Remarks on some newest liquid drop models

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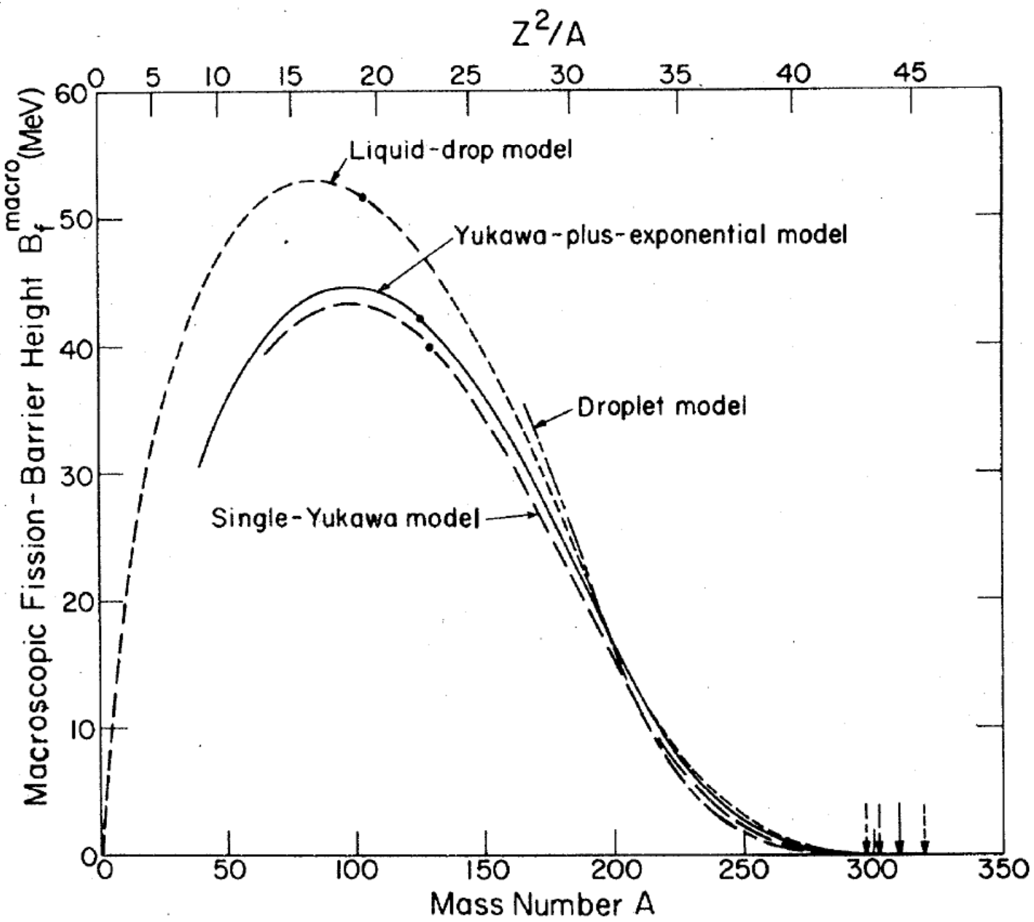
Contents:

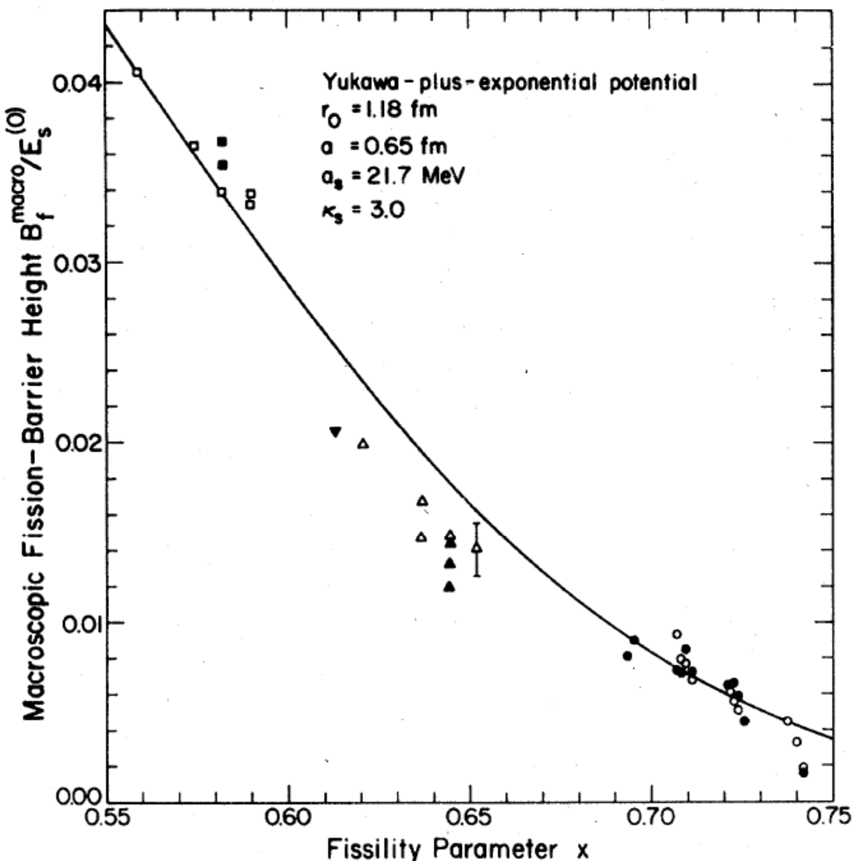
- Historical remarks
- Topographical theorem of Swiatecki
- Newest parametrisation of the liquid drop model
- Barrier heights in different models
- A small surprise for you
- Conclusion

Selected papers of the liquid drop model:

- C. F. v. Weizsäcker, Z. Phys. **96**, 431 (1935)
- H. A. Bethe, R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936)
- L. Meitner, O. R. Frisch, Nature **143**, 239 (1939)
- N. Bohr, J. A. Wheeler, Phys. Rev. **56**, 426 (1939)
- W. D. Myers, W. J. Świątecki, Nucl. Phys. **81**, 1 (1966)
- H. v. Groote, E. Hilf, Nucl. Phys. **A129**, 513 (1969)
- ....
- K. Pomorski, J. Dudek, Phys. Rev. C **67**, 044316 (2003)
- M. W. Kirson, Nucl. Phys. **A 798**, 29 (2008)
- G. Royer, Nucl. Phys. **A 807**, 105 (2008)
- L. G. Moretto, P. T. Lake, L. Phair, J. B. Elliot, Phys. Rev. C **86**, 021303(R) (2012)

Macroscopic fission barriers in different models:

