About the existance of a Poincaré transition in rotating nuclei

J. Bartel
IPHC and Université de Strasbourg,

K. Pomorski
Uniwersytet Marie Curie-Skłodowskiej, Lublin

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Breaking of left-right symmerty -

a reminder of a famous publication

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Equilibrium Configurations of Rotating Charged or Gravitating Liquid Masses with Surface Tension. II*

S. COHEN

Argonne National Laboratory, Argonne, Illinois 60440

F. PLASIL

Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

AND

W. J. SWIATECKI

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

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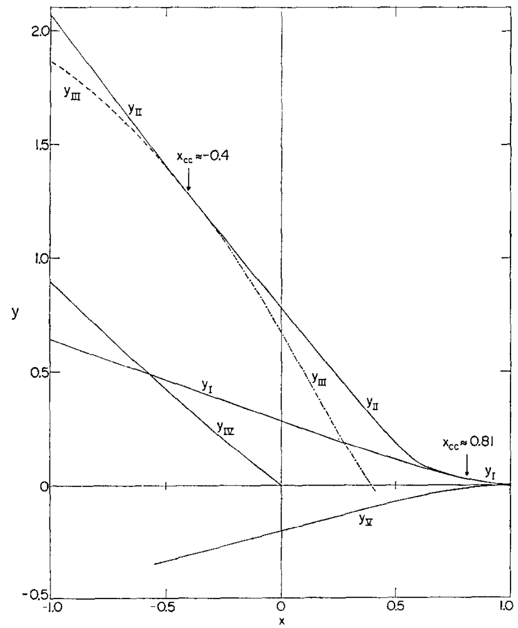


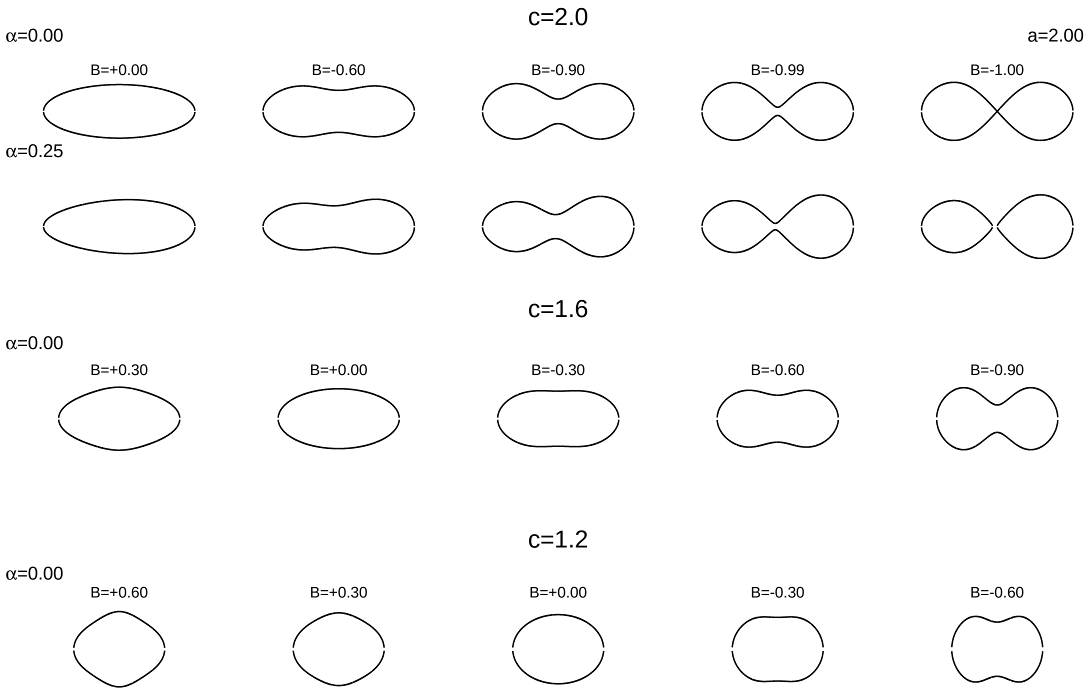
FIG. 2a. Various critical rotational parameters y in their dependence on the fissility parameter x. Triaxial shapes appear between y_I and y_{II}. Saddle shapes are stable against reflection asymmetric distortions to the right of the dot–dashed portion of y_{III}. Triaxial shapes are unstable against asymmetry between the dashed portion of y_{III} and y_{II}. The critical curves y_{IV} and y_V will be discussed in future installments of this series of papers.

