Nuclear vorticity and second-order E1 giant resonances

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Kazimierz Dolny, Poland 26-30.09.2012

Motivation

Nuclei demonstrate both

- irrotational flow (most of electric GR) $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) = 0$

- vortical flow (toroidal GR, s-p excitations) $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \neq 0$

Vorticity $\vec{w}(\vec{r})$ **is a fundamental quantity:**

- does not contribute to the continuity equation, $\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$

- represents an independent part of charge-current distribution beyond the continuity equation.

Vorticity is related to the exotic E1 modes of high interest:

- toroidal

- compression

- pygmy

(toroidal, compression) second-order GR ⟺ ISGDR

N. Paar, D. Vretenar, E. Kyan, G. Colo, RPP, 70 691 (2007).

Beyond long-wave approximation:

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}\left[1 - \frac{(kr)^2}{2(2\lambda+3)} + \ldots\right]$$

Leading dipole modes in T=0 channel

Theoretical studies:

Many publications on toroidal and compressional (ISGDR) modes and manifestations of vorticity:

V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).
M.N. Harakeh et al, PRL 38, 676 (1977).
S.F. Semenko, SJNP 34 356 (1981).
J. Heisenberg, Adv. Nucl. Phys. 12, 61 (1981).
S. Stringari, PLB 108, 232 (1982).
E. Wust et al, NPA 406, 285 (1983).
E.E. Serr, T.S. Dumitrescu, T.Suzuki, NPA 404 359 (1983).
D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).
E.B. Balbutsev and I.N. Mikhailov, JPG 14, 545 (1988).
S.I. Bastrukov, S. Misicu, A. Sushkov, NPA 562, 191 (1993).
I. Hamamoto, H.Sagawa, X.Z. Zang, PRC 53 765 (1996).
E.C.Caparelli, E.J.V.de Passos, JPG 25, 537 (1999).
N.Ryezayeva et al, PRL 89, 272502 (2002).
G.Colo, N.Van Giai, P.Bortignon, M.R.Quaglia, PLB 485, 362 (2000).
D. Vretenar, N. Paar, P. Ring,T. Nikshich, PRC 65, 021301(R) (2002).
V.Yu. Ponomarev, A.Richter, A.Shevchenko, S.Volz, J.Wambach, PRL 89, 272502 (2002).
J. Kvasil, N. Lo Iudice, Ch. Stoyanov, P. Alexa, JPG 29, 753 (2003).
A. Richter, NPA 731, 59 (2004).
X. Roca-Maza et al, PRC 85, 024601 (2012).
……….

N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007). Recent review

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

Our previous results:

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

- **derivation of vorticity operator and its relation to toroidal and compression operators** → D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

$$\hat{M}^{j+}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)$$

- **role of convection and magnetization (spin) nuclear current:**

  - j_c **dominates in IS(T=0) E1 modes**
  - j_m **dominates in in IV(T=1) E1 modes**

New results to be presented:

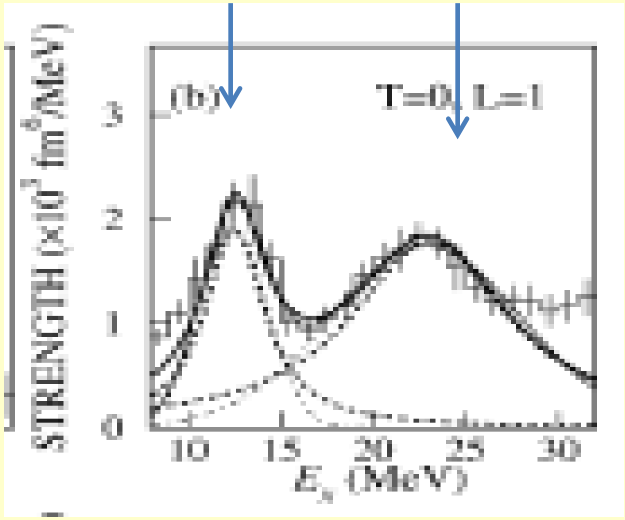
- analysis of LE (tor) and HE (com) modes of ISGDR
  in terms of
  - rot(j) ⟶ vortical motion (toroidal mode)
  - div(j) ⟶ irrotational motion (compression mode)

- deep relation between:
- vortical toroidal,
- irrotational compression,
- irrotational pygmy modes

- separable Skyrme RPA ⟶ full Skyrme RPA,

Observation of ISGDR : CM and perhaps TM:

LE (toroidal) HE (compression)



(α,α')

D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)

(γ,γ')

N.Ryezayeva et al, PRL 89, 272502 (2002).

(e,e')

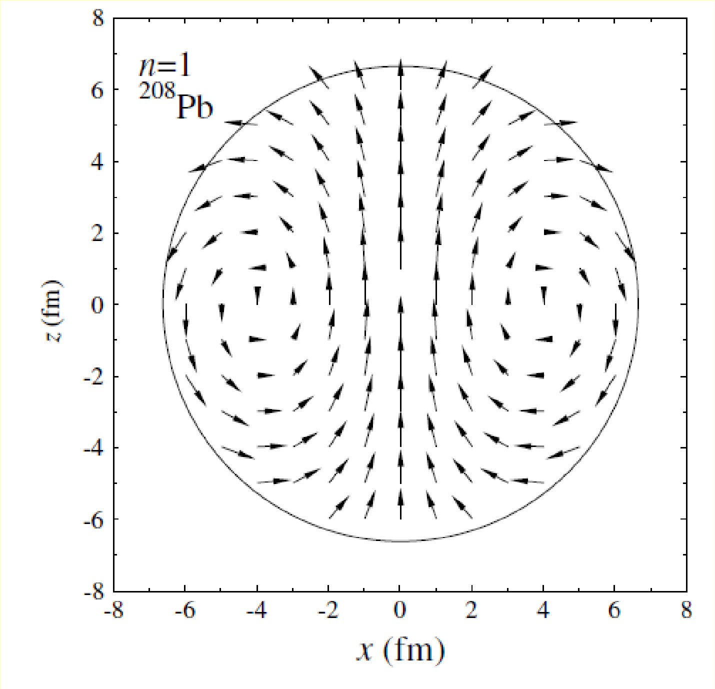
Basics of E1 (T=0) toroidal and compression modes

