

Linear response theory for D-wave terms

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Outline

- 1 Introduction
- 2 Detecting instabilities
- 3 Instabilities finite nuclei
- 4 Linear response for fitting algorithms

Skyrme functionals

We can write the total energy of the system for a general Skyrme functional

$$\mathcal{E} = \mathcal{E}_{kin} + \mathcal{E}_{Skyrme} + \mathcal{E}_{pairing} + \mathcal{E}_{Coulomb} + \mathcal{E}_{corr.}$$

Skyrme functional

$$\begin{aligned}\mathcal{E}_{Skyrme} = & \sum_{t=0,1} \int d^3\mathbf{r} \left\{ C_t^\rho [\rho_0] \rho_t^2 + C_t^{\Delta\rho} \rho_t \Delta \rho_t + C_t^\tau \rho_t \tau_t + C_t^j \mathbf{j}_t^2 + C_t^s [\rho_0] s_t^2 \right. \\ & + C_t^{\nabla s} (\nabla \cdot s_t)^2 + C_t^{\Delta s} s_t \cdot \Delta s_t + C_t^T s_t \cdot \mathbf{T}_t + C_t^F s_t \cdot \mathbf{F}_t + C_t^{\nabla J} \rho_t \nabla \cdot \mathbf{J}_t \\ & + C_t^{\nabla j} s_t \cdot (\nabla \times \mathbf{j}_t) + C_t^{J^{(0)}} (J_t^{(0)})^2 + C_t^{J^{(1)}} (\mathbf{J}_t^{(1)})^2 + C_t^{J^{(2)}} \sum_{\mu\nu=x}^z J_{t\mu\nu}^{(2)} J_{t\mu\nu}^{(2)} \left. \right\}\end{aligned}$$

[E . Perlinska et al. Phys. Rev C 69, 014316 (2004)]

The coupling constants are fitted on data.

How to determine the coupling constants?

We impose a fitting protocol (observables and pseudo-observables)

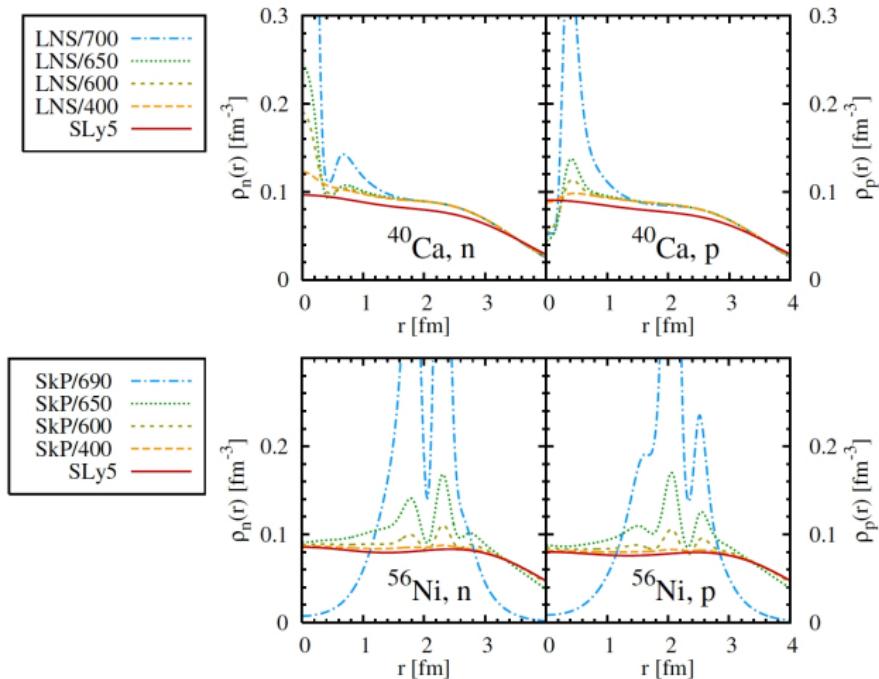
- IM properties (*i.e.* $E/A, K_\infty, m^*, \dots$)
- Ground state of some nuclei (*i.e.* $^{40}\text{Ca}, ^{48}\text{Ca}, ^{208}\text{Pb}, \dots$)
- Charge radii
- Spin orbit splitting
- ...

