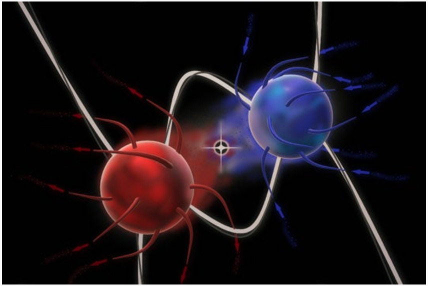
Electron pairing and correlations: insights from nanoscopic systems

Tadeusz Domański

M. Curie-Skłodowska University
LUBLIN



Concepts in Strongly Correlated Quantum Matter Kraków, 20-22 Nov. 2025

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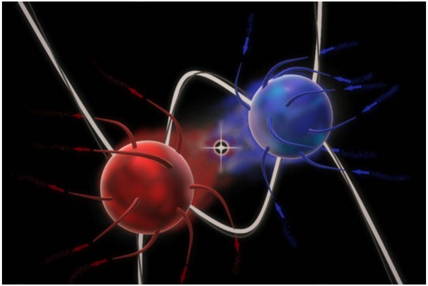

Concepts in Strongly Correlated Quantum Matter **Kraków, 20-22 Nov. 2025**

Dedicated to Prof. Józef Spałek on His birthday

OUTLINE

1. Superconductivity in nanostructures:

⋆ electron pairing / due to proximity effect /

⋆ in-gap quasiparticles / Andreev (Shiba) states /

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- ⋆ electron pairing / due to proximity effect /
- ⋆ in-gap quasiparticles / Andreev (Shiba) states /

2. Interplay with correlations:

- ⋆ quantum phase transition / parity crossing /
- ⋆ spin exchange interaction / Andreev vs Kondo /
- ⋆ bottom-up engineering / poor man's Majorana /

OUTLINE (IN BRIEF)

Correlations (on-site repulsion)$\Longrightarrow$ Superconductivity (inter-site pairing)

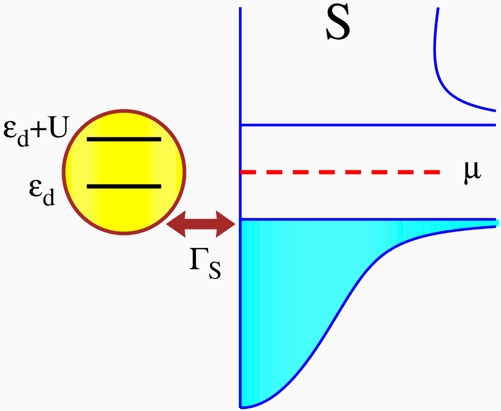
OUTLINE (IN BRIEF)

Correlations (on-site repulsion)$\Longrightarrow$ Superconductivity (inter-site pairing)

... and backwards

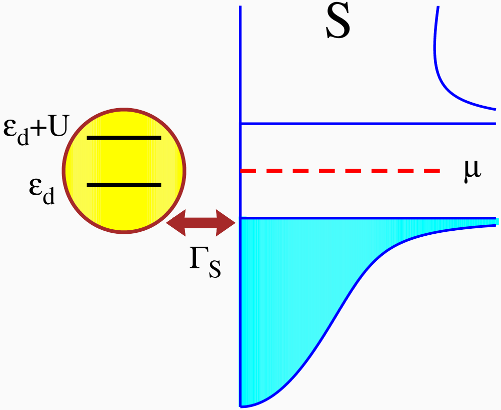
Superconductivity in nanostructures

PROXIMITY EFFECT



Quantum impurity (dot) coupled to bulk superconductor

PROXIMITY EFFECT



Quantum impurity (dot) coupled to bulk superconductor

$\Rightarrow$ **on-dot pairing**

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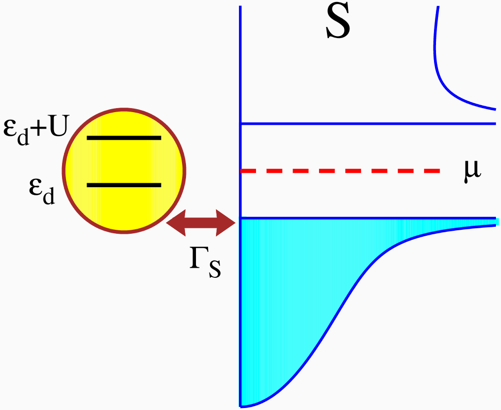


Quantum impurity (dot) coupled to bulk superconductor

$\Rightarrow$ **on-dot pairing**

$\Rightarrow$ **in-gap bound states**

MICROSCOPIC DESCRIPTION

Let us consider the Anderson-type quantum impurity (dot)