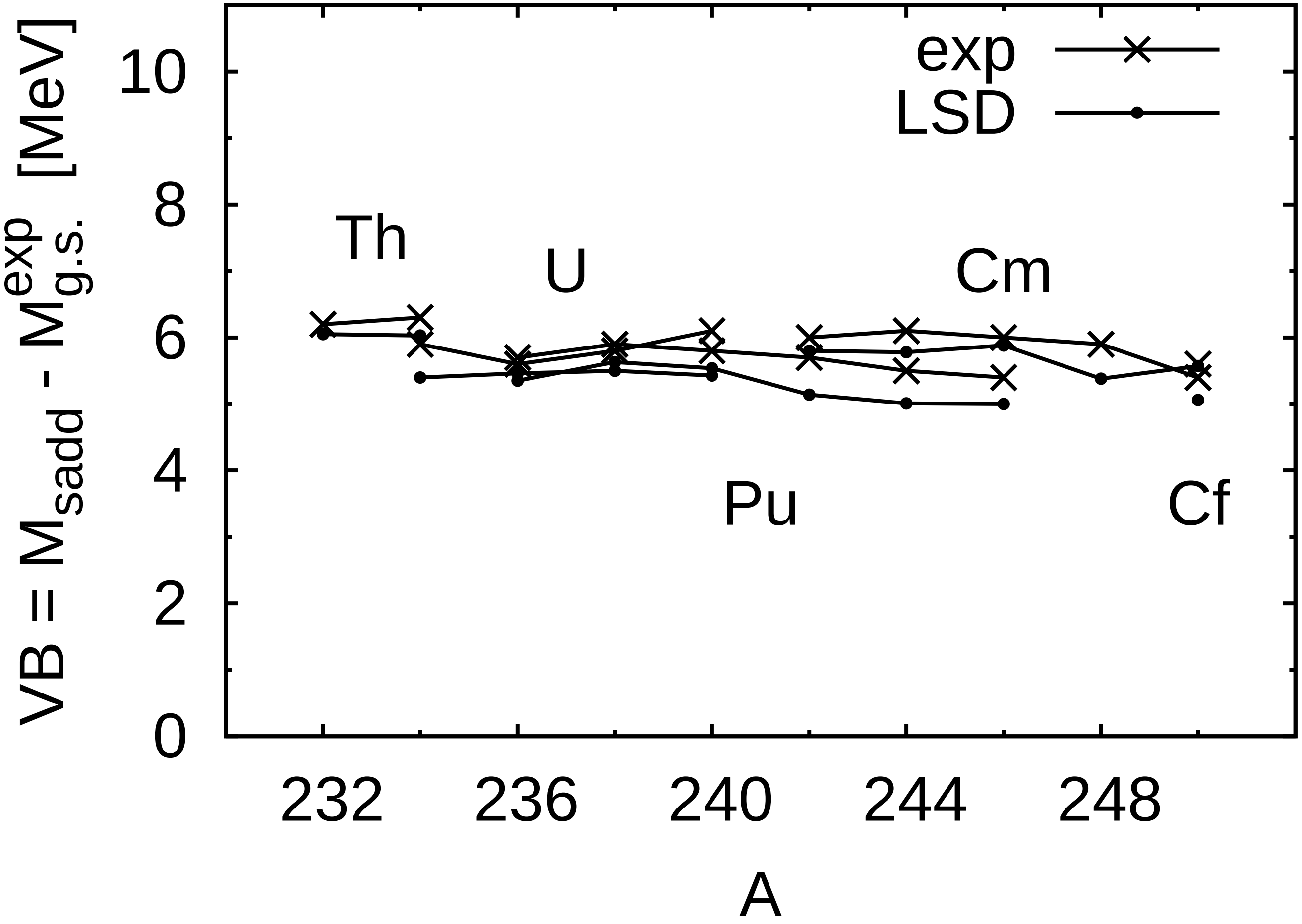




H.J. Krappe, J.R. Nix ans A.J. Sierk, Phys. Rev. C**20** (1979) 992.

$$V_{\text{sadd}} = M_{\text{mac}}(\text{sadd.}) - M_{\text{exp}}(\text{g.s.})$$

How accurate is LSD and the topographical theorem?



The rms deviation for the fission barrier heights is 0.310 MeV only!

J. Bartel, A. Dobrowolski, and K. Pomorski, IJMP **E16**, 459 (2007)

W. D. Myers, W.J. Świątecki, Nucl.Phys. **A612** (1997) 249. ← Topographical theorem

Lublin-Strasbourg Drop Model:

$$
\begin{array}{ll}
M(Z,N;\mathrm{def}) & = Z M_{\mathrm{H}} + N M_{\mathrm{n}} - 0.00001433\, Z^{2.39} \\
\textit{volume} & + b_{\mathrm{vol}}\; (1 - \kappa_{\mathrm{vol}}\, I^2\,)\, A \\
\textit{surface} & + b_{\mathrm{surf}}\, (1 - \kappa_{\mathrm{surf}} I^2\,)\, A^{2/3} B_{\mathrm{surf}}(\mathrm{def}) \\
\textit{curvature} & + b_{\mathrm{cur}}\; (1 - \kappa_{\mathrm{cur}}\, I^2\,)\, A^{1/3} B_{\mathrm{cur}}(\mathrm{def}) \\
\textit{Coulomb} & + \frac{3}{5}\, e^2 \frac{Z^2}{r_0^{ch} A^{1/3}}\, B_{\mathrm{Coul}}(\mathrm{def}) - \; C_4 \frac{Z^2}{A} \\
\textit{corrections} & + E_{\mathrm{micr}}(Z,N;\mathrm{def}) + E_{\mathrm{cong}}(Z,N)
\end{array}
$$

Liquid drop parameters – 2003

Term	Units	LDM	LSD
b_{vol}	MeV	-15.8484	-15.4920
κ_{vol}	-	1.8475	1.8601
b_{surf}	MeV	19.3859	16.9707
κ_{surf}	-	1.9830	2.2938
b_{cur}	MeV	0	3.8602
κ_{cur}	-	0	-2.3764
r_0	fm	1.18995	1.21725
C_4	MeV	1.1995	0.9181
δB	MeV	0.732	0.698

Fit to the experimental binding energies of 2766 isotopes.

