Fission Dynamics as a Probe of the Shape-Dependent Congruence Energy Term in the Macroscopic Models

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Outline

Introduction: Shape-Dependent Congruence Energy

- Fission Dynamics
- Jacobi/Poincaré Shape Transition
- New Expression for the Shape-Dependent Geometrical Factor

Summary

Macroscopic-Microscopic Method

Macroscopic Energy:

- Lublin-Strasbourg Drop (LSD),

$$M(Z,N;def) = ZM_H + NM_n - 0.00001433Z^{2.39}$$
$$+E_{LSD}(Z,N;def) + E_{micr}(Z,N;def) + E_{Congr.}$$

or Finite Range Liquid Drop Model

Microscopic Energy:

$$E_{micr} = E_{pair} + E_{shell}$$

Pairing Energy:

$$E_{pair} = E_{BCS} + \overline{E}_{pc}$$

or with Particle Number Projection method

Microscopic Energy Term with the Woods-Saxon Universal Parameter-Set Hamiltonian

Wigner/Congruence Energy: History

Wigner E, Phys. Rev. **51** *(1937) 106,947*

JUNE 1, 1937 PHYSICAL REVIEW VOLUME 51

On the Structure of Nuclei Beyond Oxygen

E. Wigner
University of Wisconsin, Madison, Wisconsin

(Received March 16, 1937)

An attempt is made to correlate the kinks in the mass defect curve with the energy differences between isobars, both as obtained from direct measurements and also from the shift of the isotopic number to higher values with increasing number of particles. Since the single-particle picture is known to be an insufficient approximation, the symmetry property of the wave function, resulting from the use of a symmetric Hamiltonian is utilized. The average interaction between symmetrically and antisymmetrically coupled particles ($L+L'$ and $L-L'$) is determined mainly from the kinks in the mass defect curve and enables one to calculate the energy differences between isobars. The energy change at the end of the shell is obtained from experimental data. It should enable one to get some idea of the probabilities with which the particles are in excited configurations. For heavier elements, the formula obtained here should naturally be identical with Weizsäcker's semi-empirical formula and the connection between both is discussed.

Congruence/Wigner Energy: History

Myers W D, Swiatecki W J, Nucl. Phys. **A 81** *(1966) 1*

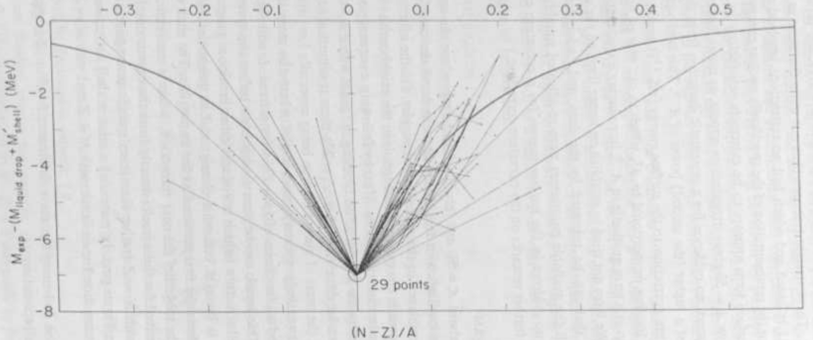


Fig. 14. The sharp trough along $N = Z$, occurring in the masses of the lighter nuclei, is illustrated. The experimental masses in the range $A = 4$ to $A = 58$ were corrected for all known effects (liquid-drop binding and shell effects deduced from nuclei with $N = Z$) and the remainder plotted as a function of $(N-Z)/A$. Lines connect even mass isobars, dots correspond to odd values of A. The smooth curve is the exponential function $-7\exp(-6|N-Z|/A)$.

