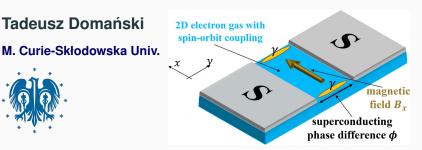
Josephson junctions: a platform for exotic superconducting phases

Tadeusz Domański





OUTLINE

1. Josephson effect

2. Josephson junctions (some applications & recent advancements)

3. Topological superconductivity
(interplay of magnetism and electron pairing)

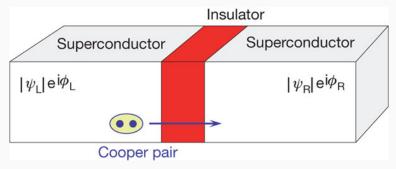
PRELIMINARIES

Part 1. Josephson effect

(prediction & discovery)

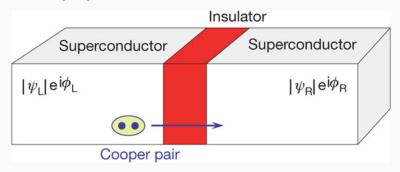
JOSEPHSON EFFECT

Two superconductors, differing in phase $\phi_L \neq \phi_R$, contacted through a narrow ($\leq \xi$) insulating region induce *super*flow of the Cooper pairs.



JOSEPHSON EFFECT

Two superconductors, differing in phase $\phi_L \neq \phi_R$, contacted through a narrow ($\leq \xi$) insulating region induce *super*flow of the Cooper pairs.



Effect has been predicted by B.D. Josephson in 1962. / 22-year-old PhD student at Cambridge, England /

In quantum mechanics the probability current is defined by

$$ec{j}(ec{r},t) = -rac{i\hbar}{2m}\left[\Psi^{\star}(ec{r},t)
abla\Psi(ec{r},t) - \Psi(ec{r},t)
abla\Psi^{\star}(ec{r},t)
ight]$$

In quantum mechanics the probability current is defined by

$$ec{j}(ec{r},t) = -\frac{i\hbar}{2m} \left[\Psi^{\star}(ec{r},t) \nabla \Psi(ec{r},t) - \Psi(ec{r},t) \nabla \Psi^{\star}(ec{r},t) \right]$$

Applying this formalism to the wave-function Φ_0 of Cooper pairs

$$\Phi_0(\vec{r},t) \equiv \underbrace{|\Phi_0(\vec{r},t)|}_{\sqrt{n(\vec{r},t)}} e^{i\phi(\vec{r},t)}$$

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$$ec{j}_{J}(ec{r},t) = - q \, rac{i\hbar}{2m} \left[\Phi_0^\star
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$$\vec{j}_{J}(\vec{r},t) = -q \frac{i\hbar}{2m} \left[\Phi_{0}^{\star} \nabla \Phi_{0} - \Phi_{0} \nabla \Phi_{0}^{\star} \right] = q n(\vec{r},t) \underbrace{\frac{\hbar}{m} \nabla \phi(\vec{r},t)}_{\vec{v}(\vec{r},t)}$$

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$$\vec{j}(\vec{r},t) = -\frac{i\hbar}{2m} \left[\Psi^{\star}(\vec{r},t) \nabla \Psi(\vec{r},t) - \Psi(\vec{r},t) \nabla \Psi^{\star}(\vec{r},t) \right]$$

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where q=2e is charge and $\vec{v}(\vec{r},t)$ is velocity of Cooper pairs.

EXPERIMENTAL EVIDENCE

VOLUME 10, NUMBER 6

PHYSICAL REVIEW LETTERS

15 March 1963

PROBABLE OBSERVATION OF THE JOSEPHSON SUPERCONDUCTING TUNNELING EFFECT

P. W. Anderson and J. M. Rowell Bell Telephone Laboratories, Murray Hill, New Jersey (Received 11 January 1963)

EXPERIMENTAL EVIDENCE

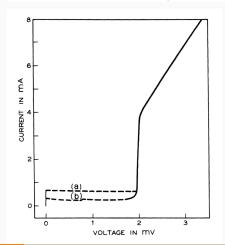
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P. W. Anderson and J. M. Rowell Bell Telephone Laboratories, Murray Hill, New Jersey (Received 11 January 1963)



Authors reported on:

"dc tunneling current at or near zero voltage in very thin tin oxide barriers between superconducting Sn and Pb"