[M . Kortelainen et al. Phys. Rev C 85 (2012)024304]

An example

Good description of masses $\sigma_{rms} = 0.582$ MeV. [S . Goriely et al. Phys. Rev Lett. 112 (2009), 152503]

... unexpected results ...



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RPA formalism

The RPA correlated Green function is the solution of Bethe-Salpeter equation

$$\begin{aligned} G_{RPA}^{(S,M,I)}(q, \omega, \mathbf{k}_1) &= G_{HF}(q, \omega, \mathbf{k}_1) \\ &+ G_{HF}(q, \omega, \mathbf{k}_1) \sum_{S',M',I'} \int \frac{d^3 k_2}{(2\pi)^3} V_{ph}^{S,M,I;S',M',I'}(q, \mathbf{k}_1, \mathbf{k}_2) G_{RPA}^{S',M',I'}(q, \omega, \mathbf{k}_2) \end{aligned}$$

The response function is now defined as

$$\chi_{RPA}^\alpha(q, \omega) = g \int \frac{d^3 k_1}{(2\pi)^3} G_{RPA}^\alpha(q, \omega, \mathbf{k}_1)$$

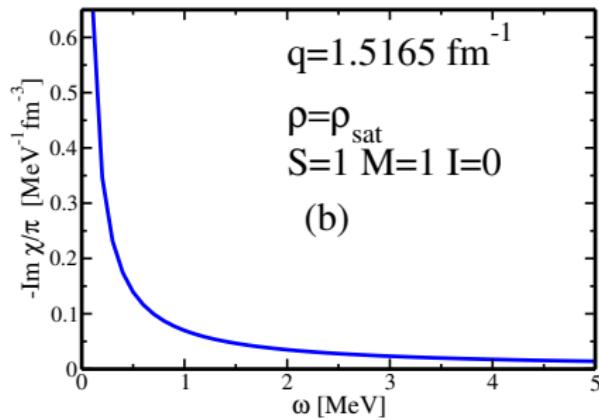
$g = 4$ is the degeneracy of SNM.

Instabilities in SNM I

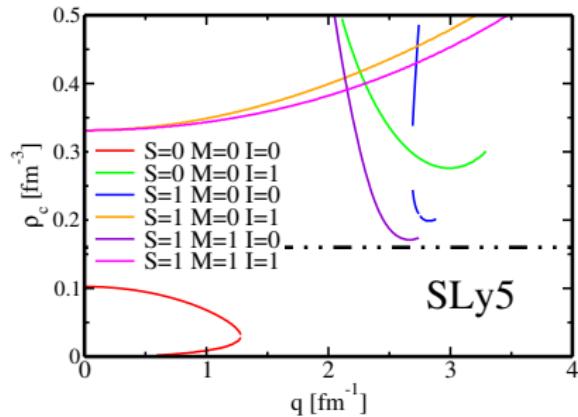
Poles

We look for the poles of the response function at zero energy.

$$1/\chi^{SMI}(\omega = 0, q) = 0$$



Instabilities in SNM II



Features

- Instabilities appear in different (S, I) channels
- Instabilities can appear for finite values of the transfer momentum q
- ≈ 1 second CPU time
- Analytic formulas simple to code

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Relations with finite nuclei

Can we relate a pole in SNM with an instability in a finite nucleus?

Protocol

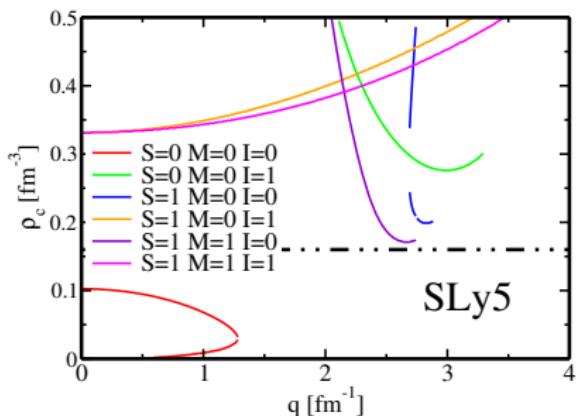
- Extensive calculations using different HF codes
 - ① Lenteur (spherical r-space code)
 - ② HOSPHE (spherical HO code)
 - ③ cr8 (3D cartesian code)
- Calculation of instabilities in IM

Result

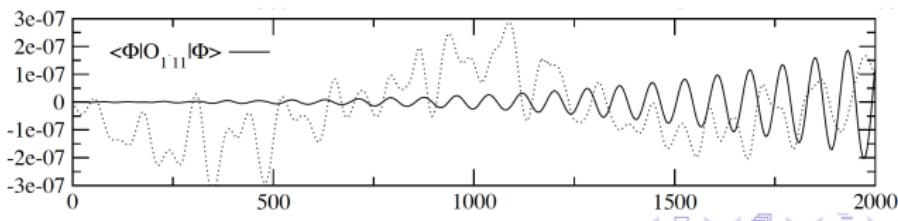
A pole in SNM at $\rho_{pole} < 1.4\rho_{sat}$ → nucleus unstable!

