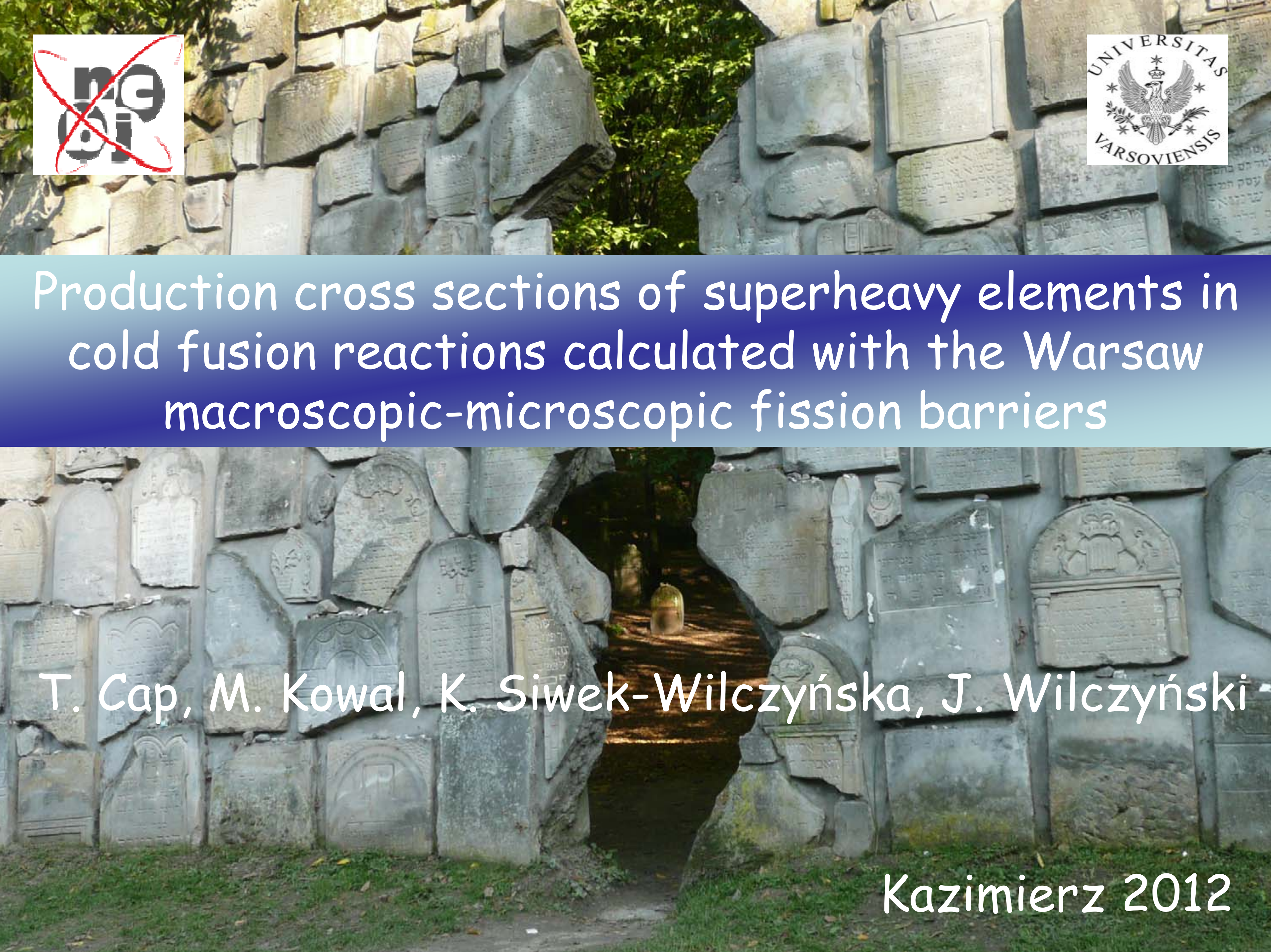
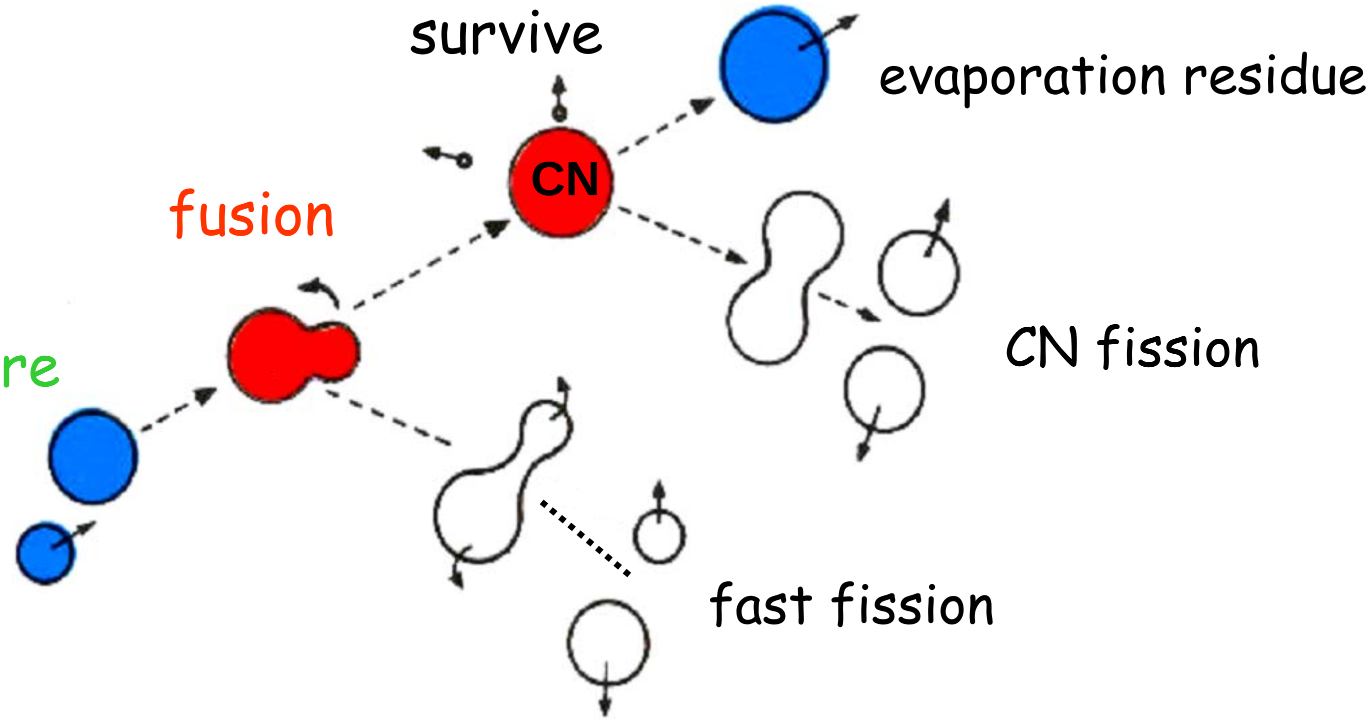


