Correlations in superconducting nano-structures under nonequilibrium cases

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Lublin, Poland

CCDS-44, Ustroń **9 September 2025**

MAIN ISSUES

1. **Pairing and correlations**

   [in nanoscopic scale]

2. **Dynamical phenomena**

   [affecting in-gap states]

3. **Out-of-equilibrium transport**

   [via Andreev scattering]

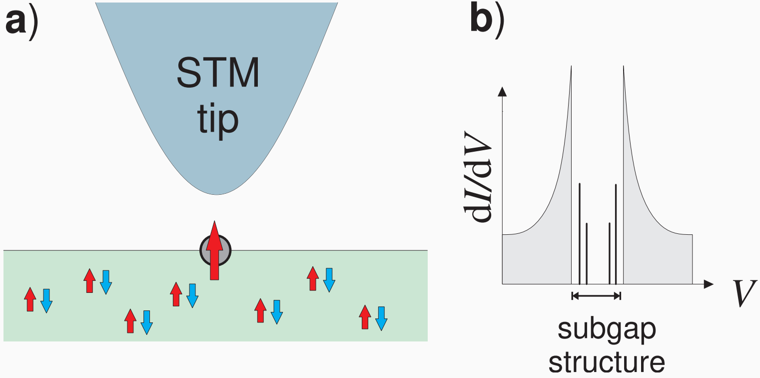
Superconducting nanostructures

Superconducting nanostructures

some examples ...

1. Local spectroscopy:

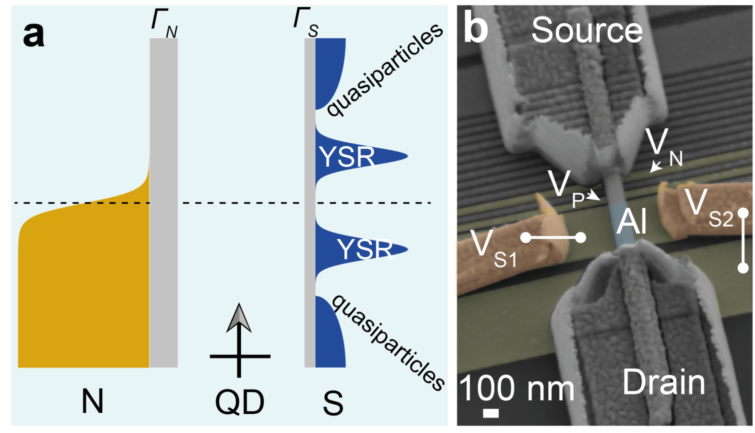
quantum impurity on a surface of superconductor + STM tip



Differential conductance probes the effective spectrum of impurity.

2. Andreev junctions:

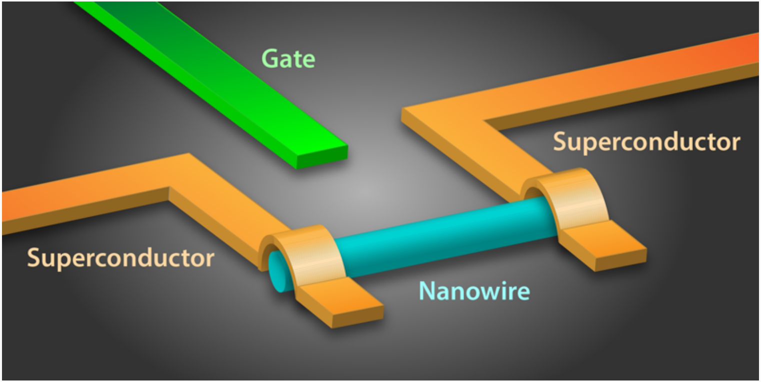
normal metal (N) - quantum dot (QD) - superconductor (S)



Subgap tunneling via the electron-to-hole (Andreev) scattering.

3. Josephson junctions:

superconductor (S) - quantum dot (QD) - superconductor (S)



Tunneling of Cooper pairs via bound states in Josephson junction.

SUPERCONDUCTING PROXIMITY EFFECT

Quantum dot (QD) coupled to bulk superconductor (SC) experiences:

$\Longrightarrow$ **on-dot pairing**

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SUPERCONDUCTING PROXIMITY EFFECT

Quantum dot (QD) coupled to bulk superconductor (SC) experiences:

$\Longrightarrow$ **on-dot pairing**

which is spectroscopically manifested by:

$\Longrightarrow$ **in-gap bound states**

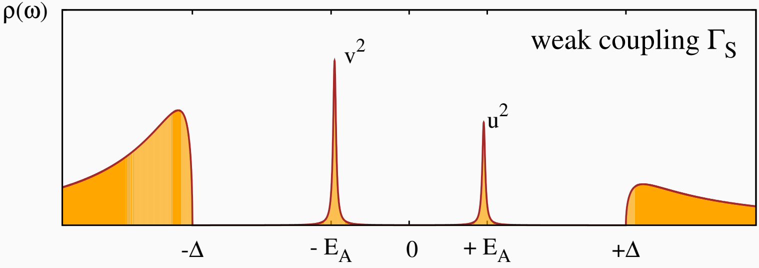
originating from:

