Wrocław, 12 Dec. 2014

Andreev and Majorana type quasiparticles in nanoscopic systems

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Motivation – experimental fact # 1

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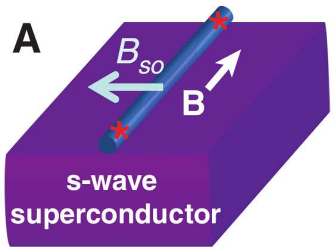
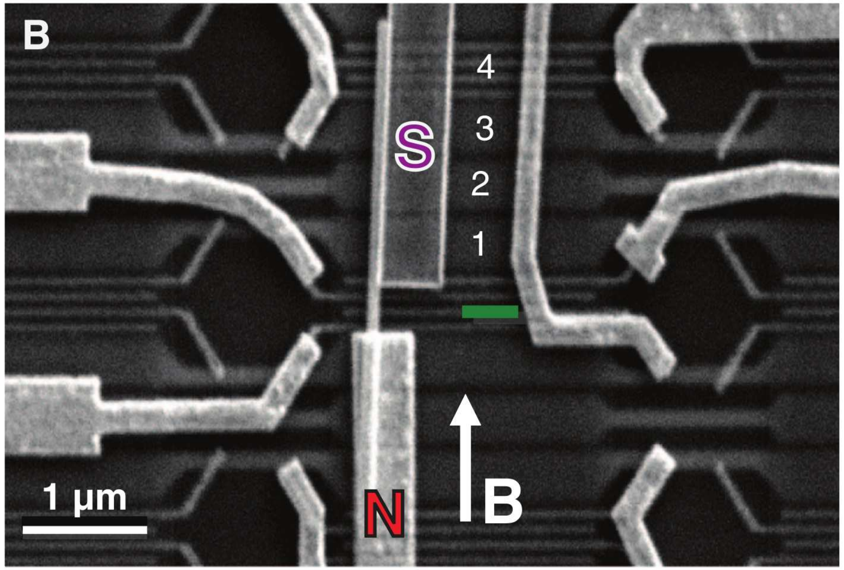
InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)





dI/dV **measured at 70 mK for varying magnetic field B indicated:**

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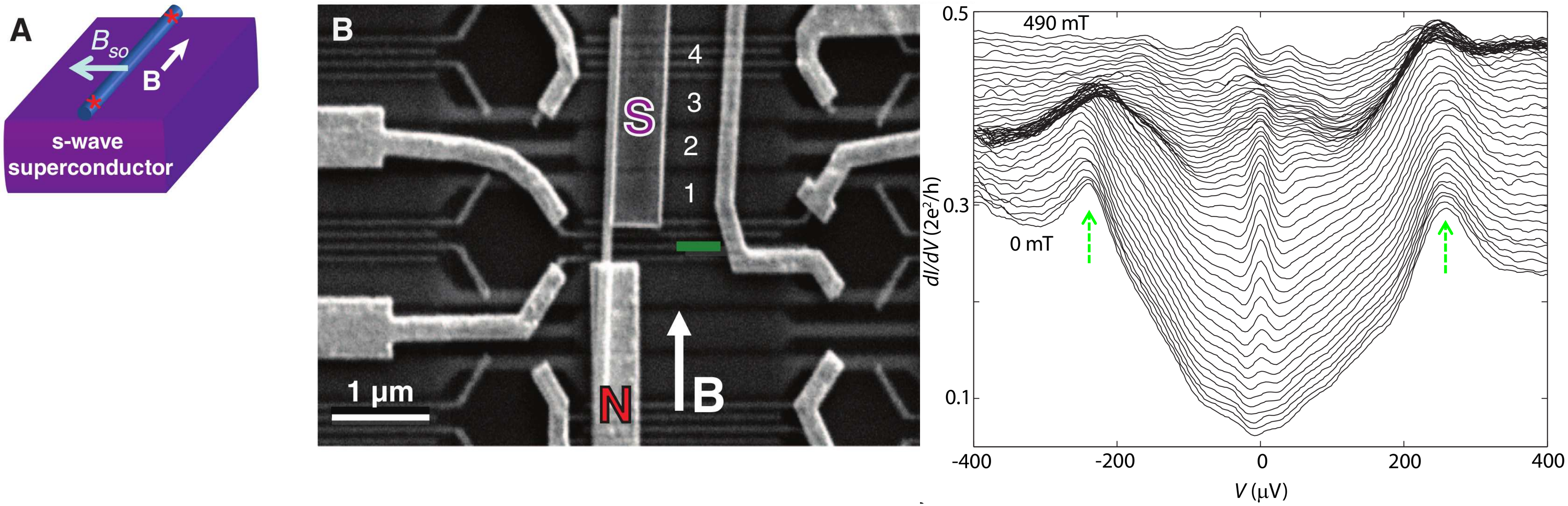

dI/dV measured at 70 mK for varying magnetic field B indicated:

$\Rightarrow$ **a zero-bias enhancement due to Majorana state**

V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

Motivation – experimental fact # 2

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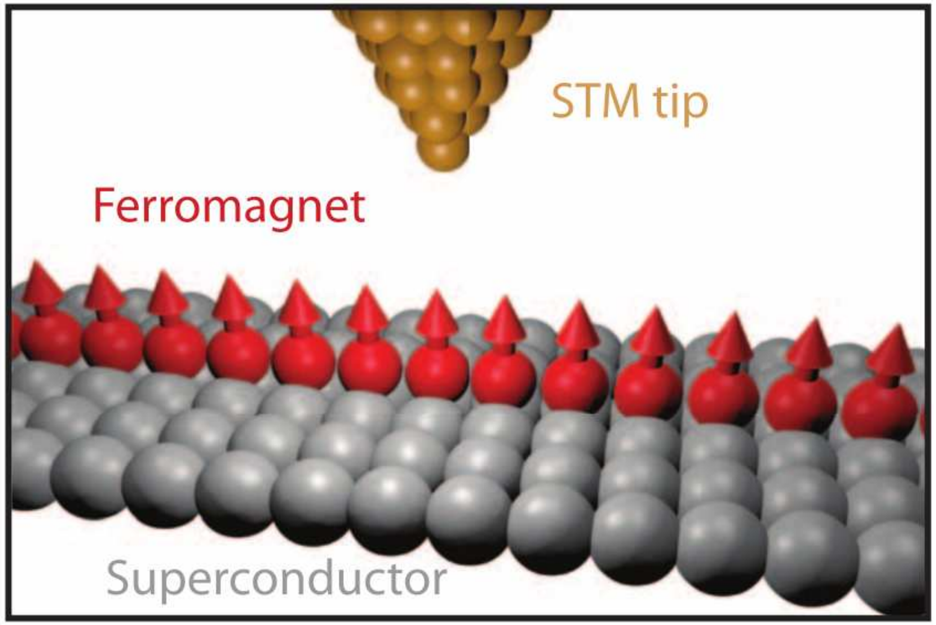
A chain of iron atoms deposited on a surface of superconducting lead



STM measurements provided evidence for:

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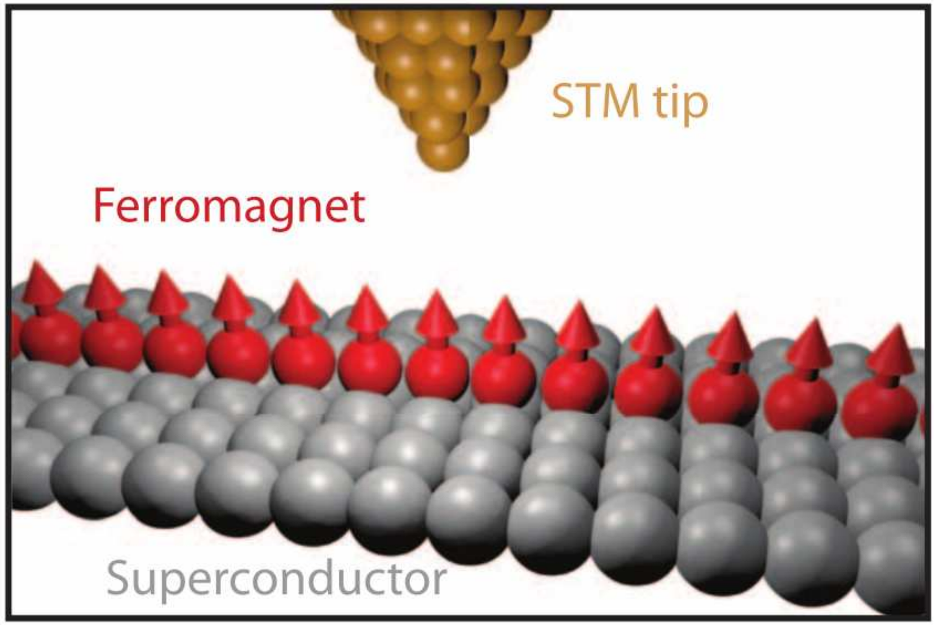

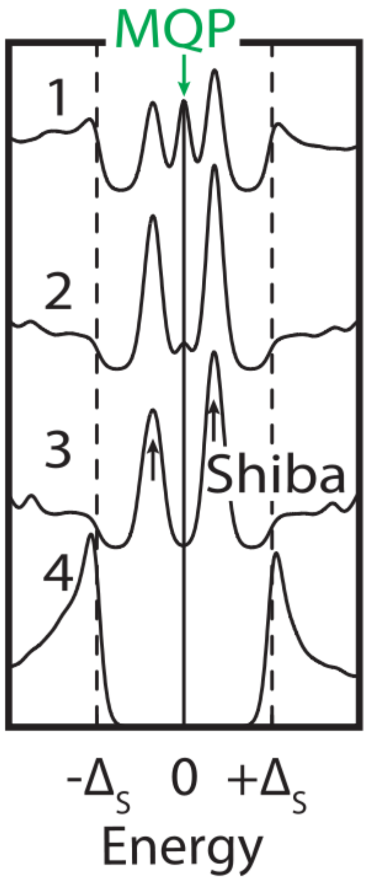

STM measurements provided evidence for:

⇒ **Majorana bound states at the edges of a chain.**

S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, Princeton (NJ), USA /

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- E. Schrödinger (1926)

$$E \longrightarrow i\frac{d}{dt} \qquad \vec{p} \longrightarrow -i\nabla$$

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$$i\dot{\psi} = (\vec{\alpha}\cdot\vec{p} + \beta m)\,\psi$$

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- E. Majorana (1937)

particle = antiparticle

E.M. noticed that particular choice of $\vec{\alpha}$ and β implies a real wave-function !

Searching for majoranas – in particle and nuclear physics

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DOUBLE BETA DECAY

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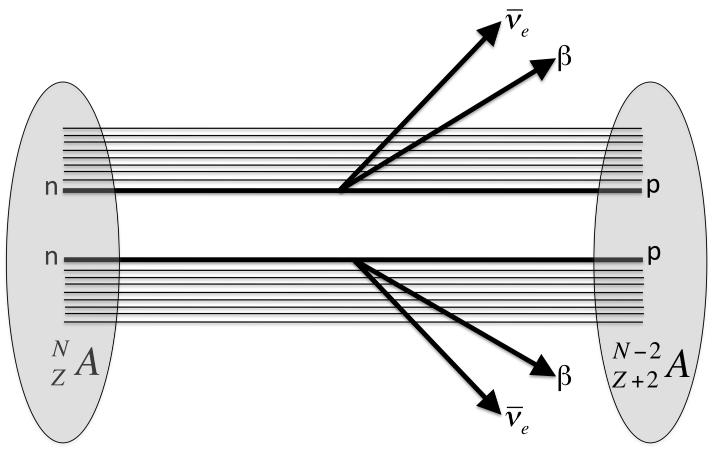

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Searching for majoranas – in particle and nuclear physics

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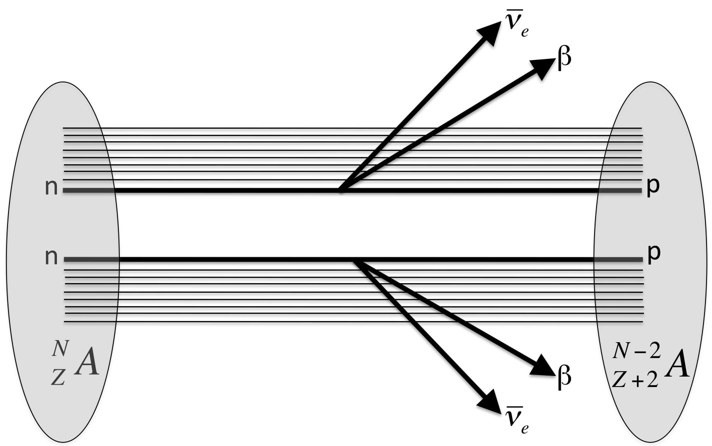

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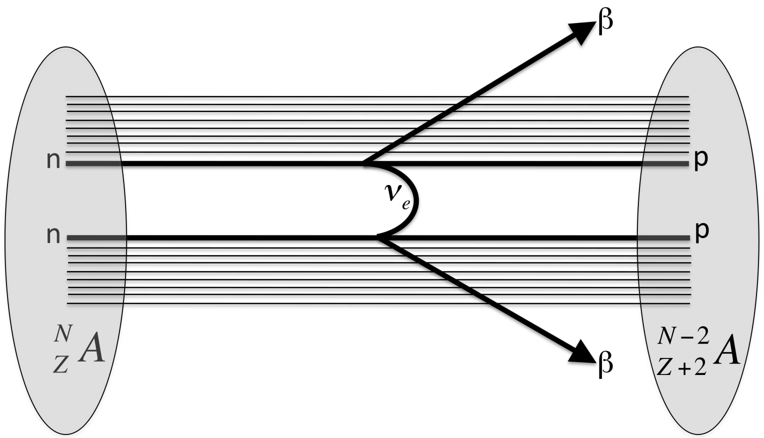

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Neutrinoless decay would imply neutrinos to be majoranas.

Searching for majoranas – **in particle and nuclear physics**

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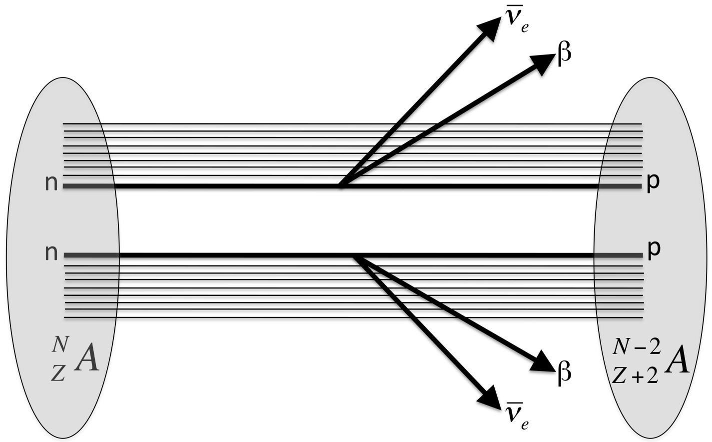

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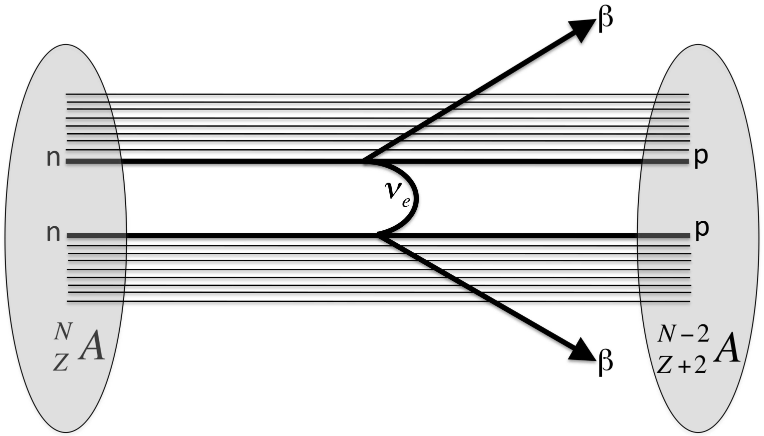

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Does it really occur ?

Search for majorana quasiparticles – in solids

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- ⋆ Formally, any Dirac fermion can be **majoranized** via
  a canonical transformation to the **Majorana basis**.

Majoranization – generic outline

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$$\left\{\hat{c}_i, \hat{c}_j^\dagger\right\} = \delta_{i,j}$$
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- $c_j^{(\dagger)}$ can be recast in terms of majoranas

$$\hat{c}_j \equiv (\hat{\gamma}_{j1} + i\hat{\gamma}_{j2})/2$$
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'real' and 'imaginary' parts

- These $\gamma_{i,n}$ operators obey unconventional algebra

$$\left\{\hat{\gamma}_{i,n}, \hat{\gamma}_{j,m}^\dagger\right\} = 2\delta_{i,j}\delta_{n,m}$$
$$\hat{\gamma}_{i,n}^\dagger = \hat{\gamma}_{i,n}$$

creation = annihilation !