Production cross sections of superheavy elements in cold fusion reactions calculated with the Warsaw macroscopic-microscopic fission barriers

T. Cap, M. Kowal, K. Siwek-Wilczyńska, J. Wilczyński

Kazimierz 2012

Nucleus-nucleus collisions which may lead to the synthesis of super-heavy nuclei



$$\sigma(\text{synthesis}) = \pi \lambda\!\!\!^{-2} \sum_{l=0}^{\infty} (2l+1) T_l P_l(\text{fusion}) P_{1n}^{\ell}(\text{survive})$$

W. J. Świątecki, K. Siwek-Wilczyńska, and J. Wilczyński, *Acta Phys. Pol.* 34, 2049 (2003).
W. J. Świątecki, K. Siwek-Wilczyńska, and J. Wilczyński, *Phys. Rev.* C 71, 014602 (2005).
T. Cap, K. Siwek-Wilczyńska, J. Wilczyński, IJMP E20, 308 (2011)
T. Cap, K. Siwek-Wilczyńska, J. Wilczyński, PR C83, 054602 (2011)
K. Siwek-Wilczyńska, T. Cap, M. Kowal, A. Sobiczewski, J. Wilczyński, PR C86, 014611 (2012)

FBD → $\sigma(\text{synthesis}) = \pi\lambda^2 \sum_{l=0}^{l\max} (2l+1) P_l(\text{fusion}) P_{1n}^{\ell}(\text{survive})$

l_{max} – calculated from the capture cross section.

$$\sigma_{cap}(E) = \pi\lambda^2 \sum_{l=0}^{\infty} (2l+1) T_l \approx \pi\lambda^2 (l_{\max}+1)^2$$

semiempirical formula

$$\sigma_{cap}(E) = \pi R_\sigma^2 \left[X\sqrt{\pi}\,(1 + erf\ X) + \exp(-X^2) \right] \frac{w}{E\sqrt{2\pi}}$$

$where: \qquad X = \frac{E - B_0}{\sqrt{2w}}, \qquad erf\ X -$ Gaussian error function

This formula derived assuming:

- Gaussian shape of the fusion barrier distribution
- Classical expression for $\sigma_{fus}(E,B)=\pi R^2(1-B/E)$

