka, ... and J.C. Davis, Nature **454***, 1072 (2008).*

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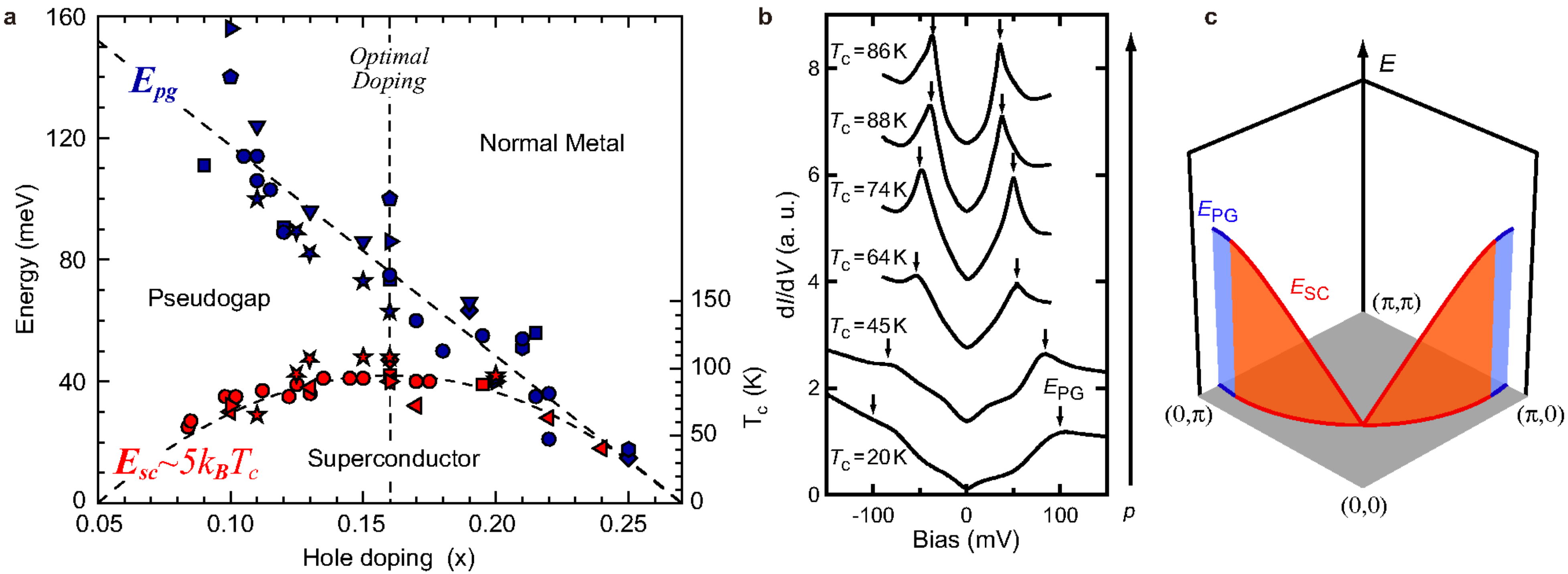

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Approaching the Mott insulator:

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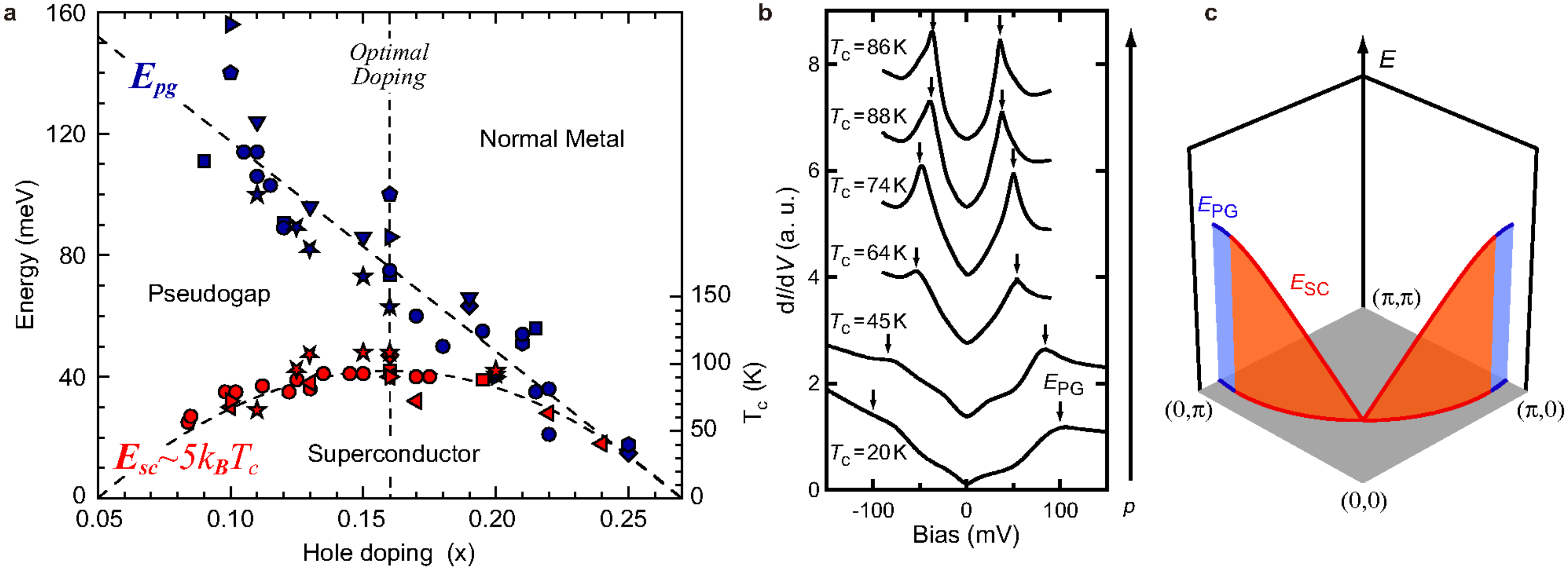

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Approaching the Mott insulator:

⇒ **the magnitude of energy gap |Δ| increases,**

two distinct gaps become gradually evident.

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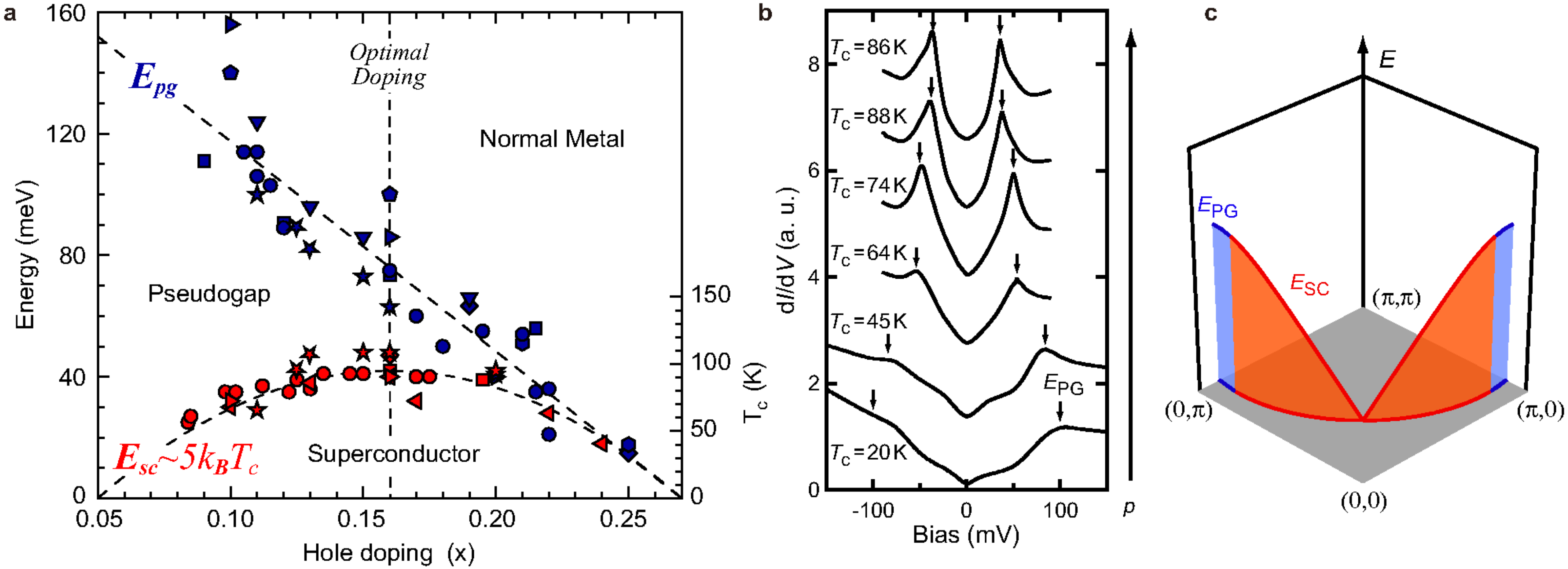

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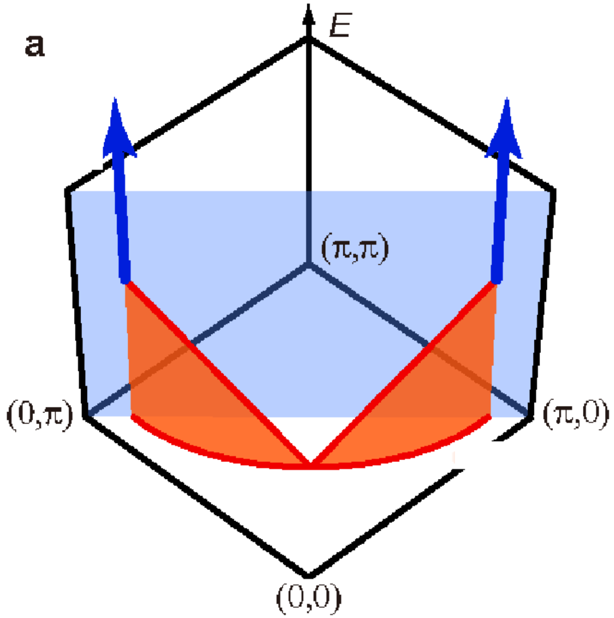

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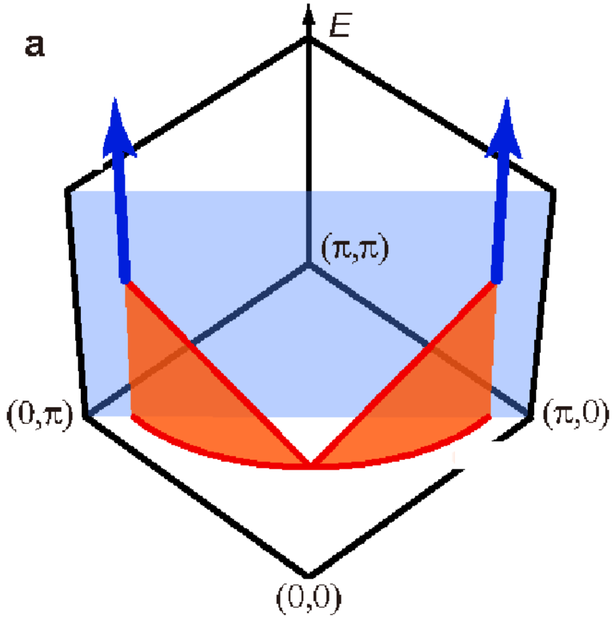

