On the Poincare instability of rotating nuclei

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- Introduction
- The Strutinsky‘s procedure for the shape of fissioning nuclei
- The shape transition in axially symmetric rotating charged liquid drop
- The shape of triaxial rotating drop
- The Poincare instability of light rotating nuclei
- The summary

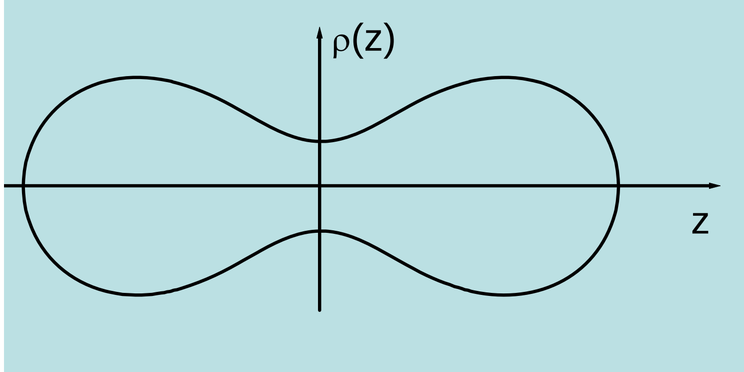
The shape parameterisations

- Expansion around sphere in terms of spherical harmonics
- Koonin-Trentalange parameterisation $y^2(z) = \sum_n a_n P_n(z/z_o)$
- (Distorted) Cassinian ovaloids
- (modified) Funny-Hills parameterisation
- Two smoothly connected spheroids
- The two center shell model

Strutinsky's optimal shapes

$$E_{LD} = E_{LD}[\rho] = E_{surf}[\rho] + E_{Coul}[\rho] + E_{curv}[\rho]$$

$$\rho = \rho(z) - \text{profile function}$$



$$E_{\text{surf}} = 2\pi\sigma \int_{z_1}^{z_2} \rho(z)\sqrt{1 + (d\rho/dz)^2}\, dz$$

$$E_{\text{Coul}} = x_{LD} \int_{z_1}^{z_2} \left[\rho^2(z) - \frac{1}{2}\frac{d\rho^2(z)}{dz} \right] \Phi_S(z, \rho(z))\, dz$$

$$\frac{\delta}{\delta\rho}\left[E_{LD} + \lambda_1 V + \lambda_2 R_{12}\right] = 0, \qquad R_{12} = \frac{2\pi}{V}\int \rho^2(z)|z|\, dz$$

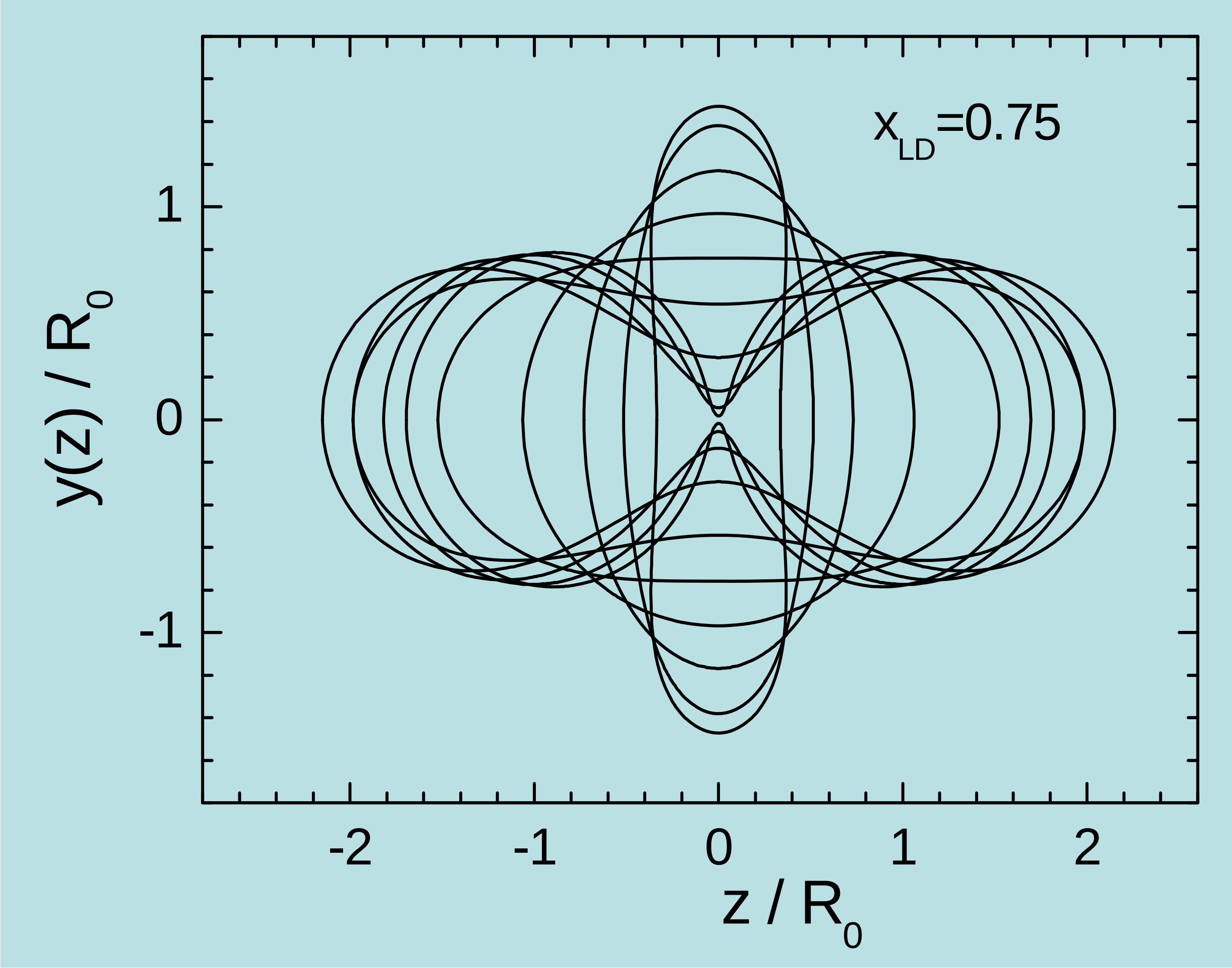
$$\rho\rho'' = 1 + (\rho')^2 + \rho\left[\lambda_1 + \lambda_2|z| + 10 x_{LD}\Phi_S(z)\right](1 + (\rho')^2)^{3/2}$$

x_{LD} – the fissility parameter, $x_{LD} \equiv E_C^{(0)} / 2E_S^{(0)} \approx (Z^2/A)/49$