$\Longrightarrow$ **leakage of the Cooper pairs onto QD** **(Andreev)**

$\Longrightarrow$ **interaction of QD spin with SC** **(Yu-Shiba-Rusinov)**

IN-GAP STATES

Spectrum of the quantum impurity coupled to superconductor



Bound states appear at $\pm E_A$ in the subgap region $E \in \langle -\Delta, \Delta \rangle$

IN-GAP STATES

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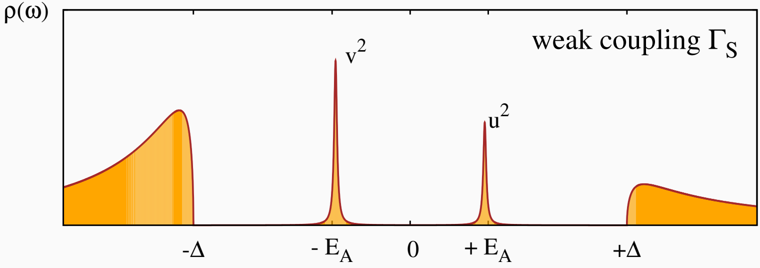
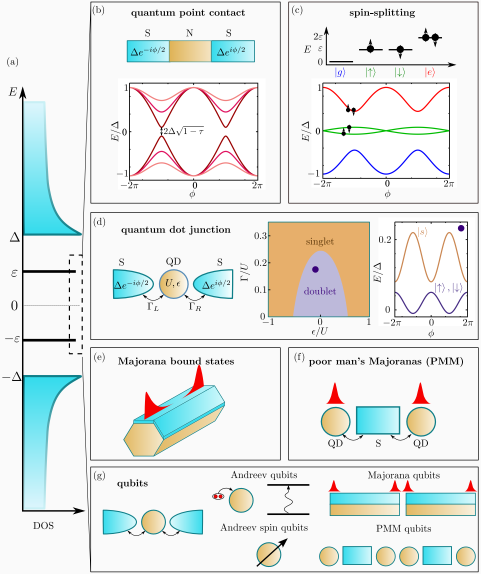


Bound states appear at $\pm E_A$ in the subgap region $E \in \langle -\Delta, \Delta \rangle$

Let's focus on such in-gap bound states ...

Why ?

VARIERY OF SUPERCONDUCTING QUBITS



R. Seoane Souto & R. Aguado, Lecture Notes in Physics, vol. 1025, Springer (2024)).

Examples ...

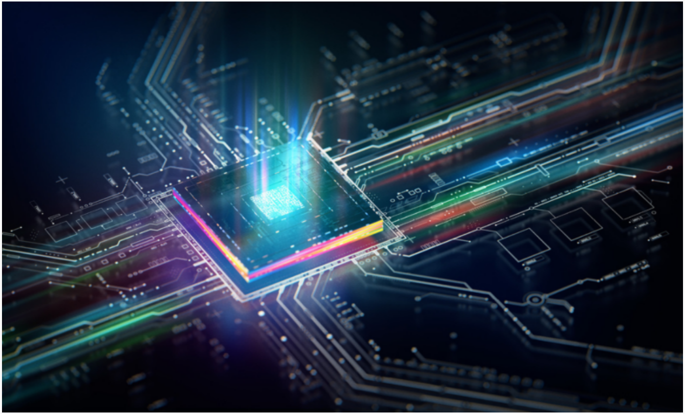
SUPERCONDUCTING PROCESSOR: WILLOW

In December 2024 Google demonstrated 105-qubit processor based on superconducting qubits (transmons).

Google Quantum AI and collaborators, Nature 638, 920 (2024).

SUPERCONDUCTING PROCESSOR: WILLOW

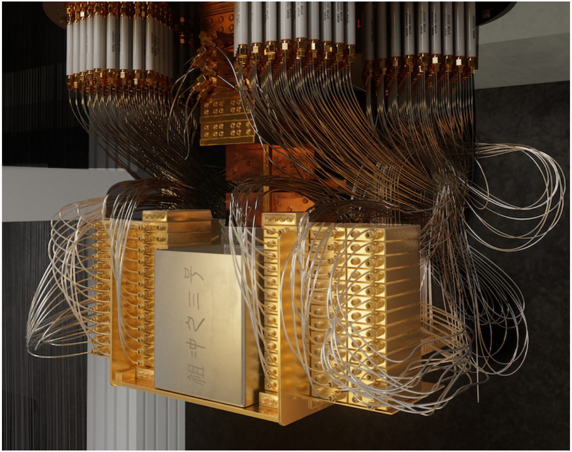
Simulation of the probability distribution obtained in 5 minutes by processor Willow would take about 10^{25} years by the fastest classical computer.



H. Neven (Google blog, 9 December 2024).

SC PROCESSOR: ZUCHONGZHI 3.0

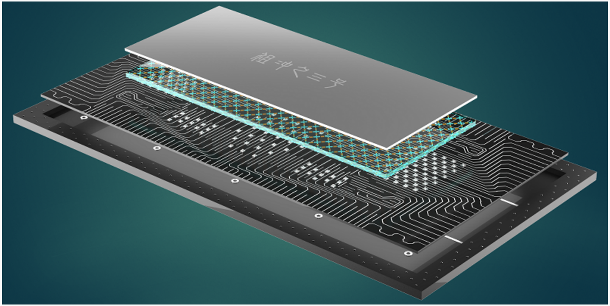
105-qubit processor constructed by the group of prof. Jian-Wei Pan (University of Science and Technology, China)



D. Gao et al, Phys. Rev. Lett. 134, 090601 (2025).

SC PROCESSOR: ZUCHONGZHI 3.0

Simulation of the probability distribution obtained in 100 seconds by processor Zuchongzhi 3.0 would take at least several 10^6 years by the fastest classical computer.