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- **Majorana-type quasiparticles**

$$\hat{\gamma}_{j1} \equiv \hat{c}_j + \hat{c}_j^\dagger$$
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must be <u>neutral</u> objects

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- **At the Fermi level the BCS coefficients $u_{k_F} = v_{k_F} = 1/\sqrt{2}$, thus**

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OK, but they must be zero-energy modes

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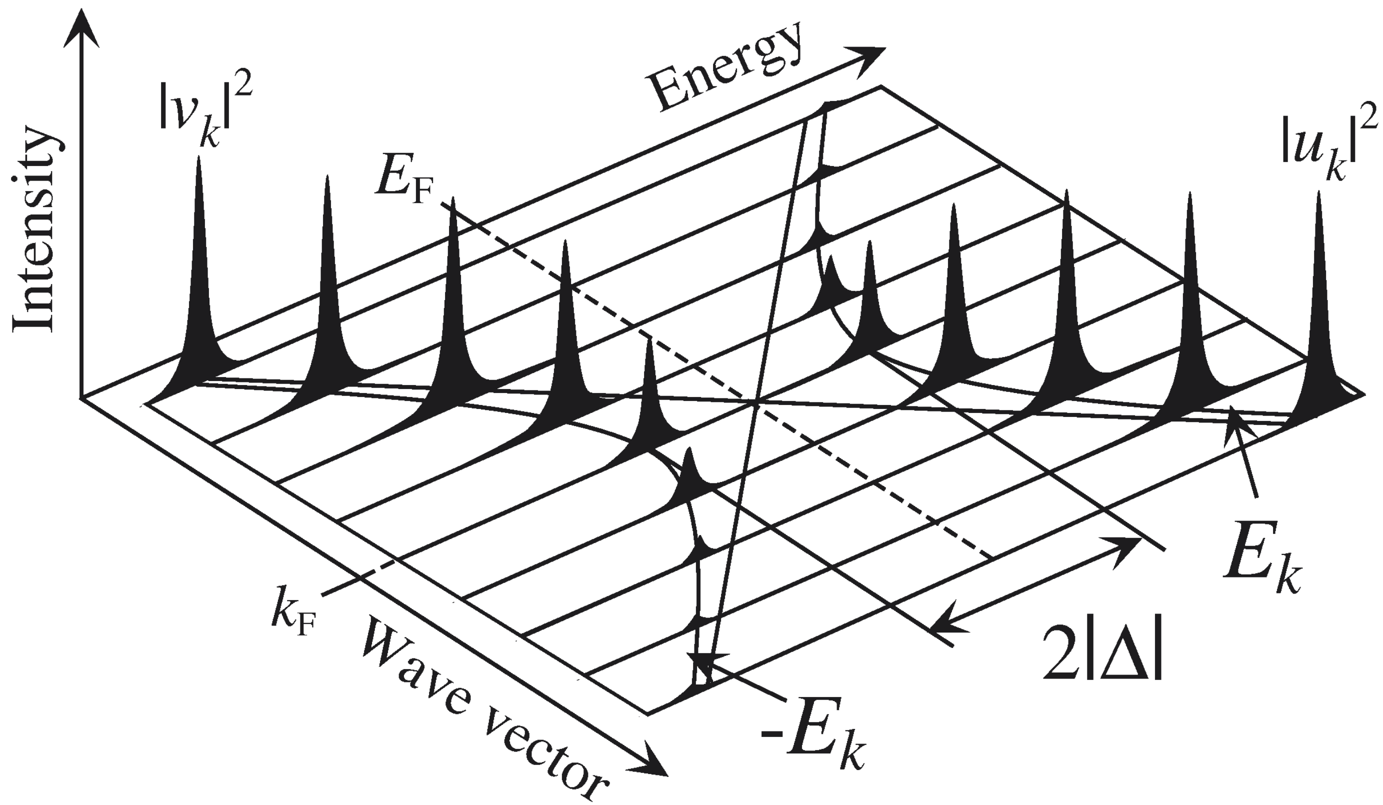

Quasiparticles of s-wave superconductors are gapped ($\pm E_k = \pm\sqrt{\varepsilon_k^2 + \Delta^2}$).

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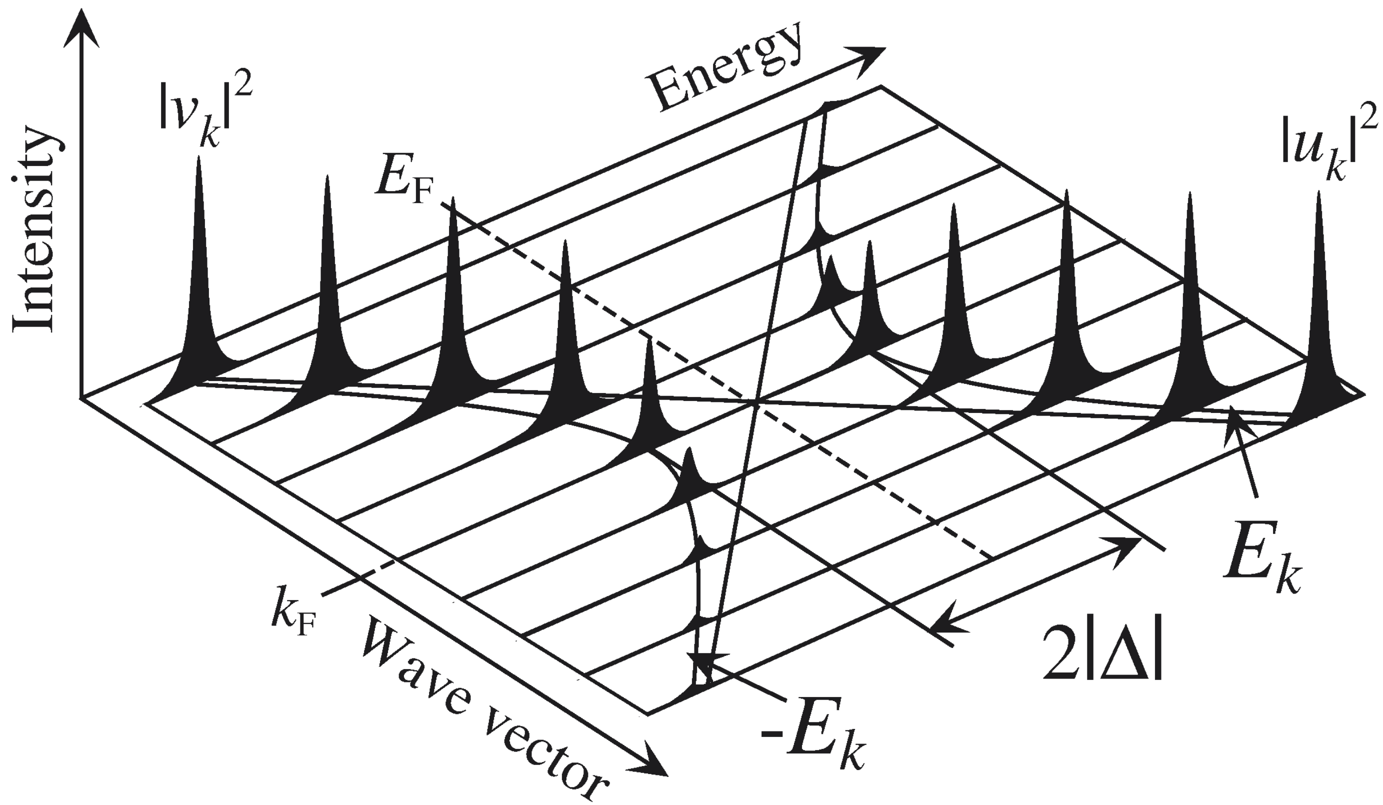

Quasiparticles of s-wave superconductors are gapped ($\pm E_k = \pm\sqrt{\varepsilon_k^2 + \Delta^2}$).

To obtain true majoranas we need the zero energy ($E_k = 0$) quasiparticles !

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- ⋆ Un-paired majoranas need **time-reversal symmetry breaking**, for instance, this can be obtained in p-wave superconductors.

- ⋆ Majorana quasiparticles obey **non-Abelian (anyon)** statistics, thus might be usefull for quantum computation.

Theoretical proposals – Fu-Kane model (2008)

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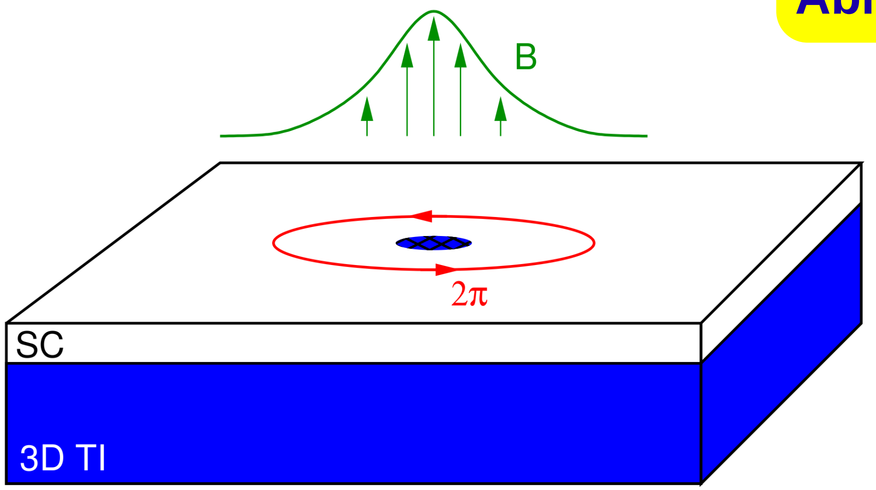
Abrikosov vortex in p-wave superconductor



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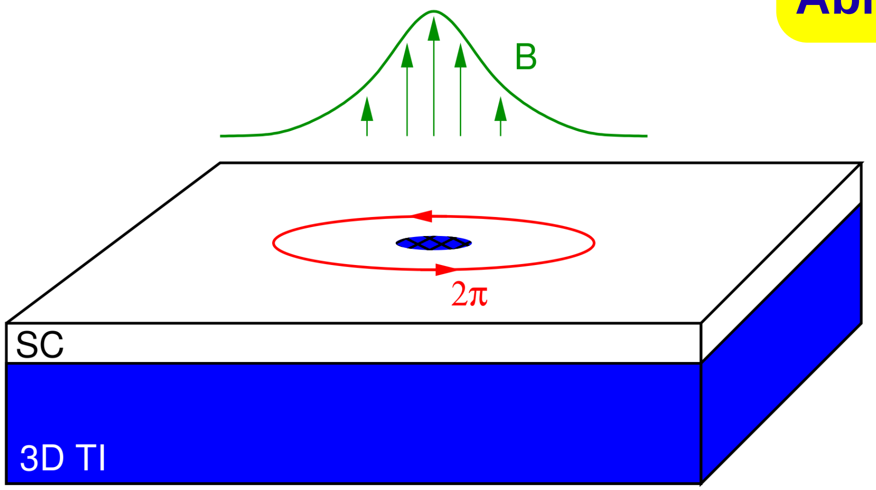

Abrikosov vortex in p-wave superconductor

$$\Delta(\vec{r}) = \Delta_0(r)e^{-i(n\phi+\alpha)}$$

n – winding number

ϕ – polar angle

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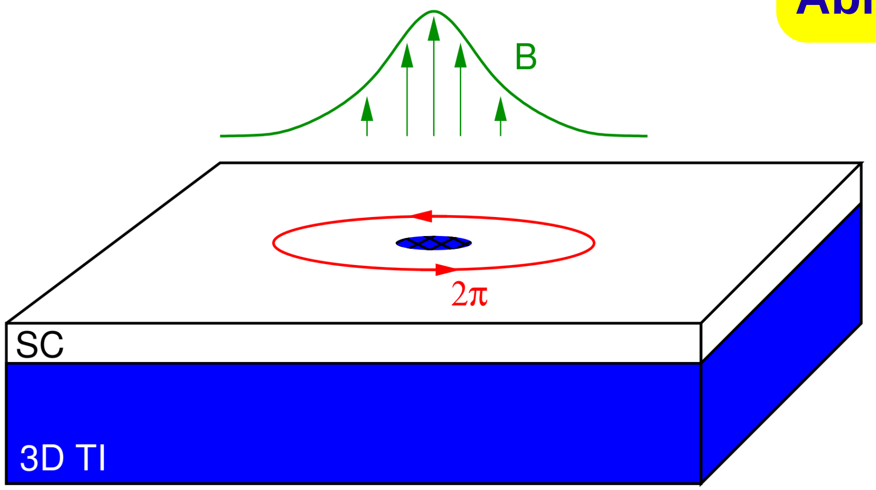

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Possible examples: SC $\Rightarrow$ Pb, Nb 3D TI $\Rightarrow$ Bi_2Se_3, Bi_2Te_3

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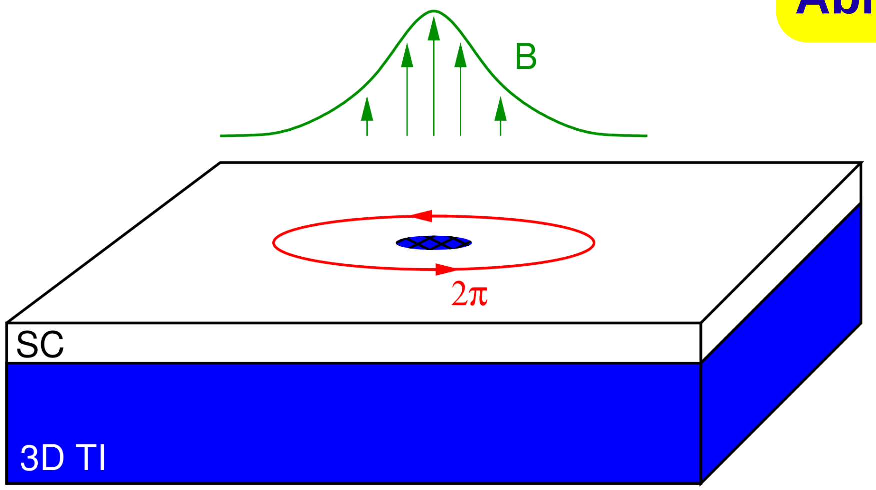

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The zero-mode Majorana quasiparticle

$$\hat{\gamma} = \frac{i}{\sqrt{2}} \int d^2\vec{r} \left[e^{i(\alpha/2-\pi/4)} \hat{c}_{\vec{r}\downarrow} - e^{-i(\alpha/2-\pi/4)} \hat{c}^{\dagger}_{\vec{r}\downarrow} \right] f(r)$$

Theoretical proposals – Kitaev model (2001)

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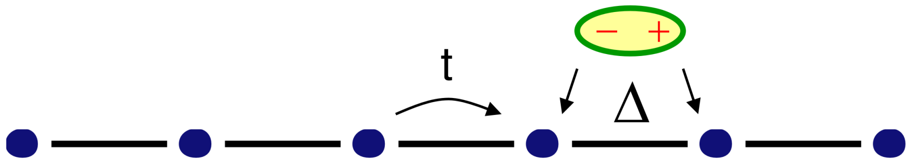

p-wave pairing of spinless 1D fermions

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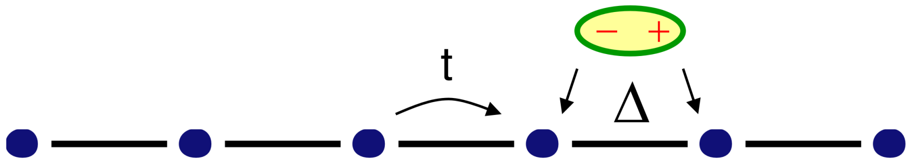

p-wave pairing of spinless 1D fermions

$$\hat{H} = t\sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.}\right) - \mu \sum_i \hat{c}_i^\dagger \hat{c}_i + \Delta \sum_i \left(\hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \text{h.c.}\right)$$

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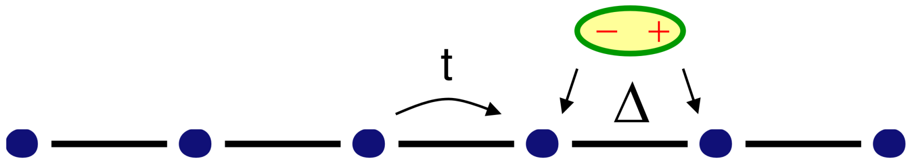

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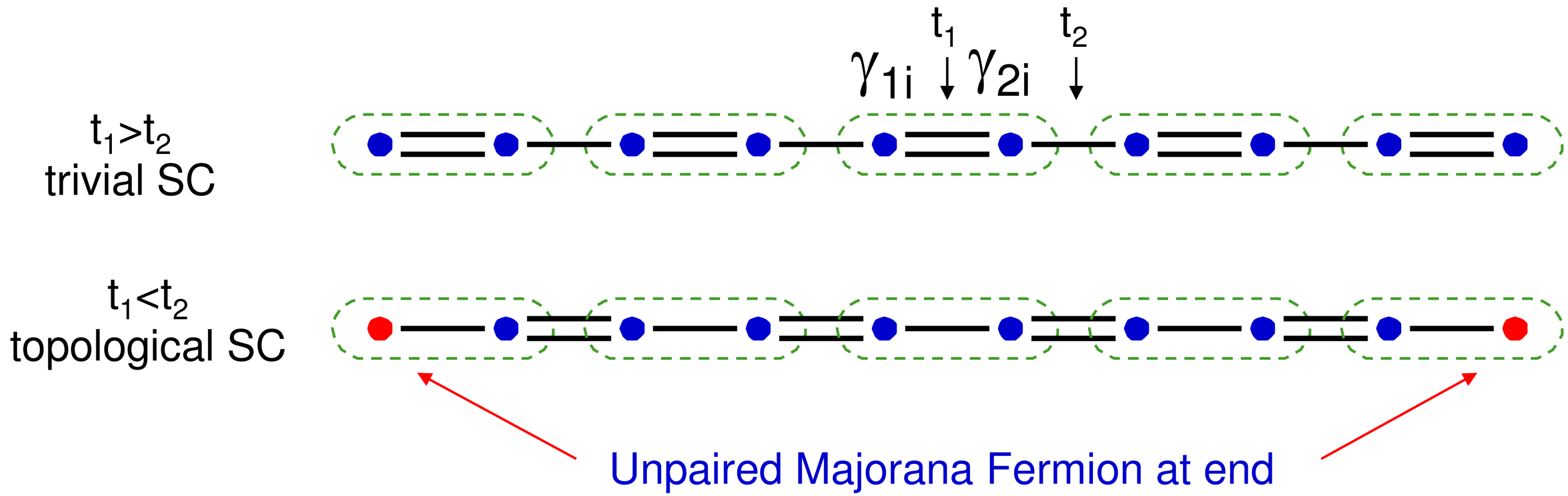
This toy-model is **exactly soluble** in Majorana basis. Two special cases:



$t_1 \equiv \mu$

$t_2 \equiv 2t$

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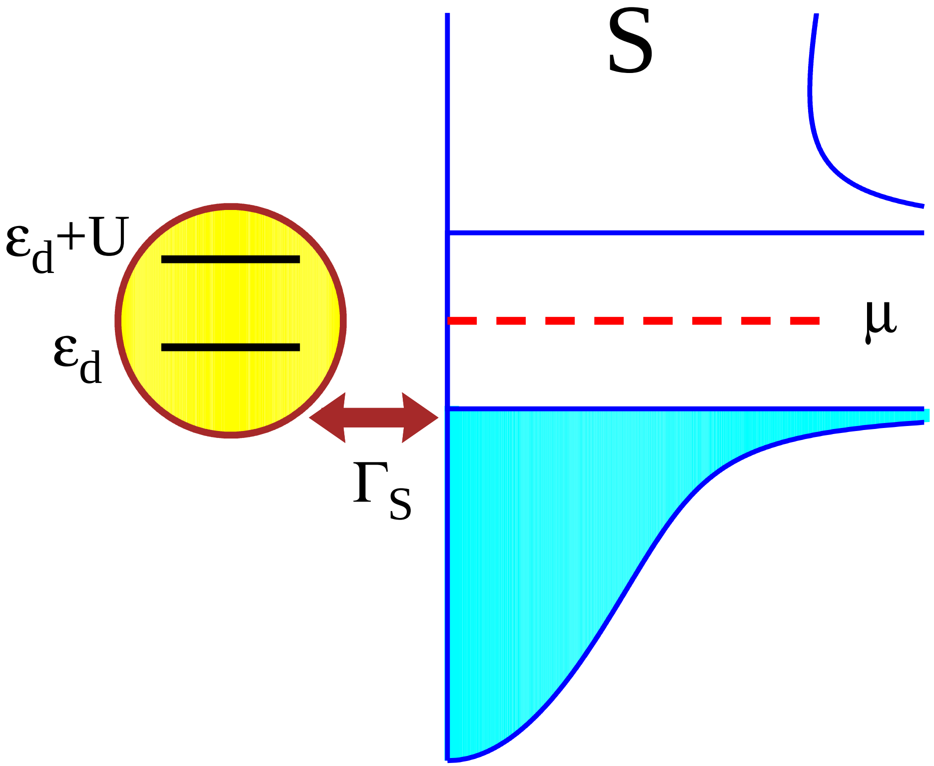
- **superconductor-double quantum dot)**