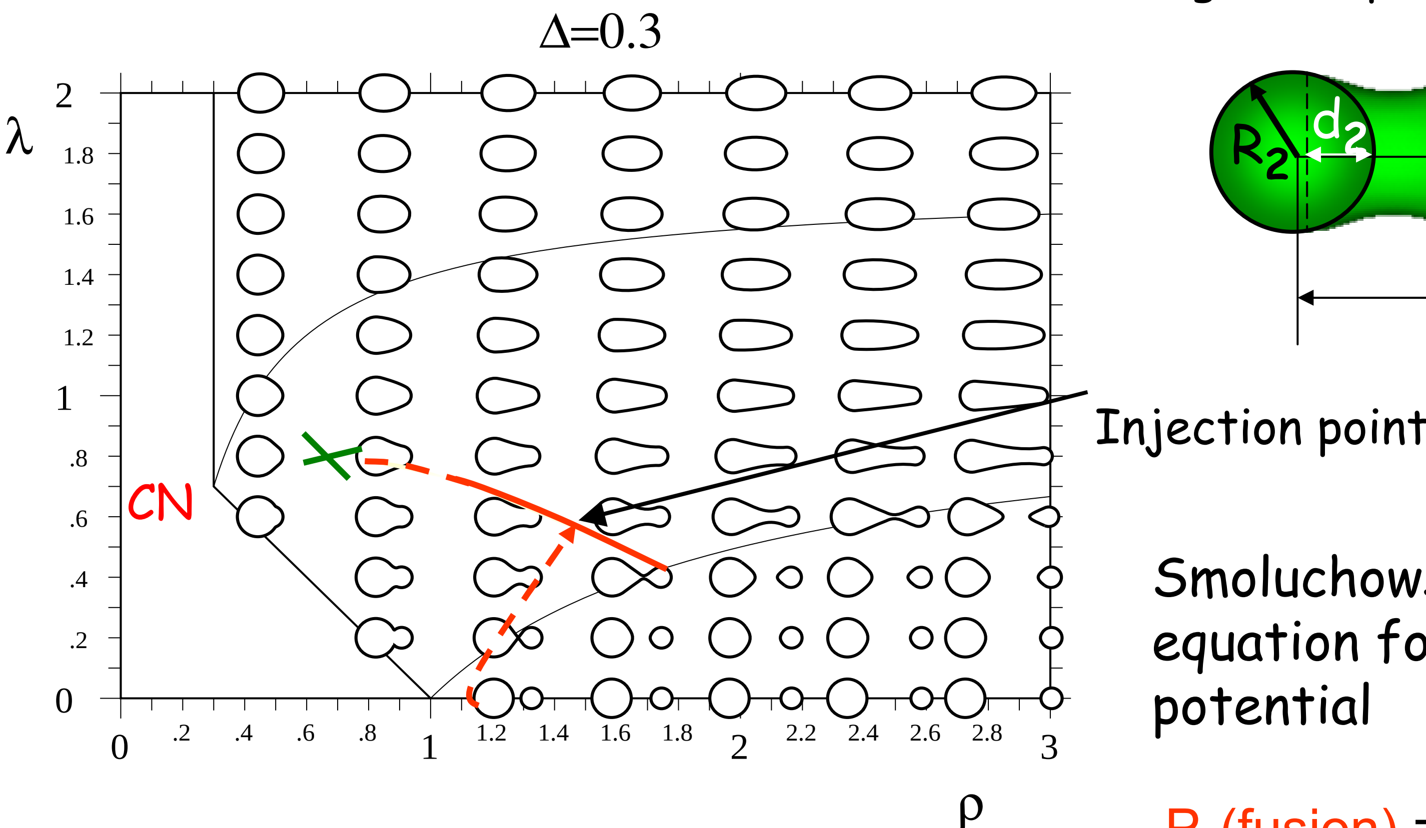
W. Świątecki, K. Siwek-Wilczyńska, J. Wilczyński Phys. Rev. C 71 (2005) 014602,
Acta Phys. Pol. B34(2003) 2049

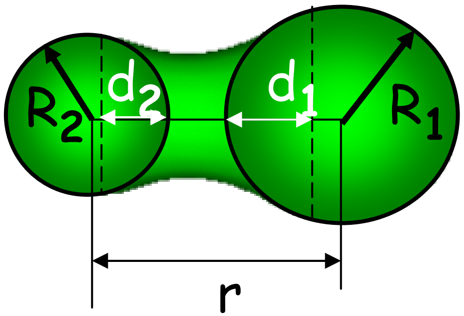
3 parameters: B_0, w, R_σ obtained from χ^2 fit to 48 experimental near-barrier fusion excitation functions for $40 < Z_{CN} < 98$

(K. Siwek-Wilczyńska, J. Wilczyński Phys. Rev. C 69 (2004) 024611)

P_l(fusion)

J. Błocki, W. J. Świątecki, Nuclear Deformation Energies, Report LBL 12811 (1982)





Smoluchowski Diffusion equation for the parabolic potential

$$P_l(\text{fusion}) = \tfrac{1}{2}\left(1-\text{erf}\sqrt{H(l)/T}\right)$$

$\lambda = (d_1 + d_2)/(R_1 + R_2)$ - neck parameter

$\rho = r/(R_1 + R_2)$ – relative distance

$\Delta = (R_1 - R_2)/(R_1 + R_2)$ – asymmetry parameter

H - the barrier opposing fusion

T - the temperature of the fusing system

$P_{1n}^{l}(survive) = [\Gamma_n / (\Gamma_n + \Gamma_f)] \times P_<$

- Partial widths for neutron emission – Weisskopf formula

$$\Gamma_{in} = \frac{m_n}{\pi^2 \hbar^2}(2s_n + 1)\int_0^{E_{in}^{\max}} \varepsilon_{in}\sigma_{in} \frac{\rho_{in}\left(E_{in}^{\max} - \varepsilon_{in}\right)}{\rho\left(E^*\right)} d\varepsilon_{in}$$

$$E_{in}^{\max} = E_{(i-1)}^{*} - E_{rot}^{in} - B_{in} - P$$

Upper limit of the final-state excitation energy after emission of a particle *i*

σ_i – cross section for the production of the compound nucleus in the inverse process
m_i, s_i, ε_i - mass, spin and kinetic energy of the emitted particle
ρ, ρ_i – level densities of the parent and daughter nuclei

- The fission width – The transition state method

$$\Gamma_{ifiss} = \frac{1}{2\pi}\int_0^{E_{if}^{\max}} \frac{\rho_{fiss}\left(E_{if}^{\max} - K\right)}{\rho\left(E^*\right)} dK$$

$$E_{if}^{\max} = E_{i-1}^{*} - B_{if} - E_{rot}(saddle) - P$$

Upper limit of the thermal excitation energy at the saddle

- The level density is calculated using the Fermi-gas-model formula including shell efects

included as proposed by Ignatyuk
(A.V. Ignatyuk et al., Sov. J. Nucl. Phys. 29 (1975) 255)

$$a = a_{macro}\left[1 + \frac{\delta_{shell}}{U}\left(1 - e^{-U/E_d}\right)\right]$$

To calculate the survival probability we need to know (for all nuclei in the deexcitation cascade):

- ground state masses,
- fission barriers,
- shell correction energies and deformations (in the ground state and saddle).