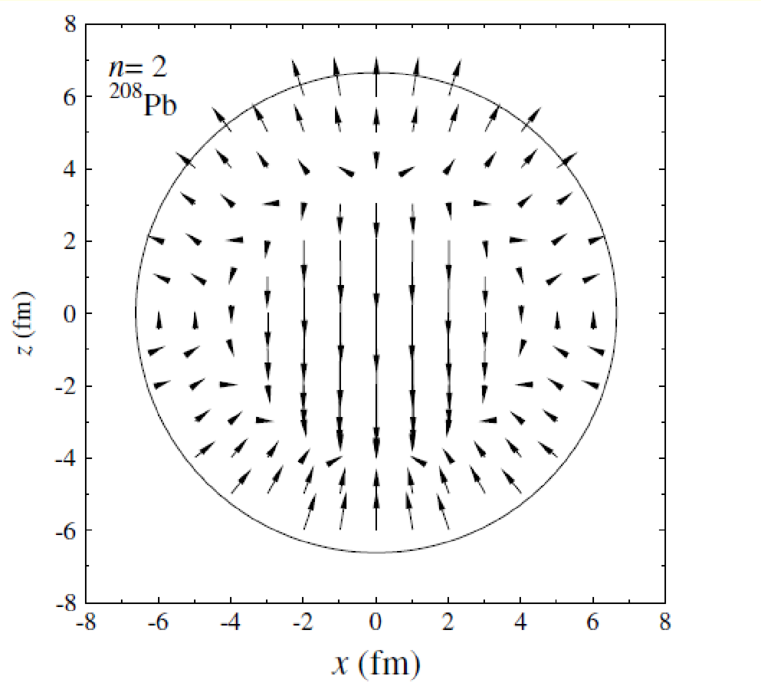
V.M. Dubovik (1975)
S.F. Semenko (1981)

TM

M.N. Harakeh (1977)
S. Stringari (1982)

CM





	TM	CM
dependence on nuclear incompress.:	no	yes
energy:	$E = 68\, A^{-1/3} MeV$	$E = 132\, A^{-1/3} MeV$
vorticity:	- yes, vortical mode	- no, irrotational mode

S. Misiku, PRC, 73, 024301 (2006)

Basics of E1 (T=0) toroidal and compression modes

V.M. Dubovik (1975)
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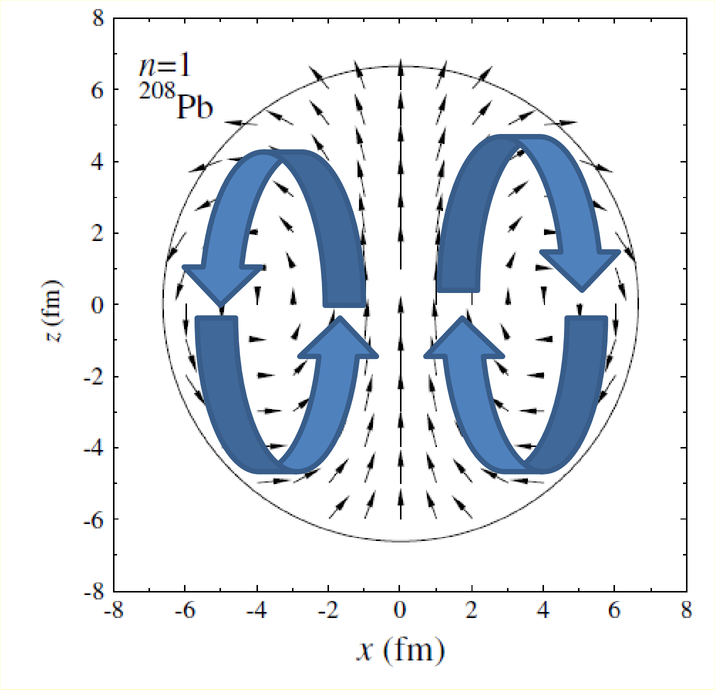

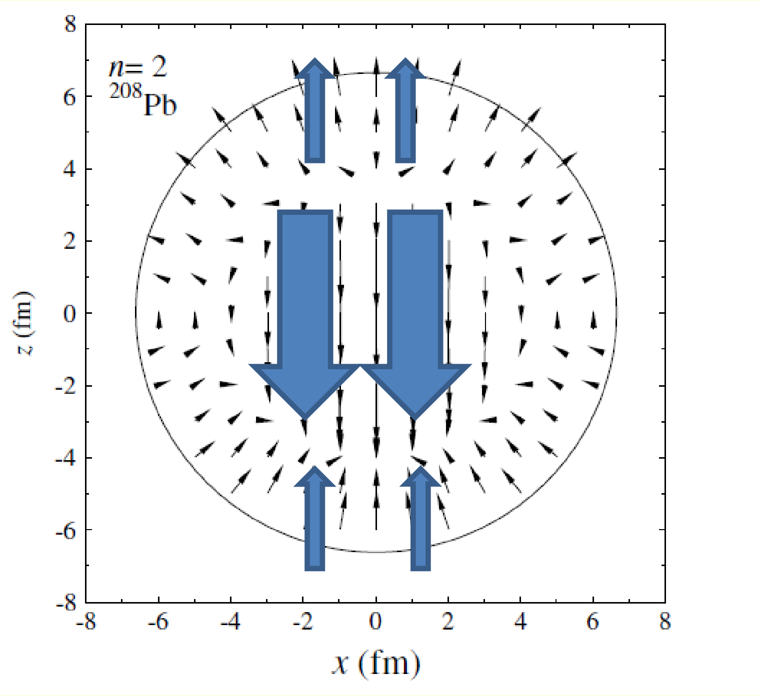

dependence on nuclear incompress.: no | yes

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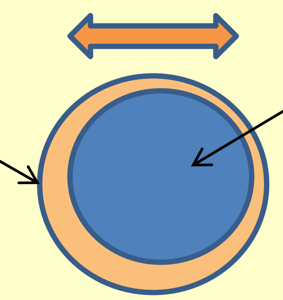
S. Misiku, PRC, 73, 024301 (2006)

vorticity: - yes, vortical mode | - no, irrotational mode - similarity to pygmy

E1 pygmy resonance (PDR)



