Non-yrast quasi parity-doublet spectra in odd-mass nuclei

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Contents

General Hamiltonian. Collective part

$$H = H_{\mathrm{qo}} + H_{\mathrm{s.p.}} + H_{\mathrm{pair}} + H_{\mathrm{Coriol}}$$

$$H_{\mathrm{qo}} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + U(\beta_2, \beta_3, I)$$

$$U(\beta_2, \beta_3, I) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{d_0 + \hat{I}^2 - \hat{I}_z^{\,2}}{2\mathcal{J}(\beta_2, \beta_3)}$$

$$H_{\mathrm{Coriol}} = -\frac{(\hat{I}_+\hat{j}_- + \hat{I}_-\hat{j}_+)}{2\mathcal{J}(\beta_2, \beta_3)} \qquad \mathcal{J}(\beta_2, \beta_3) = (d_2\beta_2^2 + d_3\beta_3^2)$$

- β_2, β_3 axial deformation variables
- B_2, B_3 mass; C_2, C_3 stiffness; d_2, d_3 inertia parameters

[N. M. et al, Phys. Rev. C **73**, 044315 (2006); **76**, 034324 (2007)]

Single particle Hamiltonian. BCS pairing

$$H_{\mathrm{sp}} = T + V_{\mathrm{ws}}(\beta_2, \beta_3, \ldots) + V_{\mathrm{s.o.}} + V_{\mathrm{c}}$$

$$\mathcal{F}_{\Omega} = \sum_{N n_z \Lambda} C^{\Omega}_{N n_z \Lambda} | N n_z \Lambda \Omega \rangle \overset{\Omega = K}{=} \mathcal{F}_K^{(+)} + \mathcal{F}_K^{(-)}$$

$$\hat{\pi}_{\mathrm{sp}} \mathcal{F}_K^{(\pm)} = \pm \mathcal{F}_K^{(\pm)} \qquad \langle \hat{\pi}_{\mathrm{sp}} \rangle = \langle \mathcal{F}_K | \hat{\pi}_{\mathrm{sp}} | \mathcal{F}_K \rangle$$

$-1 \leq \langle \hat{\pi}_{\mathrm{sp}} \rangle \leq 1$, $\beta_3 \neq 0 \Rightarrow$ **parity-mixed s.p. states**

$$H_{qp} \equiv H_{\mathrm{s.p.}} + H_{\mathrm{pair}} \qquad (\mathrm{DSM+BCS}) \qquad \Rightarrow$$

$$\epsilon^K_{\mathrm{qp}} = \sqrt{(E^K_{\mathrm{sp}} - \lambda)^2 + \Delta^2}$$

Coherent quadrupole-octupole mode (CQOM) in the even core

$$U(\beta_2,\beta_3,I)=\frac{1}{2}C_2{\beta_2}^2+\frac{1}{2}C_3{\beta_3}^2+\frac{X(I)}{d_2\beta_2^2+d_3\beta_3^2}$$

$$X(I)=[d_0+I(I+1)]/2 \qquad (K=0)$$

$$\beta_2=\sqrt{d/d_2}\eta\cos\phi\ ,\quad \beta_3=\sqrt{d/d_3}\eta\sin\phi\ ,\quad d=(d_2+d_3)/2$$

Coherent mode: $\omega=\sqrt{C_2/B_2}=\sqrt{C_3/B_3}\equiv\sqrt{C/B}$

Energy spectrum:

$$E_{n,k}(I,\pi)=\hbar\omega\left[2n+1+\sqrt{k^2+bX(I)}\right],\quad b=\frac{2B}{\hbar^2 d},\quad n=0,1,2,...$$