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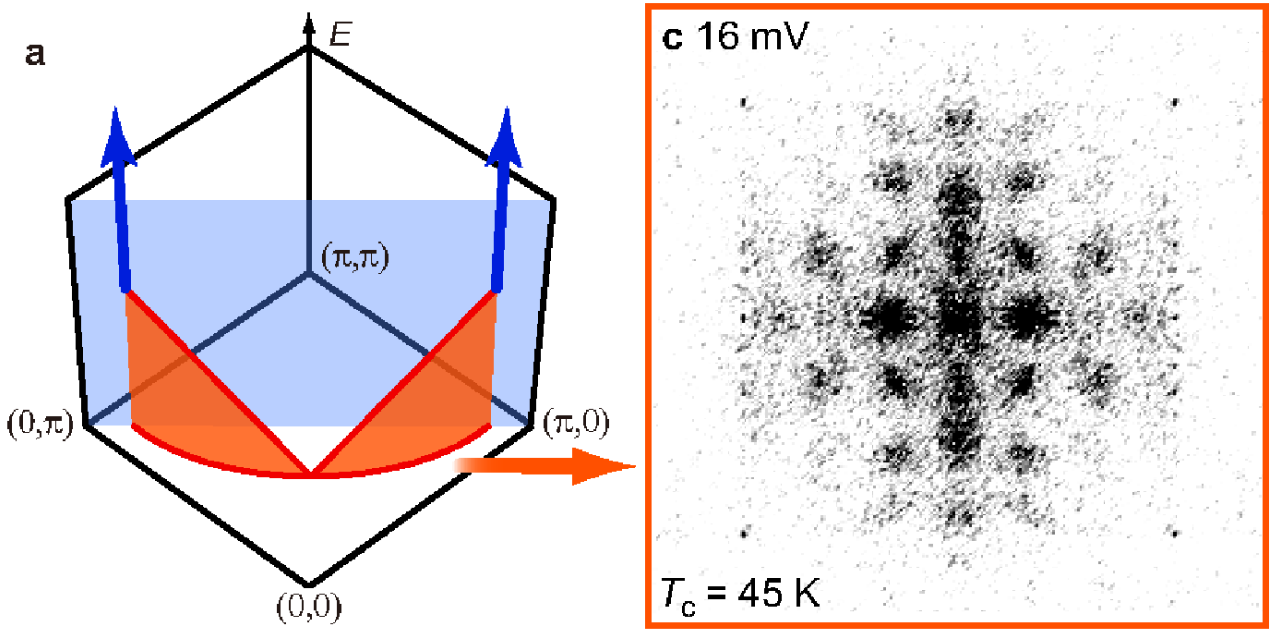

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Electrons from various parts of the Brillouin zone are responsible for:

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Cu-O-Cu centered patterns / *localized features in the $\vec{r}$-space /*

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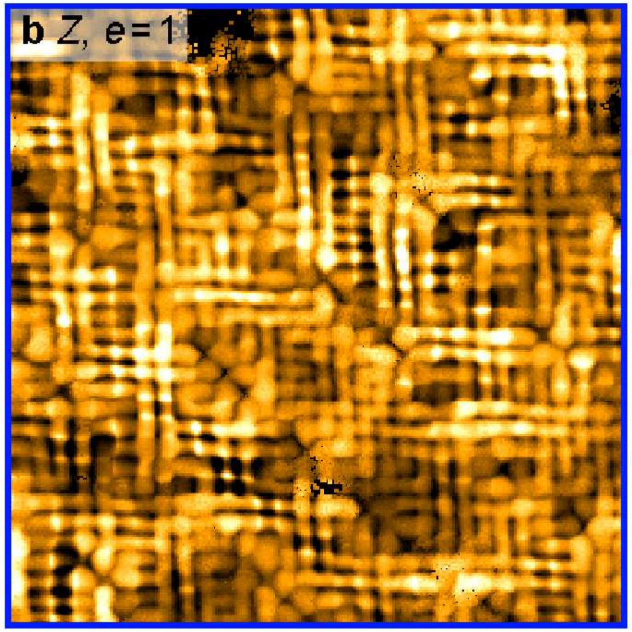

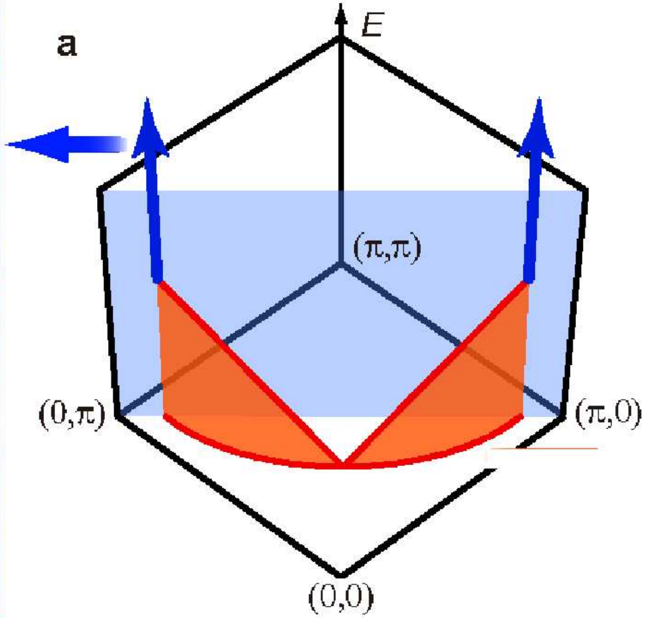

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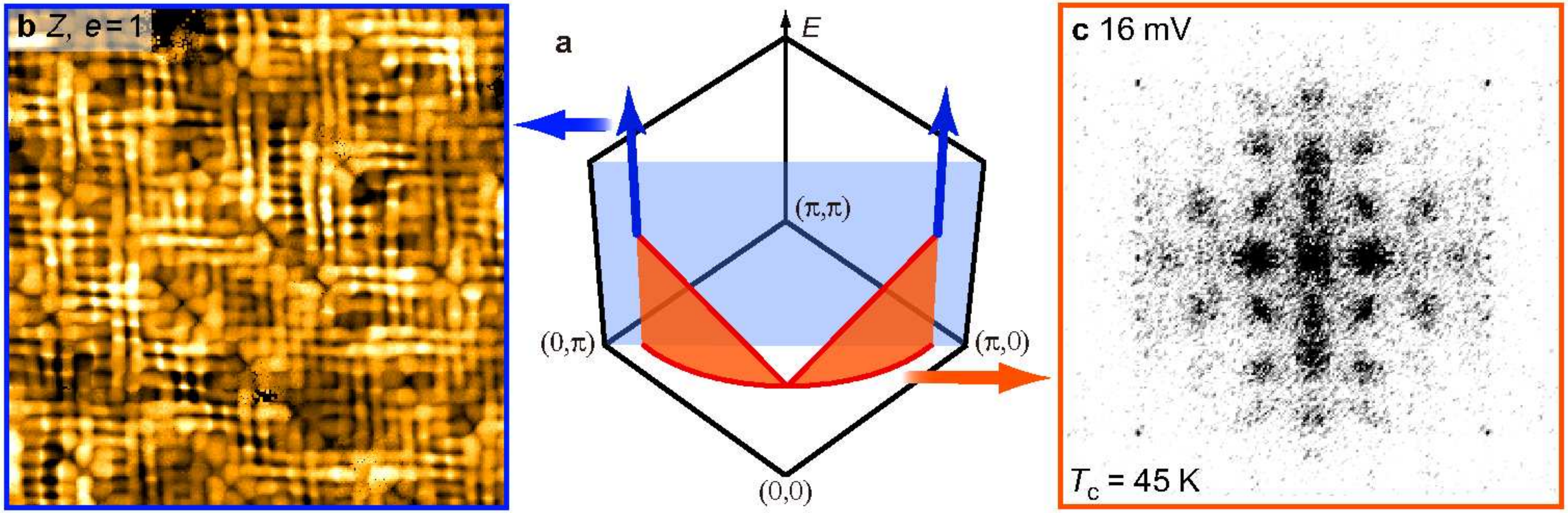

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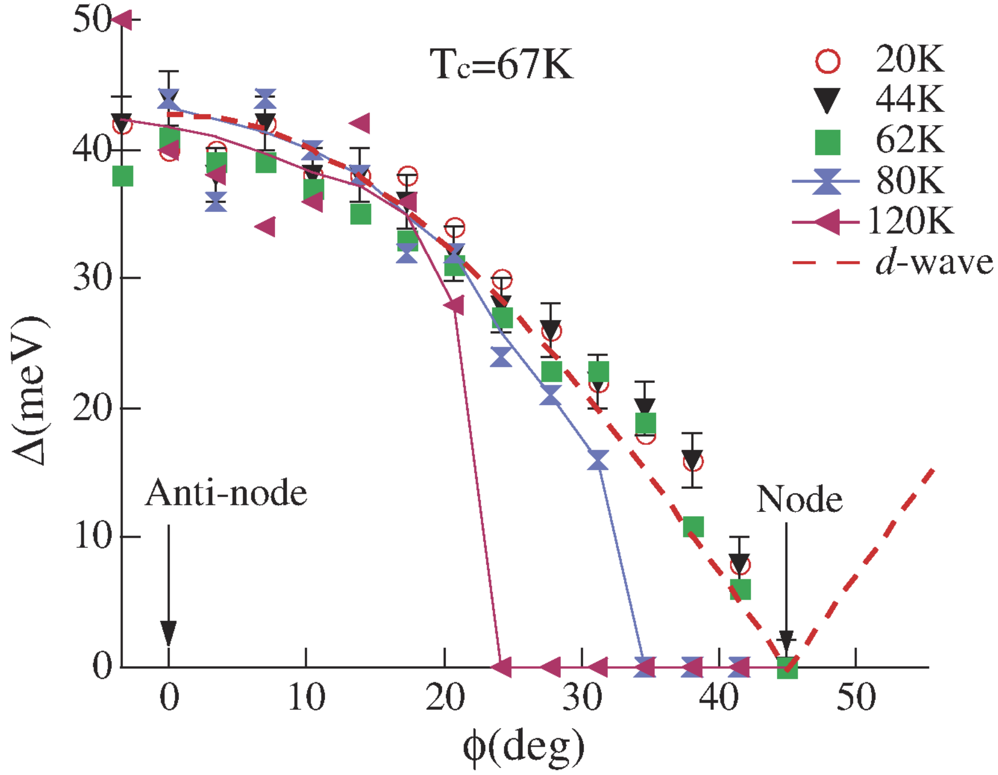