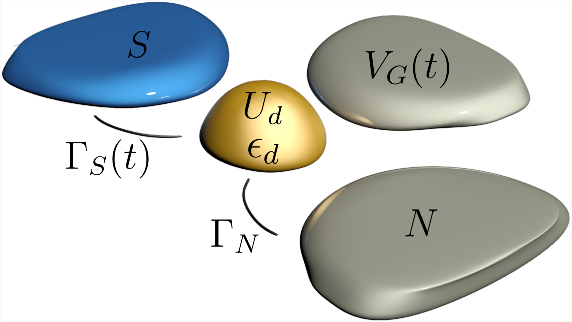
Zuchongzhi 3.0 processor consists of 105 qubits: 15 qubits in 7 arrays.

D. Gao et al, Phys. Rev. Lett. 134, 090601 (2025).

Characteristic time-scales

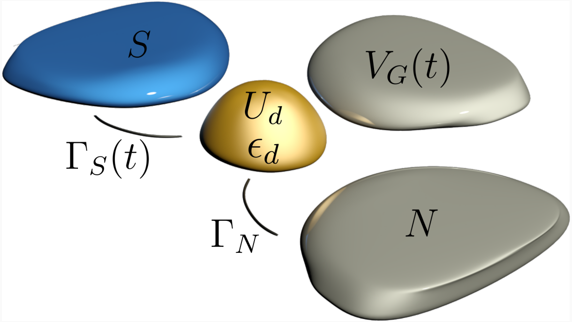
Characteristic time-scales

relevant to operations on bound states



Empirical protocols for time-resolved phenomena:

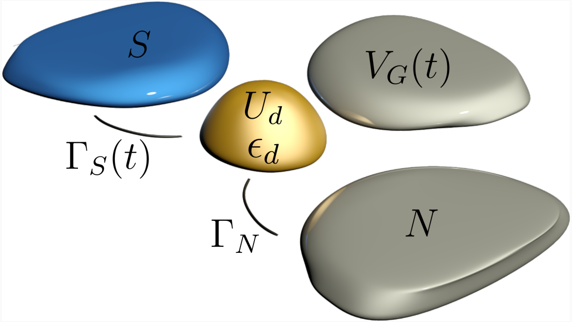
DYNAMICS OF IN-GAP STATES



Empirical protocols for time-resolved phenomena:

$\Rightarrow$ **variation of the coupling Γ_S to superconductor**

DYNAMICS OF IN-GAP STATES

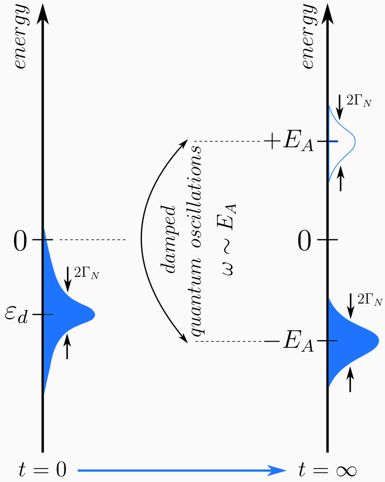


Empirical protocols for time-resolved phenomena:

$\Longrightarrow$ **variation of the coupling Γ_S to superconductor**

$\Longrightarrow$ **change of the gate potential V_G**

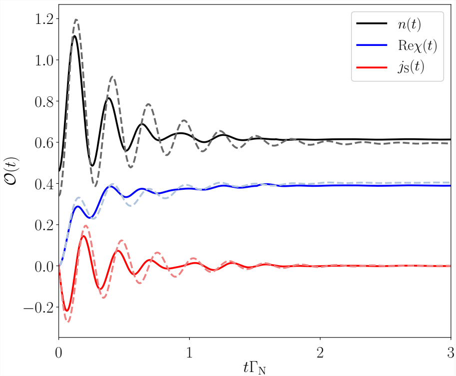
Sudden coupling of QD to superconductor $0 \rightarrow \Gamma_S$



K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

BUILDUP OF IN-GAP STATES

Time-dependent observables driven by the quantum quench $0 \to \Gamma_S$



$n(t)$ **electron number**

$\chi(t) = \left\langle \hat{d}_\downarrow \hat{d}_\uparrow \right\rangle$ **on-dot pairing**

