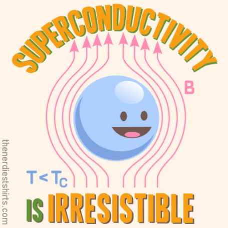
# Constructive feedback of superconductivity on the Kondo effect in nanoscopic heterostructures

Bad Honnef, 16/10/2019

T. Domański

**M. Curie-Skłodowska University (Lublin, Poland)**



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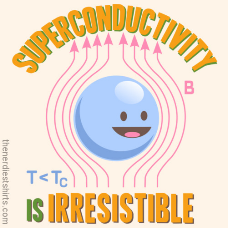


Cooperation: **Maciek Maśka (UŚ Katowice) & Irek Weymann (UAM Poznań)**

# OUTLINE

- **dualities in electron systems**

$\Rightarrow$ **localized vs itinerant**

$\Rightarrow$ **particle vs hole**

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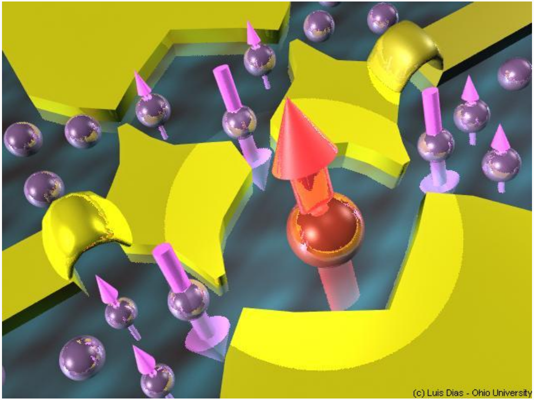
- role of topological effects

$\Longrightarrow$ **Majorana vs Kondo**

# Preliminaries

# 1. ITINERANT VS LOCALIZED ELECTRONS

**Correlated quantum impurity embedded into the Fermi sea**




**can develop the many-body Kondo state with itinerant electrons (at $T < T_K$) due to the exchange (screening) interactions.**

# 1. ITINERANT VS LOCALIZED ELECTRONS

**Prototype (single impurity Anderson) model:**

$$\hat{H} = \hat{H}_{imp} + \hat{H}_c + \hat{V}_{d-c}$$

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**where**

$\hat{H}_{imp} = \sum_\sigma \epsilon_d \, \hat{d}^\dagger_\sigma \, \hat{d}_\sigma \; + \; U \, \hat{n}_{d\uparrow} \, \hat{n}_{d\downarrow}$ **correlated impurity**

$\hat{H}_c = \sum_{k,\sigma} (\varepsilon_k - \mu) \, \hat{c}^\dagger_{k\sigma} \hat{c}_{k\sigma}$ **itinerant electrons**

$\hat{V}_{d-c} = \sum_{\mathbf{k},\sigma} \left( V_\mathbf{k} \, \hat{d}^\dagger_\sigma \hat{c}_{\mathbf{k}\sigma} + V^*_\mathbf{k} \, \hat{c}^\dagger_{\mathbf{k}\sigma} \hat{d}_\sigma \right)$ **hybridization**

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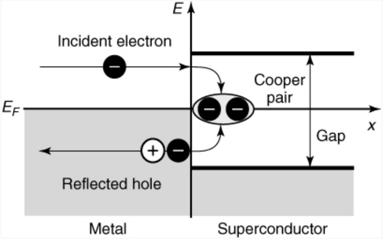


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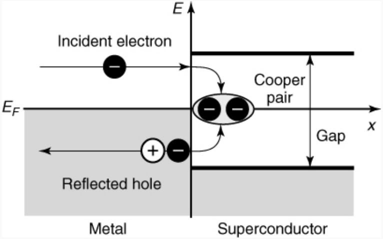
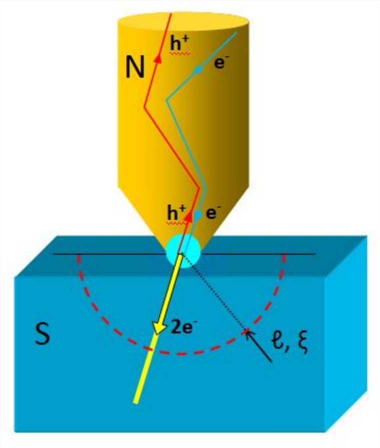



⇒ **upon injecting an electron to superconductor**

⇒ **there is reflected a hole (Andreev scattering).**

# 2. PARTICLE VS HOLE

**Particle to hole conversion (Andreev scattering mechanism).**



# 2. PARTICLE VS HOLE

**BCS ground state :**

$$|\mathbf{BCS}\rangle = \prod_k \left( u_k + v_k\ \hat{c}^\dagger_{k\uparrow}\ \hat{c}^\dagger_{-k\downarrow} \right)\ |\mathbf{vacuum}\rangle$$

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**Effective (Bogoliubov) quasiparticles**

$$\begin{aligned} \hat{\gamma}_{k\uparrow} &= u_k \hat{c}_{k\uparrow} + v_k \hat{c}^\dagger_{-k\downarrow} \\ \hat{\gamma}^\dagger_{-k\downarrow} &= -v_k \hat{c}_{k\uparrow} + u_k \hat{c}^\dagger_{-k\downarrow} \end{aligned}$$

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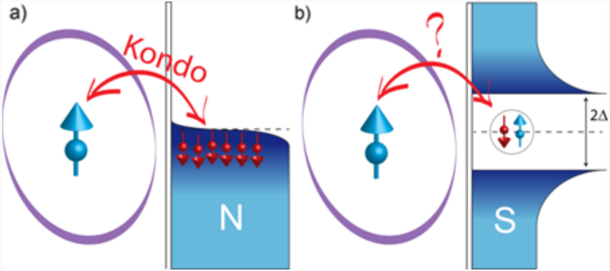
**formally due to**

$$\begin{aligned} \hat{\gamma}_{k\uparrow} &= u_k \hat{c}_{k\uparrow} + \tilde{v}_k \ \hat{b}_{q=0} \hat{c}^\dagger_{-k\downarrow} \\ \hat{\gamma}^\dagger_{-k\downarrow} &= -\tilde{v}_k \ \hat{b}^\dagger_{q=0} \ \hat{c}_{k\uparrow} + u_k \hat{c}^\dagger_{-k\downarrow} \end{aligned}$$

