

Wrocław, 2 Oct. 2014

Residual Meissner effect and other pre-pairing phenomena in the cuprate superconductors

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<http://kft.umcs.lublin.pl/doman/lectures>

Outline

Outline



Preliminaries

/ pairing vs coherence /

Outline



Preliminaries

/ pairing vs coherence /



Pre-pairing

/ experimental evidence /

Outline



Preliminaries

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Selected results

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Selected results

$\Rightarrow$ *Bogoliubov quasiparticles above T_c*

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Selected results

- *Bogoliubov quasiparticles above T_c*

$\Rightarrow$ *Diamagnetism above T_c*

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Preliminaries

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Selected results

- *Bogoliubov quasiparticles above T_c*
- *Diamagnetism above T_c*



Summary

1. Preliminaries

Superconducting state

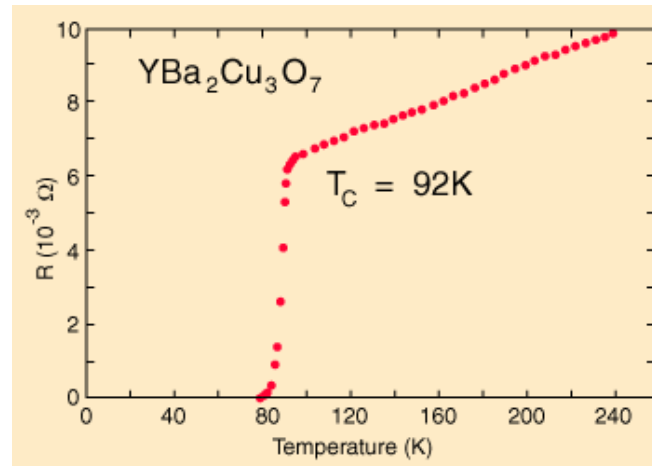
– properties

Superconducting state

– properties



ideal d.c. conductance

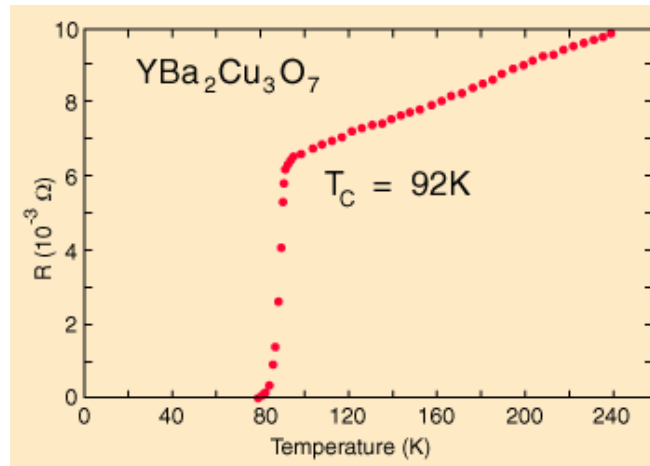


Superconducting state

– properties



ideal d.c. conductance



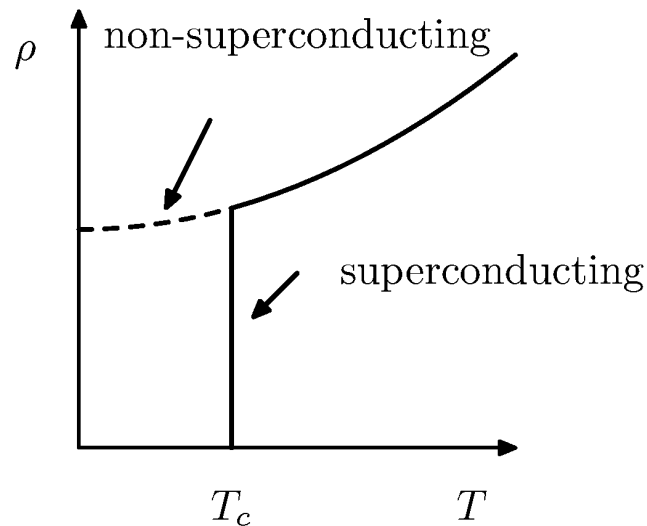
Normal conductors:

resistance $R = \rho \frac{l}{S}$

where $\rho \equiv 1/\sigma$

and $\sigma = \frac{ne^2\tau}{m}$

$\tau(T)$ – relaxation time

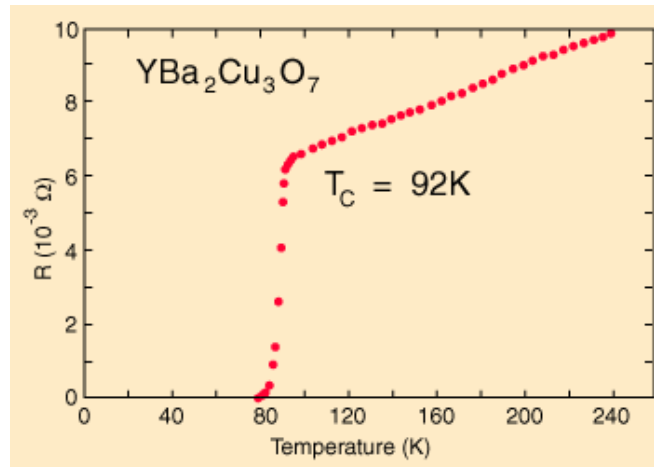


Superconducting state

– properties



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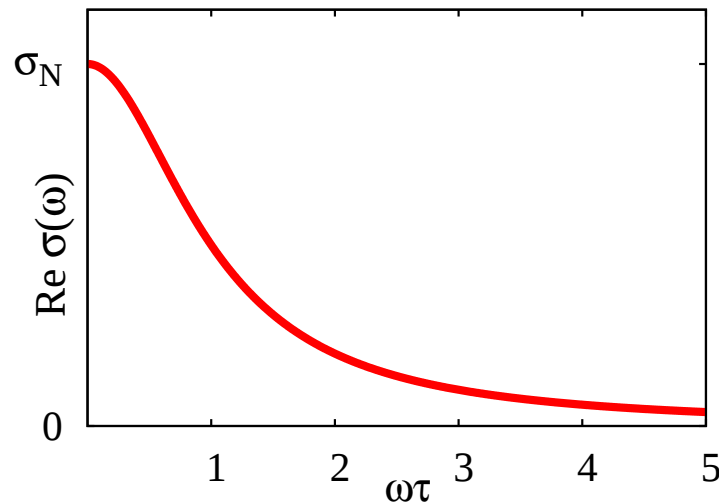
a.c. conductance

Drude conductance

$$\sigma(\omega) = \frac{ne^2\tau}{m} \frac{1}{1-i\omega\tau}$$

obeys the f-sum rule

$$\int_{-\infty}^{\infty} \text{Re } \sigma(\omega) d\omega = \pi \frac{ne^2}{m}$$

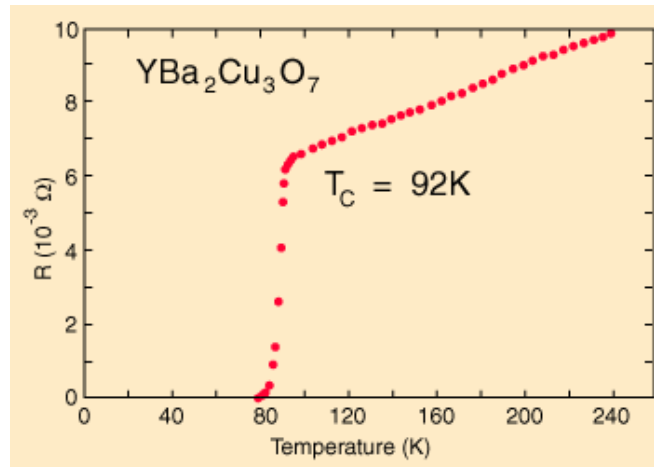


Superconducting state

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ideal d.c. conductance



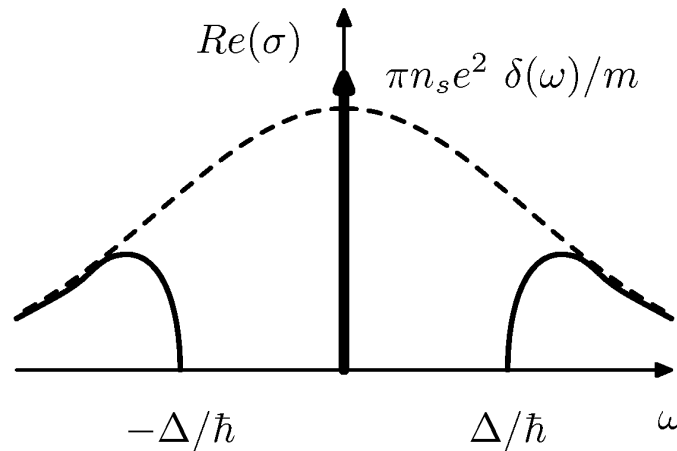
a.c. conductance

This f-sum rule

$$\int_{-\infty}^{\infty} \text{Re } \sigma(\omega) d\omega = \pi \frac{ne^2}{m}$$

must be obeyed also
below T_c , where

$$n = n_n + n_s$$



n_s – superfluid density

Superconducting state

– properties (continued)

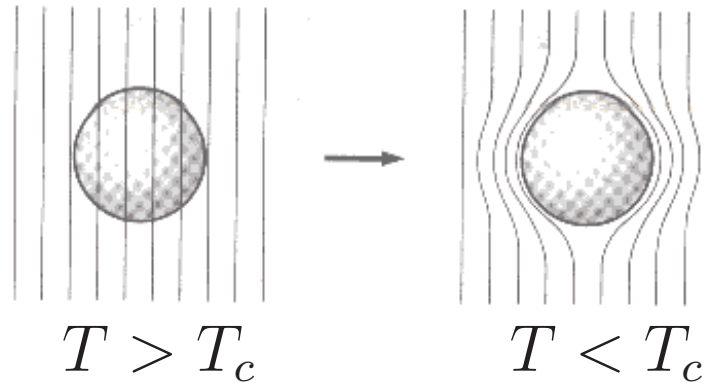
Superconducting state

– properties (continued)



ideal diamagnetism

/perfect screening of d.c. magnetic field/



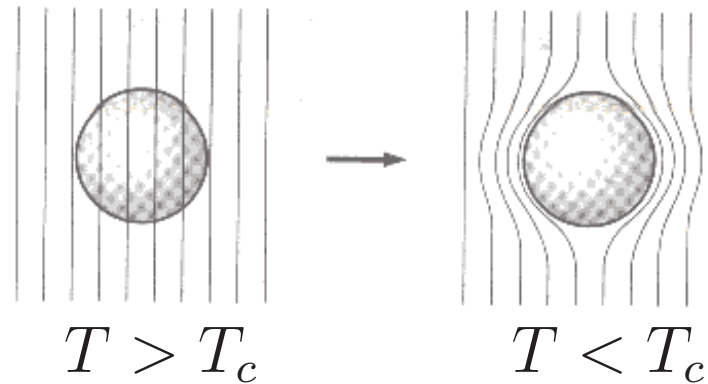
Superconducting state

– properties (continued)



ideal diamagnetism

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Meissner effect is described
by the Londons' equation

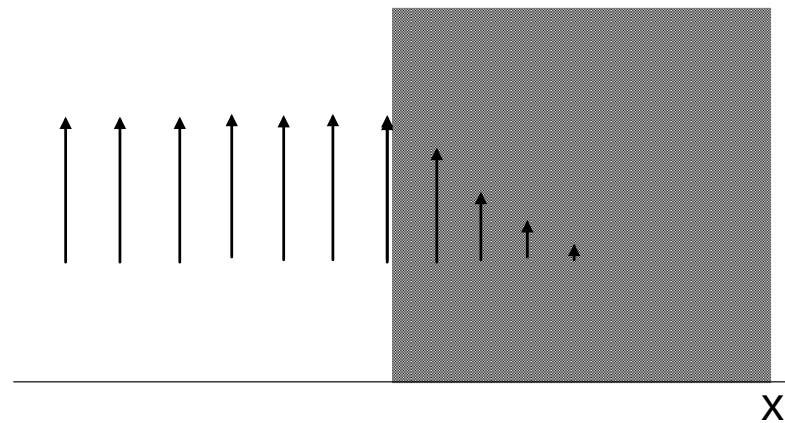
$$\vec{j} = - \frac{e^2 n_s(T)}{mc^2} \vec{A}$$

where the coefficient

$$\frac{e^2 n_s(T)}{mc^2} \equiv \rho_s(T) = \frac{1}{\lambda^2}$$

$\rho_s(T)$ – superfluid stiffness

$\lambda(T)$ – penetration depth



$$B(x) = B_0 e^{-x/\lambda}$$

Superconducting state

– **basic concepts**

Superconducting state – basic concepts

⇒ ideal d.c. conductance

Superconducting state

– basic concepts

⇒ ideal d.c. conductance

⇒ ideal diamagnetism (Meissner effect)

Superconducting state

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can be regarded as two sides of the same coin.

Superconducting state – basic concepts

⇒ ideal d.c. conductance

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can be regarded as two sides of the same coin.

Both effects are caused by the **superfluid** fraction

$$n_s(T)$$

Such superfluid fraction marks the coherence between paired electrons.

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/ classical superconductors, MgB_2 , ... /

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.. **other exotic processes**

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Onset of the fermion pairing often goes hand in hand with appearance of the **superconductivity/superfluidity** but it doesn't have to be a rule.

Formal issues

– generalities

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The order parameter

$$\chi \equiv \langle \hat{c}_{\downarrow}(\vec{r}_i) \hat{c}_{\uparrow}(\vec{r}_j) \rangle$$

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It has the following physical implications:

$$|\chi| \neq 0 \longrightarrow \text{amplitude causes the energy gap}$$

$$\nabla \theta \neq 0 \longrightarrow \text{phase slippage induces supercurrents}$$

Critical temperature

– **classification**

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Critical temperature – classification

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1. closing the gap [conventional BCS superconductors]

$$\lim_{T \rightarrow T_c} |\chi| = 0$$

Critical temperature

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The complex order parameter

$$\chi = |\chi| e^{i\theta}$$

can vanish at $T \rightarrow T_c$ by:

1. closing the gap [conventional BCS superconductors]

$$\lim_{T \rightarrow T_c} |\chi| = 0$$

2. disordering the phase [HTSC compounds, URh₂Si₂ (?)]

$$\lim_{T \rightarrow T_c} \langle \theta \rangle = 0$$

Amplitude vs phase driven transition

scenario # 1

Amplitude vs phase driven transition

scenario # 1



amplitude transition / *classical superconductors* /

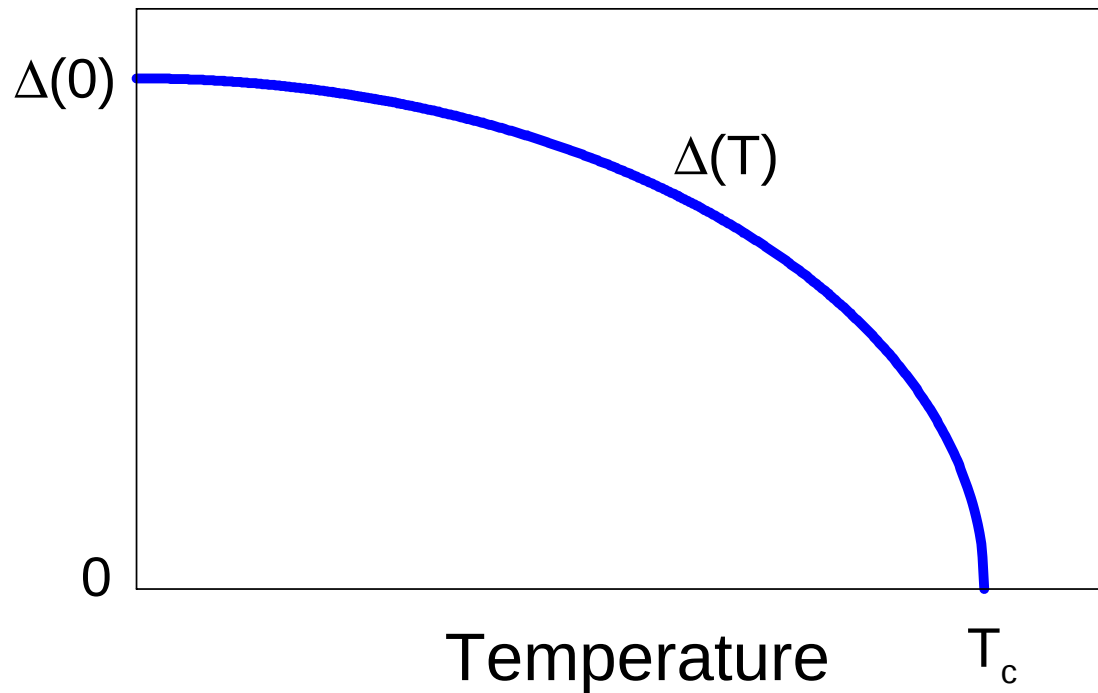
$$k_B T_c \simeq \frac{\Delta(0)}{1.76}$$