$\Phi_S(z)$ – the Coulomb potential on the surface

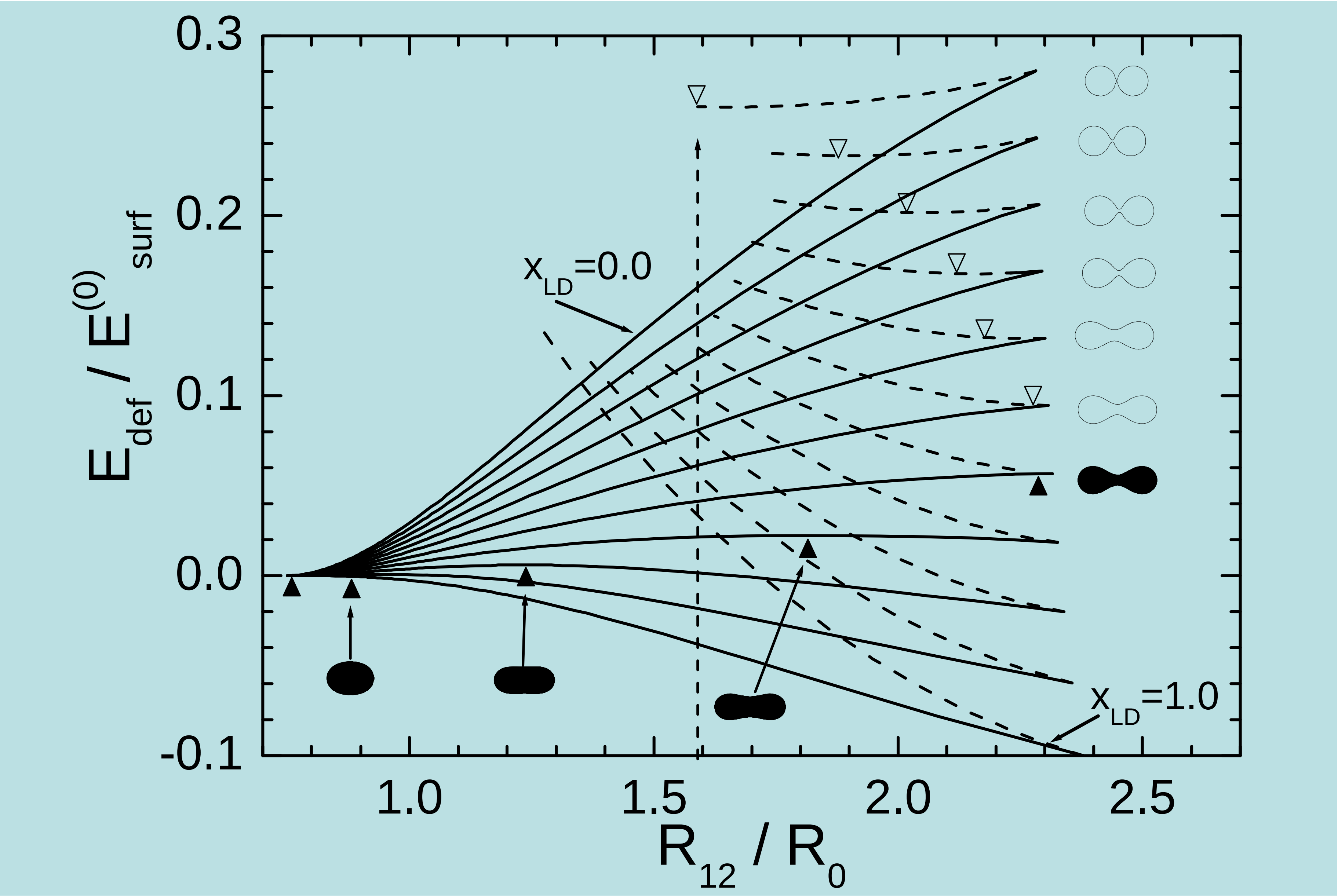
V.M.Strutinsky et al, Nucl. Phys. 46, 659 (1963)

Optimal shapes

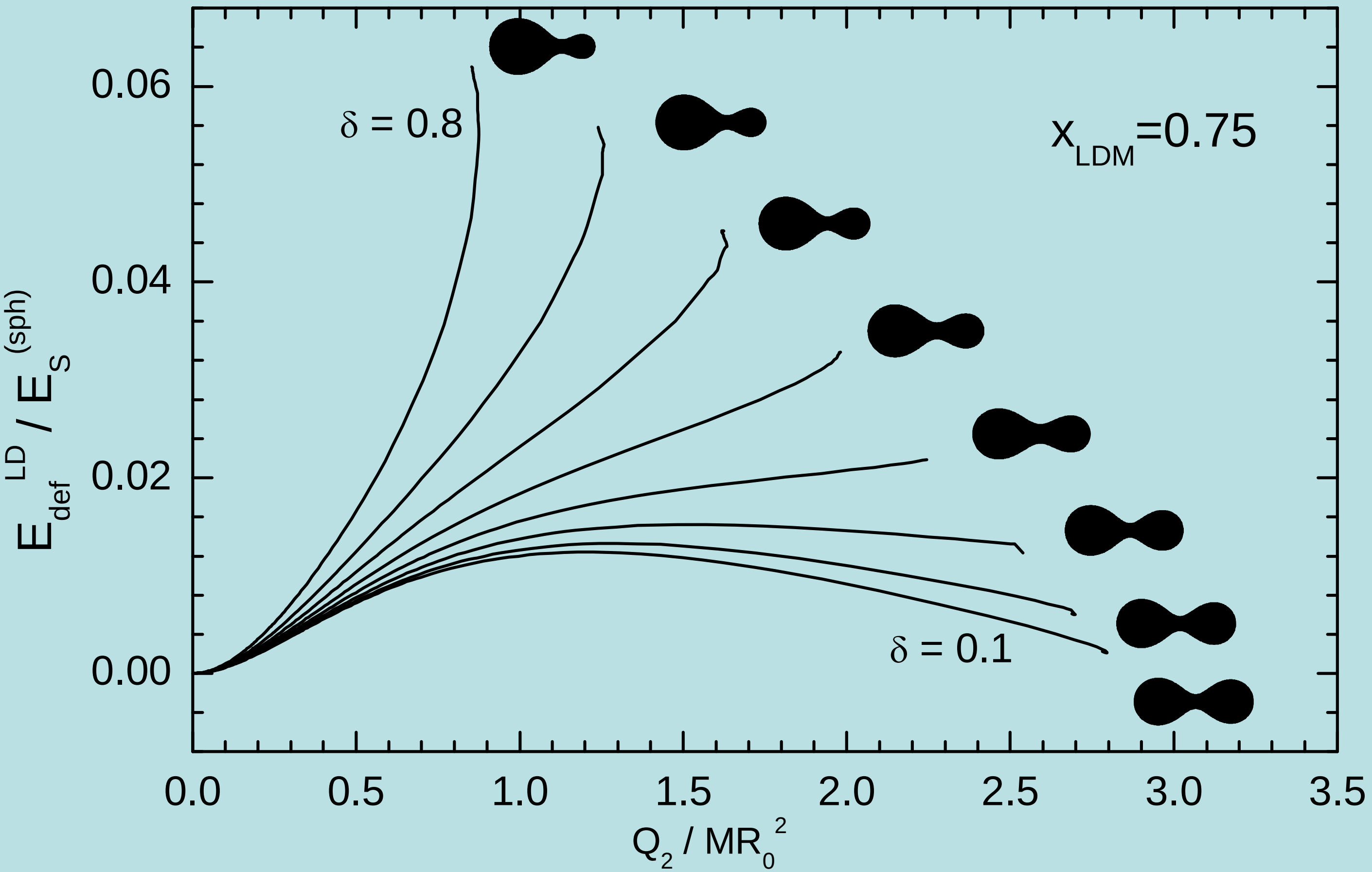


$$\rho\rho'' = 1 + (\rho')^2 + \rho\left[\lambda_1 + \lambda_2 |z| + 10 x_{LD} \Phi_S(z)\right](1 + (\rho')^2)^{3/2}$$

Deformation energy, $(R_{12})_{crit} = 2.3\ R_0$



The scission shapes



shape is divided in parts by the point of maximal curvature

F. Ivanyuk, Int. Jour. Mod. Phys. E18 (2009)879

The shape of axially symmetric rotating drop

the energy: $E_{RLD} = E_{surf}\left[\rho\right] + E_{Coul}\left[\rho\right] + E_{rot}\left[\rho\right]$

$E_{rot}\left[\rho\right] = \hbar^2 L^2 / 2J$, J – rigid-body moment of inertia

var. principle: $\frac{\delta}{\delta\rho}\left[E_{LD} + \lambda_1 V + \lambda_2 R_{12}\right] = 0 \Rightarrow \frac{\delta}{\delta\rho}\left[E_{LD} + \lambda V + E_{rot}\right] = 0$

