

# Puzzle of the third minima in actinides

*M. Kowal, J. Skalski (NCBJ)*



*P. Jachimowicz (UZ)*



1. Importance of the subject.
2. Recent status: deep IIIrd minima in U seen in experiment vs conflicting th. Predictions.
3. Change of prediction: M. Kowal, J. Skalski, PRC 85, 061302(R) 2012: no longer any deep IIIrd wells in theory.
4. How is it really?

# Status of third minimum in actinides:

**Shallow minima  
(0.5 MeV or less )**

?

**Deep minima  
(3 - 4 MeV)**

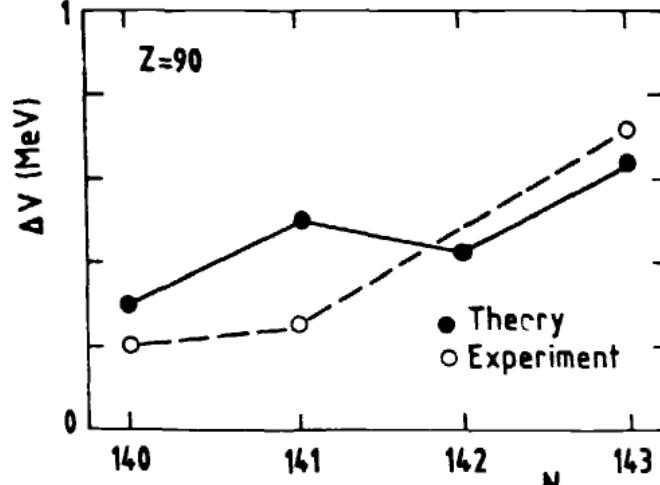
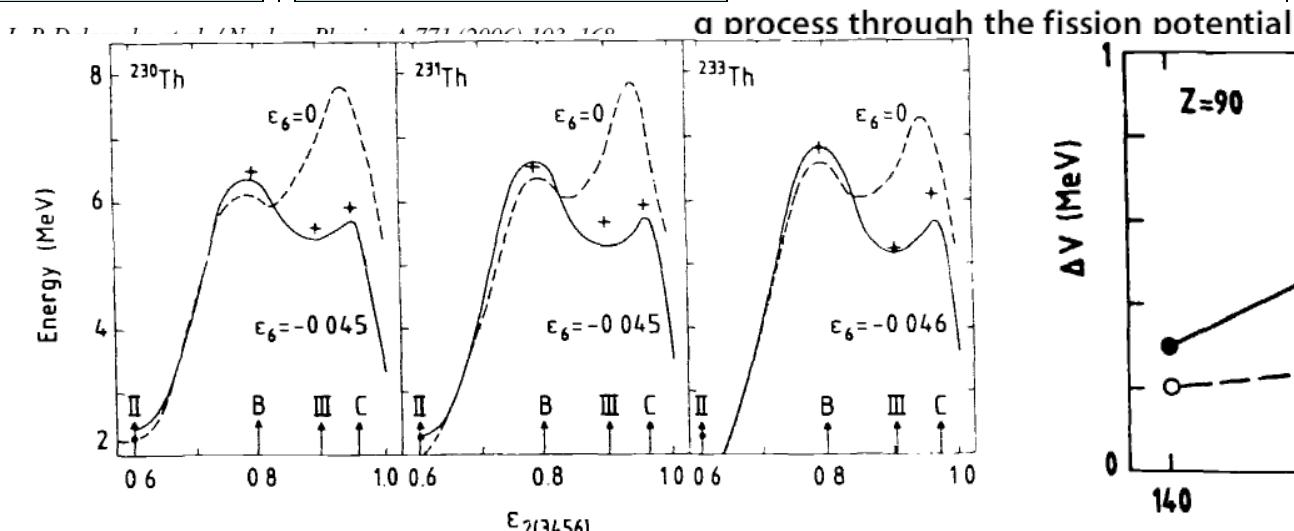
**Theory:**

**self-consistent  
models**

**mac-mic model  
P. Moller et, al.**

?

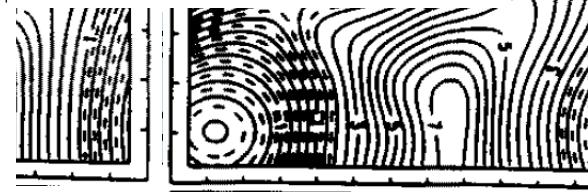
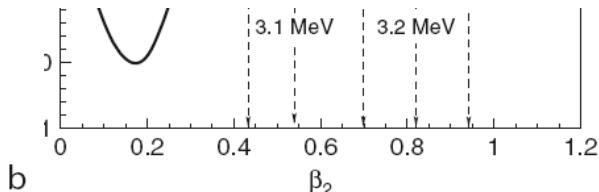
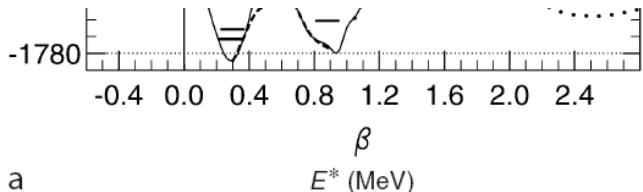
**mac-mic model  
S. Ćwiok et, al.**