$j_S(t)$ **charge current**

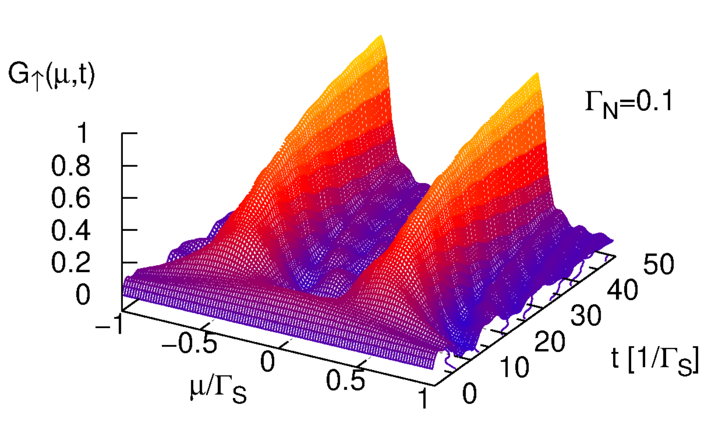
solid lines - time dependent NRG

dashed lines - Hartree-Fock-Bogolubov

K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

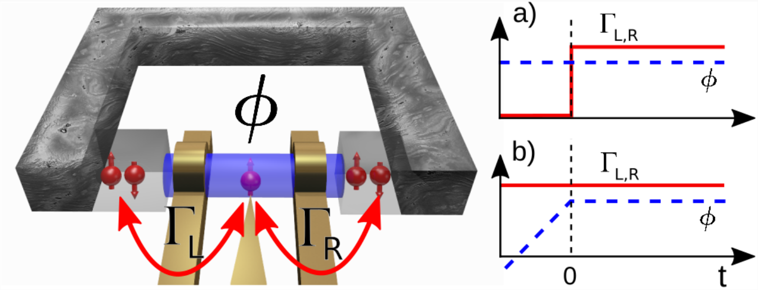
TIME-DEPENDENT TUNNELING CONDUCTANCE

Subgap tunneling conductance $G_\sigma = \frac{\partial I_\sigma(t)}{\partial \mu}$ vs time (t) and voltage (μ)



K. Wrześniewski, B. Baran, R. Taranko, T. Domański & I. Weymann, PRB 103, 155420 (2021).

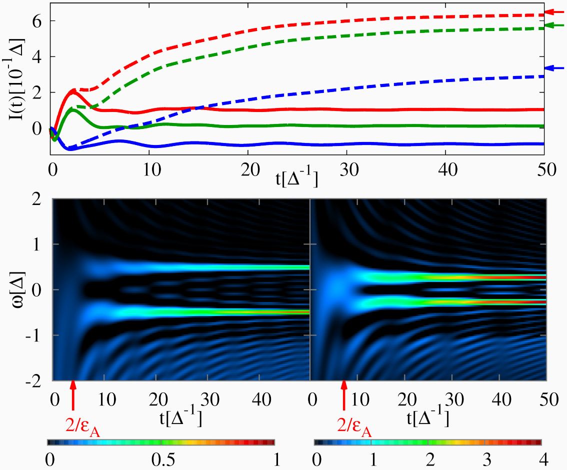
Quantum quench imposed on QD in Josephson junction geometry



R. Seoane Souto, A. Martín-Rodero, A. Levy Yeyati, Phys. Rev. Lett. 117, 267701 (2016).

BOUND STATES IN JOSEPHSON JUNCTION

Transient current and quasiparticle spectrum obtained for different ratios of Γ/Δ (from top to bottom: 10, 5 and 1).



R. Seoane Souto, A. Martín-Rodero, A. Levy Yeyati, Phys. Rev. Lett. 117, 267701 (2016).

YU-SHIBA-RUSINOV STATES OF CLASSICAL IMPURITY

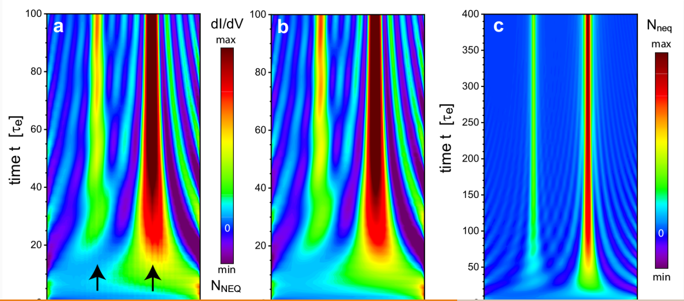
communications
physics

ARTICLE

https://doi.org/10.1038/s42005-022-01050-7 OPEN

Emergence and manipulation of non-equilibrium Yu-Shiba-Rusinov states

Jasmin Bedow[1], Eric Mascot[1,2] & Dirk K. Morr[1]



Correlation effects

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[singlet-doublet (quantum phase) transition]

SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

Quantum dot proximitized to superconductor can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Gamma_S \, \hat{d}^{\dagger}_{\uparrow} \hat{d}^{\dagger}_{\downarrow} + \text{h.c.} \right)$$

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Eigen-states of this problem are represented by:

$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left. \begin{array}{l} u\,|0\rangle - v\,|\uparrow\downarrow\rangle \\ v\,|0\rangle + u\,|\uparrow\downarrow\rangle \end{array} \right\} \Leftarrow$$

singlet states (spin 0)