# KONDO VS ELECTRON PAIRING

**'To screen or not to screen ?'**




**R. Maurand and Ch. Schönenberger, Physics 6, 75 (2013).**

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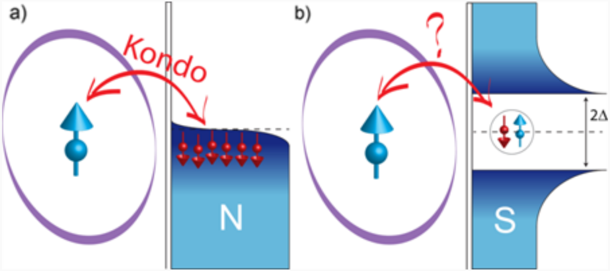


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$\Rightarrow$ electronic states near the Fermi level are missing,

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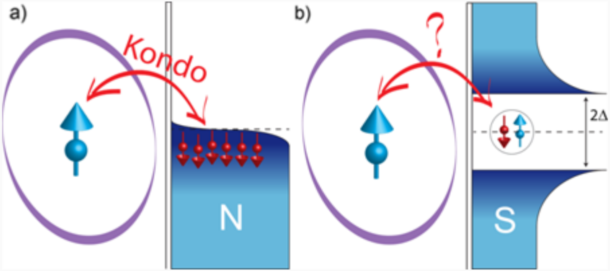


R. Maurand and Ch. Schönenberger, Physics 6, 75 (2013).

**Relevant issues:**

⇒ electronic states near the Fermi level are missing,

⇒ pairing and correlations can switch on/off the effective exchange potential due to quantum phase transition.

# KONDO VS ELECTRON PAIRING

*Impurity on a superconducting surface:*

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**K.J. Franke, G. Schulze, J.I. Pascual, Science 332, 940 (2011).**

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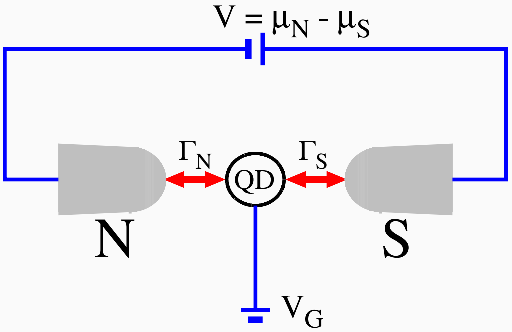
$\Rightarrow$ ***Scaling of Yu-Shiba-Rusinov energies in the weak-coupling Kondo regime***

**N. Hatter, B.W. Heinrich, D. Rolf, K.J. Franke, Nature Communications 8, 2016 (2017).**

# Part I: pairing vs Kondo

**For probing the subgap states one can measure conductance of tunneling current flowing via the quantum dot (QD) coupled between the normal (N) and superconducting (S) electrodes**



# ANDREEV TUNNELING SPECTROSCOPY

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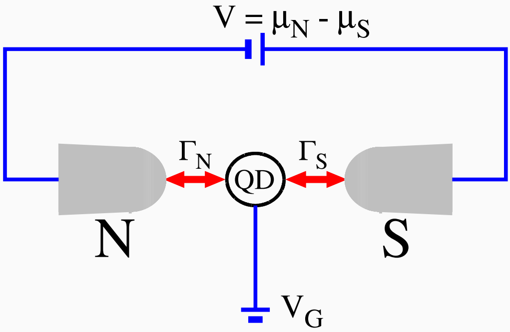


**This is a particular realization of the single-electron-transistor.**

# CORRELATIONS VS PAIRING

**The proximitized quantum dot can described by**

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow}\hat{n}_{d\downarrow} - \left( \Delta_d \, \hat{d}^{\dagger}_{\uparrow}\hat{d}^{\dagger}_{\downarrow} + \text{h.c.} \right)$$

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**Eigen-states of this problem are represented by:**

$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left. \begin{array}{l} u\,|0\rangle - v\,|\uparrow\downarrow\rangle \\ v\,|0\rangle + u\,|\uparrow\downarrow\rangle \end{array} \right\} \Leftarrow$$ **singlet states (spin 0)**

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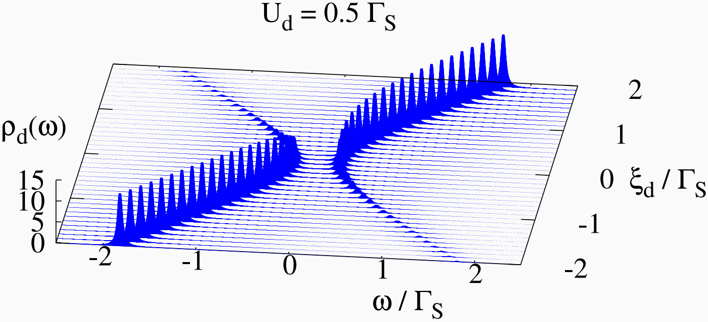
**Upon varrying the parameters $\varepsilon_d$, $U_d$ or $\Gamma_S$ there can be induced quantum phase transition between these doublet/singlet states.**

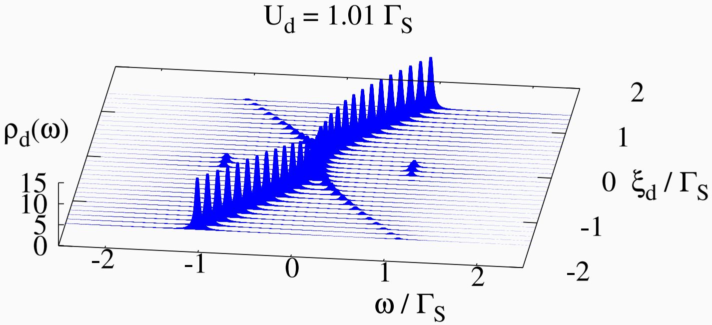
**Subgap spectrum of the correlated QD**