Amplitude vs phase driven transition

scenario # 1

⇒ **amplitude transition** / *classical superconductors* /

$$k_B T_c \simeq \frac{\Delta(0)}{1.76}$$



*Electrons' pairing
is responsible for
the energy gap
 $\Delta(T)$ in a single
particle spectrum*

$$\Delta(T_c) = 0$$

Appearance of the electron pairs is simultaneous with onset of their coherence

Amplitude vs phase driven transition

scenario # 2

Amplitude vs phase driven transition

scenario # 2

⇒ phase-driven transition / *high T_c cuprate oxides* /

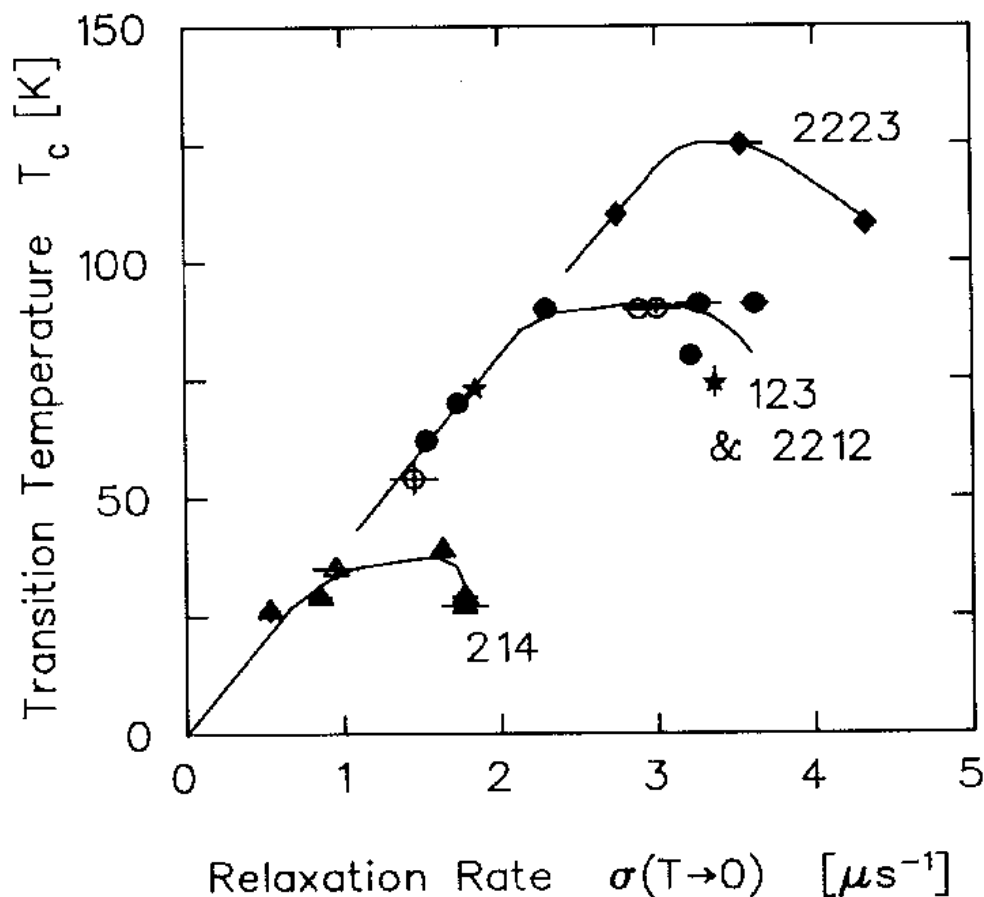
$$T_c \not\propto \Delta(0)$$

Amplitude vs phase driven transition

scenario # 2

⇒ **phase-driven transition** / high T_c cuprate oxides /

$$T_c \not\propto \Delta(0)$$



Early experiments using the muon-spin relaxation indicated that in HTSC

$$T_c \propto \rho_s(0)$$

/ Uemura scaling /

The superfluid stiffness $\rho_s(T)$ is here defined by

$$\rho_s(T) \equiv \frac{1}{\lambda^2(T)} = \frac{4\pi e^2}{m^* c^2} n_s(T)$$

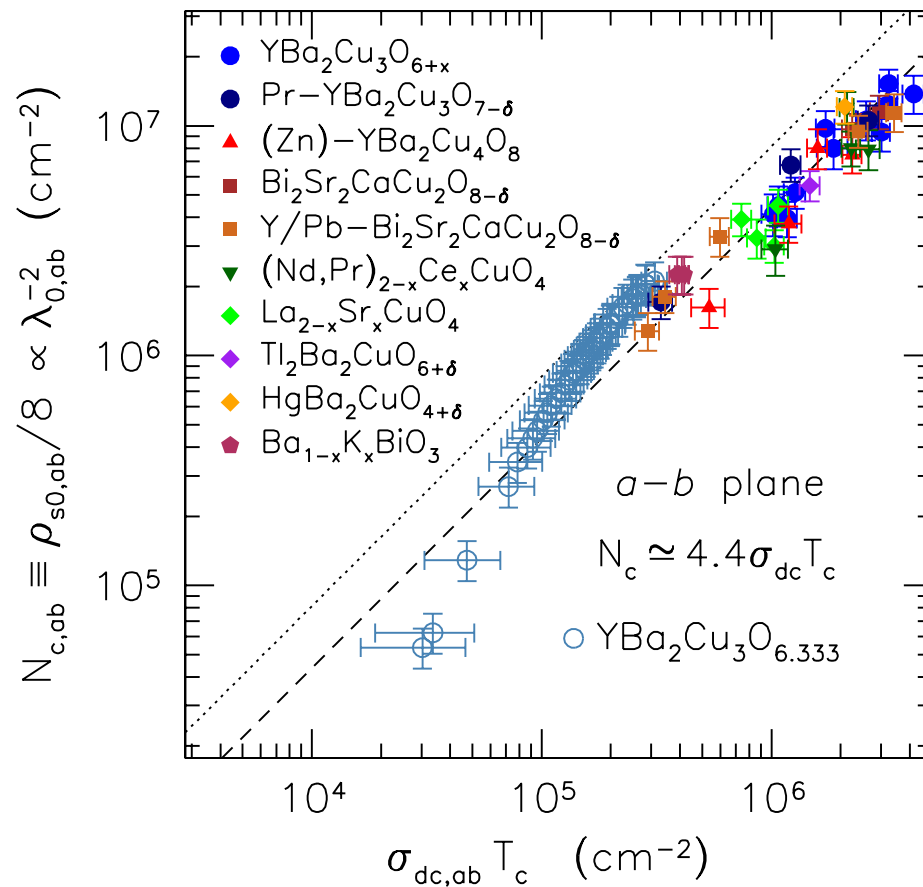
Y.J. Uemura et al, Phys. Rev. Lett. **62**, 2317 (1989).

Amplitude vs phase driven transition

scenario # 2

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Recently such scaling
has been updated from
transport measurements

$$\frac{1}{8}\rho_s = 4.4\sigma_{dc} T_c$$

/ **Homes scaling** /

This new relation is valid for
all samples ranging from the
underdoped to overdoped region.

C.C. Homes, *Phys. Rev. B* **80**, 180509(R) (2009).

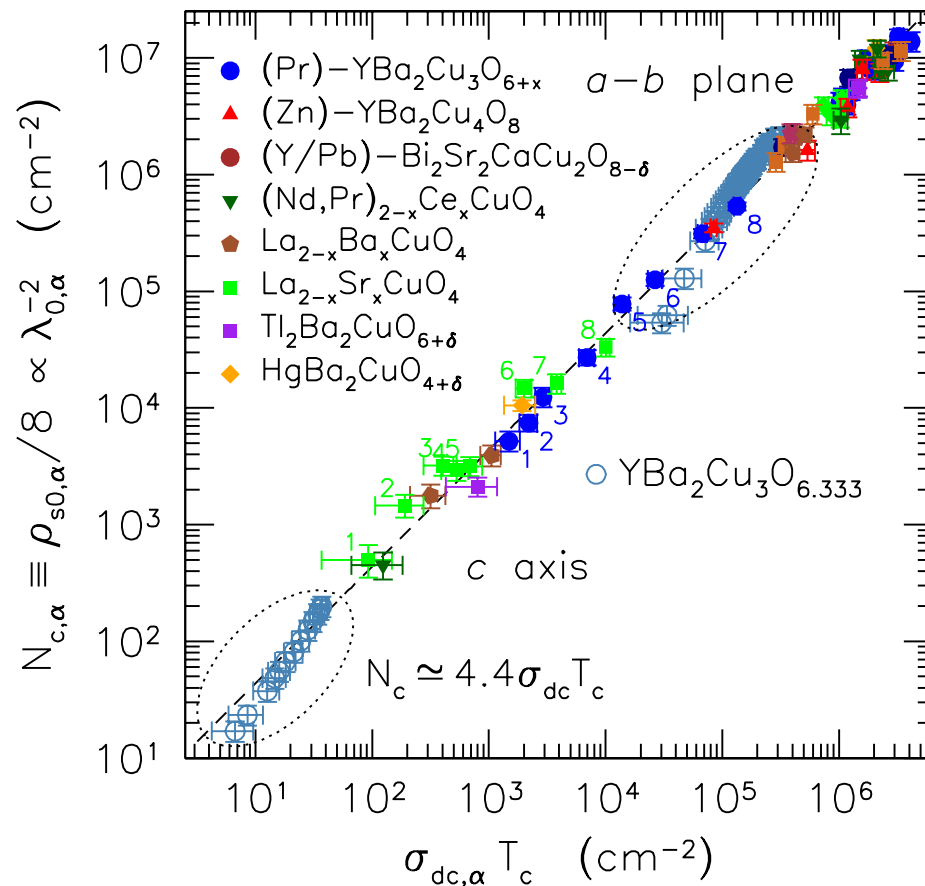
/ **ab - plane** /

Amplitude vs phase driven transition

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/ c - axis /

2. Pre-pairing

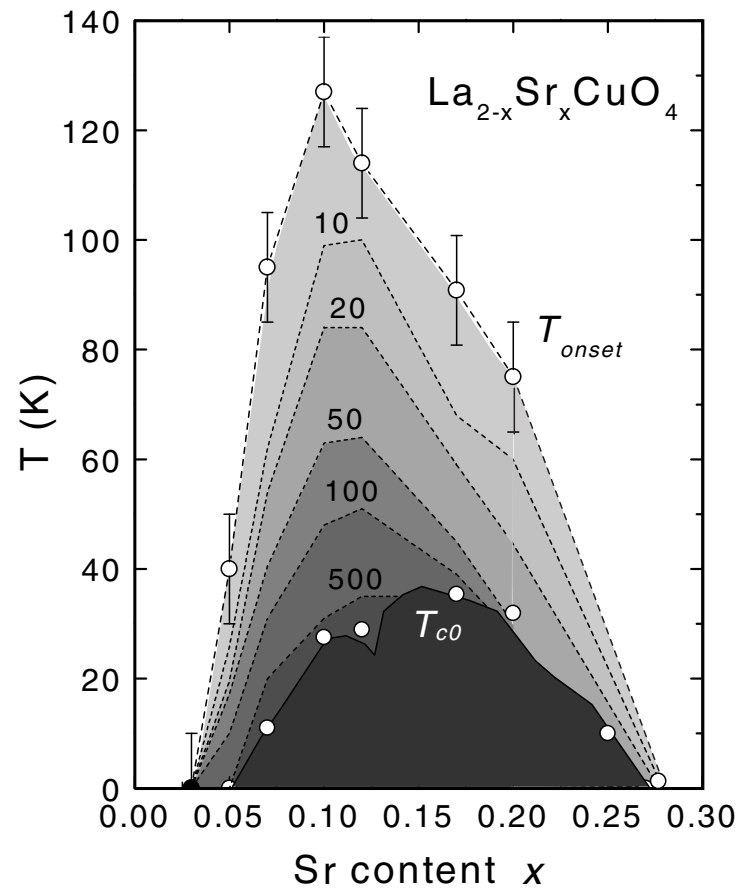
/experimental evidence/

Incoherent pairs above T_c

experimental fact # 1

Incoherent pairs above T_c

experimental fact # 1

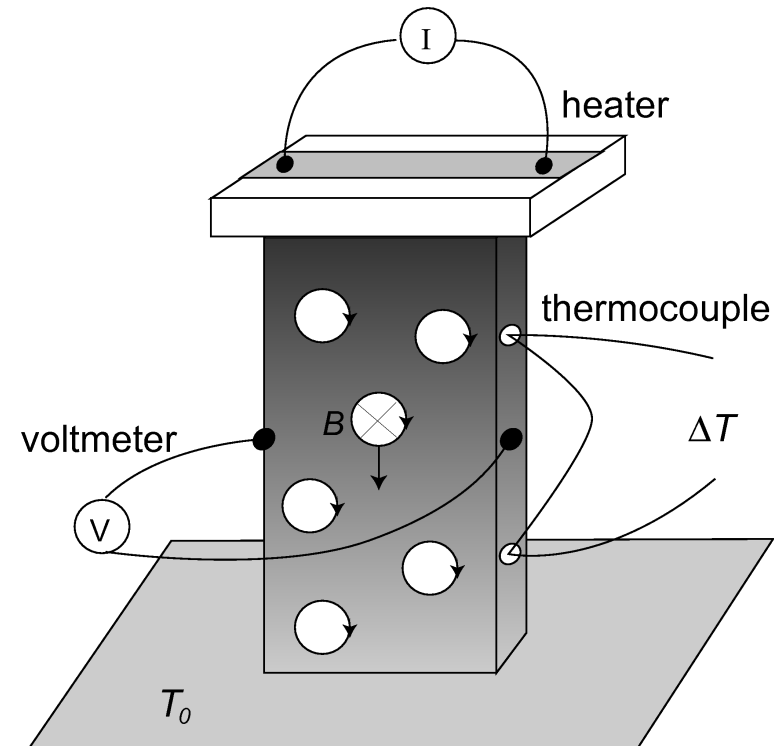
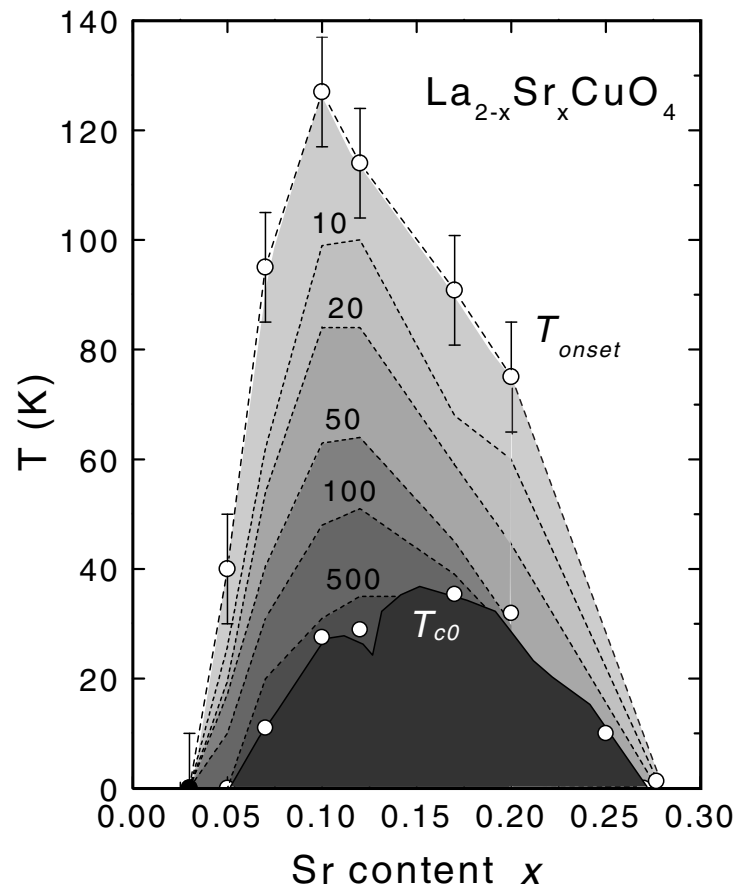


Phase slippage detected in the large Nernst effect.

Y. Wang et al, *Science* **299**, 86 (2003).

Incoherent pairs above T_c

experimental fact # 1



Phase slippage detected in the large Nernst effect.

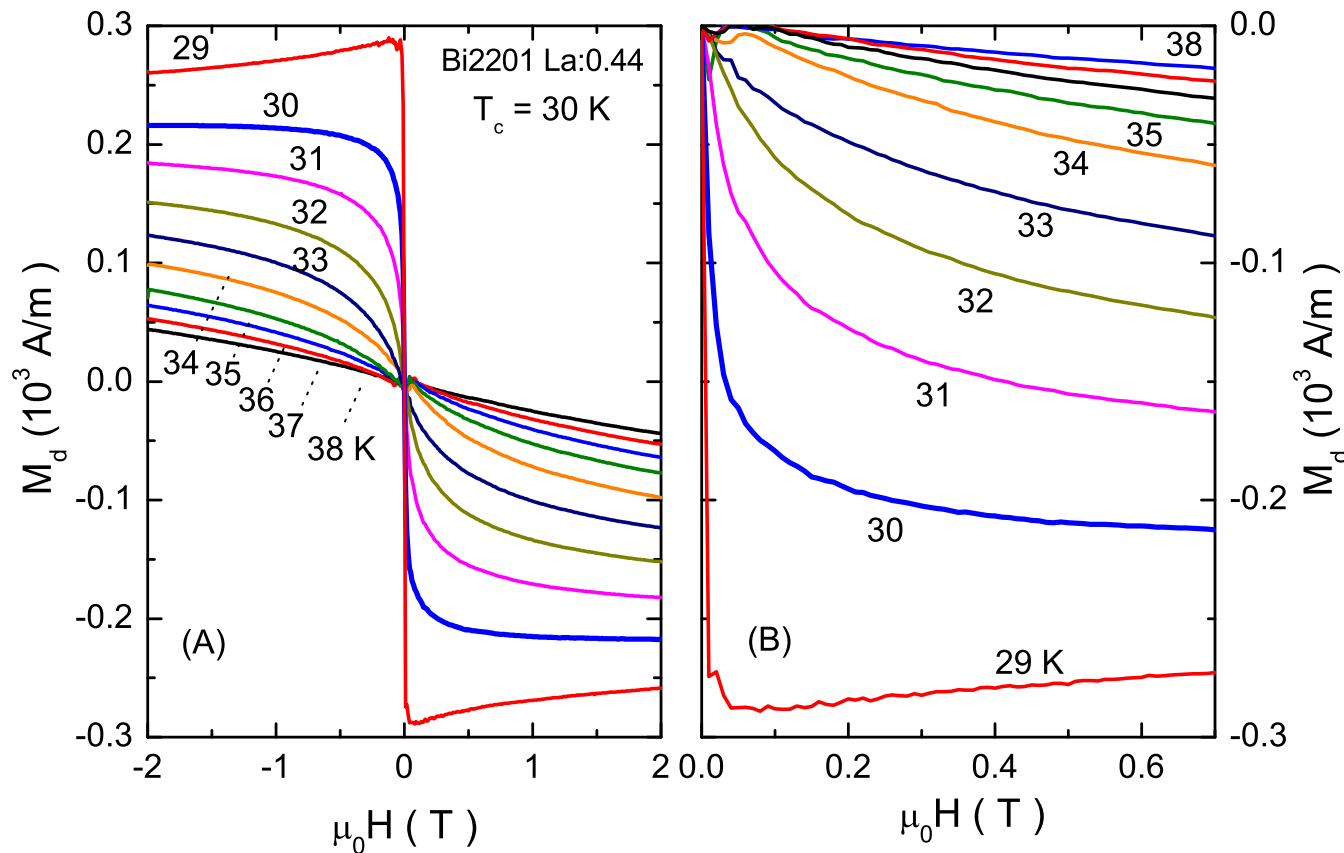
Y. Wang et al, *Science* **299**, 86 (2003).