Coherent quadrupole-octupole core function

$$\Phi^{\pi}_{n,k,I}(\eta,\phi) = \psi^{I}_{nk}(\eta)\varphi^{\pi}_{k}(\phi)$$

$$\psi^{I}_{nk}(\eta) = \sqrt{\frac{2c\Gamma(n+1)}{\Gamma(n+2s+1)}}\, e^{-c\eta^2/2} c^s \eta^{2s} L^{2s}_{n}(c\eta^2)$$

$$c = \sqrt{BC}/\hbar, \quad s = \sqrt{k^2 + bX(I)}/2$$

$$\beta_2 > 0 \Rightarrow \varphi(-\pi/2) = \varphi(\pi/2) = 0$$

$$\varphi^{+}_{\mathbf{k}}(\phi) = \sqrt{2/\pi}\cos(\mathbf{k}\phi)\ , \quad \mathbf{k} = \mathbf{1}, \mathbf{3}, \mathbf{5}, \ldots \ \rightarrow \ \pi_c = (+)$$

$$\varphi^{-}_{\mathbf{k}}(\phi) = \sqrt{2/\pi}\sin(\mathbf{k}\phi)\ , \quad \mathbf{k} = \mathbf{2}, \mathbf{4}, \mathbf{6}, \ldots \ \rightarrow \ \pi_c = (-)$$

Total core plus particle wave function

$$\Psi^{\pi}_{nkIMK}(\eta,\phi) = \frac{1}{2}\sqrt{\frac{2I+1}{16\pi^2}}(1+\mathcal{R}_1)D^I_{M\,K}(\boldsymbol{\theta})(1+\pi\hat{P})\Phi^{\pi_c}_{n,k,I}(\eta,\phi)\mathcal{F}_K$$

$$\mathcal{R}_1 D^I_{M\,K} = (-1)^{I-K} D^I_{M\,-K} \qquad \hat{P} = \hat{\pi}_c \cdot \hat{\pi}_{\text{sp}}$$

$$\mathcal{R}_1\mathcal{F}_K = \overline{\mathcal{F}}_K = \mathcal{F}_{-K} \qquad \hat{\pi}_{\text{sp}}\mathcal{F}_K = \mathcal{F}^{(+)}_K - \mathcal{F}^{(-)}_K$$

$$\mathcal{R}_1\Phi^{\pi_c}_{\text{core}} = \hat{\pi}_c\Phi^{\pi_c}_{\text{core}} = \pm\Phi^{\pm}_{\text{core}} \qquad \Phi^{\pi_c}_{\text{core}} \equiv \Phi^{\pi_c}_{n,k,I}(\eta,\phi)$$

$\pi_c = (+) \rightarrow \Phi^{+}_{\text{core}} \Rightarrow$ downwards shifted levels

$\pi_c = (-) \rightarrow \Phi^{-}_{\text{core}} \Rightarrow$ upwards shifted energy sequence

$(1+\pi\hat{P}) \rightarrow$ projects out $\mathcal{F}^{(+)}_K$ or $\mathcal{F}^{(-)}_K$

Total core plus particle wave function, expanded

$$\begin{aligned}\Psi^{\pi}_{nkIMK}(\eta,\phi) &= \frac{1}{2}\sqrt{\frac{2I+1}{16\pi^2}}\Phi^{\pi_c}_{n,k,I}(\eta,\phi)\\ &\times \Big\{D^{I}_{M\,K}(\theta)[(1+\pi\cdot\pi_c)\mathcal{F}^{(+)}_{K}+(1-\pi\cdot\pi_c)\mathcal{F}^{(-)}_{K}]\\ &+ \pi_c(-1)^{I+K}D^{I}_{M\,-K}(\theta)[(1+\pi\cdot\pi_c)\mathcal{F}^{(+)}_{-K}+(1-\pi\cdot\pi_c)\mathcal{F}^{(-)}_{-K}]\Big\}\end{aligned}$$

Case of a good s.p. parity ($\beta_3 \neq 0$) $\pi_{\text{sp}} = \pi\cdot\pi_c$:

$$\begin{aligned}\Psi^{\pi}_{nkIMK}(\eta,\phi) &= \sqrt{\frac{2I+1}{16\pi^2}}\Phi^{\pi_c}_{n,k,I}(\eta,\phi)\\ &\times \Big[D^{I}_{M\,K}(\theta)\mathcal{F}_K+\pi_c(-1)^{I+K}D^{I}_{M\,-K}(\theta)\mathcal{F}_{-K}\Big]\end{aligned}$$