NOBEL PRIZE IN PHYSICS

1973

B.D. Josephson (with L. Esaki & I. Giaver)

NOBEL PRIZE IN PHYSICS

1973

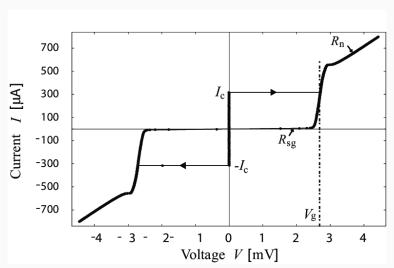
B.D. Josephson (with L. Esaki & I. Giaver)

1972

J. Bardeen, L.N. Cooper, J.R. Schrieffer

I(V) CHARACTERISTICS

Typical current-voltage plot, where $V_g=2\Delta$

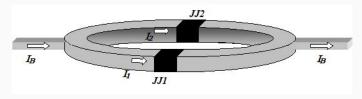


Part 2. Josephson junctions

(applications & recent advancements)

1. QUANTUM INTERFEROMETER DEVICE

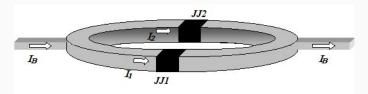
SQUID - superconducting quantum interferemoreter device



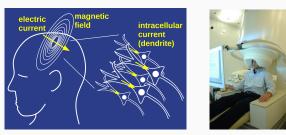
This device allows for an extremely precise detection of magnetic fields

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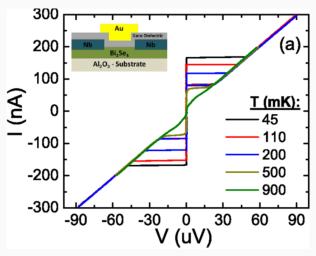
This device allows for an extremely precise detection of magnetic fields



capable to probe the neural currents in a human brain!

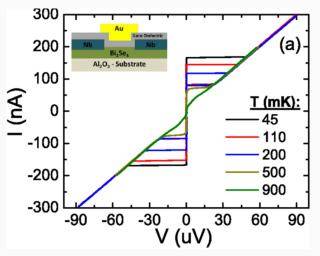
2. RESISTIVE TRANSITION

Critical dc current I_c diminishes upon increasing temperature,



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Critical dc current I_c diminishes upon increasing temperature,



when Josephson current switches from superflow to resistive behaviour.

2. ULTRAFAST THERMOMETRY

A weak link (Aluminum Dayem nanobridge) probed with nanosecond current (pump-and-probe) pulses can serve as a temperature-sensing element

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PHYSICAL REVIEW APPLIED **10**, 044068 (2018)

Nanosecond Thermometry with Josephson Junctions

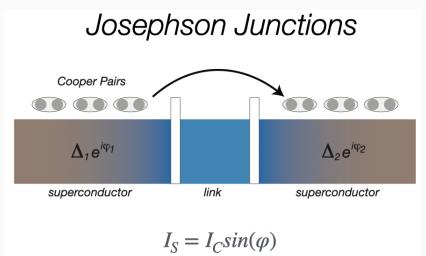
M. Zgirski, ^{1,*} M. Foltyn, ¹ A. Savin, ² K. Norowski, ¹ M. Meschke, ² and J. Pekola ²

¹ Institute of Physics, Polish Academy of Sciences, Aleja Lotnikow 32/46, 02-668 Warsaw, Poland ² Low Temperature Laboratory, Department of Applied Physics, Aalto University School of Science, P.O. Box 13500, FI-00076 Aalto, Finland

Measurements delivered resolution better than 10 ns!

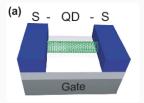
3. PHASE-PERIODICITY OF JOSEPHSON CURRENT

Superflow of the Copper pairs is driven by phase difference, therefore dc current has (usually) 2π -periodicity with respect to $\phi=\phi_L-\phi_R$.



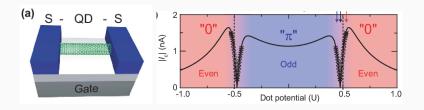
3. CURRENT REVERSAL IN NANOJUNCTIONS

Carbon nanotube interconnecting two superconductors, differing in phase.



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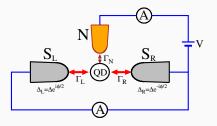


At certain gate potential the dc Josephson current abruptly changed its magnitude and direction (zero-pi transition).