K. Pomorski, J. Dudek: Phys. Rev. **C67**, 044316 (2003).

Modified Funny-Hills parametrisation:

Funny-Hills shape definition reads:

$$\tilde{\rho}_s^2(z) = \begin{cases} R_0^2 c^2 \left(1-u^2\right)\left(A+\alpha u+Bu^2\right), & \text{for} \quad B \geq 0 \\ R_0^2 c^2 \left(1-u^2\right)\left(A+\alpha u\right)\exp\left(Bc^3u^2\right) & \text{for} \quad B \leq 0\,, \end{cases}$$

where $u = \frac{z-z_{sh}}{z_0}$, $z_0 = c\,\mathrm{R}_0$ and $z_{sh} = -c^3\alpha z_0/5$.
The parameter c describes the elongation of nucleus.

Our new shape definition with the Gaussian neck reads:

$$\varrho_s^2\,(z,\varphi) = \frac{R_0^2}{c\,f(a,B)}\,(1-u^2)\,(1-\gamma\alpha u)\,\left[1 - B\,e^{-a^2(u-\alpha)^2}\right]\;\frac{1-\eta^2}{1+\eta^2+2\eta\cos(\varphi)}$$

where the factor $f(a,B)$ ensures the volume conservation.

B is the neck parameter and $B=1$ corresponds to the scission point. The deformation α gives the left-right asymmetry, while $\eta = \frac{a_y-a_x}{a_y+a_x}$ describes the nonaxiality.

J. Bartel, K.Pomorski, B. Nerlo-Pomorska, Int. J. Mod. Phys. **E21** (2012) 1250023.

Reexamination and extension of the liquid drop model: Correlation between liquid drop parameters and curvature term

L. G. MORETTO, P. T. LAKE, L. PHAIR, AND J. B. ELLIOTT PHYSICAL REVIEW C **86**, 021303(R) (2012)

TABLE II. Fits from the four mass equations described in the text. All parameters are in units of mega–electron volts. The value in parentheses is the uncertainty in the last digit.

Fit	a_v	a_s	a_r	k	a_c	δ	r_n (fm)	χ^2
i	15.597(7)	17.32(2)	$a_r = 0$	1.8048(9)	0.7060(4)	11.4(2)	–	0.58
ii	14.843(3)	$a_v = a_s$	$a_r = 0$	1.7196(16)	0.6585(4)	10.1(6)	–	4.24
iii	15.25(3)	15.17(17)	3.8(3)	1.779(2)	0.6932(11)	11.3(2)	0.60(5)	0.54
iv	15.264(4)	$a_v = a_s$	3.60(3)	1.7805(8)	0.6938(3)	11.3(2)	0.566(5)	0.54

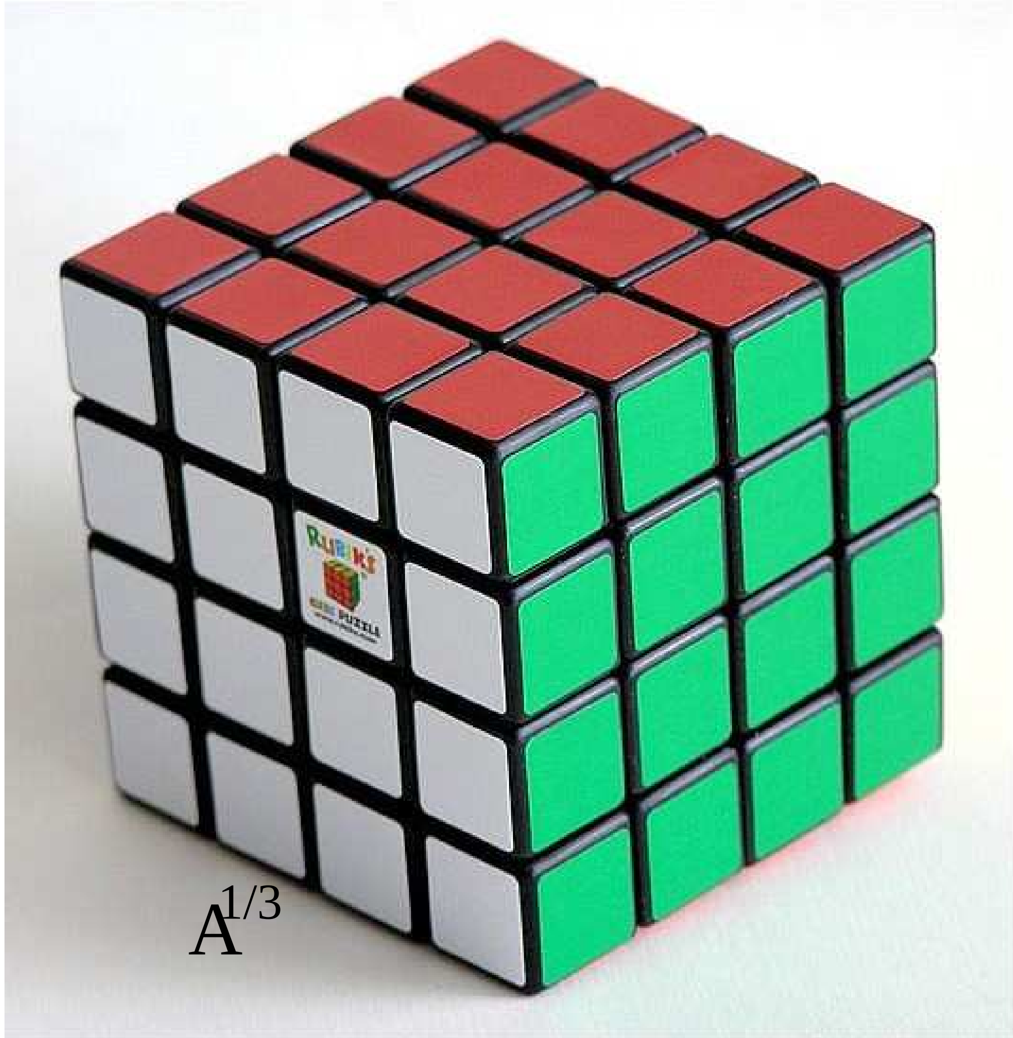
x

$$E_B = (-a_v A + a_s A^{2/3} + a_r A^{1/3})\left(1 - k\left(\frac{|I|(|I|+2)}{A^2}\right)\right) + a_c \frac{Z(Z-1)}{A^{1/3}} \pm \frac{\delta}{\sqrt{A}},$$

where $I = N - Z$

	a_v	a_r	k	x	a_c	δ	χ^2
iv	15.264(4)	3.60(3)	1.7805(8)	2	0.6938(3)	11.3(2)	0.54
v	15.247(4)	3.76(3)	1.7944(10)	1.51(3)	0.6913(3)	11.3(2)	0.46