Incoherent pairs above T_c

experimental fact # 2

Incoherent pairs above T_c

experimental fact # 2



Van Vleck
background
 $(A + BT)H$
is subtracted

$T_c = 30$ K

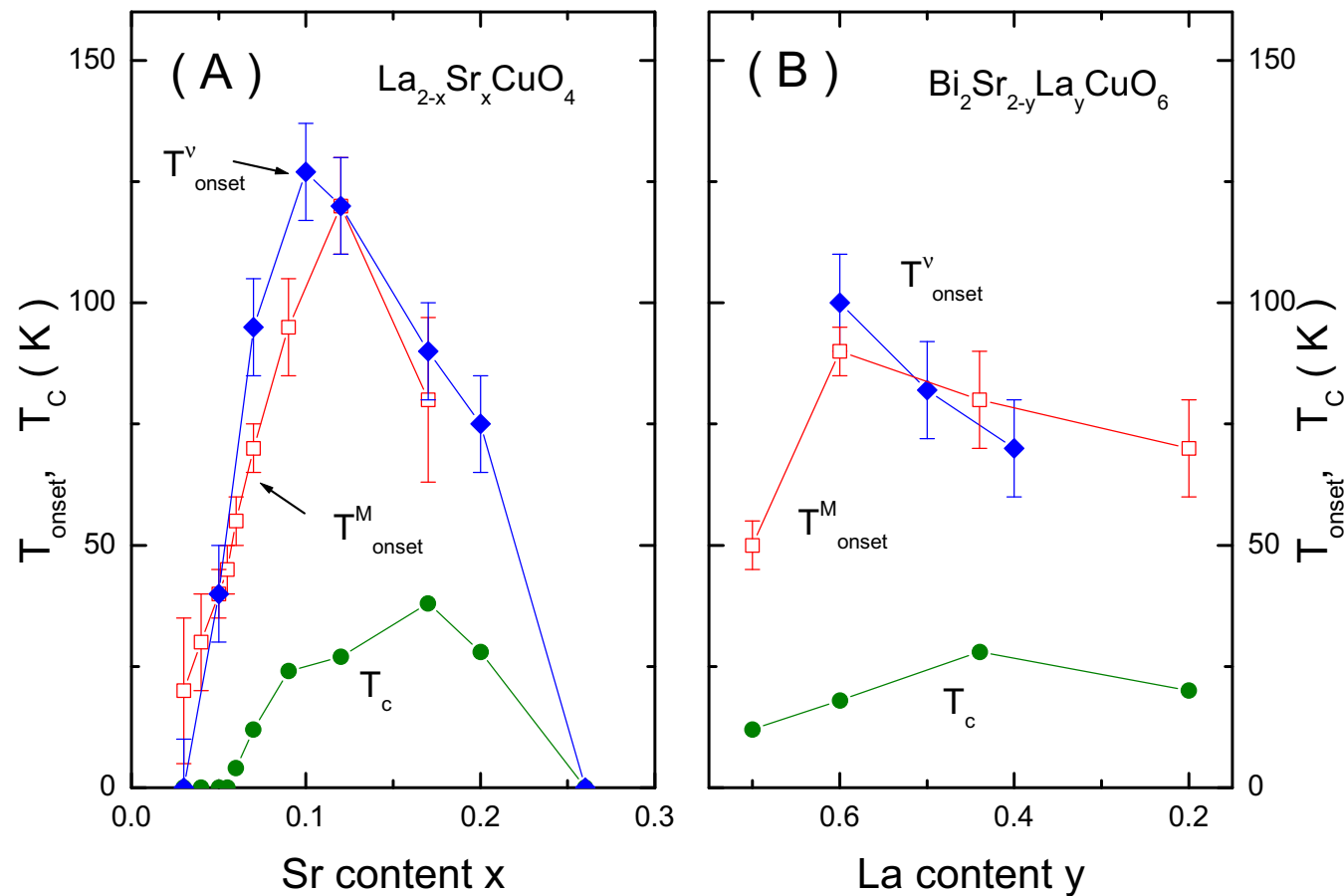
Enhanced diamagnetic response revealed above T_c
by the high precision torque magnetometry.

Incoherent pairs above T_c

experimental fact # 1 + 2

Incoherent pairs above T_c

experimental fact # 1 + 2



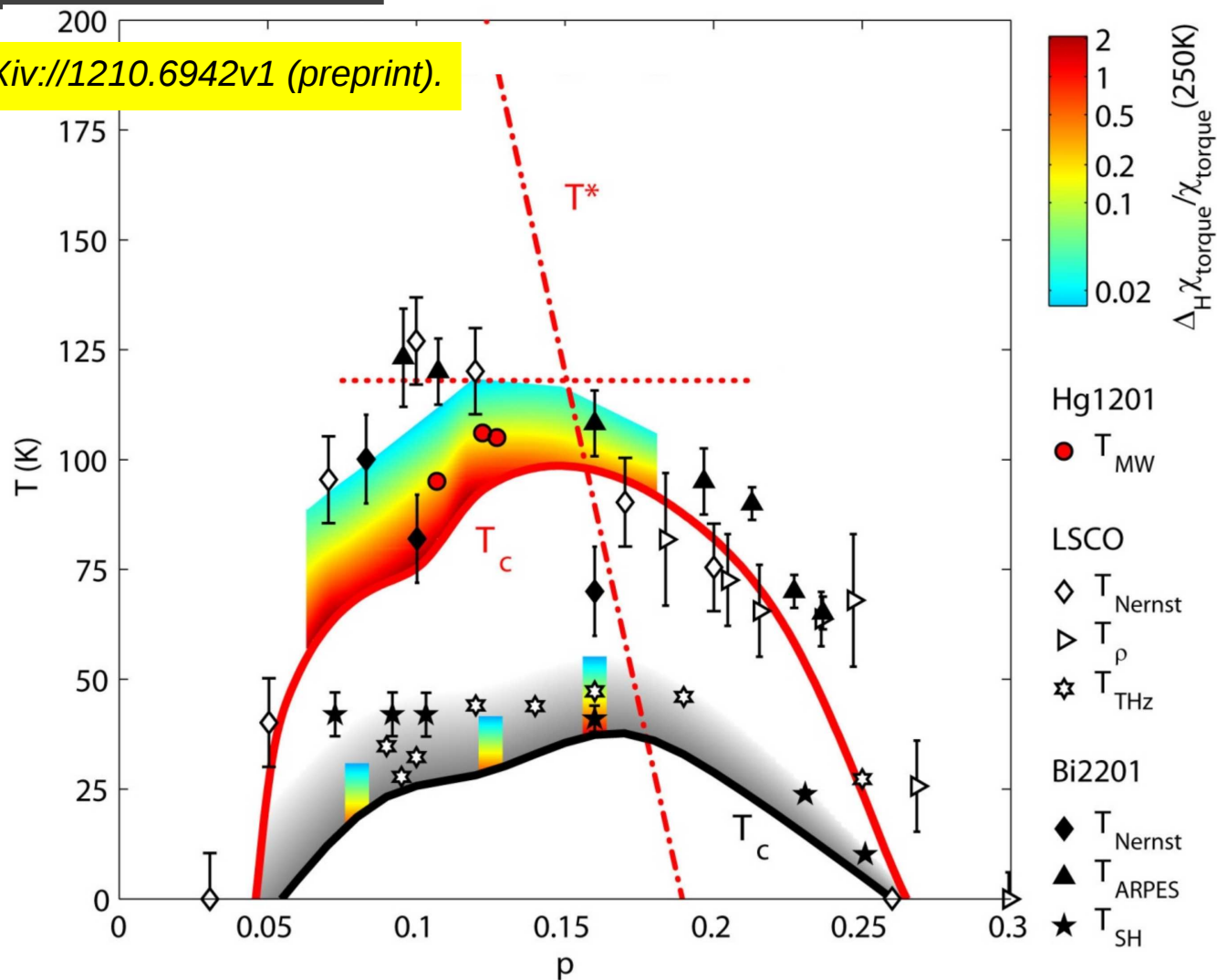
T^v – onset of the Nernst effect
 T^M – onset of the diamagnetism

L. Li, ... and N.P. Ong, *Phys. Rev. B* **81**, 054510 (2010).

Incoherent pairs above T_c

experimental fact # 1 + 2

G. Yu et al, arXiv://1210.6942v1 (preprint).



T_c – onset of the sc fluctuations

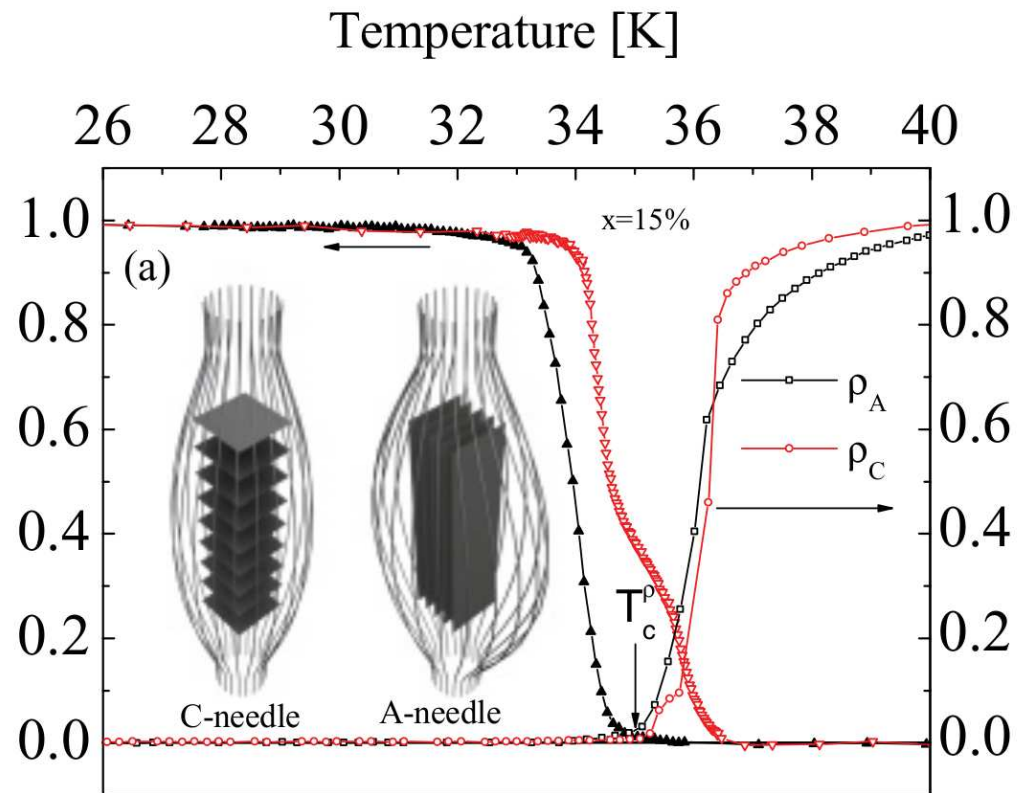
T_c – onset of the superconductivity

Incoherent pairs above T_c

experimental fact # 3

Incoherent pairs above T_c

experimental fact # 3



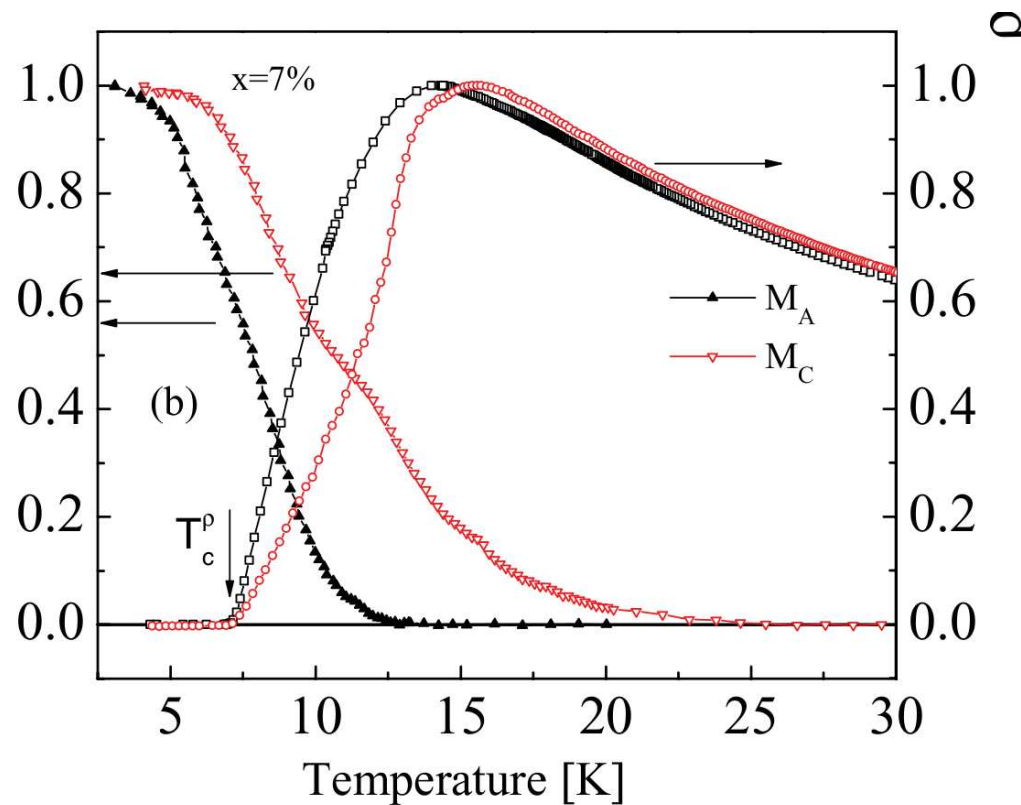
G. Drachuck et al, *Phys. Rev. B* **85**, 184518 (2012).

/ Technion Group /

Magnetization measurements for the needle
shape $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ single crystals.

Incoherent pairs above T_c

experimental fact # 3



G. Drachuck et al, *Phys. Rev. B* **85**, 184518 (2012).

/ Technion Group /

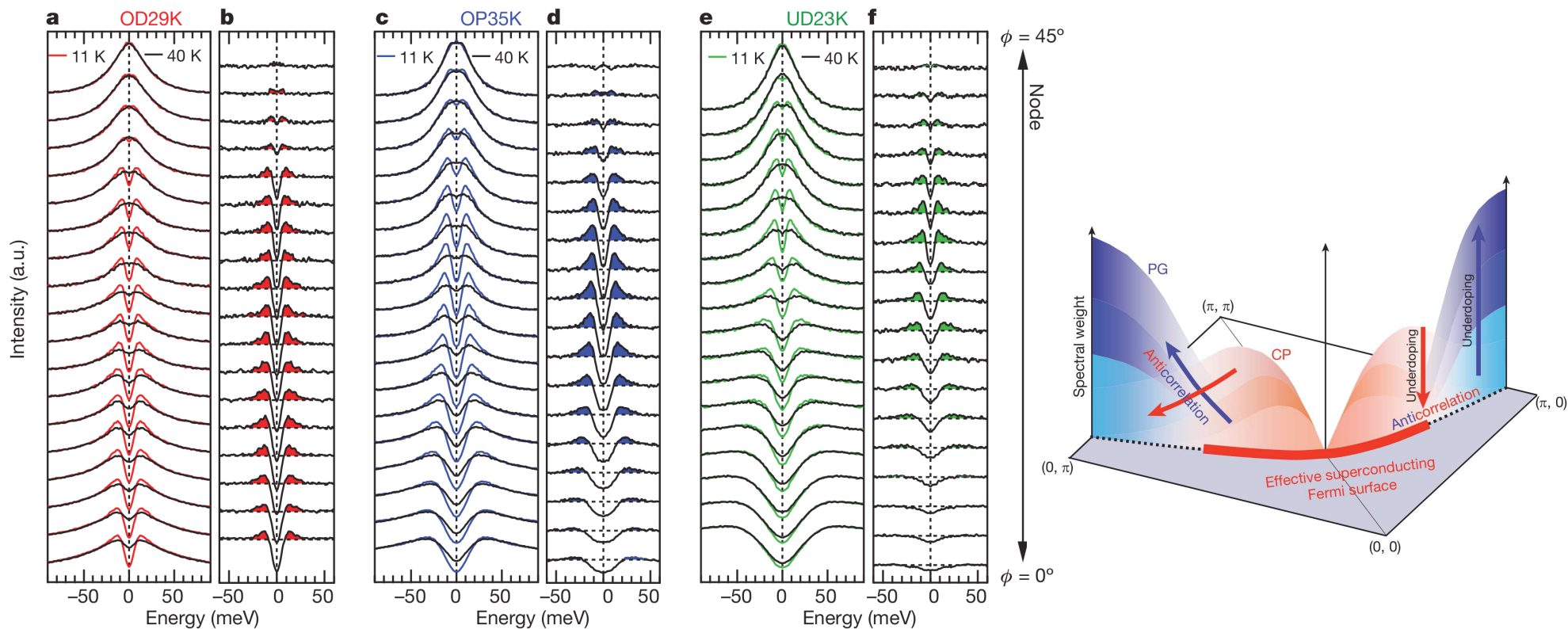
Magnetization measurements for the needle
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Incoherent pairs above T_c