diff. equation

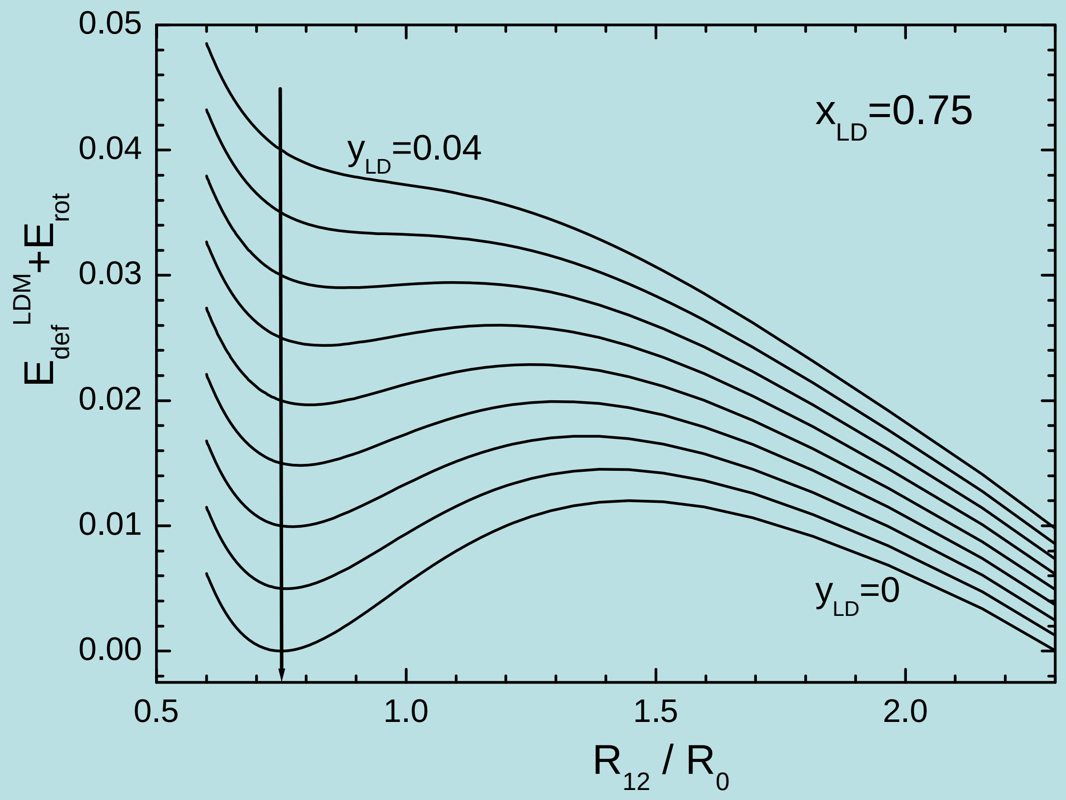
$$\rho\rho'' = 1 + (\rho')^2 - \rho\left[\lambda - 10 x_{LD}\Phi_S(z) + \frac{15}{4} B_{rot}{}^2 \, y_{LD}\Phi_{rot}(z)\right](1 + (\rho')^2)^{3/2}$$

$y_{LD} \equiv E_{rot}{}^{(0)} / E_S{}^{(0)}$, $\Phi_{rot}(z) = \delta B_{rot} / \delta\rho$, $B_{rot} \equiv E_{rot} / E_{rot}{}^{(0)}$

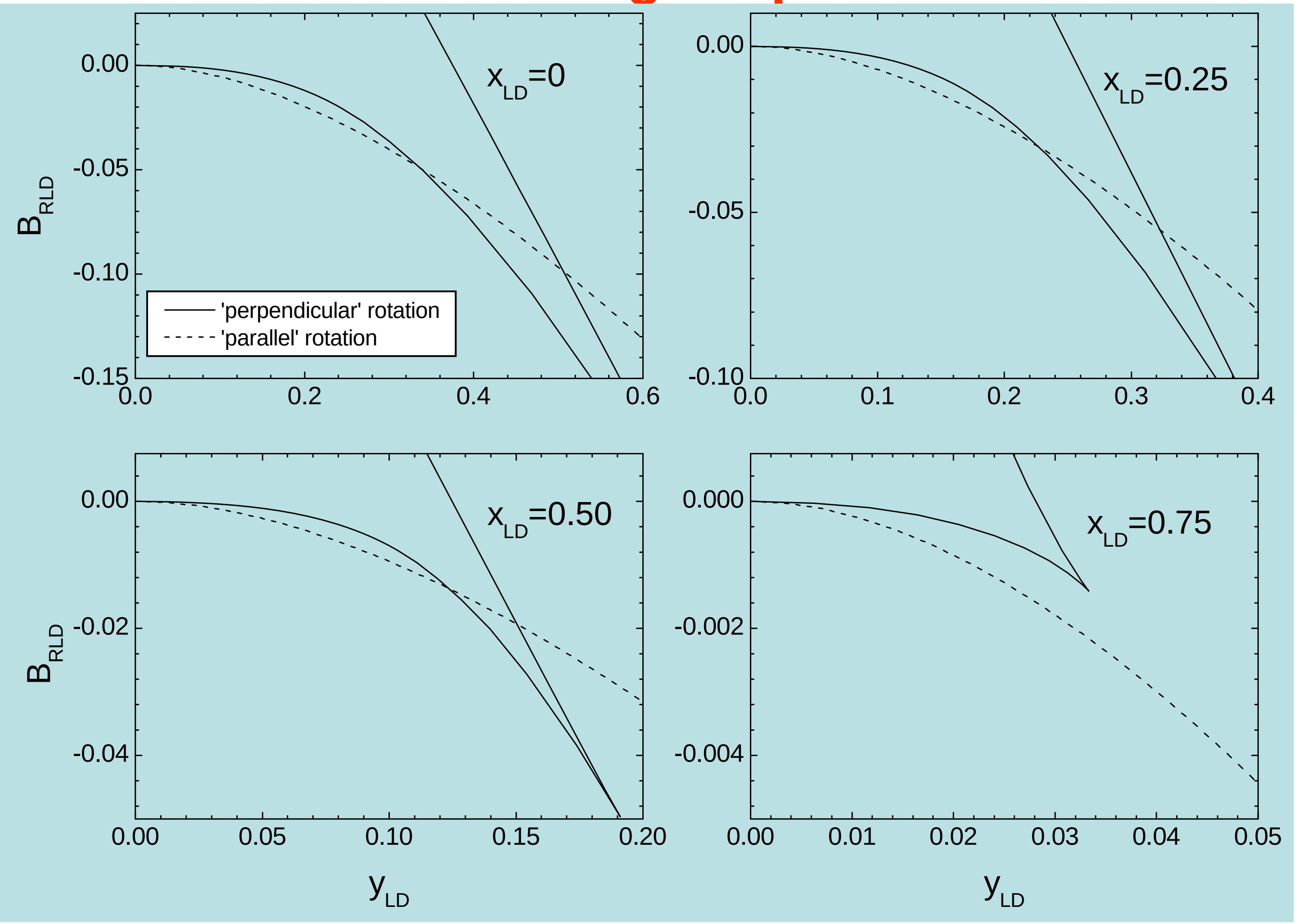
$$\Phi_{rot}(z) = \begin{cases} \rho^2(z) + 2z^2, & \text{for the perpendicular rotation} \\ 2\rho^2(z), & \text{for the parallel rotation} \end{cases}$$

J. Bartel, F. Ivanyuk and K.Pomorski, Int. Jour. Mod. Phys. E19 (2010) 601

The deformation energy of rotating drop



The energy and shape transition in axially symmetric rotating drop



$$B_{RLD} = (B_{surf} - 1) + 2x_{LD}(B_{Coul} - 1) + y_{LD}(B_{rot} - 1)$$

The non-axial rotating drops

$$\rho(z) \Rightarrow \rho(z,\varphi), \quad E_{RLD} = \int dz \int d\varphi\, \varepsilon(\rho, \rho_z, \rho_\varphi; z, \varphi)$$

Euler-Lagrange equation: $\frac{\partial \varepsilon}{\partial \rho} - \frac{d}{dz}\frac{\partial \varepsilon}{\partial \rho_z} - \frac{d}{d\varphi}\frac{\partial \varepsilon}{\partial \rho_\varphi} = 0$