Modified Funny-Hills Shape parametrisation

- **Huge variety of nuclear shapes (g.s. $\Longrightarrow$ scission point)**
- **Only a very few relevant deformation parameters**

$$\varrho_s^2(z) = \mathcal{N}\,(1-u^2)\left[1 - B\,e^{-\beta(u-\alpha)^2}\right] \qquad u = \frac{z - z_{sh}}{z_0} \qquad z_0 = c\,R_0$$

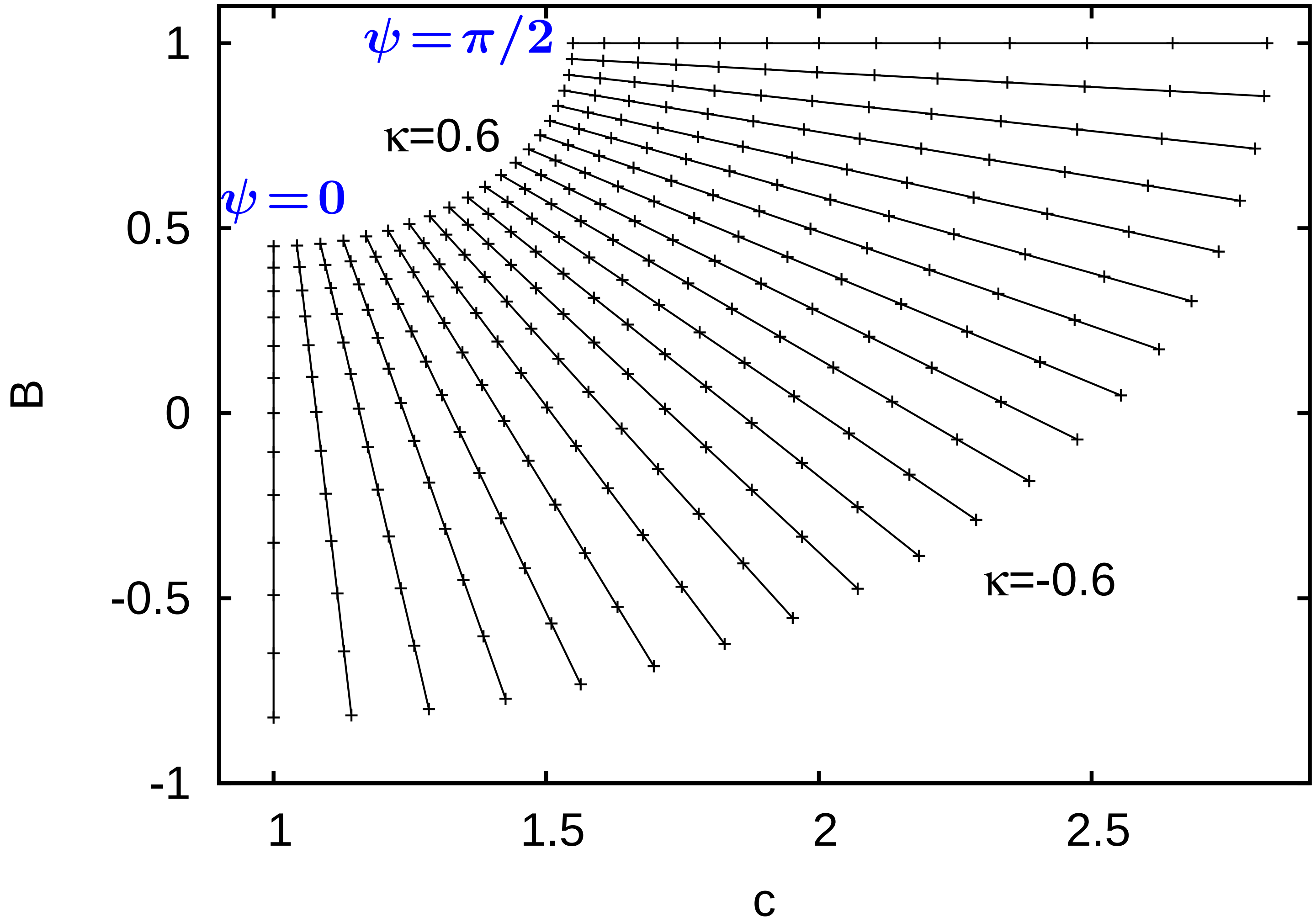
$\mathcal{N}$ normalization constant that depends on c, B and α