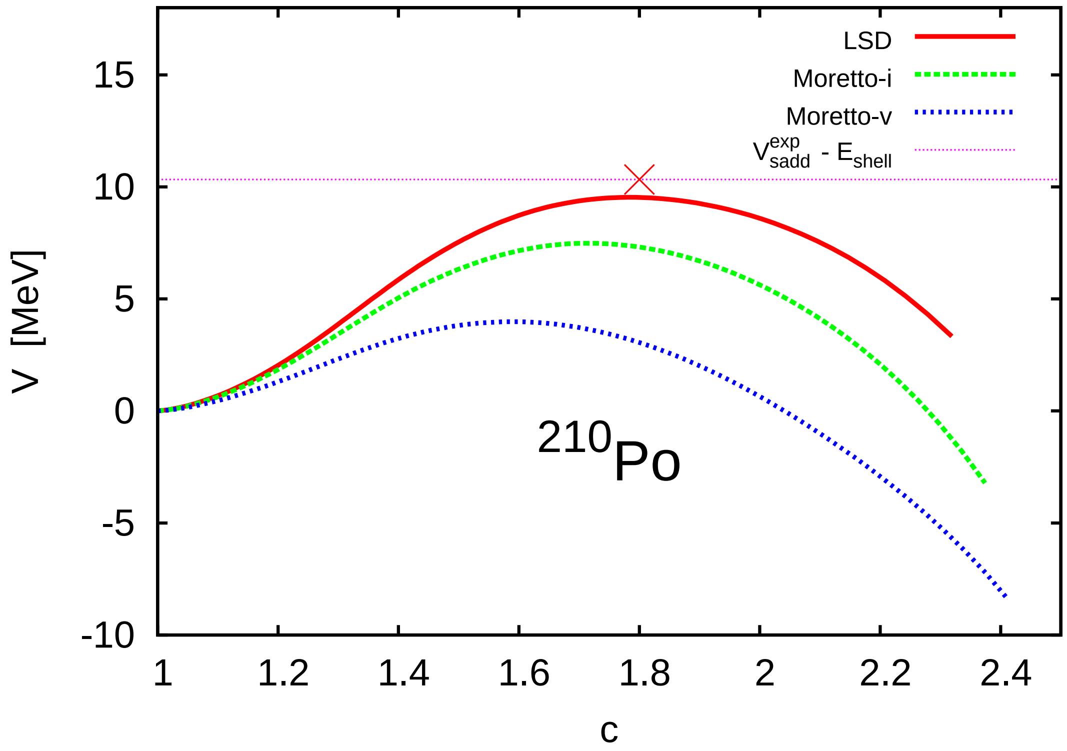
Qubic model of a nucleus (Moretto et al.)

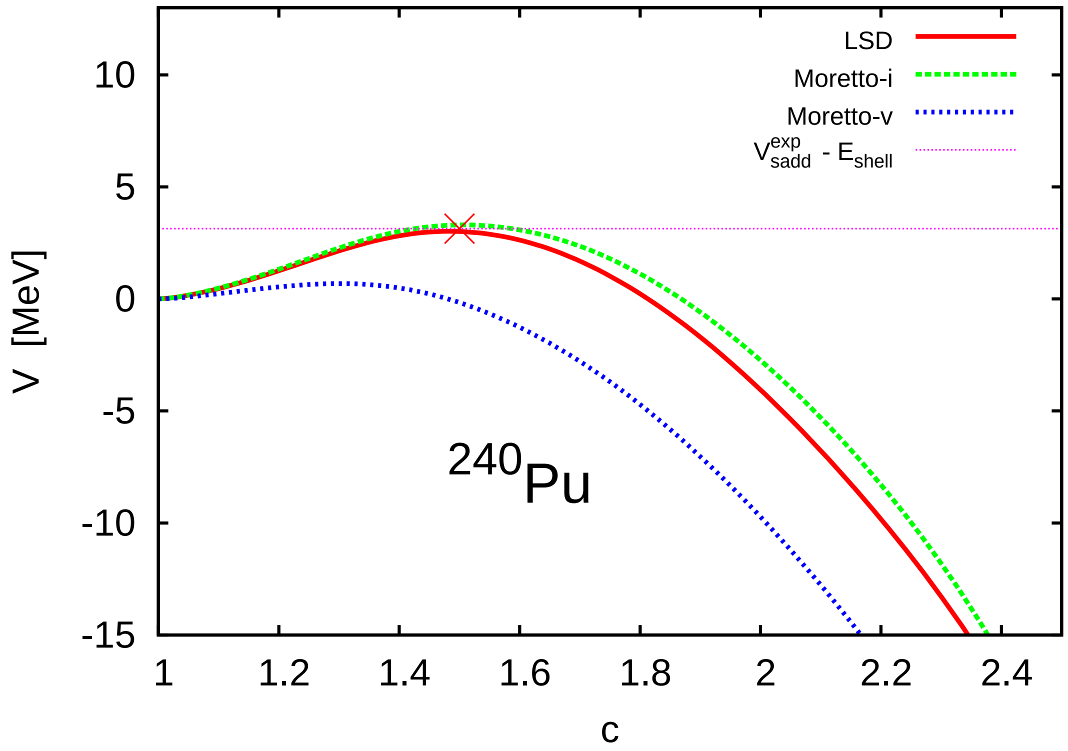


- b energy of a single bond,
- $E_{\mathrm{vol}} = 6 \cdot b/2 \times A$ – volume energy,
- $6A^{2/3}$ number of nucleons at the surface,
- $E_{\mathrm{surf}} = -b/2 \cdot 6A^{2/3}$ – surface energy