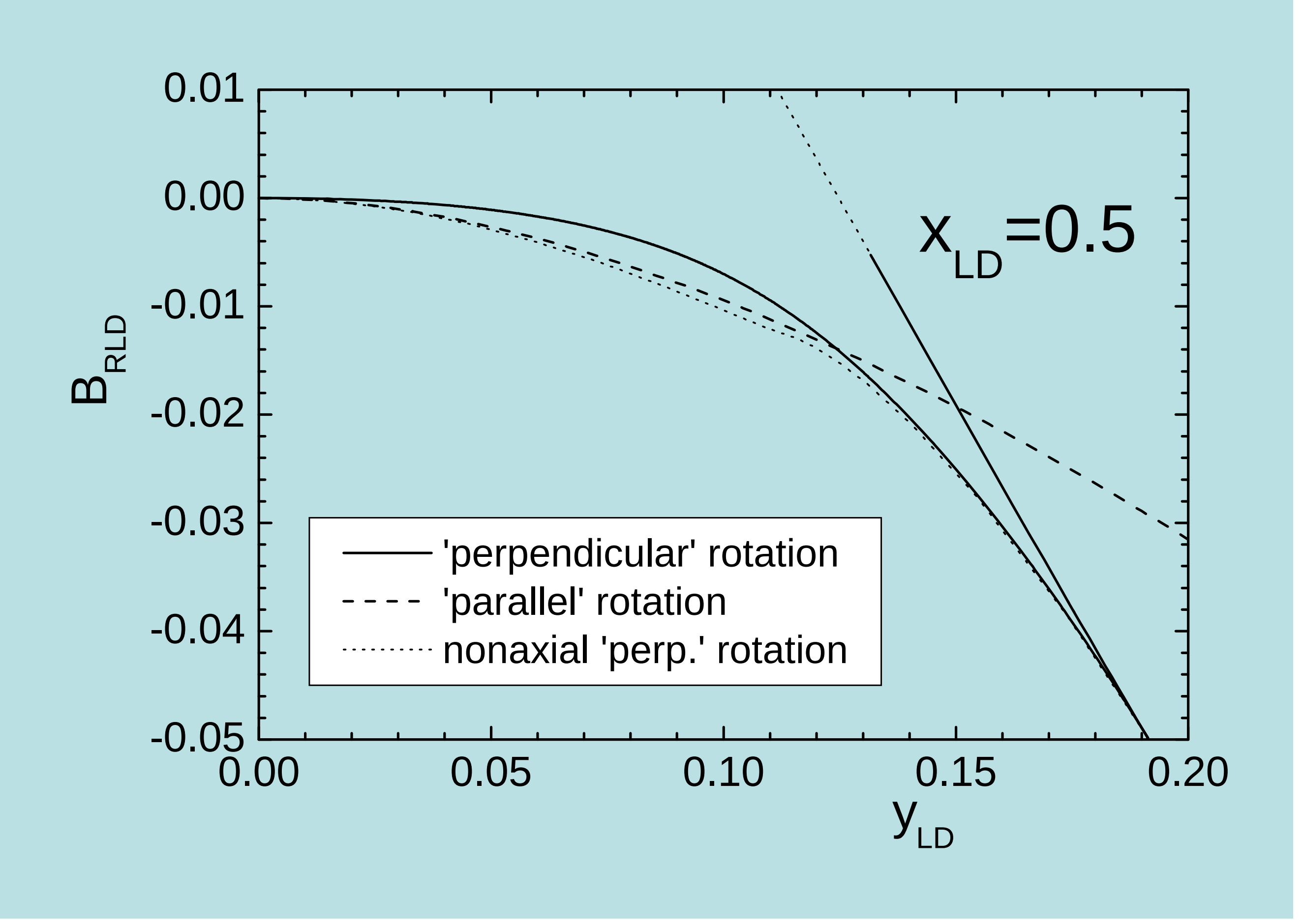
Fourier series: $\rho^2(z,\varphi) = \sum_n \rho_n(z) \cos(n\varphi)$

Approximations: $\rho^2(z,\varphi) = \zeta(z)[1 + \eta(z)\cos(2\varphi)]$

$$\rho^2(z,\varphi) = \frac{(1 - z^2/c^2)\sqrt{1-\eta^2}}{1 + \eta\ \cos(2\varphi)} \Rightarrow \rho^2(z,\varphi) = \frac{\zeta(z)\sqrt{1-\eta^2(z)}}{1 + \eta(z)\ \cos(2\varphi)}$$

K. Pomorski, F. Ivanyuk and J. Bartel, Acta Phys. Polon. B42 (2011) 455

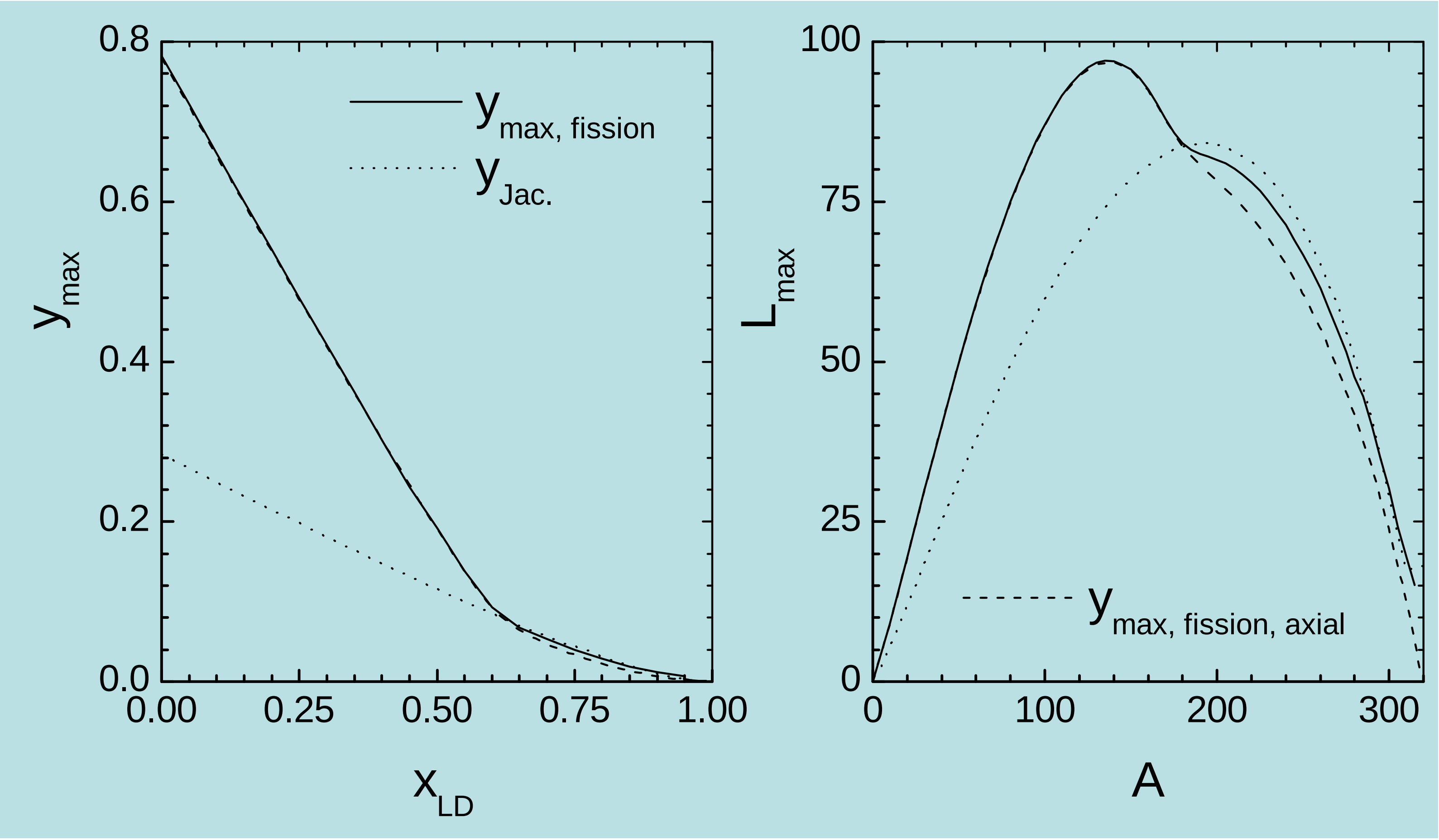
The energy of axially non-symmetric rotating drop



$$B_{RLD} = (B_{surf} - 1) + 2x_{LD}(B_{Coul} - 1) + y_{LD}(B_{rot} - 1)$$

F. Ivanyuk, K. Pomorski and J. Bartel, Int. Jour. Mod. Phys. E21 (2012) 1250032

The limiting values of rotational velocity



K.T.R.Davies and A.J.Sierk, Phys.Rev.C 31 (1985) 915

Conditional saddle-point configurations

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(Received 9 November 1984)

A general method is presented for determining an equilibrium point on a potential energy surface subject to an arbitrary number of constraints. The method is then specialized to the calculation of a conditional saddle point in the liquid-drop model for which the constraint is the mass-asymmetry degree of freedom. This approach is useful for cases in which the mass asymmetry is not one of the chosen coordinates but instead is a function of these coordinates. Conditional saddle points are calculated for the liquid-drop and Yukawa-plus-exponential nuclear energy models, with the nuclear shape parametrized using both a three-quadratic-surface model and a Legendre polynomial expansion of the nuclear surface function. We show how the conditional saddle-point shapes and energies change as the fissility x and the mass asymmetry value α are varied. As α increases for fixed x, the saddle-point configurations effectively behave like lighter (less fissile) nuclei. For fissilities less than the Businaro-Gallone value (x_{BG}), the conditional saddle-point energy always decreases with increasing α. For $x > x_{BG}$, with increasing α the conditional saddle-point energy increases until it reaches the limit of the Businaro-Gallone peak, after which the energy decreases.