$\xi_d = \varepsilon_d + \frac{1}{2} U_d$



# QUANTUM PHASE TRANSITION

**Subgap spectrum of the correlated QD** $\xi_d = \varepsilon_d + \frac{1}{2}U_d$



**Subgap spectrum of the correlated QD**

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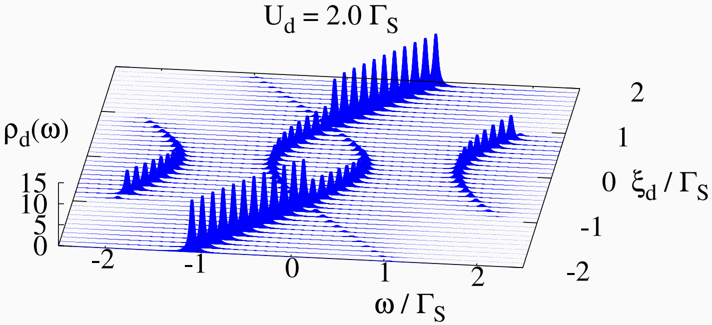
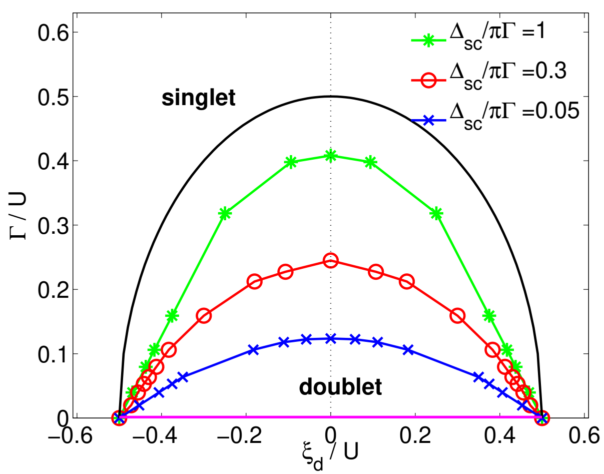


# QUANTUM PHASE TRANSITION

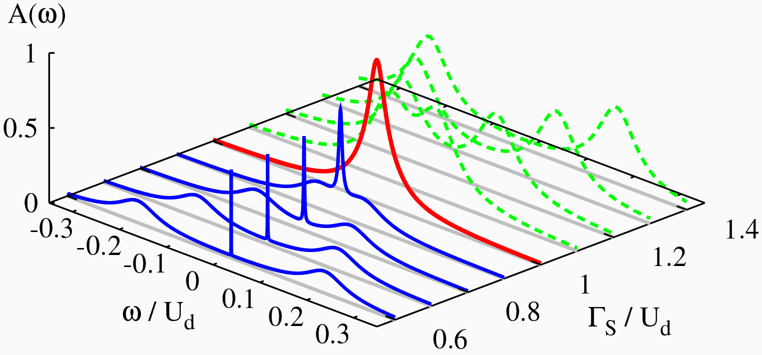
## Singlet-doublet phase transition: NRG results




**J. Bauer, A. Oguri & A.C. Hewson, J. Phys.: Condens. Matter 19, 486211 (2007).**

# SUBGAP KONDO EFFECT

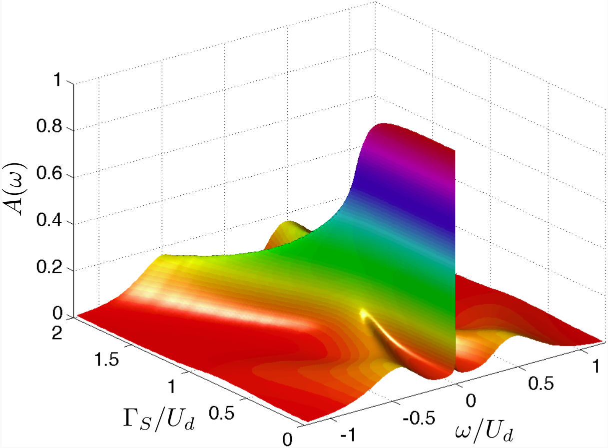
**Enhancement of the Kondo peak while approaching the quantum phase transition from the doublet side (for the half-filled QD).**




**T. Domański *et al*, Scientific Reports 6, 23336 (2016).**

# SUBGAP KONDO EFFECT

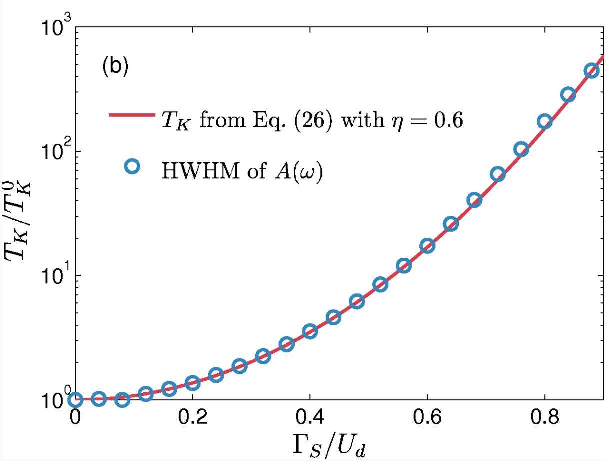
**Spectral function obtained from the NRG calculations (Budapest code).**




**T. Domański *et al*, Scientific Reports 6, 23336 (2016).**

# SUBGAP KONDO EFFECT

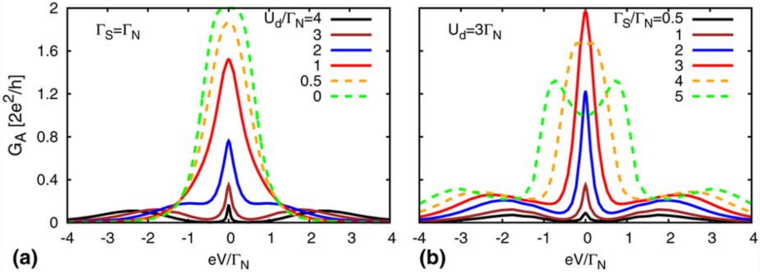
**Kondo temperature** $T_K \simeq 0.3\,\sqrt{\Gamma_N U_d}\,\exp\left[\frac{\pi\varepsilon_d(\varepsilon_d+U_d)+(\Gamma_S/2)^2}{\Gamma_N U_d}\right]$