A. Kanigel et al, *Phys. Rev. Lett.* **99**, *157001 (2007).*

Duality 4: **nodal antinodal dichotomy**

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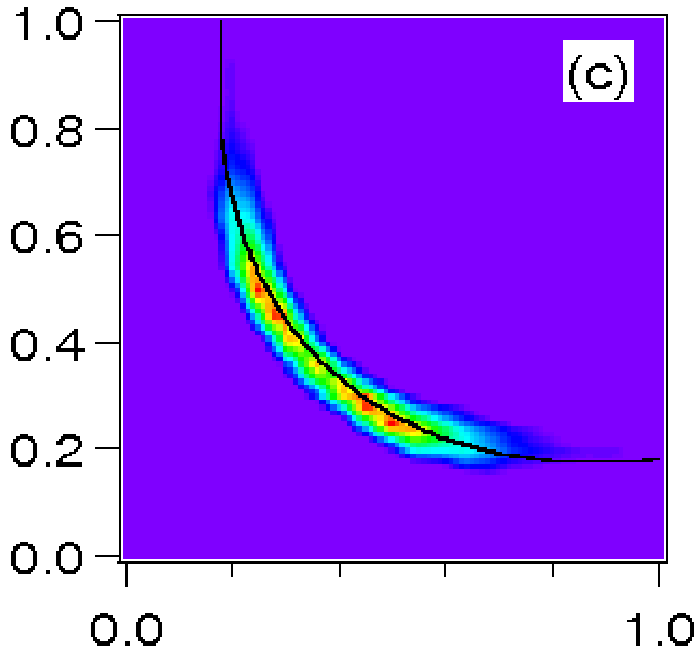

In antinodal parts the missing parts of Fermi surface are recovered at T^*.

"Death of a Fermi surface" *K. McElroy, Nature Physics* **2**, *441 (2006)* .

Duality 4: **nodal antinodal dichotomy**

Perhaps the time resolved ARPES would allow to identify the regions where (inhoherent) pairs survive above T_c.

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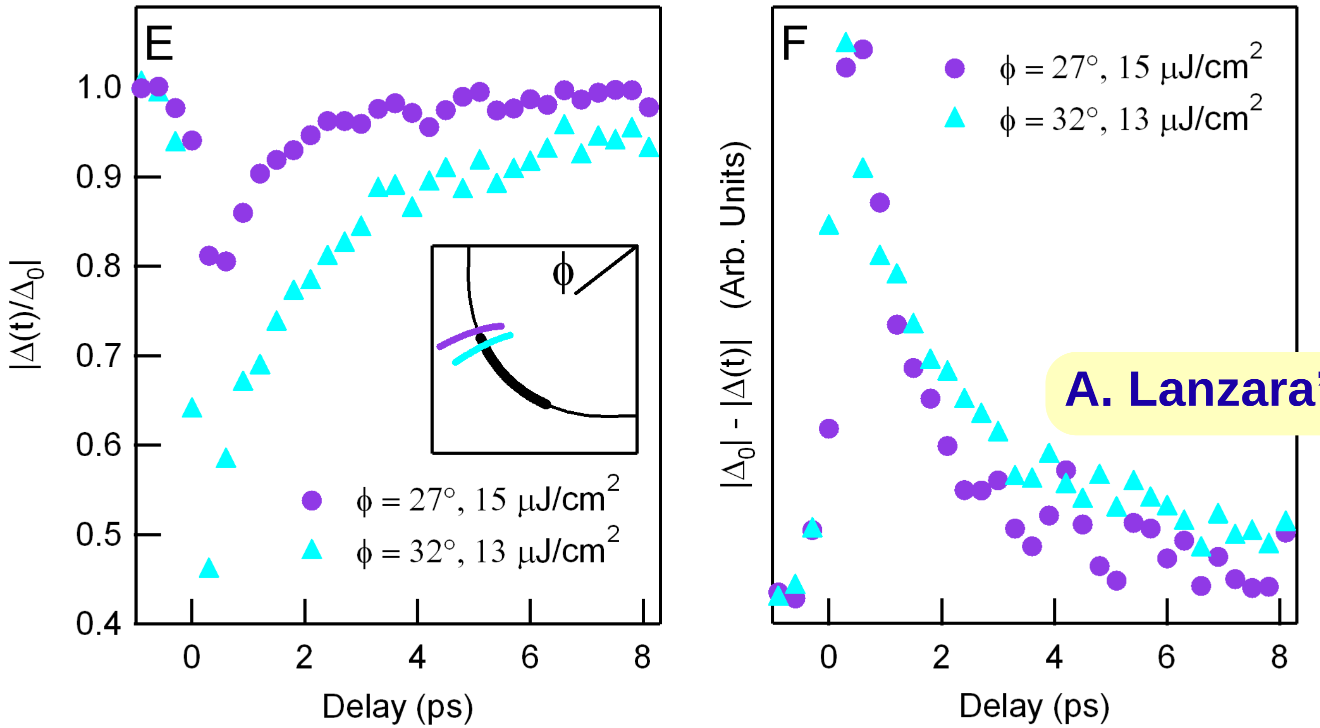

A. Lanzara's group

For $T < T_c$ the quasiparticle recovery time is $\sim$ 2 ps.

Ch.L. Smallwood et al, *Science* **336**, *1137 (2012).*

3. Methodology

Strongly correlated systems / Hubbard-Stratonovich transf. /

We consider the strongly correlated fermion system

$$\hat{H} = \hat{T}_{kin} + U \int d\vec{r} \;\; \hat{c}^{\dagger}_{\uparrow}(\vec{r}) \; \hat{c}^{\dagger}_{\downarrow}(\vec{r}) \; \hat{c}_{\downarrow}(\vec{r}) \; \hat{c}_{\uparrow}(\vec{r})$$

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$\hat{c} \, |\psi\rangle = \psi \, |\psi\rangle$ and $\langle\psi| \, \hat{c}^{\dagger} = \langle\psi| \, \bar{\psi}$

we can express the partition function by the path integral

$$Z = \int D \left[\bar{\psi}, \psi\right] e^{-S[\bar{\psi},\psi]}$$

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where the imaginary-time fermionic action

$$S[\bar{\psi},\psi] = \int_0^{\beta} d\tau \int d\vec{r} \Big[\sum_{\sigma} \bar{\psi}_{\sigma}(\vec{r},\tau) \left(\partial_{\tau} + \hat{\xi}\right) \psi_{\sigma}(\vec{r},\tau) \\ - g\, \bar{\psi}_{\uparrow}(\vec{r},\tau)\, \bar{\psi}_{\downarrow}(\vec{r},\tau)\, \psi_{\downarrow}(\vec{r},\tau) \psi_{\uparrow}(\vec{r},\tau) \Big]$$

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and $\hat{\xi} \equiv -\hbar^2\nabla^2/2m - \mu, \qquad g = -U$.

Hubbard-Stratonovich – continued

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To eliminate the quartic term we introduce the auxiliary pairing fields

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simplifying the action to a bi-linear form

$$S = \int_0^\beta d\tau \int d\vec{r} \left[\sum_\sigma \bar{\psi}_\sigma(\vec{r},\tau)\left(\partial_\tau + \hat{\xi}\right)\psi_\sigma(\vec{r},\tau) + \frac{|\Delta(\vec{r},\tau)|^2}{g} - \bar{\Delta}(\vec{r},\tau)\ \psi_\downarrow(\vec{r},\tau)\psi_\uparrow(\vec{r},\tau) - \Delta(\vec{r},\tau)\ \bar{\psi}_\uparrow(\vec{r},\tau)\bar{\psi}_\downarrow(\vec{r},\tau)\right]$$

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The mean field (*saddle point*) solution usually relies on assumption of the static and uniform pairing field

$$\Delta(\vec{r},\tau) = \Delta \quad , \qquad \bar{\Delta}(\vec{r},\tau) = \bar{\Delta} \quad .$$

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We shall try to go beyond this scheme treating the fermionic and bosonic degrees of freedom on an equal footing !

Boson-Fermion scenario

[in the lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\, \delta_{i,j})\, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu \right) \hat{b}^{\dagger}_{l} \hat{b}_{l}$$

$$+ \sum_{i,j} g_{ij} \left[\hat{b}^{\dagger}_{l} \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.} \right]$$

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For some more specific derivation see for instance:

E. Altman and A. Auerbach, Phys. Rev. B **65**, *104508 (2002).*

or *Y. Yildirim and Wei Ku, Phys. Rev. X* **1**, *011011 (2011).*

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T. Domański and J. Ranninger, Phys. Rev. **B 63**, *134505 (2001).*

4. *Pre*-pairing above T_c

a) Bogoliubov quasiparticles

BCS excitation spectrum

/ in conventional superconductors /

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/ in conventional superconductors /

The effective (Bogoliubov) quasiparticles :

$$\hat{\gamma}_{\mathbf{k}\uparrow} = u_{\mathbf{k}}\, \hat{c}_{\mathbf{k}\uparrow} + v_{\mathbf{k}}\, \hat{c}^{\dagger}_{-\mathbf{k}\downarrow}$$

$$\hat{\gamma}^{\dagger}_{-\mathbf{k}\downarrow} = -v_{\mathbf{k}}\, \hat{c}_{\mathbf{k}\uparrow} + u_{\mathbf{k}}\, \hat{c}^{\dagger}_{-\mathbf{k}\downarrow}$$

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$$A(\mathbf{k},\omega) = |u_{\mathbf{k}}|^2\, \delta(\omega - E_{\mathbf{k}}) + |v_{\mathbf{k}}|^2\, \delta(\omega + E_{\mathbf{k}})$$