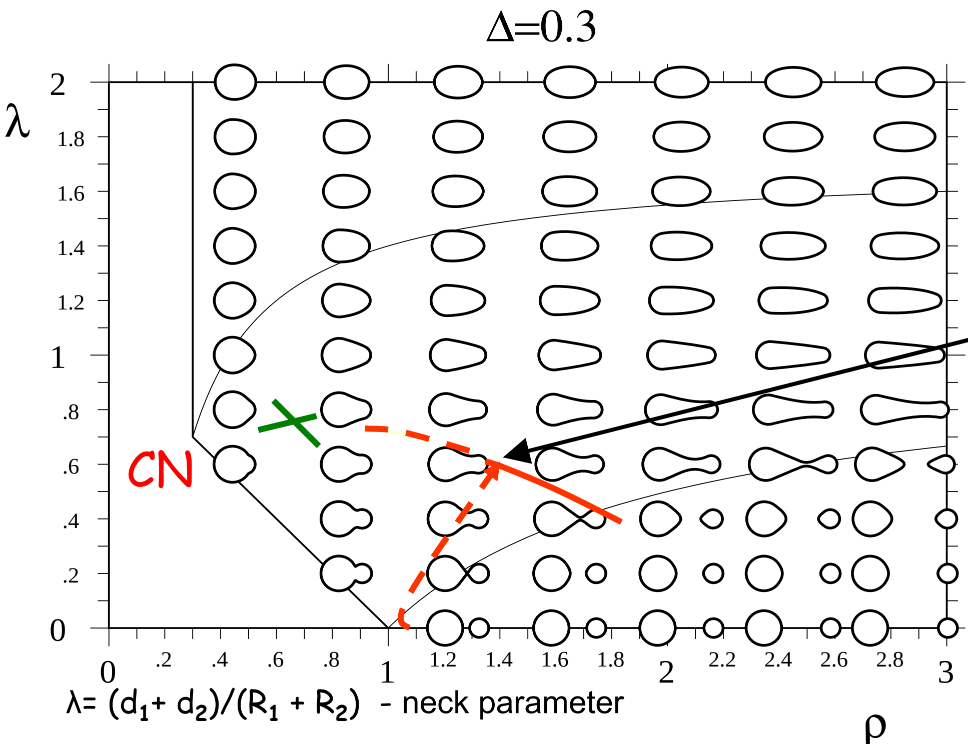
Those values were calculated using the Warsaw macroscopic-microscopic model including the nonaxial shapes.

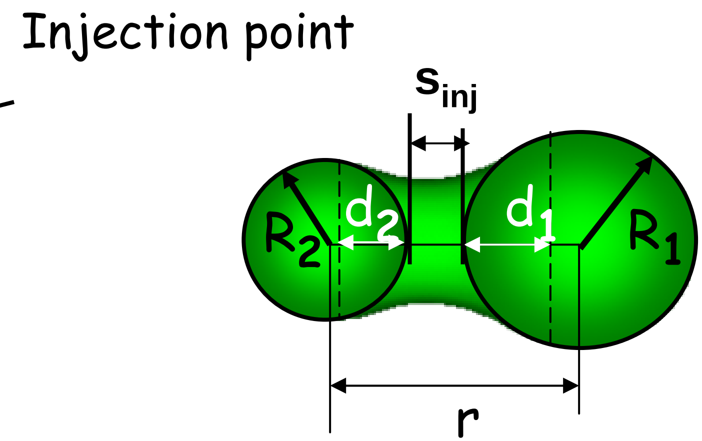
M. Kowal, P. Jachimowicz, J. Skalski, arXiv:1203.5013
M. Kowal, P. Jachimowicz, A. Sobiczewski, Phys. Rev. C82 (2010) 014303
M. Kowal (unpublished).

$$\sigma(\text{synthesis}) = \pi \lambda\!\!\!^{-2} \sum_{l=0}^{l\max} (2l+1) P_l(\text{fusion}) P_{1n}^{\ell}(\text{survive})$$

In FBD model there is only one adjustable parameter.

$$S_{inj} = L - 2(R_1 + R_2) = r - (R_1 + R_2)$$





$\lambda = (d_1 + d_2)/(R_1 + R_2)$ - neck parameter

$\rho = r/(R_1 + R_2)$ – relative distance

$\Delta = (R_1 - R_2)/(R_1 + R_2)$ – asymmetry parameter

For each studied reaction the excitation function was calculated assuming different values of $s_{inj.}$ The s_{inj} – values for which the cross section at maximum of the calculated excitation function agreed with the maximum of the experimental function was adopted for a given 1n reaction.