experimental fact # 4

Incoherent pairs above T_c

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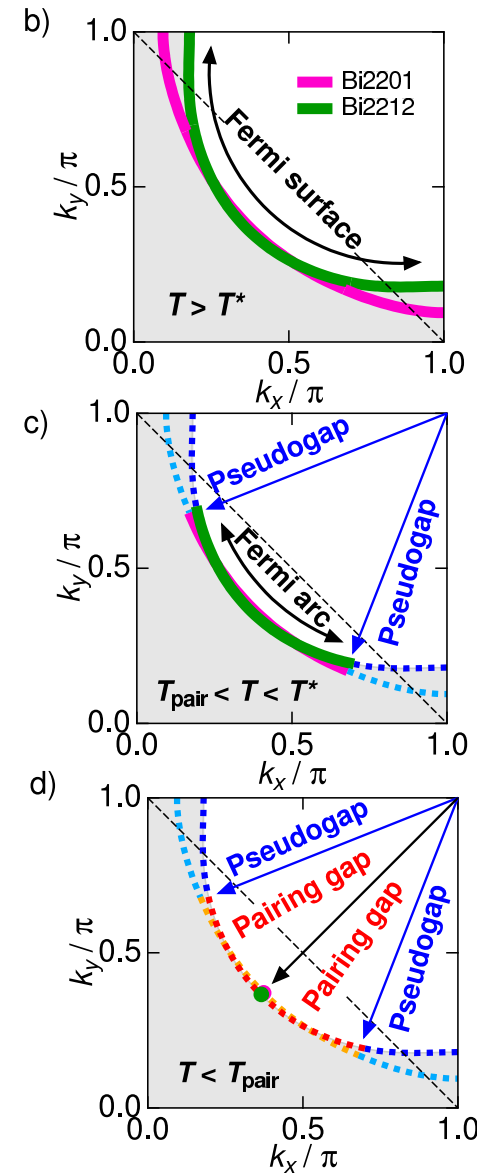
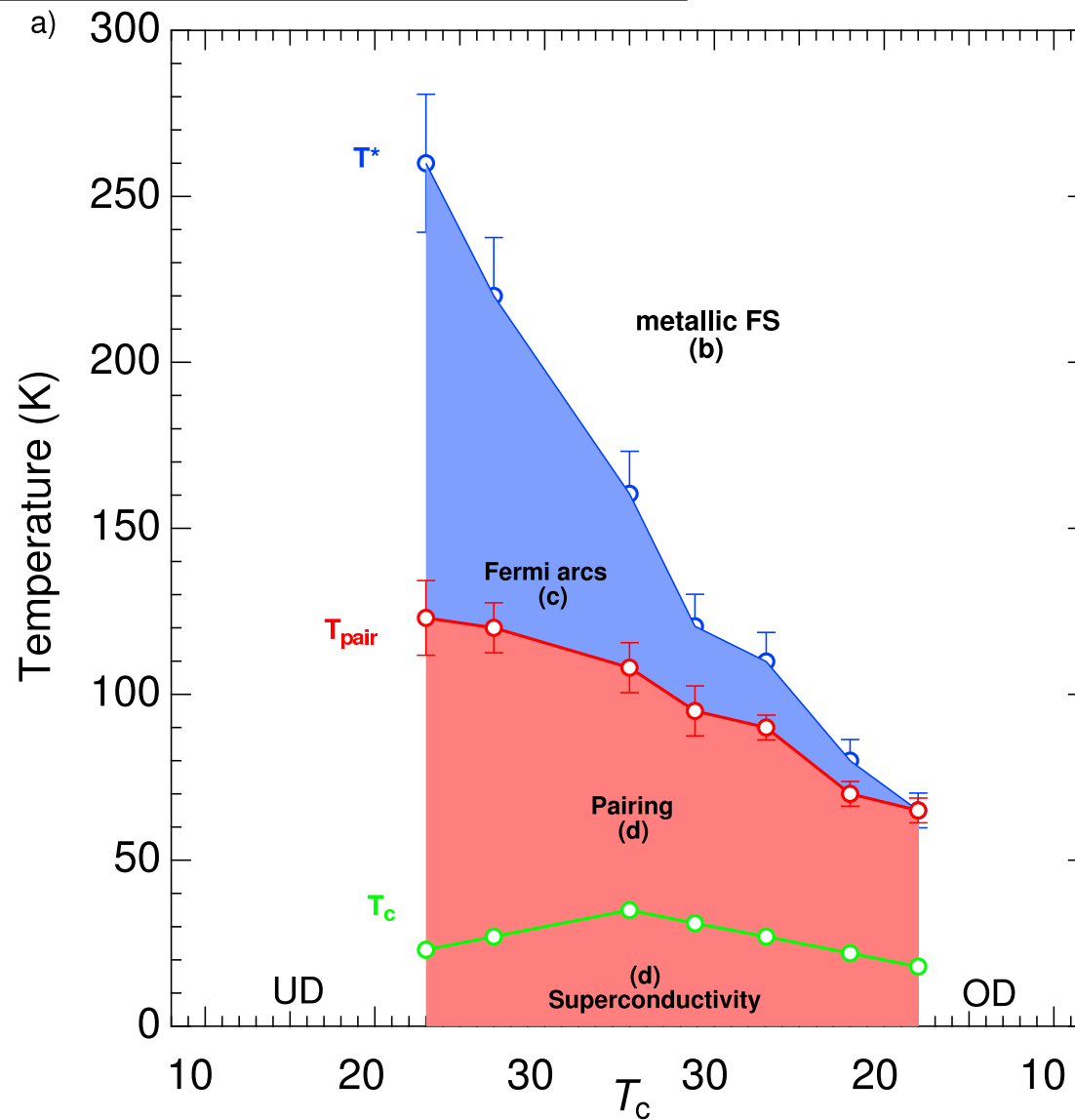


Transfer of the spectral weight in the single particle spectrum.

T. Kondo, R. Khasanov, T. Takeuchi, J. Schmalian & A. Kamiński, Nature **457**, 296 (2009).

Incoherent pairs above T_c

experimental fact # 4

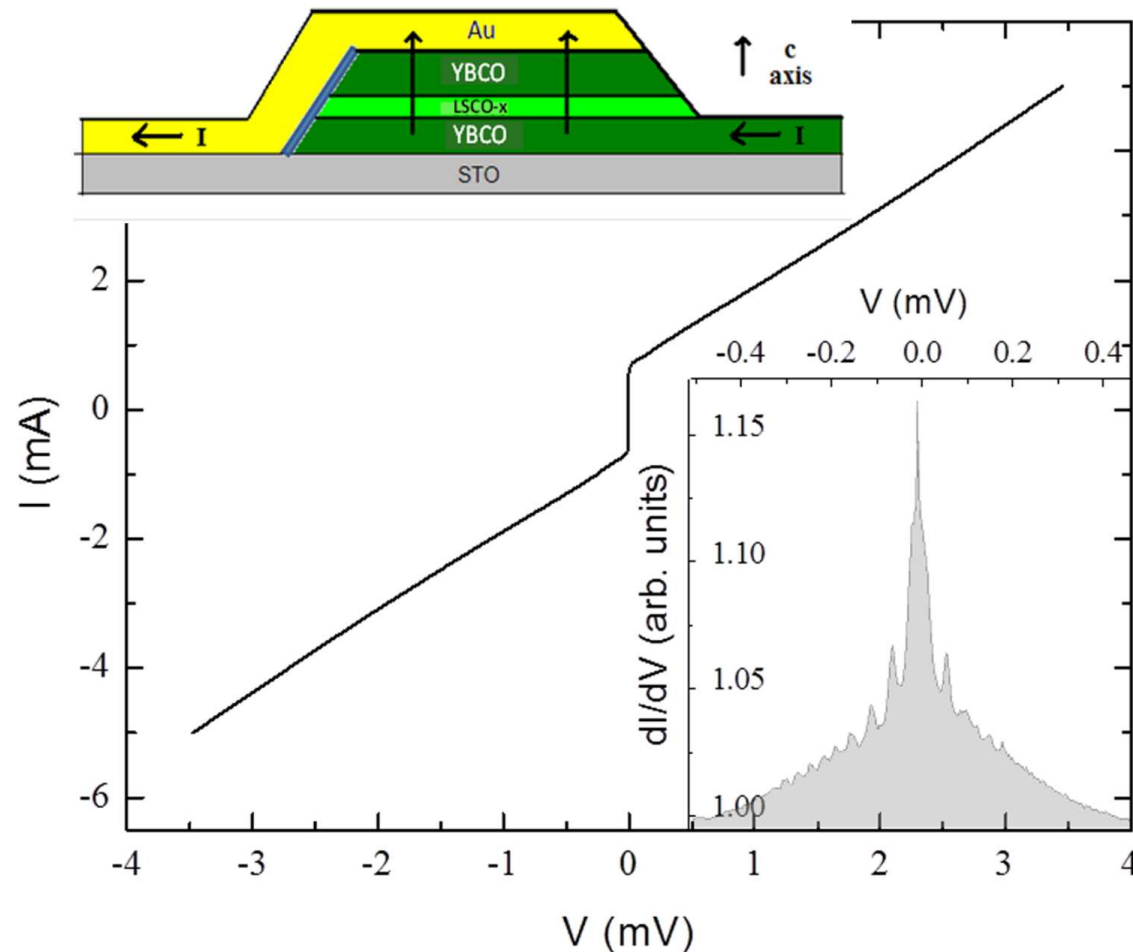


Incoherent pairs above T_c

experimental fact # 5

Incoherent pairs above T_c

experimental fact # 5

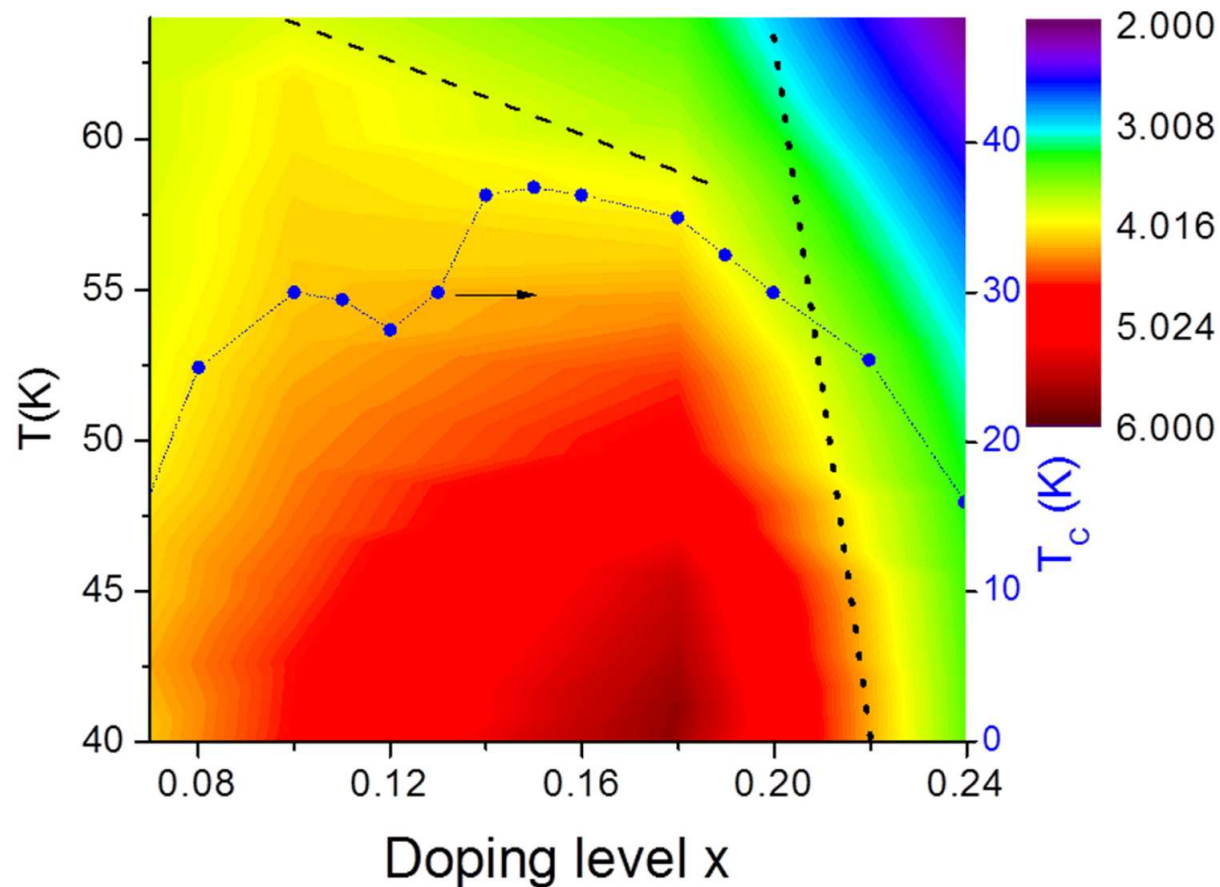


Josephson current in YBaCuO - LaSrCuO - YBaCuO junction with LaSrCuO being in the pseudogap state well above T_c

T. Kirzhner and G. Koren, Scientific Reports 4, 6244 (2014).

Incoherent pairs above T_c

experimental fact # 5



Phase diagram for the coherence length $\xi(T, x)$ appearing in the empirical law $I_c \propto \exp[-d/\xi]$ for Josephson current I_c .

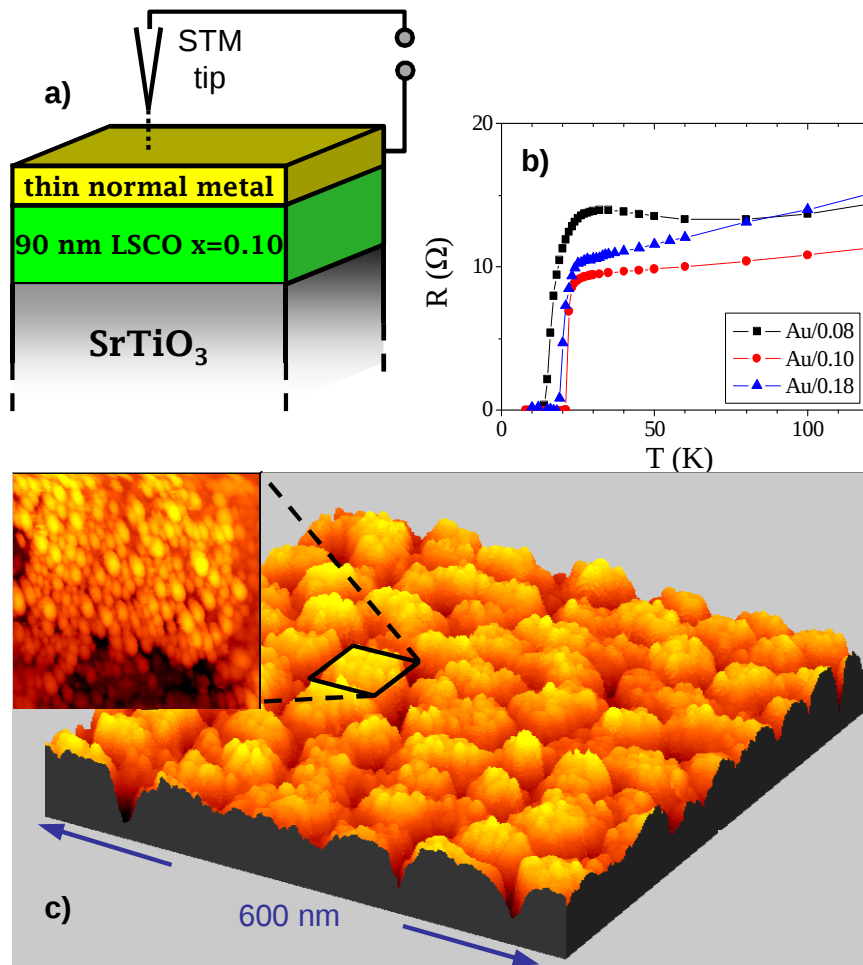
T. Kirzhner and G. Koren, Scientific Reports 4, 6244 (2014).

Incoherent pairs above T_c

experimental fact # 6

Incoherent pairs above T_c

experimental fact # 6



Pseudogap induced
well above T_c
in ultrathin metallic
slab deposited on
 $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$.

O. Yuli et al, *Phys. Rev. Lett.* **103**, 197003 (2009).

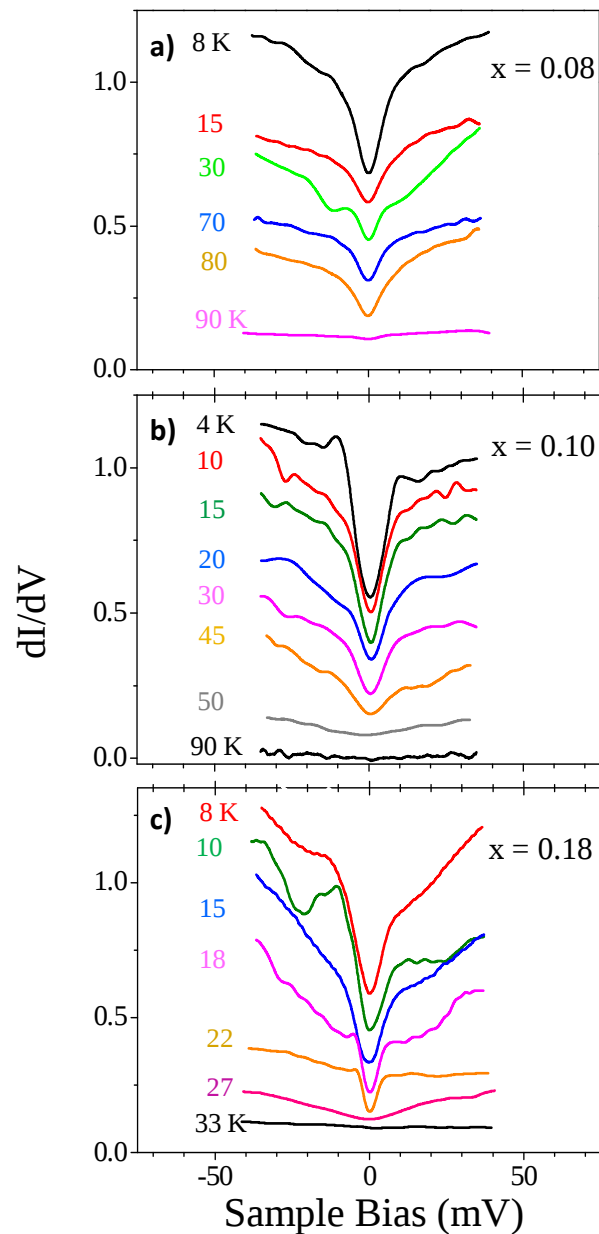
Incoherent pairs above T_c

experimental fact # 6

Measured DOS $\longrightarrow$

Proximity
induced
pseudogap
observed
far above T_c .