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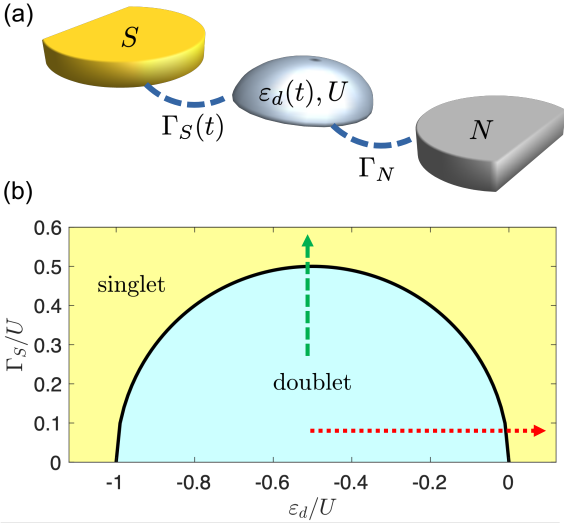
Upon varrying the ratio ε_d/U_d or Γ_S/U_d the doublet-singlet transition can be induced between these ground states.

QUENCH ACROSS STATIC QPT BOUNDARY



K. Wrześniewski, I. Weymann, N. Sedlmayr & T. Domański, Phys. Rev. B 105, 094514 (2022).

GENERAL OUTLINE

For $t < 0$ we assume the system $\hat{H}_0$ to be in its ground state:

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Fidelity (similarity) of these states is:

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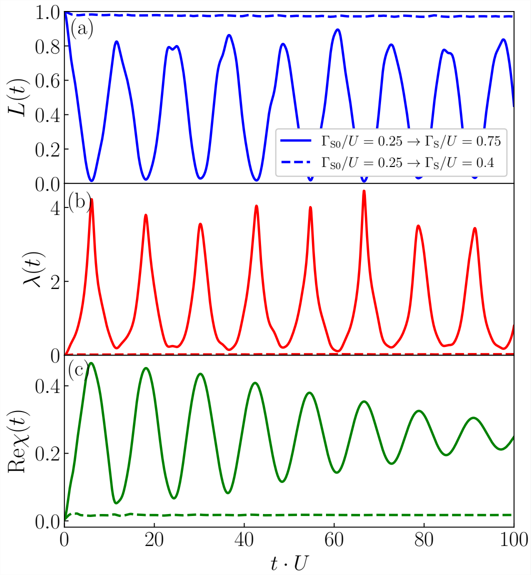
Loschmidt amplitude

tNRG RESULTS: ABRUPT CHANGE OF Γ_S



$$\varepsilon_d = -U/2$$

$$\Gamma_N = U/100$$

K. Wrześniewski, I. Weymann, N. Sedlmayr & T. Domański, Phys. Rev. B 105, 094514 (2022).

Triplet blockade

in junctions with two quantum dots

ANDREEV BLOCKADE: CONCEPT

SciPost Phys. 11, 081 (2021)

Theory of Andreev blockade in a double quantum dot with a superconducting lead

David Pekker, Po Zhang and Sergey M. Frolov

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA, 15260

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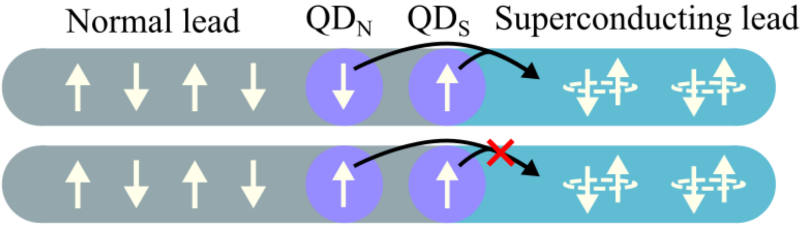
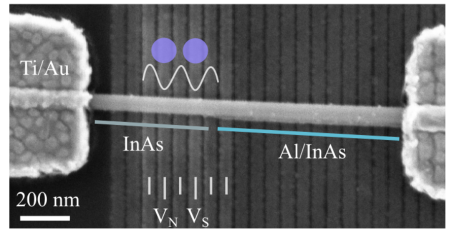
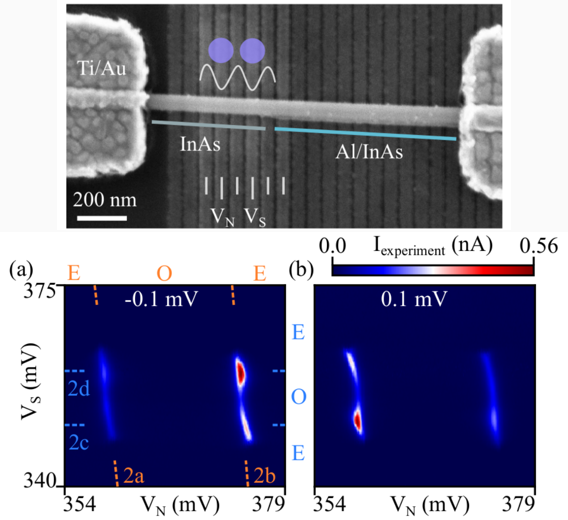


Superconducting proximity effect would be blocked by triplet configuration of the quantum dots (Andreev current forbidden).