Difficulty: localize precisely the scission configuration

Solution: introduce new parameters ψ and κ

$$c = 1 + e^{-\kappa} \sin\psi \quad \text{and} \quad B = 1 - e^{-\kappa} \cos\psi$$



breaking axial symmetry

supose ellipsoidal shape $\perp$ to z axis

and introduce the non-axiality parameter

$$\eta = \frac{a_y - a_x}{a_y + a_x}$$

assume that η is independent of z

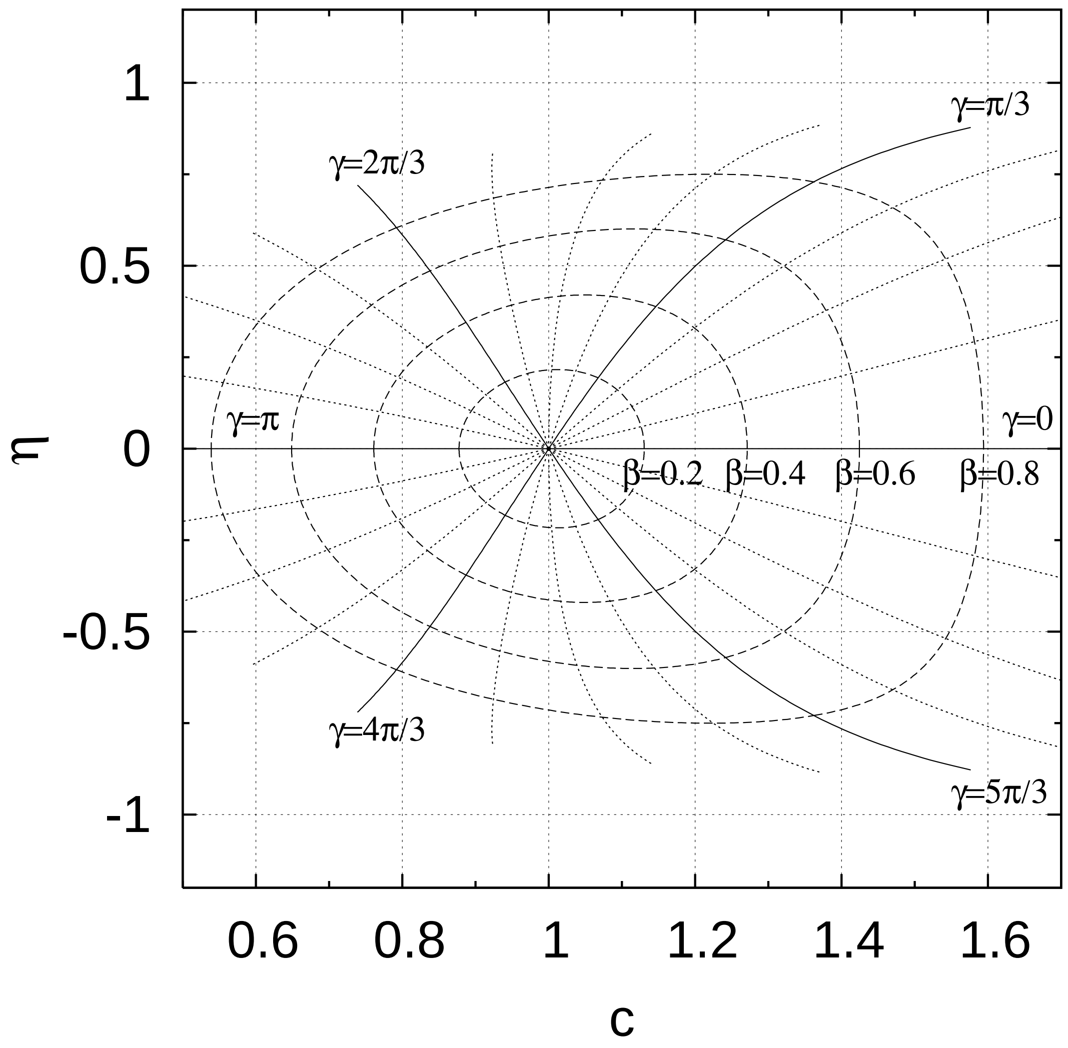
$$a_x(z) = \varrho_s(z) \left(\frac{1-\eta}{1+\eta}\right)^{1/2}$$