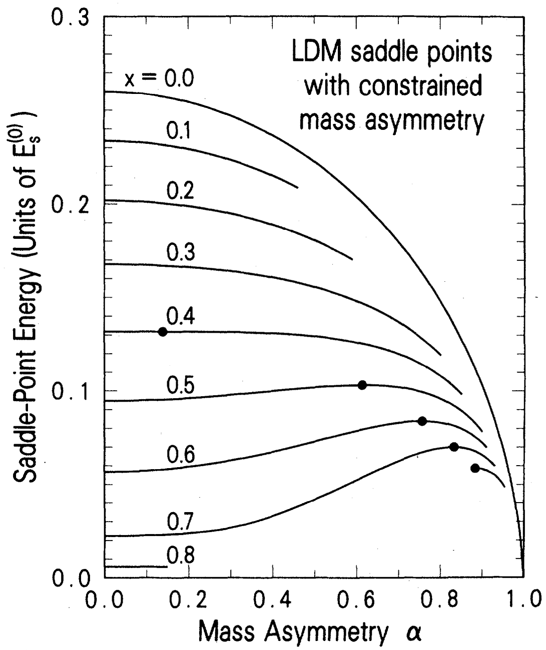
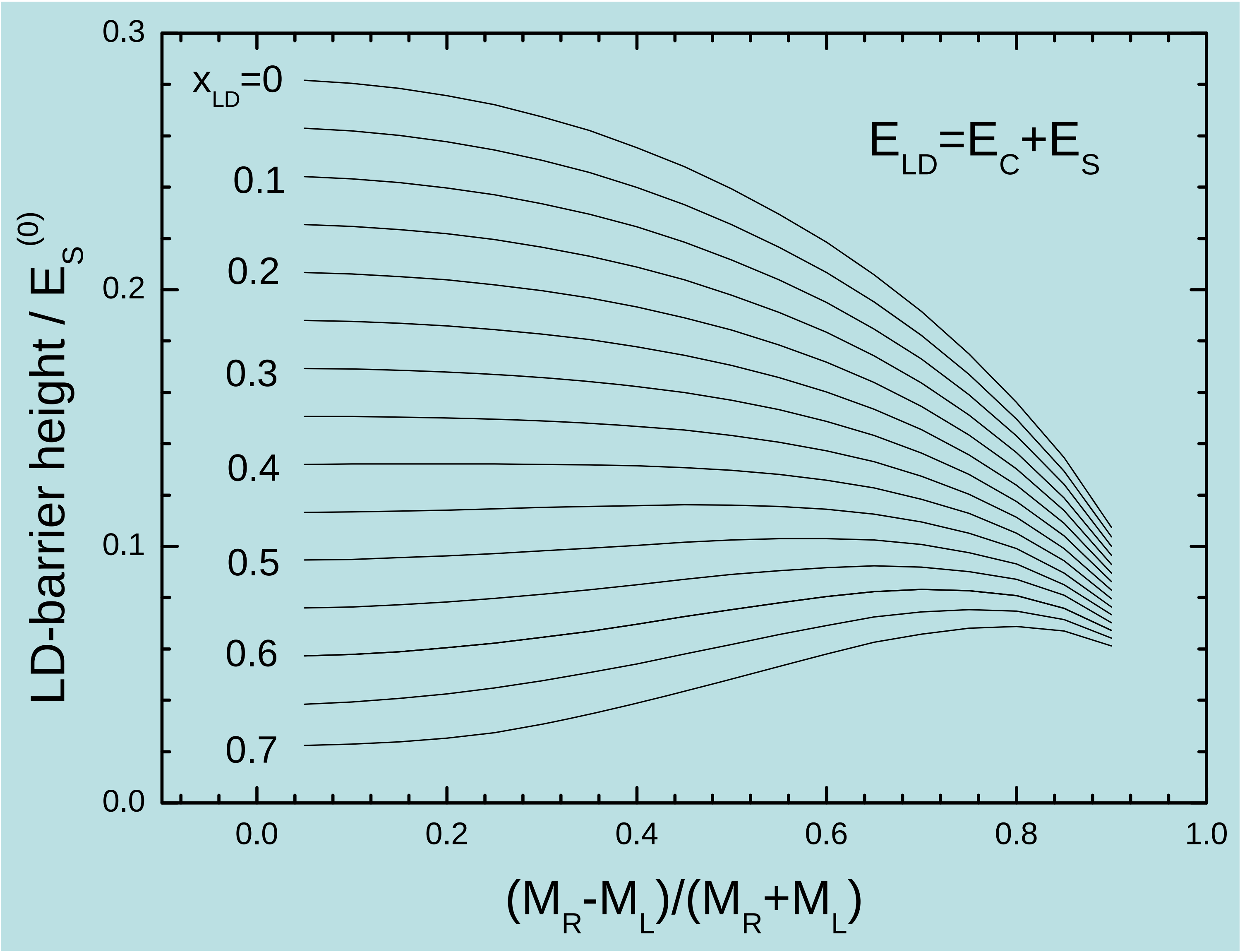


FIG. 1. Saddle-point energies as a function of constrained mass asymmetry for various values of the liquid-drop-model fissility parameter x. The solid points correspond to the Basinaro-Gallone family of asymmetric saddle-point shapes with two unstable degrees of freedom. The solid lines terminate to the right at shapes with very small neck radii, beyond which we cannot calculate. The curve for $x = 0.8$ is not drawn where no constrained saddle point exists.

Businaro-Gallone point



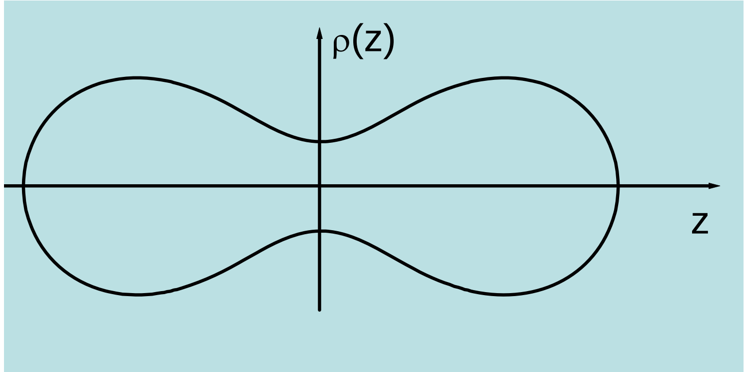
The shape of left-right asymmetric rotating drop

the energy: $E_{RLD} = E_{surf}[\rho] + E_{Coul}[\rho] + E_{rot}[\rho]$

$E_{rot}[\rho] = \hbar^2 L^2 / 2J$, J – rigid-body moment of inertia

var.principle: $\frac{\delta}{\delta\rho}[E_{LD} + E_{rot} + \lambda V] = 0 \Rightarrow \frac{\delta}{\delta\rho}[E_{LD} + E_{rot} + \lambda_1 V + \lambda_2 Q_3] = 0$

$$\rho\rho'' = 1 + (\rho')^2 - \rho\left[\lambda - 10 x_{LD}\Phi_S(z) + \frac{15}{4} B_{rot}{}^2\, y_{LD}\Phi_{rot}(z)\right](1 + (\rho')^2)^{3/2}$$