Z=104

$^{208}Pb(^{48}Ti,n)^{255}Rf$ LBNL
$^{208}Pb(^{50}Ti,n)^{257}Rf$ GSI
$^{208}Pb(^{50}Ti,n)^{257}Rf$ LBNL

Z=105

$^{209}Bi(^{50}Ti,n)^{258}Db$ GSI
$^{209}Bi(^{50}Ti,n)^{258}Db$ LBNL
$^{208}Pb(^{51}V,n)^{258}Db$ LBNL

Z=106

$^{208}Pb(^{54}Cr,n)^{261}Sg$ GSI
$^{208}Pb(^{52}Cr,n)^{259}Sg$ LBNL

Z=107

$^{209}Bi(^{54}Cr,n)^{262}Bh$ GSI
$^{209}Bi(^{54}Cr,n)^{262}Bh$ LBNL
$^{209}Bi(^{52}Cr,n)^{260}Bh$ LBNL
$^{208}Pb(^{55}Mn,n)^{262}Bh$ LBNL

Z=108

$^{208}Pb(^{58}Fe,n)^{265}Hs$ GSI
$^{208}Pb(^{56}Fe,n)^{263}Hs$ LBNL

Z=109

$^{209}Bi(^{58}Fe,n)^{266}Mt$ GSI
$^{208}Pb(^{59}Co,n)^{266}Mt$ LBNL

Z=110

$^{208}Pb(^{64}Ni,n)^{271}Ds$ GSI
$^{208}Pb(^{64}Ni,n)^{271}Ds$ LBNL
$^{208}Pb(^{64}Ni,n)^{271}Ds$ RIKEN
$^{207}Pb(^{64}Ni,n)^{270}Ds$ GSI
$^{208}Pb(^{62}Ni,n)^{269}Ds$ GSI

Z=111

$^{209}Bi(^{64}Ni,n)^{272}Rg$ GSI
$^{209}Bi(^{64}Ni,n)^{272}Rg$ RIKEN
$^{208}Pb(^{65}Cu,n)^{272}Rg$ LBNL