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}$$

coupled to bulk superconductor

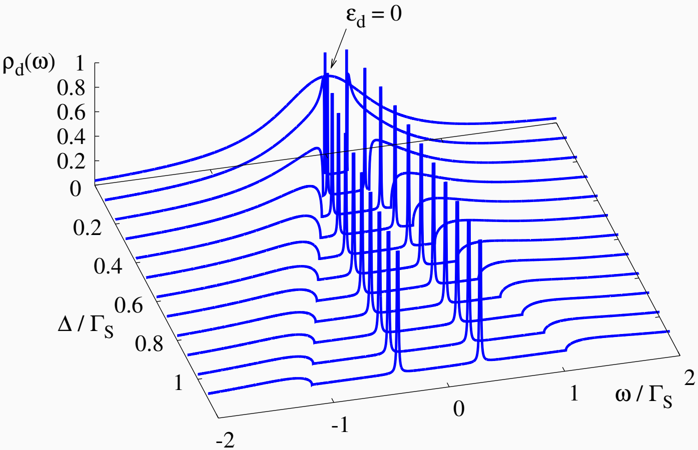
$$\hat{H} \; = \; \hat{H}_{QD} + \hat{H}_S + \sum_{\mathbf{k},\sigma} \left(V_{\mathbf{k}} \, \hat{d}^{\dagger}_{\sigma} \hat{c}_{\mathbf{k}\sigma} + V^{*}_{\mathbf{k}} \, \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{d}_{\sigma} \right)$$

described by the BCS model

$$\hat{H}_S = \sum_{k,\sigma} (\varepsilon_k - \mu) \, \hat{c}^{\dagger}_{k\sigma} \hat{c}_{k\sigma} - \sum_{k} \left(\Delta \hat{c}^{\dagger}_{k\uparrow} \; \hat{c}^{\dagger}_{k\downarrow} + \text{h.c.} \right)$$

UNCORRELATED QUANTUM DOT

Spectrum $\rho_d(\omega)$ of uncorrelated QD **[where $\Gamma_S = 2\pi|V_{k_F}|^2\rho_s(\epsilon_F)$]**



In-gap (Andreev/Shiba) bound states appear:

⇒ **in pairs,**

⇒ **symmetrically**

Pairing vs Coulomb repulsion

PAIRING VS CORRELATIONS

Quantum dot proximitized to superconductor can described by

$$\hat{H}_{QD} = \sum_{\sigma} \epsilon_d \, \hat{d}^{\dagger}_{\sigma} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow}\hat{n}_{d\downarrow} \; + \; \hat{V}_{QD-S} + \hat{H}_S$$

Eigen-states of this problem are represented by:

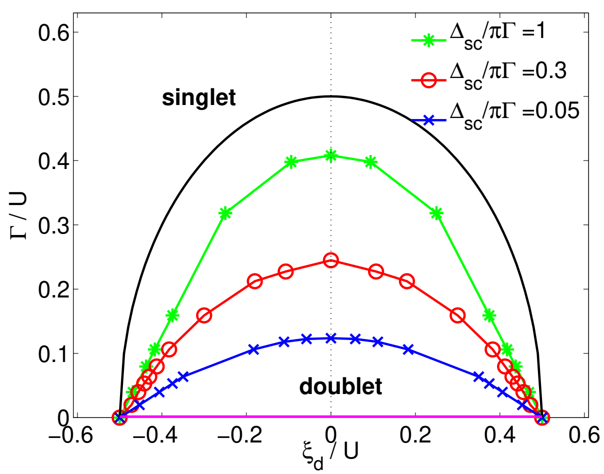
$|\uparrow\rangle$ **and** $|\downarrow\rangle$ $\Leftarrow$ **doublet states (spin $\frac{1}{2}$)**

$$\left.\begin{array}{l} u\,|0\rangle - v\,|\uparrow\downarrow\rangle \\ v\,|0\rangle + u\,|\uparrow\downarrow\rangle \end{array}\right\} \Leftarrow$$ **singlet states (spin 0)**

Upon varying the ratio ε_d/U_d or Γ_S/U_d the doublet-singlet transition can be induced between these ground states.