Congruence/Wigner Energy: History

250 W.D. Myers, W.J. Swiatecki / Nuclear Physics A 612 (1997) 249–261

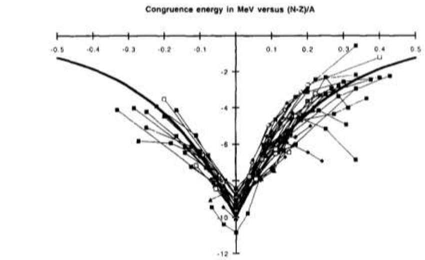


Fig. 1. The difference between measured binding energies and those calculated using the shell- and pairing-corrected Thomas–Fermi model of Ref. [5]. The points (denoted by a variety of symbols) refer to the 28 isobaric chains with $A = 6, 8, \ldots, 60$ that straddle the locus $N = Z$ in the chart of nuclei. The curve is the semi-empirical fit $C(I) = \sim 10 \exp(-4.2|I|)$/MeV.

refer to as the congruence energy – is provided by the following semi-empirical formula:

$$C(I) = -C_0 \exp(-W|I|/C_0) \quad (1a)$$

$$\approx -C_0 + W|I| + \cdots \quad \text{for small } |I|, \quad (1b)$$

with $C_0 = 10$ MeV, $W = 42$ MeV.

A *positive* contribution to nuclear masses proportional to $|I|$ (like the second term in Eq. (1b)) is usually referred to as a Wigner term (Refs. [2,3]). Its origin goes back to Wigner's supermultiplet theory, introduced into nuclear physics at a time when the independent-particle model was not believed to be a good approximation (Ref. [6]). By contrast, the *negative* contribution of the type of Eq. (1a), which is the subject of the present paper is, we believe, a direct consequence of the model of independent quantized particles in a mean-field potential well. (We shall come back to the relation between the congruence energy and the Wigner term in Section 6.)

Myers W D, Swiatecki W J, Nucl. Phys. **A 612** (1997)

$$\omega = 2 - \sqrt{(\text{Neck area})/(\text{Mean fragment cross-section})}$$

$$= 2 - R_n/\bar{R}_f, \quad \text{for necked-in shapes,}$$

$$\omega = 1, \quad \text{for convex shapes,}$$

where R_n is an effective neck radius and $\bar{R}_f$ is a mean of the effective transv of the two nascent fragments. Thus we arrive at a tentative semi-empirical expr the congruence energy of necked-in shapes:

$$C(I, \text{shape}) = -\left(2 - R_n/\bar{R}_f\right)C_0 \exp(-W|I|/C_0).$$

Lublin - Strasbourg Drop model

K. Pomorski and J. Dudek, PRC 67 (2003) 044316

J. Dudek, K. Pomorski, et al. EPJ **A 20** (2004) 165

$$W_0^{LSD} = -C_0 \exp(-W|I|/C_0)$$

Finite Range Liquid Drop Model

Sierk A J, PRC 33 (1986) 2039

Möller P, Sierk A J, Iwamoto A, PRL 92 (2004) 072501

$$W_0^{FRLDM} = -C_0 + W|I|$$

Langevin equations

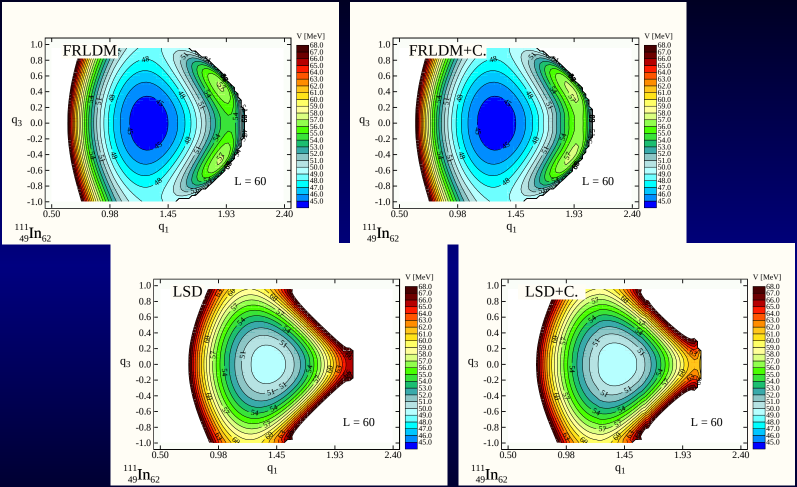
P.N. Nadtochy, G.D. Adeev, Phys. Rev. **C 72**, (2005) 054608