**T. Domański *et al*, Scientific Reports 6, 23336 (2016).**

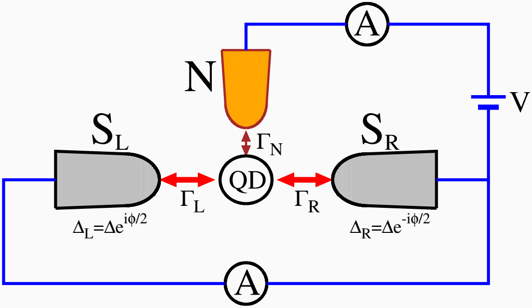
# SUBGAP KONDO EFFECT

**Physical observability in the Andreev differential conductance**



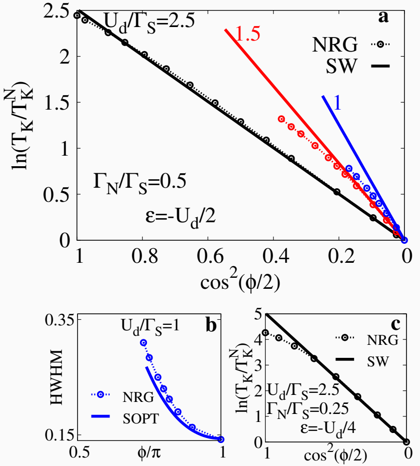

**T. Domański *et al*, Scientific Reports 6, 23336 (2016).**

## Quantum dot embedded in Josephson & Andreev circuits.




**T. Domański ... V. Janiš & T. Novotný, Phys. Rev. B 95, 045104 (2017).**

## Phase-controlled scaling of the Kondo temperarture $T_K$

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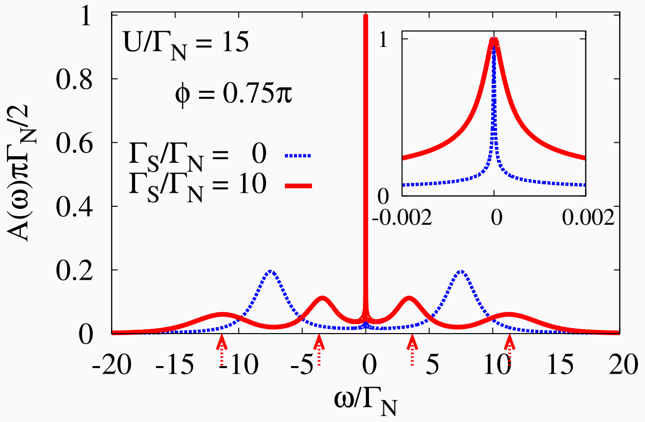
## Spectrum of the half-filled quantum dot

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# MID-SUMMARY

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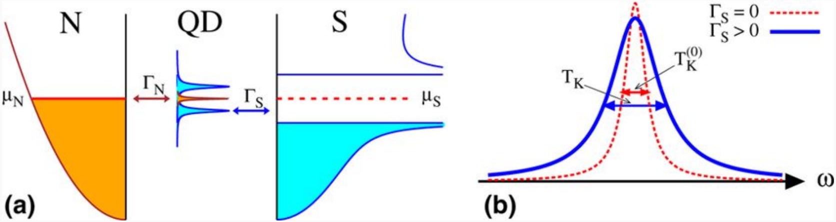
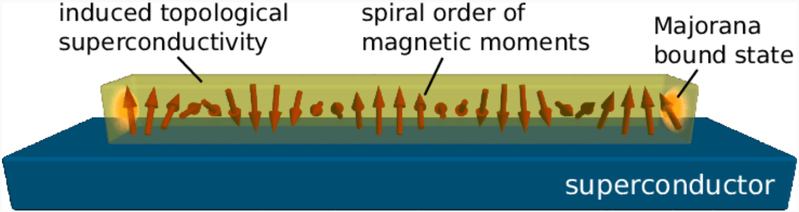



**Constructive feedback of electron pairing on the Kondo effect !**

# Part II: role of topological effects

# FEASIBLE REALIZATION OF TOPOLOGICAL PHASE

**Topological superconductivity can be driven e.g. by the spin-orbit Rashba interaction combined with the external magnetic field.**

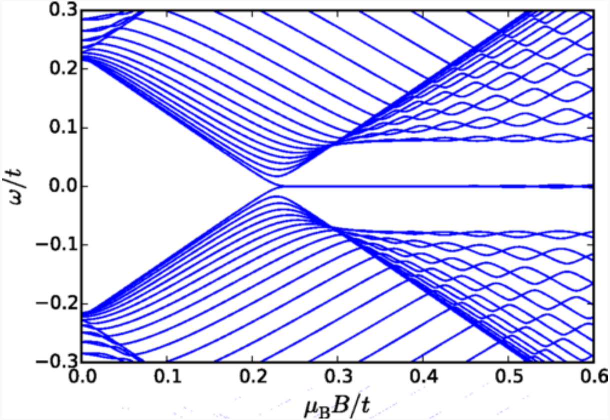



**R. Lutchyn, J. Sau, S. Das Sarma, Phys. Rev. Lett. 105, 077001 (2010).**

**Y. Oreg, G. Refael, F. von Oppen, Phys. Rev. Lett. 105, 177002 (2010).**

# TRANSITION FROM TRIVIAL TO TOPOLOGICAL PHASE

**A pair of the Shiba (Andreev) states evolve into the Majorana qps**




**Mutation of the trivial bound states into the nontrivial Majorana modes**

**M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).**

# PROPERTIES OF MAJORANA QPS

- particle = antiparticle $\hat{\gamma}^{\dagger}_{i,n} = \hat{\gamma}_{i,n}$

$\Rightarrow$ **neutral in charge**

$\Rightarrow$ **of zero energy**

- fractional character $\hat{\gamma}^{\dagger}_{i,n}\,\hat{\gamma}_{i,n} = 1/2$

$\Rightarrow$ **half occupied/empty**

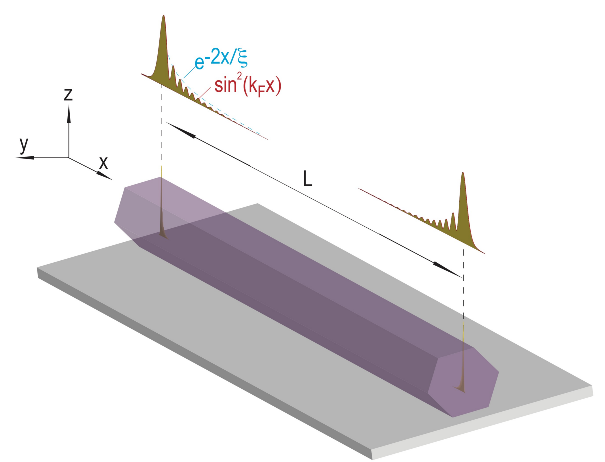
- spatially nonlocal

$\Rightarrow$ **exist in pairs near boundaries/defects**

- topologically protected

$\Rightarrow$ **immune to dephasing/decoherence**

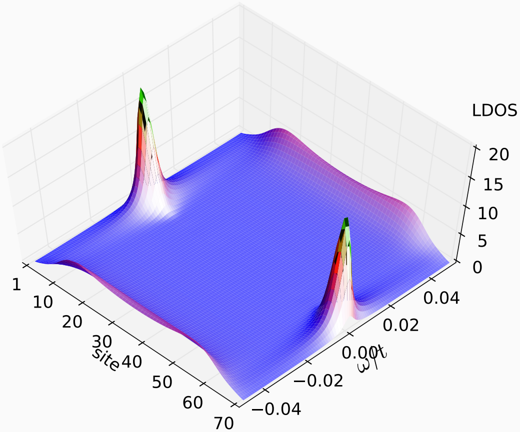
## Majorana qps are localized near the nanowire edges