  A.R. Wright, M. Velthosrt, Phys. Rev. Lett. (2013)

Quantum impurity (dot or wire) coupled to a superconductor

Electronic spectrum



Microscopic model

Anderson-type Hamiltonian

Quantum impurity (dot)

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U \, \hat{n}_{d\uparrow} \, \hat{n}_{d\downarrow}$$

coupled with a superconductor

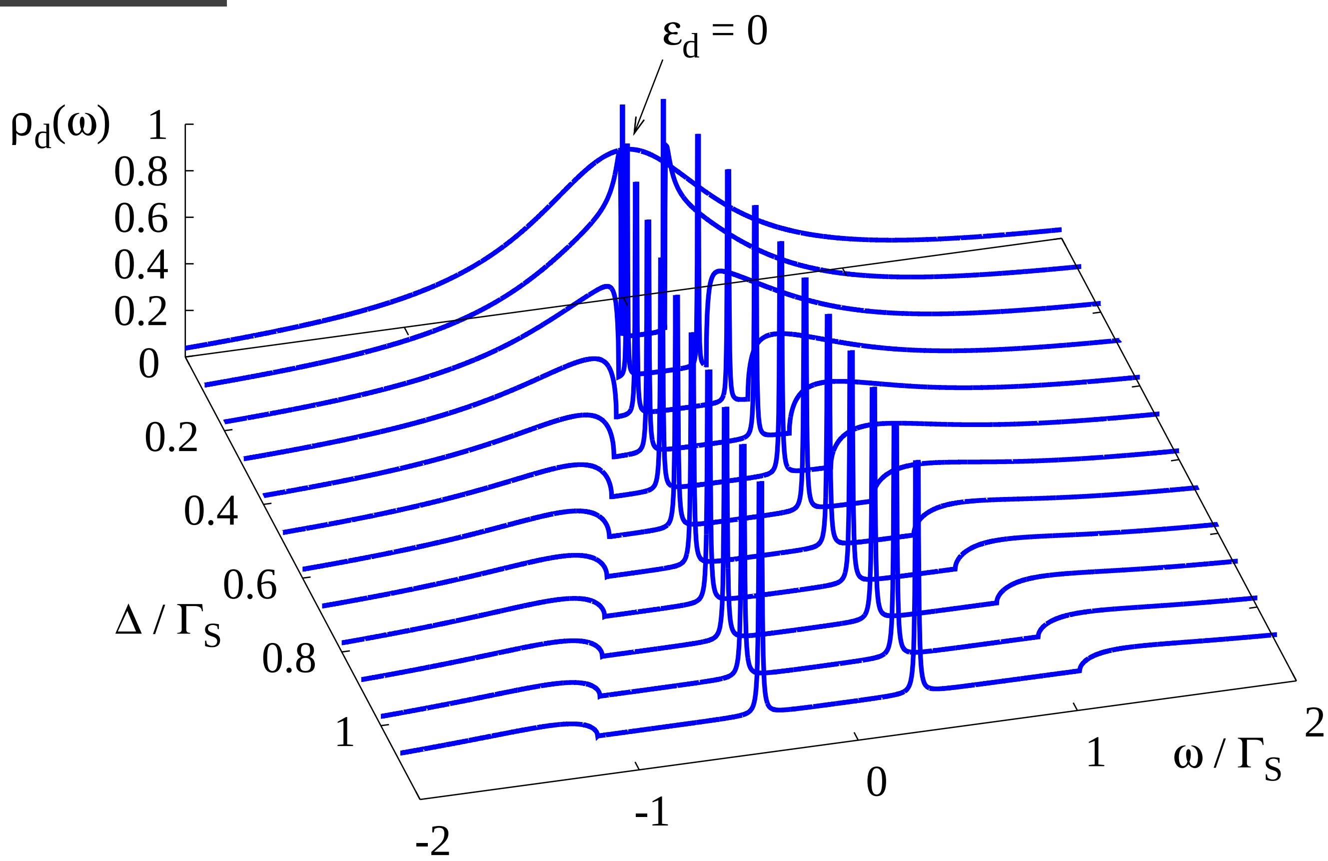
$$\begin{aligned} \hat{H} &= \sum_{\sigma} \epsilon_d \hat{d}^{\dagger}_{\sigma} \hat{d}_{\sigma} + U \, \hat{n}_{d\uparrow} \, \hat{n}_{d\downarrow} + \hat{H}_S \\ &+ \sum_{\mathbf{k},\sigma} \left(V_{\mathbf{k}} \, \hat{d}^{\dagger}_{\sigma} \hat{c}_{\mathbf{k}\sigma} + V^{*}_{\mathbf{k}} \, \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{d}_{\sigma} \right) \end{aligned}$$

where

$$\hat{H}_S = \sum_{k,\sigma} \left(\varepsilon_k - \mu \right) \hat{c}^{\dagger}_{k\sigma} \hat{c}_{k\sigma} - \sum_{k} \left(\Delta \hat{c}^{\dagger}_{k\uparrow} \, \hat{c}^{\dagger}_{k\downarrow} + \mathrm{h.c.} \right)$$

Uncorrelated QD – exactly soluble $U_d = 0$ case

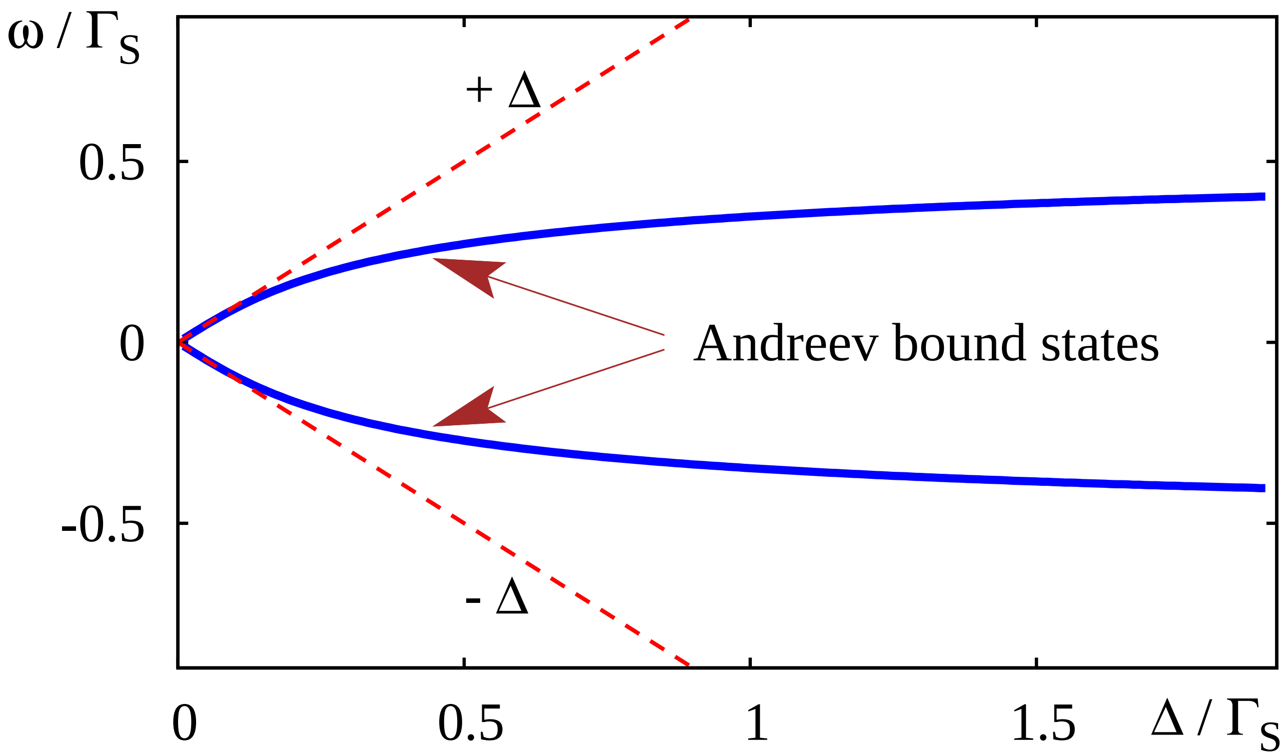
Uncorrelated QD – **exactly soluble $U_d = 0$ case**



In-gap resonances: Andreev bound states.

J. Barański and T. Domański, J. Phys.: Condens. Matter **25**, 435305 (2013).

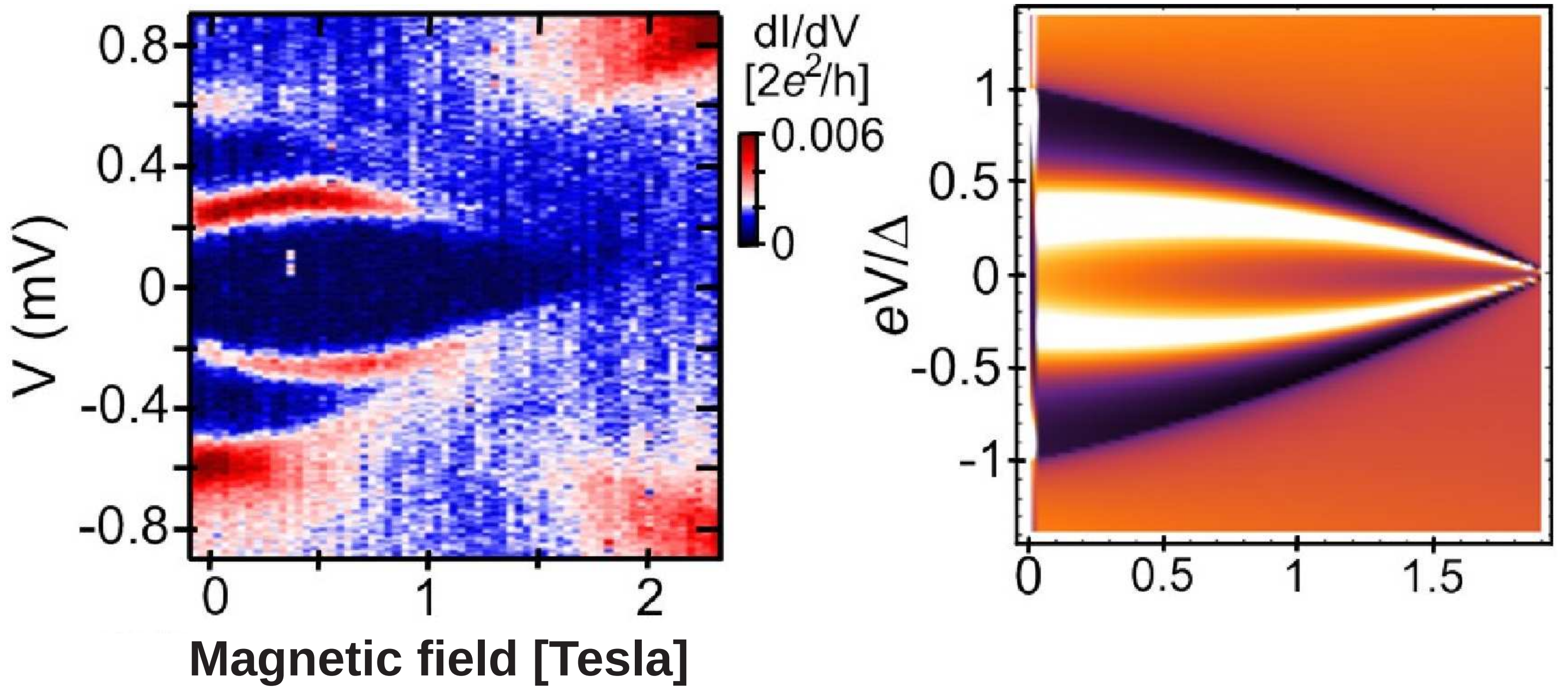
Uncorrelated QD – exactly soluble $U_d = 0$ case



Energies of the in-gap resonances (Andreev bound states)

J. Barański and T. Domański, J. Phys.: Condens. Matter **25**, 435305 (2013).

Subgap states – experimental data



Differential conductance of nanotubes coupled to vanadium (S) and gold (N)

/ external magnetic field changes the magnitude of pairing gap $\Delta(B)$ /

Eduardo J.H. Lee, ..., S. De Franceschi, Nature Nanotechnology **9**, 79 (2014).

Andreev vs Majorana states – a story of mutation

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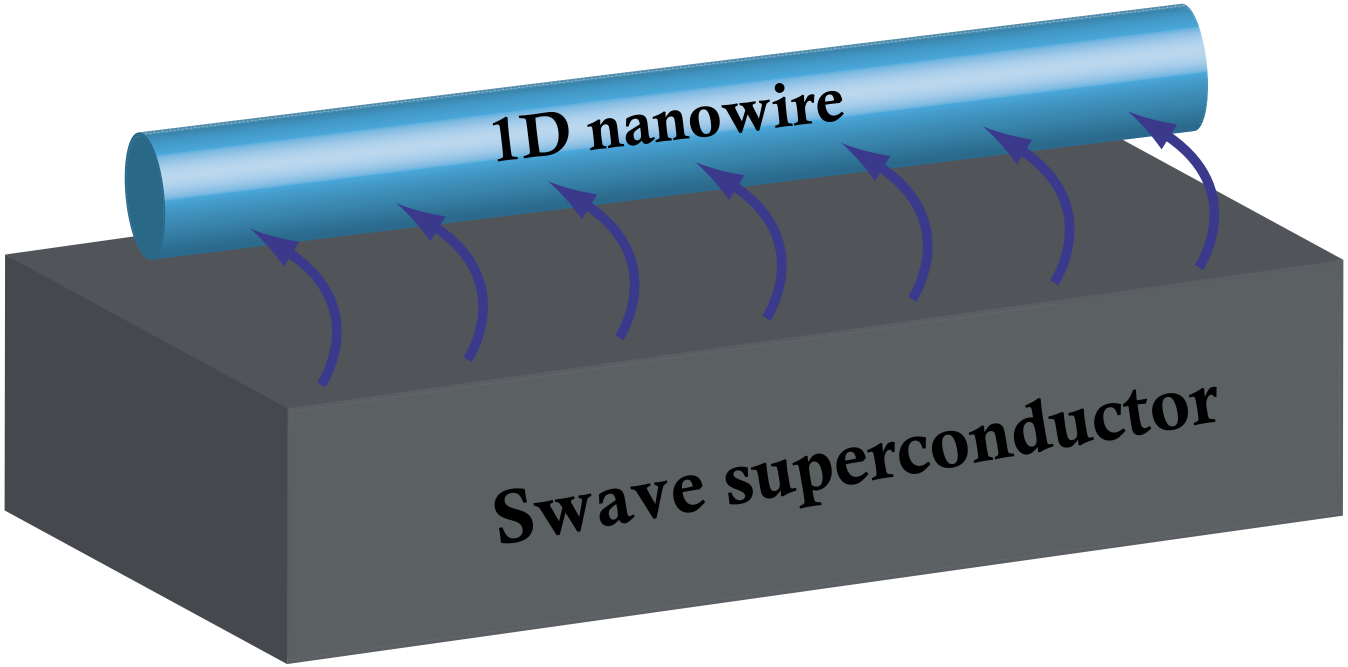
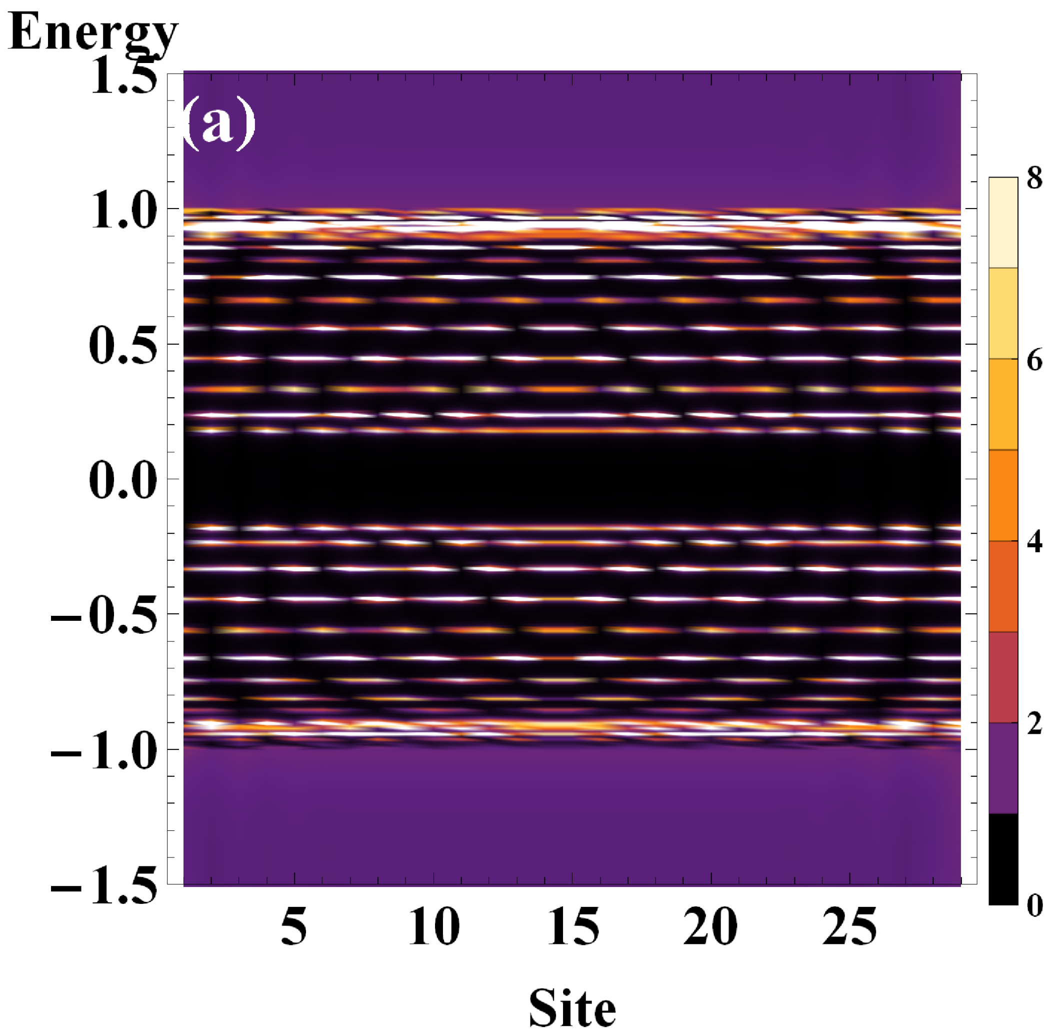