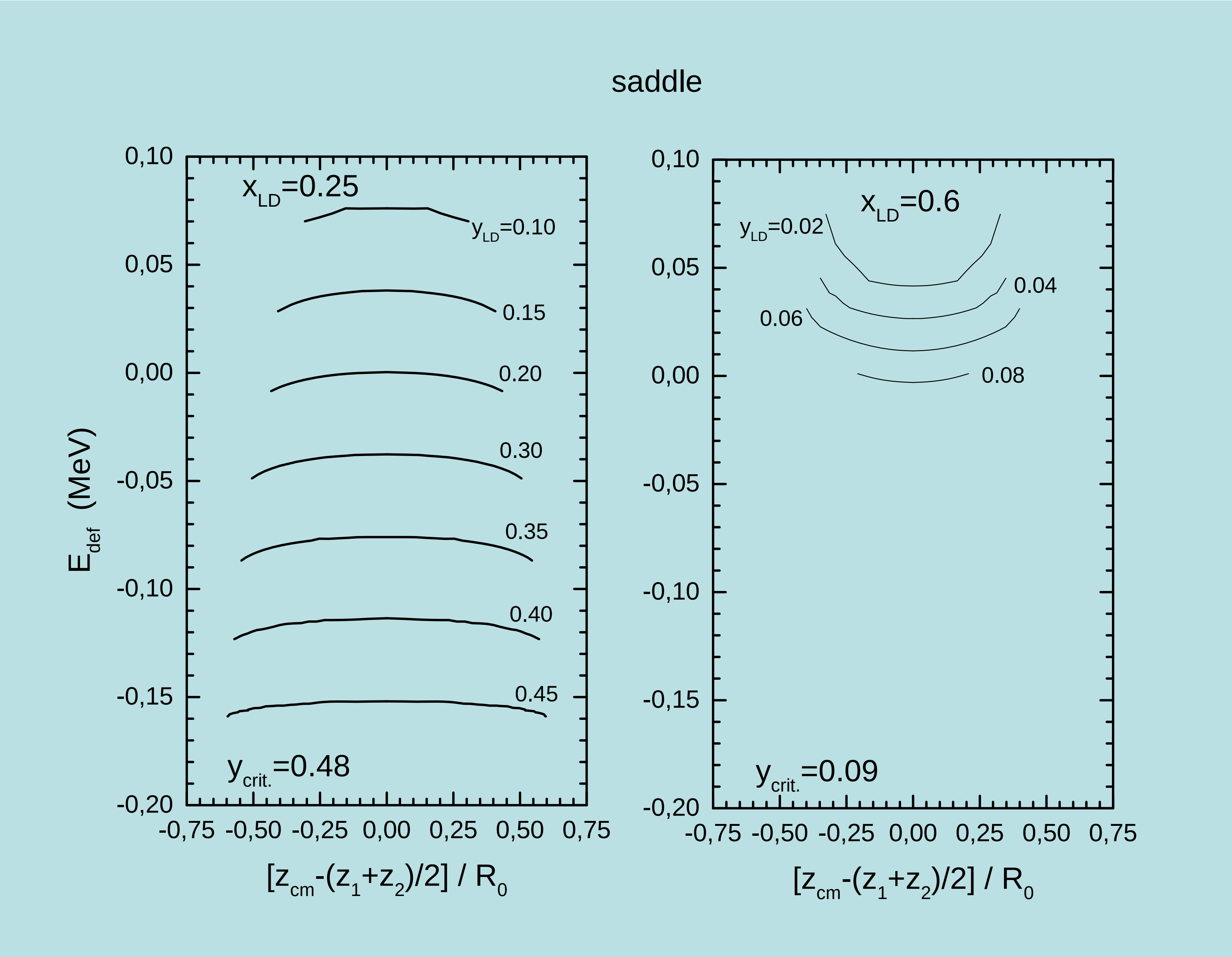
$y_{LD} \equiv E_{rot}{}^{(0)} / E_S{}^{(0)}$, $B_{rot} \equiv E_{rot} / E_{rot}{}^{(0)}$

$\Phi_{rot}(z) \equiv \delta B_{rot} / \delta\rho =$

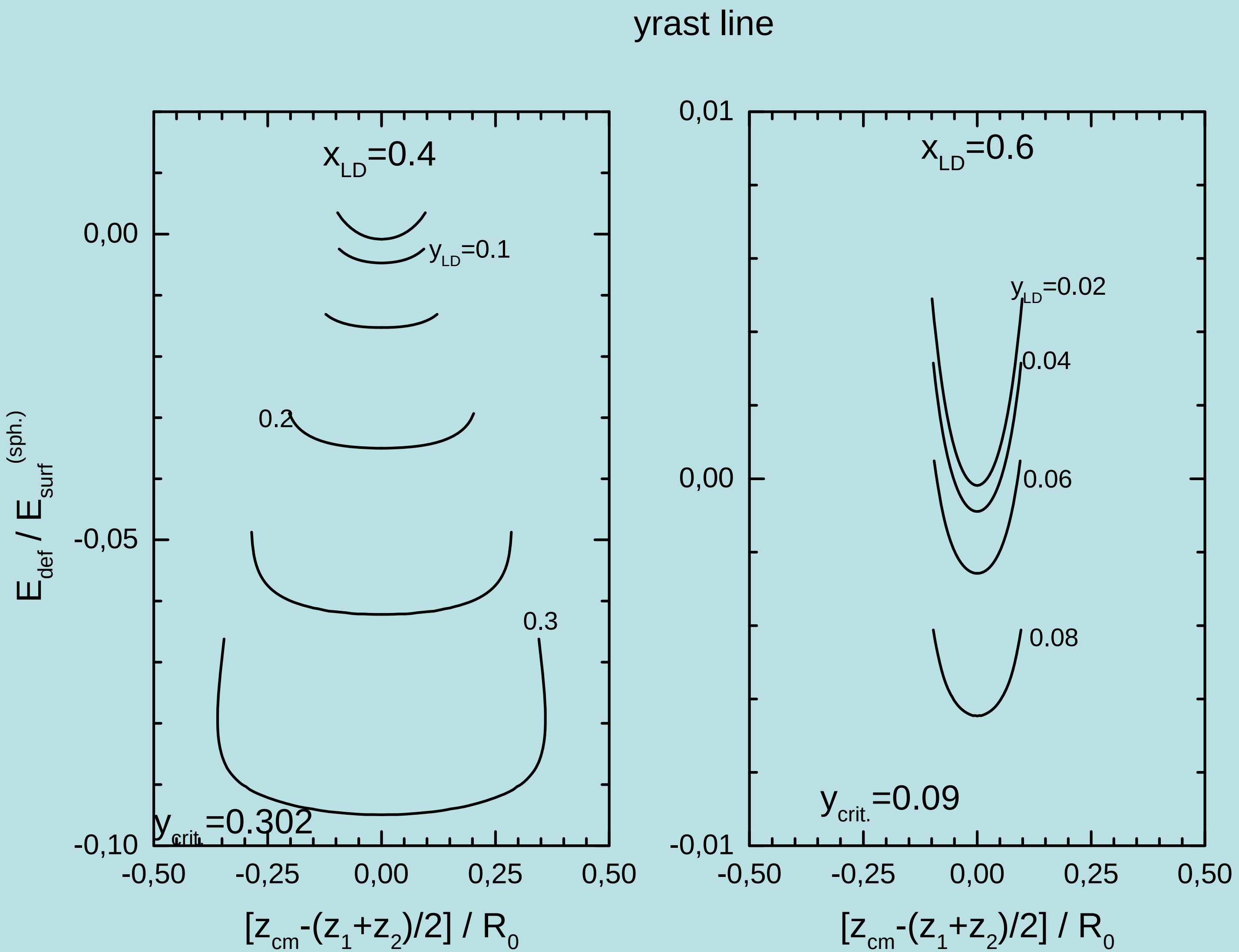
$\rho^2(z) + 2z^2$, for the perp. rotation