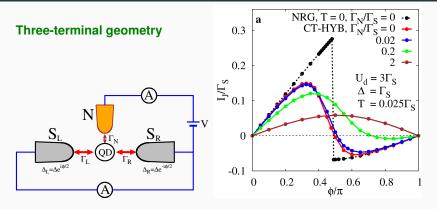
H.I. Jorgensen, T. Novotný, K. Grove-Rasmussen, K. Flensberg, P.E. Lindelof, NanoLett. <u>7</u>, 2441 (2007).

3. JOSEPHSON CURRENT REVERSAL

Three-terminal geometry



3. JOSEPHSON CURRENT REVERSAL



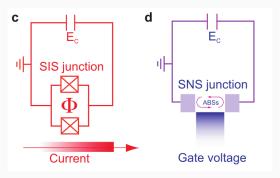
Reversal of dc Josephson current at certain phase difference ϕ is driven by parity change of the Andreev bound states of QD.

T. Domański, M. Žonda, V. Pokorný, G. Górski, V. Janiš,

T. Novotný Phys. Rev. B 95, 045104 (2017).

4. SUPERCONDUCTING QUBITS

Schematical view of the superconducting quantum bits in realization of: <u>transmon</u> (left) and gatemon (right h.s. panel).

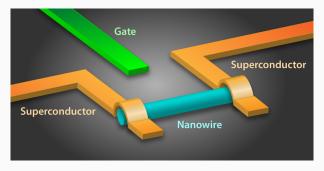


R. Aguado, Appl. Phys. Lett. 117, 240501 (2020).

Superconducting island circuit based on Josephson junction, which is capacitively shunted (E_C is the charging energy).

4. SUPERCONDUCTING QUBITS: GATEMON

Idea: Electrical control over the Josephson supercurrent through semiconducting nanowire accomplished by a side-gate potential.



J.M. Nichol, Physics 8, 87 (2015).

The field-free Josephson diode in a van der Waals heterostructure

https://doi.org/10.1038/s41586-022-04504-8 Received: 29 March 2021

Accepted: 2 February 2022

Published online: 27 April 2022



Heng Wu^{12.36™}, Yaojia Wang^{1,36}, Yuanfeng Xu^{1,4}, Pranava K. Sivakumar¹, Chris Pasco⁵, Ulderico Filippozzi³, Stuart S. P. Parkin¹, Yu-Jia Zeng², Tyrel McQueen⁵ & Mazhar N. Ali^{1,3™}

The superconducting analogue to the semiconducting diode, the Josephson diode, has long been sought with multiple avenues to realization being proposed by theorists $^{1-3}$. Showing magnetic-field-free, single-directional superconductivity with Josephson coupling, it would serve as the building block for next-generation superconducting circuit technology. Here we realized the Josephson diode by fabricating an inversion symmetry breaking van der Waals heterostructure of NbSe $_{\! /\! }$ Nb $_{\! BFg}$ /NbSe $_{\! /\! }$. We demonstrate that even without a magnetic field, the junction can be superconducting with a positive current while being resistive with a negative current.

Nature | Vol 604 | 28 April 2022 | 653.

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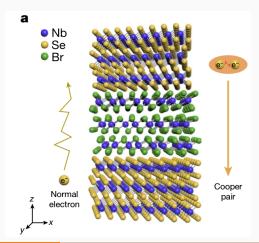
Heng Wu^{12,3,6,53}, Yaojia Wang^{1,3,6}, Yuanfeng Xu^{1,4}, Pranava K. Sivakumar¹, Chris Pasco⁵, Ulderico Filippozzi³, Stuart S. P. Parkin¹, Yu-Jia Zeng², Tyrel McQueen⁵ & Mazhar N. Ali^{1,3,53}

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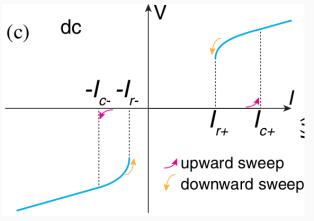
Nature | Vol 604 | 28 April 2022 | 653.

Discovery of the magnetic field-free superconducting diode in van der Waals heterostructure of NbSe₂/Nb₃Br₈/NbSe₂.

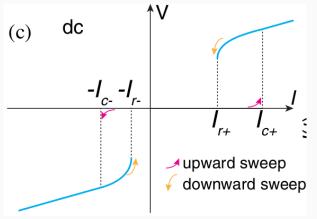
Niobium bromide (just a few atoms thick) placed between layers of superconducting niobium diselenide does conduct electricity without resistance solely in one direction of the applied voltage.



