FROM BOGOLUBOV TO MAJORANA QPS IN TOPOLOGICAL SUPERCONDUCTORS



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Bogolubov quasiparticles in superconductors

 \Rightarrow particle vs hole dilemma

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 - N. Bogolubov J. Bardeen E. Majorana







N.N. BOGOLUBOV

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I. On the theory of superfluidity N.N. Bogolubov, J. Phys.(USSR) <u>11</u>, 23 (1947) [Izv. Akad. Nauk Ser.Fiz. <u>11</u>, 77 (1947)] Seminal contributions to the quantum field theory:

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II. On a new method in the theory of superconductivity N.N. Bogolubov, Nuovo Cim. <u>7</u>, 794 (1958)

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Effective spectrum of the superfluid ⁴He

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Bulk superconductors

SUPERCONDUCTOR

Perfect conductor



SUPERCONDUCTOR



HALLMARKS OF ELECTRON PAIRING

BCS ground state :

$$|\mathrm{BCS}
angle = \prod_k \left(u_k + v_k \ \hat{c}^\dagger_{k\uparrow} \ \hat{c}^\dagger_{-k\downarrow}
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Effective (Bogoliubov) quasiparticles

formally due to

$$\hat{\gamma}_{k\uparrow} = u_k \hat{c}_{k\uparrow} + \tilde{v}_k \hat{b}_{q=0} \hat{c}^{\dagger}_{-k\downarrow}$$

 $\hat{\gamma}^{\dagger}_{-k\downarrow} = -\tilde{v}_k \hat{b}^{\dagger}_{q=0} \hat{c}_{k\uparrow} + u_k \hat{c}^{\dagger}_{-k\downarrow}$

BOGOLIUBOV QUASIPARTICLES

Quasiparticle spectrum of conventional superconductors consists of the Bogoliubov (p/h) branches gaped around E_F



Let us consider the interface of metal ${f N}$ and superconductor ${f S}$



where incident electron ...

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Practical evidence:



- \Rightarrow upon injecting an electron to superconductor
- \Rightarrow a hole is reflected back (Andreev scattering).

Topological superconductors

Itinerant 1-dimensional fermions with intersite (p-wave) pairing

$$\hat{H} = t \sum_{i} \left(\hat{c}_{i}^{\dagger} \hat{c}_{i+1} + \text{h.c.} \right) - \mu \sum_{i} \hat{c}_{i}^{\dagger} \hat{c}_{i} + \Delta \sum_{i} \left(\hat{c}_{i}^{\dagger} \hat{c}_{i+1}^{\dagger} + \text{h.c.} \right)$$

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Alexei Y. Kitaev, Phys. Usp. 44, 131 (2001).

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They manifest themselves by very exotic phenomena !

• particle = antiparticle

$$\hat{\gamma}_{i,n}^{\dagger}=\hat{\gamma}_{i,n}$$

- \Rightarrow neutral in charge
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- topologically protected
- \Rightarrow immune to dephasing/decoherence

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REALIZATIONS OF KITAEV SCENARIO

Intersite pairing of the same spin electrons can be driven e.g. by the spin-orbit (Rashba) interaction in presence of the external magnetic field, using nanowires proximitized to *s-wave* superconductor.



R. Lutchyn, J. Sau, S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010).Y. Oreg, G. Refael, F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010).

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Roman Lutchyn was born in Lviv and received his M.Sc. in Kyiv !

Nanowire



A. Das et al, Nature Phys. 8, 887 (2012).

Nanowire + Rashba



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Nanowire + Rashba + magnetic field



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Nanowire + Rashba + magnetic field + superconductor



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 $B < B_{cr} \rightarrow$ trivial superconducting phase

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 $B > B_{cr} \rightarrow nontrivial$ superconducting phase

TRANSITION FROM TRIVIAL TO TOPOLOGICAL PHASE

Effective quasiparticle states of the Rashba nanowire



SPATIAL PROFILE OF MAJORANA QPS

Majorana qps are localized near the edges



R. Aguado, Riv. Nuovo Cim. 40, 523 (2017).

 $t_{35}/t = 1.0$ LDOS 20 15 10 5 0 1 0.04 10 20 0.02 30 ^Sit_e40 0.gqt 50 -0.02 60 -0.04 70

 $t_{35}/t = 0.8$ LDOS 20 15 10 5 0 1 0.04 10 20 0.02 30 ^Sit_e40 0.gqt 50 -0.02 60 -0.04 70

 $t_{35}/t = 0.6$



 $t_{35}/t = 0.4$



 $t_{35}/t = 0.2$



 $t_{35}/t = 0.1$



 $t_{35}/t = 0.0$



Differential conductance dI/dV obtained for InSb nanowire at 70 mK upon varying a magnetic field.



V. Mourik, ..., and L.P. Kouwenhoven, Science 336, 1003 (2012).

/ Technical Univ. Delft, Netherlands /

STM measurements for the nanochain of Fe atoms self-organized on a surface of superconducting Pb.