H. Poincar\'e, Acta Math. {\bf 7}, 259 (1885)

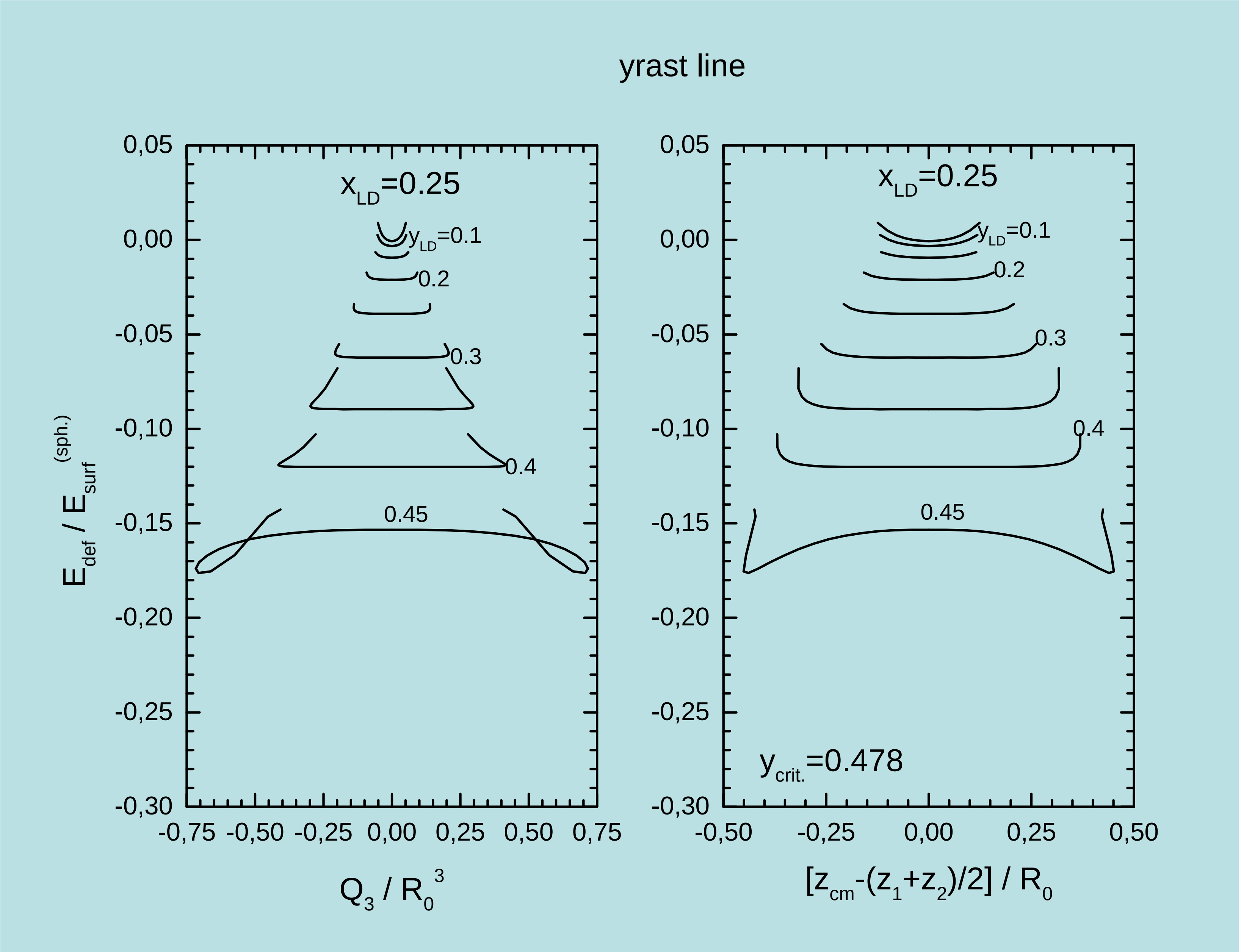
$$E_{def} = (B_{surf} - 1) + 2x_{LD}(B_{Coul} - 1) + y_{LD}(B_{rot} - 1), \; y_{LD} \equiv E_{rot}^{(0)} / E_{surf}^{(0)}$$



$$E_{def} = (B_{surf} - 1) + 2x_{LD}(B_{Coul} - 1) + y_{LD}(B_{rot} - 1),\ y_{LD} \equiv E_{rot}{}^{(0)} / E_{surf}{}^{(0)}$$



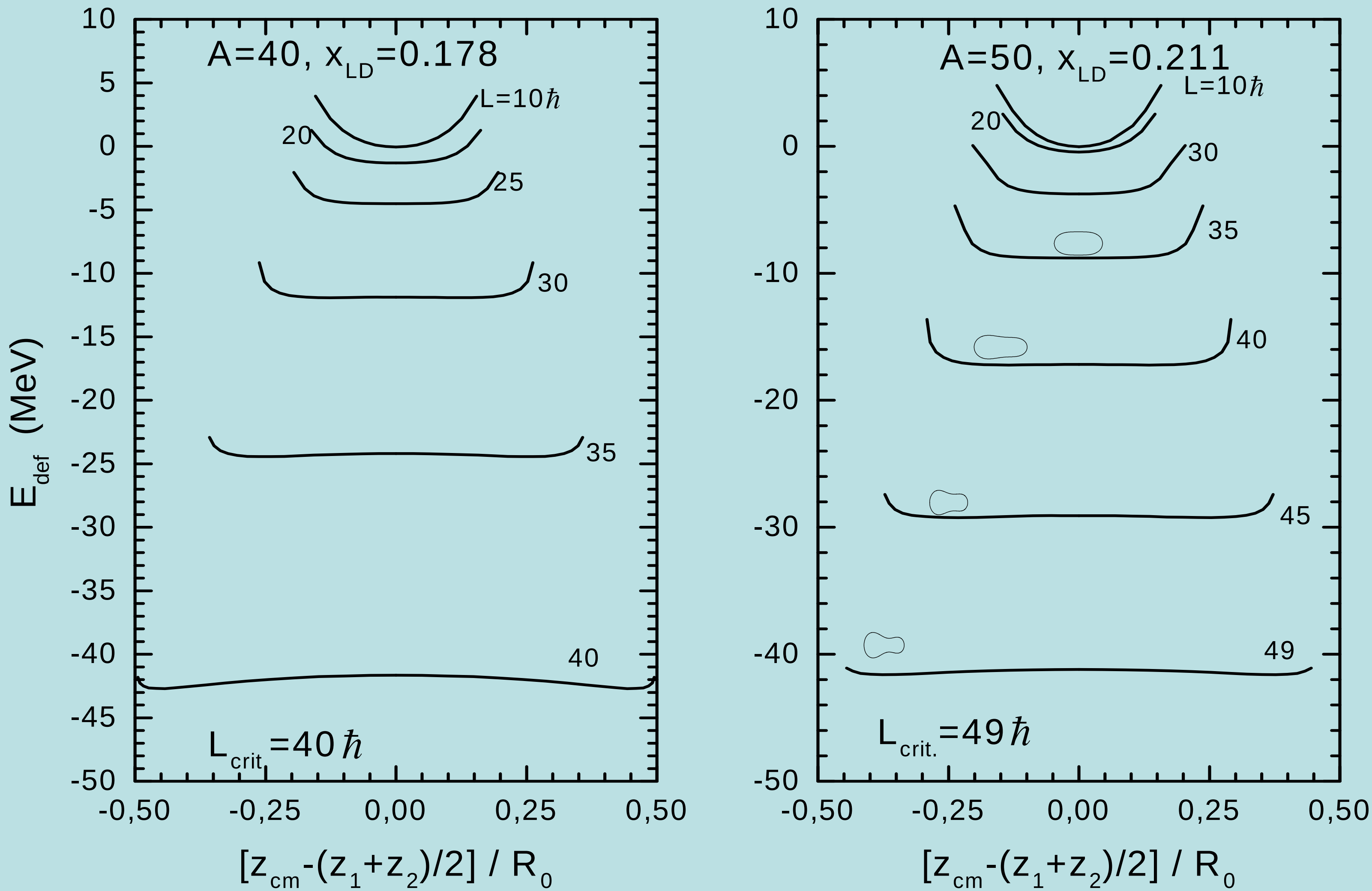
$$E_{def} = (B_{surf} - 1) + 2x_{LD}(B_{Coul} - 1) + y_{LD}(B_{rot} - 1),\ y_{LD} \equiv E_{rot}^{(0)} / E_{surf}^{(0)}$$