Imagine a quantum wire deposited on s-wave superconductor

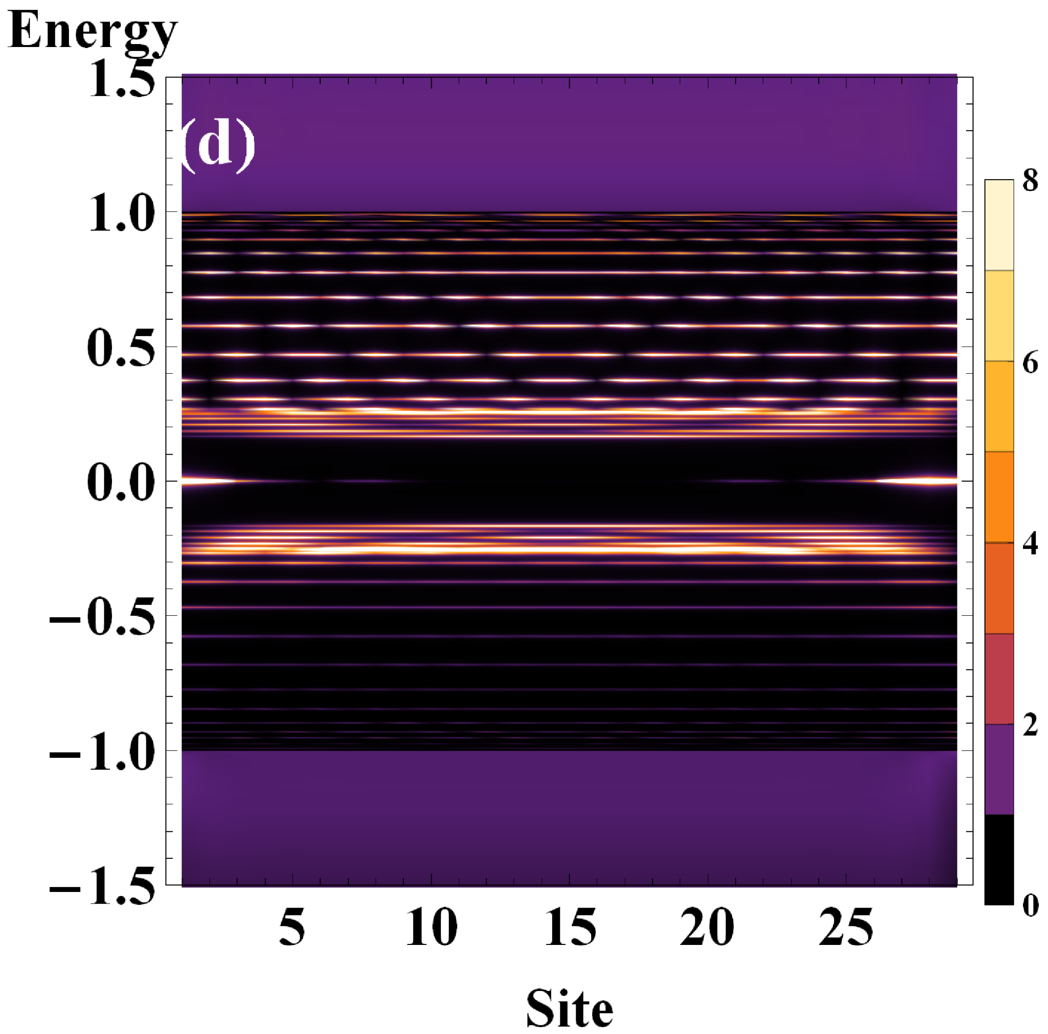
D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B **88***, 165401 (2013).*

Andreev vs Majorana states – a story of mutation



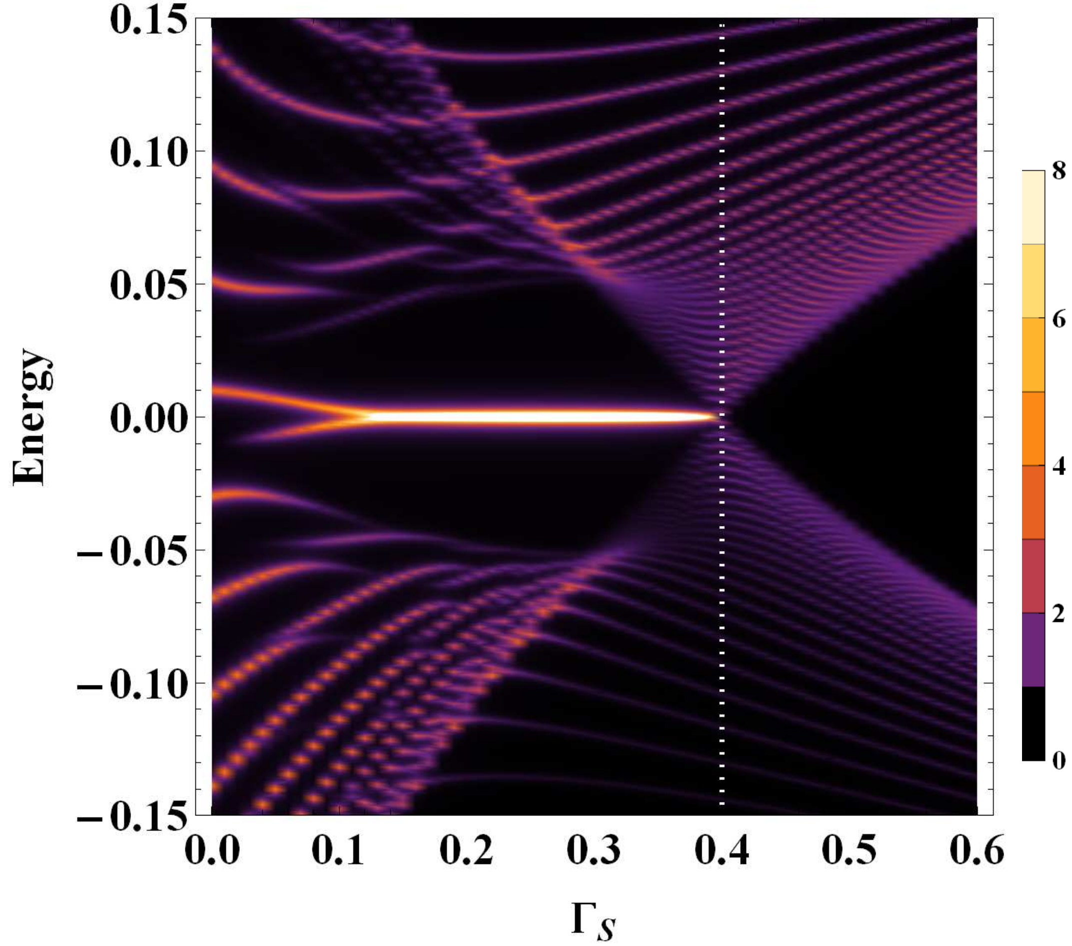
Spectrum of a quantum wire reveals a number of Andreev states.

Andreev vs Majorana states – a story of mutation



Spin-orbit coupling can induce the Majorana-type quasiparticles.

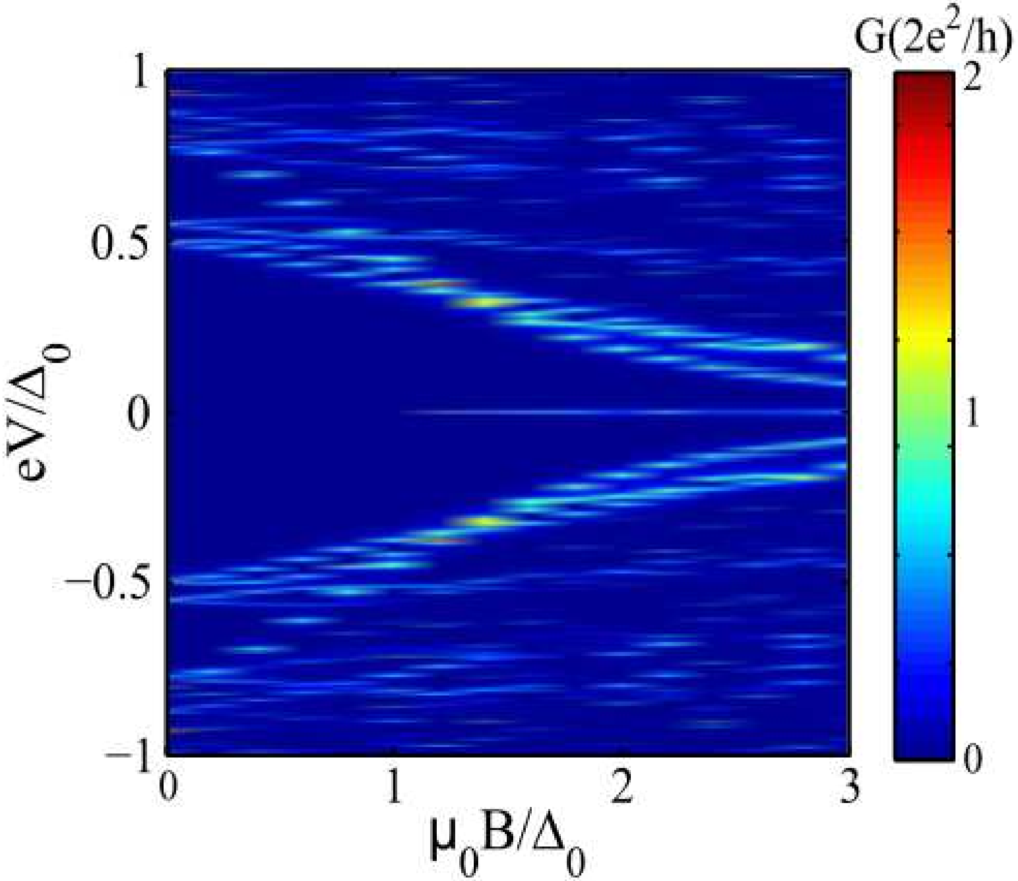
Andreev vs Majorana states – a story of mutation



Majorana quasiparticles appear at the edges of a quantum wire.

D. Chevallier, P. Simon, and C. Bena, Phys. Rev. B **88**, *165401 (2013).*

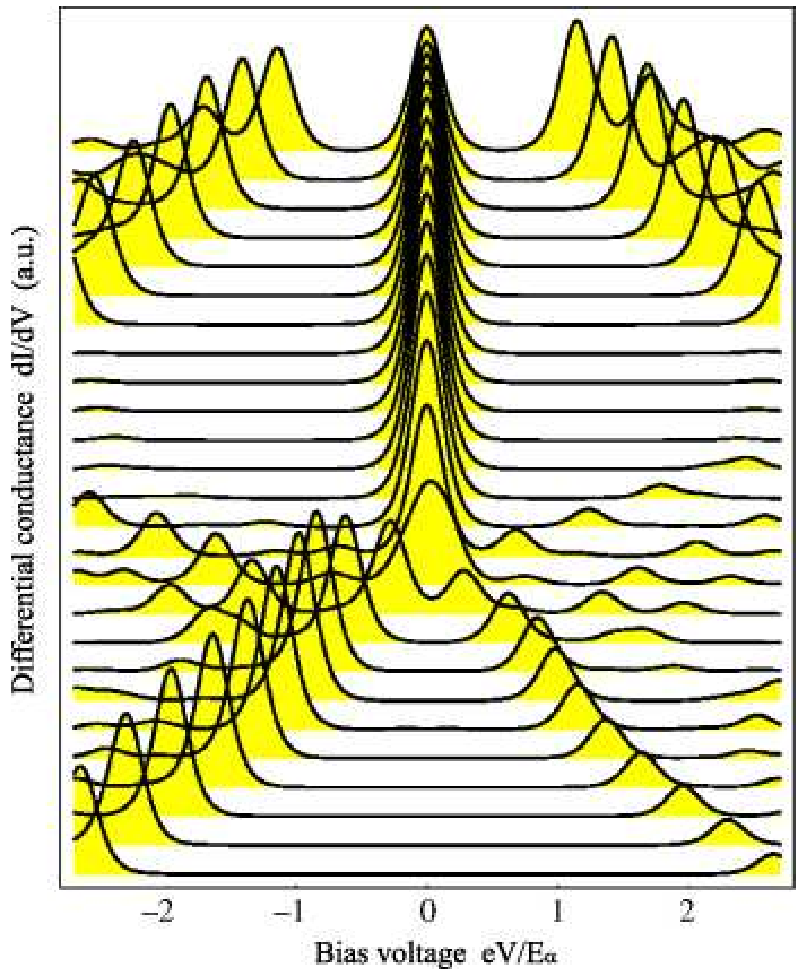
Andreev vs Majorana states – a story of mutation



Quasiparticles at the edge of a quantum wire for varying magnetic field.

J. Liu, A.C. Potter, K.T. Law, and P.A. Lee, Phys. Rev. Lett. **109***, 267002 (2012).*

Andreev vs Majorana states – a story of mutation



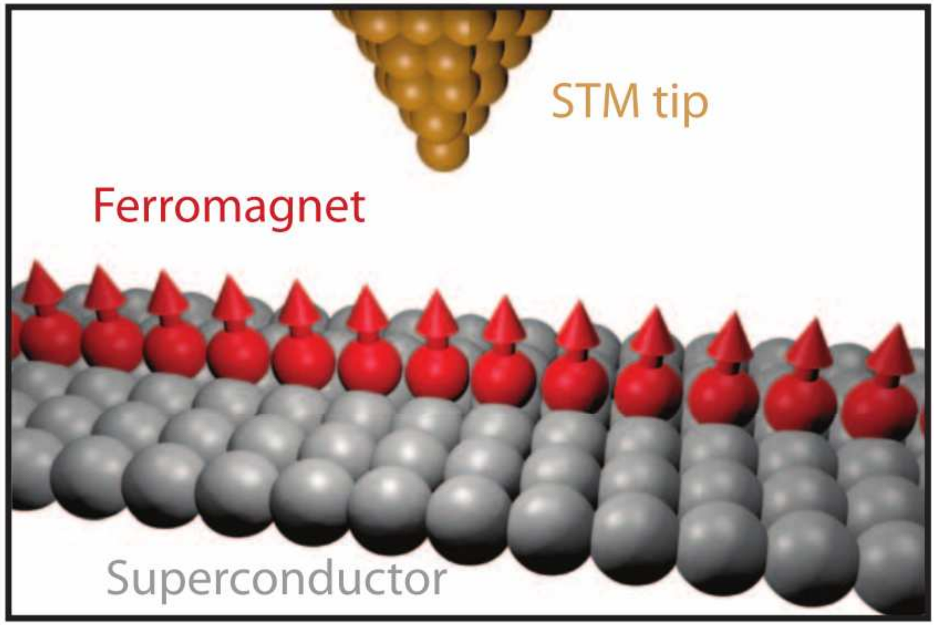
Quasiparticles at the edge of a quantum wire for varying magnetic field.