$$\pi_c = \pi\cdot\pi_{\text{sp}} = (\pm)$$

Coriolis interaction, $H_{\text{Coriol}} \rightarrow H^{\text{c}}_{K}$

$$
\begin{aligned}
H^{\text{c}}_{K} &= \frac{1}{2\mathcal{J}(\beta_2,\beta_3)}(-1)^{I+1/2}\left(I+\frac{1}{2}\right)a_{1/2}\delta_{K,\frac{1}{2}} \\
&- \frac{1}{[2\mathcal{J}(\beta_2,\beta_3)]^2}(I\mp K)(I\pm K+1)\sum_{K'=K\pm1}\frac{P^2_{K'K}\left|a_{K'K}\right|^2}{\epsilon^{K'}_{qp}-\epsilon^{K}_{qp}}
\end{aligned}
$$

$$
P_{K'K} = U_K U_{K'} + V_K V_{K'}
$$

$$
a_{1/2} = \tfrac{1}{2}\pi_c\left[(1+\pi\pi_c)a^{(+)}_{\frac{1}{2}-\frac{1}{2}} + (1-\pi\pi_c)a^{(-)}_{\frac{1}{2}-\frac{1}{2}}\right] = \pi\pi_0 a^{(\pi_0)}_{\frac{1}{2}-\frac{1}{2}}
$$

$$
a_{K'K} = \tfrac{1}{2}\left[(1+\pi\pi_c)a^{(+)}_{K'K} + (1-\pi\pi_c)a^{(-)}_{K'K}\right] = a^{(\pi_0)}_{K'K}, \quad \pi_0 \equiv \pi_{\text{gs/bh}}
$$

$$
\begin{aligned}
a^{(+)}_{K'K} &= \langle\mathcal{F}^{(+)}_{K'}|\hat{j}_+|\mathcal{F}^{(+)}_{K}\rangle = \langle\mathcal{F}^{(+)}_{K}|\hat{j}_-|\mathcal{F}^{(+)}_{K'}\rangle \\
a^{(-)}_{K'K} &= \langle\mathcal{F}^{(-)}_{K'}|\hat{j}_+|\mathcal{F}^{(-)}_{K}\rangle = \langle\mathcal{F}^{(-)}_{K}|\hat{j}_-|\mathcal{F}^{(-)}_{K'}\rangle.
\end{aligned}
$$

Final form of the model Hamiltonian

$$
H_K = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{\widetilde{X}(I)}{\mathcal{J}(\beta_2,\beta_3)}
$$

$$
\begin{aligned}
\widetilde{X}(I,K) &= \frac{1}{2}\Bigg[d_0 + I(I+1) - K^2 + (-1)^{I+\frac{1}{2}}\left(I+\frac{1}{2}\right)a_{1/2}\delta_{K,\frac{1}{2}} \\
&- A(\beta_2^0,\beta_3^0)(I\mp K)\,(I\pm K+1)\sum_{K'=K\pm 1}\frac{P^2_{K'K}\left|a_{K'K}\right|^2}{\epsilon_{qp}^{K'}-\epsilon_{qp}^{K}}\Bigg]
\end{aligned}
$$

$A(\beta_2^0,\beta_3^0) \equiv 1/[2\mathcal{J}(\beta_2^0,\beta_3^0)] \rightarrow K$- mixing constant

Yrast and non-yrast quasi parity-doublet spectrum

$$\textbf{CQOM:}\quad E_{n,k}(I,K) = \hbar\omega\left[2n+1+\sqrt{k^2+b\widetilde{X}(I,K)}\right]$$

$I^{\pi=\pi_0}$, $k^{(+)} = 1, 3, 5, \ldots$
$I^{\pi=-\pi_0}$, $k^{(-)} = 2, 4, 6, \ldots$

$\Rightarrow$ **split parity-doublet sequences:** $k_n^{(+)}$, $k_n^{(-)}$, $n = 0, 1, 2, \ldots$