BLONS J.- PAYA D.  
CEA Centre d'Etudes Nucléaires de Saclay, 91 - Gif-sur-Yvette  
Dept. de Physique Nucléaire

Communication présentée à : International Conference on High Spin Physics and Gamma-Soft Nuclei

Pittsburg, PA (US)  
17-21 Sep 1990



1. IIIrd minima in actinides, if exist, are low-spin hyperdeformed states (axis ratio close to 3:1)
  - maybe the only ones in both medium and heavy nuclei.
2. Their large quadrupole deformation & mass-assymetry makes them unique (collective E1 ca 10keV rotational transitions)
3. Experiments confirming predicted minima may validate nuclear models.
4. S.p. orbits at the Fermi level in super- and hyper-deformed actinides are those occupied at normal shape in SHN; they can provide a test of a model.

# Good methods should give similar predictions.

- Micro-macro, as a simpler one, is better tested/fitted against various data, eg. fission half-lives.
- Selfconsistent methods could (if constructed properly) give better extrapolations. But it is not guaranteed at present. Hence, a prudent idea is to see whether both methods give similar results.

# Woods-Saxon model with the Yukawa+exponential macroscopic energy

Deformation: in terms of nuclear surface;  
Multipole expansion parameters (not very bright for  
the IIIrd minima);

For the IIIrd minima important are axially-symmetric multipoles: beta2-beta8, ...

Model tested in many regions, in particular:

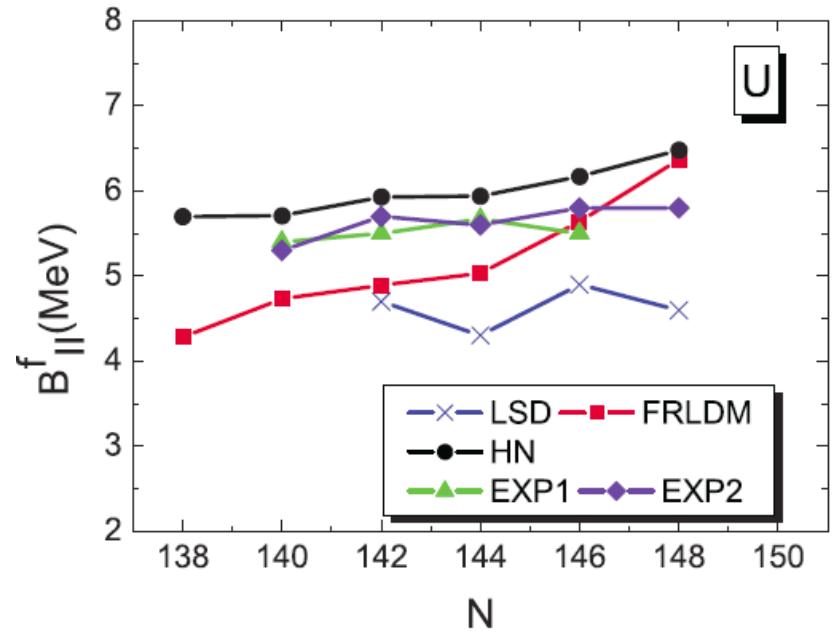
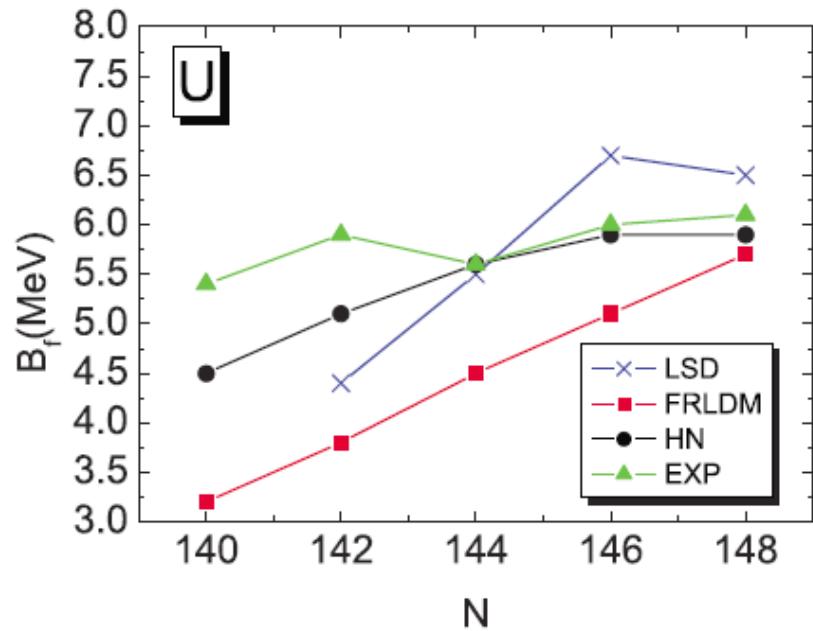
- First & second barriers in actinides,
- Second minima in actinides,
- Fission barriers for SHN