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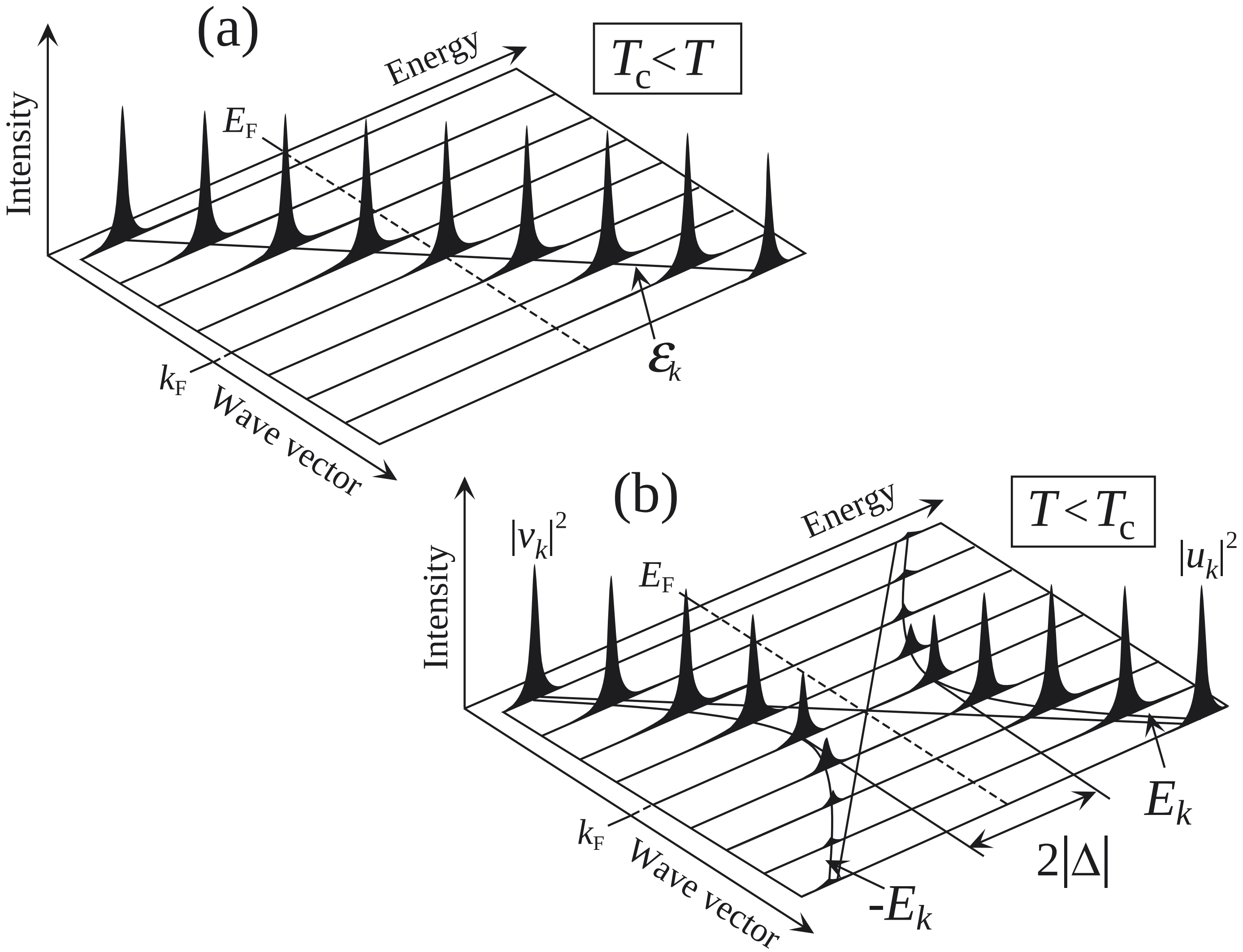
$$A(\mathrm{k},\omega) = |u_{\mathrm{k}}|^2\,\delta(\omega - E_{\mathrm{k}}) + |v_{\mathrm{k}}|^2\,\delta(\omega + E_{\mathrm{k}})$$

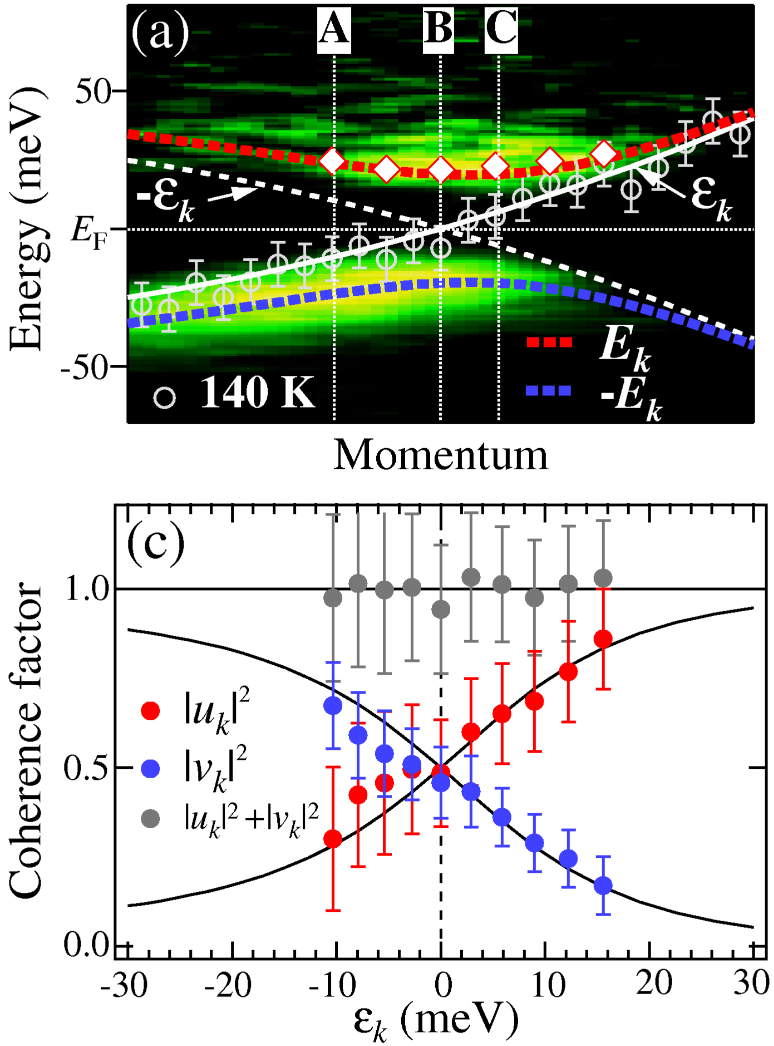
Occupancy of the momentum k is given by

$$n_{\mathrm{k}} = |u_{\mathrm{k}}|^2\, f_{FD}(E_{\mathrm{k}}) + |v_{\mathrm{k}}|^2\, \underbrace{f_{FD}(-E_{\mathrm{k}})}_{1-f_{FD}(E_{\mathrm{k}})}$$



The single particle spectrum (in conventional superconductors) consists of two Bogoliubov branches gaped around E_F.

Experimental data for cuprates – $T < T_c$



H. Matsui, T. Sato, and T. Takahashi et al, *Phys. Rev. Lett.* **90**, *217002 (2003).*

Beyond the BCS approximation

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We have generalized the Bogoliubov ansatz, taking into account the non-condensed (preformed) pairs

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$$\begin{aligned}
\hat{c}_{\mathbf{k}\uparrow}(l) &= u_{\mathbf{k}}(l)\,\hat{c}_{\mathbf{k}\uparrow} + v_{\mathbf{k}}(l)\,\hat{c}^{\dagger}_{-\mathbf{k}\downarrow} + \\
&\quad \frac{1}{\sqrt{N}}\sum_{\mathbf{q}\neq 0}\left[u_{\mathbf{k},\mathbf{q}}(l)\,\hat{b}^{\dagger}_{\mathbf{q}}\hat{c}_{\mathbf{q}+\mathbf{k}\uparrow} + v_{\mathbf{k},\mathbf{q}}(l)\,\hat{b}_{\mathbf{q}}\hat{c}^{\dagger}_{\mathbf{q}-\mathbf{k}\downarrow}\right], \\
\hat{c}^{\dagger}_{-\mathbf{k}\downarrow}(l) &= -v^{*}_{\mathbf{k}}(l)\,\hat{c}_{\mathbf{k}\uparrow} + u^{*}_{\mathbf{k}}(l)\,\hat{c}^{\dagger}_{-\mathbf{k}\downarrow} + \\
&\quad \frac{1}{\sqrt{N}}\sum_{\mathbf{q}\neq 0}\left[-v^{*}_{\mathbf{k},\mathbf{q}}(l)\,\hat{b}^{\dagger}_{\mathbf{q}}\hat{c}_{\mathbf{q}+\mathbf{k}\uparrow} + u^{*}_{\mathbf{k},\mathbf{q}}(l)\,\hat{b}_{\mathbf{q}}\hat{c}^{\dagger}_{\mathbf{q}-\mathbf{k}\downarrow}\right],
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\end{aligned}
$$

with the boundary conditions

$u_{\mathbf{k}}(0) = 1$ **and** $v_{\mathbf{k}}(0) = v_{\mathbf{k},\mathbf{q}}(0) = u_{\mathbf{k},\mathbf{q}}(0) = 0.$

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with the boundary conditions

$u_{\mathbf{k}}(0) = 1$ **and** $v_{\mathbf{k}}(0) = v_{\mathbf{k},\mathbf{q}}(0) = u_{\mathbf{k},\mathbf{q}}(0) = 0.$

The corresponding fixed point values $\lim_{l\to\infty} u_{\mathbf{k}}(l)$ **(and other parameters) have been determined solving the set of coupled flow equations**

$\frac{\partial}{\partial l} u_{\mathbf{k}}(l)$, $\frac{\partial}{\partial l} v_{\mathbf{k}}(l)$, $\frac{\partial}{\partial l} u_{\mathbf{k},\mathbf{q}}(l)$, $\frac{\partial}{\partial l} v_{\mathbf{k},\mathbf{q}}(l)$.

Bogoliubov QPs above T_c

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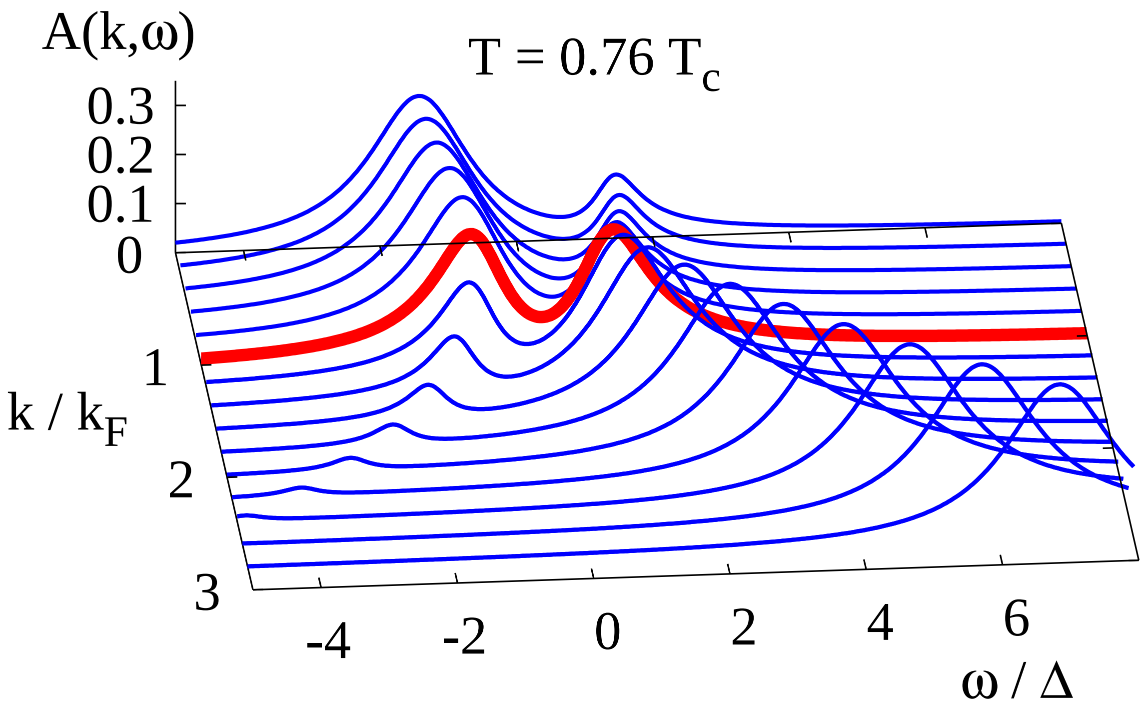