$$a_y(z) = \varrho_s(z) \left(\frac{1+\eta}{1-\eta}\right)^{1/2}$$

volume conservation then leads to

$$\widetilde{\varrho}_s^2(z,\varphi) = \varrho_s^2(z) \, \frac{1-\eta^2}{1+\eta^2+2\eta\cos(2\varphi)}$$

J. Bartel, F. Ivanyuk, K. Pomorski, IJMPE E19 (2010) 601



Transformation from (c, η) to (β, γ) in the pure spheroidal case

Lublin-Strasbourg Drop Model

According to the Strutinski theorem

$$E_{\text{tot}} = \int \mathcal{H}(\rho)\, d^3 r = E_{\text{mac}} + \delta E_{\text{mic}}$$

with the Lublin–Strasbourg drop

$$E_{\text{mac}}(Z,N,def) = a_{\text{vol}}\,(1-\kappa_{\text{vol}} I^2)\,A + a_{\text{surf}}\,(1-\kappa_{\text{surf}} I^2)\,A^{2/3}\,B_{\text{surf}}(def) + a_{\text{cur}}\,(1-\kappa_{\text{cur}} I^2)\,A^{1/3}\,B_{\text{cur}}(def) + \frac{3\,e^2}{5\,r_{\text{o}}}\,\frac{Z^2}{A^{1/3}}\,B_{\text{Coul}}(def) - C_4\,\frac{Z^2}{A} - E_{\text{cong}}$$

with shell and pairing corrections

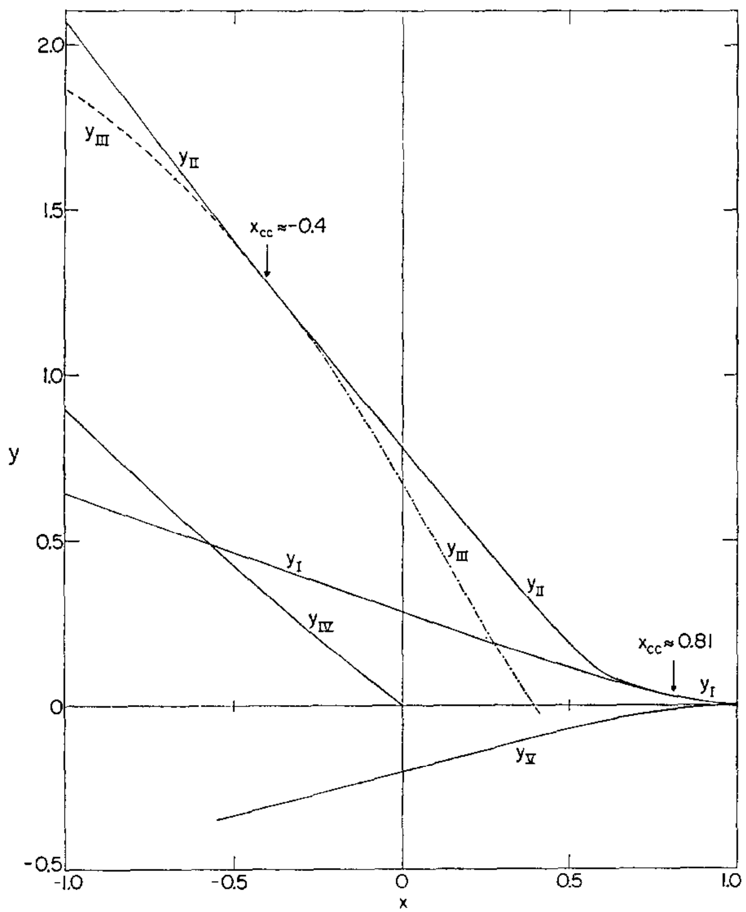
$$\delta E_{\text{mic}} = \delta E_{\text{shell}} + \delta E_{\text{pair}}$$

$$\delta E_{\text{shell}} = \sum_{\text{occ}} \varepsilon_\nu - \widetilde{E}$$

$$\delta E_{\text{pair}} = E_{\text{BCS}} - \sum_{\text{occ}} \varepsilon_\nu - \langle E_{\text{pair}} \rangle$$

