Mean Field Fluctuations at Scission

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1. Presentation of the problem

For the fissioning nucleus the total number of particles is conserved, for the two fragments (at large distance) each conserves its particle number.

For the corresponding canonically conjugated variables we have therefore one independent overall phase angle for the compound state, for the two fragments there are two independent phase angles, i.e. there is an increase in the symmetry in gauge space along the fission path, which implies a phase transition somewhere around scission.

Phase transitions are connected with larger fluctuations of the mean field. What fluctuates in this case?

It is the distribution of the constant total number of particles N over states belonging to the right N_r or the left fragment N_l. This requires a more precise definition of what “right” and “left” levels means.

2. Definition of "left" and "right" levels along the fission path

- construct a selfconsistent HF or HFB solution with the constraint of representing two clusters with particle numbers N_r and N_l with

$$N_r + N_l = N, \qquad N_r - N_l = \Delta N$$

  and center-of-mass distance D, chosen just large enough that no particles can be exchanged between the clusters.

- decrease D in small steps, follow all single-particle levels contiuously in the diabatic scheme, and indicate whether the level comes from the right or the left cluster. In this way all states can be classified as either r or l states all along the fission path.

- use the constraint that N_r particles shall occupy the lowest "r" levels and N_l particles the lowest "l" levels.

In this way right and left cluster are defined for all distances D, independent of the geometry of the mean-field potential.

Note that the correlation scheme depends only on N and ΔN, or equivalently, N_r and N_l !

3. The static generator-coordinate wave-function, based on constrained self-consistent HF or HFB states

To account for the expected fluctuations of the mean field along the fission path we propose a Hill-Wheeler ansatz for the wave function

$$|\Psi\rangle = \sum_{\boldsymbol{q}} f(\boldsymbol{q})|HF\rangle_{\boldsymbol{q}}$$

or - to account for pairing correlations -

$$|\Psi\rangle = \sum_{\boldsymbol{q}} f(\boldsymbol{q})|HFB\rangle_{\boldsymbol{q}}$$

where

$$\boldsymbol{q} \equiv \{D_i, N, \Delta N\}$$

and D is assumed to be discretized, D_i, in small steps.

The Hill-Wheeler equation for the weight funcion $f(\boldsymbol{q})$ is

$$\sum_{\boldsymbol{q}'}[\mathcal{H}(\boldsymbol{q}',\boldsymbol{q}) - E\mathcal{N}(\boldsymbol{q}',\boldsymbol{q})]f(\boldsymbol{q}') = 0$$

in terms of the overlap matrix elements

$$\mathcal{H}(\boldsymbol{q}',\boldsymbol{q}) =_{\boldsymbol{q}'} \langle HF|\hat{H}|HF\rangle_{\boldsymbol{q}} \quad \text{and} \quad \mathcal{N}(\boldsymbol{q}',\boldsymbol{q}) =_{\boldsymbol{q}'} \langle HF|HF\rangle_{\boldsymbol{q}}$$

.

(and similarly for HFB states)

The Hill-Wheeler equation is transformed in the usual way into the hermitian eigenvalue equation

$$\mathcal{N}^{-1/2}\mathcal{H}\mathcal{N}^{-1/2}\boldsymbol{g}_k = E_k\boldsymbol{g}_k$$

with the kth eigenvector

$$\boldsymbol{g}_k = \mathcal{N}^{1/2}\boldsymbol{f}_k$$

(we assume that the matrix $\mathcal{N}$ has been renormalized so that $\mathcal{N}^{-1}$ exists)

4. The time-dependent problem

We seek a time-dependent Hill-Wheeler wave function

$$|\Psi(t)\rangle = \sum_{\boldsymbol{q}} f(t, \boldsymbol{q})|HF\rangle_{\boldsymbol{q}}$$

which describes the adiabatic evolution of the system in time over the generating states $|HF\rangle_{\boldsymbol{q}}$. For the weight function the ansatz

$$\boldsymbol{f}(t) = \mathcal{N}^{-1/2}\boldsymbol{g}(t) = \mathcal{N}^{-1/2}\sum_k a_k(t)\boldsymbol{g}_k$$

is made. Insertion into the time-dependent Schrödinger equation

$$\hat{H}\Psi(t) = i\hbar\partial_t\Psi(t)$$

yields a set of differential equations for the expansion coefficients $a_k(t)$

$$a_k E_k = i\hbar\dot{a}_k$$

with the solution

$$a_k(t) = a_k(0)e^{-iE_k t/\hbar}$$

and

$$g(\boldsymbol{q}, t) = \sum_k e^{-iE_k t/\hbar} a_k(0) g_k(\boldsymbol{q}).$$

The initial condition

Assume that at $t = 0$ the system is well described by one HF or HFB state $|HF\rangle_{\boldsymbol{q}_{\text{init}}}$, i.e.

$$|\Psi(t=0)\rangle = |HF\rangle_{\boldsymbol{q}_{\text{init}}}$$

This implies

$$g(\boldsymbol{q}, t=0) = \mathcal{N}^{1/2}(\boldsymbol{q}, \boldsymbol{q}_{\text{init}}) = \sum_k a_k(0) g_k(\boldsymbol{q})$$

Therefore

$$a_k(0) = \sum_{\boldsymbol{q}} \mathcal{N}^{1/2}(\boldsymbol{q}, \boldsymbol{q}_{\text{init}}) g_k(\boldsymbol{q})$$