T. Domański and J. Ranninger, Phys. Rev. Lett. **91**, *255301 (2003).*

T. Domański, Phys. Rev. A **84**, *023634 (2011).*

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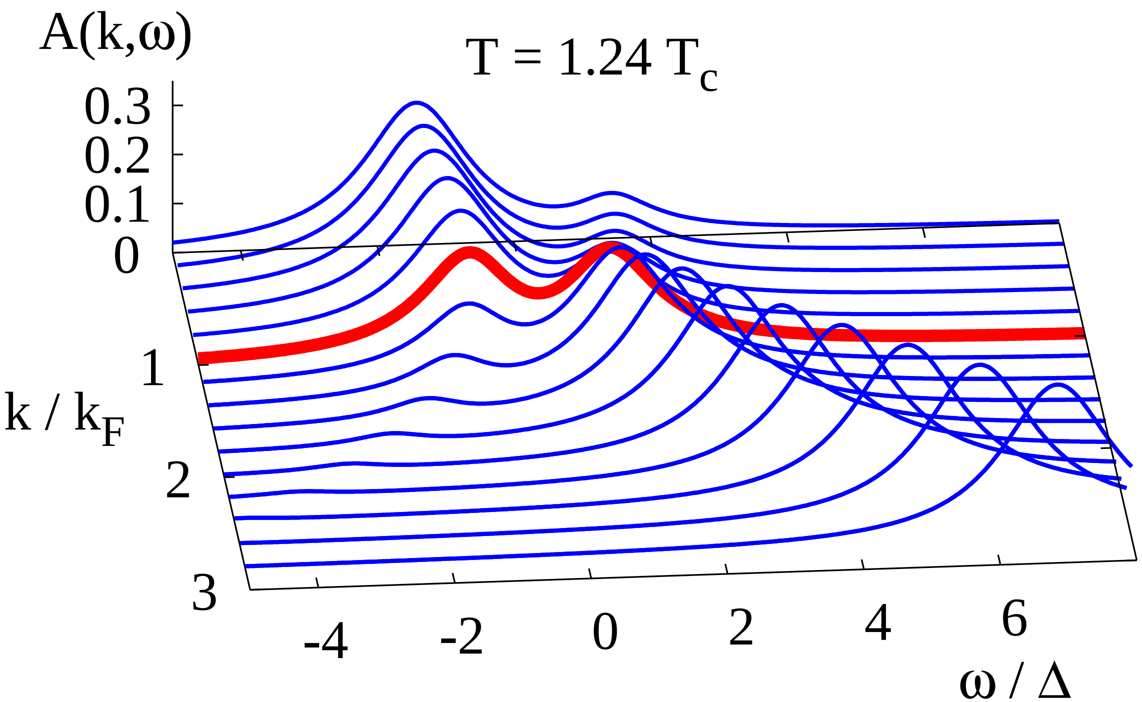

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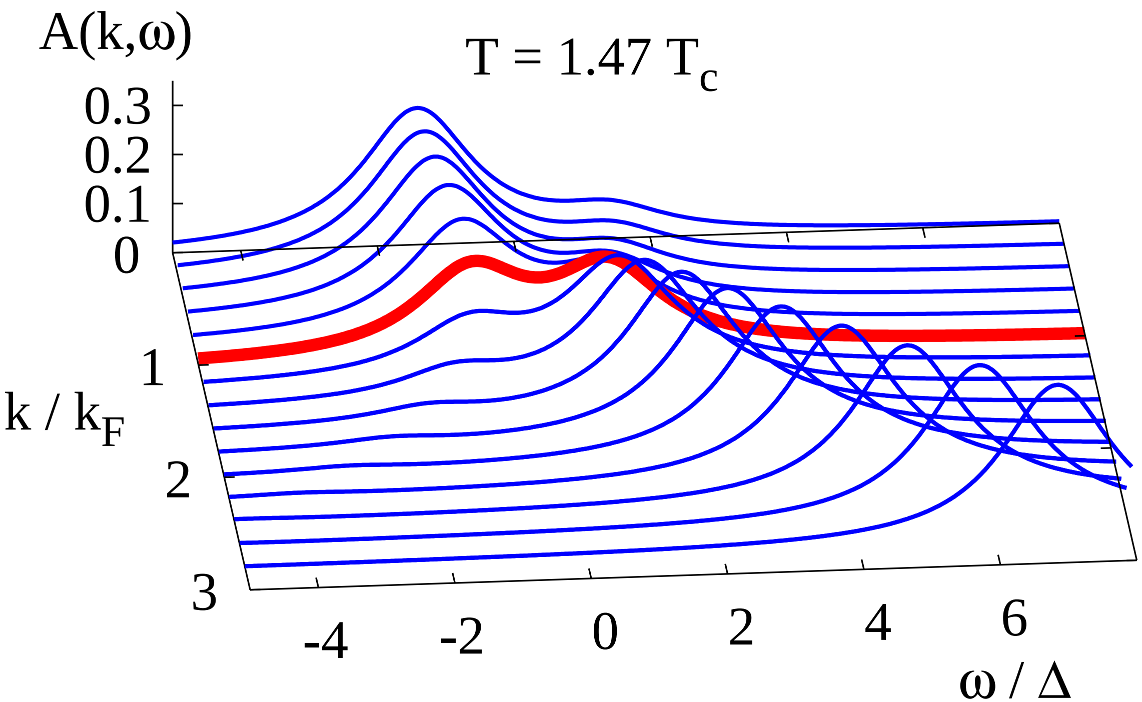

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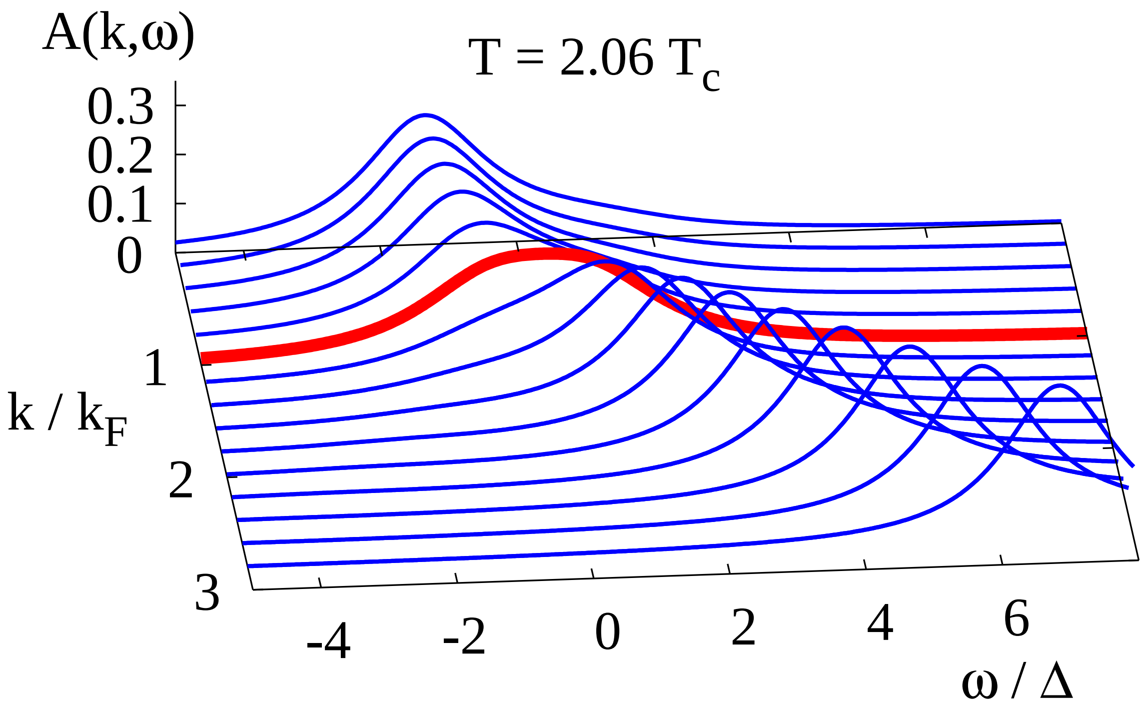

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Evidence for Bogoliubov QPs above T_c

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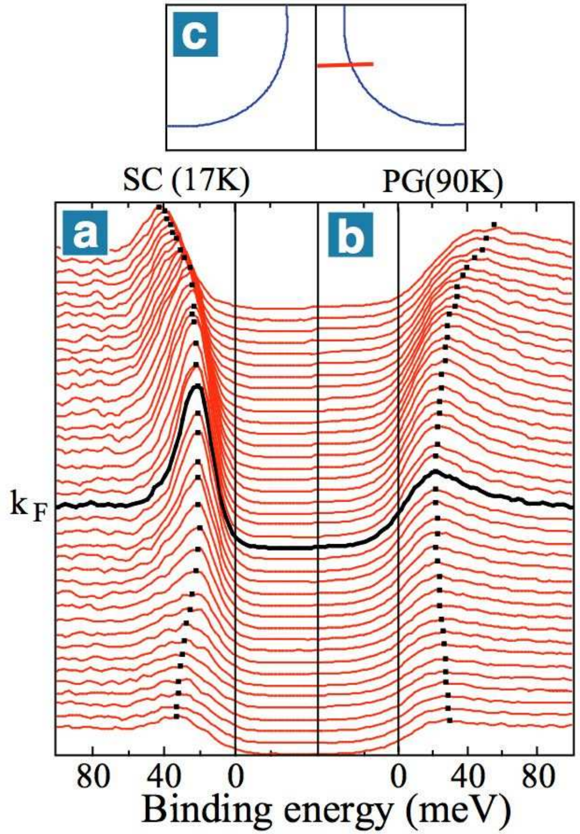
J. Campuzano group (Chicago, USA)