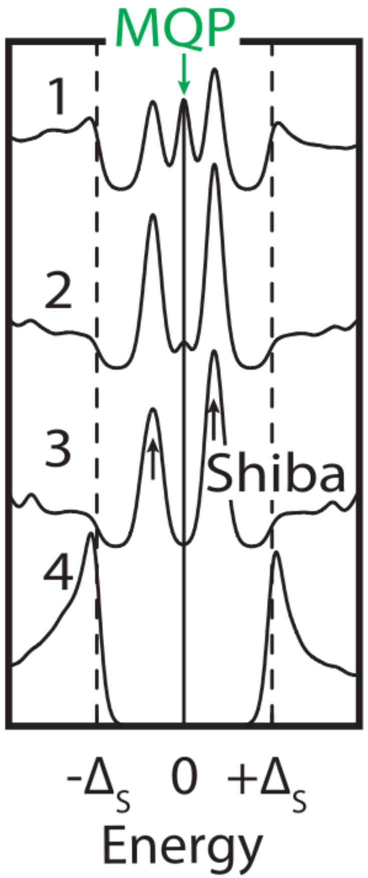
T.D. Stanescu, R.M. Lutchyn, and S. Das Sarma, Phys. Rev. B **84**, *144522 (2011).*

Experimental results – for Majorana quasiparticles

Experimental results – for Majorana quasiparticles

A chain of iron atoms deposited on a surface of superconducting lead





STM measurements provided evidence for:

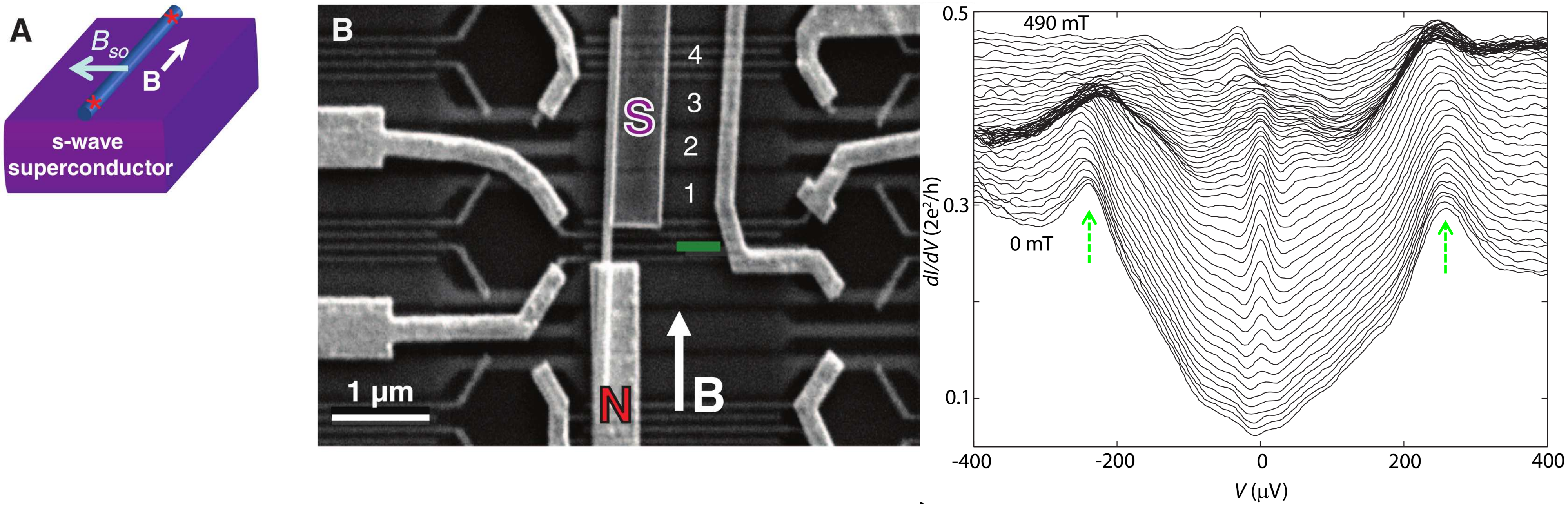
$\Rightarrow$ **Majorana bound states at the edges of a chain.**

S. Nadj-Perge, ..., and A. Yazdani, Science **346**, 602 (2014).

/ Princeton University, Princeton (NJ), USA /

Experimental results – for Majorana quasiparticles

InSb nanowire between a metal (gold) and a superconductor (Nb-Ti-N)



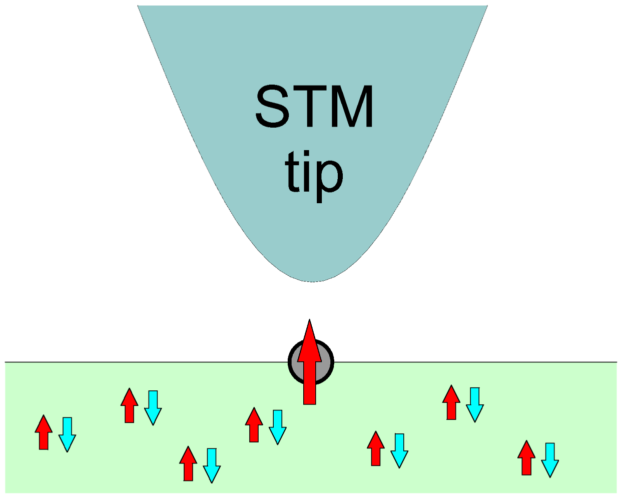
dI/dV measured at 70 mK for varying magnetic field B indicated:

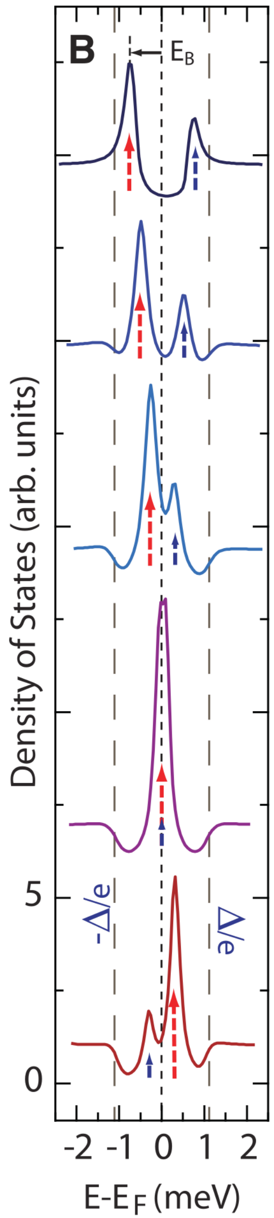
$\Rightarrow$ **a zero-bias enhancement due to Majorana state**

V. Mourik, ..., and L.P. Kouwenhoven, Science **336**, 1003 (2012).

/ Kavli Institute of Nanoscience, Delft Univ., Netherlands /

Subgap states of the bulk materials





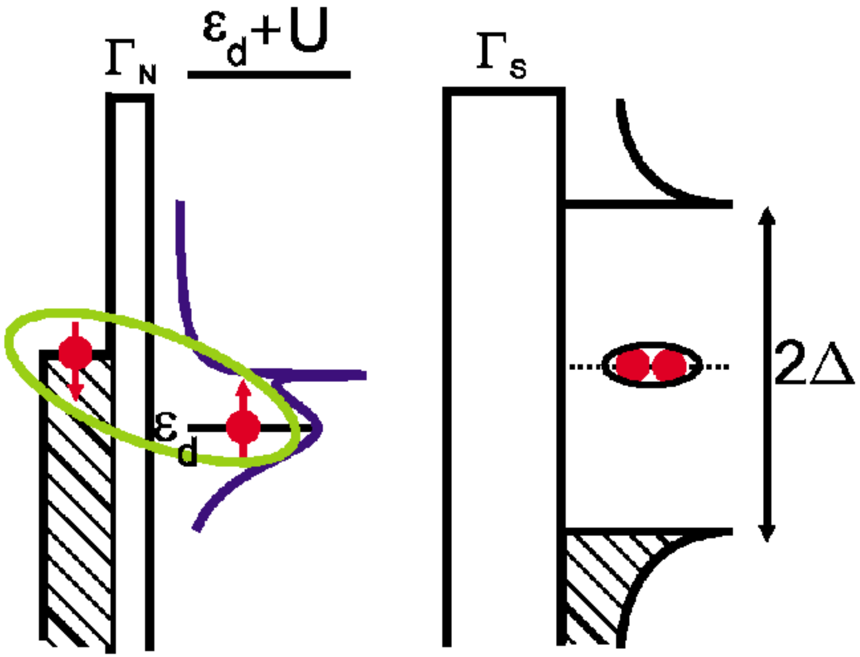
STM scheme (left) and the experimental data (right) obtained for Mn impurities on the superconducting Pb(111) surface.

K.J. Franke, G. Schulze, and J.I. Pascual, Science **332**, 940 (2011).

Other relevant physical effects

Relevant problems : issue # 1

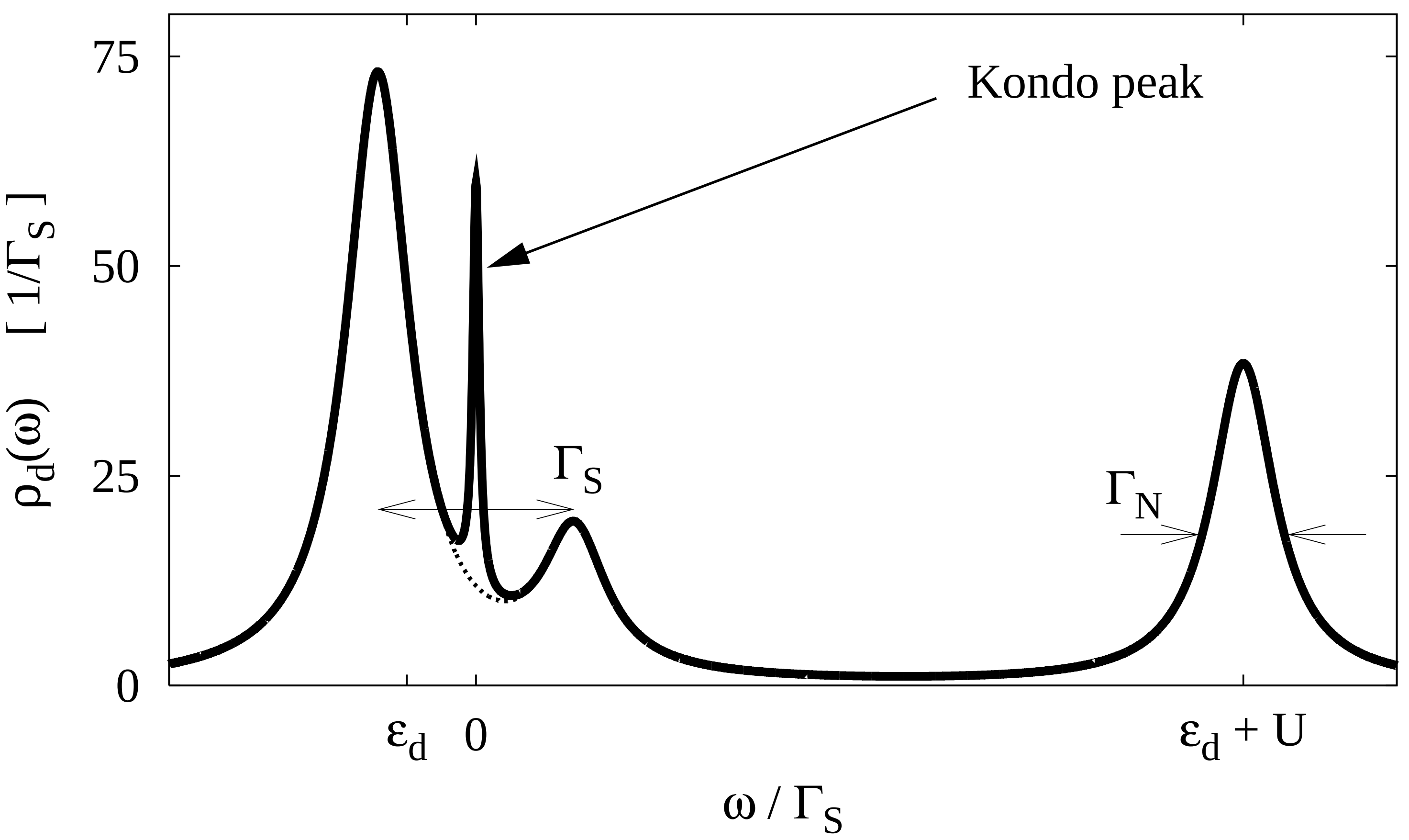
Coupling of the QD with a metallic electrode:



- ★ **broadens the QD levels** $(\sim \Gamma_N)$
- ★ **induces the Kondo resonance** (below T_K).

Relevant problems : issue $\#$ 2

Additional hybridization Γ_S with superconductor induces on-dot pairing



Theoretical background: – various many-body techniques

EOM	R. Fazio and R. Raimondi (1998)
slave bosons	P. Schwab and R. Raimondi (1999)
NCA	A.A. Clerk, V. Ambegaokar, and S. Hershfield (2000)
IPT	J.C. Cuevas, A. Levy Yeyati, and A. Martin-Rodero (2001)
constrained sb	M. Krawiec and K.I. Wysokiński (2004)
NRG	Y. Tanaka, N. Kawakami, and A. Oguri, (2007)
EOM revised	T. Domański et al, (2007)
NRG	J. Bauer, A. Oguri, and A.C. Hewson, (2007)
f-RG	C. Karrasch, A. Oguri, and V. Meden, (2008)
QMC	A. Koga, (2013)
CUT	M. Zapalska and T. Domański, (2014)
NRG	R. Žitko et al, (2014)

Andreev conductance – theoretical results

Subgap conductance $G_A(V)$ obtained for: $U = 10\Gamma_N$

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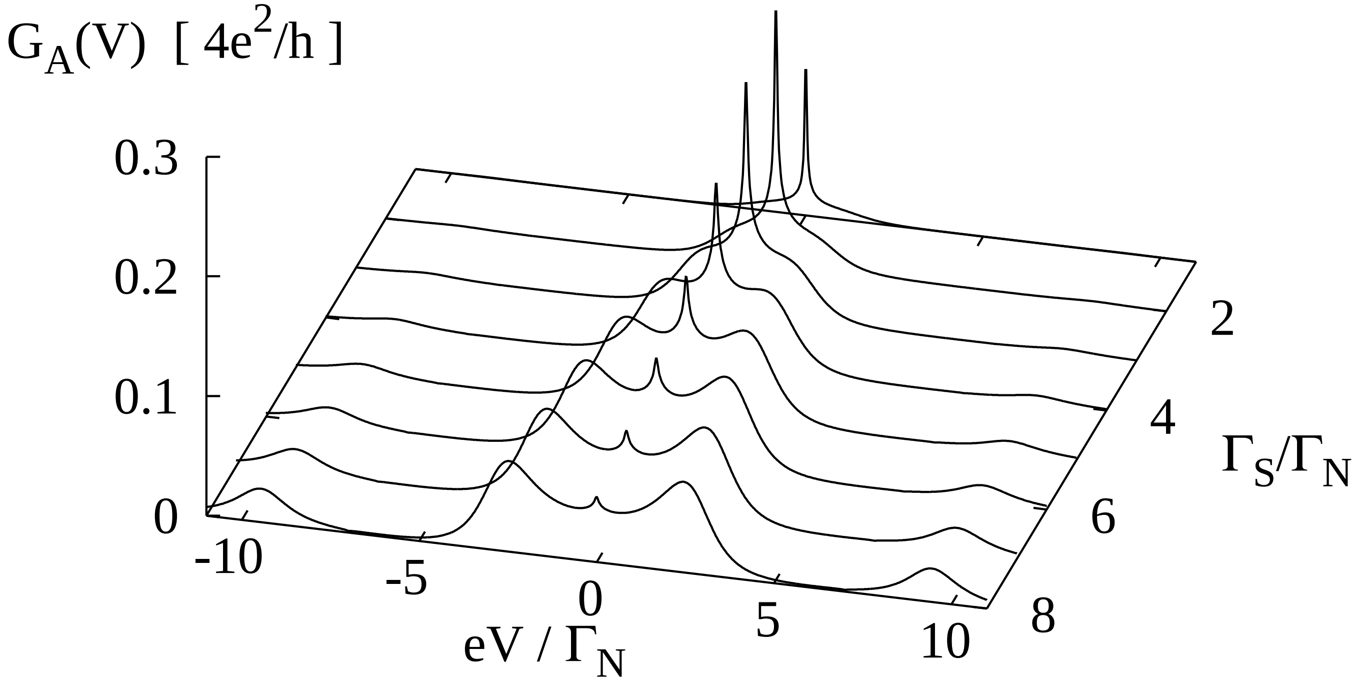

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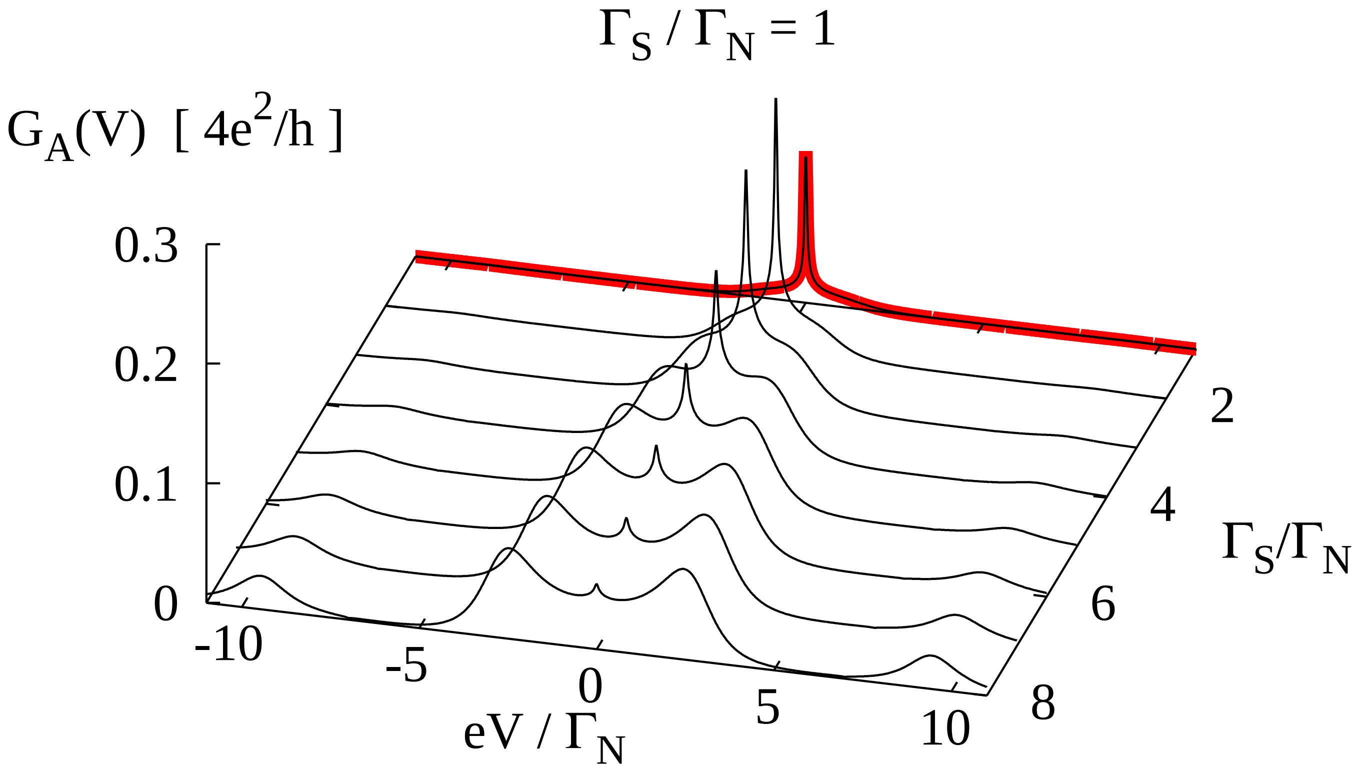