Mechanism behind the Josephson diode effect in not fully understood.



Mechanism behind the Josephson diode effect in not fully understood.



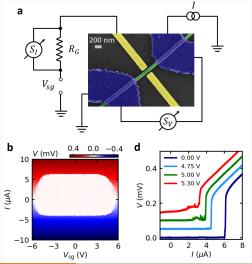
Speculations:

Major role plays the asymmetric sc proximity process inside the tunneling barrier due to: (a) inversion symmetry breaking, (b) time-reversal breaking.

6. JOSEPHSON TRANSISTOR

T. Elalaily, M. Berke, I. Lilja, A. Savin, G. Fülöp, L. Kupás, T. Kanne, J. Nygard,

P. Makk, P. Hakonenm, S. Csonka, arXiv:2312.15453 (2023).

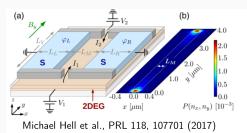


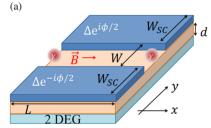
TOPOLOGICAL MATTER

Part 3. Topological superconductivity (in Josephson junctions)

Theoretical concept (2017)

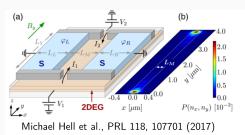
<u>Idea</u>: Narrow metallic region with the strong spin-orbit interaction and in presence of magnetic field embedded between external superconductors.

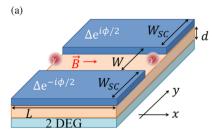




F. Pientka et al., Phys. Rev. X 7,021032 (2017)

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F. Pientka et al., Phys. Rev. X 7,021032 (2017)

Benefit:

Phase-tunable topological superconductivity induced in the metallic stripe.

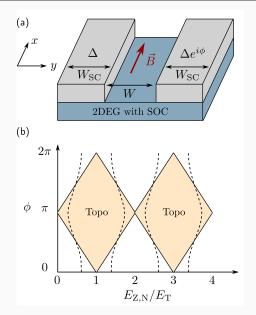
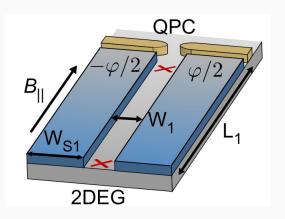


Diagram of topological superconducting state vs

- phase difference ϕ , magnetic field E_z .

Experimental realization (2019)

Two-dimensional electron gas of InAs epitaxially covered by a thin Al layer



Width:

 $W_1 = 80 \text{ nm}$

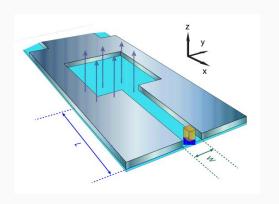
Length:

 L_1 = 1.6 μ m

A. Fornieri, ..., <u>Ch. Marcus</u> and F. Nichele, Nature <u>569</u>, 89 (2019).

Niels Bohr Institute (Copenhagen, Denmark)

Two-dimensional HgTe quantum well coupled to 15 nm thick Al film



Width:

W = 600 nm

Length:

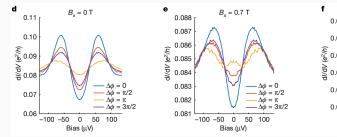
L = 1.0 μ m

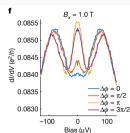
H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature <u>569</u>, 93 (2019).

Würzburg Univ. (Germany) + Harvard Univ. (USA)

PLANAR JOSEPHSON JUNCTION: EXPERIMENT

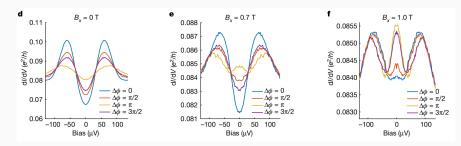
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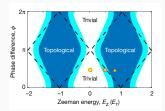


PLANAR JOSEPHSON JUNCTION: EXPERIMENT

H. Ren, ..., L.W. Molenkamp, B.I. Halperin & A. Yacoby, Nature 569, 93 (2019).



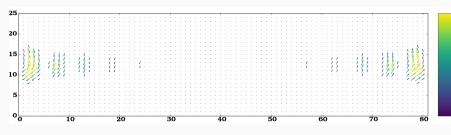
Experimental data obtained for three different magnetic fields indicated by the symbols in phase diagram \Rightarrow .



