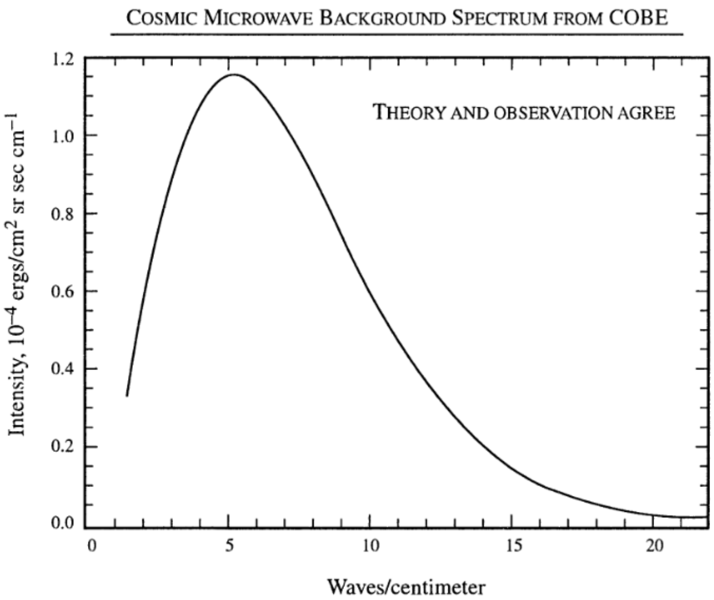
— Cosmic Micro Background Radiation (CMBR) —

The specific intensity of a gas of photons with a blackbody spectrum is

$$I_\nu = \frac{4\pi\hbar\nu^3/c^2}{\exp\{2\pi\hbar\nu/k_B T\} - 1}. \tag{1.8}$$

Convert the specific intensity in Eq. (1.8) into an expression for what is plotted in the Figure, the energy per square centimeter per steradian per second. Note that the x-axis is I/λ, the inverse wavelength of the photons. Show that the peak of a 2.73K blackbody spectrum does lie at 1/λ = 5.38 cm.



Źródło danych:
http://lambda.gsfc.nasa.gov/product/cobe/firas_monopole_get.cfm
COBE/FIRAS CMB monopole spectrum
Reference1 = Table 4 of Fixsen et al. 1996 ApJ 473, 576.
Reference2 = Fixsen & Mather 2002 ApJ 581, 817.

Exercise 4 An inverse wavelength is ν/c, so replacing ν everywhere in Eq. (1.8) by c/λ leads to

$$I_\nu = \frac{4\pi\hbar c}{\lambda^3} \frac{1}{\exp\{2\pi\hbar c/\lambda k_B T\} - 1}. \tag{A.7}$$

This is energy per Hz; we want energy per cm^{-1}, so we need to multiply by c, leaving

$$I_{1/\lambda} = \frac{4\pi\hbar c^2}{\lambda^3} \frac{1}{\exp\{2\pi\hbar c/\lambda k_B T\} - 1}. \tag{A.8}$$

Plugging in numbers leads to

$$I_{1/\lambda} = 1.2 \times 10^{-5}\text{erg sec}^{-1}\text{ cm}^{-1}\text{sr}^{-1} \left(\frac{\text{cm}}{\lambda}\right)^3 \frac{1}{\exp\{0.53\text{cm}/\lambda\} - 1}. \tag{A.9}$$

A quick check verifies that this agrees with Figure 1.10.

To find the peak, differentiate I with respect to $1/\lambda$ and set equal to zero. This leaves

$$\lambda = \frac{1}{3} \frac{(2\pi\hbar c/k_B T)}{1 - \exp\{-2\pi\hbar c/\lambda k_B T\}}. \tag{A.10}$$

So $1/\lambda_{\text{peak}}$ is $3/.53\text{cm}^{-1}$. The exact coefficient, accounting for the exponential is 2.82, so $1/\lambda_{\text{peak}} = 5.3\text{cm}^{-1}$, exactly where it occurs in Figure 1.10.