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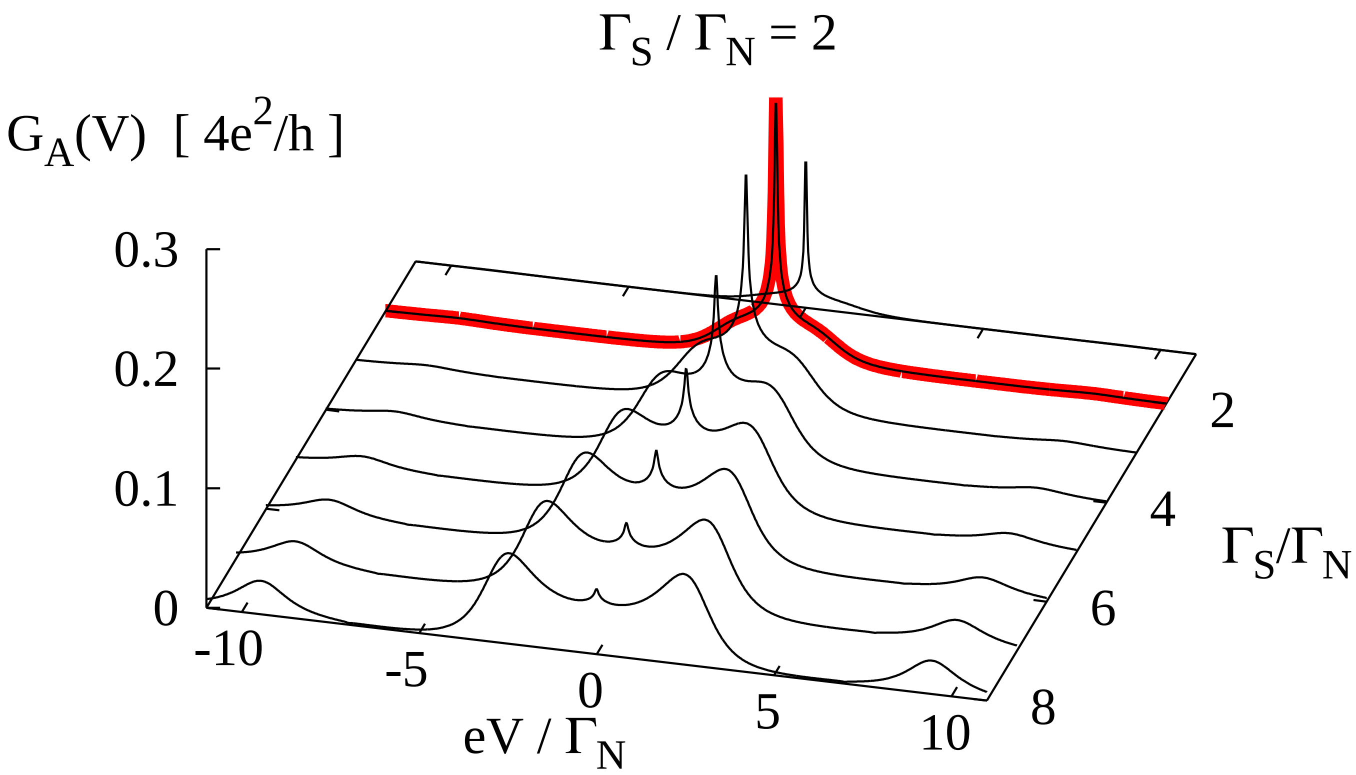

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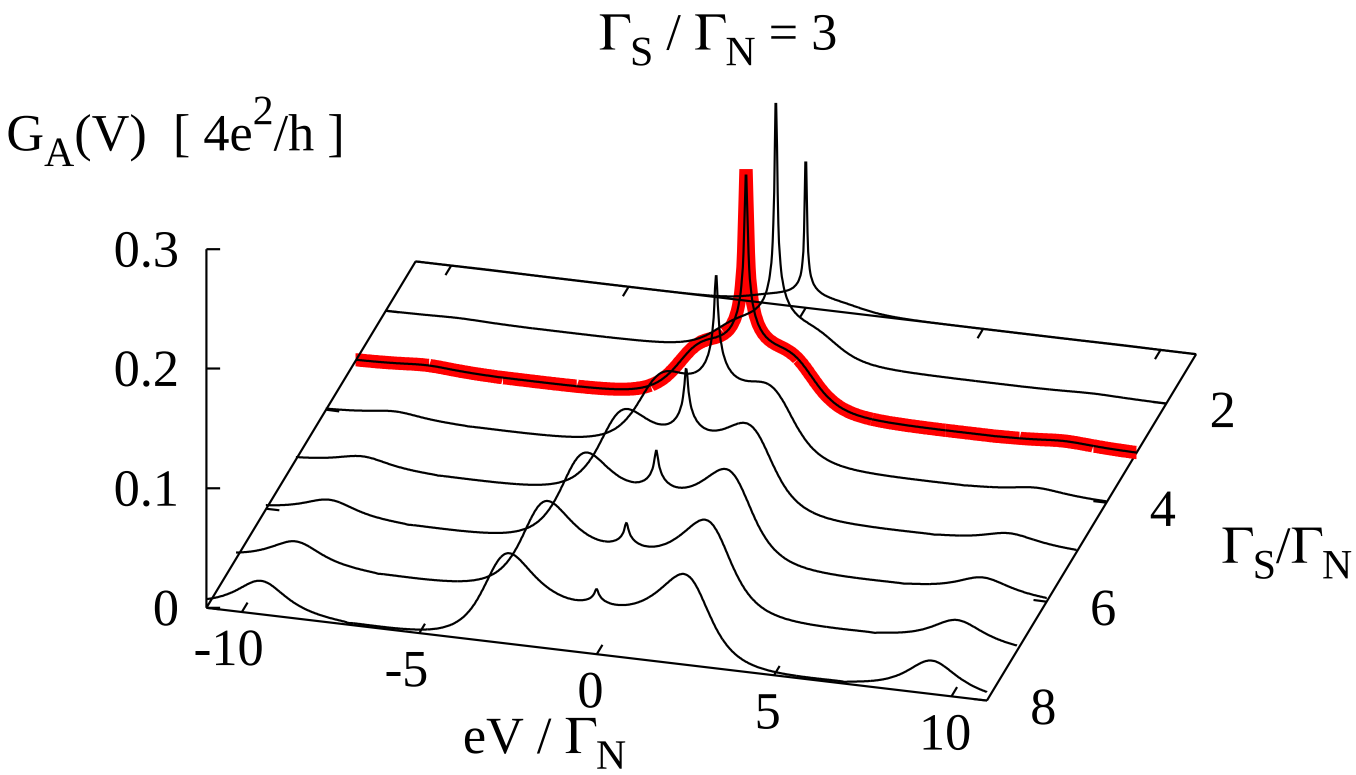

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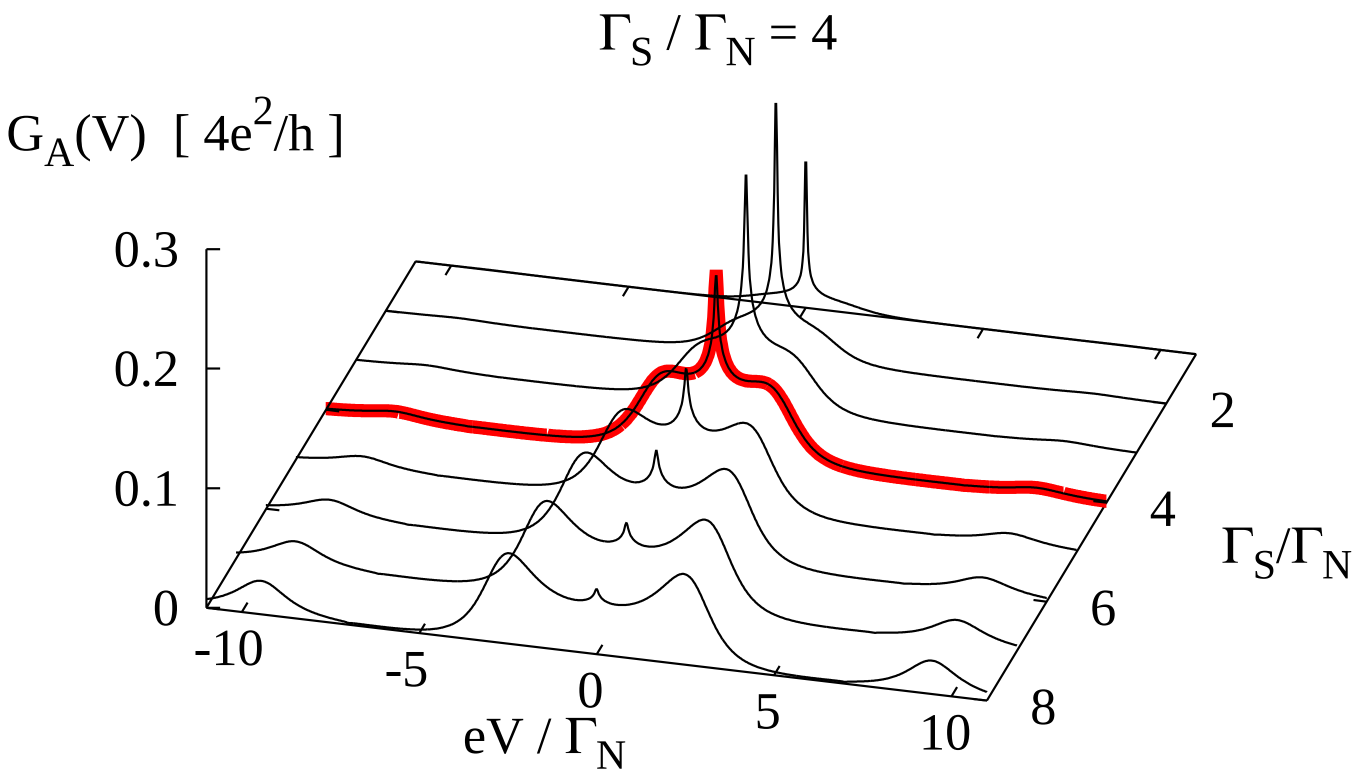

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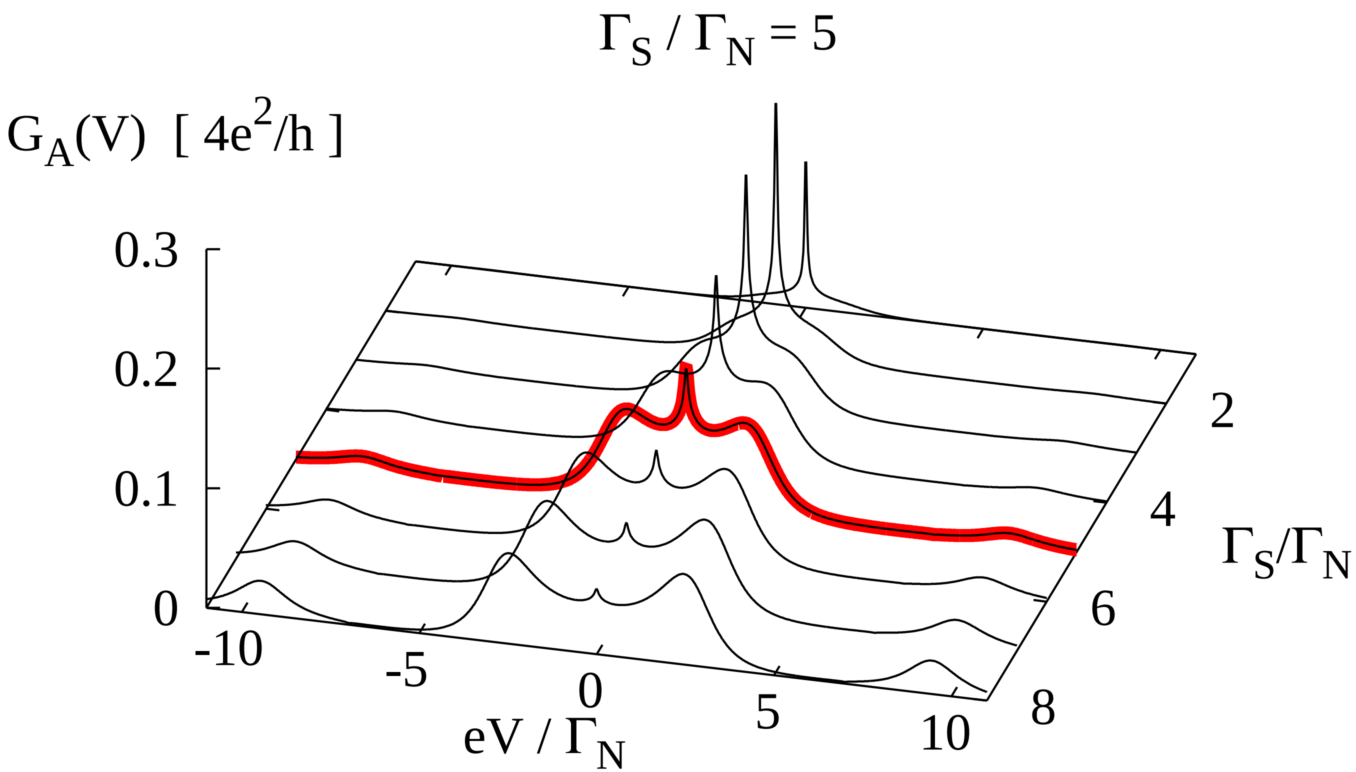

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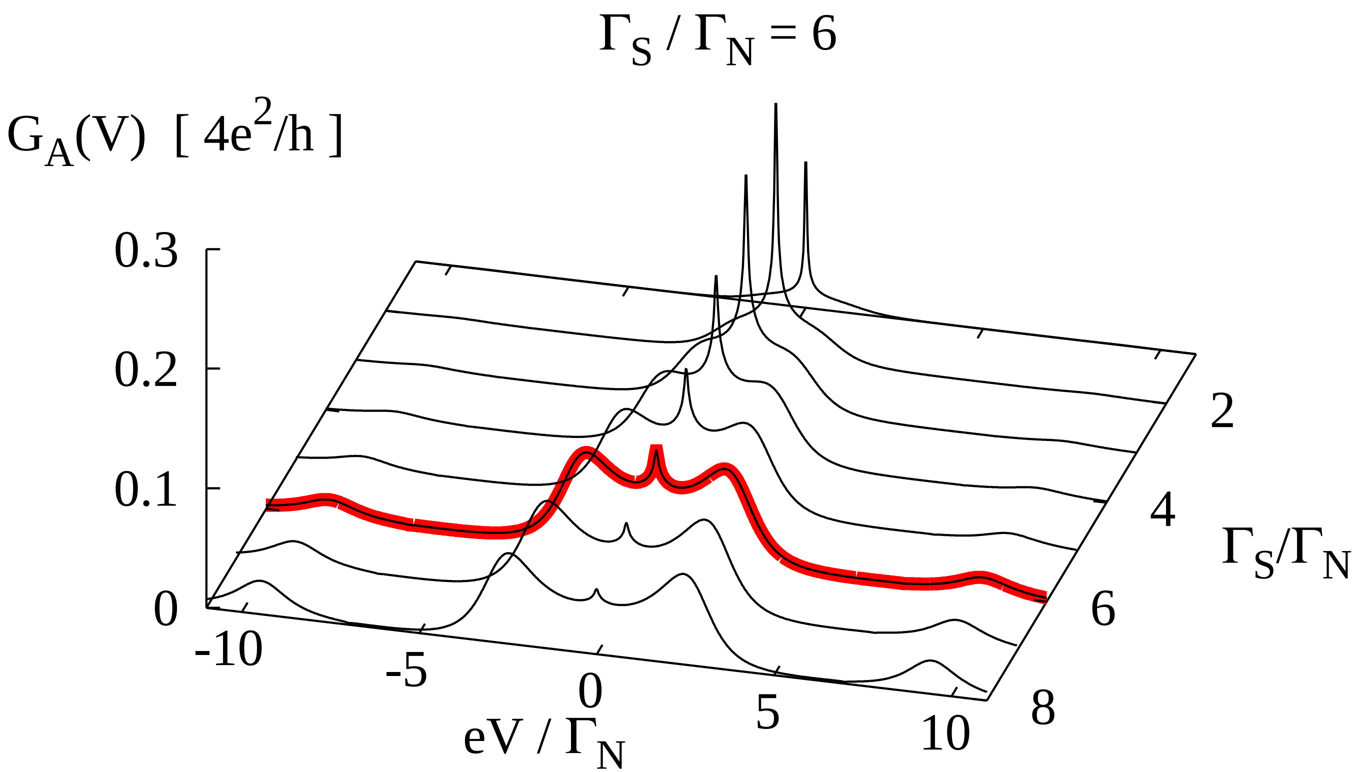

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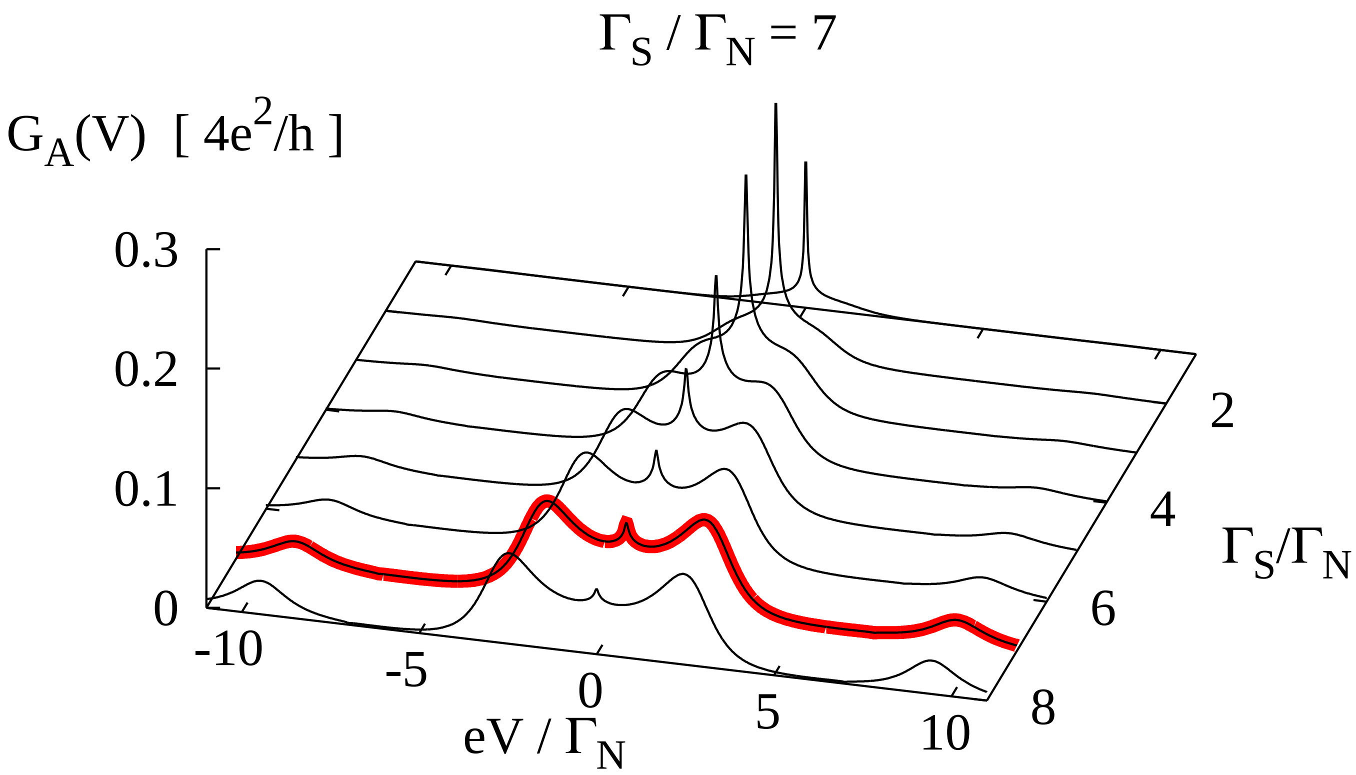