Topography of Majorana modes

TOPOGRAPHY OF MAJORANA MODES

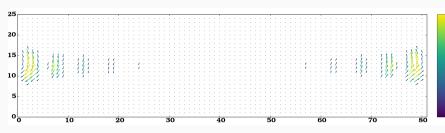
Spatial profile of the zero-energy quasiparticles of a homogeneous metallic strip embedded into the Josephson junction obtained for the phase difference $\phi=\pi$, that is optimal for topological state.



"Majorana polarization vector" $u_{\uparrow,n}v_{\uparrow,n}-u_{\downarrow,n}v_{\downarrow,n}$ obtained for the quasiparticle eigenvalue $E_n=0$.

TOPOGRAPHY OF MAJORANA MODES

Spatial profile of the zero-energy quasiparticles of a homogeneous metallic strip embedded into the Josephson junction obtained for the phase difference $\phi=\pi$, that is optimal for topological state.

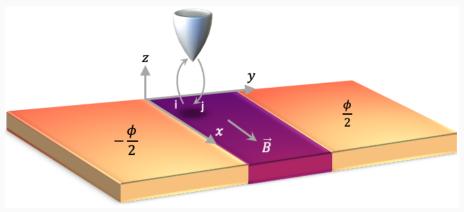


"Majorana polarization vector" $u_{\uparrow,n}v_{\uparrow,n} - u_{\downarrow,n}v_{\downarrow,n}$ obtained for the quasiparticle eigenvalue $E_n = 0$. Magnitude of this quantity is measurable by the conductance of SESAR spectroscopy.

Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

TOPOGRAPHY OF MAJORANA MODES

Selective Equal Spin Andreev Reflection (SESAR) spectroscopy:

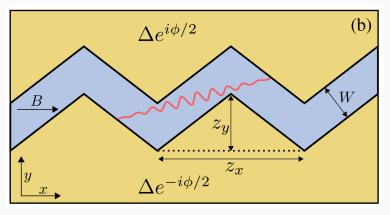


Sz. Głodzik, N. Sedlmayr & T. Domański, PRB <u>102</u>, 085411 (2020).

Means to localize Majoranas

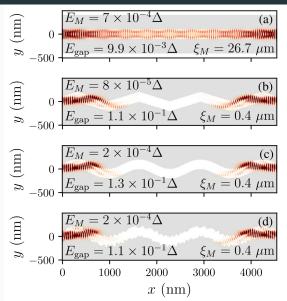
I. DESHAPED JOSEPHSON JUNCTION

To reduce spatial extent of the Majorana modes and increase the topological gap one can use zigzag-shape metallic stripe.



T. Laeven, B. Nijholt, M. Wimmer & A.R. Akhmerov, PRL <u>102</u>, 086802 (2020).

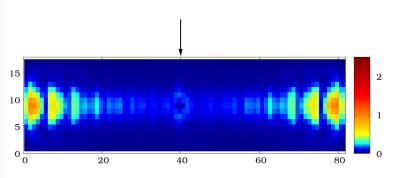
I. DESHAPED JOSEPHSON JUNCTION



T. Laeven, B. Nijholt, M. Wimmer & A.R. Akhmerov, PRL 102, 086802 (2020).

II. LOCAL DEFECT IN JOSEPHSON JUNCTION

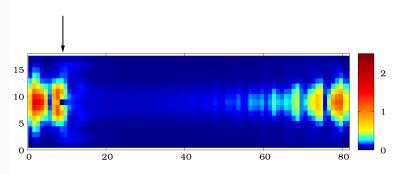
Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed in the center.



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB <u>102</u>, 085411 (2020).

II. LOCAL DEFECT IN JOSEPHSON JUNCTION

Spatial profile of the Majorana modes in presence of the strong electrostatic defect placed near the edge.



Sz. Głodzik, N. Sedlmayr & T. Domański, PRB <u>102</u>, 085411 (2020).