Good News! ...

According to our prescription SLy5 is unstable in the spin channel ($\rho_{crit}^{110} = 1.07\rho_0$)



SLy5 unstable in TDHFB calculations! [S. Fracasso, E. B. Suckling, and P. D. Stevenson, arXiv:1206.0056](#)



... Bad news

We analyze 236 standard Skyrme functionals

Channel	Number of EDF ($\rho_{crit} < 1.4\rho_{sat}$)
S=0, l=1	15
S=1 l=0	191
S=1 l=1	28

A catastrophe!

86% of analyzed EDF have at least one pole in SNM at $\rho_{crit} < 1.4\rho_{sat}$

And now?

It is mandatory to abandon such functionals!

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D wave

Example: we introduce a new term on top of a standard Skyrme functional

$$v^D(\mathbf{R}, \mathbf{r}) = t_D(1 + x_D \hat{P}_\sigma) [\mathbf{k}'^2 \delta(\mathbf{r}) \mathbf{k}^2 - (\mathbf{k}' \delta(\mathbf{r}) \mathbf{k})^2]$$

[K.Bennaceur et al., *in preparation*]

Not so easy to fit!!!

- Highly unstable and difficult to fit
- Naturalness analysis: 4th order c.c. smaller than 2 order

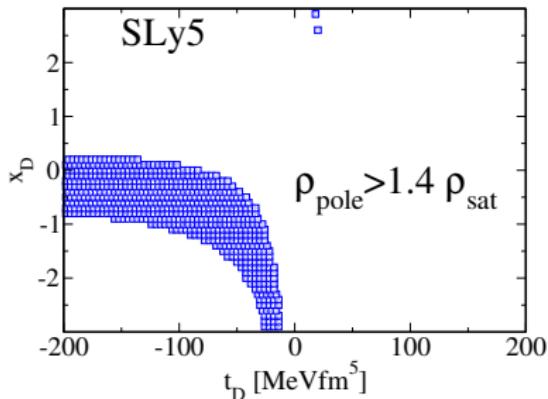
[M. Kortelainen et al., Phys.Rev C 82, 011403, (2010)]

- Density Matrix Expansion can be a starting point
- New couplings among spin channels (*i.e.* spin-orbit, tensor, ...)

New fitting protocol

1 step

- Starting point SLy5 functional (perturbative).
- Identification of a stable region of parameters x_D, t_D

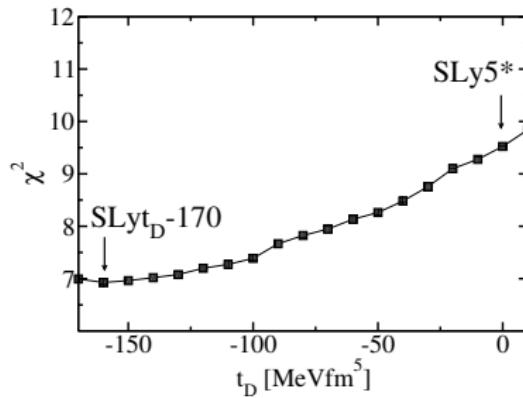
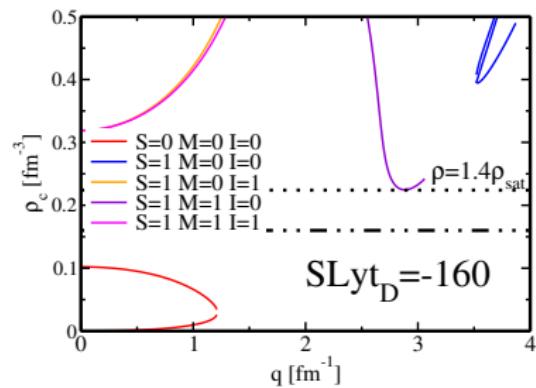