$$\frac{dq_i}{dt} = \sum_j [M^{-1}(\vec{q})]_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{1}{2}\sum_{j,k} \frac{d[M^{-1}(\vec{q})]_{jk}}{dq_i} p_j p_k - \frac{dV(\vec{q})}{dq_i} - \sum_{j,k} \gamma_{ij}(\vec{q})[M^{-1}(\vec{q})]_{jk} p_k + \sum_j g_{ij}(\vec{q})\Gamma_j(t)$$

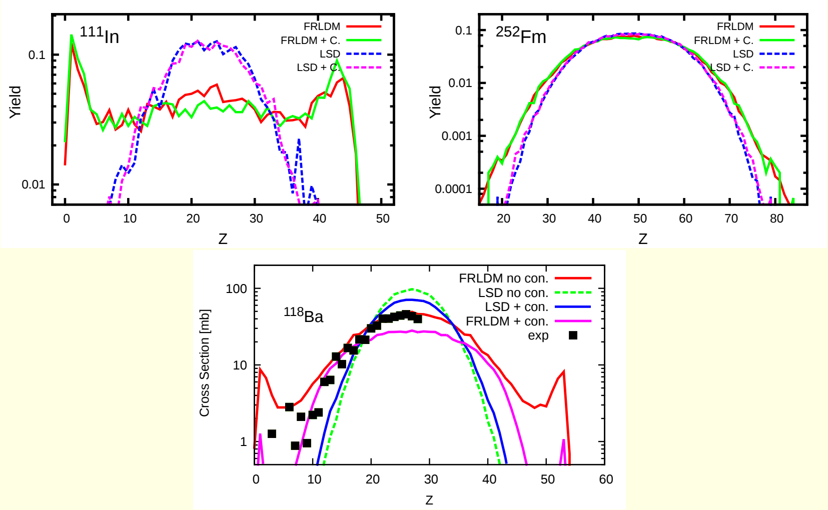
- $[M^{-1}(\vec{q})]_{ij}$ - tensor ot inertia, M_{ij} - tensor of mass
- $V(\vec{q})$ - potential energy
- $\vec{q} = (q_1, q_2, g_3)$ - collective coordinates (funny hills parametrisation)
- $\vec{p} = (p_1, p_2, p_3)$ - conjugate momenta
- $\Gamma_i(t)$ - random variable: $\langle \Gamma_i \rangle = 0$, $\langle \Gamma_i(t_1)\Gamma_j(t_2)\rangle = 2\delta_{ij}\delta(t_1 - t_2)$
- $D_{ij} = g_{ik}g_{kj} \equiv T\gamma_{ij}$ - diffusion tensor
- $T = [E_{int}/a(\vec{q})]^{1/2}$ - Temperature from Fermi gas model
- E_{int} -internal excitation energy
- $E_{coll}(\vec{q}, \vec{(p)}) = \frac{1}{2}[M^{-1}(\vec{q})]_{ij} p_i p_j$ - the kinetic energy of the collective degrees of freedom
- $a(\vec{q}) = a_v A + a_s A^{2/3} B_s(\vec{q})$ - Ignatyuk level density parameter