QUANTUM PHASE TRANSITION

Singlet-doublet (quantum phase) transition: NRG results



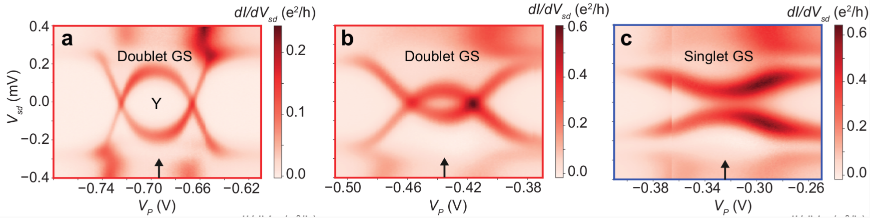
$\xi_d \equiv \varepsilon_d - U/2$

J. Bauer, A. Oguri & A.C. Hewson, J. Phys.: Condens. Matter 19, 486211 (2007).

SINGLET VS DOUBLET: EXPERIMENT

Differential conductance vs source-drain bias V_{sd} (vertical axis) and gate potential V_p (horizontal axis) measured for various Γ_S/U

$U \gg \Gamma_s$ $\quad$ $U \geq \Gamma_s$ $\quad$ $U < \Gamma_s$



J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, Commun. Phys. 3, 125 (2020).

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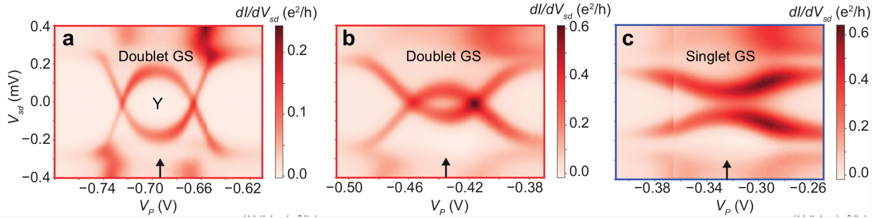
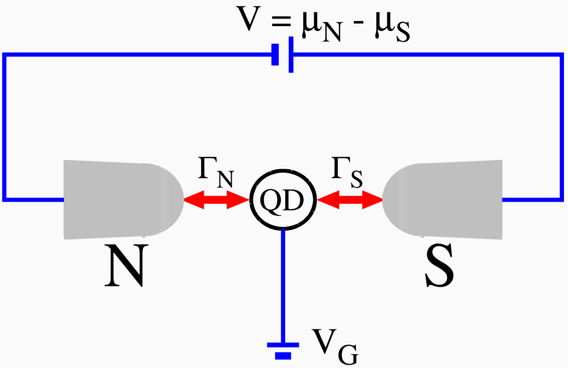


J. Estrada Saldaña, A. Vekris, V. Sosnovtseva, T. Kanne, P. Krogstrup, K. Grove-Rasmussen and J. Nygård, Commun. Phys. 3, 125 (2020).

Crossings of in-gap states correspond to the singlet-doublet QPT.

EMPIRICAL DETECTION OF IN-GAP SATES

Subgap states can be probed by measuring the charge transport through a quantum dot (QD) coupled between the normal (N) and superconducting (S) electrodes



Inside the pairing gap the current is due to electron-to-hole scattering.

CANONICAL TRANSFORMATION

N-QD-S junction can be described by

$$\hat{H} \;=\; \hat{H}_N + \hat{V}_{N-QD} + \underbrace{\hat{H}_{QD} + \hat{V}_{QD-S} + \hat{H}_S}_{\hat{H}_{QD}^{prox}}$$

where $\hat{H}_{QD} + \hat{V}_{QD-S} + \hat{H}_S$ **can be treated as**

$$\hat{H}_{QD}^{prox} = \sum_{\sigma} \epsilon_d \, \hat{d}_{\sigma}^{\dagger} \, \hat{d}_{\sigma} \; + \; U_d \, \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} - \left(\Gamma_S \, \hat{d}_{\uparrow}^{\dagger} \hat{d}_{\downarrow}^{\dagger} + \text{h.c.} \right)$$

We used canonical transformation to eliminate $\hat{V}_{N-QD}$**,**

deriving the effective model

$$\hat{H}_{eff} = \hat{H}_N - \sum_{k,p} J_{k,p} \hat{\vec{S}}_{k,p} \cdot \hat{\vec{S}}_d + \hat{H}_{QD}^{prox}$$