Familiar treatment:
- oscillation of the neutron skin against the n+p core
- mixed isospin,
- mainly observed in **T=1** channel,
- related to:
  - neutron skin
  - density derivative of the symmetry energy,
- fully irrotational, no vorticity

PDR(T=0)

- low-energy part of the E1(T=1) compression mode produced by the external field

$$F(E1\mu) \propto [r^3 - \frac{5}{3} r < r^2 >_0] Y_{1\mu}$$

-embraces both torodial and compression (vortical and irrotational) flows

Two conceptions of vorticity in nuclear theory:

1. Hydrodynamical vorticity:

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \qquad \delta\vec{v}(\vec{r}) = \frac{\delta\vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$

$$(\vec{\nabla} \times \delta\vec{j}_{nuc}) \to \rho_0 \boxed{(\vec{\nabla} \times \delta\vec{v})} \to \rho_0(\vec{r})\,\vec{w}(\vec{r})$$

2. Wambach vorticity $\longleftrightarrow$ j_+ vorticity

D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$ - **continuity equation**

$$\delta\vec{j}_{(fi)}(\vec{r}) = \left\langle j_f m_f \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, j_i m_i \right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}} [j^{(fi)}_{\lambda\lambda-1}(r)\vec{Y}^{*}_{\lambda\lambda-1\mu} + j^{(fi)}_{\lambda\lambda+1}(r)\vec{Y}^{*}_{\lambda\lambda+1\mu}]$$

$$\delta\vec{j}^{\nu}_{1\mu}(\vec{r}) = \left\langle \nu \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, 0 \right\rangle = -\frac{i}{\sqrt{3}} [\underbrace{j^{\nu}_{10}(r)\vec{Y}^{*}_{10\mu}}_{j_-} + \underbrace{j^{\nu}_{12}(r)\vec{Y}^{*}_{12\mu}}_{j_+}]$$ - **current transition density**

$j^{\nu}_{+}(r)$ - independent part of charge-current distribution, decoupled to CE
- may be the measure of the vorticity

HD and j+ prescriptions give opposite conclusions on CM vorticity!

E1(T=0)

$$\langle \nu | \hat{M}^{j+}_{vor}(E1\mu) | 0\rangle = -\frac{1}{5\sqrt{2}c}\int dr^3 \; r^2 \vec{Y}_{12\mu} \cdot \delta\vec{j}_{nuc,+}(\vec{r})$$

$$\langle \nu | \hat{M}_{com}(E1\mu) | 0\rangle = -\frac{1}{10\sqrt{2}c}\int dr^3 \; [r^3 - \frac{5}{3} r\langle r^2\rangle_0] Y_{1\mu} \, [\vec{\nabla} \cdot \delta\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\langle \nu | \hat{M}_{tor}(E1\mu) | 0\rangle = -\frac{1}{10\sqrt{2}c}\int dr^3 \; [r^3 - \frac{5}{3} r\langle r^2\rangle_0] \vec{Y}_{11\mu} \cdot [\vec{\nabla} \times \delta\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\langle \nu | \hat{M}^{HD}_{vor}(E1\mu) | 0\rangle = -\frac{1}{10\sqrt{2}c}\int dr^3 \; [r^3 - \frac{5}{3} r\langle r^2\rangle_0] \vec{Y}_{11\mu} \cdot \rho_0 [\vec{\nabla} \times \delta\vec{v}(\vec{r})]$$

$$\langle \nu / \hat{M}^{j+}_{vor}(E1\mu) / 0\rangle = -\frac{1}{5\sqrt{2}c}\int dr \, r^4 \; j^{\nu}_{+}(r)$$

$$\langle \nu / \hat{M}_{com}(E1\mu) / 0\rangle = -\frac{1}{6c}\int dr \, r^2 \, [\frac{2\sqrt{2}}{5} r^2 j^{\nu}_{+}(r) - (r^2 - \langle r^2\rangle_0) j^{\nu}_{-}(r)]$$

$$\langle \nu / \hat{M}_{tor}(E1\mu) / 0\rangle = -\frac{1}{6c}\int dr \, r^2 \, [\frac{\sqrt{2}}{5} r^2 j^{\nu}_{+}(r) + (r^2 - \langle r^2\rangle_0) j^{\nu}_{-}(r)]$$

The model:

P.-G. Reinhard,
Ann. Physik, 1, 632 (1992)

- self-consistent full Skyrme RPA ,
  - no separable ansatz
  - wide configuration space

- Q,P hybrid configuration space:
  - ~ 260 1ph states until ~ 35 MeV,
  - 14 local operators to simulate the effect of higher shells

→ effective extends the configuration space up to 200 MeV

- SLy6 force
- only T=0 channel
- only convection nuclear current
- ^{208}Pb

Strength function

$$S(E1;\omega) = \sum_{\nu \neq 0} |< \Psi_\nu | \hat{M}_{E1} | 0 >|^2 \, \varsigma(\omega - \omega_\nu)$$