M. Kowal, P. Jachimowicz, and A. Sobiczewski, *Phys. Rev. C* **82**, 014303 (2010).

M. Kowal and J. Skalski,, *Phys. Rev. C* **82**, 054303 (2010).

P. Jachimowicz, M. Kowal, and J. Skalski, *Phys. Rev. C* **85**, 034305 (2012).

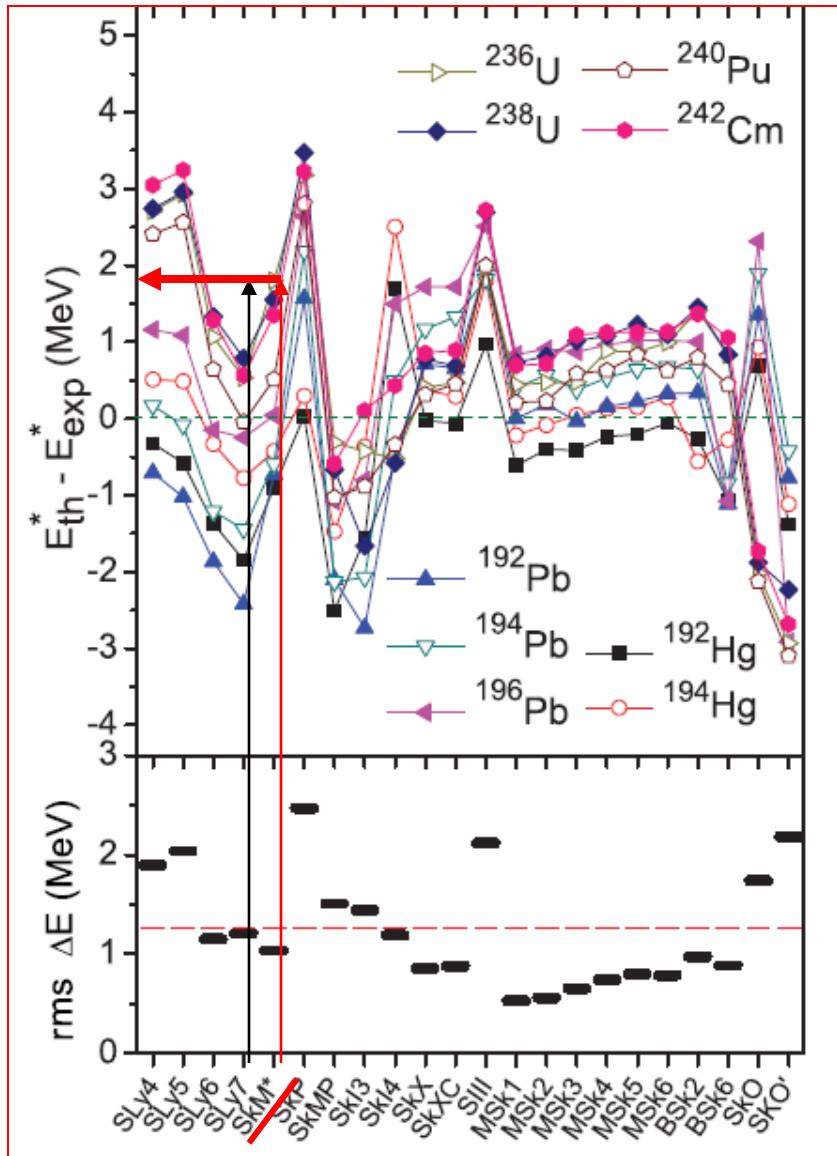
# First & second barriers in actinides,



Models	LSD	FRLDM	HN
$N$	16	18	18
$\langle  B_f^{\text{th}} - B_f^{\text{expt}}  \rangle$	0.9	1.0	0.4
Max $ B_f^{\text{th}} - B_f^{\text{expt}} $	1.8	2.2	1.0
rms	1.0	1.1	0.5