III. RANDOM DISORDER

"Benefits of Weak Disorder in dim=1 Topological Superconductors"

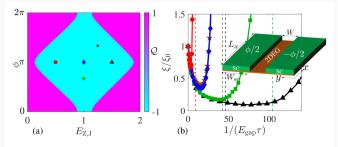
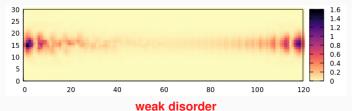


FIG. 1. (a) Phase diagram of the planar Josephson junction Eq. (1) in the clean limit. In the topological phase (Q = -1), the system supports zero-energy MBSs at each end of the junction. (b) The Majorana localization length ξ versus the disorder-induced inverse mean free time τ^{-1} for different points inside the topological phase [see markers in (a)].

A. Haim & A. Stern, Phys. Rev. Lett. 122, 126801 (2019).

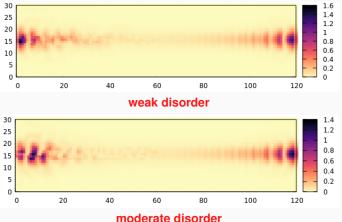
III. RANDOM DISORDER

The left-hand-side part of the metallic stripe is randomly disordered



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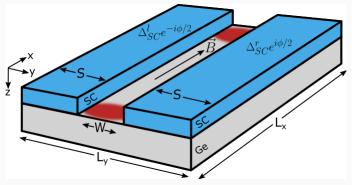
Sz. Głodzik, N. Sedlmayr & T. Domański, PRB 102, 085411 (2020).

TOPOLOGICAL JOSEPHSON JUNCTIONS

New proposals

1. GERMANIUM BASED PLANAR JJ

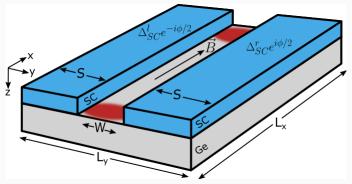
2D hole gas of germanium (Ge) exhibits strong and tunable spin-orbit interaction (cubic in momentum). Such Ge structures can be compatible with the existing metal-oxide-semiconductor (CMOS) technology.



M. Luethi, K. Laubscher, S. Bosco, D. Loss & J. Klinovaja, PRB <u>107</u>, 035435 (2023).

1. GERMANIUM BASED PLANAR JJ

2D hole gas of germanium (Ge) exhibits strong and tunable spin-orbit interaction (cubic in momentum). Such Ge structures can be compatible with the existing metal-oxide-semiconductor (CMOS) technology.

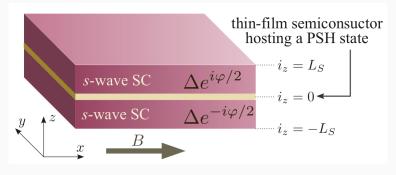


M. Luethi, K. Laubscher, S. Bosco, D. Loss & J. Klinovaja, PRB <u>107</u>, 035435 (2023).

 \Longrightarrow topological phase is asymmetric on phase reversal $\phi \longrightarrow -\phi$

2. VERTICAL JOSEPHSON JUNCTION

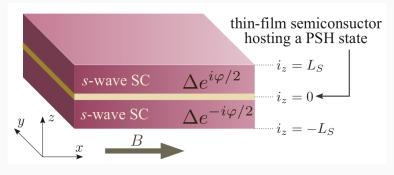
Josephson junction, comprising a thin semiconducting film sandwiched between conventional s-wave superconductors.



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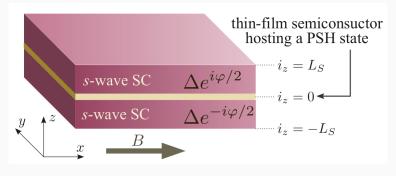
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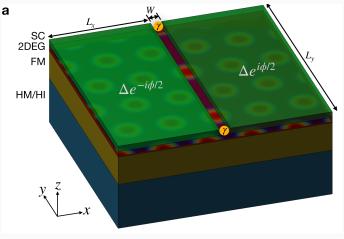
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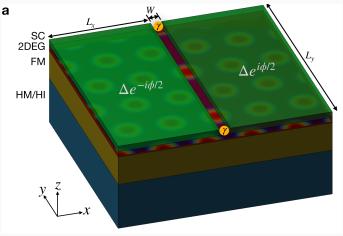
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 - ⇒ semiconducting region hosts a Persistent Spin-Helix state
 - \Rightarrow a weak Zeeman field can induce p_x-wave superconductivity

Josephson junction deposited on top of a skyrmion crystal.