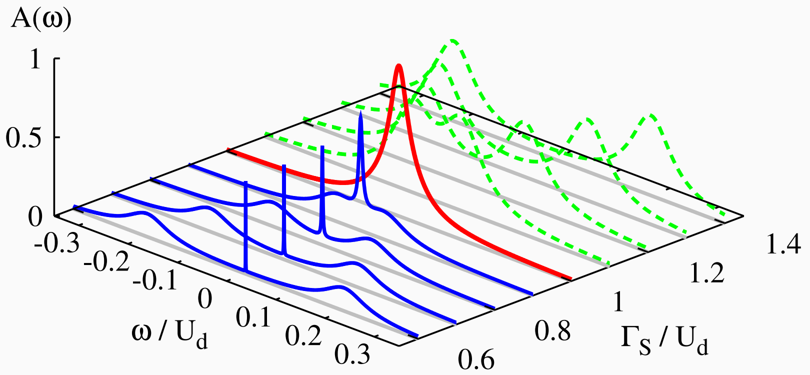
with the exchange interaction $J_{k,p}$

KONDO EFFECT VS PAIRING

In particular, for the half-filled QD ($\varepsilon_d = -U_d/2$) we obtain

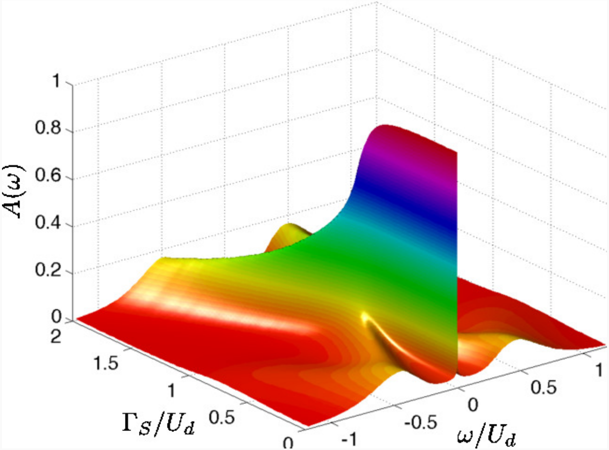
$$\lim_{k,p \to k_F} J_{k,p} = \frac{-4U_d |V_{k_F}|^2}{U_d^2 - \Gamma_S^2}$$

which diverges upon aproaching the parity crossing



NONPERTURBATIVE RESULTS

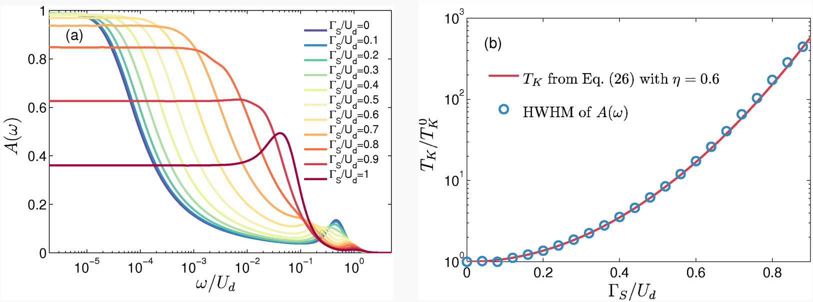
Similar results for the half-filled QD ($\varepsilon_d = -U_d/2$) have been also obtained by NRG calculations (using Ljubljana code)



T. Domański, I. Weymann, M. Barańska & G. Górski, Scientific Reports 6, 23336 (2016).

Kondo temperature estimated from the canonical transformation

$T_K \simeq 0.3\sqrt{\Gamma_N U_d}\,\exp\left[\frac{\pi\varepsilon_d(\varepsilon_d+U_d)+(\Gamma_S/2)^2}{\Gamma_N U_d}\right]$ **is consistent with NRG**



Conclusion:

Kondo temperature T_K is amplified by on-dot pairing !

T. Domański, I. Weymann, M. Barańska & G. Górski, Scientific Reports 6, 23336 (2016).

Dynamical phase transition

Dynamical phase transition

[in time-domain]

GENERAL OUTLINE

For $t < 0$ we assume the system $\hat{H}_0$ to be in its ground state:

$$\hat{H}_0 |\Psi_0\rangle = E_0 |\Psi_0\rangle$$

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Fidelity (similarity) of these states is:

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Loschmidt amplitude

STATIONARY VS DYNAMICAL PHASE TRANSITION

Idea: M. Heyl, A. Polkovnikov, S. Kehrein, Phys. Rev. Lett. 110, 135704 (2013).

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$$\mathcal{Z} = \left\langle e^{-\beta \hat{H}} \right\rangle$$

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$$L(t) = \left| \left\langle \Psi_0 | e^{-it\hat{H}} | \Psi_0 \right\rangle \right|^2$$