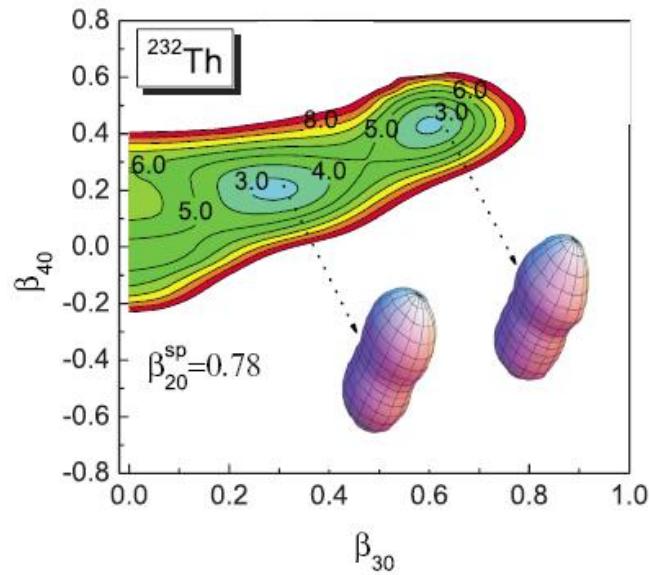
Theoretical models	LSD [30]	[32]	FRLDM [30]	[32]	HN [30]	[32]
$N$	12	18	14	22	14	22
$\langle  B_f^{\text{th}} - B_f^{\text{exp}}  \rangle$	0.78	0.84	0.79	0.90	0.56	0.58
Max $ B_f^{\text{th}} - B_f^{\text{exp}} $	1.50	1.50	1.85	2.33	1.34	1.19
$\delta_{\text{RMS}}$	0.92	0.94	0.95	1.11	0.66	0.69

# Second minima in actinides,



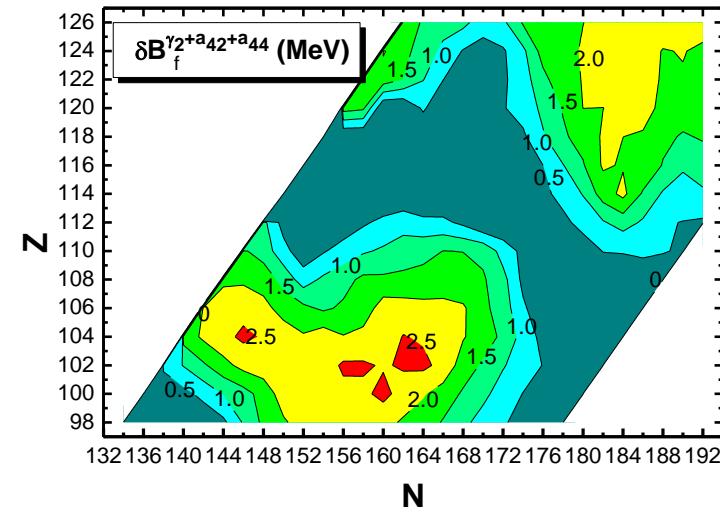
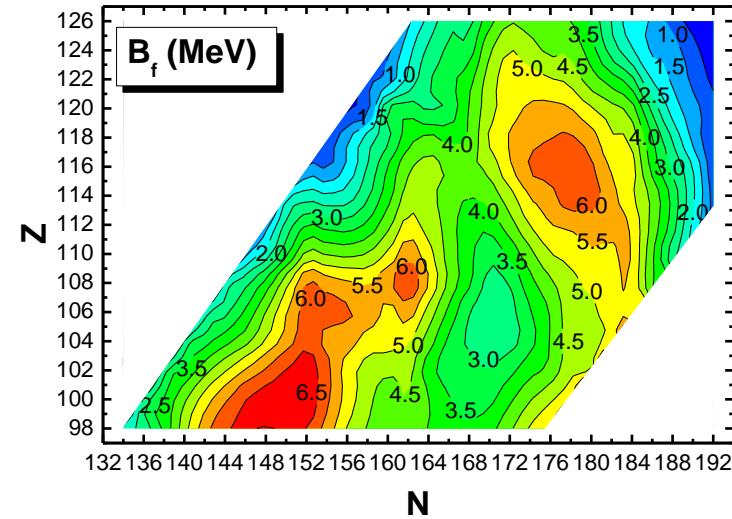
Z	N	A	$E_{II}^{min}(th)^*$	$E_{II}^{min}(exp)^*$
92	144	236	2.04	2.75
92	146	238	1.94	2.56
94	142	236	2.43	3.00
94	144	238	2.05	2.40
94	146	240	1.95	2.80
94	148	242	1.99	2.20
96	144	240	1.69	2.00
96	146	242	1.64	1.90
96	148	244	1.68	2.20(?)

