APPENDIX

TwoStars, A Binary Star Code

Binary star systems play a very important role in determining various stellar properties, including masses and radii. In addition, analyses using sophisticated binary-star modeling codes can provide information about variations in surface flux such as limb darkening and the presence of star spots or reflective heating. Advanced codes can also detail the effects of gravitational tidal interactions and centrifugal forces that result in stars that deviate (sometimes significantly) from spherical symmetry.

A simple binary star code is developed in this appendix that incorporates a number of the basic features of more sophisticated codes. `TwoStars` is designed to provide position, radial-velocity, and binary light curve information that can be used to determine masses (m_1 and m_2 from determination of the semimajor axes and periods of the orbits), radii (R_1 and R_2 by measuring eclipse times), effective temperature ratios (from the relative depths of the primary and secondary minima), limb darkening, orbital eccentricity (e), orbital inclination (i), and orientation of periastron (ϕ). However, in order to greatly simplify the code, it is assumed that the two stars are strictly spherically symmetric, that they do not collide with one another, and that their surface fluxes vary only with stellar radius (i.e., there are no anomalous star spots or localized heating).

To begin, assume that the orbits of the two stars lie in the x–y plane with the center of mass of the system located at the origin of the coordinate system, as shown in Fig. 1 (the z-axis is out of the page). In order to generalize the orientation of the orbit, periastron for Star 1 (the point in the orbit closest to the center of mass) is at an angle ϕ measured counterclockwise from the positive x-axis and in the direction of the orbital motion. It is also assumed that the orbital plane is inclined an angle i with respect to the plane of the sky (the y'–z' plane) as shown in Fig. 2. The line of sight from the observer to the center of mass is along the x'-axis, and the center of mass is located at the origin of the primed coordinate system. Finally, the y'-axis is directed out of the page and is aligned with the y-axis of Fig. 1.

It is a straightforward process to show that the transformation between the two coordinate systems is given by

$$x' = z\cos i + x\sin i \tag{1}$$

$$y' = y \tag{2}$$

$$z' = z\sin i - x\cos i, \tag{3}$$

which of course simplifies significantly for the case where the centers of mass lie along the x–y plane (i.e., $z = 0$).

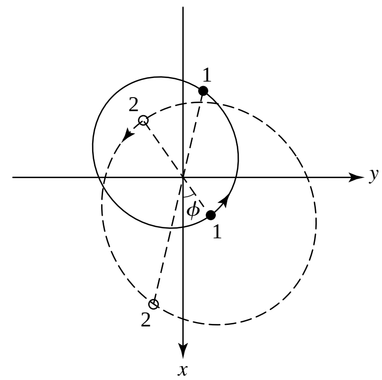

FIGURE 1 The orbits of the stars in the binary system lie in the x–y plane, with the z-axis directed out of the page. The center of mass of the system is located at the origin of the coordinate system. In this example $m_2/m_1 = 0.68$, $e = 0.4$, and $\phi = 35°$. The two positions of Stars 1 and 2 are separated by $P/4$, where P is the orbital period.

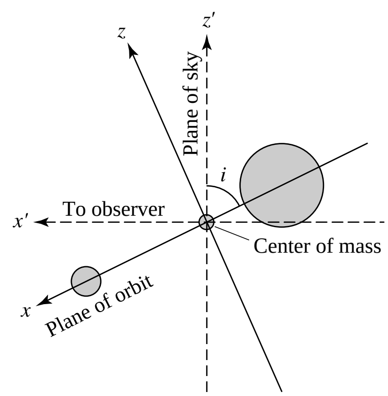

FIGURE 2 The plane of the orbit is inclined an angle i with respect to the plane of the sky (the y'–z' plane). The line of sight from the observer to the center of mass is along the x'-axis, and the center of mass is located at the origin of the primed coordinate system. The y'-axis is aligned with the y-axis of Fig. 1, and both are directed out of the page. The foreground star in this illustration is the smaller star.

The motions of the stars in the x–y plane are determined directly by using Kepler's laws and invoking the concept of the reduced mass. The approach is similar to what was used in `Orbit`, except that no assumption is made about the relative masses of the two objects in the system [in `Orbit` it was assumed that one object (a planet) was much less massive than the other object (the parent star)].