2 step

- Starting point: perturbative approach
- We re-fit all coupling constant
- LR criterium checked at every iteration

Preliminary results

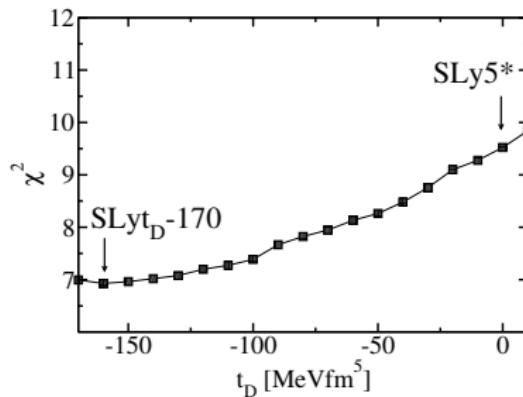
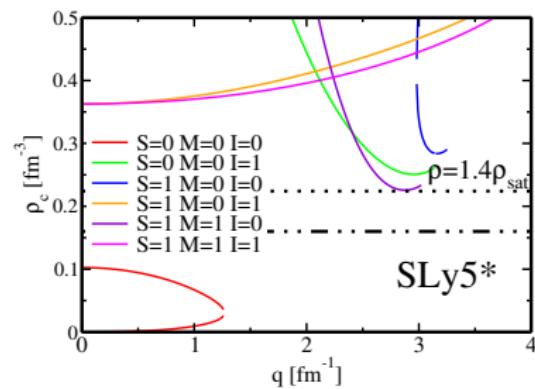
Exploratory analysis: we impose $x_D = 0$ and vary t_D .



The effect of the D-wave is small.

Preliminary results

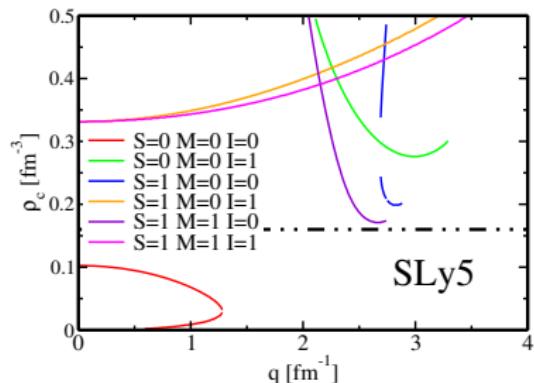
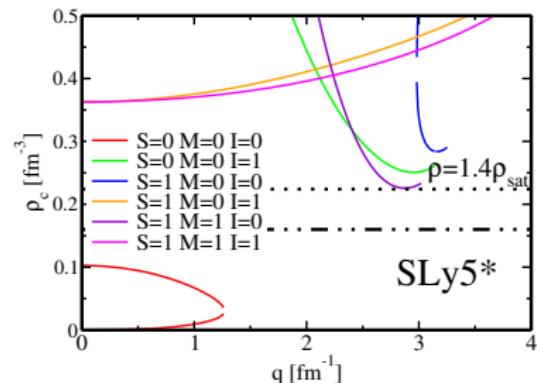
An interesting results: a stable version of SLy5!



A *modern* Skyrme functional free from pathologies.

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A *modern* Skyrme functional free from pathologies.

Conclusions

Some open questions:

- Why so many functionals close to instability?
- Is it the limit of the zero range force?
- It is possible to use LR to cure instabilities (no time consuming!)
- We can now explore a new generation of functionals

Status of LR calculations

- 3-body terms have been added [[J. Sadoudi et al. , private communication](#)]
- Pure Neutron Matter (Neutrino mean free path) [[A. Pastore et al. , arXiv:1207.4006 \(2012\)](#)]
- LR code now included in Saclay-Lyon fitting protocol
- Extension to asymmetric matter at finite temperature

Thank you!!!

I thank for collaboration and/or discussions

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SLy5* vs SLy5

	SLy5*	Sly5
t_0	-2495.31	-2484.880
t_1	484.02	483.130
t_2	-469.48	-549.400
t_3	13867.43	13763.000
x_0	0.620	0.778
x_1	-0.086	-0.328
x_2	-0.9469	-1.000
x_3	0.9343	1.267
α	1/6	1/6
W	120.25	126
J^2	yes	yes