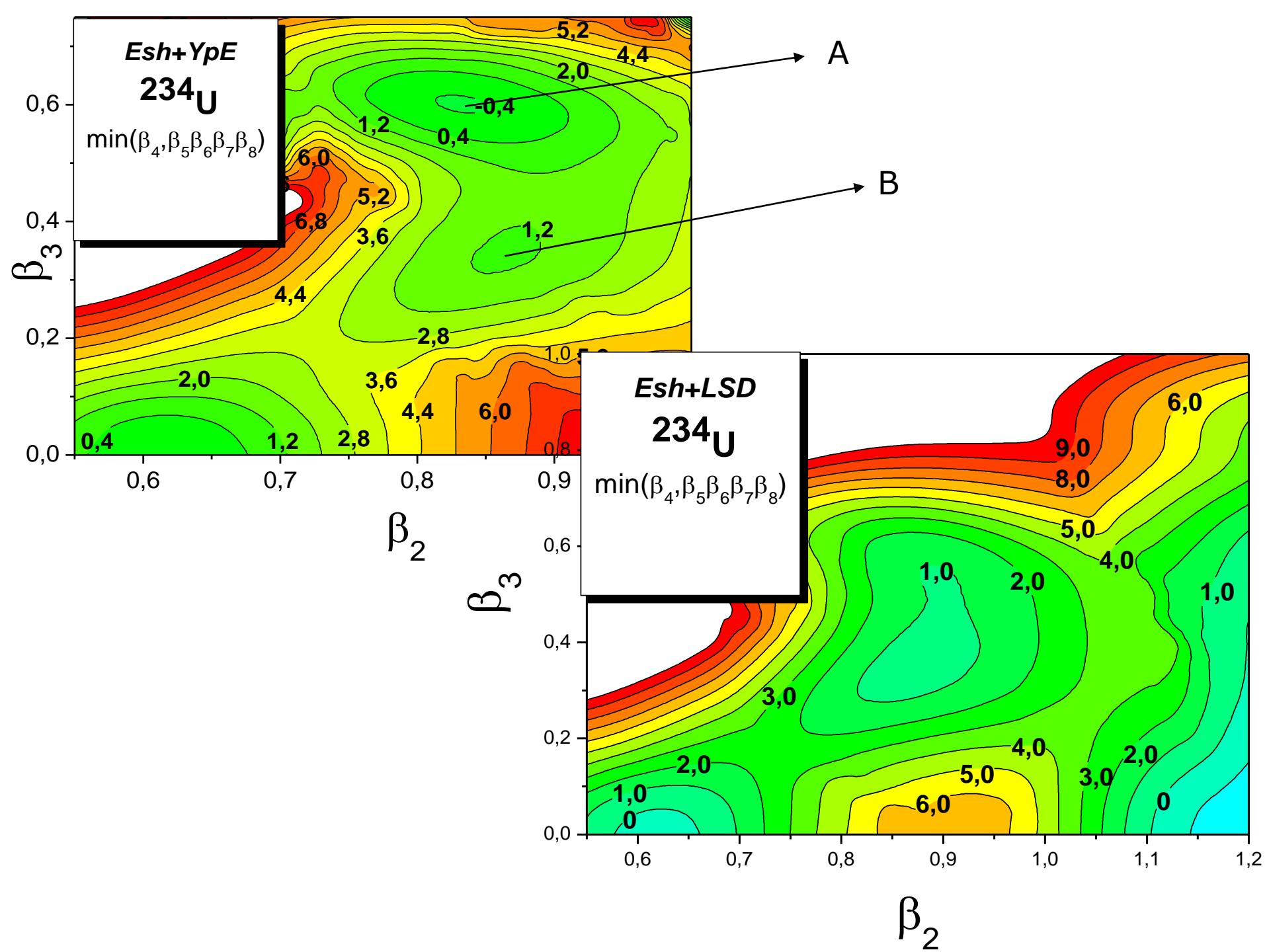
Max diff = ~800 KeV!