Z=112

$^{208}Pb(^{70}Zn,n)^{277}Cn$ GSI
$^{208}Pb(^{70}Zn,n)^{277}Cn$ RIKEN

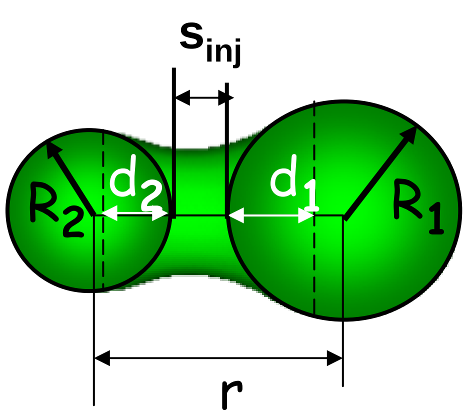
Z=113

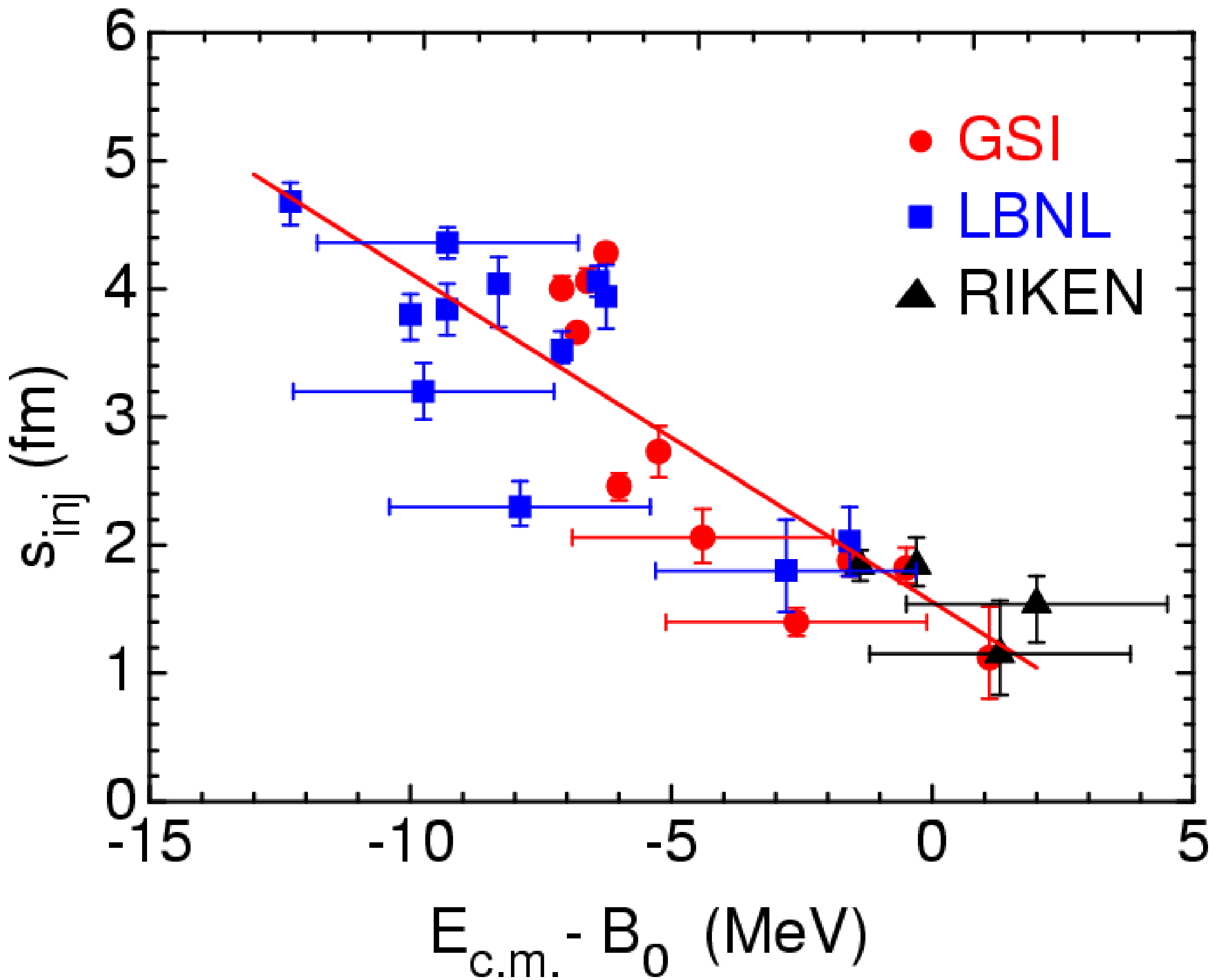
$^{209}Bi(^{70}Zn,n)^{278}113$ RIKEN

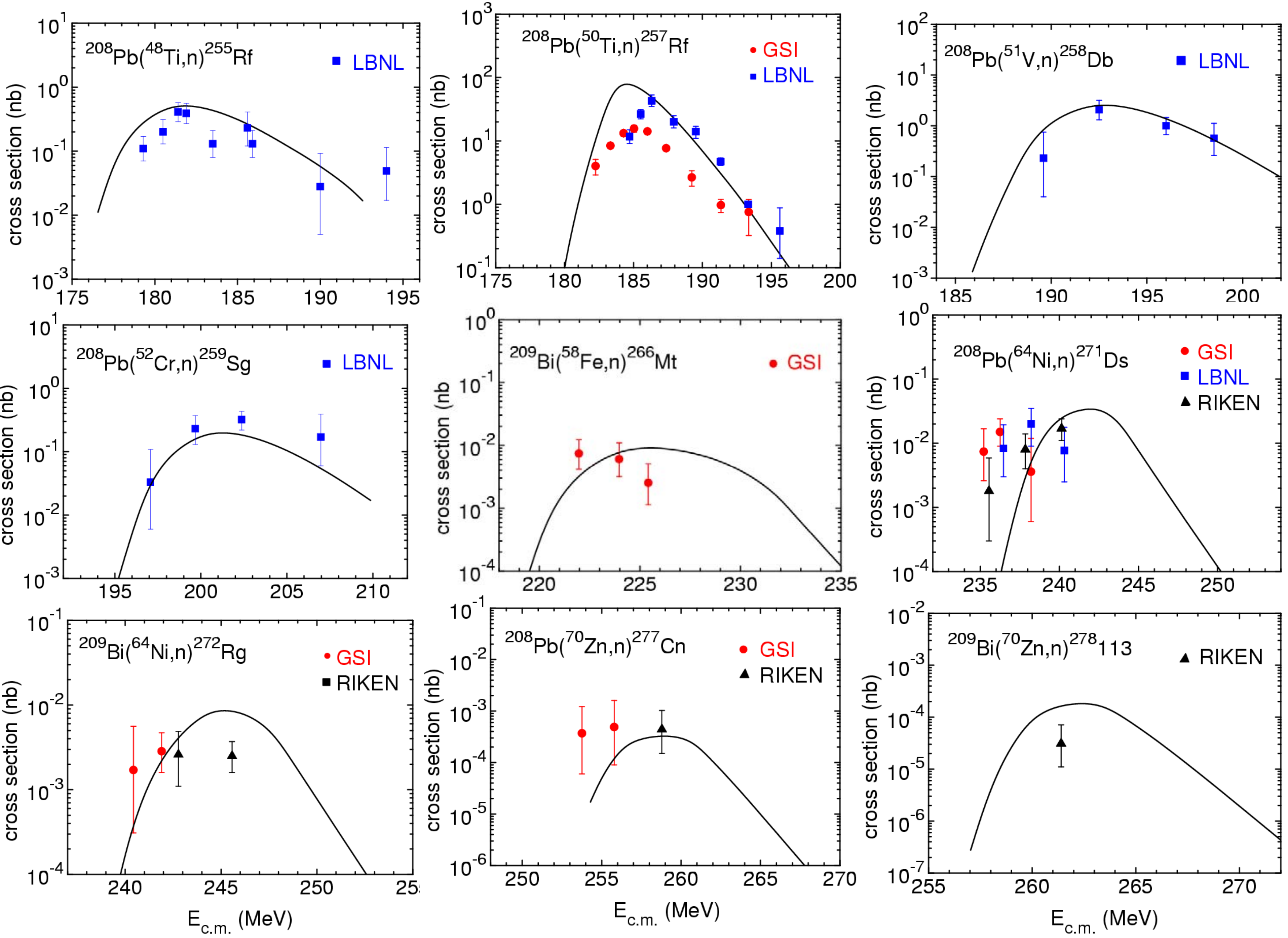
Systematics of the s_{inj} parameter from the fit to the maximum values of the experimental cross sections (complete set of existing data, 27 cold fusion reactions were studied)