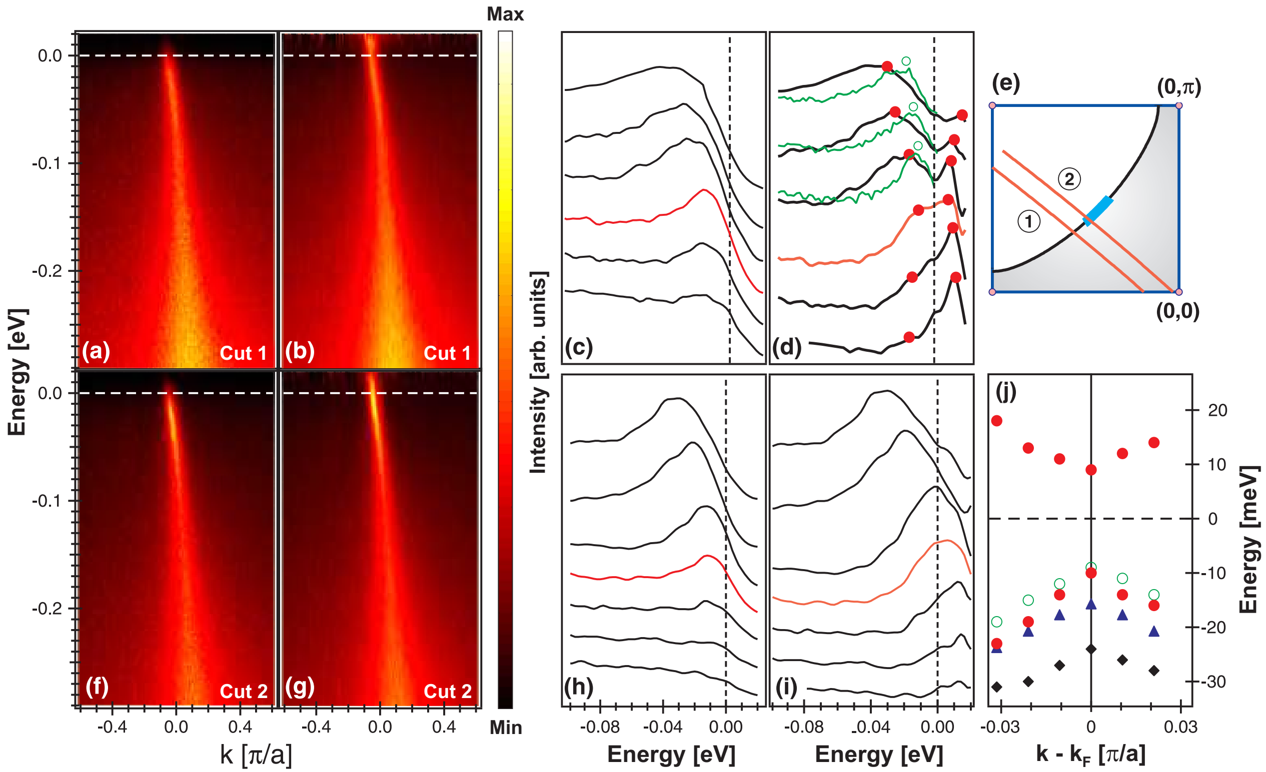


Results for: $Bi_2Sr_2CaCu_2O_8$

A. Kanigel et al, Phys. Rev. Lett. **101**, *137002 (2008).*

Evidence for Bogoliubov QPs above T_c

PSI group (Villigen, Switzerland)

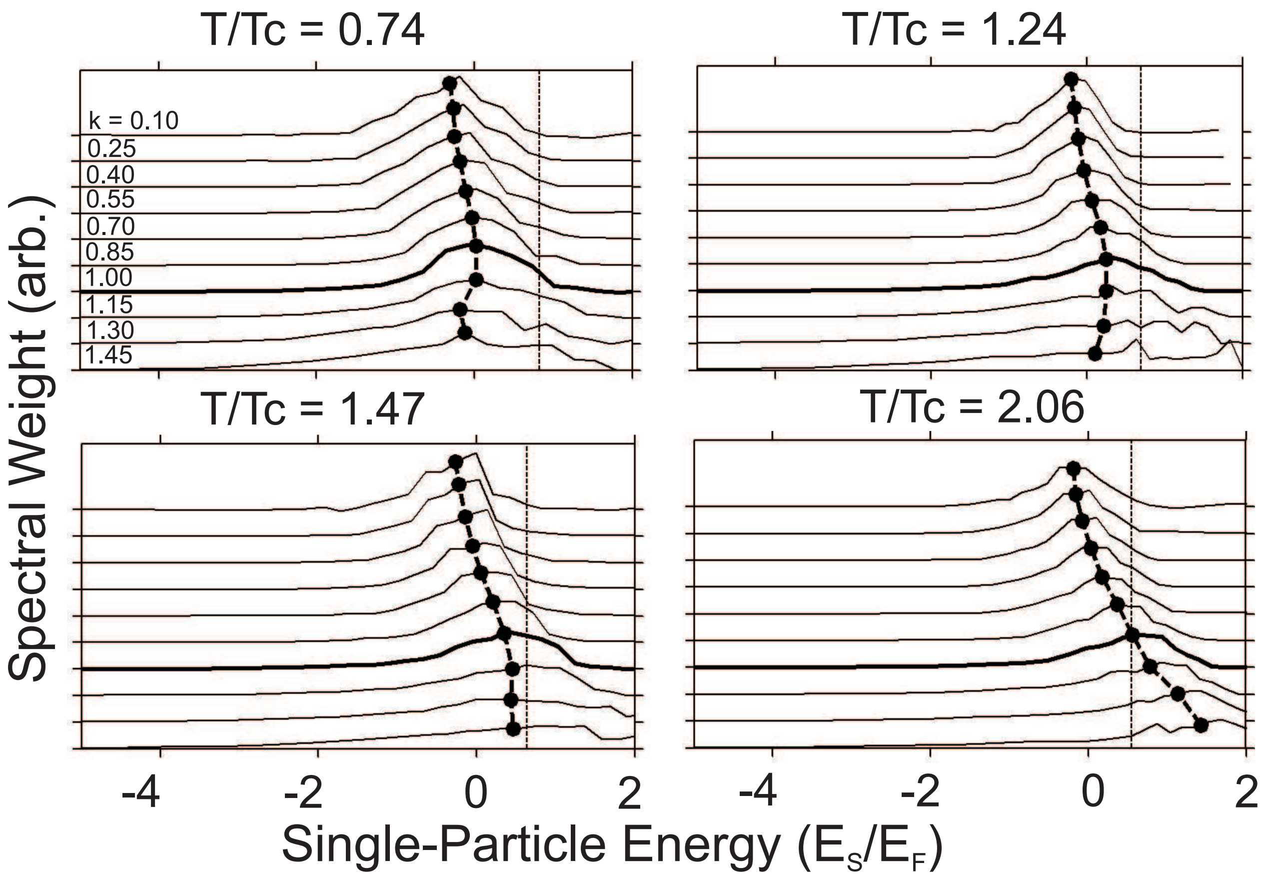


Results for: $La_{1.895}Sr_{0.105}CuO_4$

M. Shi et al, Eur. Phys. Lett. **88**, *27008 (2009).*

Evidence for Bogoliubov QPs above T_c

D. Jin group (Boulder, USA)



Results for: ultracold ^{40}K atoms

J.P. Gaebler et al, Nature Phys. **6**, *569 (2010).*

Local spectrum

experimental STM data

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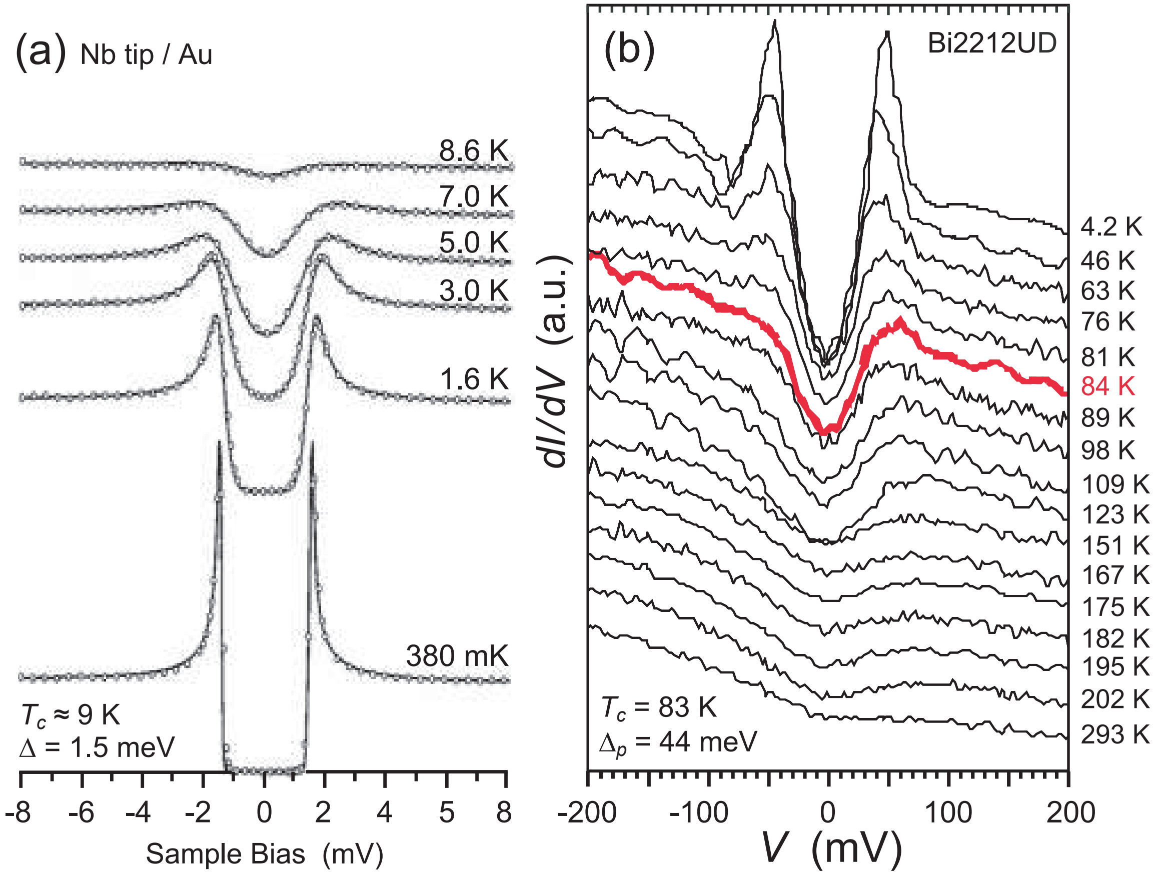
conv. sc. — the high T_c cuprates