ANDREEV BLOCKADE: REALIZATION



ANDREEV BLOCKADE: REALIZATION



P. Zhang, H. Wu, J. Chen, S.A. Khan, P. Krogstrup, D. Pekker, and S.M. Frolov, Phys. Rev. Lett. 128, 046801 (2022).

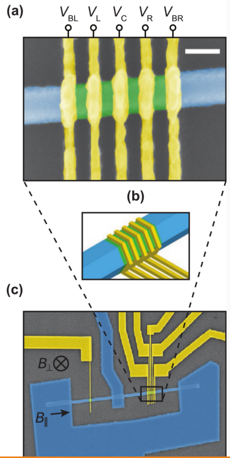
BLOCKADE IN JOSEPHSON JUNCTION

PHYSICAL REVIEW B **102**, 220505(R) (2020)

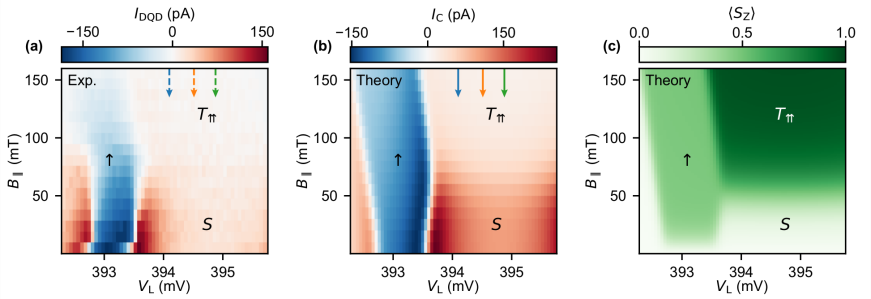
Rapid Communications

Triplet-blockaded Josephson supercurrent in double quantum dots

Daniël Bouman,[1] Ruben J. J. van Gulik,[1] Gorm Steffensen,[2] Dávid Pataki,[3] Péter Boross,[4] Peter Krogstrup,[2] Jesper Nygård,[2] Jens Paaske,[2] András Pályi,[3] and Attila Geresdi[1,5,*]



BLOCKADE IN JOSEPHSON JUNCTION



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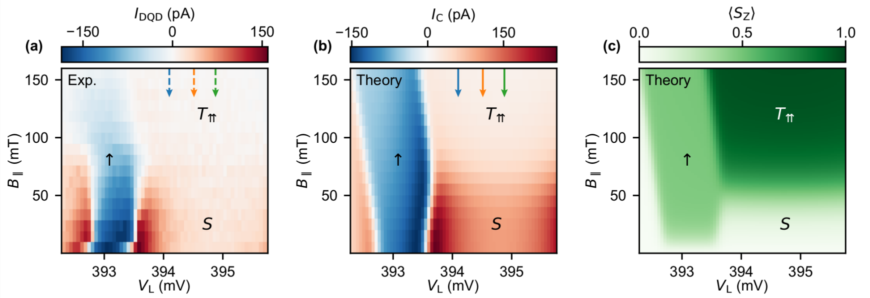


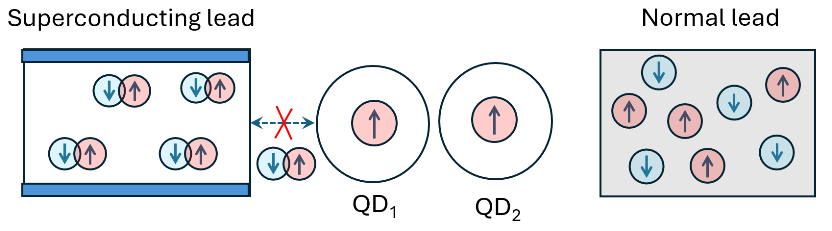
Experimental observation:

"magnetic field dependence of the supercurrent amplitude in the even occupied state reveals the presence of supercurrent blockade in the spin-triplet ground state"

Andreev blockade

[dynamical realizations]

DYNAMICAL ANDREEV BLOCKADE



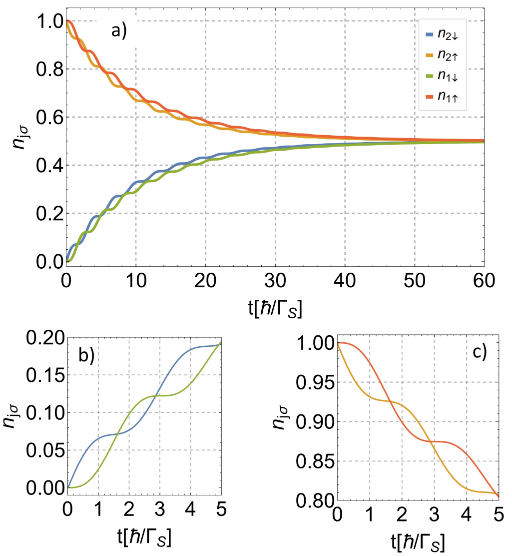
Superconducting proximity effect is blocked:

$\Rightarrow$ **when both quantum dots are singly occupied**

$\Rightarrow$ **by the same spin (for example $\uparrow$) electrons**

R. Taranko, J. Barański, A. Jankiewicz, K. Wrześniewski, I. Weymann & T. Domański [submitted (2025)].

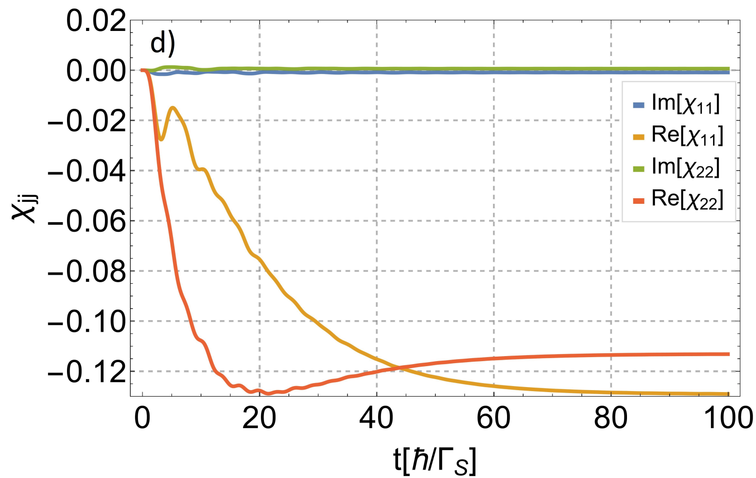
TRANSIENT BLOCKADE



Occupancy of the quantum dots initially occupied by ↑ electrons.

TRANSIENT BLOCKADE

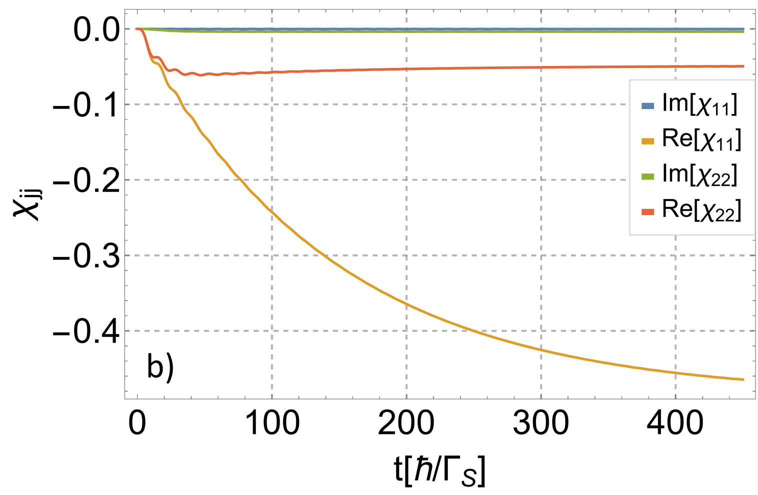
Results for $\chi_{jj}(t) \equiv \langle \hat{d}_{j\downarrow} \hat{d}_{j\uparrow} \rangle$ in the strong inter-dot coupling $V_{12} = \Gamma_S$.



Pairing in the quantum dots initially occupied by $\uparrow$ electrons

TRANSIENT BLOCKADE

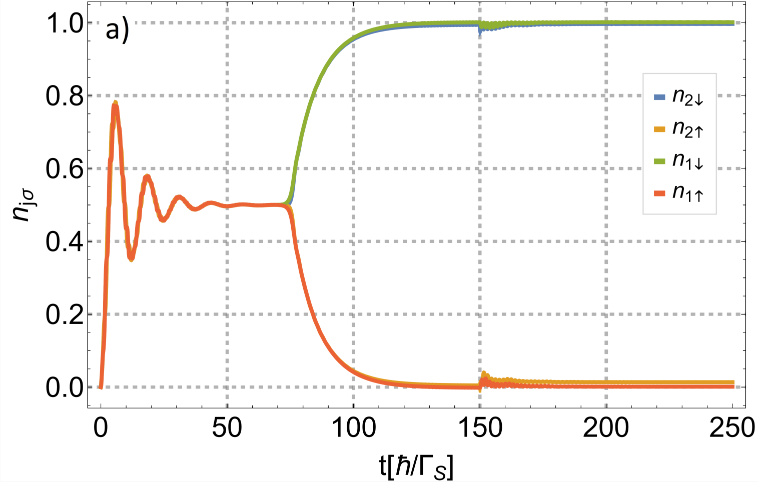
Results for $\chi_{jj}(t) \equiv \langle \hat{d}_{j\downarrow}\hat{d}_{j\uparrow} \rangle$ in the weak inter-dot coupling $V_{12} = 0.1\Gamma_S$.



Pairing in the quantum dots initially occupied by ↑ electrons

ZEEMAN INDUCED BLOCKADE

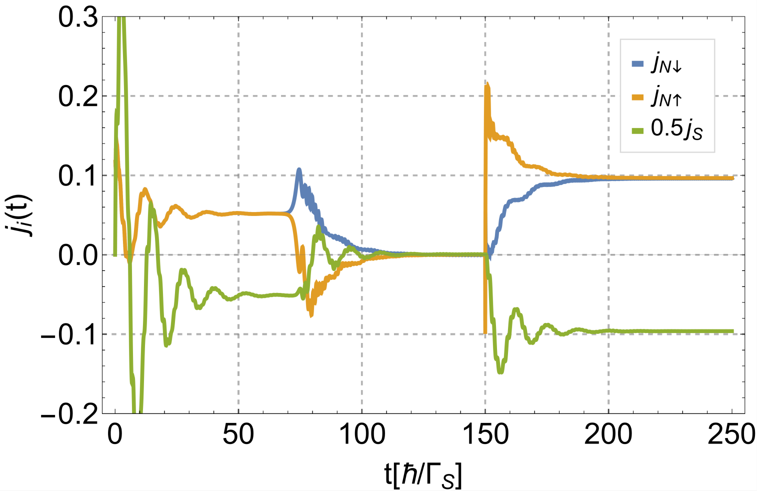
Magnetic field $B = 10\Gamma_S/\mu_B$ is switched on at $t = 75\Gamma_S/\hbar$