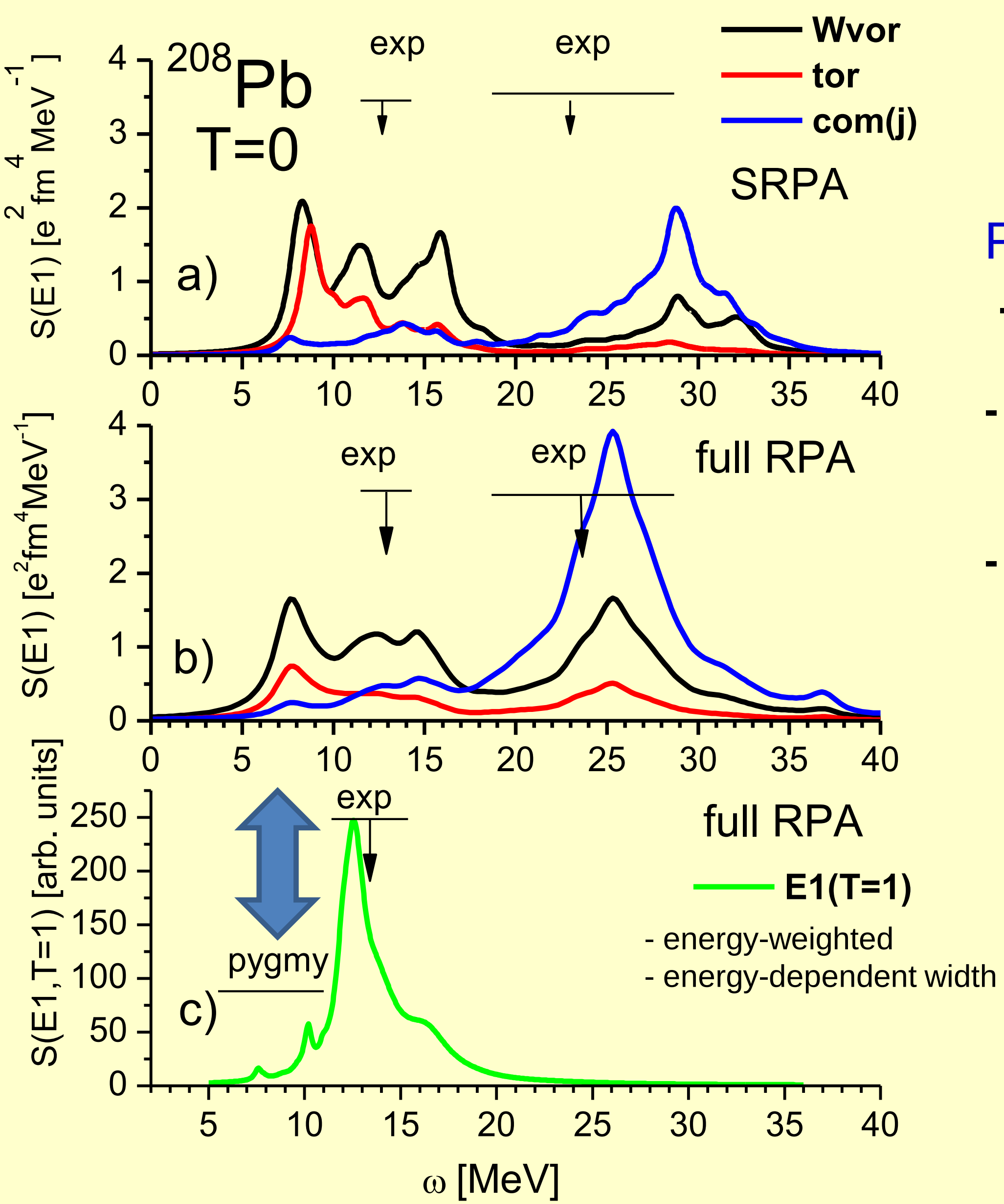
Toroidal, compressional, vortical operators

with the Lorentz weight

$$\varsigma(\omega - \omega_\nu) = \frac{1}{2\pi} \frac{\Delta}{[(\omega - \omega_\nu)^2 + \frac{\Delta^2}{4}]}$$

$$\Delta = 1 \; MeV$$

Separable and full RPA results vs experiment



Exp:

ISGDR(LE)	ISGDR(HE)
E=12.7 MeV	E=23.0 MeV
Γ=3.5 MeV	Γ=10.3 MeV

Full RPA:

- mainly reproduces SRPA results,

- downshifts com(j) by 3MeV (because of larger config. space),

- better describes HE exper. data, already reasonable agreement

Perhaps:

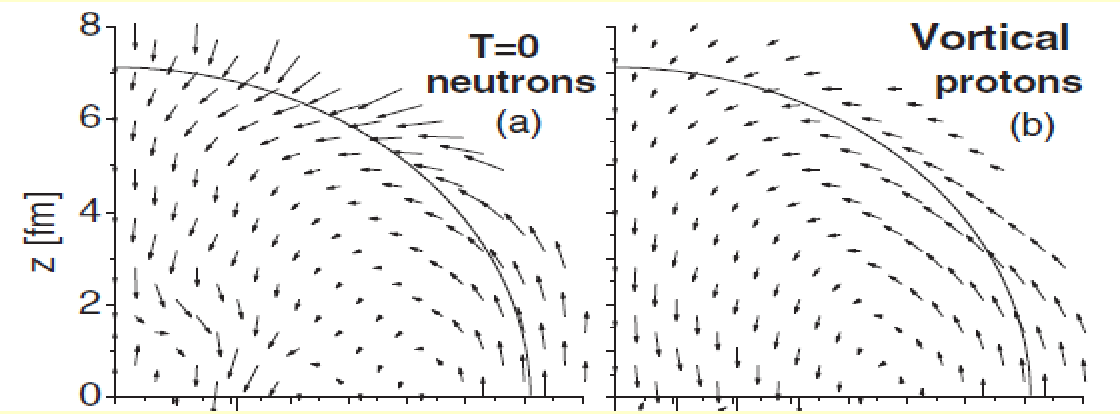
- ISGDR(LE) is not TM but low-energy CM branch,

- to look for TM at the pygmy resonance energy ~ 6-8 MeV, while the experiment covers 8-32 MeV.