Potential Energy Surfaces



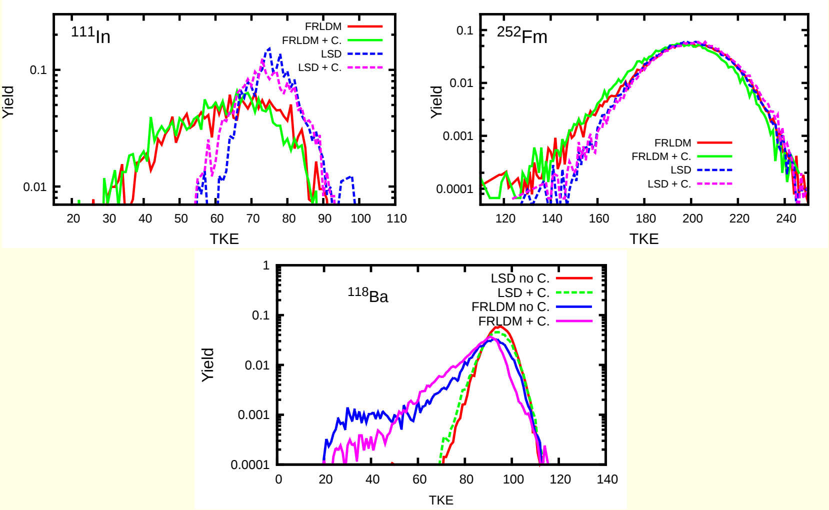
The potential energy surfaces for the ^{111}In calculated with the LSD (bottom) and the FRLDM model (top) with (right) and without congruence energy in the plane (q_1, q_3)-(elongation versus mass asymmetry) for $h = 0.0$.

Results: Fission-Fragment Charge-Distributions



Charge distributions for ^{84}Kr+^{27}Al(10.6AMeV)$\rightarrow$ ^{111}In, ^{78}Kr+^{40}Ca(5.5AMeV)$\rightarrow$ ^{118}Ba and ^{20}Ne+^{232}Th (215MeV)$\rightarrow$ ^{252}Fm for fission channel. Exp. - G. Ademard et al., Phys. Rev.**C 83**, 054619 (2011)

Results: Total Kinetic Energy Distributions



The mean TKE distributions for ^{111}In, ^{118}Ba and ^{252}Fm for fission channel calculated with the FRLDM and LSD.

Particle Emission Multiplicity

$^{111}_{49}In_{62}$

	FRLDM	FRLDM+C.	LSD	LSD+C.	Exp.[1]
n_{pre}	2.95	2.91	2.77	3.06	-
n_{post}	0.81	0.85	1.29	1.18	-
n_{tot}	4.58	4.64	5.34	5.44	-
p_{pre}	0.75	0.75	0.69	0.67	1.3
p_{post}	0.14	0.15	0.15	0.17	0.16
α_{pre}	0.62	0.58	0.35	0.49	0.7
α_{post}	0.22	0.24	0.60	0.54	0.3
$<M>$	52.41	52.50	53.07	52.65	-
FWHM(M)	73.32	78.13	34.04	34.50	-
T	2.46	2.48	2.41	2.34	-
$<TKE>$	63.45	61.03	76.10	74.19	-
FWHM(TKE)	36.38	35.56	18.29	20.96	-

$^{252}_{100}Fm_{152}$

	FRLDM	FRLDM+C.	LSD	LSD+C.	Exp.[2]
n_{pre}	6.48	6.11	6.41	6.36	6.36
n_{post}	2.25	2.44	3.30	3.28	3.83
n_{tot}	10.90	11.00	12.90	12.81	14.6
p_{pre}	0.09	0.10	0.09	0.09	-
p_{post}	0.01	0.01	0.02	0.02	-
α_{pre}	0.14	0.16	0.15	0.15	-
α_{post}	0.01	0.01	0.03	0.03	-
$<M>$	122.43	122.59	122.45	122.47	126
FWHM(M)	55.12	56.32	48.50	49.39	60
T	2.06	2.10	2.09	2.11	-
$<TKE>$	198.96	195.87	199.75	201.02	-
FWHM(TKE)	36.58	37.06	32.60	33.11	48

1. Y. Futami *et al.*, Nucl. Phys. **A607** (1996) 85,

2. D.J. Hinde *et al.*, Phys. Rev. **C 39**, (1989) 2268

A Few Remarks

The Congruence Energy:

- *... should double at the scission point*
- *... could be used to help reproducing the fission barrier heights since ...*
- *... it changes the potential energy surfaces at large elongation*
- *... does not change fission fragment charge distributions for heavy nuclei*
- *... does not change significantly the particle multilicity*

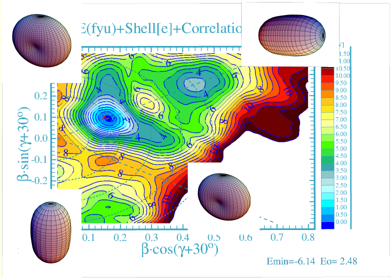
A Question:

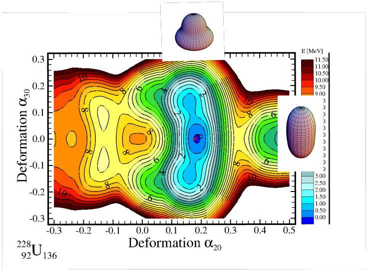
- *... does it have any influence on the other fenomena such as e. g. shape coexistence ?*

Macroscopic-Microscopic Method

Nuclear surface parametrization:

$$\mathcal{R}(\vartheta,\varphi) = R_0\, c(\{\alpha\}) \sum_{\lambda,\mu} [1 + \alpha_{\lambda,\pm\mu}\, Y_{\lambda,\pm\mu}(\vartheta,\varphi)]$$

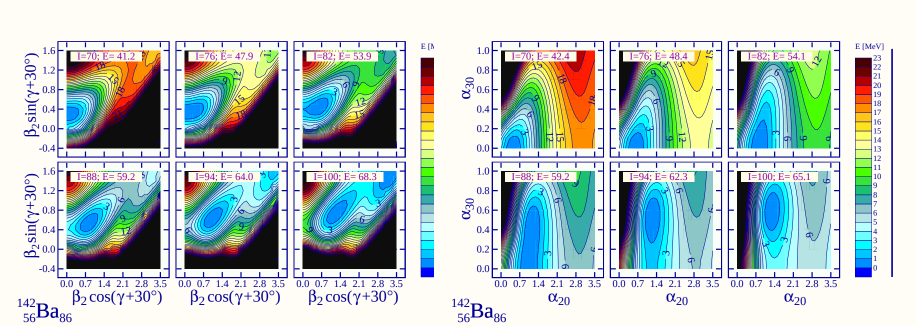




$\{\beta,\gamma\} \to \{\alpha_{20},\alpha_{22}\} \leftrightarrow$ Att.: Shapes out of scale

Jacobi/Poincaré Shape Transition

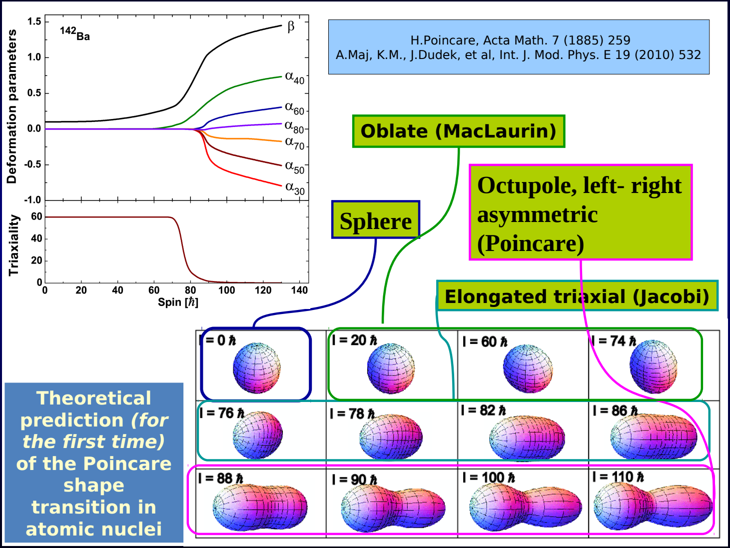
Jacobi shape transition | Poincaré shape transition



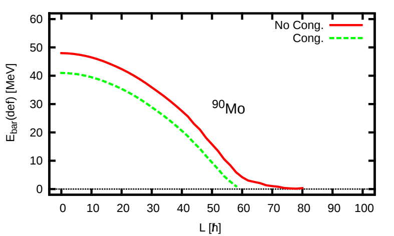
The ^{142}Ba with spin $I = 0 - 110\ \hbar$ A. Maj et al. Int.J.Mod.Phys.E 19 (2010).