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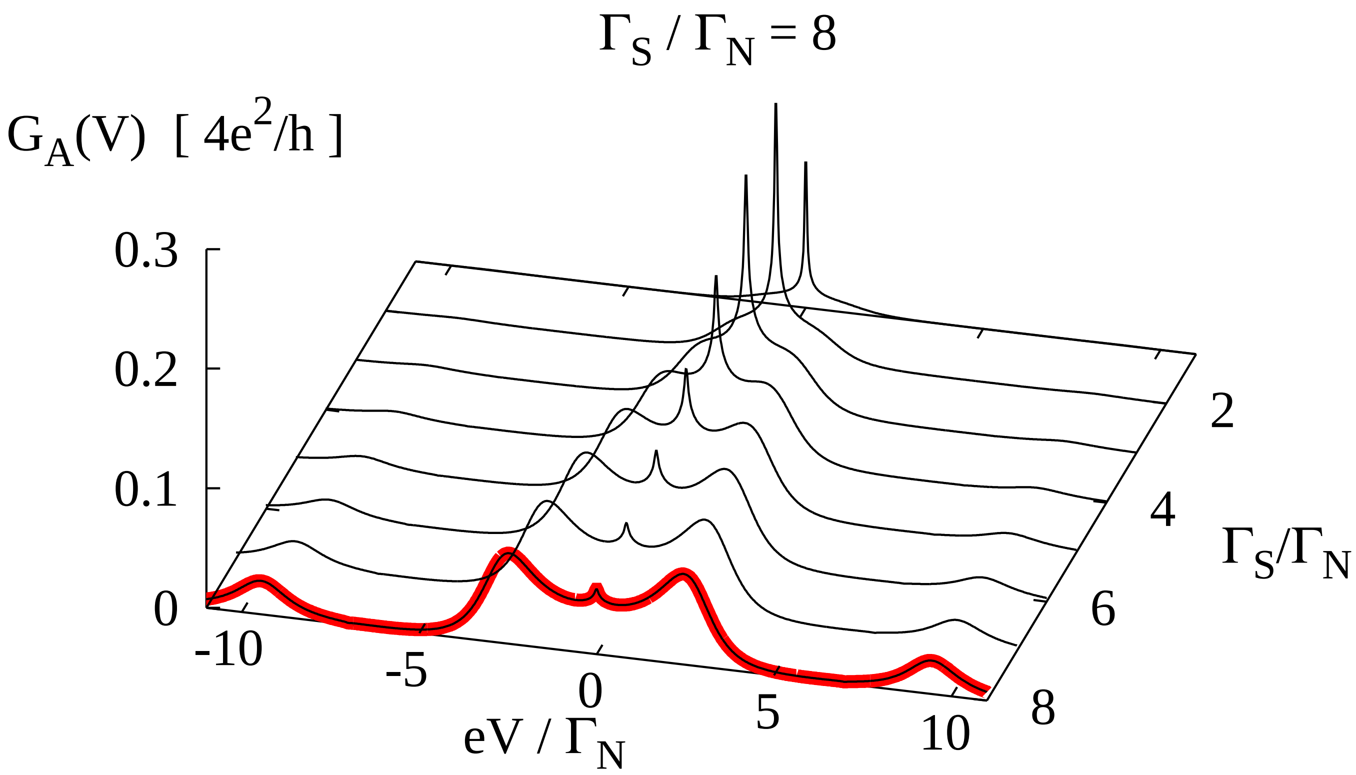

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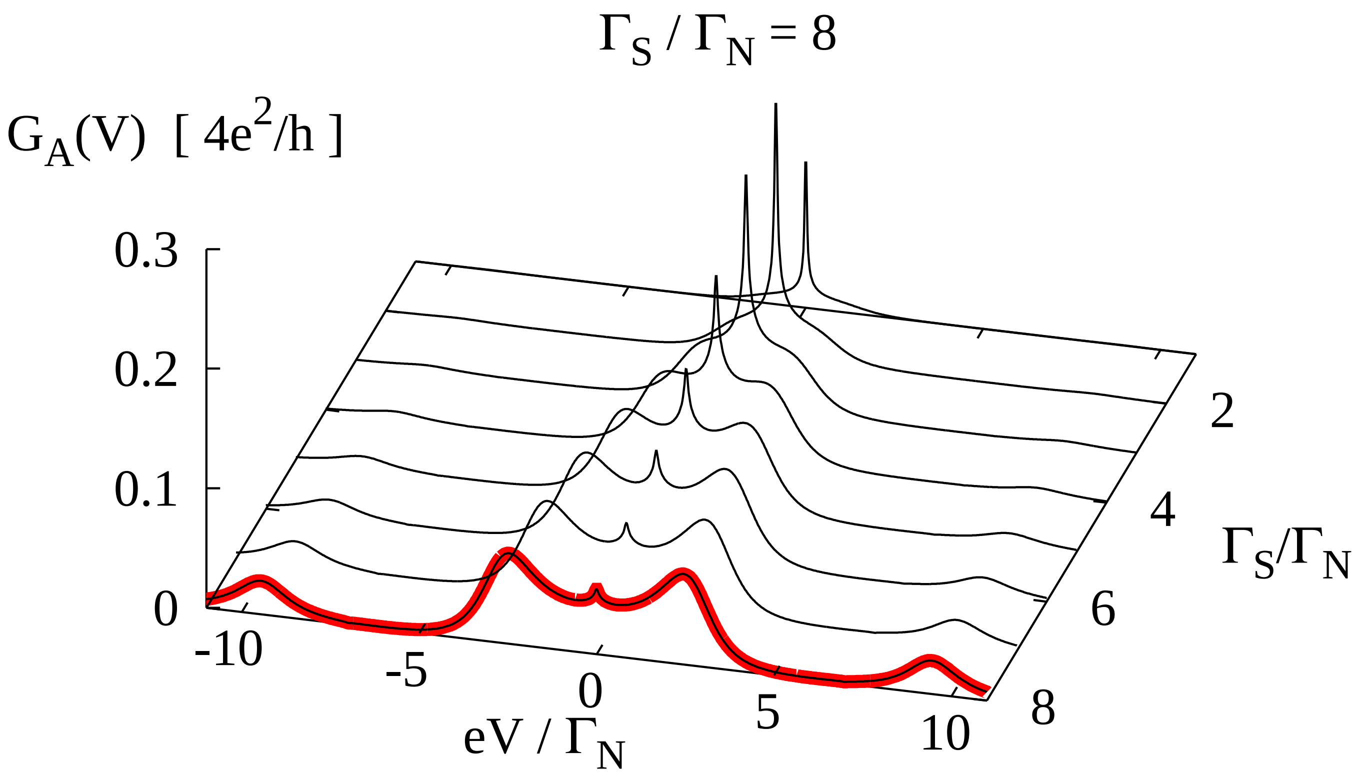

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Kondo state <u>enhances</u> the zero-bias conductance !

Andreev conductance – experimental data

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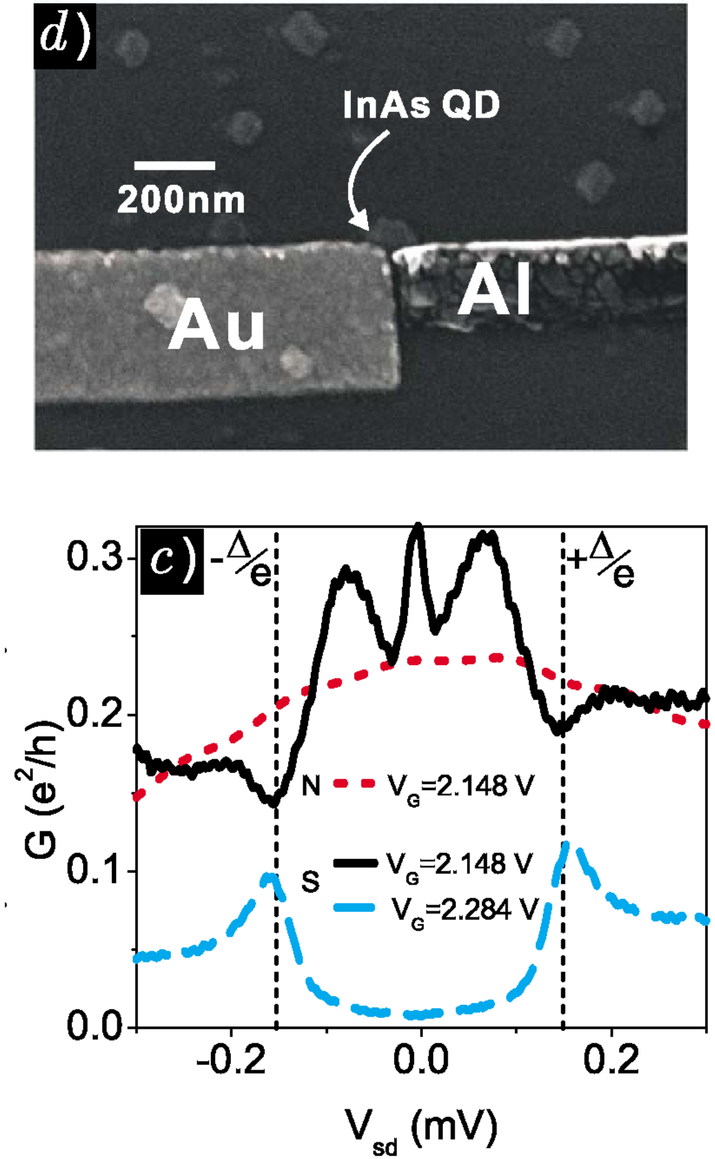

R.S. Deacon et al, *Phys. Rev. B* **81**, *121308(R) (2010).*

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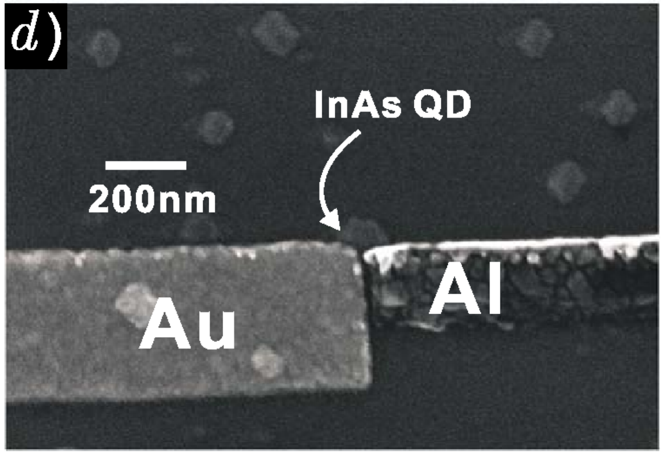
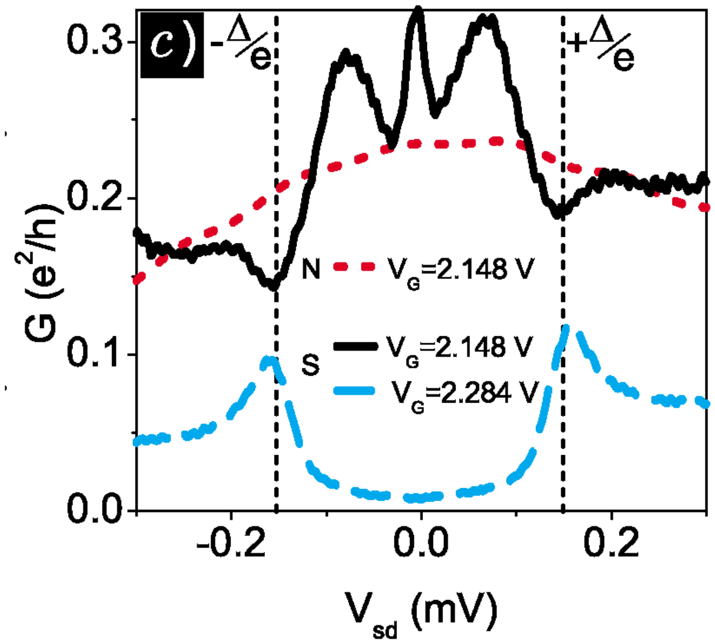


"The zero-bias conductance peak is consistent with Andreev transport enhanced by the Kondo singlet state"



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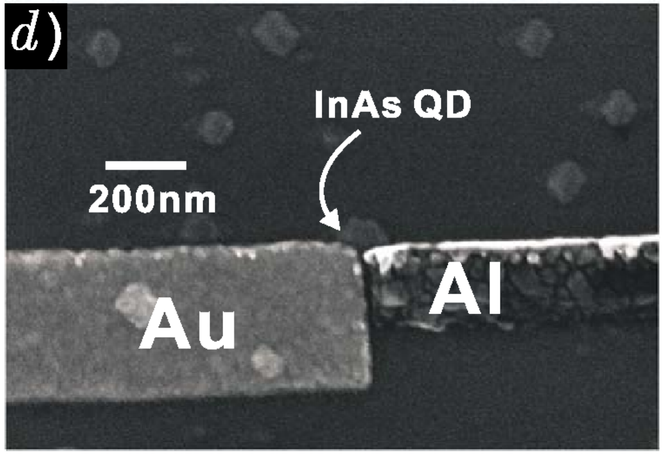

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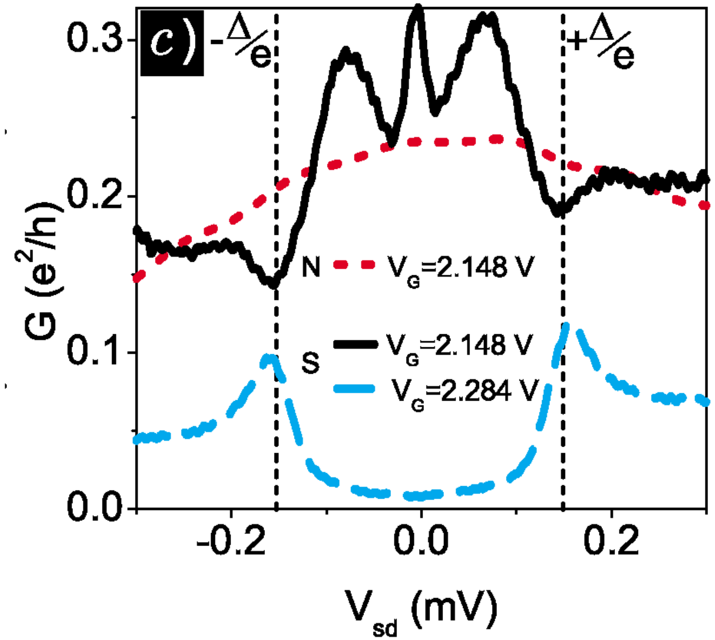
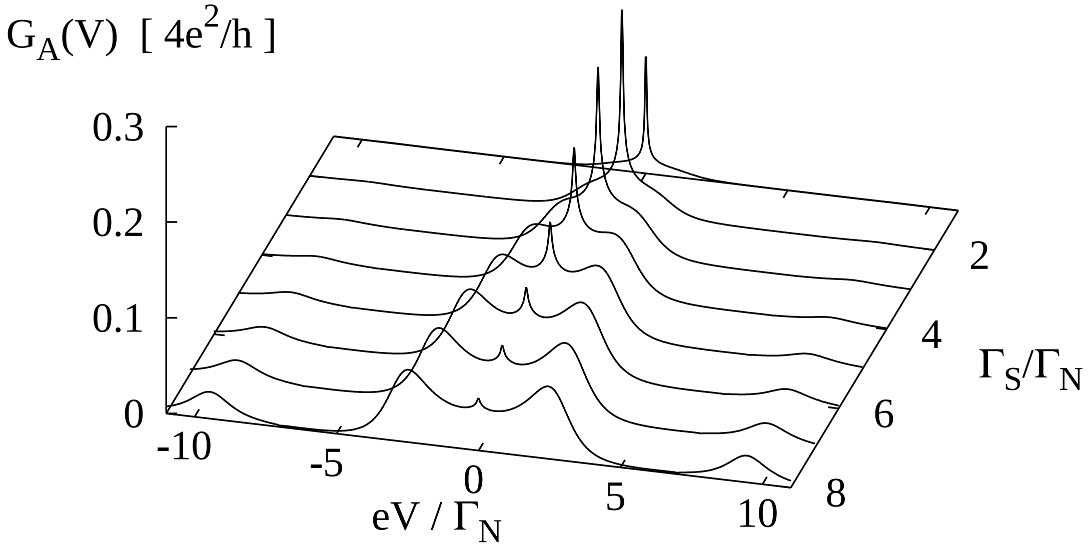


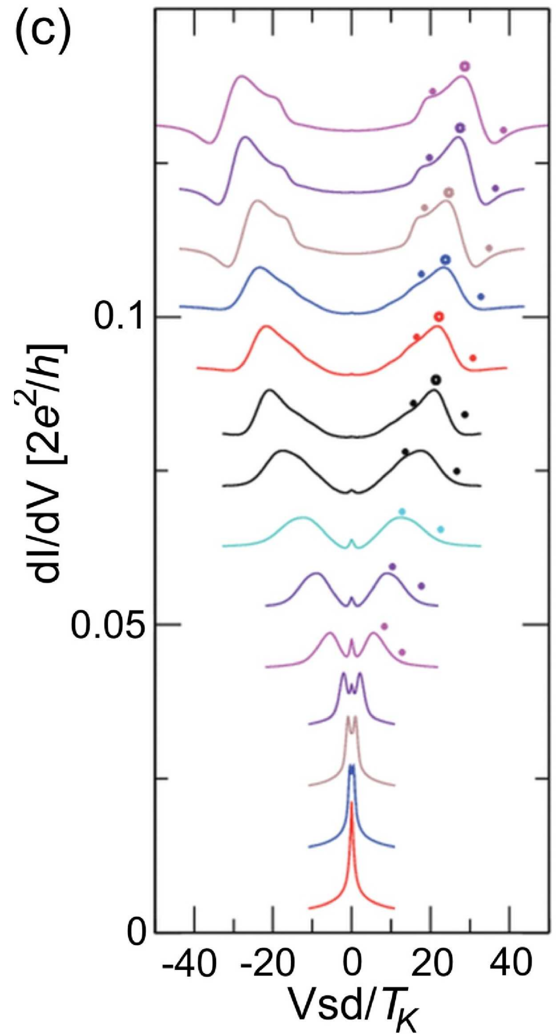
"We note that the feature exhibits excellent qualitative agreement with a recent theoretical treatment by Domanski et al"

R.S. Deacon et al, *Phys. Rev. B* **81**, *121308(R) (2010).*

Kondo effect vs induced pairing



Theory: T. Domański, A. Donabidowicz, PRB **78**, 073105 (2008).