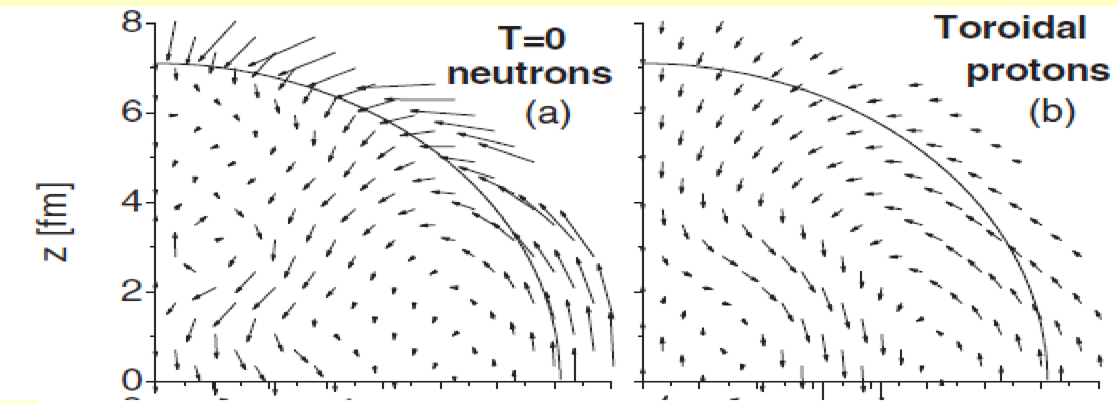
E1(T=0) velocity fields in most collective RPA states in PDR region

J. Kvasil et al
PRC, 84, 034303 (2011)

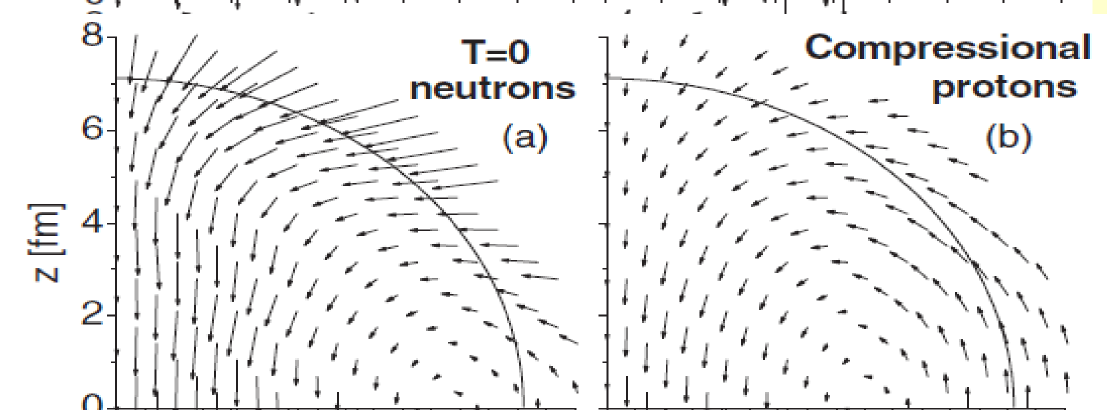
SRPA calculations



vortical probe,
E= 8.3 MeV



toroidal probe,
E= 8.7 MeV



compression probe,
E= 7.1 MeV

All the probes give a similar toroidal-like picture!

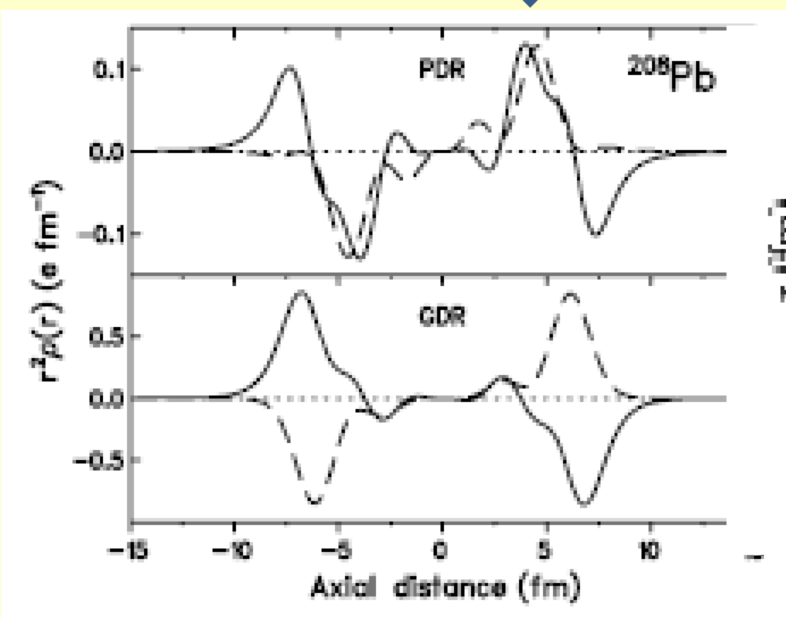
E1 transition densities and velocity fields

N.Ryezayeva et al, PRL 89, 272502 (2002).