O. Yuli et al, PRL **103**, 197003 (2009).

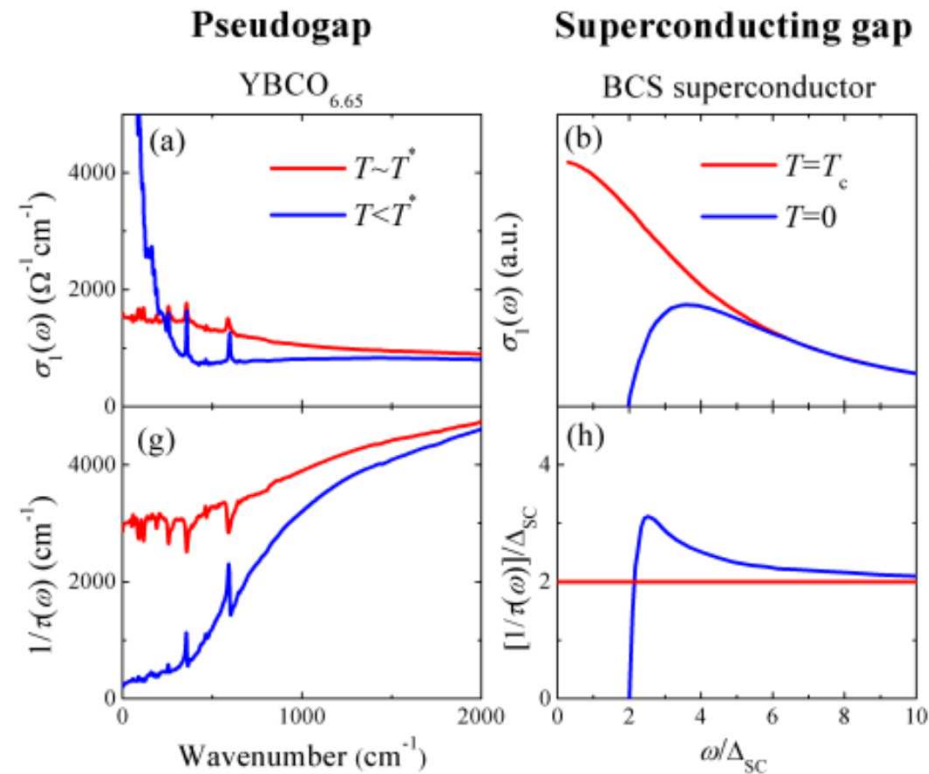


Incoherent pairs above T_c

experimental fact # 7

Incoherent pairs above T_c

experimental fact # 7

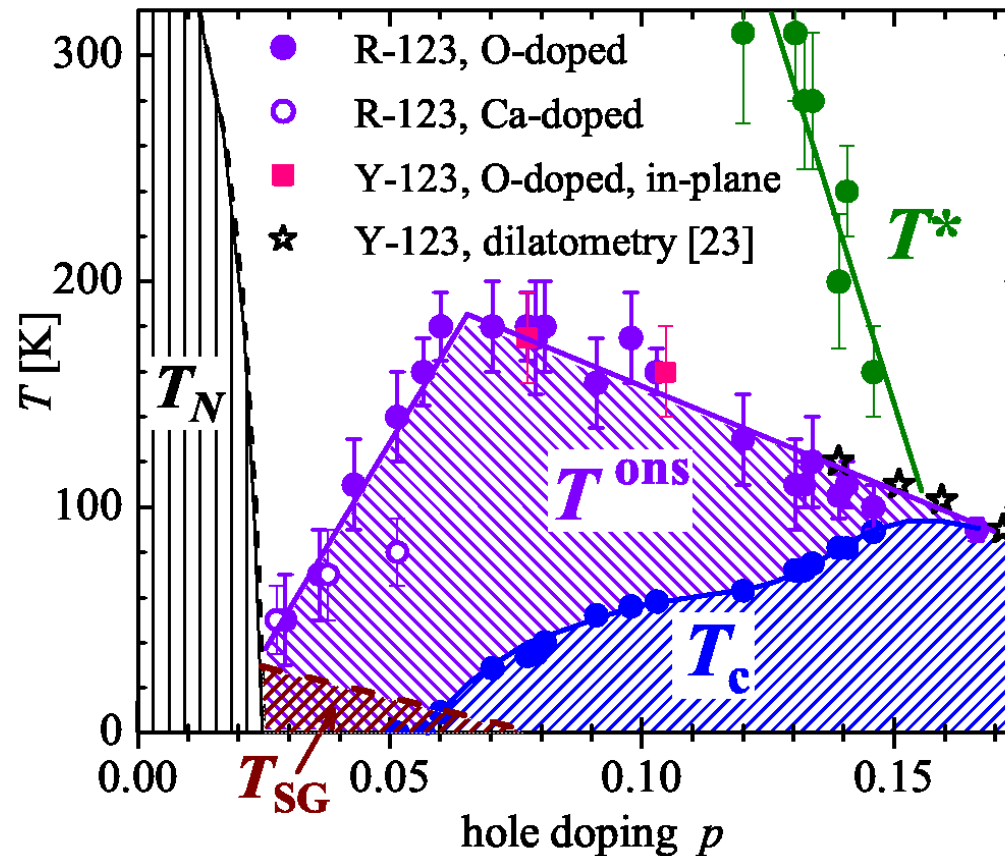


S.J. Moon et al, *Phys. Rev. B* **90**, 014503 (2014).

Schematic view of the optical conductance in the pseudogap state of YBaCuO superconductor compared with conventional BCS materials.

Incoherent pairs above T_c

experimental fact # 7



A. Dubroka et al, *Phys. Rev. Lett.* **106**, 047006 (2011).

Onset of the superfluid fraction observed in the c-axis optical measurements $\text{Re}\sigma_c(\omega)$.

Incoherent pairs above T_c

... *continued*

Incoherent pairs above T_c

... continued

⇒ **Josephson-like features seen above T_c in the tunneling**

N. Bergeal et al, Nature Phys. 4, 608 (2008).

Incoherent pairs above T_c

... continued

⇒ **Josephson-like features seen above T_c in the tunneling**

N. Bergeal et al, Nature Phys. 4, 608 (2008).

⇒ **Bogoliubov quasiparticles detected above T_c in ARPES measurements for YBaCuO and LaSrCuO compounds**

Argonne (2008), Villigen (2009).

Incoherent pairs above T_c

... continued

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N. Bergeal et al, Nature Phys. 4, 608 (2008).

⇒ Bogoliubov quasiparticles detected above T_c in ARPES measurements for YBaCuO and LaSrCuO compounds

Argonne (2008), Villigen (2009).

⇒ Bogoliubov-type interference patterns in pseudogap state as the *fingerprint* of phase incoherent d-wave superconductivity preserved up to $1.5 T_c$

J. Lee, ... and J.C. Davis, Science 325, 1099 (2009).

3. Selected results

/ below and above T_c /

Boson and fermion degrees of freedom

[spatial representation]

$$\begin{aligned}\hat{H} &= \sum_{i,j,\sigma} (t_{ij} - \mu \delta_{i,j}) \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \sum_l \left(E_l^{(B)} - 2\mu \right) \hat{b}_l^\dagger \hat{b}_l \\ &+ \sum_{i,j} g_{ij} \left[\hat{b}_l^\dagger \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.} \right]\end{aligned}$$

Boson and fermion degrees of freedom

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$$\vec{R}_l = (\vec{r}_i + \vec{r}_j)/2$$

Boson and fermion degrees of freedom

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This Hamiltonian describes a two-component system consisting of:

Boson and fermion degrees of freedom

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This Hamiltonian describes a two-component system consisting of:

$\hat{c}_{i\sigma}^{(\dagger)}$ itinerant fermions (e.g. holes near the Mott insulator)

Boson and fermion degrees of freedom

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$\hat{b}_l^{(\dagger)}$ immobile local pairs (RVB defines them on the bonds)

Boson and fermion degrees of freedom

[spatial representation]

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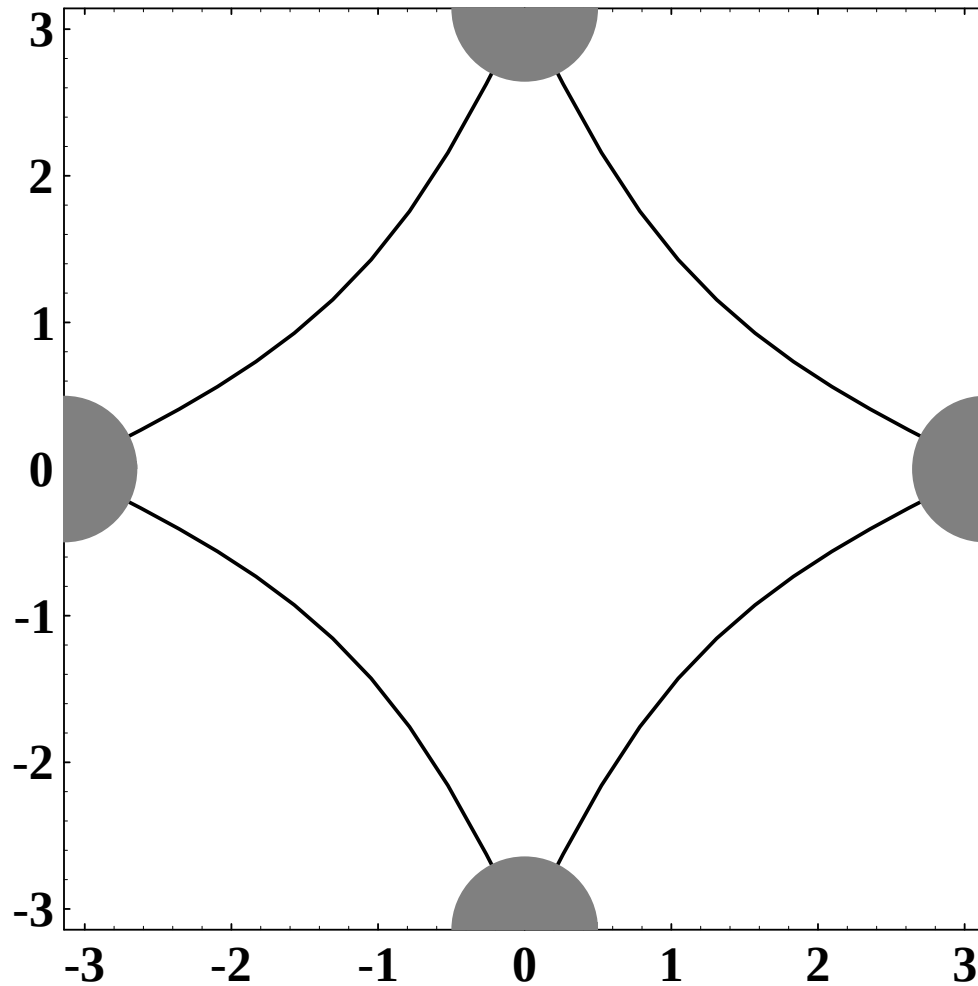
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Phenomenological arguments

– supporting BF scenario

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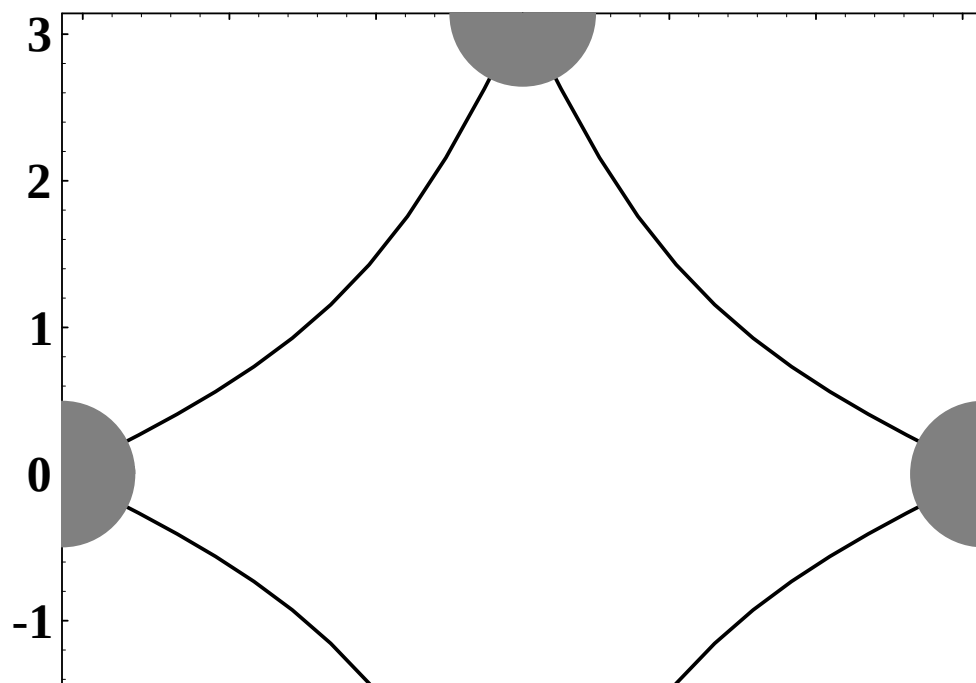


paired & unpaired carriers

V.G. Geshkenbein, L.B. Ioffe, A.I. Larkin, *Phys. Rev. B* **55**, 3173 (1997).

Phenomenological arguments

– supporting BF scenario



paired & unpaired carriers

Received 27 Dec 2013 | Accepted 7 Jun 2014 | Published 11 Jul 2014

DOI: 10.1038/ncomms5353

OPEN

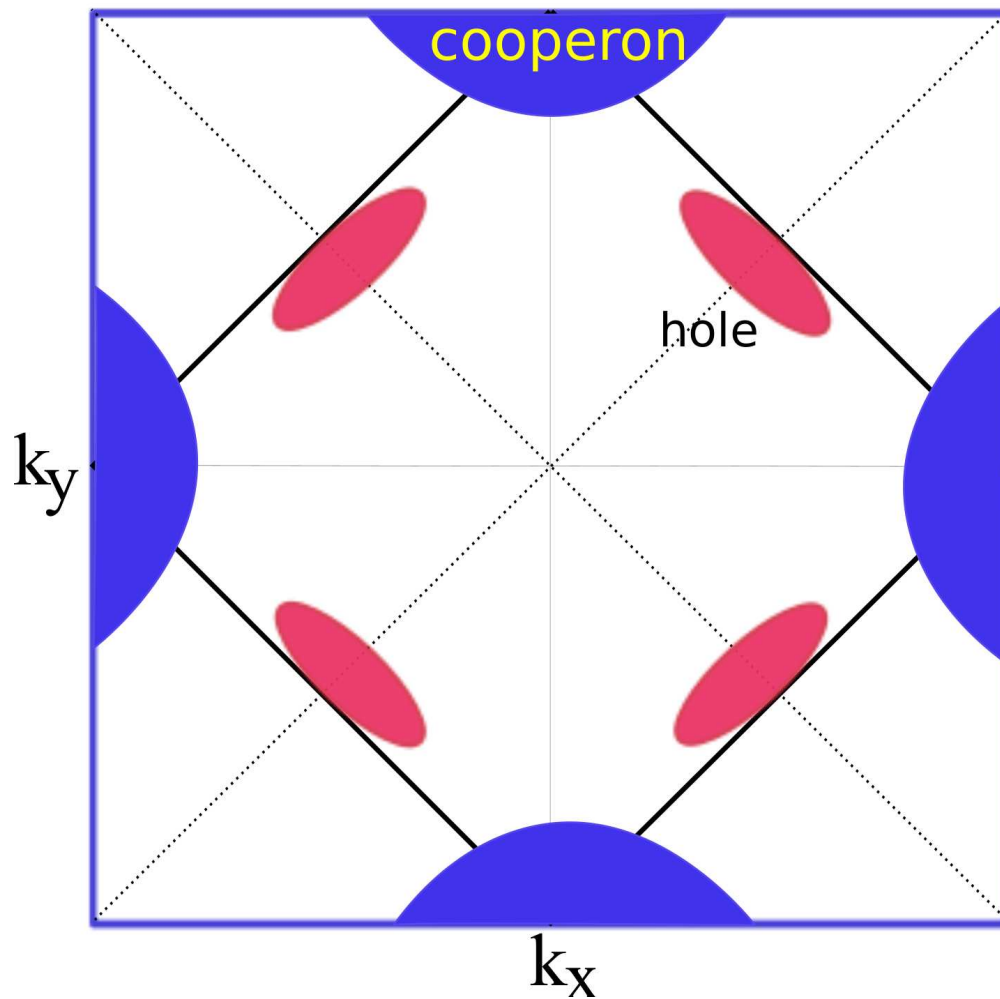
Photo-enhanced antinodal conductivity in the pseudogap state of high- T_c cuprates

F. Cilento¹, S. Dal Conte^{2,3,†}, G. Coslovich^{4,†}, S. Peli^{2,5}, N. Nembrini^{2,5}, S. Mor², F. Banfi^{2,3}, G. Ferrini^{2,3}, H. Eisaki⁶, M.K. Chan⁷, C.J. Dorow⁷, M.J. Veit⁷, M. Greven⁷, D. van der Marel⁸, R. Comin^{9,10}, A. Damascelli^{9,10}, L. Rettig^{11,†}, U. Bovensiepen¹¹, M. Capone¹², C. Giannetti^{2,3} & F. Parmigiani^{1,4}

Phenomenological arguments

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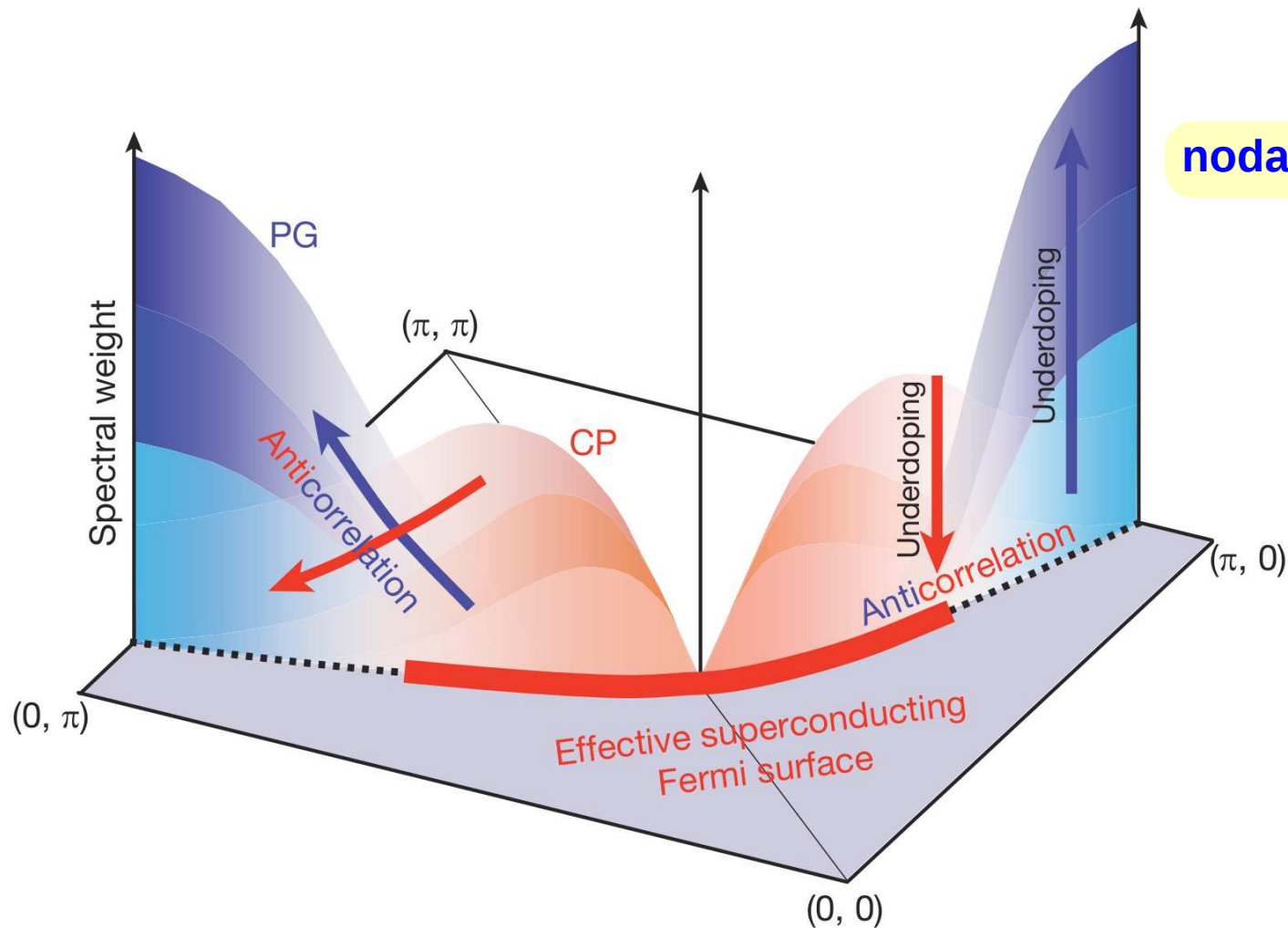
*K.-Y. Yang, E. Kozik, X. Wang, and M. Troyer, Phys. Rev. B **83**, 214516 (2011).*



cooperons & holes

Phenomenological arguments

– supporting BF scenario



nodal-antinodal dichotomy

T. Kondo, R. Khasanov, T. Takeuchi, J. Schmalian & A. Kamiński, Nature **457**, 296 (2009).

Bogoliubov quasiparticles

below and above T_c

BCS physics

The BCS ground state:

BCS physics

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$$|\text{BCS}\rangle = \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} e^{i\theta_{\mathbf{k}}} \hat{c}_{\mathbf{k}\uparrow}^{\dagger} \hat{c}_{-\mathbf{k}\downarrow}^{\dagger} \right) |\text{vac}\rangle$$

BCS physics

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where

$$|v_{\mathbf{k}} e^{i\theta_{\mathbf{k}}}|^2 = \frac{1}{2} \left[1 - \frac{\epsilon_{\mathbf{k}} - \mu}{\sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2}} \right] = 1 - |u_{\mathbf{k}}|^2$$

BCS excitation spectrum

BCS excitation spectrum

The effective (Bogoliubov) quasiparticles :

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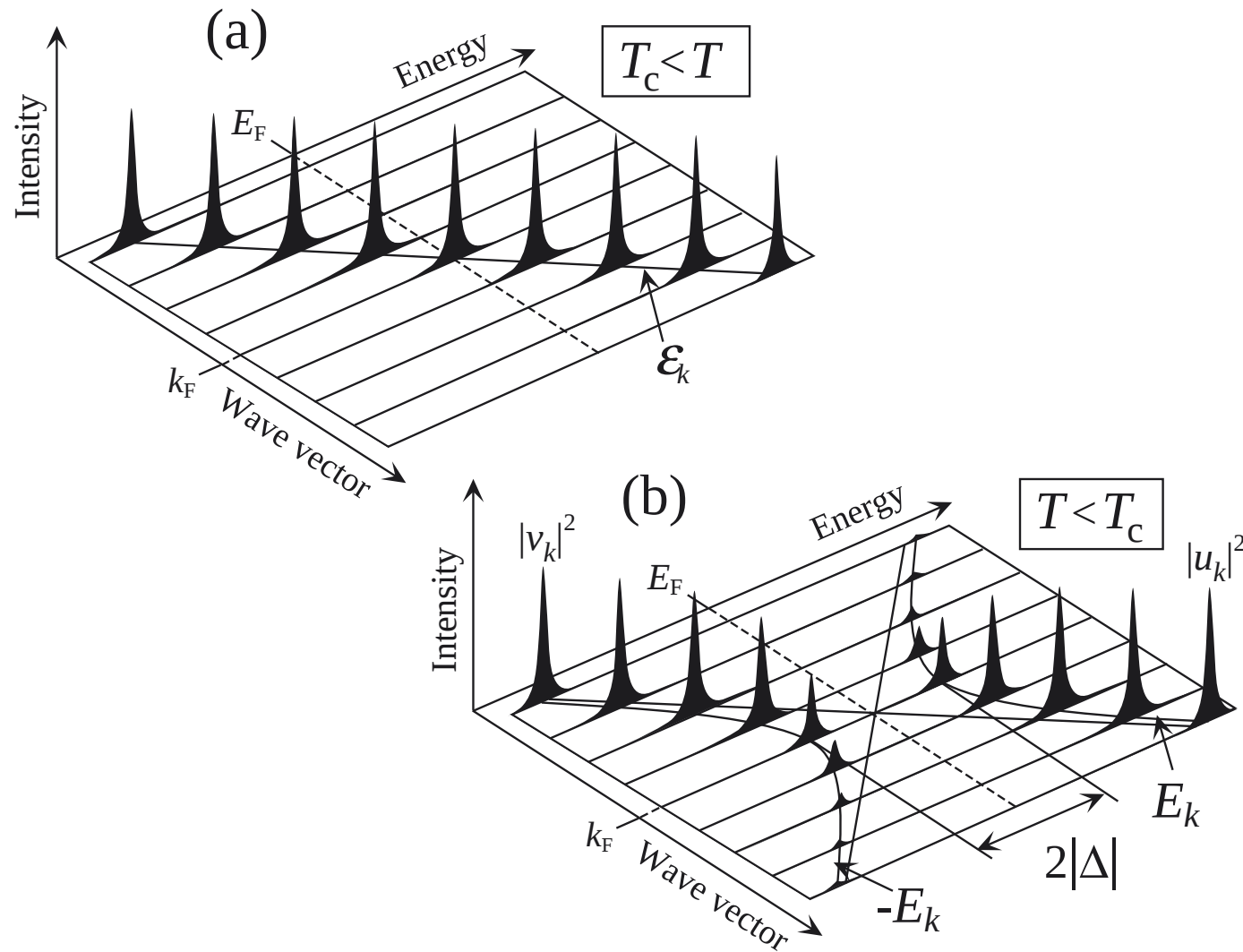
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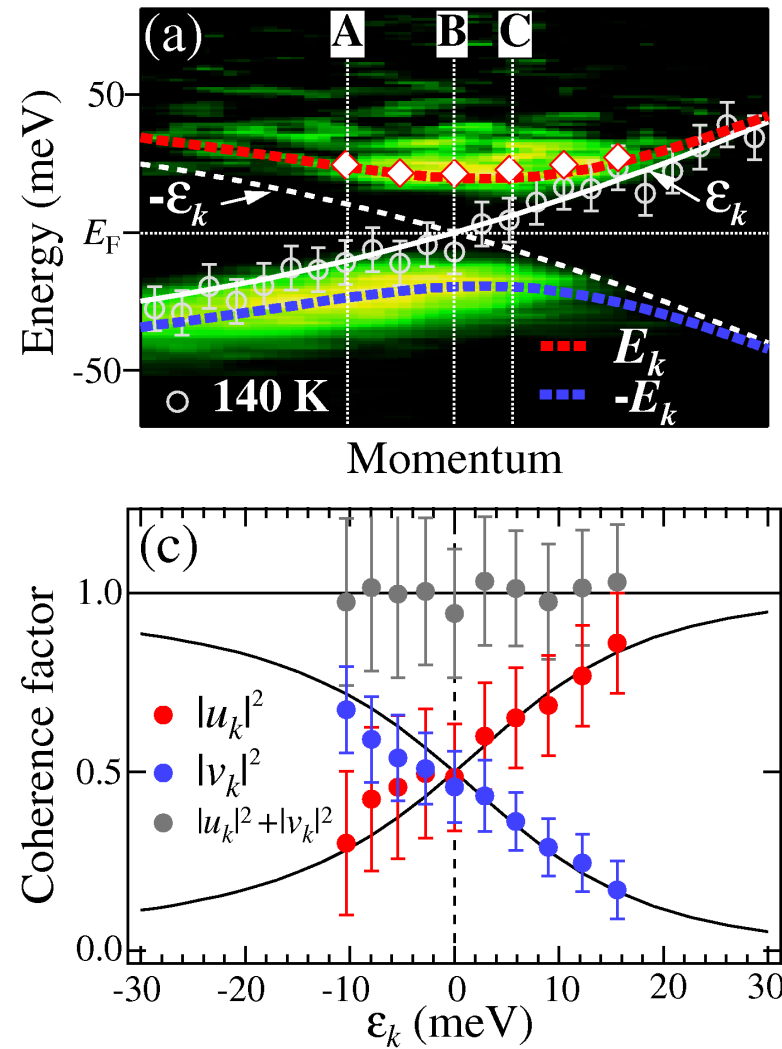
$$n_{\mathbf{k}} = |u_{\mathbf{k}}|^2 f_{FD}(E_{\mathbf{k}}) + |v_{\mathbf{k}}|^2 \underbrace{f_{FD}(-E_{\mathbf{k}})}_{1-f_{FD}(E_{\mathbf{k}})}$$



Single particle spectrum of conventional superconductors consists of two Bogoliubov branches gaped around E_F (no fluctuation effects are here taken into account).

Experimental data for cuprates

below T_c



H. Matsui, T. Sato, and T. Takahashi et al, *Phys. Rev. Lett.* **90**, 217002 (2003).

Beyond the BCS approximation

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We have generalized the Bogoliubov ansatz taking into account the non-condensed (preformed) pairs

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$$\begin{aligned}\hat{c}_{\mathbf{k}\uparrow}(l) &= u_{\mathbf{k}}(l) \hat{c}_{\mathbf{k}\uparrow} + v_{\mathbf{k}}(l) \hat{c}_{-\mathbf{k}\downarrow}^{\dagger} \\ &\quad + \frac{1}{\sqrt{N}} \sum_{\mathbf{q} \neq 0} \left[u_{\mathbf{k},\mathbf{q}}(l) \hat{b}_{\mathbf{q}}^{\dagger} \hat{c}_{\mathbf{q}+\mathbf{k}\uparrow} + v_{\mathbf{k},\mathbf{q}}(l) \hat{b}_{\mathbf{q}} \hat{c}_{\mathbf{q}-\mathbf{k}\downarrow}^{\dagger} \right],\end{aligned}\tag{1}$$

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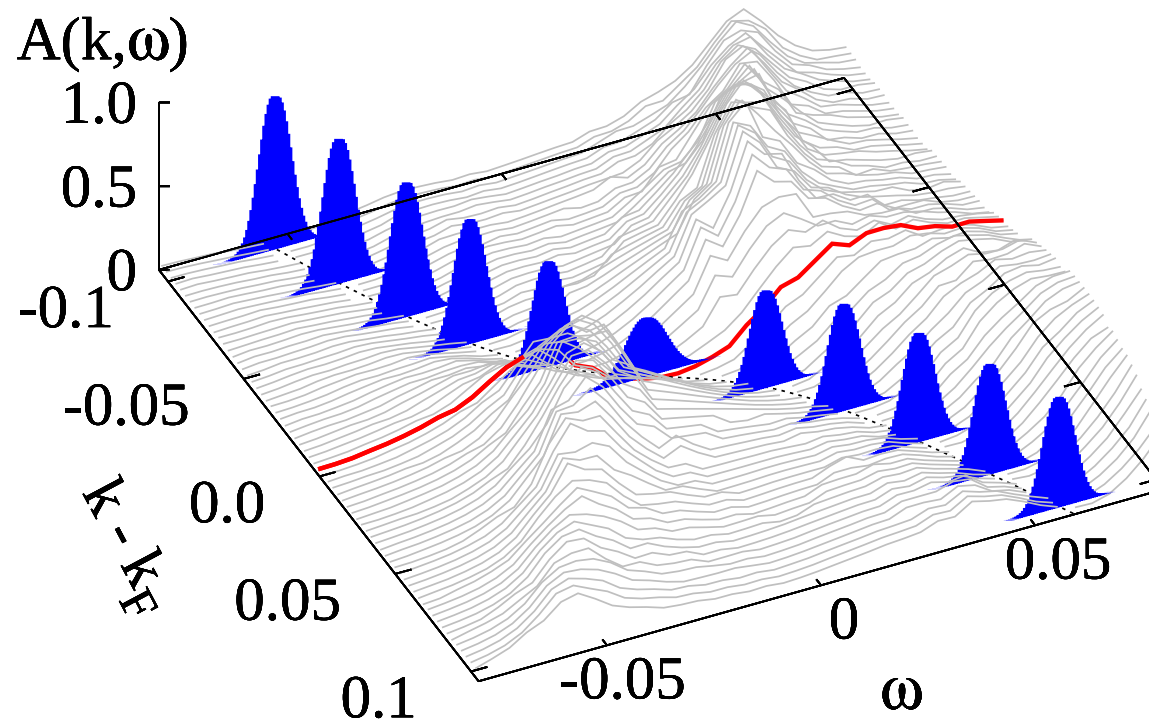
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and we determined the **fixed point** values $\lim_{l \rightarrow \infty} u_{\mathbf{k}}(l) \equiv \tilde{u}_{\mathbf{k}}$
etc from the coupled set of flow equations

$$\frac{\partial}{\partial l} u_{\mathbf{k}}(l), \quad \frac{\partial}{\partial l} v_{\mathbf{k}}(l), \quad \frac{\partial}{\partial l} u_{\mathbf{k},\mathbf{q}}(l), \quad \frac{\partial}{\partial l} v_{\mathbf{k},\mathbf{q}}(l).$$

Effective spectrum above T_c

$$T_c < T < T^*$$

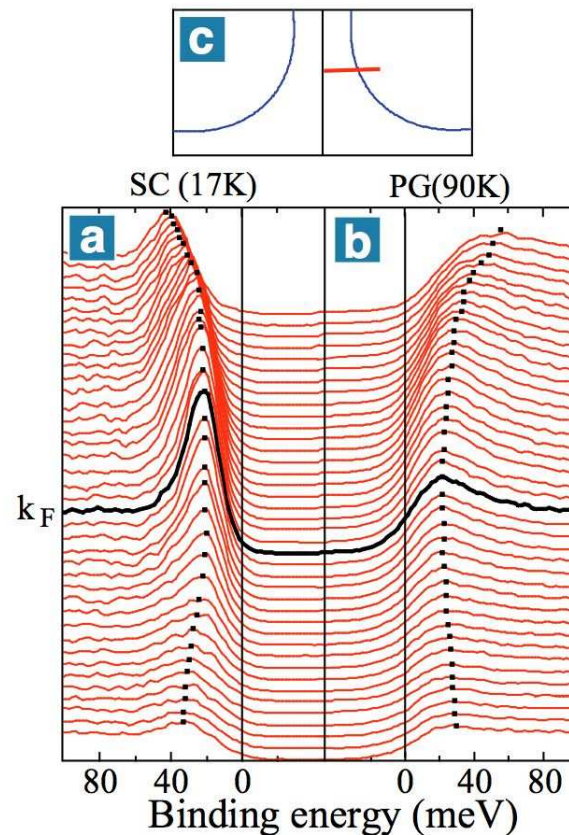


*T. Domański and J. Ranninger, Phys. Rev. Lett. **91**, 255301 (2003).*

Evidence for Bogoliubov QPs above T_c

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J. Campuzano group (Chicago, USA)

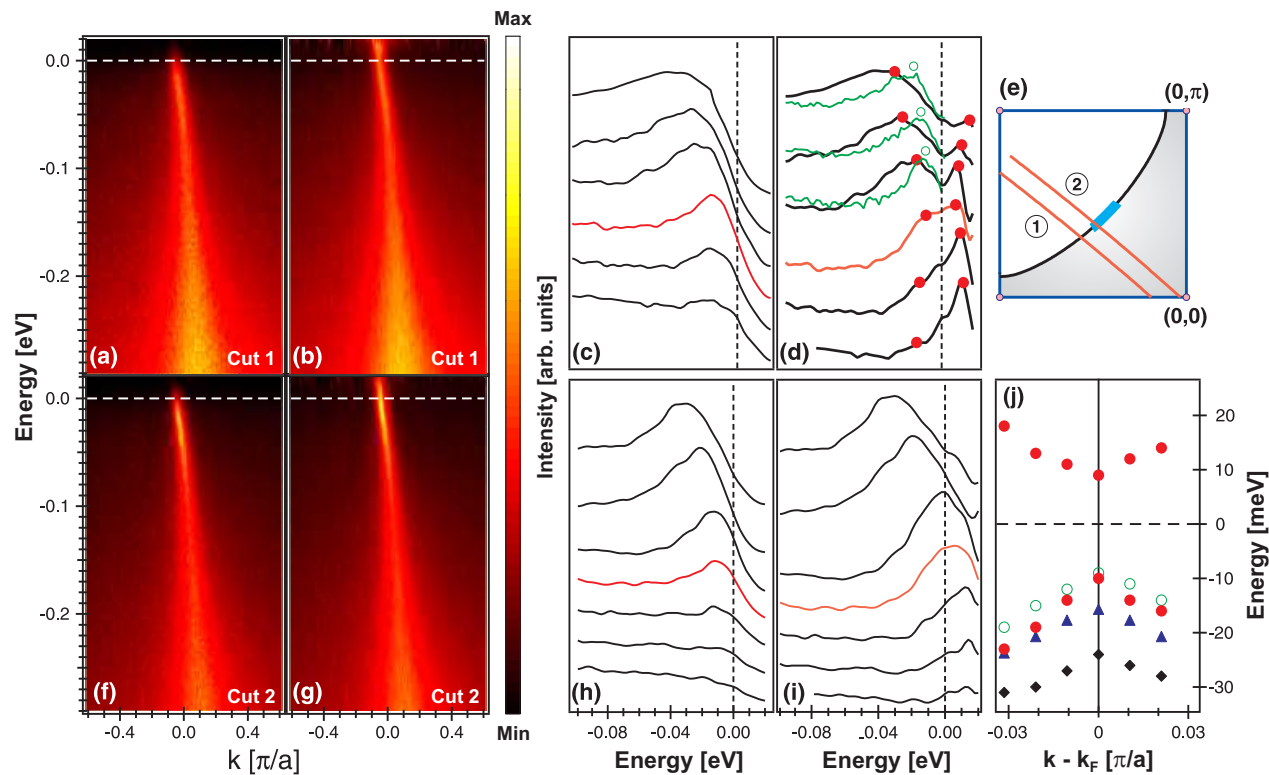


Results for: $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$

A. Kanigel et al, *Phys. Rev. Lett.* **101**, 137002 (2008).

Evidence for Bogoliubov QPs above T_c

PSI group (Villigen, Switzerland)