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Critical temperature T_c

nonanalytical $\lim_{T \to T_c} F(T)$

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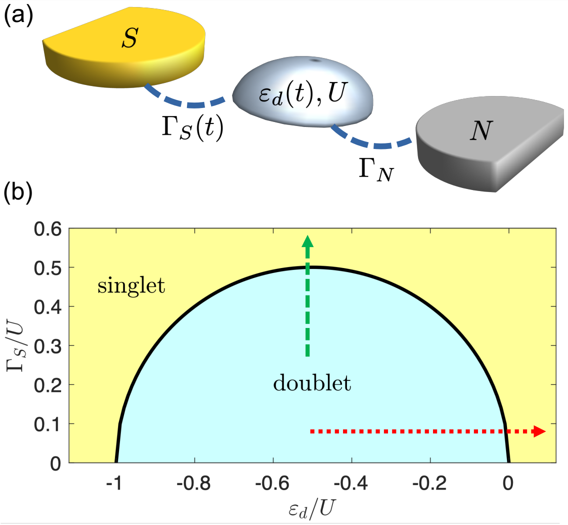
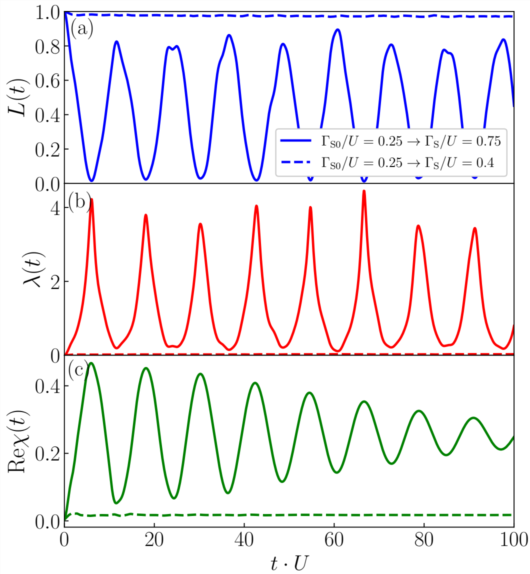
QUENCH ACROSS QPT BOUNDARY



K. Wrześniewski, I. Weymann, N. Sedlmayr & T. Domański, Phys. Rev. B 105, 094514 (2022).

ABRUPT CHANGE OF Γ_S: t-NRG RESULTS



$\varepsilon_d = -U/2$

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Triplet blockade

in junctions with two quantum dots

ANDREEV BLOCKADE: CONCEPT

SciPost Phys. 11, 081 (2021)

Theory of Andreev blockade in a double quantum dot with a superconducting lead

David Pekker, Po Zhang and Sergey M. Frolov

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA, 15260

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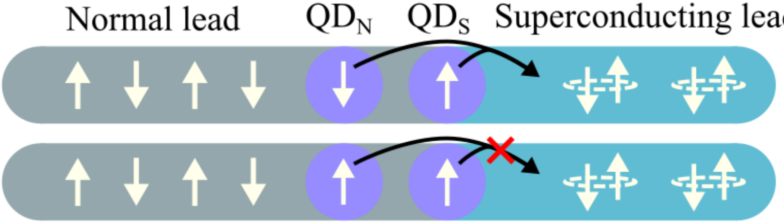
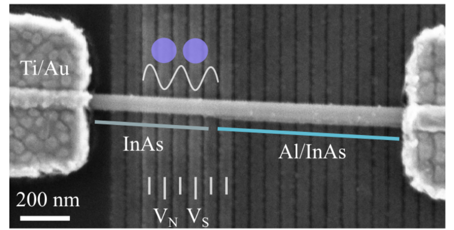


Superconducting proximity effect would be blocked by triplet configuration of the quantum dots (Andreev current forbidden).

ANDREEV BLOCKADE: REALIZATION



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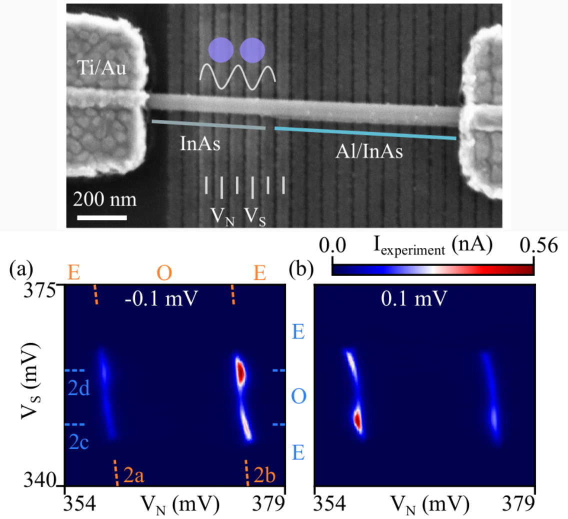

P. Zhang, H. Wu, J. Chen, S.A. Khan, P. Krogstrup, D. Pekker, and S.M. Frolov, Phys. Rev. Lett. 128, 046801 (2022).

STATIC REALIZATION: S-DQD-S

PHYSICAL REVIEW B **102**, 220505(R) (2020)