A careful reading of the code available on the companion website will identify several explicit instances of plus and minus signs associated with the variables `vr`, `v1r`, `v2r`, `x1`, `y1`, `x2`, and `y2`. The choice of minus signs corresponds to the choice of the coordinate system and its relationship to the observer. For instance, if the inclination angle is $i = 90°$, then

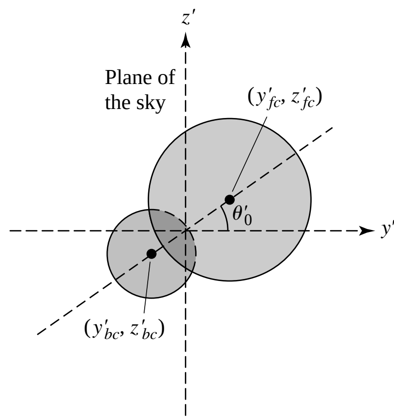

FIGURE 3 The disks of the two stars projected onto the plane of the sky. The foreground star (assumed in this illustration to be the larger star) has $x'_c > 0$, where x'_c is the x' coordinate of its center of mass. The angle between the y'-axis and the line connecting the centers of the disks of the two stars projected onto the plane of the sky is θ'_0.

the x and x' axes are aligned and motion in the positive x direction corresponds to motion toward the observer (a negative radial velocity).

In order to compute the light curve for the eclipse, it is necessary to integrate the luminous flux over the portion of each star's surface that is visible to the observer. This is done by first determining which star is in front of the other. Given that the plane of the sky corresponds to the y'–z' plane, the center of mass of the star that is closest to the observer has the coordinate value $x' > 0$ (see Fig. 3).

If the star in front is partially or entirely eclipsing the background star, then the distances between their centers of mass projected onto the y'–z' plane must be less than the sum of their radii; or, for an eclipse to be taking place,

$$\sqrt{\left(y'_{fc} - y'_{bc}\right)^2 + \left(z'_{fc} - z'_{bc}\right)^2} < R_f + R_b, \tag{4}$$

where (y'_{fc}, z'_{fc}) and (y'_{bc}, z'_{bc}) are the locations of the centers of mass of the foreground and background stars, respectively, as projected onto the plane of the sky.

To optimize the computation of the integrated luminous flux, it is appropriate to locate the line of symmetry between the centers of mass of the two stars. Again referring to Fig. 3, we see that the angle between the y'-axis and the line connecting the projected centers of mass is given by

$$\theta'_0 = \tan^{-1}\left(\frac{z'_{fc} - z'_{bc}}{y'_{fc} - y'_{bc}}\right). \tag{5}$$

Once the background star has been identified and the line of symmetry determined, the decrease in the amount of light due to the eclipse can be computed by first finding out which parts of the background star are behind the foreground star. If a point on the eclipsed disk is within a distance R_f of the center of the foreground star's disk as projected onto the y'–z'

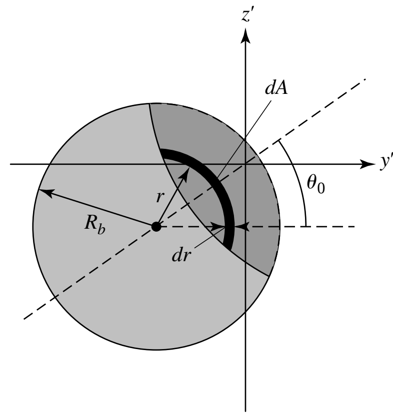

FIGURE 4 The region of the background star being eclipsed is shown in dark gray. Numerical integration of the flux over arcs of various radii r and thickness dr makes it possible to determine how much light is blocked by the foreground star.

plane, then that point on the star's surface is behind the foreground star. In other words, the condition for a point (y'_b, z'_b) on the disk of the background star to be behind the disk of the foreground star is

$$\sqrt{\left(y'_b - y'_{fc}\right)^2 + \left(z'_b - z'_{fc}\right)^2} < R_f. \tag{6}$$