R. Aguado, Riv. Nuovo Cim. 40, 523 (2017).

## Low energy quasiparticles of the Rashba nanowire

$t_{35}/t = 1.0$




**M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).**

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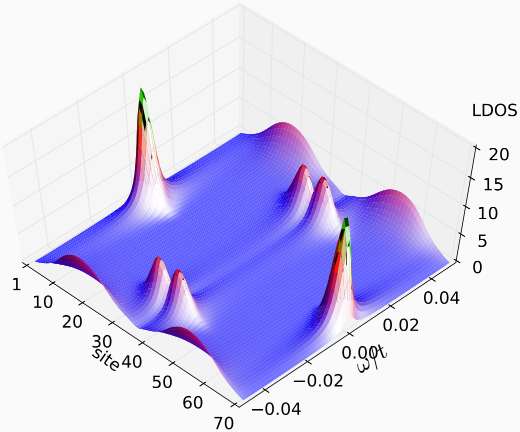
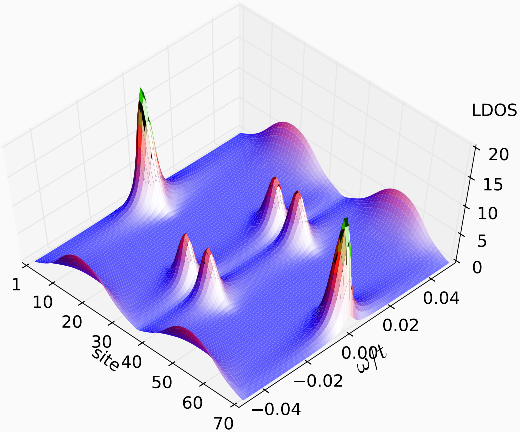



M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

## Low energy quasiparticles of the Rashba nanowire

$t_{35}/t = 0.6$

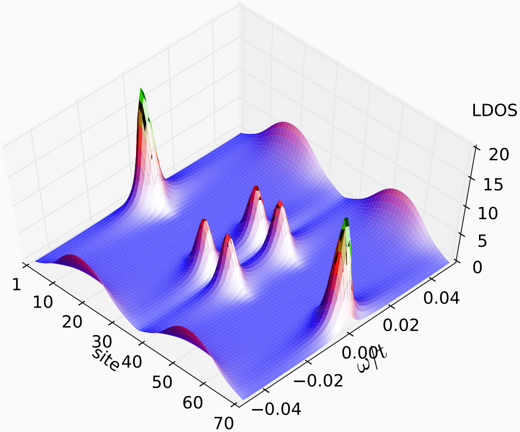



M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

# TOPOLOGICAL PROTECTION

## Low energy quasiparticles of the Rashba nanowire

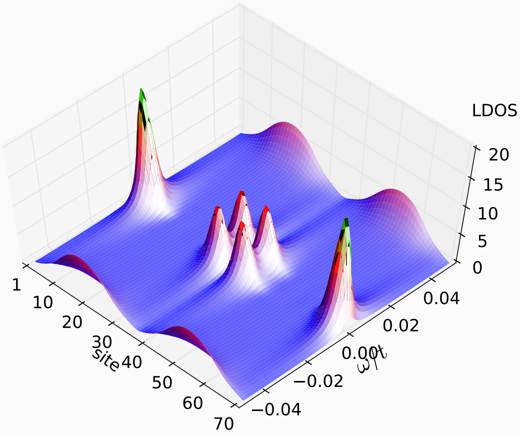
$t_{35}/t = 0.4$




**M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).**

## Low energy quasiparticles of the Rashba nanowire

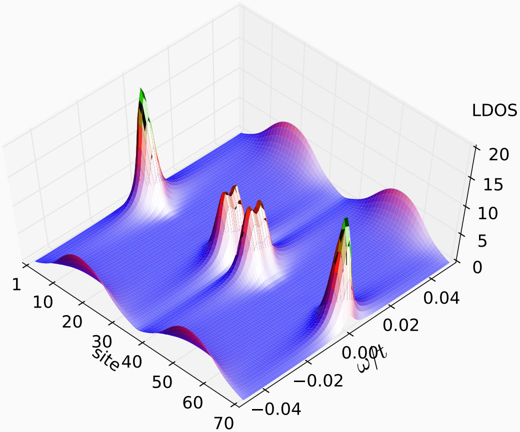
$t_{35}/t = 0.2$




M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).

## Low energy quasiparticles of the Rashba nanowire

$t_{35}/t = 0.1$

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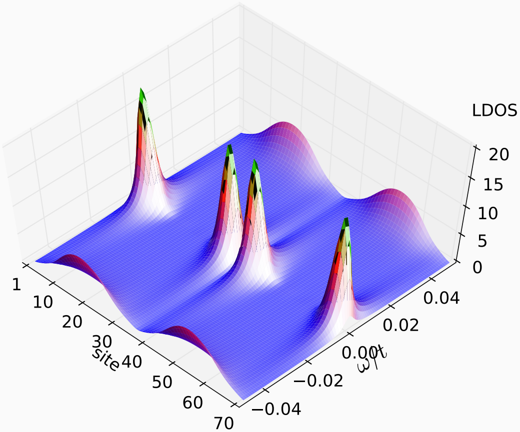
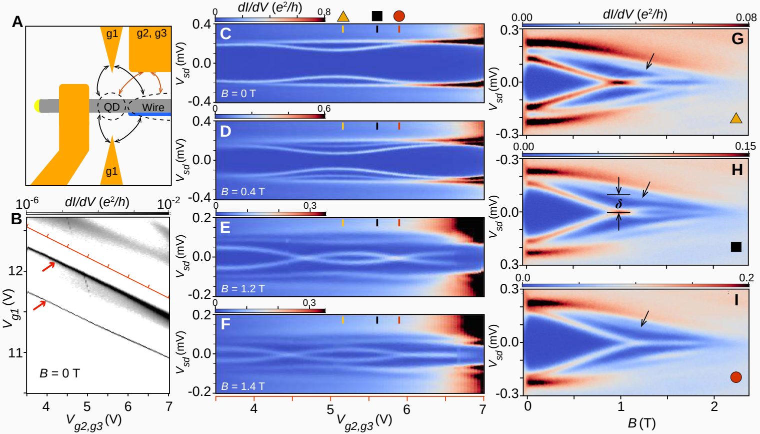



**M.M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, T. Domański, PRB 95, 045429 (2017).**

## Coalescence of the Andreev into Majorana states

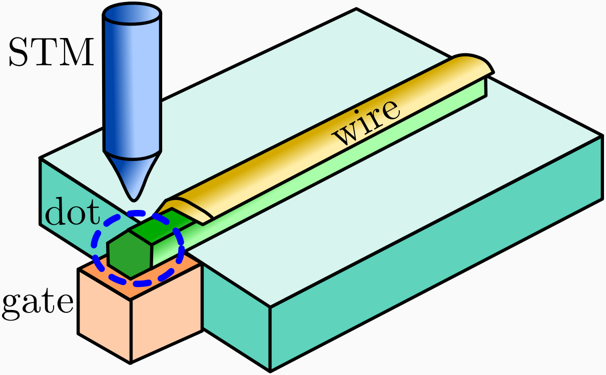



M.T. Deng, ..., and Ch. Marcus, Science **354**, 1557 (2016).