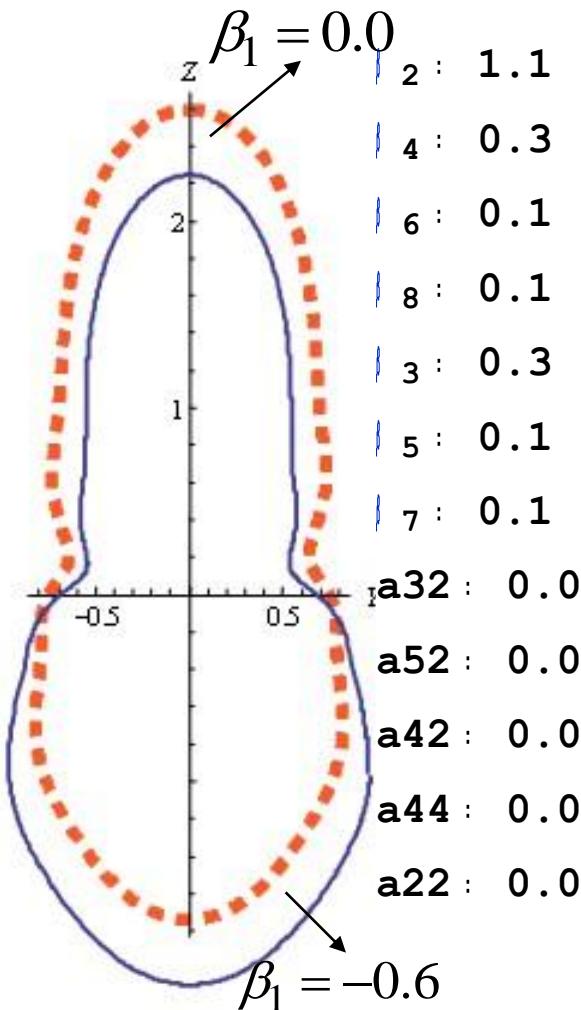


# Fission barriers for SHN

Nucleus	SHF	FRLDM	ETFSI	HN	EXP
$^{284}\text{112}_{172}$	6.06	7.41	2.2	4.29	5.5
$^{286}\text{112}_{174}$	6.91	8.24	3.6	5.01	5.5
$^{288}\text{114}_{174}$	8.12	9.18	6.1	5.53	6.7
$^{290}\text{114}_{176}$	8.52	9.89	6.6	5.83	6.7
$^{292}\text{114}_{178}$	—	9.98	7.2	6.34	6.7
$^{292}\text{116}_{176}$	9.35	9.26	6.5	6.22	6.4
$^{294}\text{116}_{178}$	9.59	9.46	7.2	6.28	6.4
$^{296}\text{116}_{180}$	—	9.10	7.2	6.07	6.4
$^{294}\text{118}_{176}$	—	8.48	6.6	5.99	—
$^{296}\text{118}_{178}$	—	8.36	7.0	6.04	—
$^{298}\text{118}_{180}$	—	8.05	7.4	5.72	—
$^{296}\text{120}_{176}$	—	7.69	6.2	5.64	—
$^{298}\text{120}_{178}$	—	7.33	6.6	5.50	—
$^{300}\text{120}_{180}$	—	7.01	6.8	5.05	—
$^{302}\text{120}_{182}$	—	6.07	7.2	4.66	—
$^{304}\text{120}_{184}$	—	4.86	6.8	4.20	—

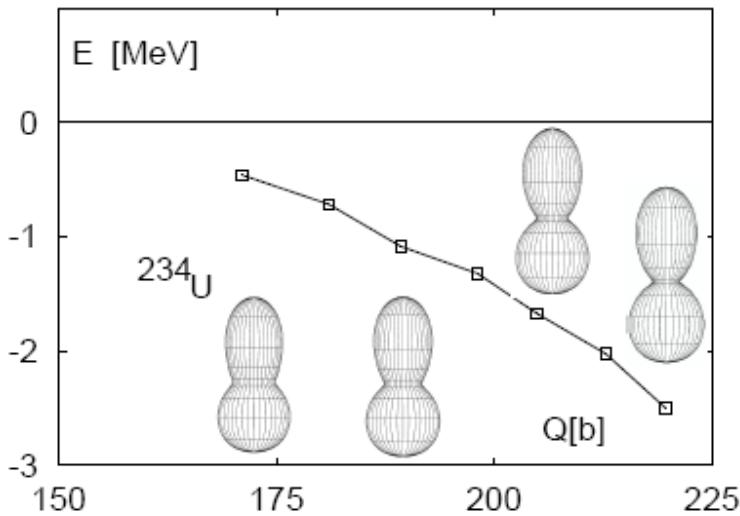






- The dipole deformation 1 is omitted there, as corresponding to a shift of the origin of coordinates which leaves energy (always calculated in the center of mass frame) invariant. However, this is true only for weakly deformed shapes. **For large elongations,  $b_1$  acquires a meaning of a real shape variable.**

# III minima – type: A



One can find continuous 8D paths starting at the supposed IIIrd minimum and leading to scission, along which energy decreases gradually.

- minima with larger octupole deformations (A) have quadrupole moments  $Q=170$  b, disturbingly close to the scission region.
- minima (A) are just intermediate configurations on the scission path, whose energy was calculated erroneously because of limitations of the admitted class of shapes.