$$E_{\mathrm{LD}} = a_{\mathrm{vol}}(A - A^{2/3}) + E_{\mathrm{LD}}$$

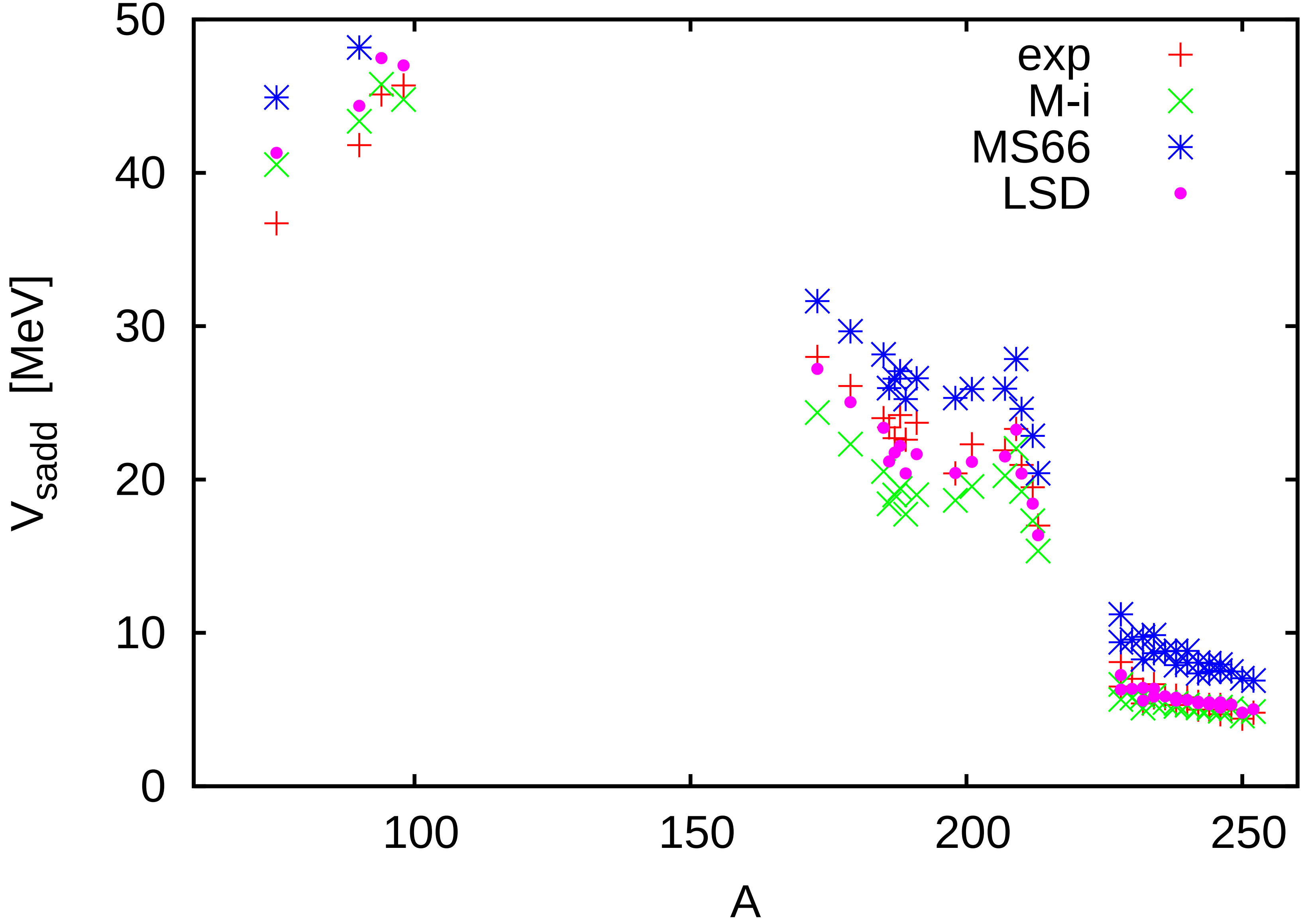
Fission barriers in LSD and Moretto (i) and (v) models





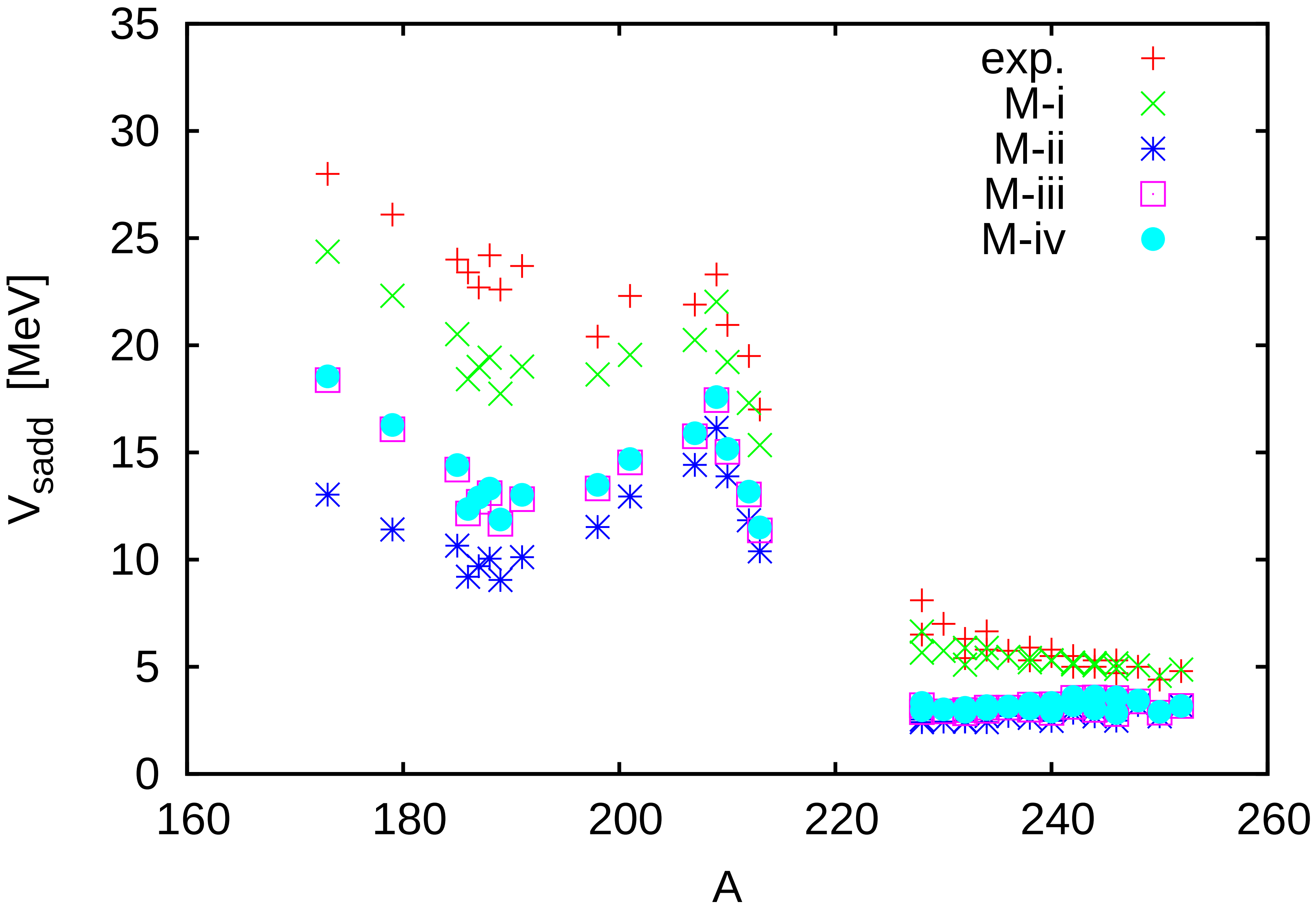
Topographical theorem of Świątecki was used here to renormalise the experimental barrier heights.