Projectiles:
^{48}Ti – ^{70}Zn

Targets:
^{208}Pb or ^{209}Bi



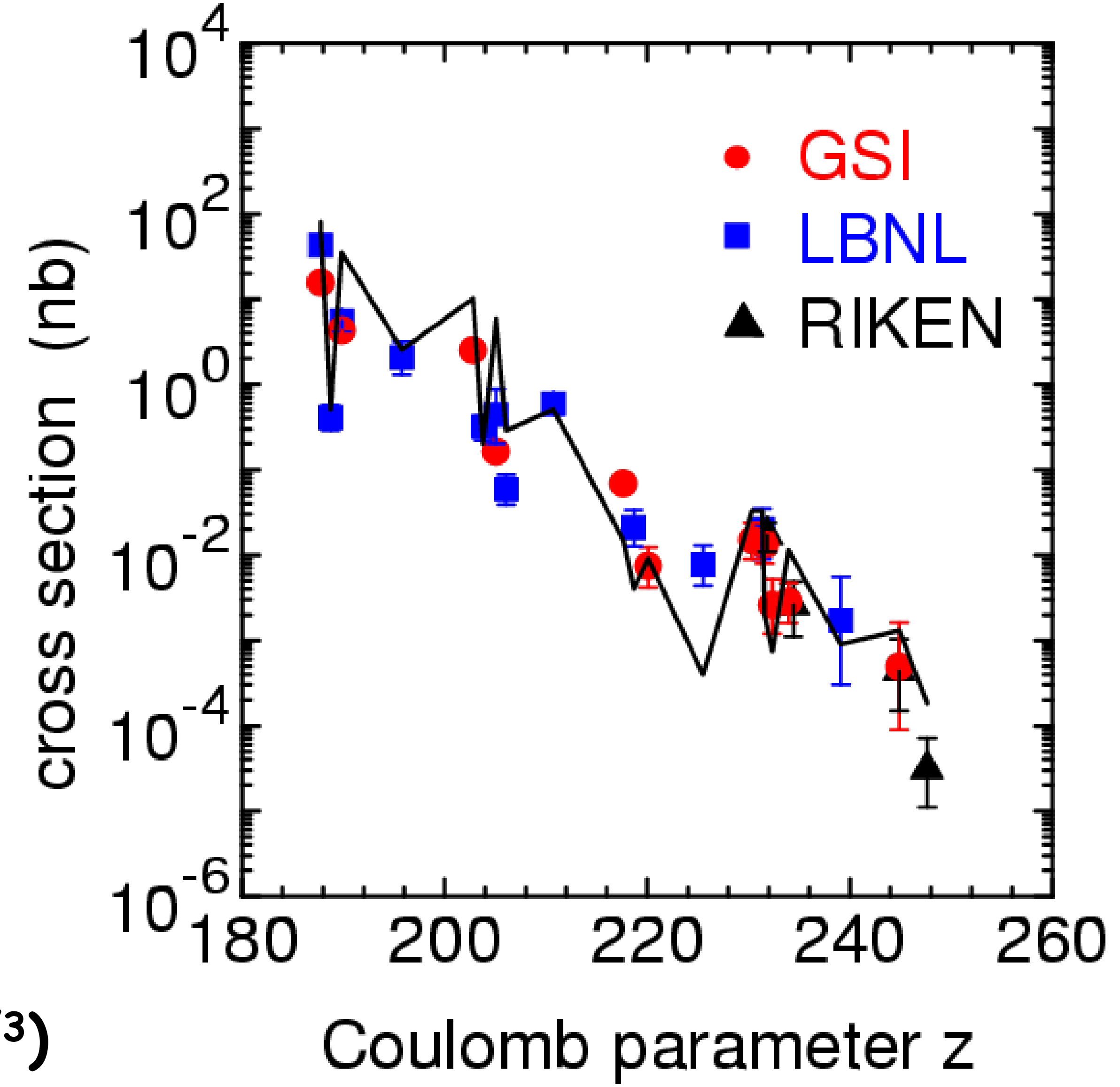


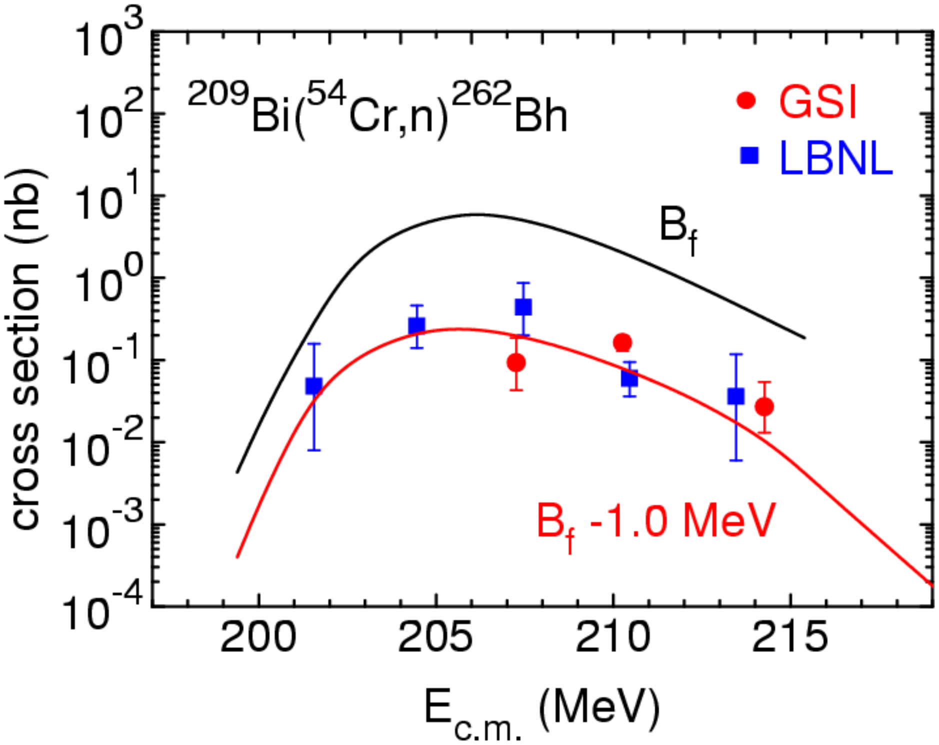


$^{208}Pb(^{48}Ti,n)^{255}Rf$
LBNL
$^{208}Pb(^{50}Ti,n)^{257}Rf$
GSI
LBNL
$^{208}Pb(^{51}V,n)^{258}Db$
LBNL
$^{208}Pb(^{52}Cr,n)^{259}Sg$
LBNL
$^{209}Bi(^{58}Fe,n)^{266}Mt$
GSI
$^{208}Pb(^{64}Ni,n)^{271}Ds$
GSI
LBNL
RIKEN
$^{209}Bi(^{64}Ni,n)^{272}Rg$
GSI
RIKEN
$^{208}Pb(^{70}Zn,n)^{277}Cn$
GSI
RIKEN
$^{209}Bi(^{70}Zn,n)^{278}113$
RIKEN
cross section (nb)
$E_{c.m.}$ (MeV)

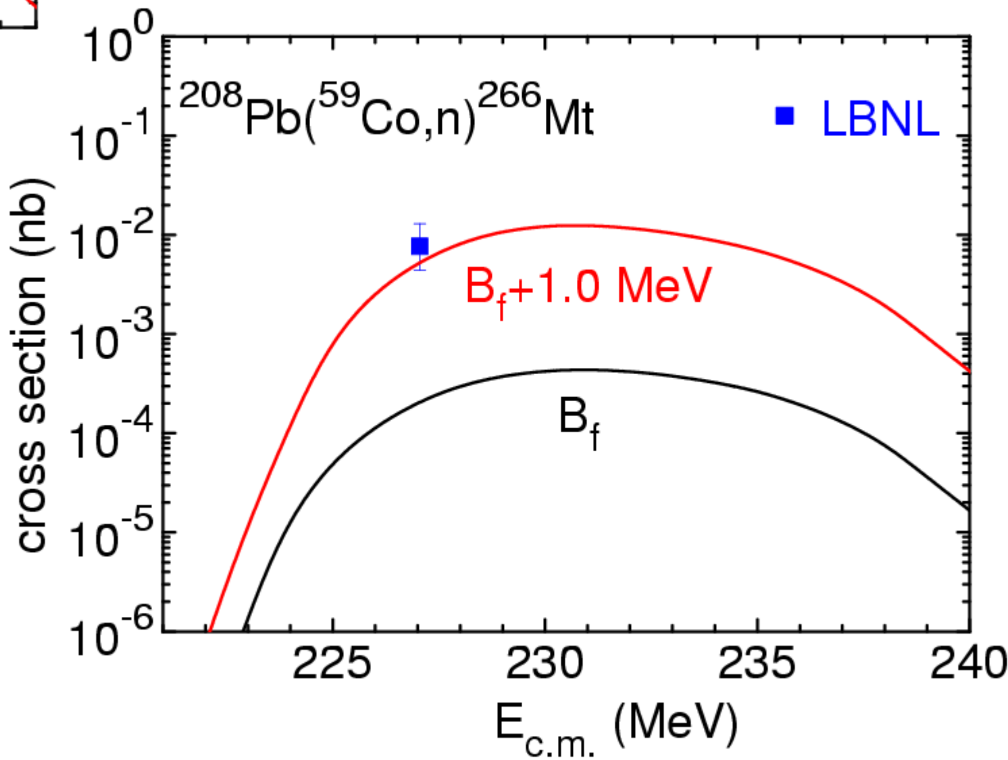
- Good reproduction of the experimental trend in cross sections – within 7 orders of magnitude.
- Reproduction of the deviations from a smooth dependence of the cross section on z caused by structure effects of projectile, target and CN.

$z = Z_p Z_t / (A_p^{1/3} + A_t^{1/3})$





The cross section is very sensitive to the value of the fission barrier.



Sugestion: cold fusion excitation function as a tool to extract the „experimental" value of fission barrier.

SUMMARY

The Fusion by Diffusion model was applied to calculate synthesis cross sections of superheavy nuclei in cold fusion reactions (Z =104 - 113).

Fission barriers and ground state masses calculated with the Warsaw macroscopic-microscopic model (including nonaxial shapes) were applied. Good agreement with experimental cross sections was obtained.

The sugested systematics of s_{inj} can be used to calculate cross section for unexplored yet cold fusion reactions.

Uncertainties of calculated cross sections

$$\sigma(\text{synthesis}) = \pi \lambda\!\!\!^{-2} \sum_{l=0}^{l\max} (2l+1) P_l(\text{fusion}) P_{xn}^{\ell}(\text{survive})$$

σ(capture) does not change significantly from one system to another. Resulting uncertainties are not large unless the deeply sub-barrier reactions are studied (e.q. cold fusion)

P(fusion) depends on the asymmetry of the colliding system and the entrance channel energy. Theoretical (or phenomenological) predictions may result in large uncertainties of several orders of magnitude only for unexplored heavy and symmetric systems.

P(survival) Very strong dependence on B_f-B_n easily resulting in orders of magnitude differences of the cross section (1 MeV difference - about 1 order of magnitude on each step of the deexcitation cascade).

<u>It is very important to do systematic studies and use well tested theoretical predictions for both, ground state and saddle properties.</u>