## / Niels Bohr Institute, Copenhagen, Denmark /

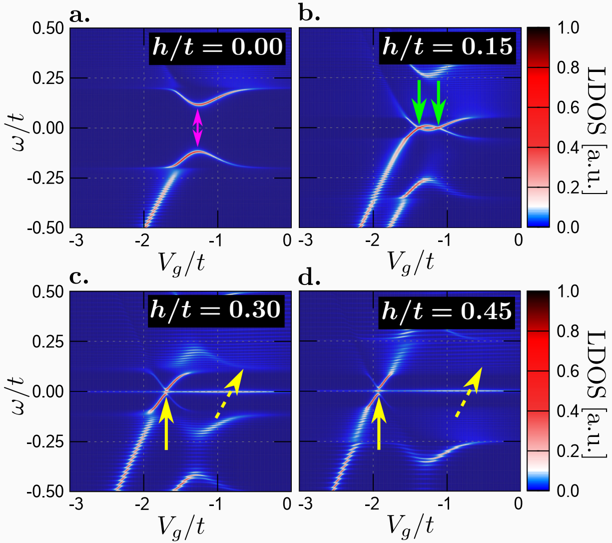
# TRIVIAL VS MAJORANA BOUND STATES

**Schematics of a quantum dot – nanowire hybrid structure.**



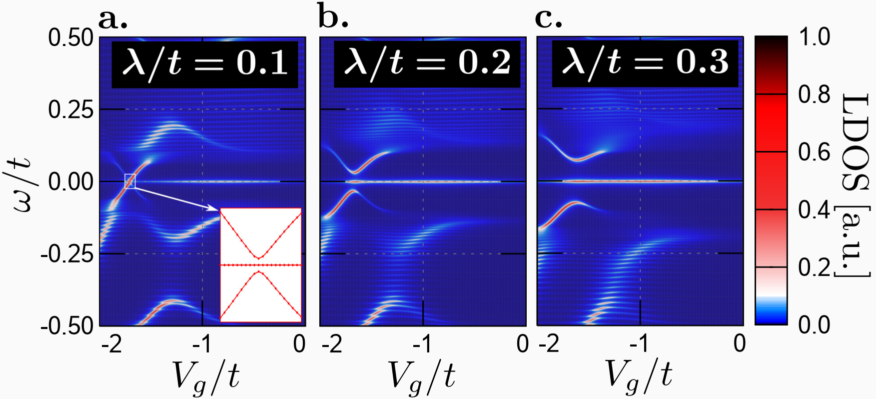

A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. 96, 195403 (2017).

# DISTINGUISHING ANDREEV FROM MAJORANA




A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. **96**, 195403 (2017).

**QD spectrum vs gate potential $V_g$ for various spin-orbit couplings $\lambda$.**




A. Ptok, A. Kobiałka & T. Domański, Phys. Rev. 96, 195403 (2017).

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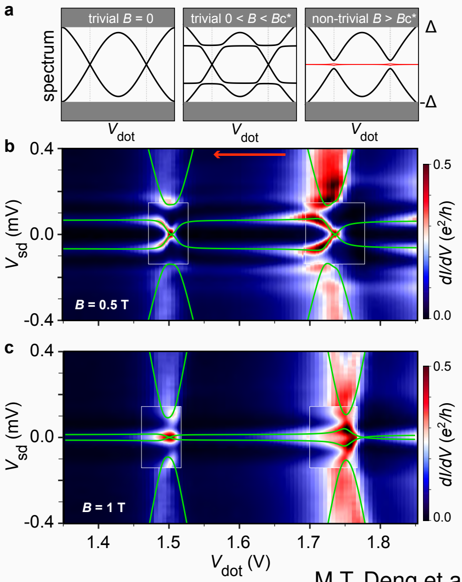


M.T. Deng et al., Phys. Rev. B **98**, 085125 (2018).

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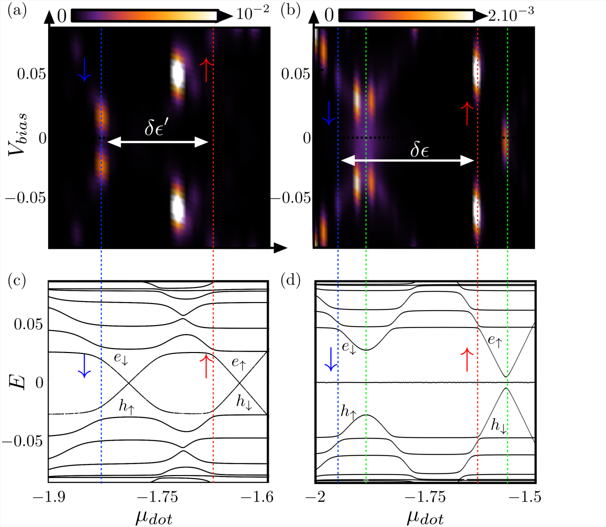
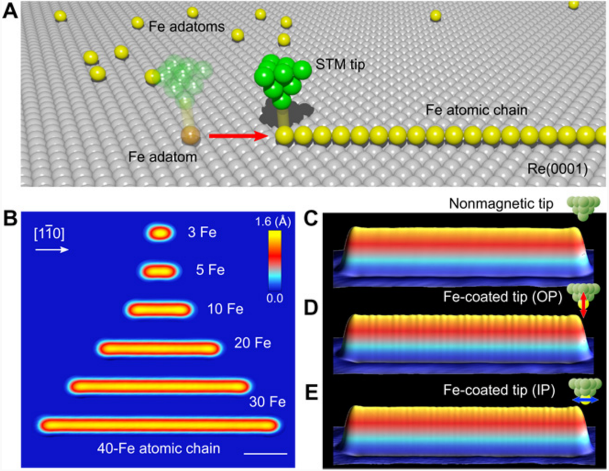