# III minima – type: B

$$-0.35 < \beta_1 < 0.00$$

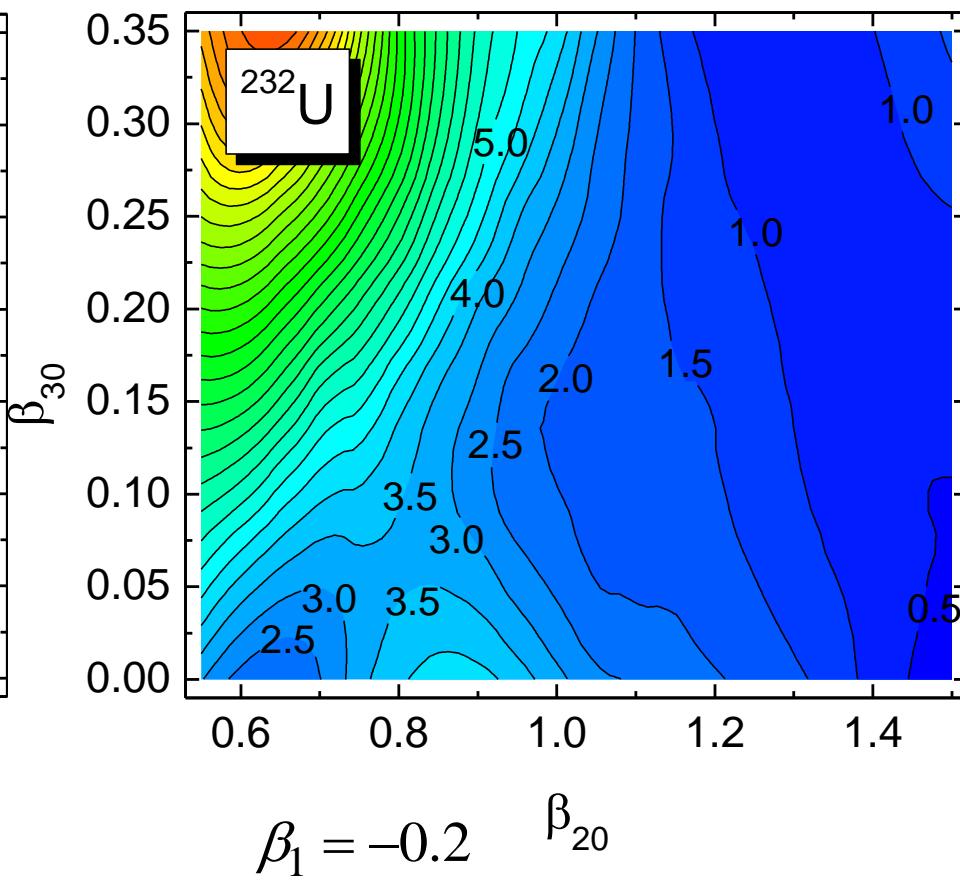
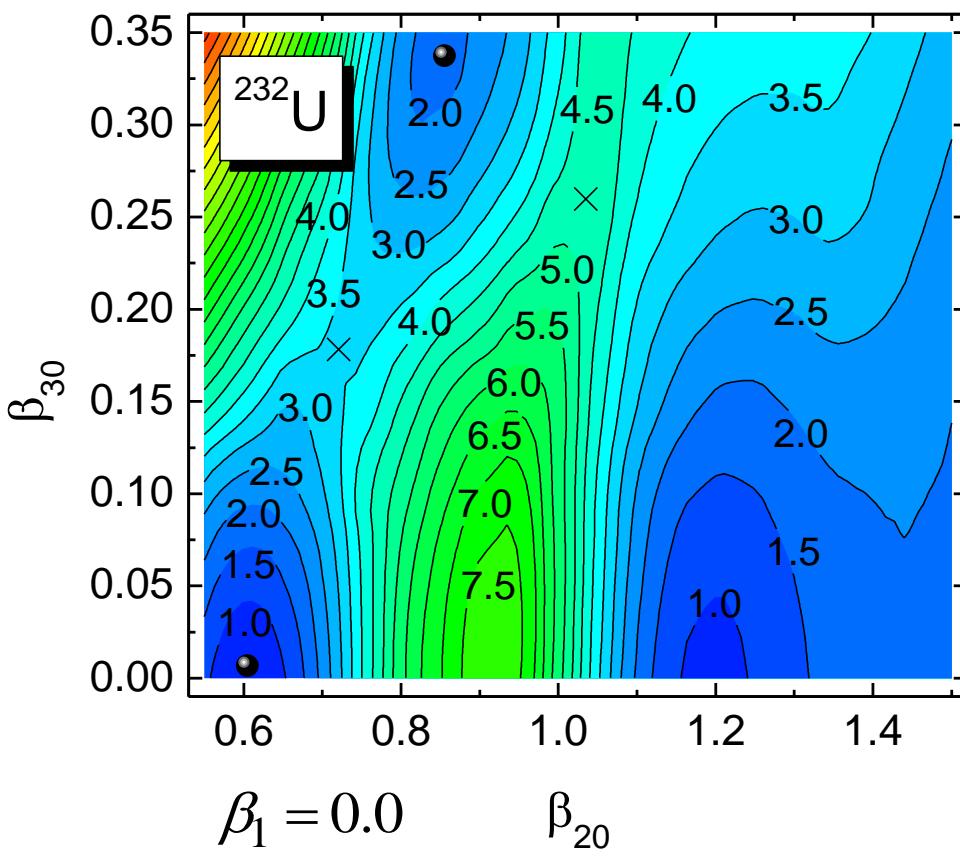
$$-0.10 < \beta_4 < 0.35$$