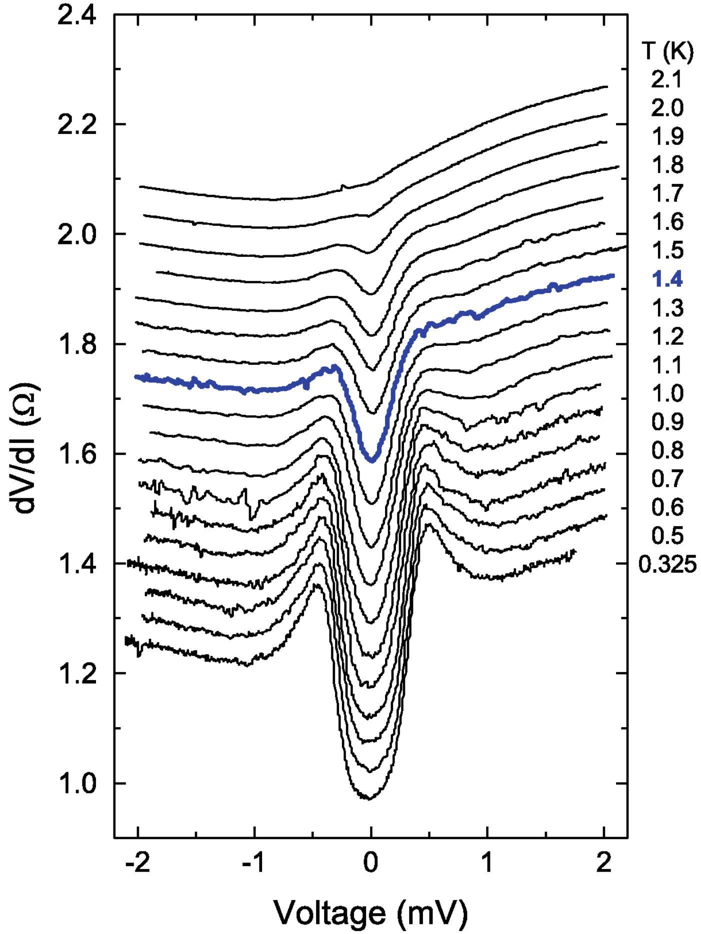


Ch. Renner et al, *Phys. Rev. Lett.* **80**, *149 (1998).*

Local spectrum

experimental STM data

URu_2Si_2



F. Morales and R. Escudero, J. Low Temp. Phys. **154**, *68 (2009).*

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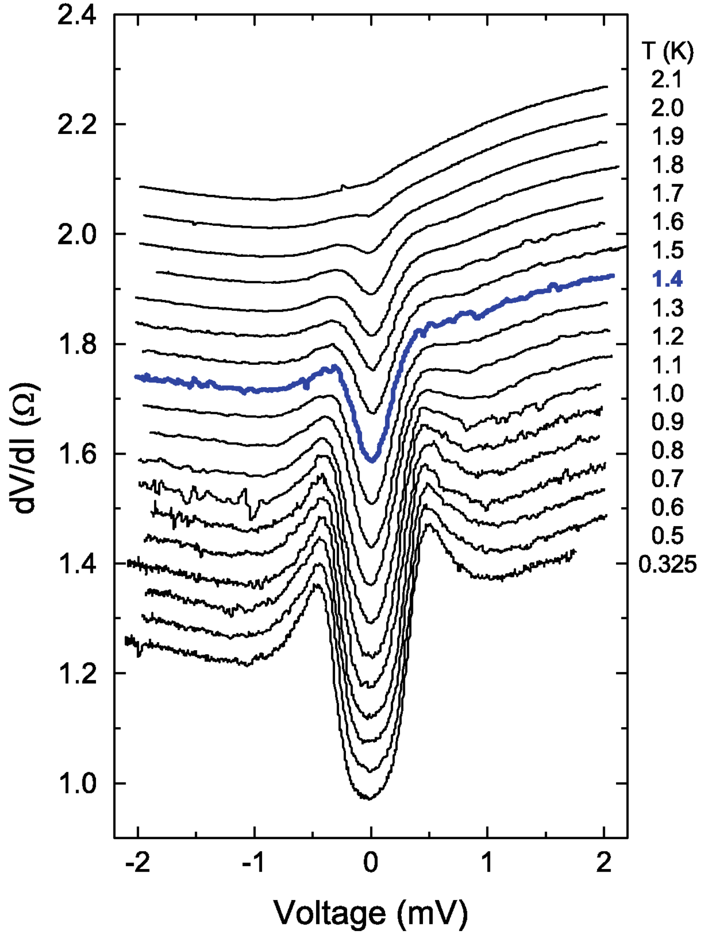
The superconducting pseudogap seems to persist up to $\sim 1.5T_c$



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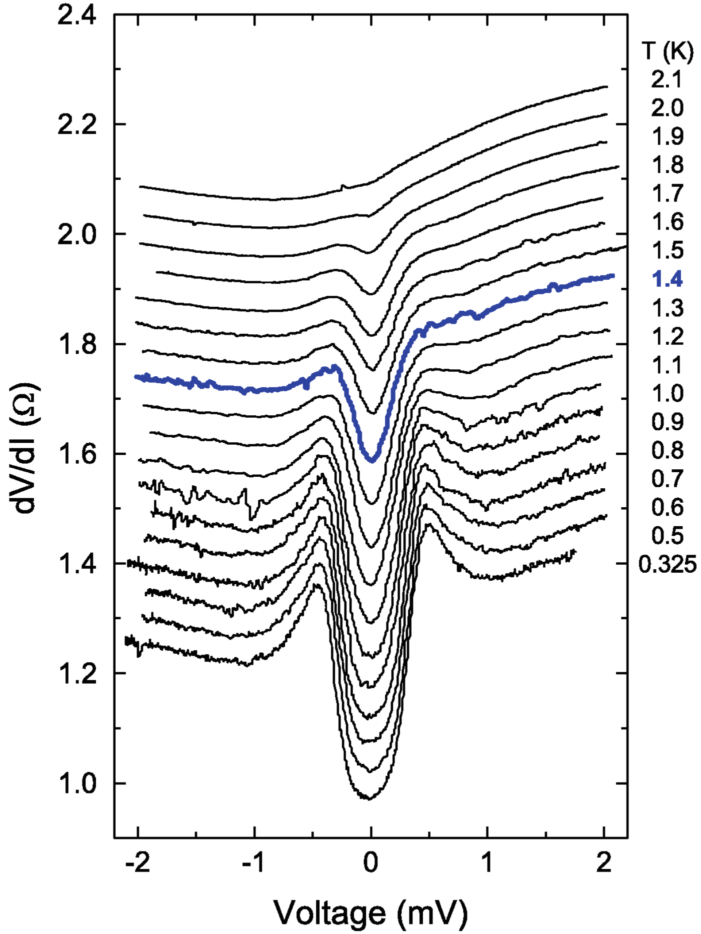
The Bogoliubov quasiparticle branches should be observable too !!!

/ by ARPES or SI-STM /



F. Morales and R. Escudero, J. Low Temp. Phys. ***154****, 68 (2009).*

4. *Pre*-pairing above T_c

b) residual Meissner effect

Correlation functions

For studying the diamagnetic response (in the Kubo formalism)
we have to determine the current-current correlation function

$$- \hat{T}_\tau \langle \hat{j}_{\rm q}(\tau) \; \hat{j}_{-\rm q}(0) \rangle$$

with statistical averaging defined as

$$\langle \ldots \rangle = \text{Tr} \left\{ e^{-\beta \hat{H}} \ldots \right\} / \text{Tr} \left\{ e^{-\beta \hat{H}} \right\}$$

and $\beta^{-1} = k_B T$.

This can be achieved using the following invariance

$$\begin{aligned}
\text{Tr} \left\{ e^{-\beta \hat{H}} \hat{O} \right\} &= \text{Tr} \left\{ e^{\hat{S}(l)} e^{-\beta \hat{H}} \hat{O} e^{-\hat{S}(l)} \right\} \\
&= \text{Tr} \left\{ e^{\hat{S}(l)} e^{-\beta \hat{H}} e^{-\hat{S}(l)} e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)} \right\} \\
&= \text{Tr} \left\{ e^{-\beta \hat{H}(l)} \hat{O}(l) \right\}
\end{aligned}$$

where

$$\hat{H}(l) = e^{\hat{S}(l)} \hat{H} e^{-\hat{S}(l)} \qquad \hat{O}(l) = e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)}$$

Technicalities

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The initial current operator

$$\hat{j}_{\mathbf{q},\sigma} = \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}+\frac{\mathbf{q}}{2}} \hat{c}^{\dagger}_{\mathbf{k},\sigma} \hat{c}_{\mathbf{k}+\mathbf{q},\sigma}$$

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is constrained (from the flow equation) in the form

$$\hat{\mathrm{j}}_{\mathbf{q},\uparrow}\ (l) = \sum_{\mathbf{k}} \mathrm{v}_{\mathbf{k}+\frac{\mathbf{q}}{2}} \Big(\mathcal{A}_{\mathbf{k},\mathbf{q}}(l) \hat{c}^{\dagger}_{\mathbf{k},\uparrow}\ \hat{c}_{\mathbf{k}+\mathbf{q},\uparrow} + \mathcal{B}_{\mathbf{k},\mathbf{q}}(l) \hat{c}_{-\mathbf{k},\downarrow} \hat{c}^{\dagger}_{-(\mathbf{k}+\mathbf{q}),\downarrow}$$

$$+ \sum_{\mathbf{p}} \Big(\mathcal{D}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) \hat{b}_{\mathbf{k}+\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{k},\uparrow}\ \hat{c}^{\dagger}_{\mathbf{p}-\mathbf{q},\downarrow} + \mathcal{F}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) \hat{b}^{\dagger}_{\mathbf{k}+\mathbf{p}} \hat{c}_{\mathbf{p},\downarrow} \hat{c}_{\mathbf{k}+\mathbf{q},\uparrow}\ \Big)\Big)$$

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$$+ \sum_{\mathbf{p}} \Big(\mathcal{D}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) \hat{b}_{\mathbf{k}+\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{k},\uparrow} \hat{c}^{\dagger}_{\mathbf{p}-\mathbf{q},\downarrow} + \mathcal{F}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) \hat{b}^{\dagger}_{\mathbf{k}+\mathbf{p}} \hat{c}_{\mathbf{p},\downarrow} \hat{c}_{\mathbf{k}+\mathbf{q},\uparrow} \Big) \Big)$$

with the boundary conditions

$$\mathcal{A}_{\mathbf{k},\mathbf{q}}(0) = 1 \quad \text{and} \quad \mathcal{B}_{\mathbf{k},\mathbf{q}}(0) = \mathcal{D}_{\mathbf{k},\mathbf{p},\mathbf{q}}(0) = \mathcal{F}_{\mathbf{k},\mathbf{p},\mathbf{q}}(0) = 0$$

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We next determine all fixed point values $\lim_{l\to\infty} \mathcal{A}_{\mathbf{k},\mathbf{q}}(l) \equiv \tilde{\mathcal{A}}_{\mathbf{k},\mathbf{q}}$ etc
from the set of flow equations

$$\frac{\partial}{\partial l}\mathcal{A}_{\mathbf{k},\mathbf{q}}(l) , \quad \frac{\partial}{\partial l}\mathcal{B}_{\mathbf{k},\mathbf{q}}(l) , \quad \frac{\partial}{\partial l}\mathcal{D}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) , \quad \frac{\partial}{\partial l}\mathcal{F}_{\mathbf{k},\mathbf{p},\mathbf{q}}(l) .$$