D. Chevallier, ... and J. Klinovaja, Phys. Rev. B **97**, 04504 (2018).

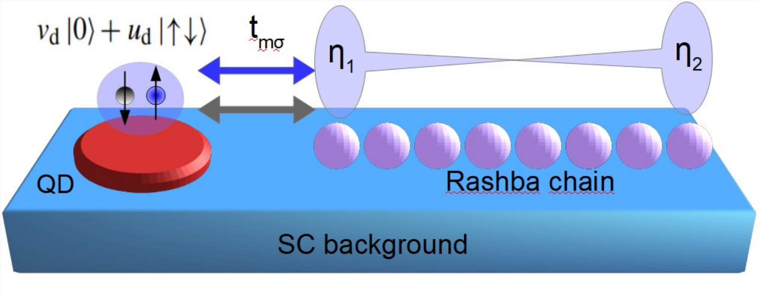
# Can one confront Majorana with Kondo ?

## STM for Fe nanchain on superconducting Re




H. Kim, ..., and R. Wiesendanger, Science Adv. **4**, eaar5251 (2018).

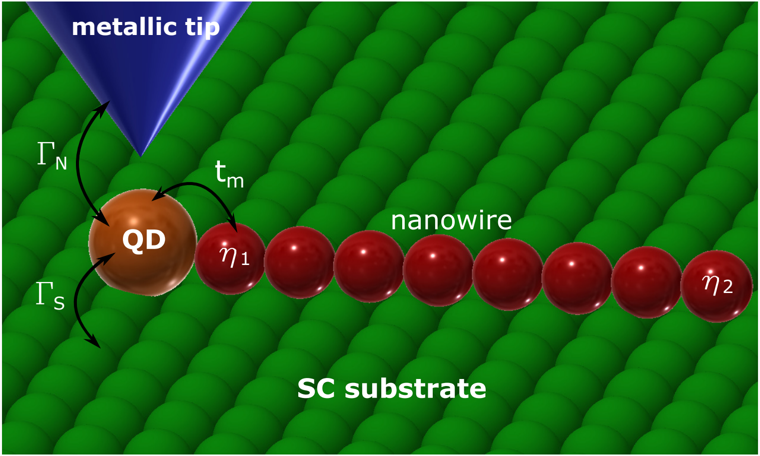
# KONDO + MAJORANA + ELECTRON PAIRING




**Schematic view of the STM-type configuration**

# KONDO AND MAJORANA PHYSICS

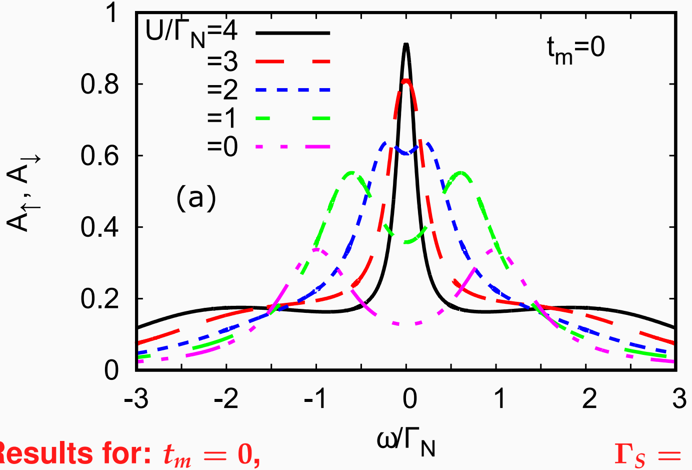
**Possible setup for probing the Kondo – Majorana – pairing effects.**




G. Górski, J. Barański, I. Weymann & T. Domański, Sci. Rep. 8, 15717 (2018).

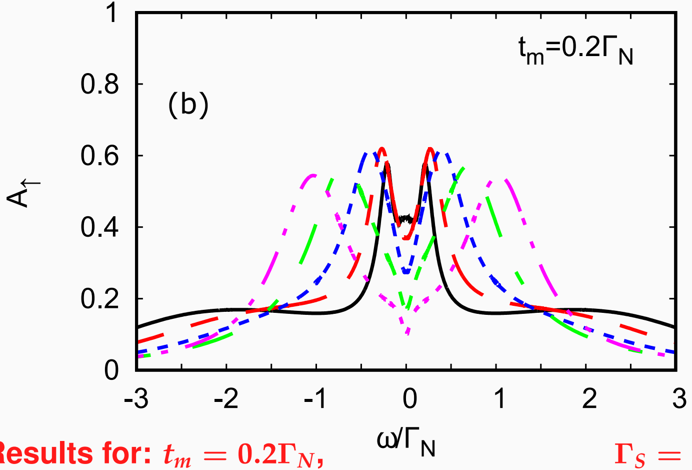
# KONDO VS PAIRING IN ABSENCE OF MAJORANA