S. Nadj-Perge, ..., and <u>A. Yazdani</u>, Science **346**, 602 (2014). / **Princeton University, USA** /

AFM & STM data for Fe chain on Pb(110) surface



R. Pawlak, M. Kisiel *et al*, npj Quantum Information **2**, 16035 (2016). / University of Basel, Switzerland /

Results for the litographically fabricated Al nanowire



F. Nichele, ..., and Ch. Marcus, Phys. Rev. Lett. 119, 136803 (2017).

/ Niels Bohr Institute, Copenhagen, Denmark /

MAGNETIC CHAINS IN SUPERCONDUCTORS

Nanochain of magnetic impurities embedded in superconductor:



T.-P. Choy, J.M. Edge, A.R. Akhmerov, and C.W.J. Beenakker, Phys. Rev. B <u>84</u>, 195442 (2011).

MAGNETIC CHAINS IN SUPERCONDUCTORS



Ground state energy vs the pitch vector *q*

Quasiparticle energies

This magnetic chain self-tunes to the topological phase / topofilia /
























Structure factor:
$$A(q) = \frac{1}{L} \sum_{jk} e^{iq(j-k)} \langle S_j \cdot S_k \rangle$$



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TEMPERATURE EFFECT ON MAJORANA QPS













THERMAL EFFECTS: CONCLUSIONS

Thermal effects are responsible for:

- \Rightarrow changeover of topological \mathbb{Z}_2 number
- \Rightarrow scaling of the coherence length $\xi(T) \propto 1/T$
- \Rightarrow closing of the topological energy gap
- \Rightarrow overdamping of the Majorana qps

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Realistically such critical temperature would be:

 $\Rightarrow T_c \approx 5 \text{ K}$

Ongoing projects & challengies

1. MAJORANA STATES LEAKING TO QUANTUM DOT

Schematics of a quantum dot – nanowire hybrid structure.



A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. 96, 195403 (2017).

1. DISTINGUISHING MAJORANA QPS

QD spectrum vs gate potential V_g for various spin-orbit couplings λ .



A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. 96, 195403 (2017).

1. EXPERIMENTAL EVIDENCE

'Coalescence' of the Andreev into Majorana qps



M.T. Deng, ..., and Ch. Marcus, Science 354, 1557 (2016).

/ Niels Bohr Institute, Copenhagen, Denmark /

2. TWO-DIMENSIONAL MAGNETIC STRUCTURES

Magnetic island of Co atoms deposited on the superconducting Pb surface

Diameter of island: 5 – 10 nm



G. Ménard, ..., and <u>P. Simon</u>, Nature Commun. **8**, 2040 (2017). / **P. & M. Curie University (Paris, France)** /

2. EVIDENCE FOR DELOCALIZED MAJORANA MODES

Majorana modes propagating along magnetic islands



G. Ménard, ..., and <u>P. Simon</u>, Nature Commun. **8**, 2040 (2017). / P. & M. Curie University (Paris, France) /

2. PROPAGATING MAJORANA EDGE MODES

Magnetic island of Fe atoms deposited on the superconducting Re surface



Chern number: C = 20

A. Palacio-Morales, ..., and <u>R. Wiesendanger</u>, arXiv:1809.04503 (preprint). / University of Hamburg (Germany) /

2. PROPAGATING MAJORANA EDGE MODES

Real space maps of the tunneling conductance (top panel) and deconvoluted DOS (bottom panel) obtained for various energies (as indicated) in the subgap regime ($\Delta = 240 \mu eV$).



A. Palacio-Morales, ..., and R. Wiesendanger, arXiv:1809.04503 (preprint).

/ University of Hamburg (Germany) /

3. MIXED-DIMENSIONALITY STRUCTURES

Our project: Majorana qps of the 1D-2D hybrid structure



A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

3. TRIVIAL VS MAJORANA MODES

Majorana/Andreev quasiparticles of a wire-plaquette hybrid



plaquette: nontopological

nanowire: topological

A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

3. TRIVIAL VS MAJORANA MODES

Majorana/Andreev quasiparticles of a wire-plaquette hybrid



Both regions are assumed to be in topological sc phase.

A. Kobiałka, T. Domański & A. Ptok, arXiv:1808.05281

3. ISLAND + NONOWIRE

Itinerant Majorana mode leaking into side-attached nanowire.



E. Mascot, S. Cocklin, S. Rachel, and D.K. Morr, arXiv:1811.06664 Univ. of Illinois at Chicago (USA)

3. DEFECTS IN MAGNETIC ISLAND

Localized Majorana at point-like defect, coexisting with itinerant

Majorana edge mode (observed in Co-Si island on disordered Pb)



PERSPECTIVES

- novel devices
- \Rightarrow e.g. fractional SQUID
- stable quantum bits
- quantum computing
- Q-stations:
- \Rightarrow USA (California & Washington)
- \Rightarrow Netherland (Delft)
- \Rightarrow Denmark (Kopenhaga)
- \Rightarrow Australia (Sydney)

Majorana spintronics

- immune to decoherence
 - non-Abelian braiding
 - **IBM** initiative

ACKNOWLEDGEMENTS

- Majorana quasiparticles
- \Rightarrow M. Maśka & A. Gorczyca-Goraj (Katowice),
- \Rightarrow A. Kobiałka (Lublin),
- \Rightarrow A. Ptok (Kraków),
- \Rightarrow J. Tworzydło (Warszawa),
- Shiba states/bands in topological phases
- \Rightarrow Sz. Głodzik (Lublin),
- Majorana vs Kondo
- \Rightarrow I. Weymann (Poznań),
- \Rightarrow G. Górski (Rzeszów),
- \Rightarrow J. Barański (Dęblin).

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http://kft.umcs.lublin.pl/doman/lectures