Energy landscapes in light + medium-mass nuclei

studied nuclei: ^{46}Ti , ^{92}Mo , ^{138}Ba



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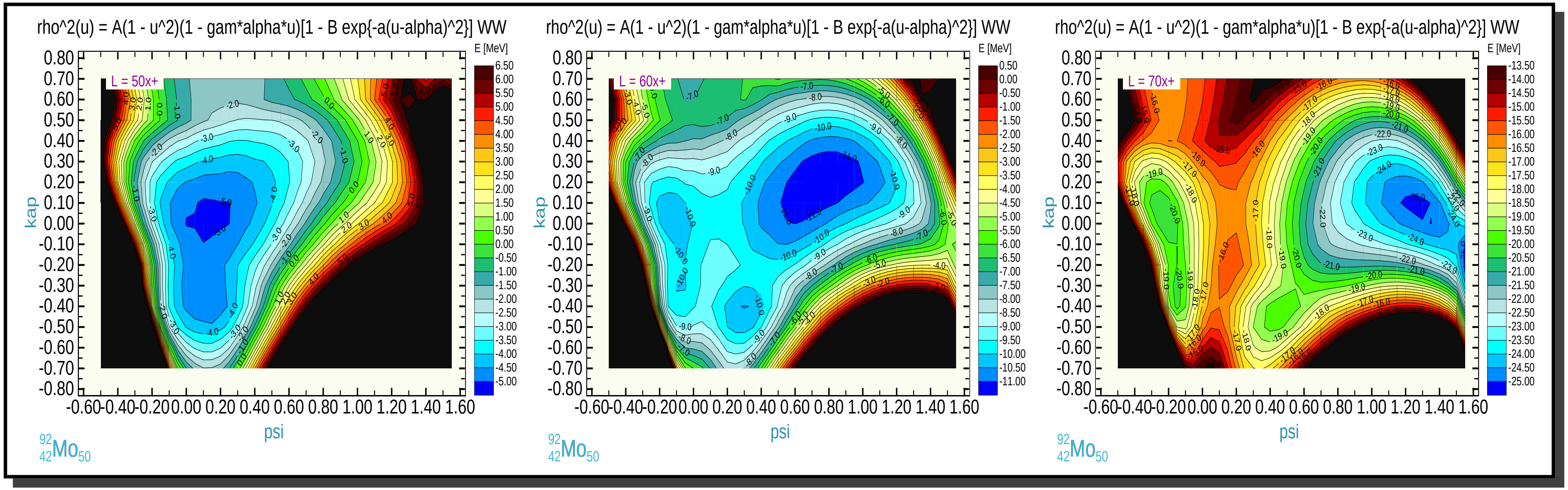
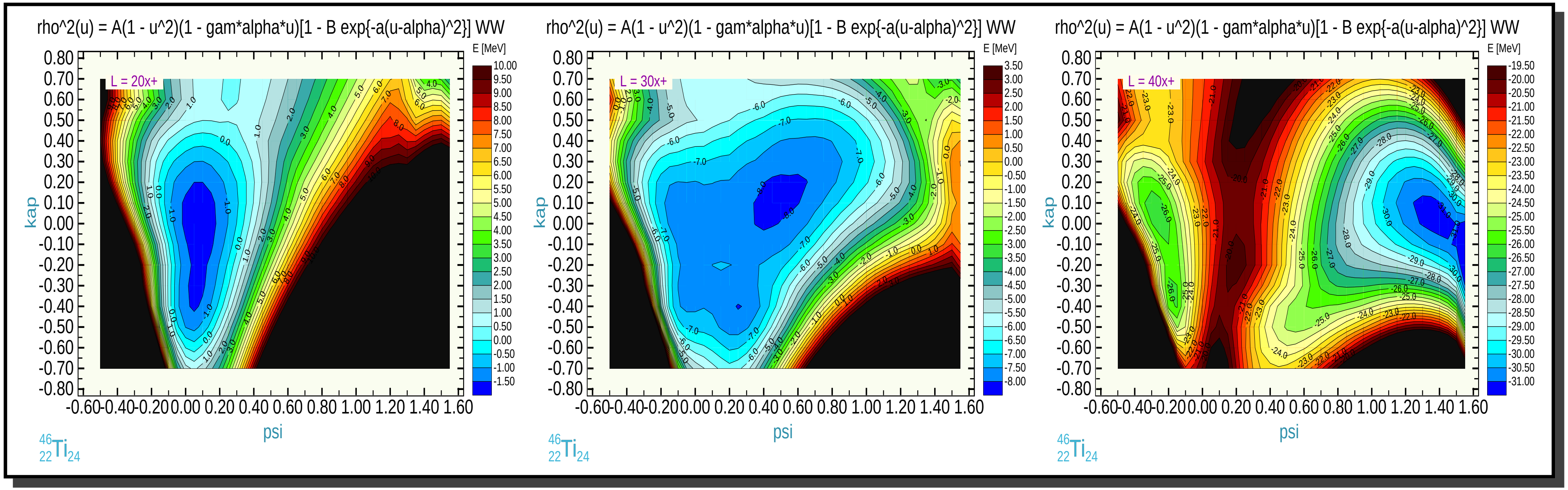
Instability against reflexion asymmetry when $x \leq 0.4$ for $L = 0$, decreasing for $L > 0$