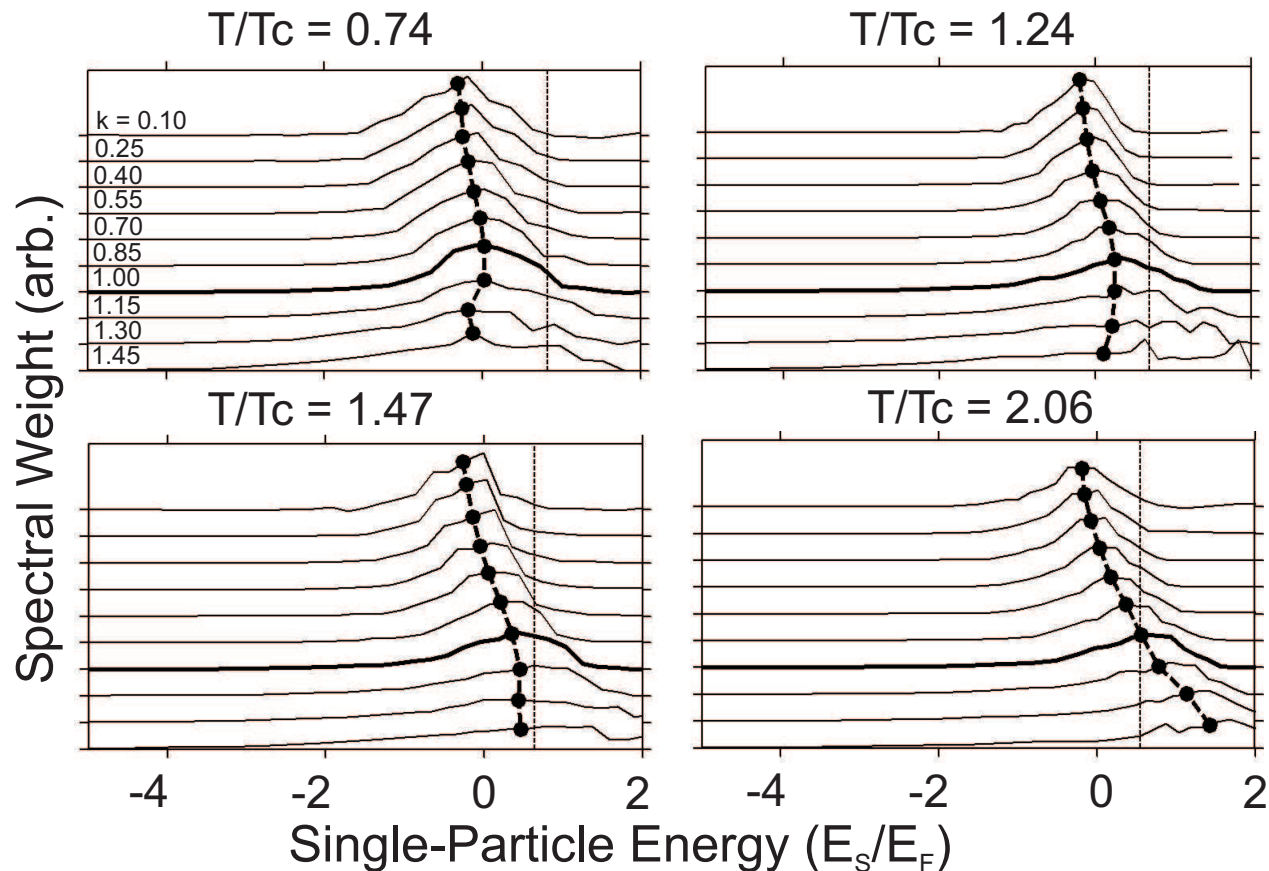


Results for: $\text{La}_{1.895}\text{Sr}_{0.105}\text{CuO}_4$

M. Shi et al, Eur. Phys. Lett. 88, 27008 (2009).

Evidence for Bogoliubov QPs above T_c

D. Jin group (Boulder, USA)



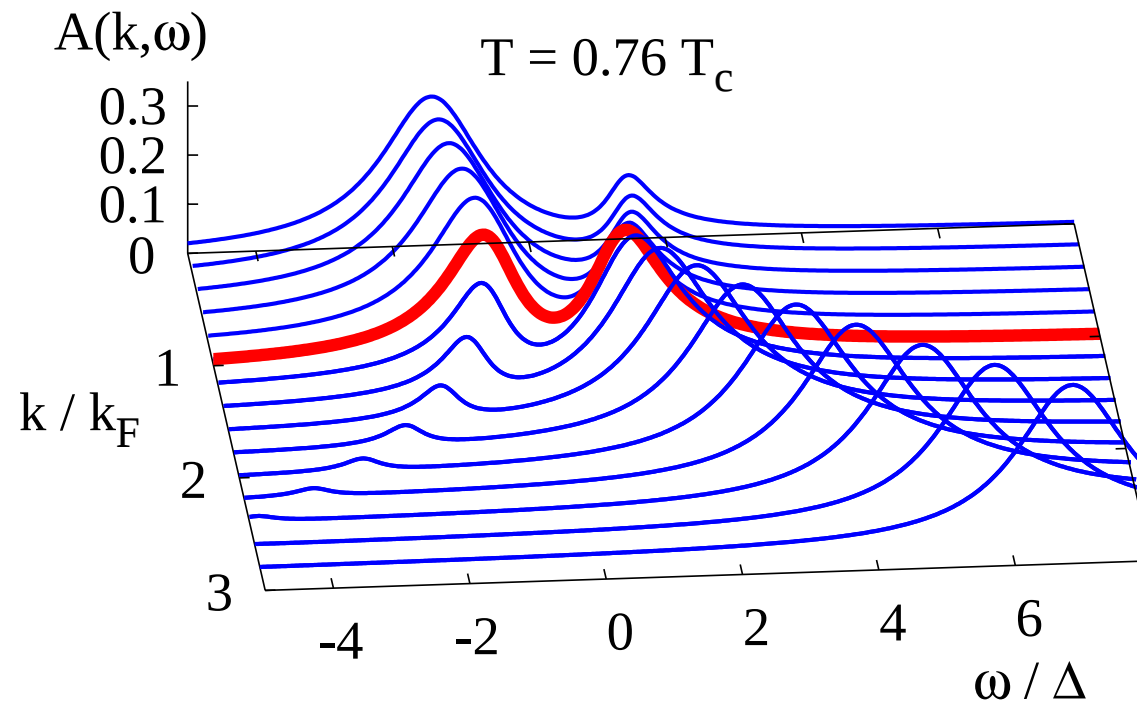
Results for: ultracold ^{40}K atoms

J.P. Gaebler et al, Nature Phys. 6, 569 (2010).

Bogoliubov QPs above T_c

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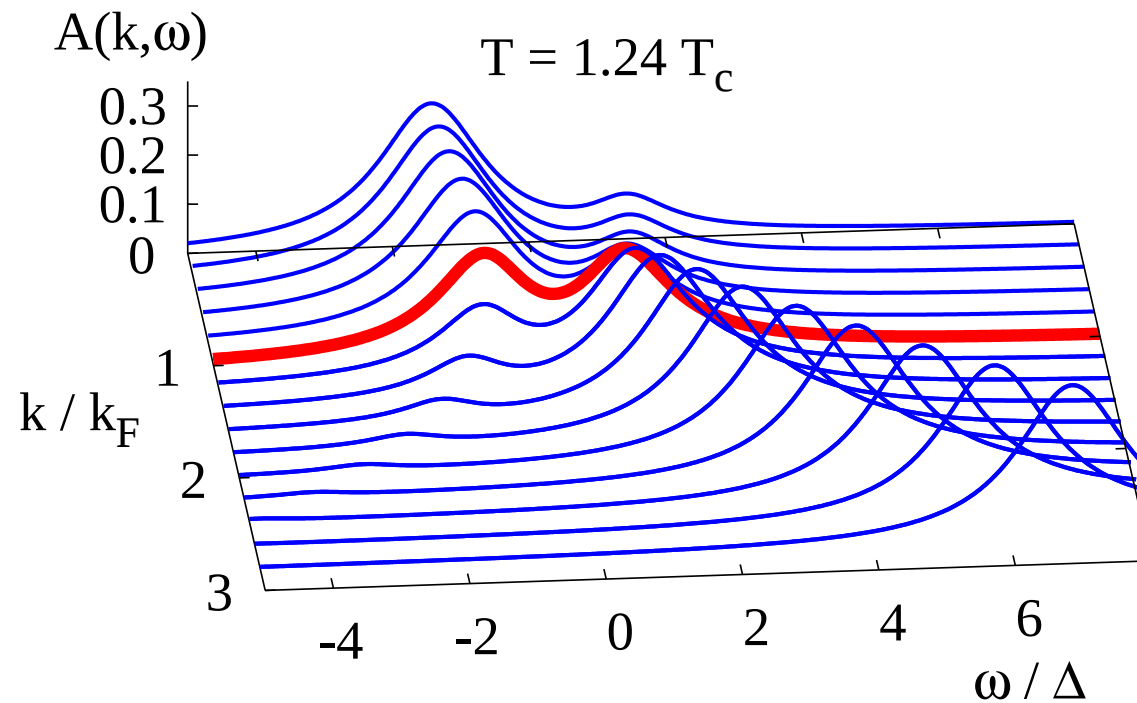
Reproducing experimental line-shapes of the ultra-cold fermion atoms



T. Domański, Phys. Rev. A **84**, 023634 (2011).

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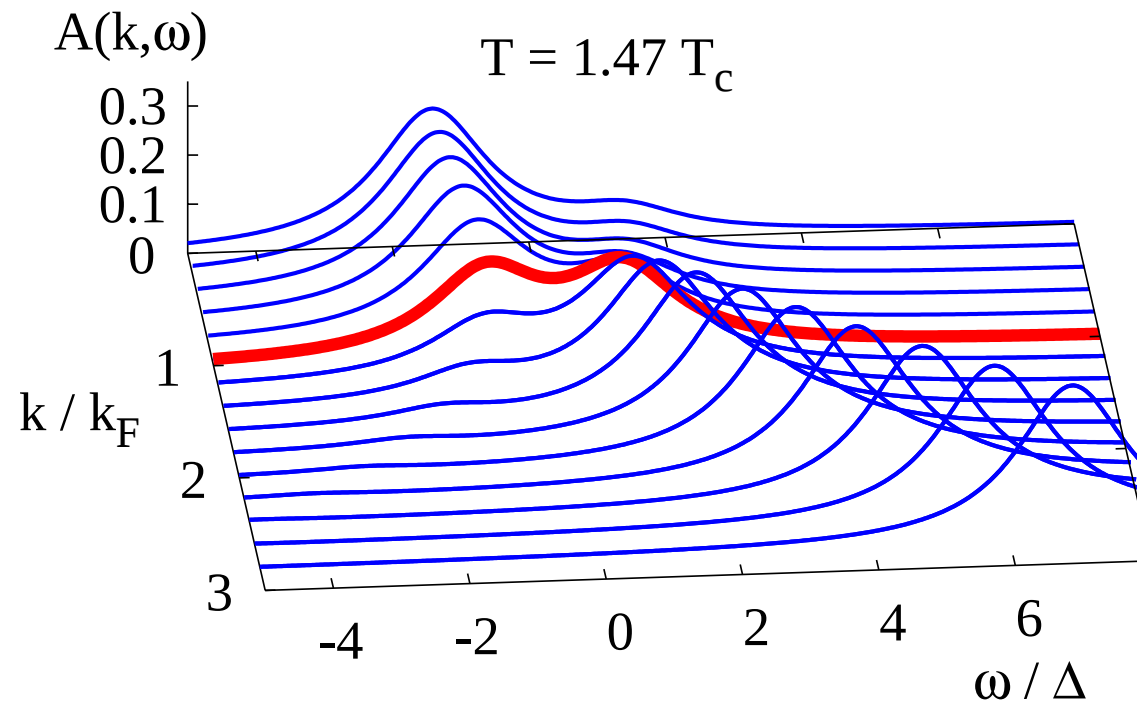
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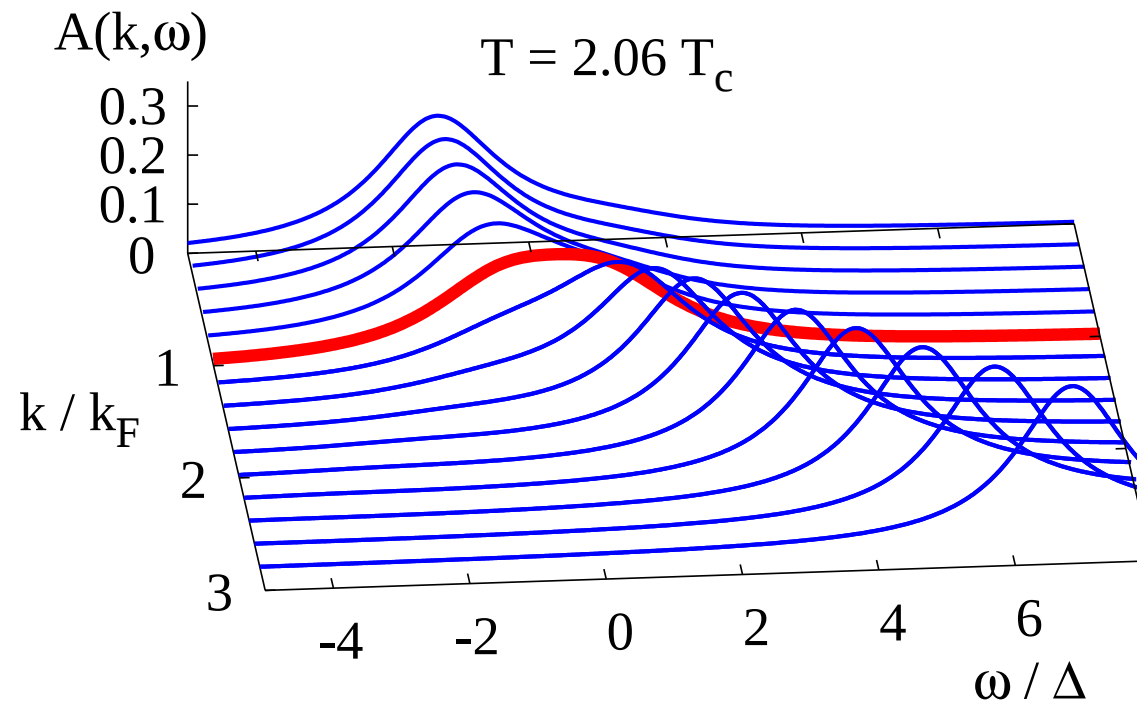
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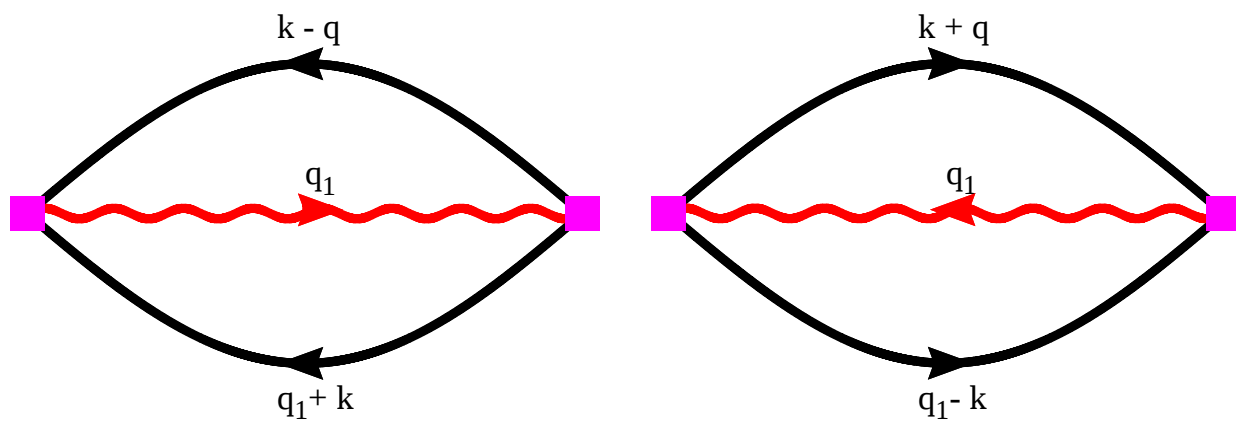
Diamagnetism

its origin above T_c

Diamagnetic response above T_c

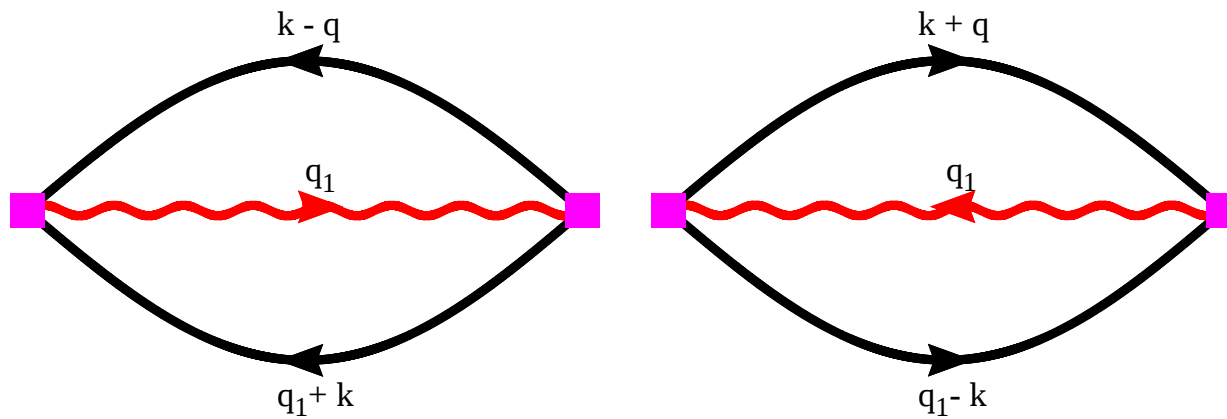
Diamagnetic response above T_c

The anomalous contributions

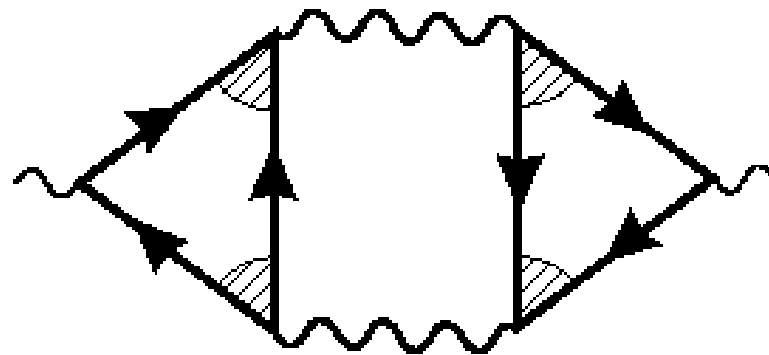


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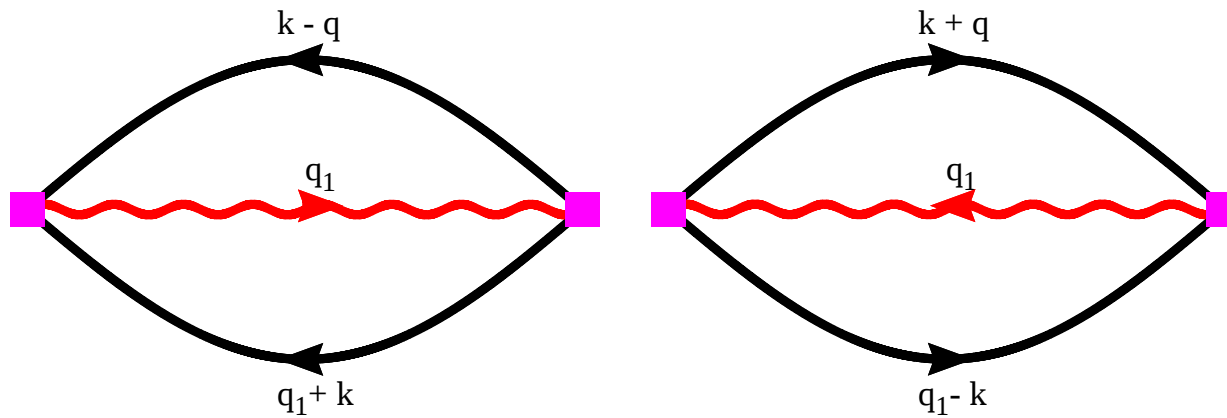


resemble the Aslamasov-Larkin diagram

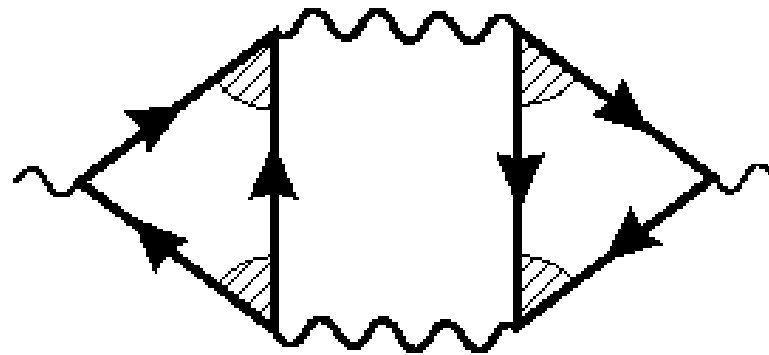


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enhancing the conductance/diamagnetism above T_c .