Experiment: E.J.H. Lee *et al*, Phys. Rev. Lett. **109**, 186802 (2012).

Further related topics – Cooper pair splitters

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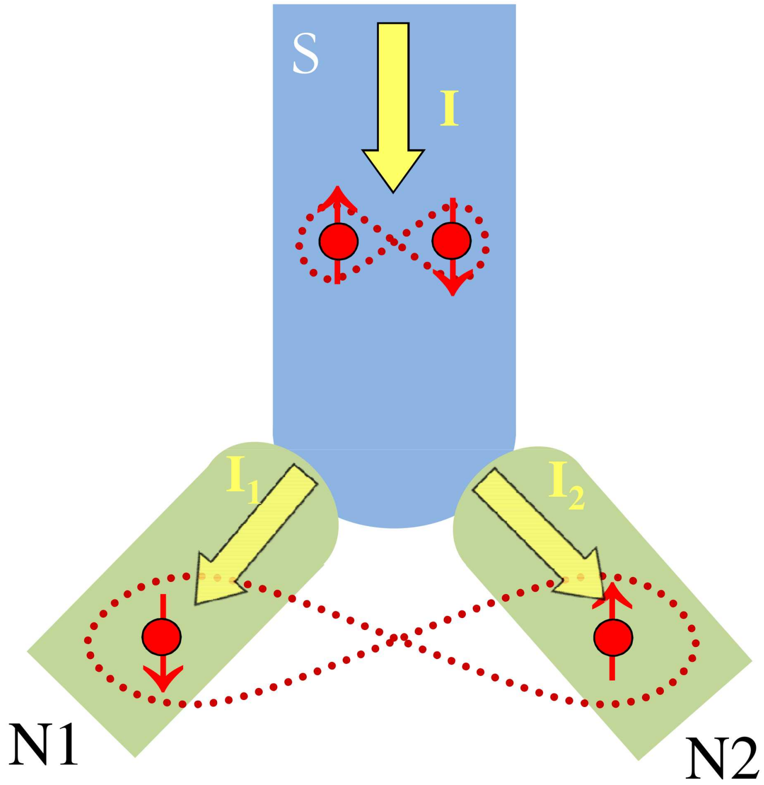

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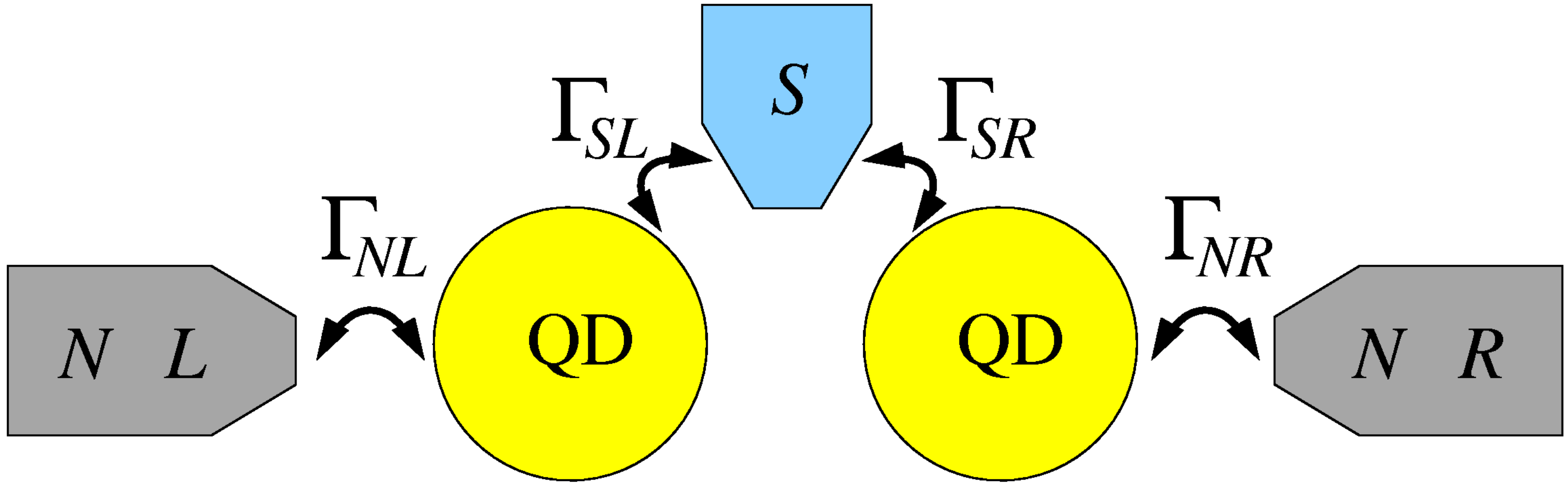

Cooper pairs can be split by two quantum dots (*quantum forks*).

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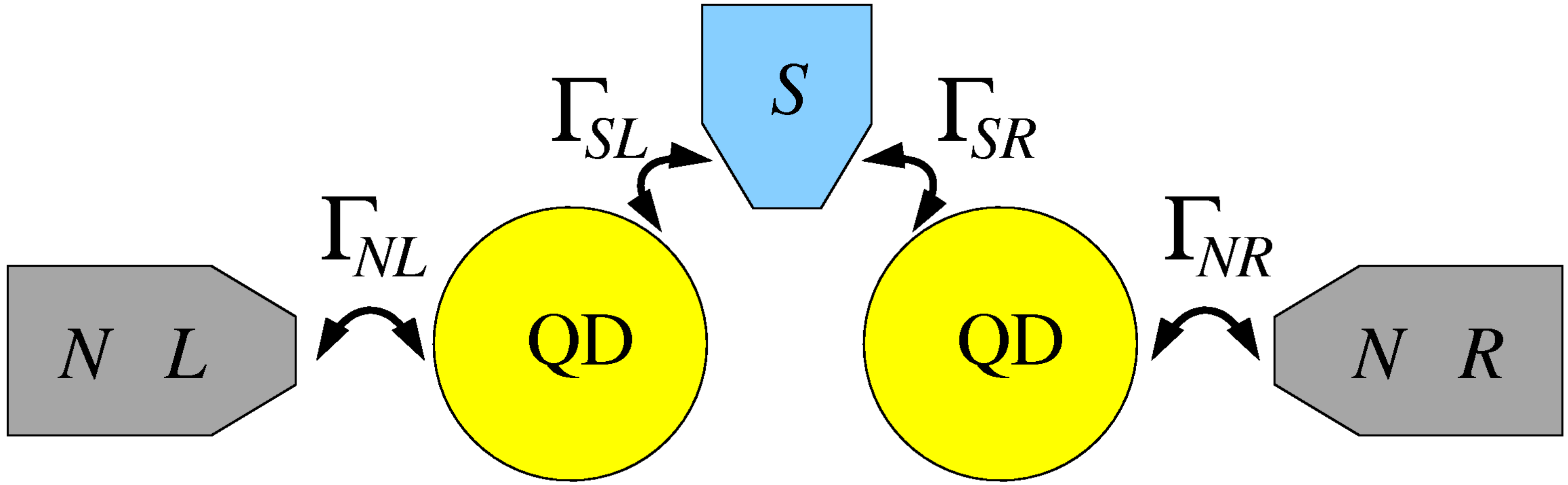

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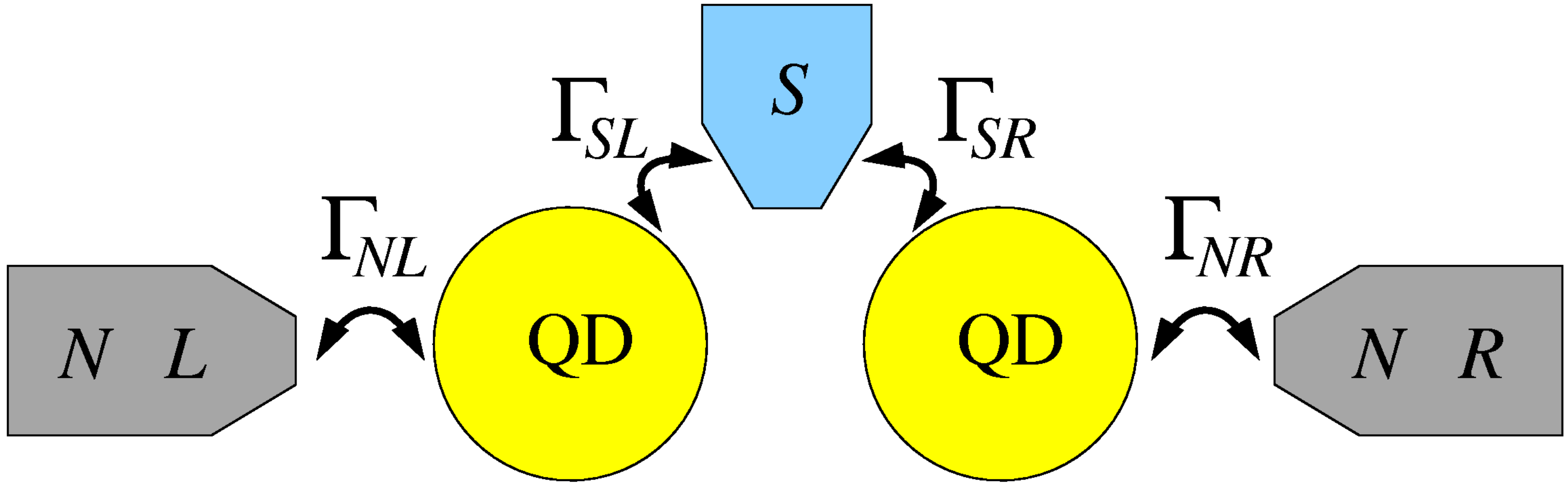

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L. Hofstetter, S. Csonka, J. Nygård, C. Schönenberger, Nature **461**, *960 (2009).*

J. Schindele, A. Baumgartner, C. Schönenberger, Phys. Rev. Lett. **109**, *157002 (2012).*

Direct vs crossed Andreev reflections

in three terminal junction

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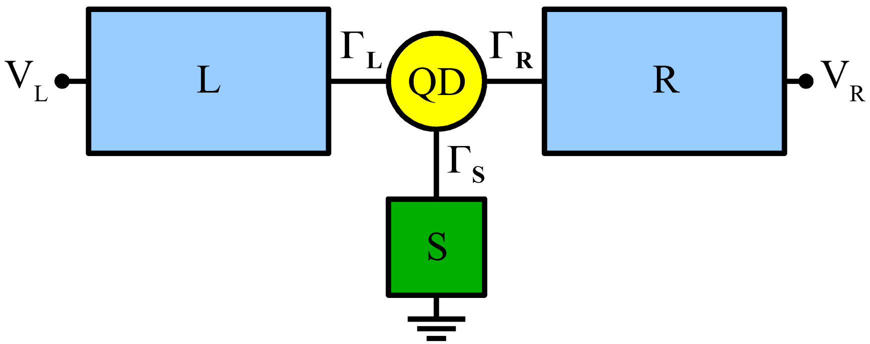

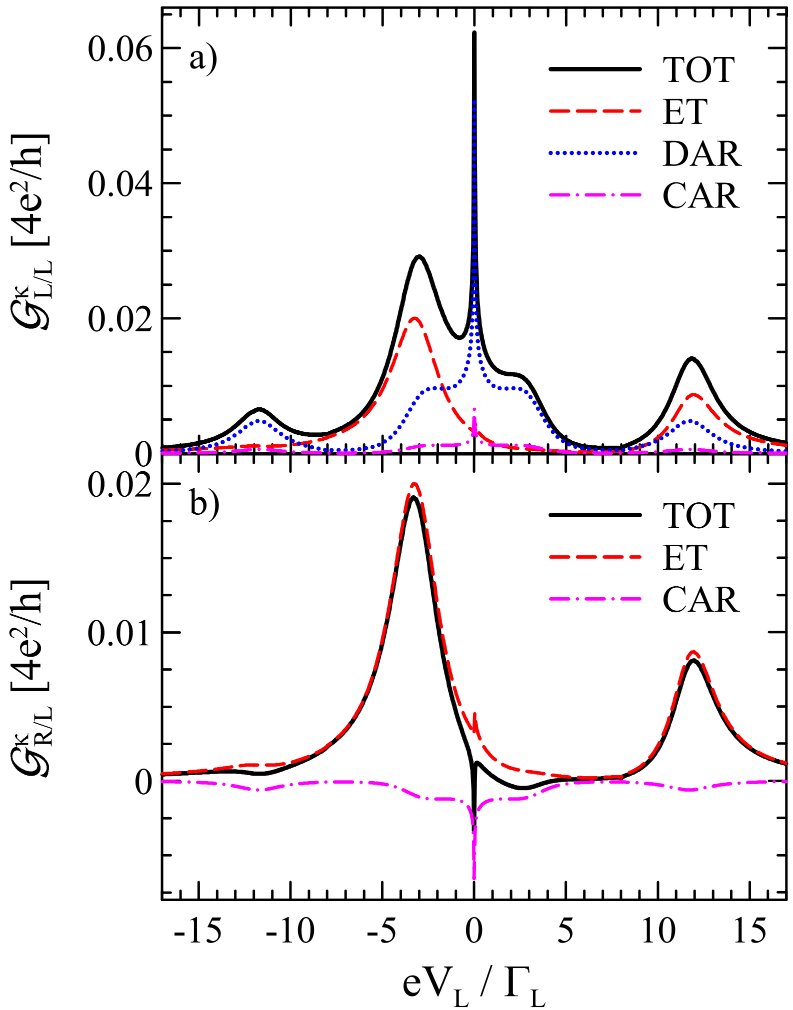

L, R – normal electrods

S – superconducting electrode

Kondo effect inverts a sign of the CAR conductance $G_{R/L}$

G. Michałek, B.R. Bułka, T. Domański, and K.I. Wysokiński, Phys. Rev. B ***88****, 155425 (2013).*

Kondo temperature vs induced pairing

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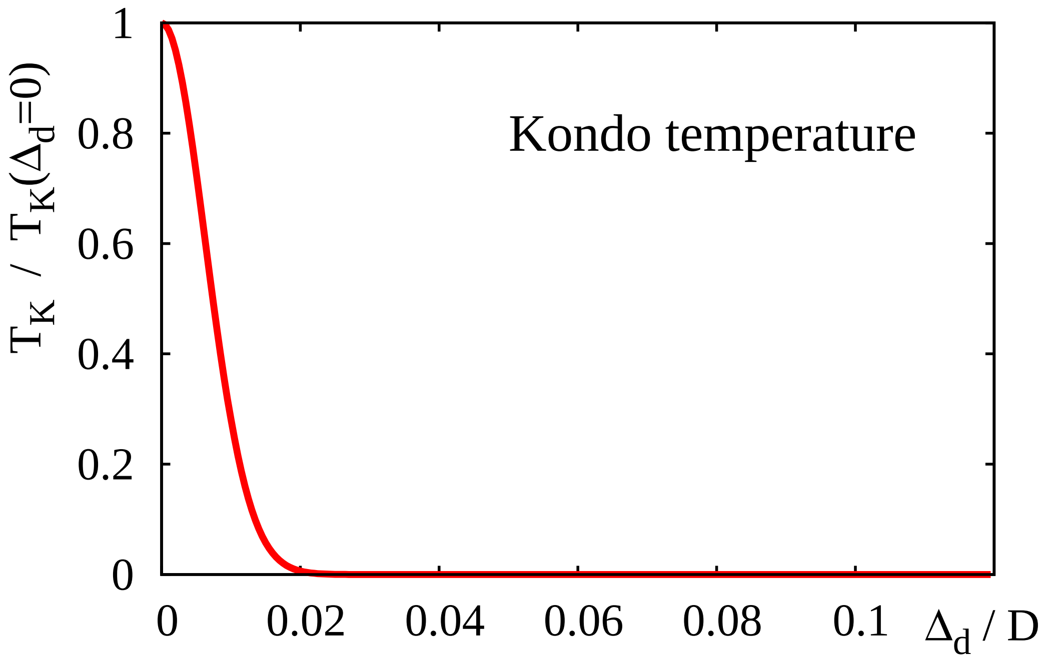

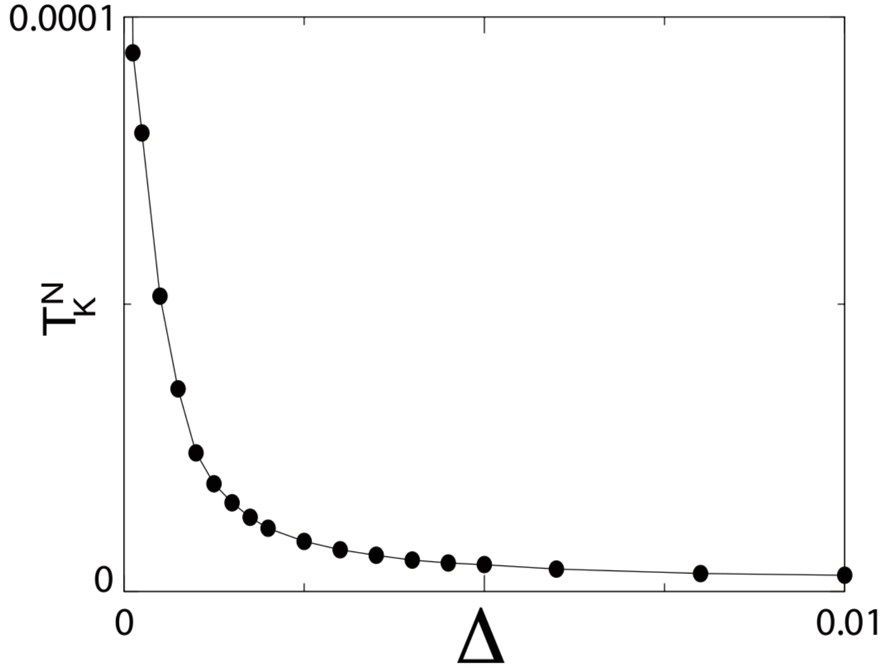

CUT: T. Domański, J. Barański, M. Zapalska, Phil. Mag. B (2014).

NRG data: T. Žitko *et al*, arXiv:1405.6084 (2014) preprint.

Summary

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http://kft.umcs.lublin.pl/doman/lectures