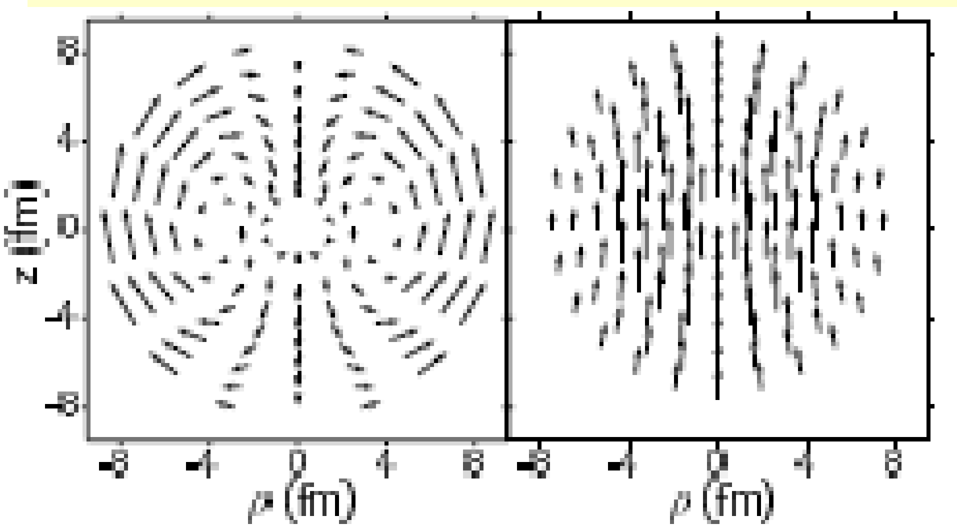
QPNM calculations

isoscalar
interior motion

Cumulative effect of RPA states
in 6.5-10.5 MeV region



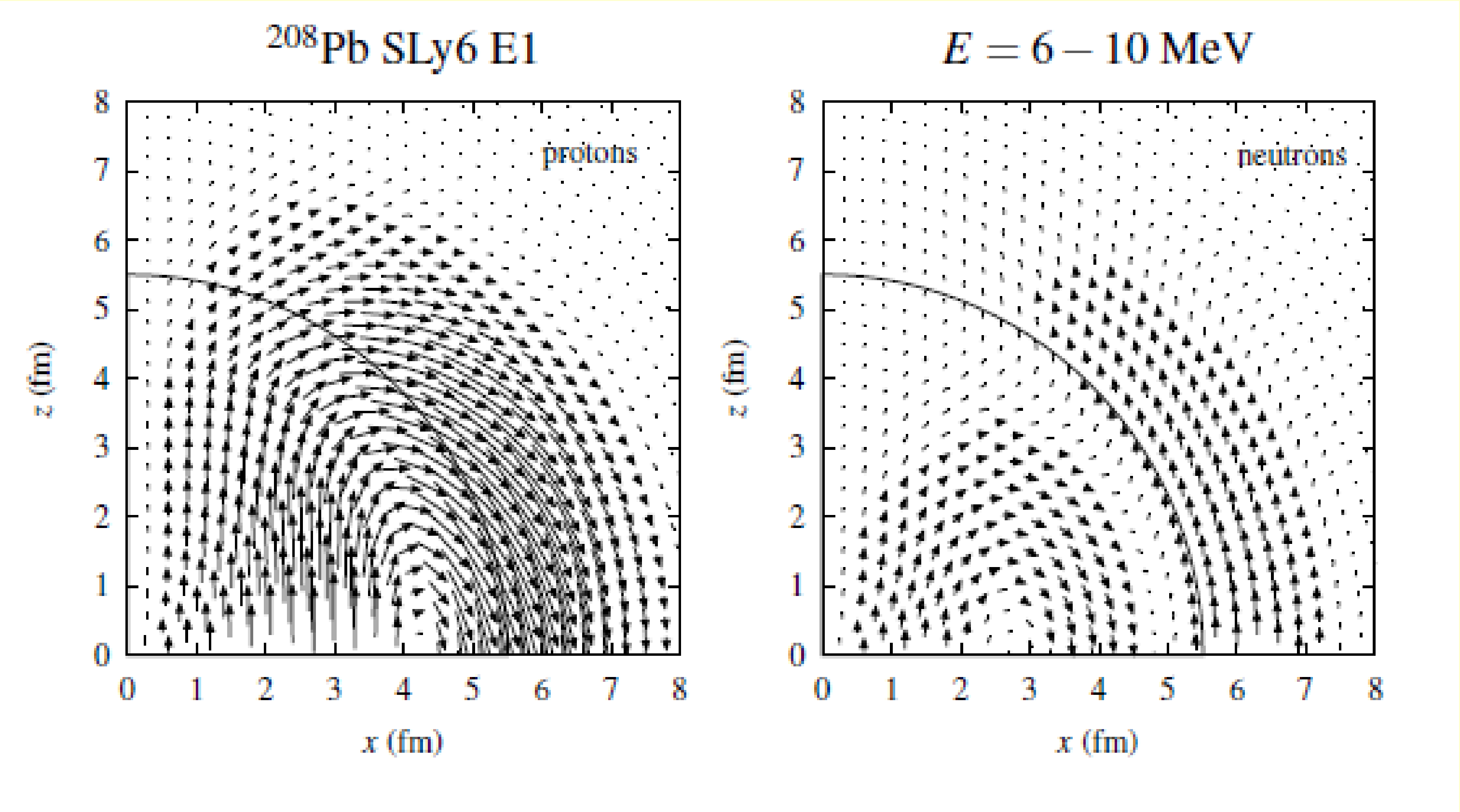
PDR GDR



Toroidal-like picture even in T=1 case!