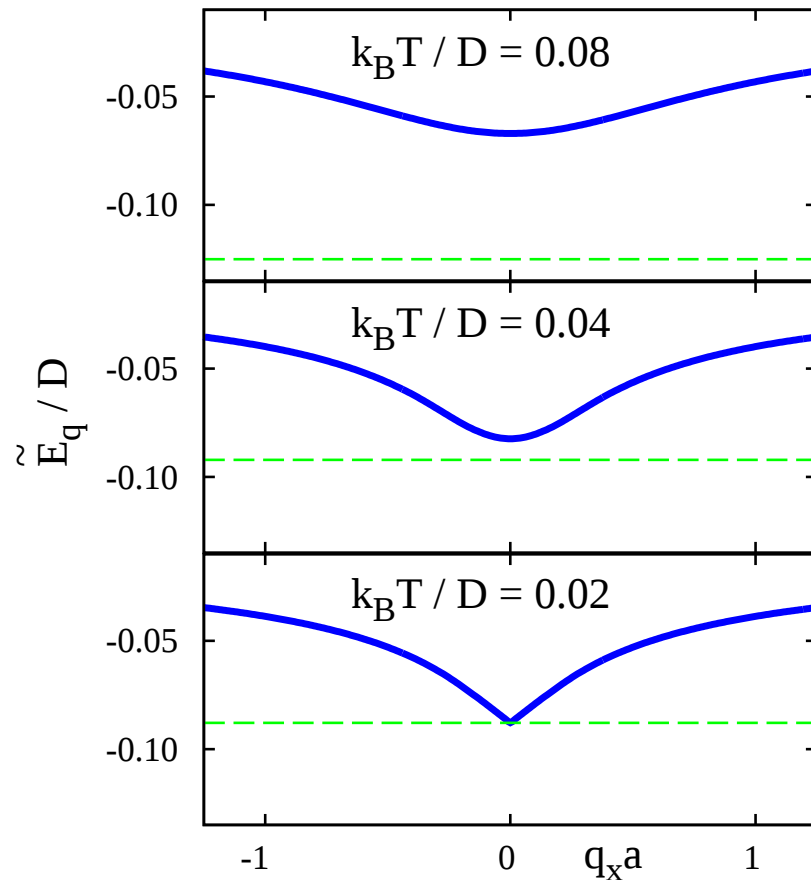
Onset of diamagnetism above T_c

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Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.

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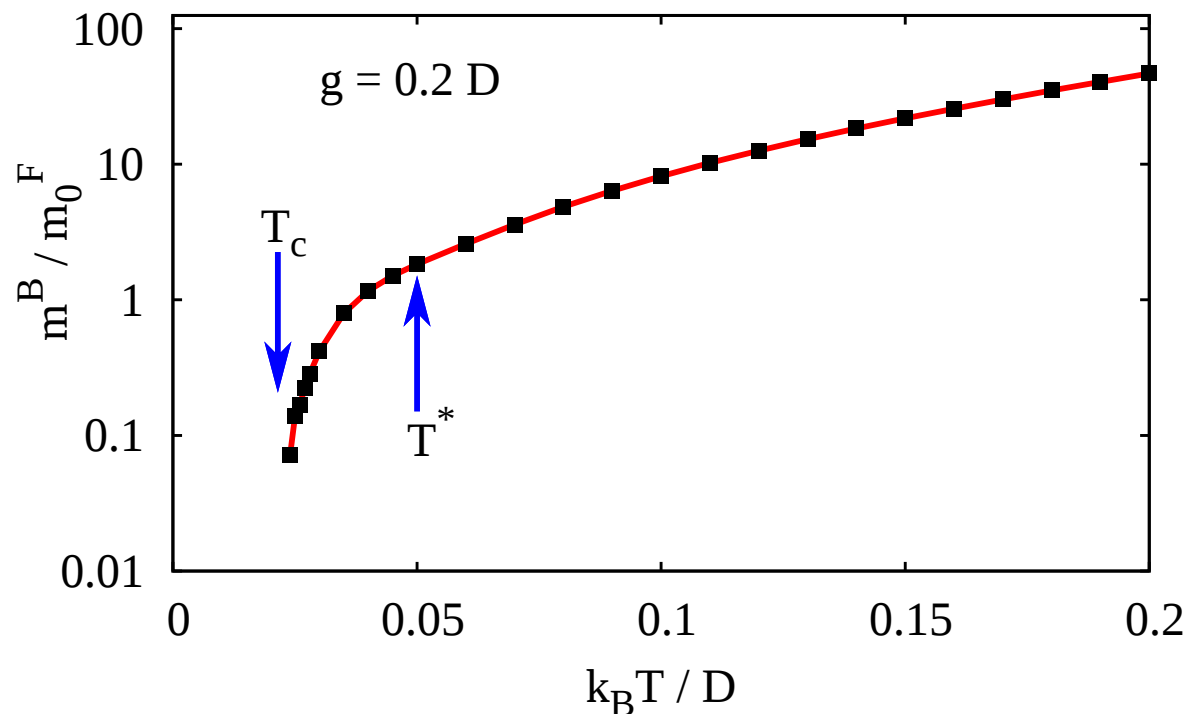
Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.



Qualitative changes of the preformed pairs' spectrum.

Onset of diamagnetism above T_c

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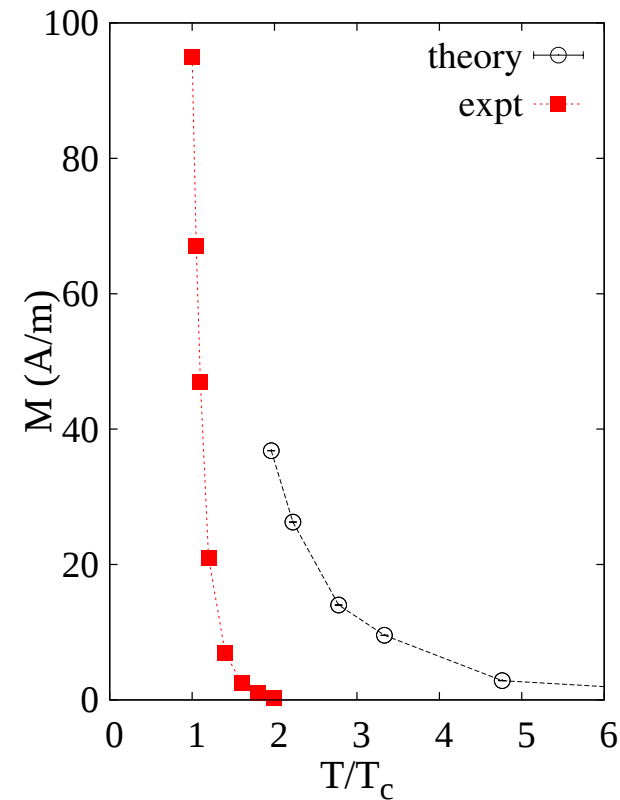
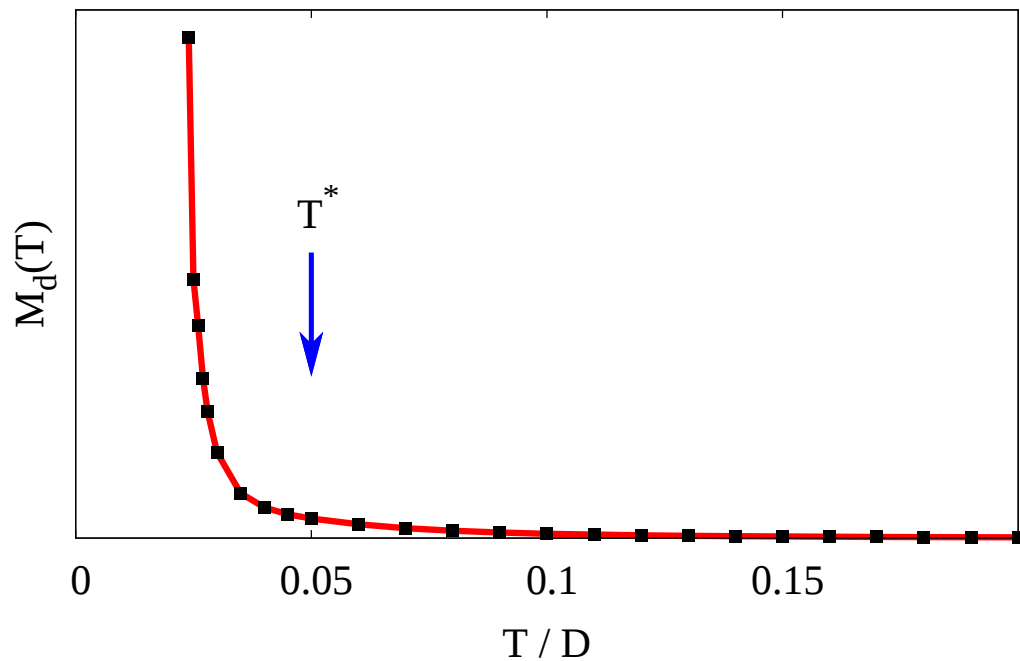
Effective mass m_B of the preformed pairs

M. Zapalska and T. Domański, Phys. Rev. B **84**, 174520 (2011).

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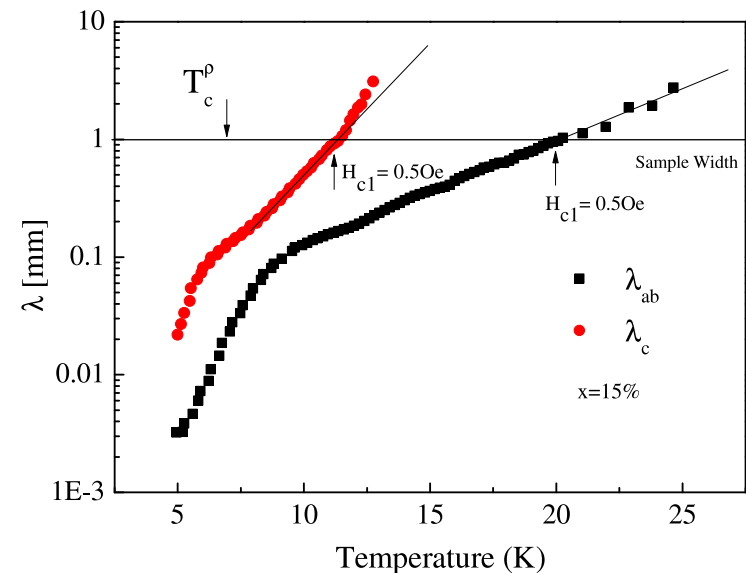
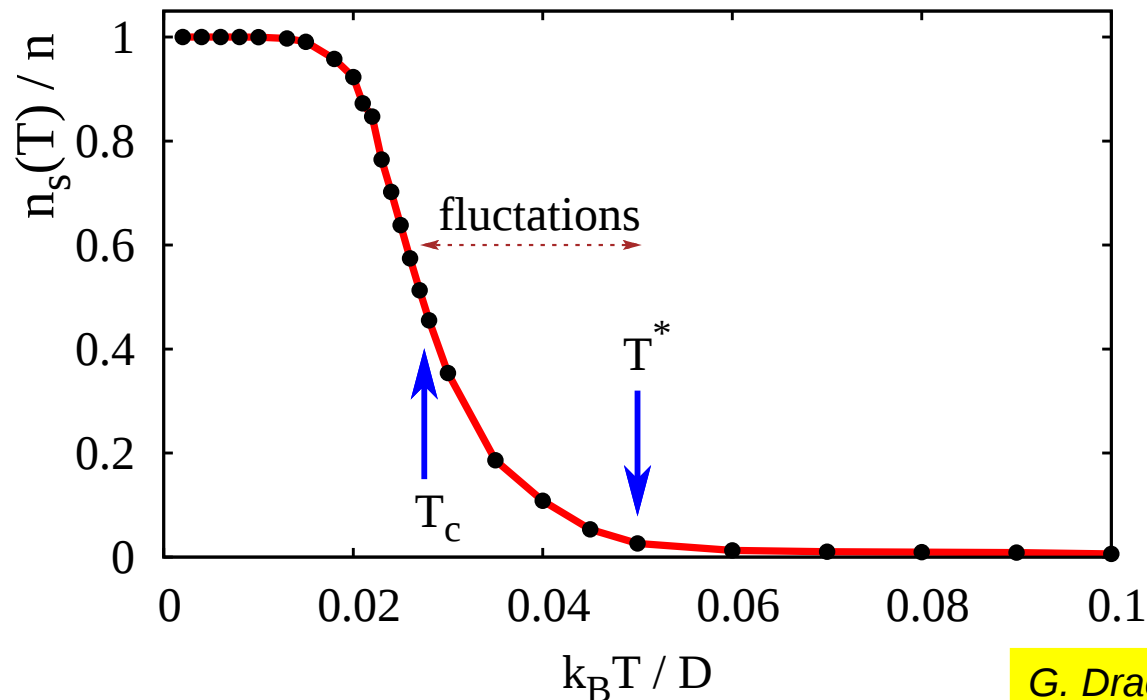
M. Zapalska and T. Domański (2014).



*K.-Y. Yang, ... and M. Troyer, Phys. Rev. B **83**, 214516 (2011).*

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G. Drachuck et al, *Phys. Rev. B* **85**, 184518 (2012).

$$J_x(\mathbf{q} \rightarrow 0, 0) = - \frac{e^2 n_s(T)}{m} A_x(\mathbf{q} \rightarrow 0, 0)$$

M. Zapalska and T. Domański, *Phys. Rev. B* **84**, 174520 (2011).

Diamagnetic response above T_c

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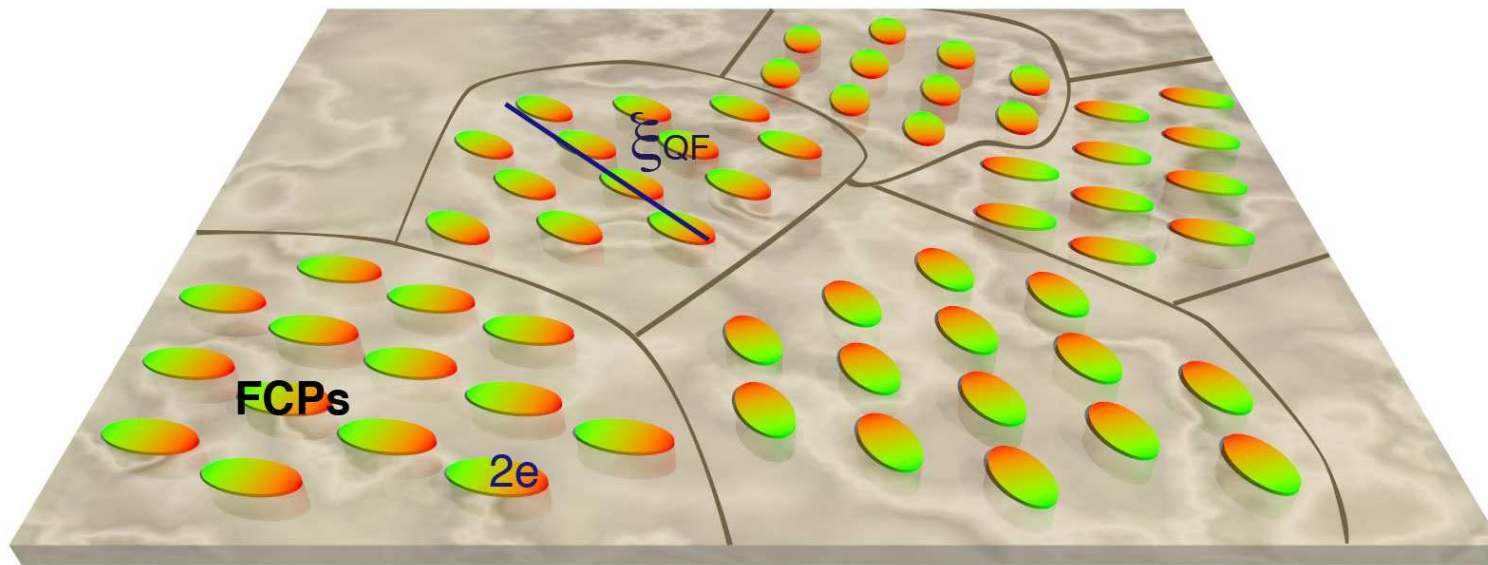
Residual diamagnetism originates from collective behavior of the pre-formed pairs.

Pair susceptibility is enhanced at T_{sc} and it ultimately diverges at some lower T_c .

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Cartoon illustrating the vortex liquid above T_c .

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Thank you