N. Mohanta, S. Okamoto & E. Dagotto, Communications Physics 4, 163 (2021).

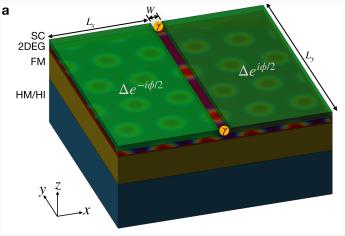
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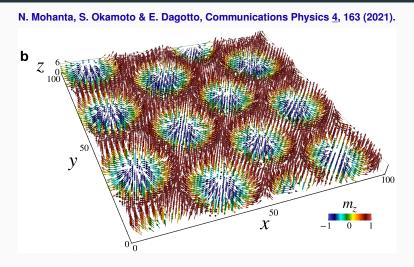
FM - ferromagnetic layer

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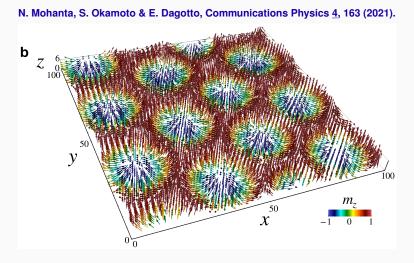


N. Mohanta, S. Okamoto & E. Dagotto, Communications Physics 4, 163 (2021).

FM - ferromagnetic layer HM/HI - heavy metal or heavy insulator

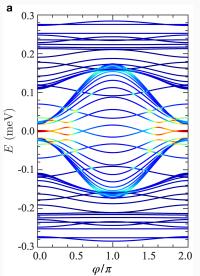


Skyrmions are driven in FM region by:



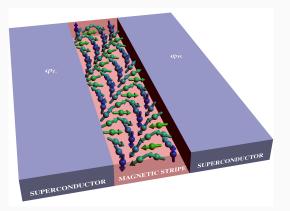
Skyrmions are driven in FM region by: \star ferromagnetic exchange \star Dzialoshinskii-Moriya interaction \star magnetic field

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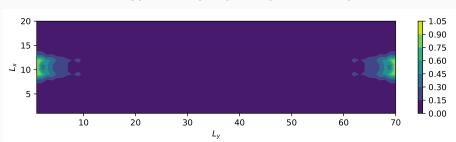
Phase difference has detrimental influence on the Majorana modes.

Narrow metallic stripe with the classical magnetic moments placed between two s-wave superconductors, differing in phase $\phi_L \neq \phi_R$.

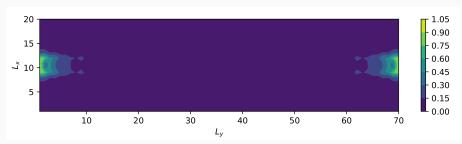


M.M. Maśka, M. Dziurawiec & N. Sedlmayr, T.D. – work in progress/ Technical University (Wrocław) & UMCS (Lublin) cooperation/

Classical moments of the metallic region develop such magnetic textures, which support the topological superconducting state.

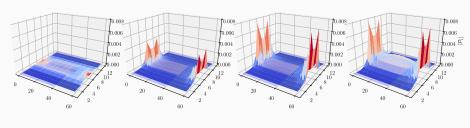


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Spatial profile of the Majorana modes – preliminary results obtained for the coplanar magnetic moments.

Topography of the Majorana quasiparticles obtained for the coplanar moments and for various Josephson phase differences $\phi_R-\phi_L$.



$$\phi_R - \phi_L = 0.6\pi$$
 $\phi_R - \phi_L = 0.4\pi$ $\phi_R - \phi_L = 0.2\pi$ $\phi_R - \phi_L = 0.0$

Details will be provided tomorrow by: M.M. Maśka

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http://kft.umcs.lublin.pl/doman/lectures

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(M. Curie-Skłodowska University, Lublin)



→ Aksel Kobiałka

(University of Uppsala, Sweden)



SINGLY OCCUPIED VS BCS-TYPE CONFIGURATIONS

The proximitized quantum dot can described by

$$\hat{H}_{QD} = \sum \epsilon_d \; \hat{d}_{\sigma}^{\dagger} \; \hat{d}_{\sigma} \; + \; U_d \; \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Delta_d \; \hat{d}_{\uparrow}^{\dagger} \hat{d}_{\downarrow}^{\dagger} + \mathrm{h.c.} \right)$$

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Eigen-states of this problem are represented by:

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Upon varrying the parameters ε_d , U_d or Γ_S there can be induced quantum phase transition between these doublet/singlet states.