Current fields in pygmy region

SRPA



Coexistence of T=0 toroidal and T=1 pygmy motion

PDR:

- is very reach and specific phenomenon where both vortical (toroidal) and irrotational (n- skin oscillations) flows coexist

- the vortical motion dominates in the interior (core) while the irrotational motion takes at the surface (neutron skin)

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[(r^3 - \frac{5}{3}r < r^2 >_0)]\,\vec{Y}_{11\mu}(\hat{\vec{r}})\cdot[\vec{\nabla}\times\vec{j}_{nuc}(\vec{r})]$$

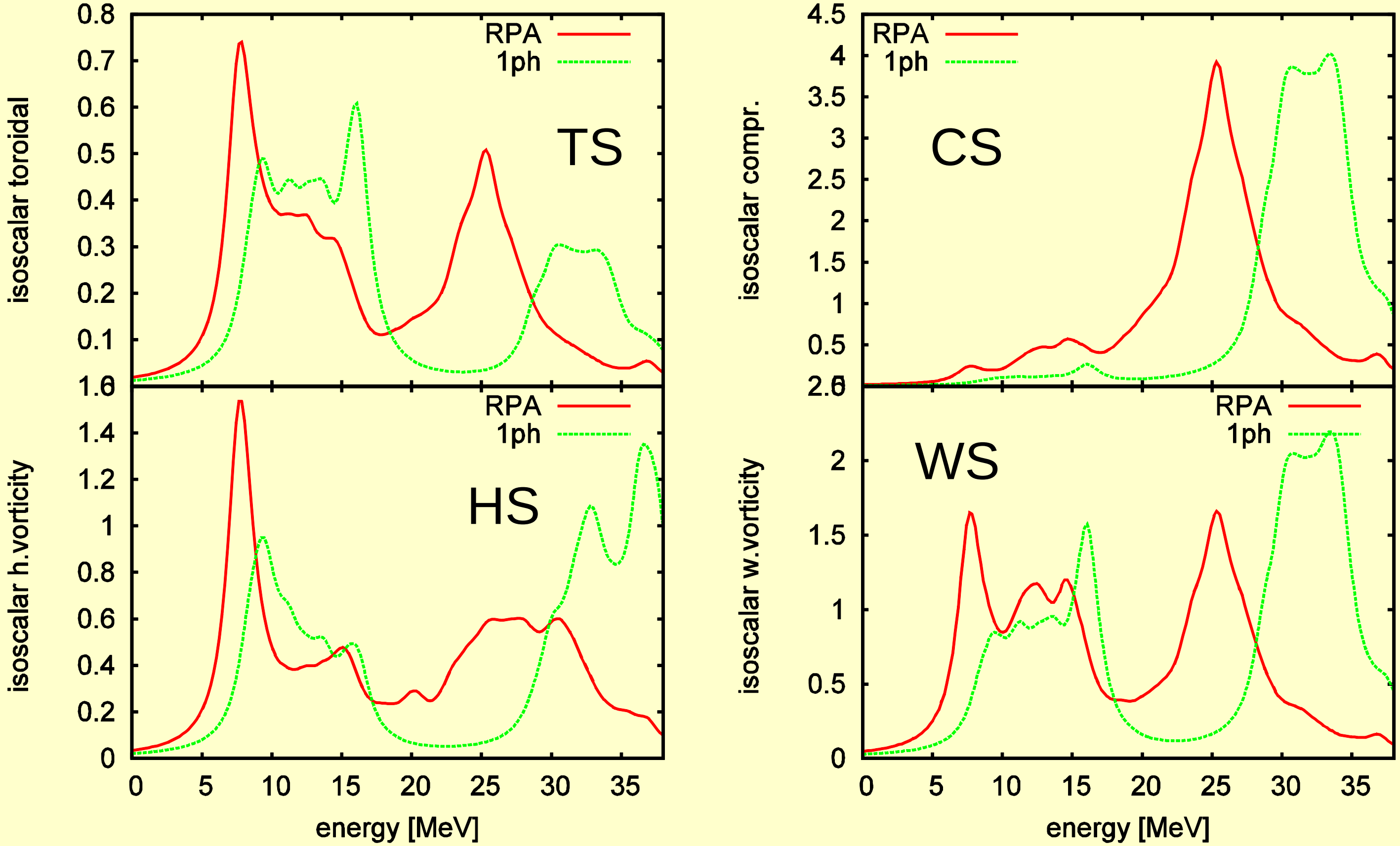
$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\,[(r^3 - \frac{5}{3}r < r^2 >_0)Y_{1\mu}]\,[\vec{\nabla}\cdot\vec{j}_{nuc}(\vec{r})]$$

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3 - \frac{5}{3} < r^2 >_0 r]Y_{1\mu} \qquad \hat{M}_{com}(E\lambda\mu) = -k\hat{M}'_{com}(E\lambda\mu)$$

$$\hat{M}_{tor}(E1\mu) = \frac{1}{20c}\int d\vec{r}\,\hat{\vec{j}}_{nuc}(\vec{r})\cdot[\vec{\nabla}\times(\vec{r}\times\vec{\nabla})\,(r^3 - \frac{5}{3}r < r^2 >_0)Y_{1\mu}]$$

Full RPA vs 1ph

^{208}Pb



- strong collective shifts: ~2-3 MeV (LE), ~9 MeV (HE)
- the LE and HE branches are indeed collective
- the most collective strengths at 6-8 MeV and 23-27 MeV