Fission barrier heights in Myers-Świątecki, LSD and Moretto models

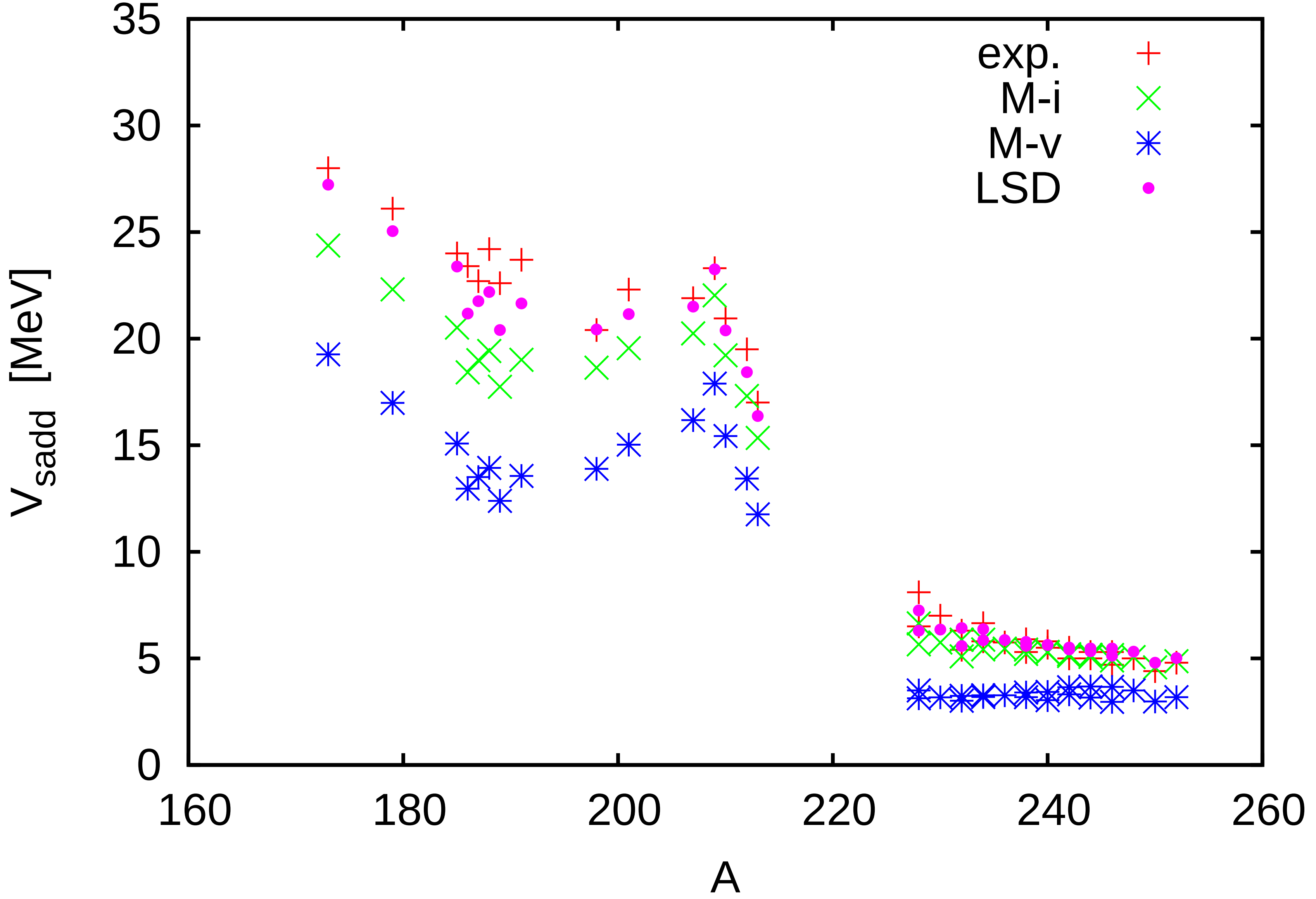


The experimental data are taken from W.D. Myers, W.J. Świątecki, Nucl.Phys. **A612** (1997) 249.

Fission barrier heights in the Moretto models



Fission barrier heights in the Moretto (i), (v) models



The LD model (i) has no curvature term while (v) has it and in addition has a free x-parameter in the isospin dependent term $I(I+x)$, what leads to the best mass fit.

On the coefficients of the liquid drop model mass formulae and nuclear radii

G. Royer

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G. Royer / Nuclear Physics A 807 (2008) 105–118 107

Different subsets of the following expansion of the nuclear binding energy in powers of $A^{-1/3}$ and $|I|$ have been considered:

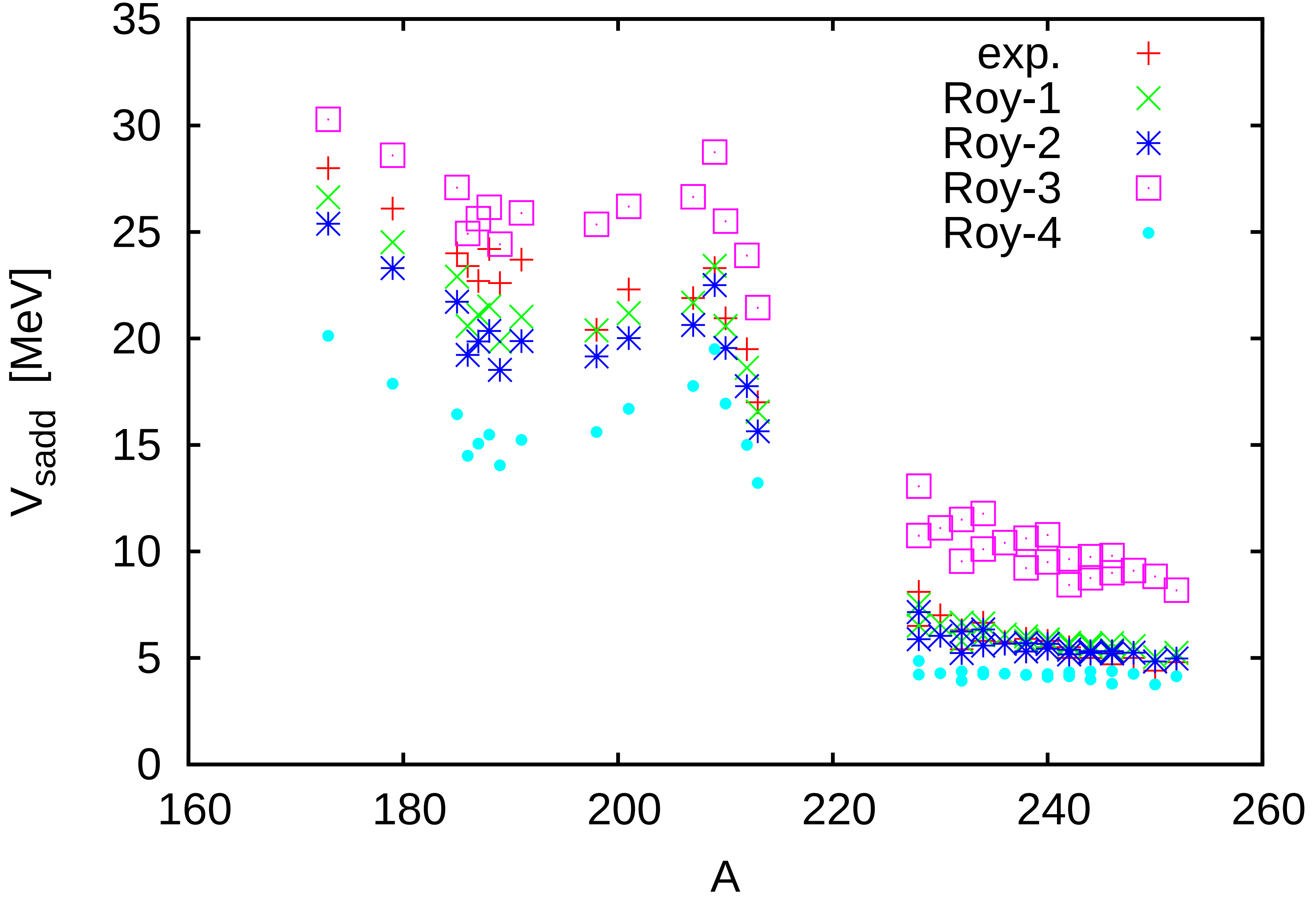
$$B = a_v\left(1 - k_{v_1}|I| - k_{v_2}I^2 - k_{v_3}I^4\right)A - a_s\left(1 - k_{s_1}|I| - k_{s_2}I^2 - k_{s_3}I^4\right)A^{2/3}$$
$$- a_k\left(1 - k_{k_1}|I| - k_{k_2}I^2 - k_{k_3}I^4\right)A^{1/3} - a_0A^0 - \frac{3}{5}\frac{e^2Z^2}{R_0} + f_p\frac{Z^2}{A} + a_{c,\text{exc}}\frac{Z^{4/3}}{A^{1/3}}$$
$$- E_{\text{pair}} - E_{\text{shell}} - E_{\text{Wigner}}. \quad (4)$$

38 different parameters sets adjusted to 2027 experimental masses are presented in four tables.

In the following we have chosen one sets from each table which gives the best mass fit.

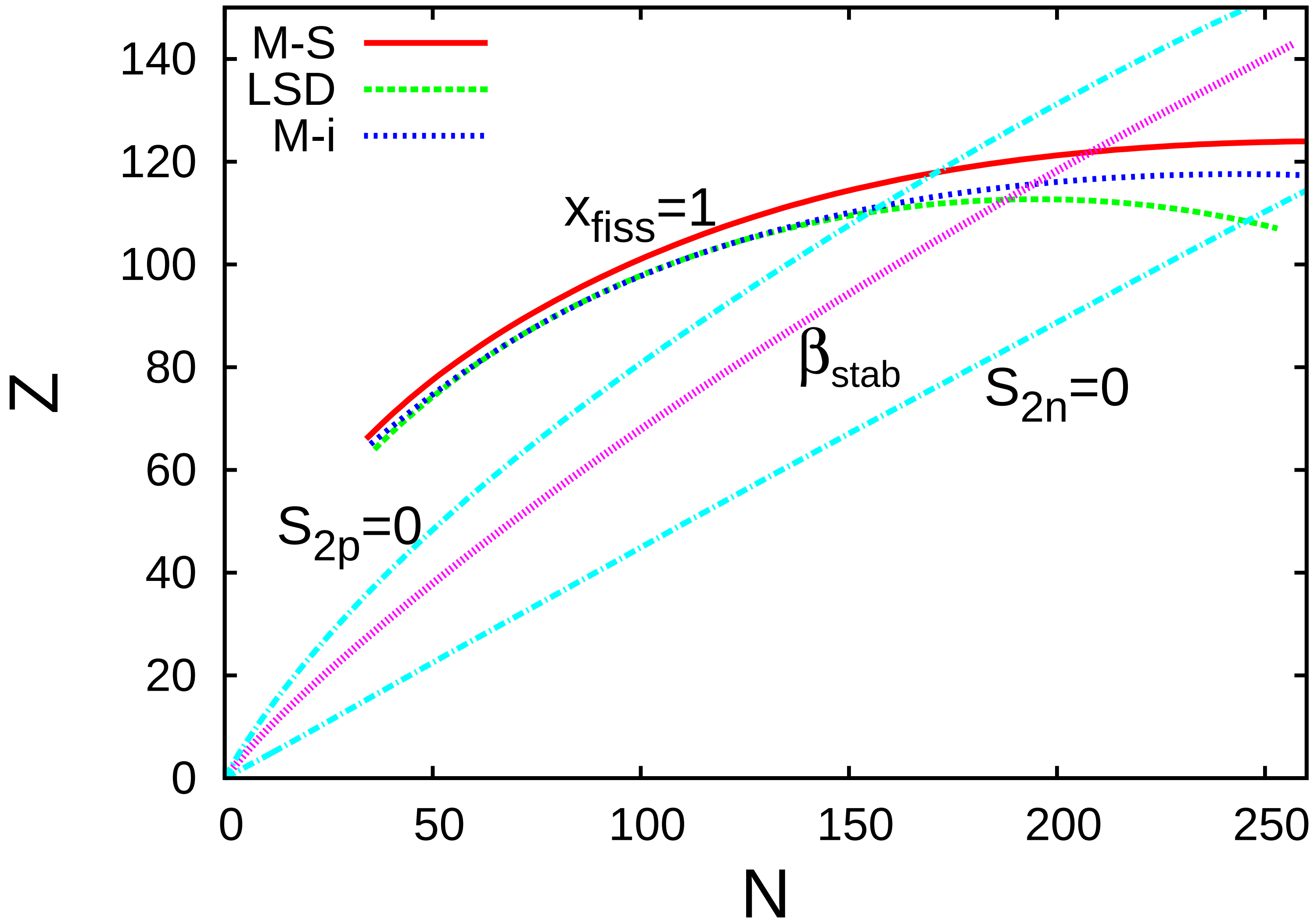
Similarly to Moretto at al. the fission barrier heights are not discussed in the paper.