A rough estimate:

$$x \approx \frac{1}{50}\frac{Z^2}{A}, \quad y \approx 2\,\frac{L^2}{A^{7/3}}$$

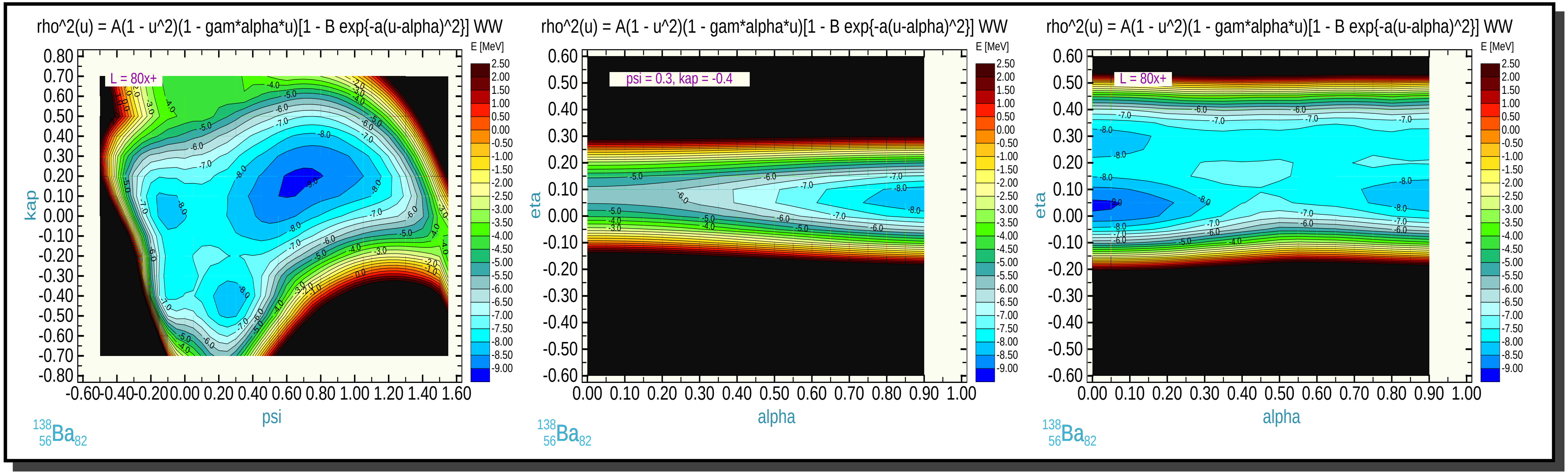

rho^2(u) = A(1 - u^2)(1 - gam*alpha*u)[1 - B exp{-a(u-alpha)^2}] WW
L = 50x+
L = 60x+
L = 70x+
E [MeV]
kap
psi
$^{92}_{42}Mo_{50}$


rho^2(u) = A(1 - u^2)(1 - gam*alpha*u)[1 - B exp{-a(u-alpha)^2}] WW
L = 20x+
L = 30x+
L = 40x+
E [MeV]
kap
psi
$^{46}_{22}Ti_{24}$