$I^{\pm} = I_n^{\pm},\ (I_n+1)^{\pm},\ (I_n+2)^{\pm},\ (I_n+3)^{\pm},\ \ldots$

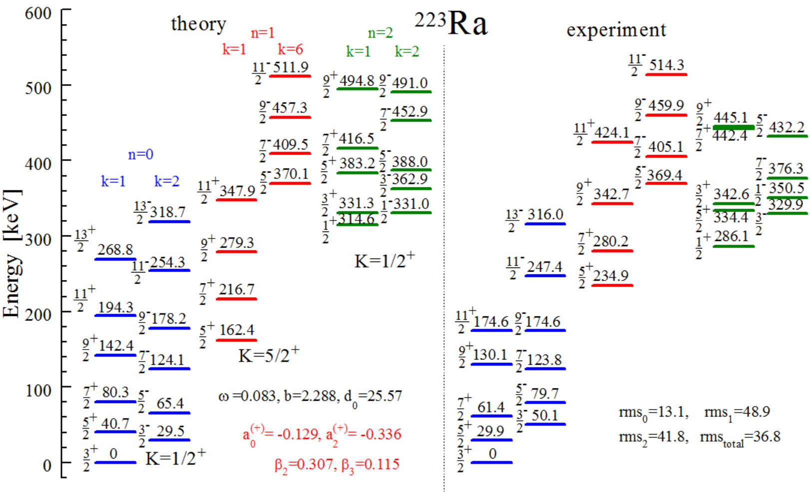
$n = 0 \rightarrow$ **yrast;** $n = 1, 2, \ldots \rightarrow$ **non-yrast sequences**

Coriolis decoupling factors (for $K = 1/2$): $a_{1/2} = a_0,\ a_1,\ a_2$
K mixing: $a_{K'K}$ calculated for β_2^0 and β_3^0 within **DSM+BCS**

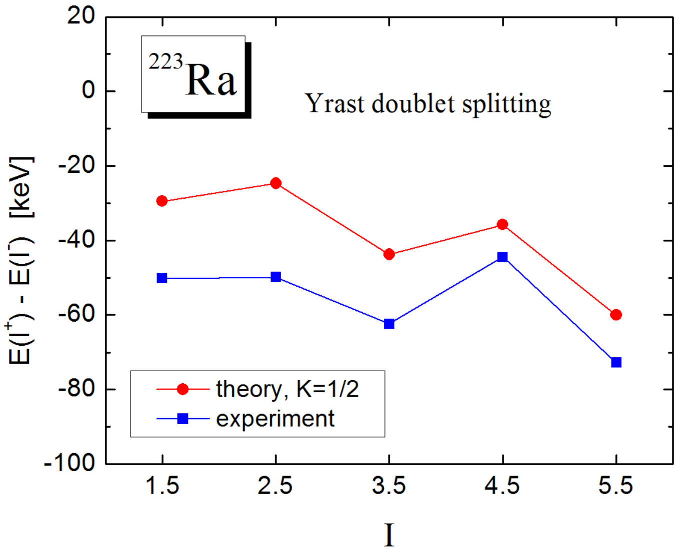
Adjustable parameters: ω, b, d_0 and A
$k^{(+)}$ and $k^{(-)}$ allowed to vary with $k^{(+)} < k^{(-)}$

Pairing constant: $G_{n/p} = \left(g_0 \mp g_1 \frac{N-Z}{A}\right)/A$,
$g_0 = 17.8\text{MeV}$, $g_1 = 7.4\text{MeV}$

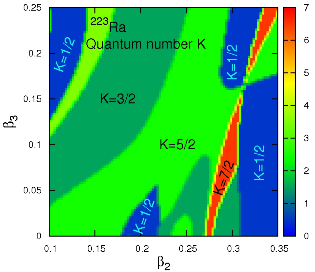
Split parity-doublets in ^{223}Ra, $K = 1/2^+$ assumed for the yrast band

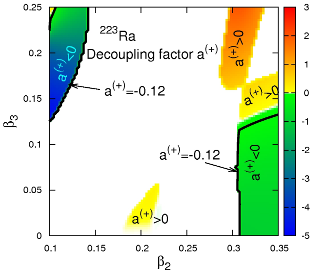


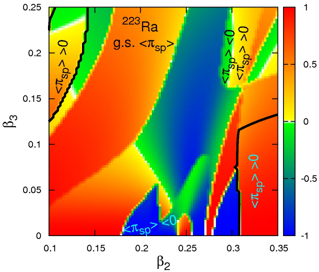
Doublet splitting in the yrast levels of ^{223}Ra