Fission barrier heights in different LD models of Royer *



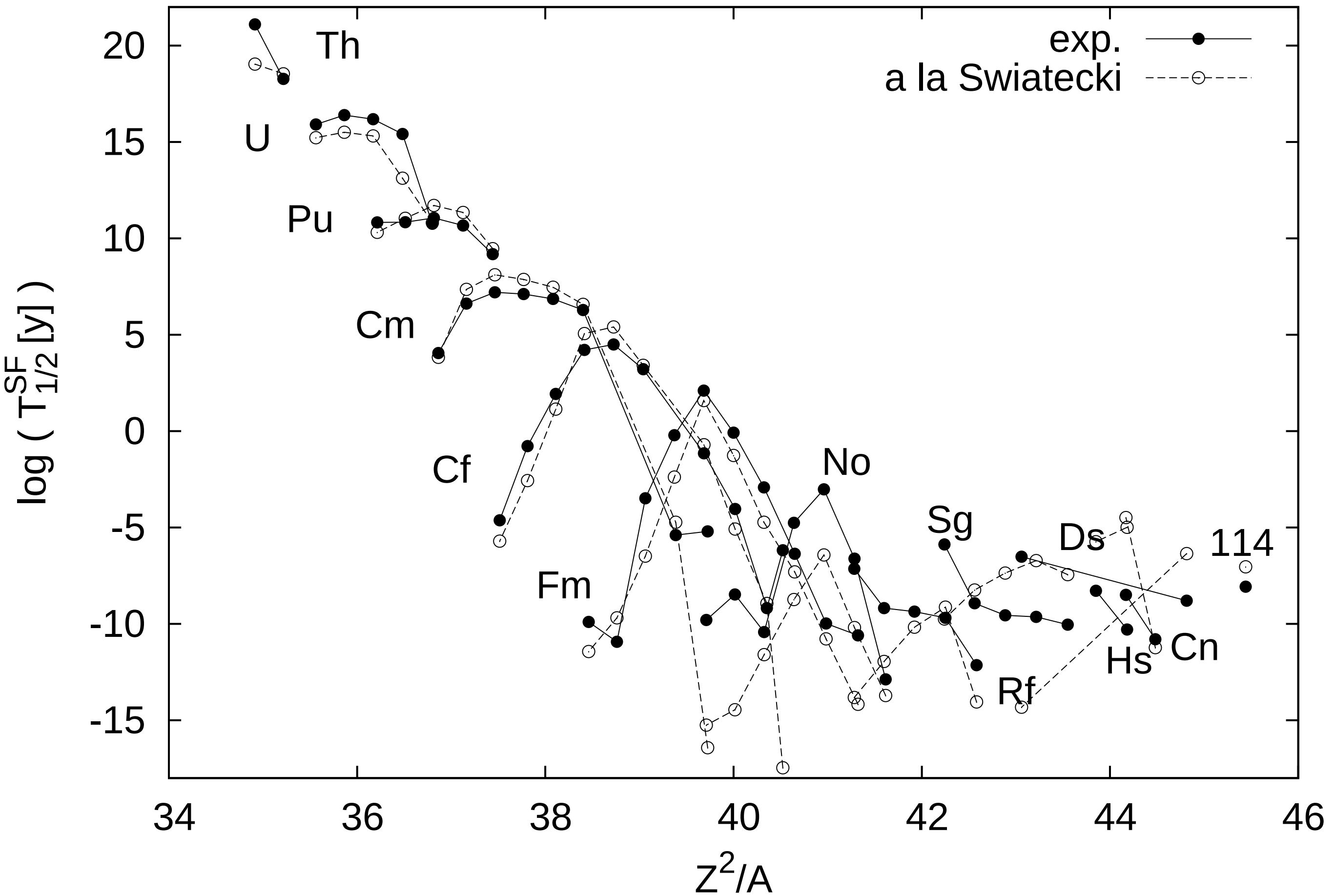
*G. Royer, Nucl. Phys. **A807** (2008) 105.

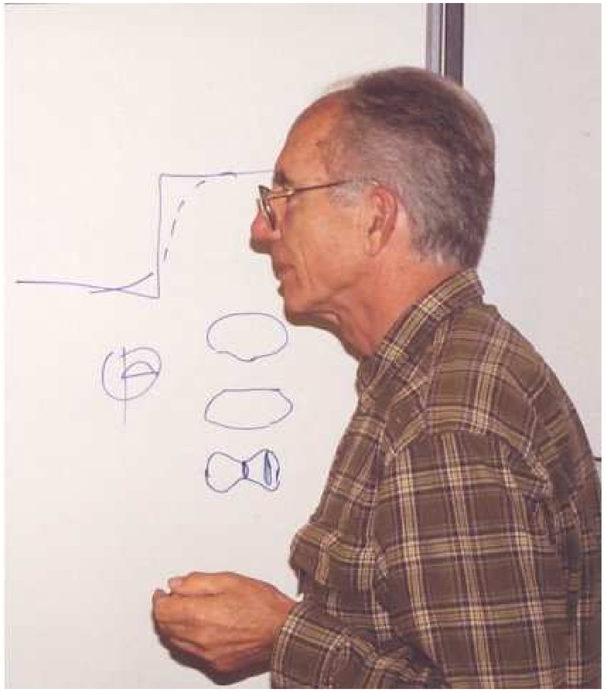
LD limits of stability



Proton, neutron drip lines and β-stability line are almost the same in all presented here liquid drop models.

Life-times à la Świątecki: Physics is simple





Experimental masses and spontaneous fission half-lives are from NUDAT. The estimates of are made in a simple one parameter model of Świątecki* using the LSD macroscopic masses:

$$\log T^{SF}_{1/2}[y] + k(M_{\rm LSD} - M_{\rm exp}) = a\,\frac{Z^2}{A} + b.$$

*W.J. Świątecki, Phys. Rev. **100** (**1955**) 937;
H.J. Krappe and K. Pomorski, *Theory of Nuclear Fission*, Lecture Notes in Physics 838, Springer, 2011.

Conclusions:

- Lublin-Strasbourg drop with the deformation dependent congruence term reproduces well all known barrier heights,

- All liquid drop models developed by Moretto et al. by adjustment to the known binding energies are unable to reproduce the experimental fission barrier heights,

- The above conclusion can be applied to all 38 LD models developed by Royer,

- It is not the best idea to remove from the LD model terms which have a clear physical meaning,

- The same remark refers to adding new terms to the LD model.

Thank you for your attention!