Rapid Communications

Triplet-blockaded Josephson supercurrent in double quantum dots

Daniël Bouman,[1] Ruben J. J. van Gulik,[1] Gorm Steffensen,[2] Dávid Pataki,[3] Péter Boross,[4] Peter Krogstrup,[2] Jesper Nygård,[2] Jens Paaske,[2] András Pályi,[3] and Attila Geresdi[1,5,*]

[1] *QuTech and Kavli Institute of Nanoscience, Delft University of Technology, NL-2600 GA Delft, The Netherlands*
[2] *Center for Quantum Devices, Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen, Denmark*
[3] *Department of Theoretical Physics and MTA-BME Exotic Quantum Phases Research Group, Budapest University of Technology and Economics, H-1111 Budapest, Hungary*
[4] *Institute for Solid State Physics and Optics, Wigner Research Centre for Physics, P.O. Box 49, H-1525 Budapest, Hungary*
[5] *Quantum Device Physics Laboratory, Department of Microtechnology and Nanoscience, Chalmers University of Technology, SE-41296 Gothenburg, Sweden*

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[5] *Quantum Device Physics Laboratory, Department of Microtechnology and Nanoscience, Chalmers University of Technology, SE-41296 Gothenburg, Sweden*

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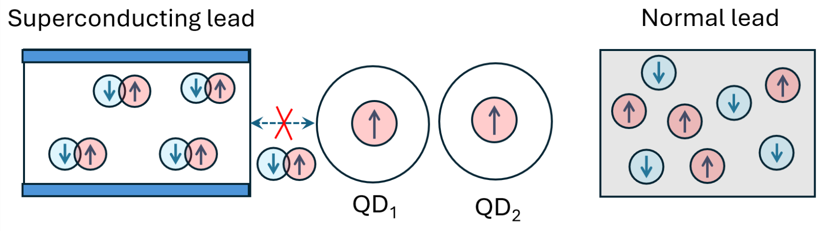
Conclusion: **"magnetic field dependence of the supercurrent amplitude in the even occupied state reveals the presence of a supercurrent blockade in the spin-triplet ground state"**

DYNAMICAL TRIPLET BLOCKADE



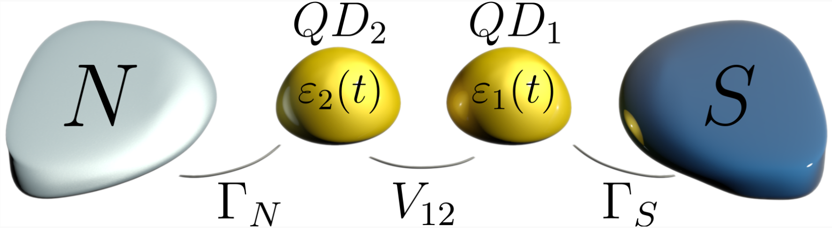
Superconducting proximity effect is blocked:

⇒ **when both quantum dots are singly occupied**

⇒ **by the same spin (for example ↑) electrons**

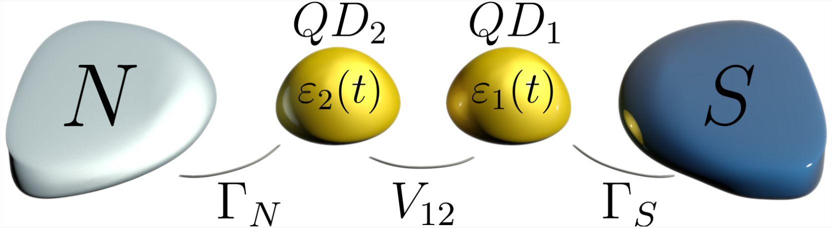
R. Taranko, J. Barański, A. Jankiewicz, K. Wrześniewski, I. Weymann & T. Domański [submitted (2025)].

TRIPLET BLOCKADE VS ON-DOT PAIRING



We inspected evolution of the triplet configuration:

TRIPLET BLOCKADE VS ON-DOT PAIRING



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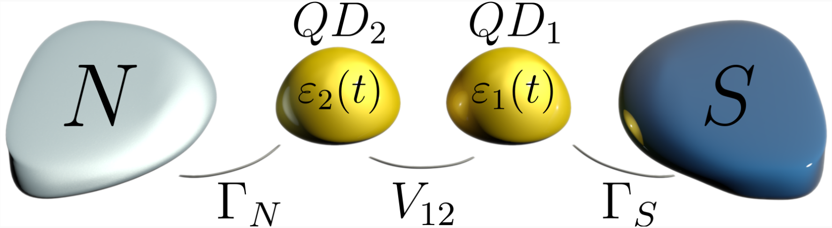
TRIPLET BLOCKADE VS ON-DOT PAIRING