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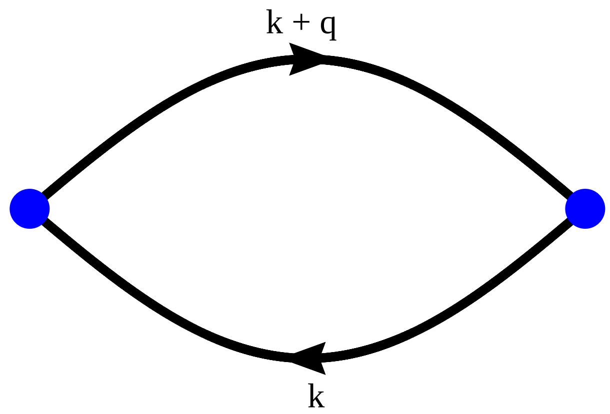

$\Longleftarrow$ **the usual bubble diagram**

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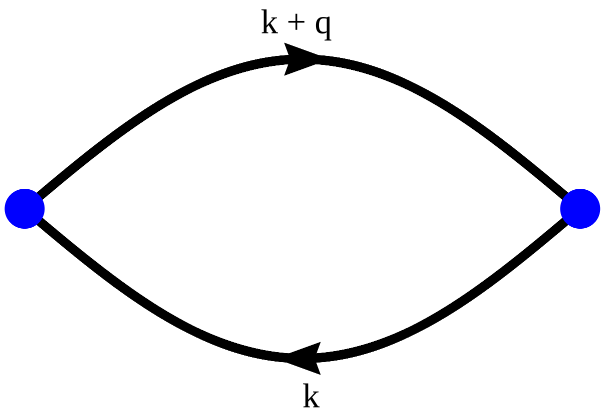

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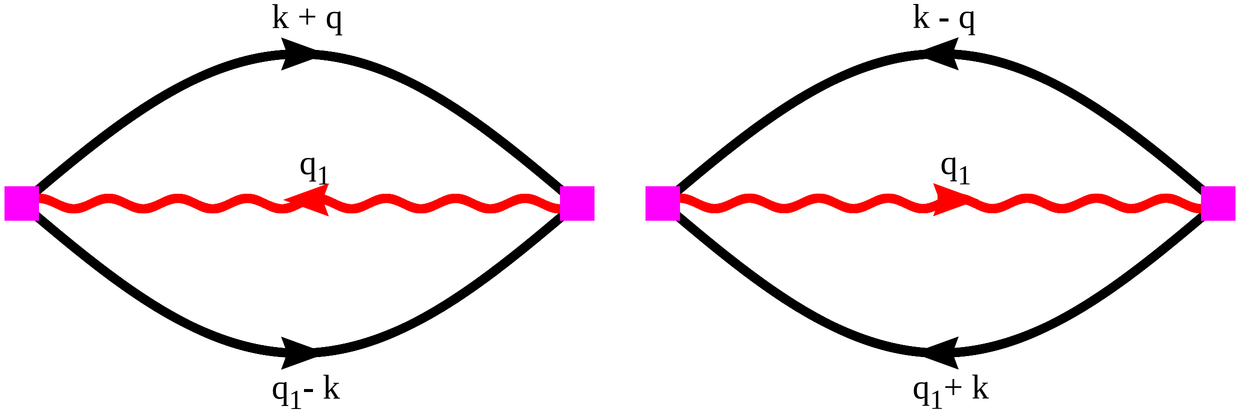

anomalous diagrams

Vertex functions have to be determined from the flow equations.

M. Zapalska, T. Domański, Phys. Rev. B **84***, 174520 (2011).*

Onset of diamagnetism above T_c

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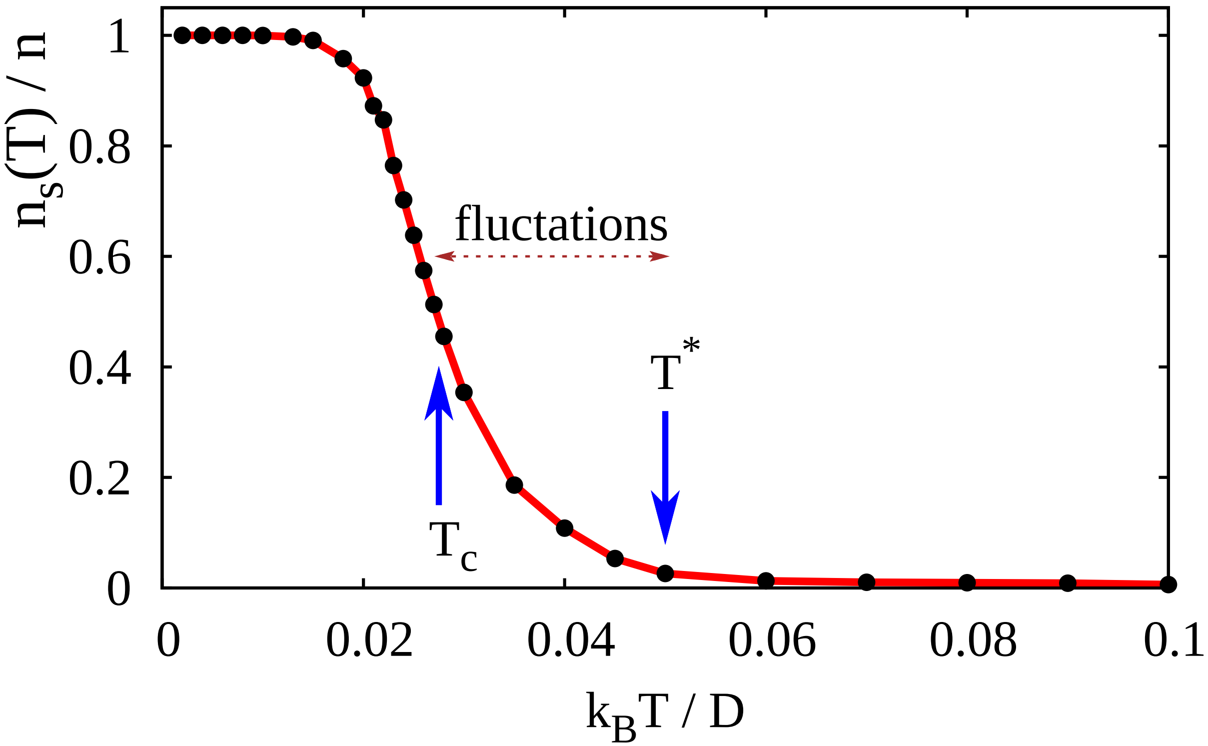
Residual diamagnetism emerges simultaneously with the collective features.

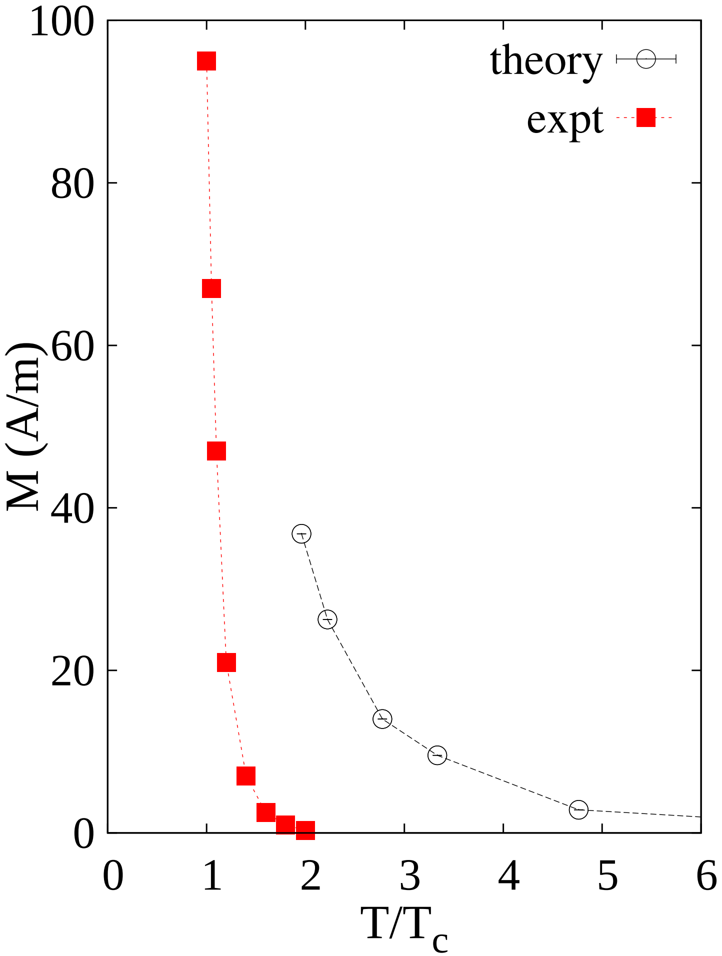


$$\vec{j} = -\frac{e^2 n_s(T)}{m} \vec{A}$$

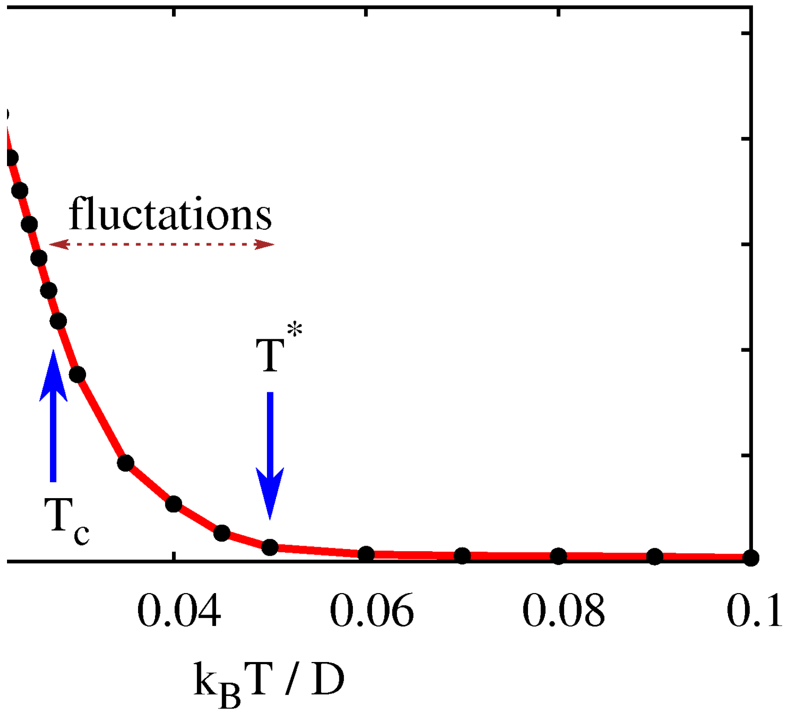
M. Zapalska, T. Domański, Phys. Rev. B **84**, *174520 (2011).*

Onset of diamagnetism above T_c

Comparison to the Quantum Monte Carlo simulations & experimental data



M. Zapalska, T.D., Phys. Rev. B ***84****, 174520 (2011).*



QMC: *K.-Y. Yang, ... and M. Troyer, Phys. Rev. B* ***83****, 214516 (2011).* **/ ETH Zürich, Switzerland /**

expt: *L. Li, ... and N.P. Ong, Phys. Rev. B* ***81****, 054510 (2010).* **/ Princeton, USA /**

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