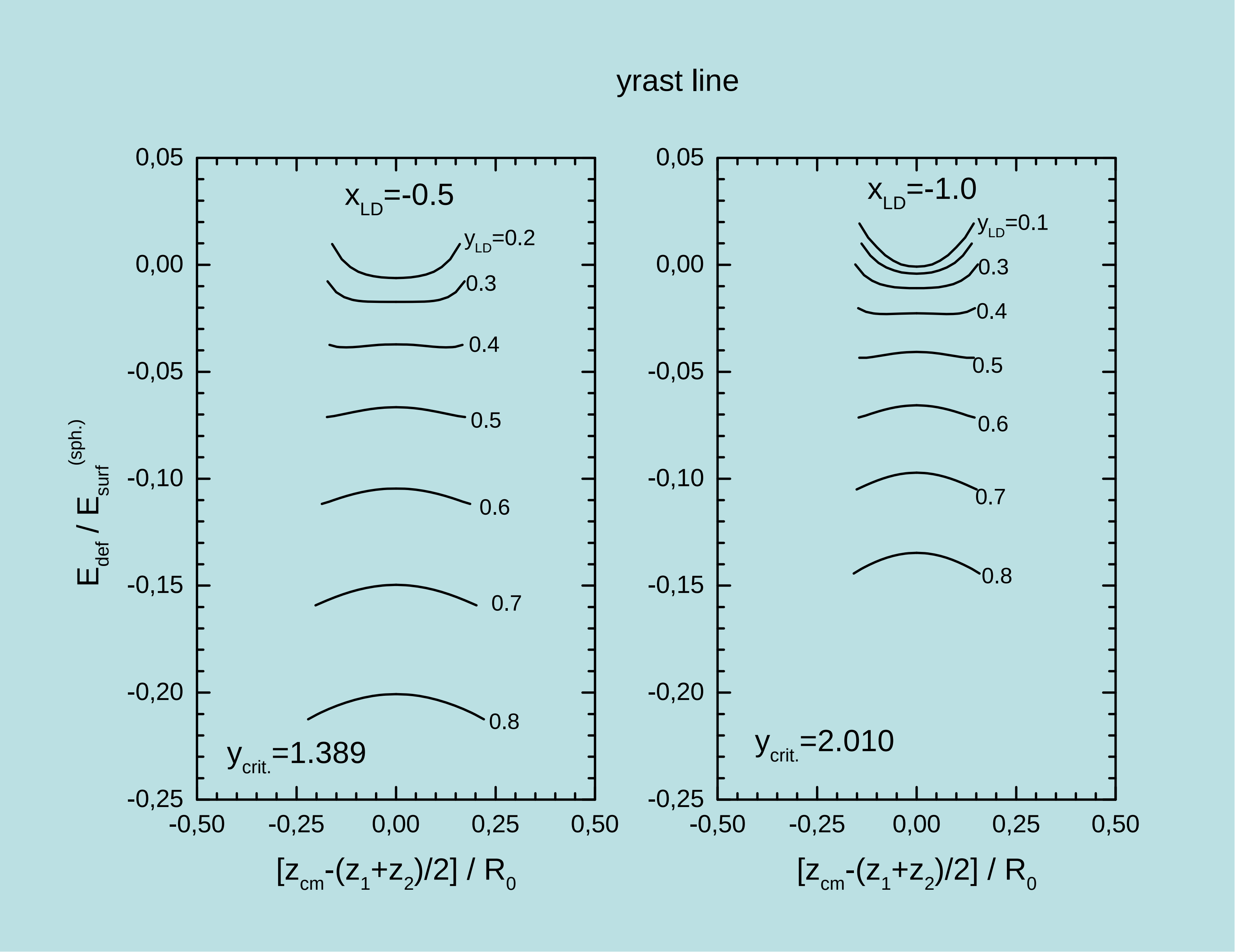
$$E_{def} = (B_{surf} - 1) + 2x_{LD}(B_{Coul} - 1) + y_{LD}(B_{rot} - 1)$$

$$y_{LD} \equiv E_{rot}{}^{(0)} / E_{surf}{}^{(0)} = \hbar^2 L^2 / [2J^{(0)} E_{surf}{}^{(0)}] \sim \hbar^2 L^2$$

$$y_{LD} = 1.9249 \;\hbar^2 L^2 / [1 - 1.7826\, I^2] / A^{7/3}, \; I \equiv (N - Z)/A$$



$$E_{def} = (B_{surf} - 1) + 2x_{LD}(B_{Coul} - 1) + y_{LD}(B_{rot} - 1), \; y_{LD} \equiv E_{rot}^{(sph)} / E_{surf}^{(sph)}$$



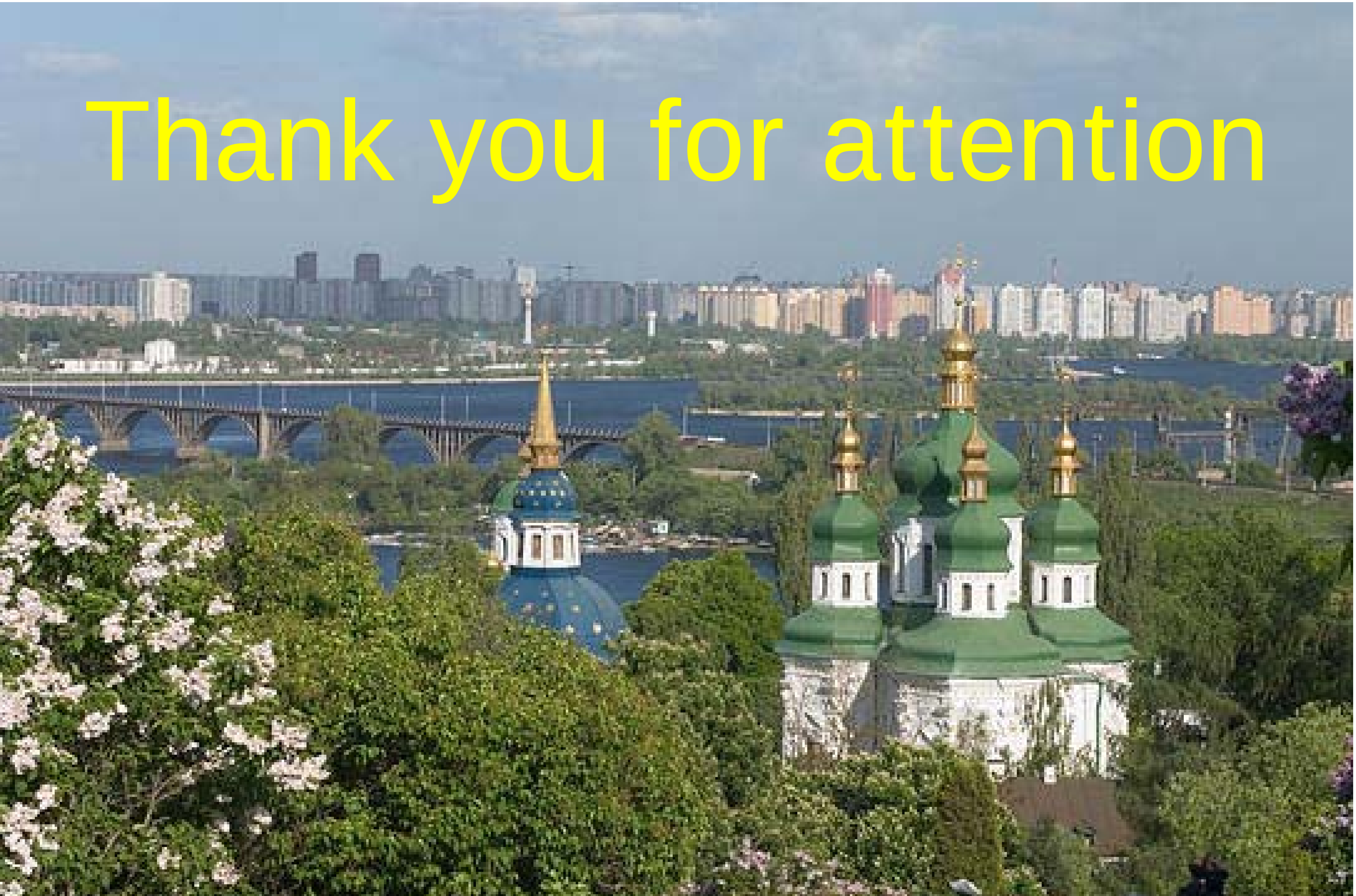
The summary

Within the Strutinsky optimal shapes procedure we have examined the evolution of the shape of light rotating nuclei.

We have found that:

- below the Businaro-Gallone point the shape of rotating nucleus at the fission saddle is unstable with respect to mass-asymmetric deformation;
- the ground state shape is getting unstable with respect to mass-asymmetric deformation at $L \approx (30\text{-}40)\,\hbar$ for the mass number A=40-50;
- the potential energy surface in this region is very flat and can be modified substantially be the shell effects.

The gravitating objects lose the stability with respect to mass-asymmetric deformation at relatively small angular momentum.


Thank you for attention