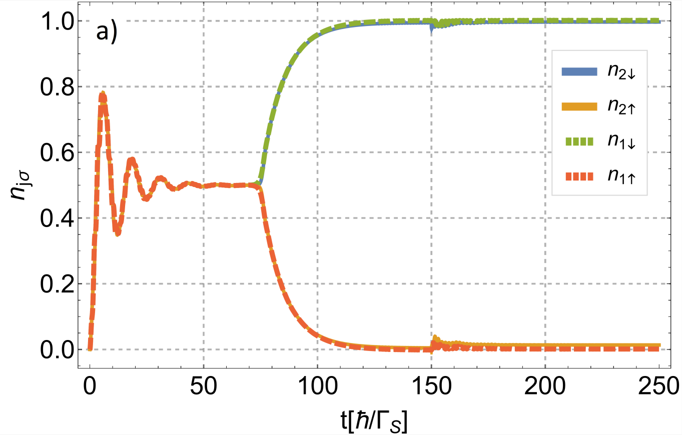
We inspected evolution of the triplet configuration:

$\Rightarrow$ **imposed by initial conditions**

$\Rightarrow$ **enforced by magnetic field**

EVOLUTION OF TRIPLET CONFIGURATION

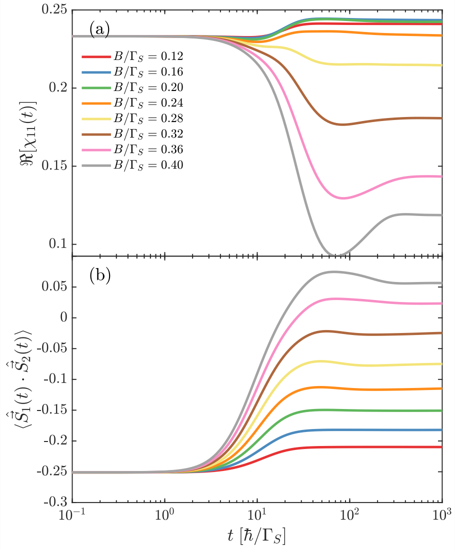
Triplet configuration cam be enforced by magnetic field



Results obtained for the uncorrelated system ($U_d = 0$)

EVOLUTION OF CORRELATED DOTS

t-NRG results obtained for the weakly coupled quantum dots



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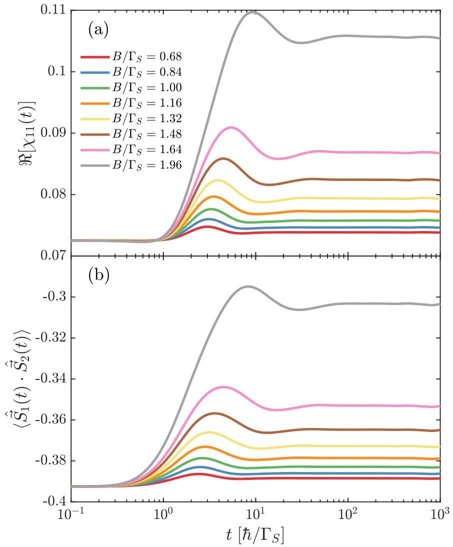

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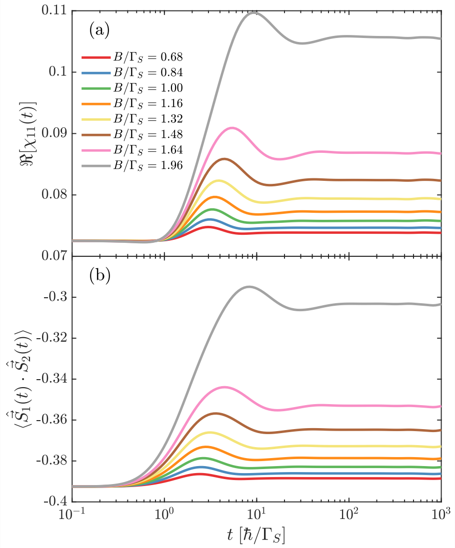
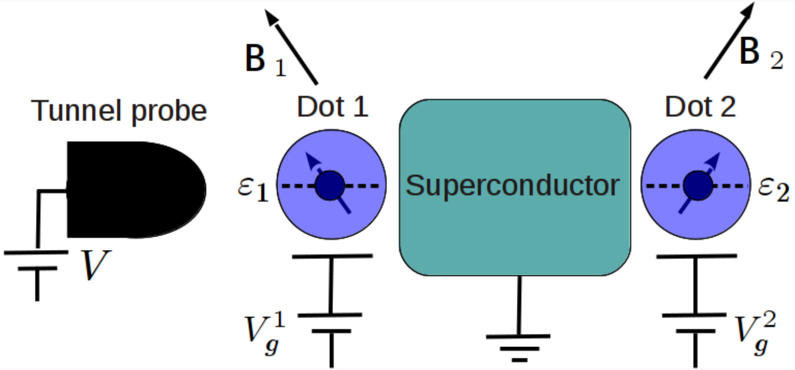


inter-dot spin exchange ?