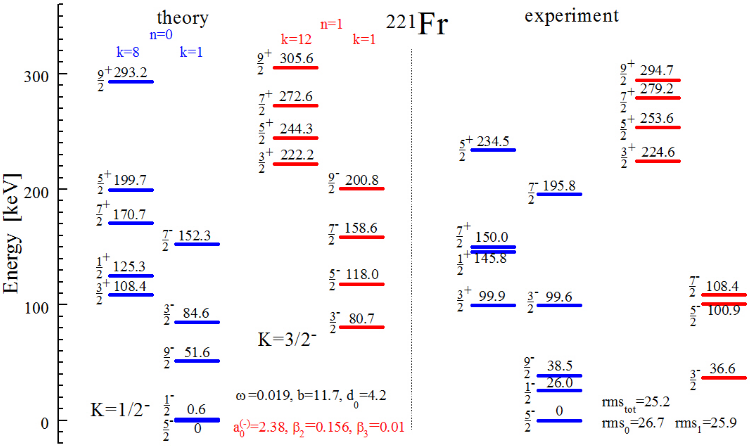
DSM evaluation of $a_0^{(+)}$ in ^{223}Ra



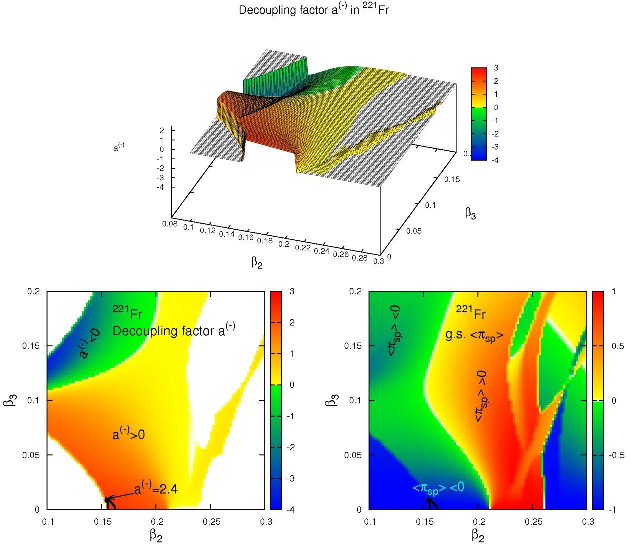




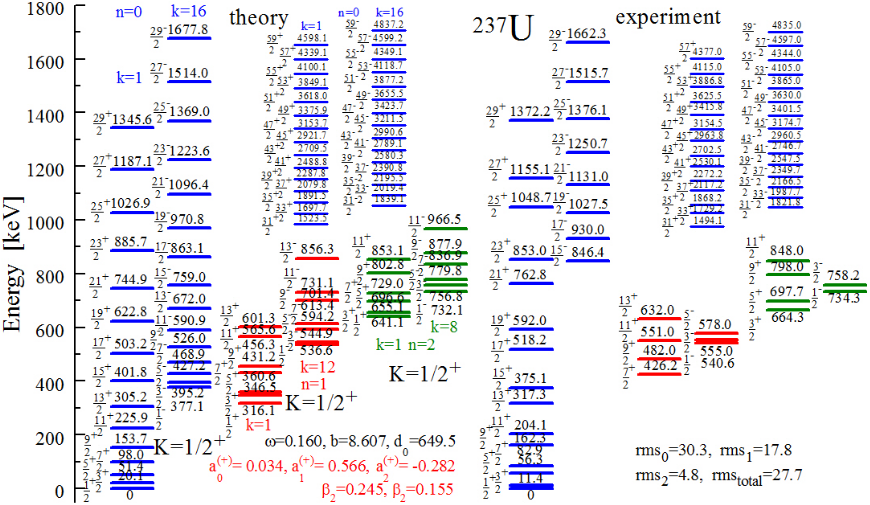
Split parity-doublets in ^{221}Fr, strong Coriolis effect in the yrast band



DSM evaluation of $a_0^{(-)}$ in ^{221}Fr



Split parity-doublets in ^{237}U



Conclusion/Perspectives

Results

- CQOM + DSM+BCS description of yrast and non-yrast split parity-doublet spectra in odd-mass nuclei
- Fully microscopic description of Coriolis decoupling and K-mixing effects

Perspectives

- Calculation of B(E1), B(E2) and B(E3) transition probabilities
- Extension beyond the limits of the coherent-mode assumption