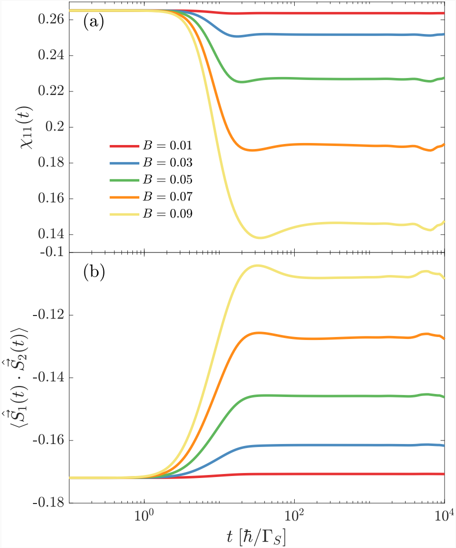
Occupancy of the initially empty quantum dots.

NON-EQUILIBRIUM CHARGE TRANSPORT

Magnetic field $B = 10\Gamma_S/\mu_B$ is switched on at $t = 75\Gamma_S/\hbar$ and bias voltage is strongly amplified at $t = 150\Gamma_S/\hbar$



Results obtained by time-dependent NRG calculations



Model parameters:

$U_1 = 0.9\Gamma_S, \quad U_2 = 0,$

$\varepsilon_1 = -U_1/2, \quad \varepsilon_2 = 0,$

$V_{12} = 0.225\Gamma_S$

SUMMARY

By attaching the quantum impurity to bulk superconductor (or when its energy level / coupling strength is varried):

- **Rabi-type oscillations are induced** **(due to particle-hole mixing)**
- **leading to buildup (re-arrangement) of the in-gap states**
- **dynamical phase transition can occur** **(changeover of ground state)**

SUMMARY

By attaching the quantum impurity to bulk superconductor
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- **Rabi-type oscillations are induced** (due to particle-hole mixing)
- **leading to buildup (re-arrangement) of the in-gap states**
- **dynamical phase transition can occur** (changeover of ground state)

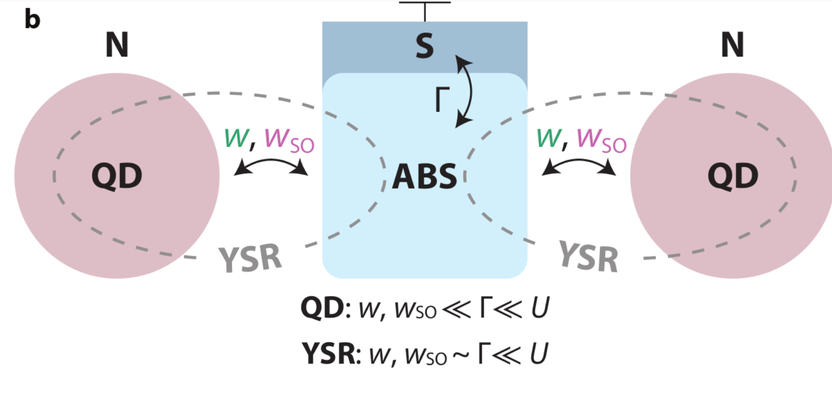
These phenomena could be detected in the charge transport measurements, using time-resolved Andreev spectroscopy.

Outlook

[triplet conf. in topological superconductors]

MINIMAL KITAEV CHAIN

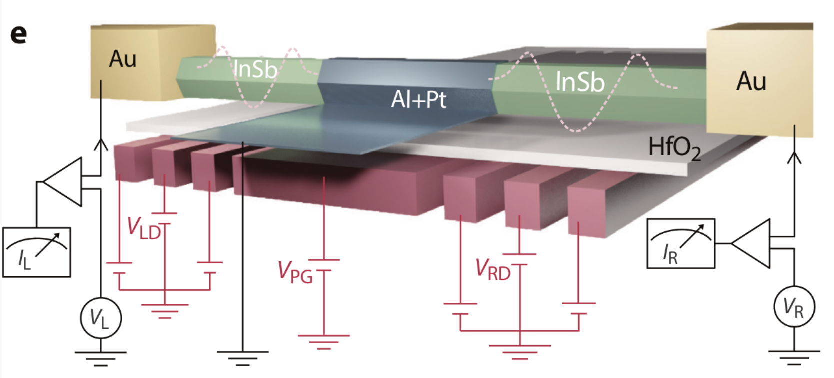
Effective triplet pairing can be realized using two quantum dots interconnected by superconductor (Poor Man's Majorana states)



T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).

MINIMAL KITAEV CHAIN

Two spin-polarized quantum dots in an InSb nanowire strongly coupled by elastic co-tunneling and crossed Andreev reflection



T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).