The Poincaré instability

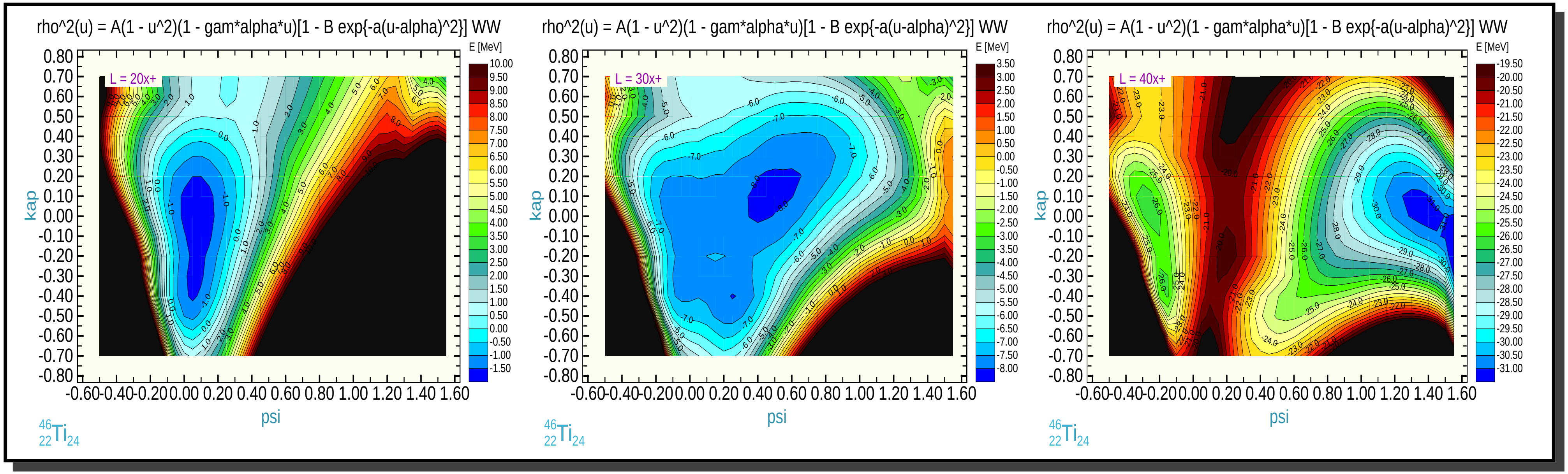
also motivated by the experimental studies of A. Maj & coworkers

(see also talk by Kasia Mazurek this conference)



in ^{138}Ba

in a light nucleus as a function of L



^{46}Ti

Remark:

rotation stabilizes the nucleus versus the Poincaré instability at fixed elongation

can we understand that?

consider moment of inertia of 2 touching spheres ($\alpha = 0$) with one of them swallowing the other one ($\alpha \to 1$) $\quad \mathcal{J} \downarrow$

$L \uparrow$ how to get more stability? $\Rightarrow$ $E_{rot} \downarrow$ $\mathcal{J} \uparrow$ $\Rightarrow$ $\alpha \downarrow$

Conclusions

- **Modified Funny-Hills (MFH) parametrisation describes nuclear shapes very well (already well known)**
- **Our LDM study with the LSD and the MFH parametrization allows for a systematic study of the Poincaré instability in light nuclei**
- **This instability against left-right asymmetry for certain stationary points already present in medium-mass nuclei with mass $A \approx 130$**
- **Higher angular momenta tend to stabilize the nucleus already pointed out by CPS 35 years ago**
- **More nuclear systems need to be studied, in particular concerning the effect of finite temperatures**
- **Comparison with Fedir Ivanyuk's result rather difficult "*optimal*" fission valley $\Leftrightarrow$ 4D energy landscape**