Minimal Kitaev chain
(platform of poor man's Majorana modes)

MINIMAL KITAEV CHAIN: CONCEPT

Topological superconductor can be obtained using only two sites (quantum dots) proximitized to bulk superconductor



This scenario enables realization of poor man's Majorana modes

M. Leijnse, and K. Flensberg, Phys. Rev. B 86, 134528 (2012).

MINIMAL KITAEV CHAIN

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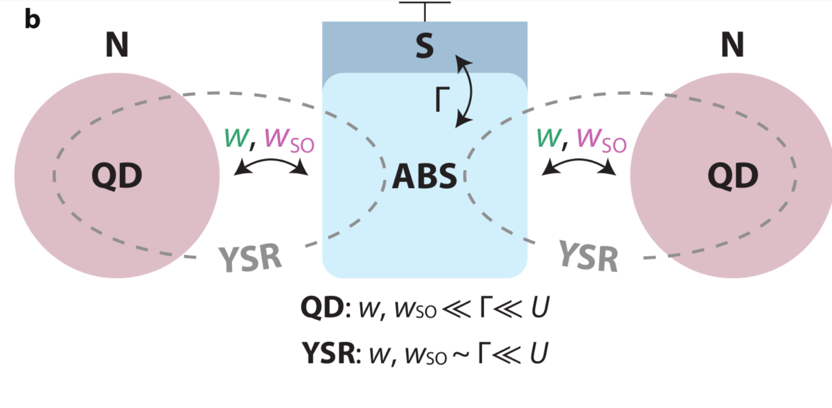
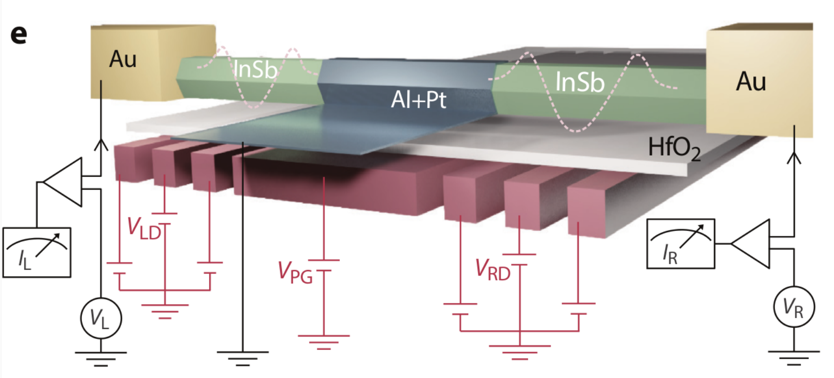


T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).

MINIMAL KITAEV CHAIN

Two spin-polarized quantum dots of InSb nanowire strongly coupled by the co-tunneling and crossed Andreev reflection

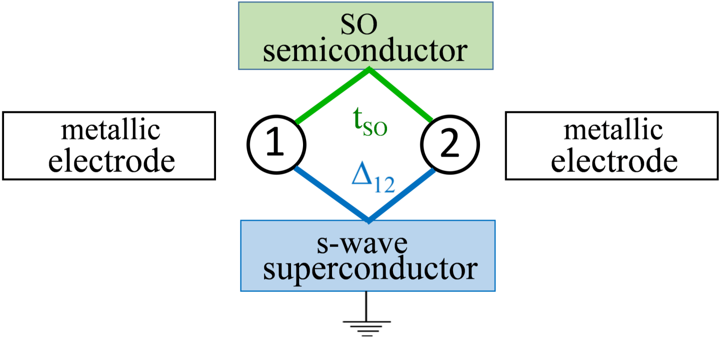


T. Dvir, ... & L.P. Kouwenhoven, Nature 614, 445 (2023).

COOPER PAIR SPLITTER GEOMETRY

Two quantum dots coupled via:

tunneling + crossed Andreev reflection + spin orbit interaction



Interplay of spin orbit interaction and Andreev reflection in proximized quantum dots

Bogdan R. Bułka,[1] Tadeusz Domański,[2] and Karol I. Wysokiński[2]

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(Dated: Received October 21, 2025)

arXiv/2510.17379 (2025)

SUMMARY

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Coulomb and Cooper can be friends !