The eclipsed region can then be mapped out by starting along the line of symmetry at some distance r from the center of the disk of the background star and moving at increasing angles of $\Delta\theta'$ from θ'_0 until the inequality of Eq. (6) is no longer satisfied or until $\Delta\theta'$ exceeds 180°. In the later case, this would imply that the entire disk within the radius r of its center is eclipsed. Given the assumption of spherical symmetry, the region of the background star's disk between $\theta'_{\rm min} = -\Delta\theta' + \theta'_0$ and θ'_0 is identical to the region between θ'_0 and $\theta'_{\rm max} = \Delta\theta' + \theta'_0$ for a fixed value of r (see Fig. 4). For an arc-shaped surface of radius r and width dr, the area of the surface is given by

$$dA = r\,dr\,\left(\theta'_{\rm max} - \theta'_{\rm min}\right) = 2r\,dr\,\Delta\theta'. \tag{7}$$

Now, if the luminous flux at that radius from the center of the background star's disk is $F(r)$, the amount of light in that arc that has been blocked is given by

$$dS = F(r)\,dA = 2F(r)\,r\,dr\,\Delta\theta'. \tag{8}$$

By subtracting the loss in light due to each eclipsed arc from the total light of the uneclipsed star, we can determine the total amount of light received from the background star during a partial or total eclipse. (Note that due to the effects of limb darkening, $F(r)$ is not constant across the entire disk.)

Finally, all that remains is to convert the total amount of light received to magnitudes.

TwoStars implements each of the ideas described. An example of the input required for TwoStars, along with the first ten lines of model output, is shown in Fig. 5.

The source code for (TwoStars), together with compiled versions of the program, is available for download from the companion website at
http://www.aw-bc.com/astrophysics.

```
Specify the name of your output file: c:\YYSgr.txt


Enter the data for Star #1
   Mass (solar masses):            5.9
   Radius (solar radii):           3.2
   Effective Temperature (K):      15200

Enter the data for Star #2
   Mass (solar masses):            5.6
   Radius (solar radii):           2.9
   Effective Temperature (K):      13700

Enter the desired orbital parameters
   Orbital Period (days):          2.6284734
   Orbital Eccentricity:           0.1573
   Orbital Inclination (deg):      88.89
   Orientation of Periastron (deg): 214.6

Enter the x', y', and z' components of the center of mass velocity vector:
Notes:  (1)  The plane of the sky is (y',z')
        (2)  If v_x' < 0, then the center of mass is blueshifted

   v_x' (km/s)                     0
   v_y' (km/s)                     0
   v_z' (km/s)                     0


The semimajor axis of the reduced mass is     0.084318 AU
   a1 =      0.040971 AU
   a2 =      0.043166 AU

         t/P         v1r (km/s)      v2r (km/s)          Mbol        dS (W)
       0.000000      112.824494     -118.868663      -2.457487   0.000000E+00
       0.000999      114.247263     -120.367652      -2.457487   0.000000E+00
       0.001998      115.660265     -121.856351      -2.457487   0.000000E+00
       0.002997      117.063327     -123.334576      -2.457487   0.000000E+00
       0.003996      118.456275     -124.802147      -2.457487   0.000000E+00
       0.004995      119.838940     -126.258884      -2.457487   0.000000E+00
       0.005994      121.211155     -127.704610      -2.457487   0.000000E+00
       0.006993      122.572755     -129.139152      -2.457487   0.000000E+00
       0.007992      123.923575     -130.562338      -2.457487   0.000000E+00
       0.008991      125.263455     -131.973998      -2.457487   0.000000E+00
```

FIGURE 5 An example of the input required for the Fortran 95 command-line version of TwoStars for the system YY Sgr. The first ten lines of model output to the screen are also shown.

SUGGESTED READING

Technical

Bradstreet, D. H., and Steelman, D. P., "Binary Maker 3.0—An Interactive Graphics-Based Light Curve Synthesis Program Written in Java," *Bulletin of the American Astronomical Society*, January 2003.

Kallrath, Josef, and Milone, Eugene F., *Eclipsing Binary Stars: Modeling and Analysis*, Springer-Verlag, New York, 1999.

Terrell, Dirk, "Eclipsing Binary Stars: Past, Present, and Future," *Journal of the American Association of Variable Star Observers*, *30*, 1, 2001.

Van Hamme, W., "New Limb-Darkening Coefficients for Modeling Binary Star Light Curves," *The Astronomical Journal*, *106*, 2096, 1993.

Wilson, R. E., "Binary-Star Light-Curve Models," *Publications of the Astronomical Society of the Pacific*, *106*, 921, 1994.