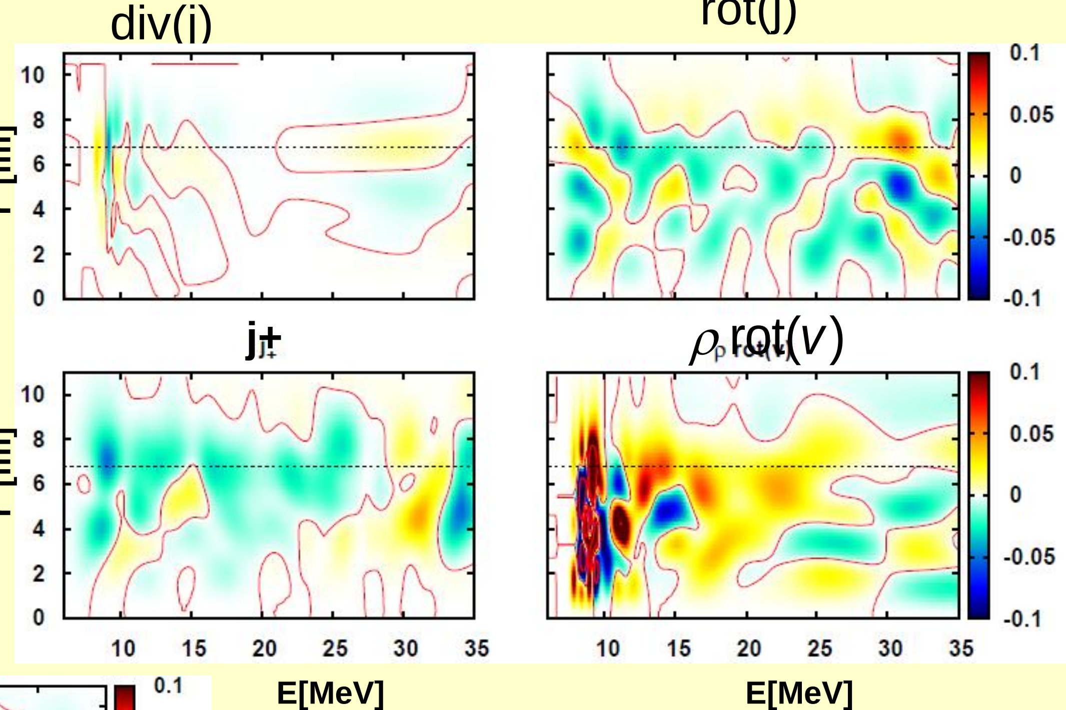
Transition densities: SLy6, ^{208}Pb

$$\vec{\nabla}\times\delta\vec{j}^{\,\nu}_{\lambda\mu}(\vec{r}) \propto [rot(j)]^{\nu}_{\lambda}(r)\,\vec{Y}_{\lambda\lambda\mu}(\hat{\vec{r}})$$

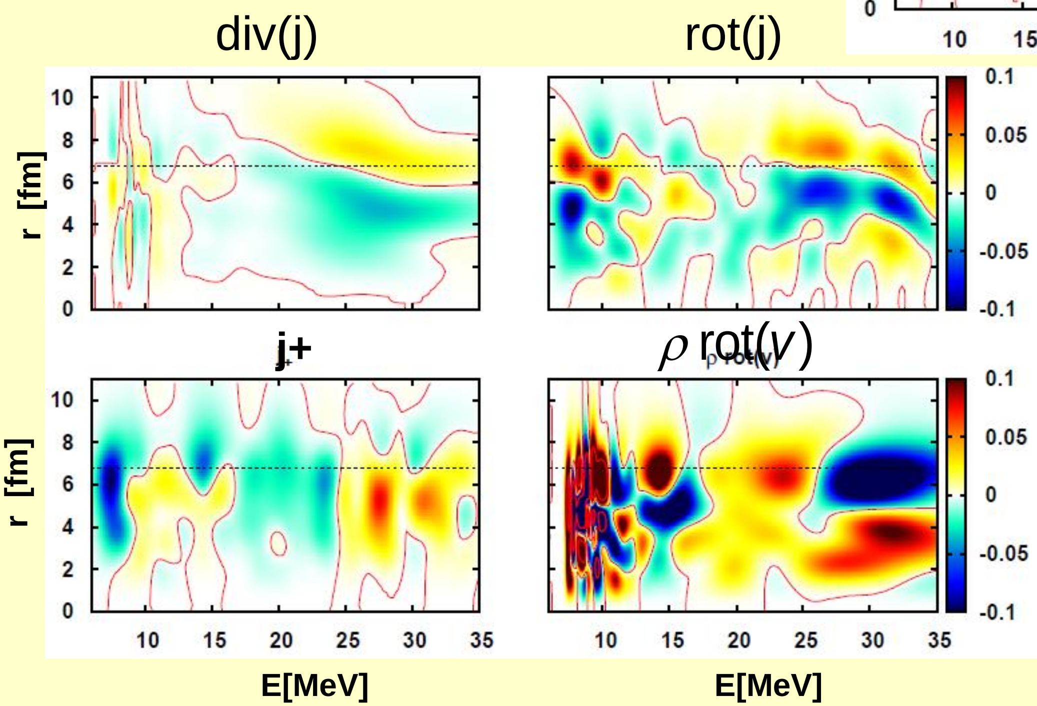
$$\vec{\nabla}\cdot\delta\vec{j}^{\,\nu}_{\lambda\mu}(\vec{r}) \propto [div(j)]^{\nu}_{\lambda}(r)\,Y_{\lambda\mu}(\hat{\vec{r}})$$

$$\delta\vec{j}^{\,\nu}_{\lambda\mu}(\vec{r}) \propto [j^{\nu}_{\lambda,\lambda-1}(r)\,\vec{Y}_{\lambda\lambda-1\mu}(\hat{\vec{r}}) + j^{\nu}_{\lambda,\lambda+1}(r)\,\vec{Y}_{\lambda\lambda+1\mu}(\hat{\vec{r}})]$$

1ph



full RPA



- **significant collectivity,**
- **both surface and interior motion,**
- rot(j) **is weaker than** $\rho\,\mathrm{rot}(v)$
- rot(j) **and** $\rho\,\mathrm{rot}(v)$ **are peaked in the PDR region**

Conclusions

★ The LE and HE branches of ISGDR are investigated within
full RPA + SRPA with Skyrme forces
The treatment of the LE/HE as toroidal/compression
modes was inspected .

★ Collective character of TM and CM.
TM as a main carrier of the vorticity.
Correlation of TM and pygmy resonance.

★ PDR:
- is interesting not only by connection to symmetry energy etc
but also as a particular mode combining vortical and irrotational flow
- both vortical (toroidal) and irrotational (n- skin oscillations) flows coexist
- vortical motion ⟶ interior (core)
irrotational ⟶ surface (neutron skin)
- optimal object to investigate the vortical motion

Thank you for attention!