**Perturbative results for the proximitized quantum dot.**



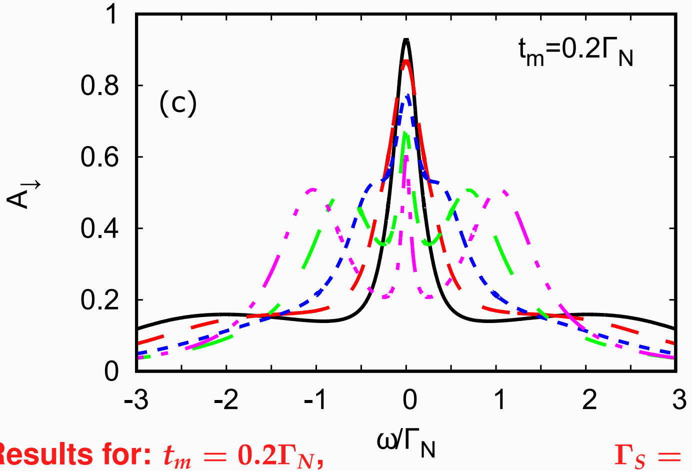

**Results for:** $\underline{t_m = 0}$**,** $\Gamma_S = 2\Gamma_N$

## Perturbative results for the proximitized quantum dot.



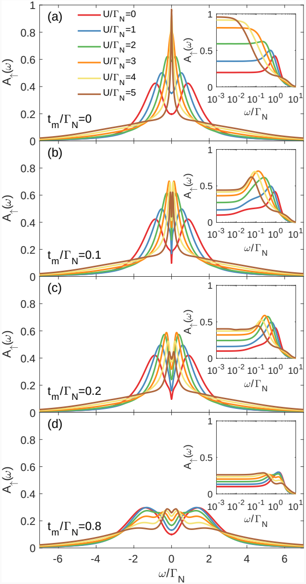

**Results for:** $t_m = 0.2\Gamma_N$, $\Gamma_S = 2\Gamma_N$

**Perturbative results for the proximitized quantum dot.**

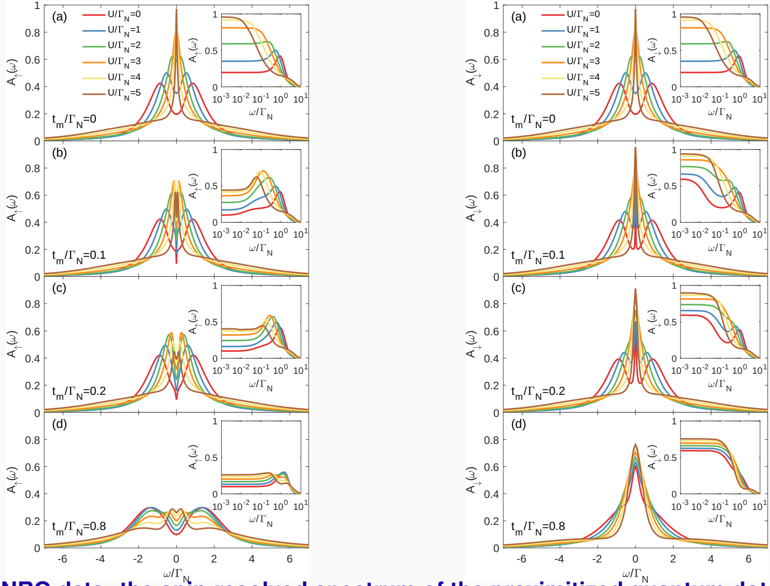



**Results for:** $\underline{t_m = 0.2\Gamma_N}$, $\Gamma_S = 2\Gamma_N$

# QUANTUM DOT SPECTRUM [SCI.REP. 8, 15717 (2018)]



# QUANTUM DOT SPECTRUM [SCI.REP. 8, 15717 (2018)]




**NRG data: the spin-resolved spectrum of the proximitized quantum dot.**

## SUMMARY: KONDO VS MAJORANA

- **spin-selective influence of Majorana on Kondo:**

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# SUMMARY: KONDO VS MAJORANA

- **spin-selective influence of Majorana on Kondo:**

$\Rightarrow$ **constructive for $\downarrow$ electrons**

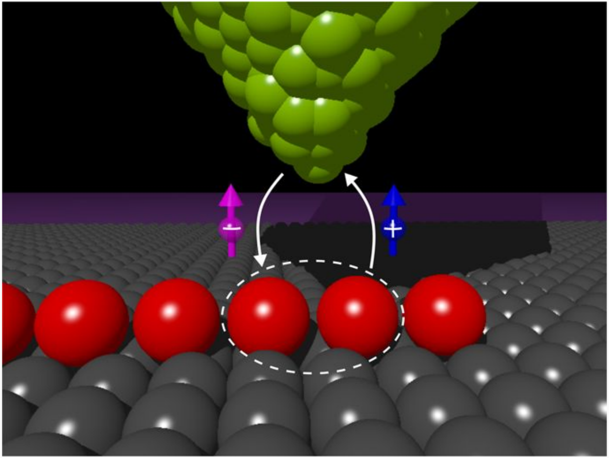
$\Rightarrow$ **destructive for $\uparrow$ electrons**

- **empirical observability via:**

$\Rightarrow$ **selective equal spin Andreev reflections (SESAR)**

$\Rightarrow$ **originating from equal-spin intersite pairing**

## Main idea of the SESAR spectroscopy



M. Maśka and T. Domański, Scientific Reports **7**, 16193 (2017).

# ACKNOWLEDGEMENTS


- **Majorana quasiparticles**

⇒ **M. Maśka & A. Gorczyca-Goraj (Katowice),**
**A. Kobiałka (Lublin), A. Ptok (Kraków),**
**Sz. Głodzik (Lublin), N. Sedlmayr (Lublin).**

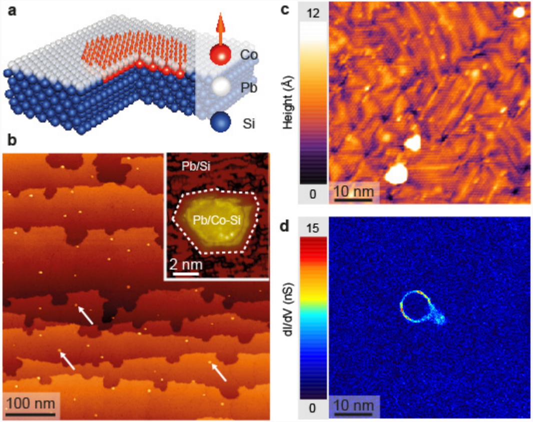
- **Quantum phase transition & Kondo effect**

⇒ **M. Barańska & J. Barański (Dęblin)**
**T. Novotný, M. Žonda & V. Janiš (Prague).**

- **Majorana vs Kondo**

⇒ **I. Weymann (Poznań), G. Górski (Rzeszów)**

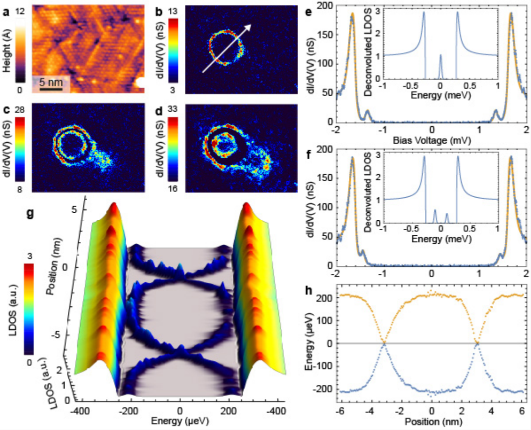
## Majorana modes propagating along magnetic islands




G. Ménard, ..., and P. Simon, Nature Commun. **8**, 2040 (2017).

**/ P. & M. Curie University (Paris, France) /**

## Majorana modes propagating along magnetic islands




G. Ménard, ..., and P. Simon, Nature Commun. **8**, 2040 (2017).

**/ P. & M. Curie University (Paris, France) /**

## ISSUE TO BE ADDRESSED

**Kondo vs dispersive Majorana mode(s)**