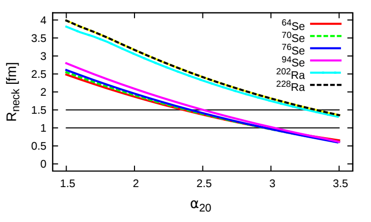
Investigation of the temperature influence on the Poincaré shape transitions is in progress

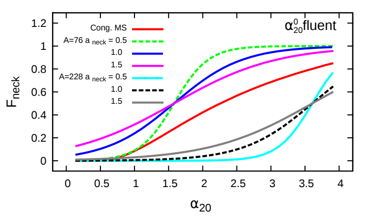
Jacobi/Poincaré Shape Transition



Shape-Dependent Congruence Energy







A new parameterization of the shape dependence [KM, J. Dudek, A. Maj]

$$E_{cong}(\alpha_{20}) = W_0(Z,N) \cdot F_{neck}(\alpha_{20})$$

$$F_{neck}(\alpha_{20}) = 1 + \frac{1}{2}\left\{1 + \tanh\left[(\alpha_{20} - \alpha^0_{20})/a_{neck}\right]\right\}.$$

$$W_0(Z,N) = -C_0 \exp(-W|I|/C_0) \quad \textit{with} \quad I \equiv (N-Z)/A$$

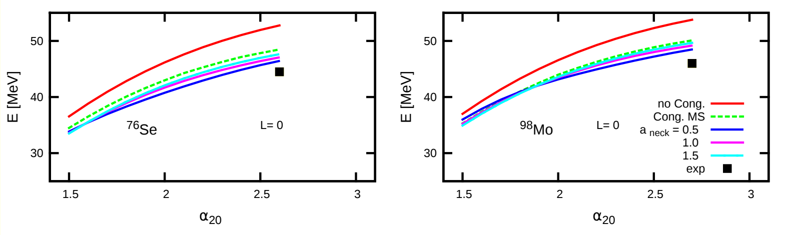
Wigner E, Phys. Rev. 51 (1937) 106,947; Myers W D, Swiatecki W J, Nucl. Phys. A 81 (1966) 1, Nucl. Phys.

Fission Barrier Heights

Nucleus	Exp/Ref	No C.	C. M.-S.	α_{20}^0 **A**-dependent		
			a_{neck} →	0.5	1.0	1.5
^{70}Se	39.4 [2]	50.618	43.337	38.973	40.393	41.825
^{76}Se	44.5 [2]	54.323	49.624	43.944	45.084	46.068
^{75}Br	41.0 [3]	52.603	47.062	42.169	43.410	44.599
^{90}Mo	42.0 [4]	50.890	45.519	40.995	42.308	43.359
^{98}Mo	46.0 [4]	54.571	50.651	46.495	47.443	48.132
^{173}Lu	29.0 [5]	28.707	25.635	27.433	26.797	26.616
^{228}Ra	6.3 [5]	6.204	6.013	6.204	6.186	6.120

Comparison of the barrier heights for nuclei listed. Columns 2-5 contain: Experimental values [Exp], reference of origin [Ref], LSD model results with congruence ignored, and Congruence effect from Myers and Świątecki [C. M.-S.],[1] . The last three columns represent the results obtained using the hypotheses: $a_{neck} = 0.5, 1.0$ and 1.5 .

1. Myers W D, Swiatecki W J, Nucl. Phys. **A 612** (1997) 249
2. Fan T S, et al, Nucl. Phys. **A 679** (2000) 121
3. Delis D N, et al, Nucl. Phys. **A 534** (1991) 403
4. Jing K X, et al, Nucl. Phys. **A 645** (1999) 203
5. Moretto L G, et al, Phys. Lett. **B 38** (1972) 471



Summary and Conclusions

- *The fission dynamics tested with four macroscopic models.*
- *The fission fragment charge distribution could be very sensitive on the adding congruence energy.*

- *A new formula for the shape-dependent congruence energy is proposed and tested [KM, J. Dudek and A. Maj; to be published].*

- *The spin range for the shape transition is influenced by the congruence energy.*