$$-0.55 < \beta_2 < 1.50$$

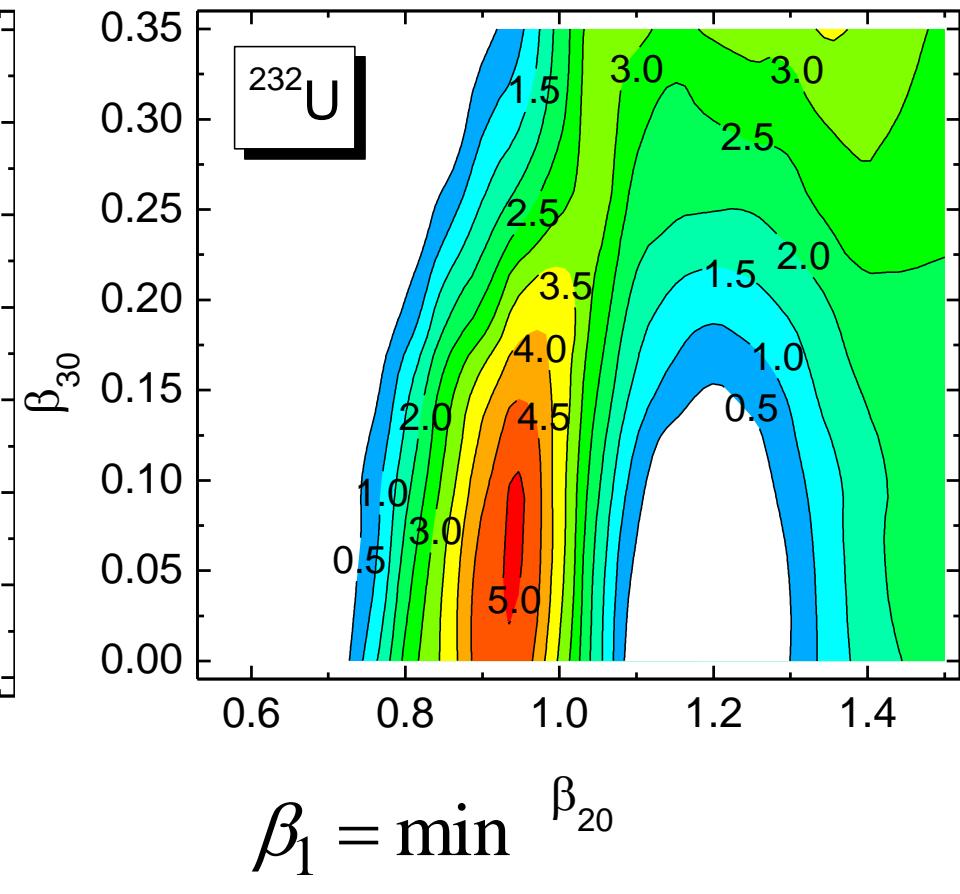
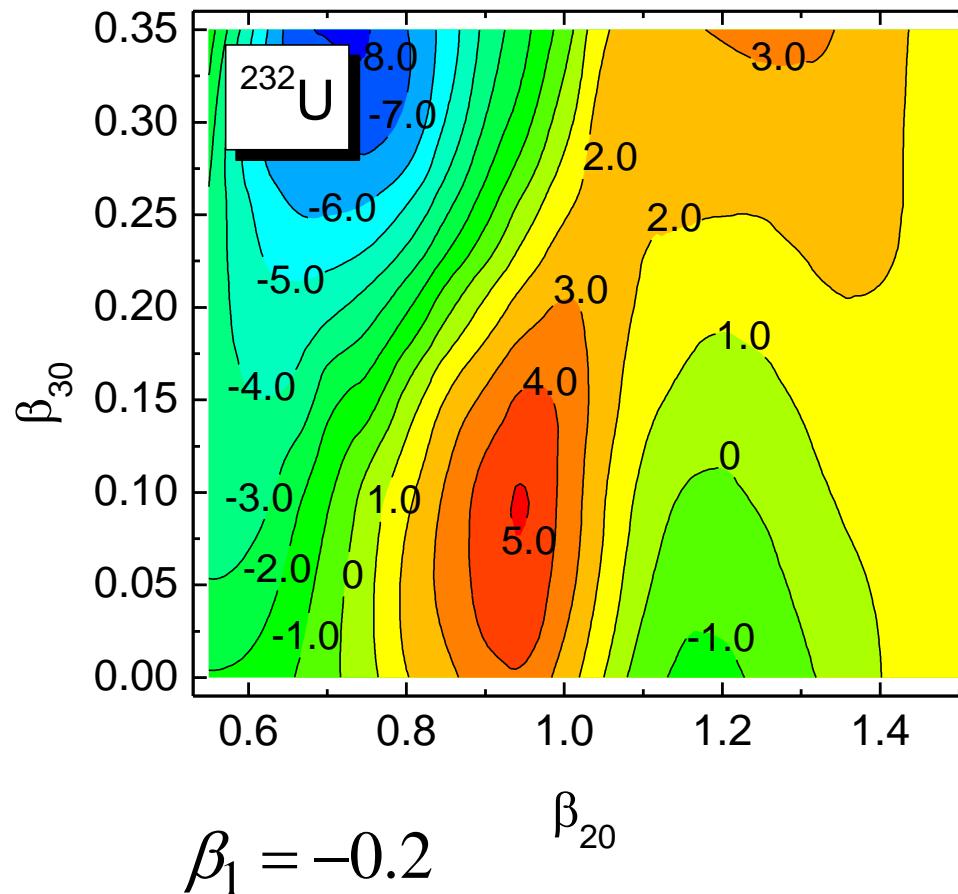
$$-0.20 < \beta_5 < 0.20$$

$$0.00 < \beta_3 < 0.35$$

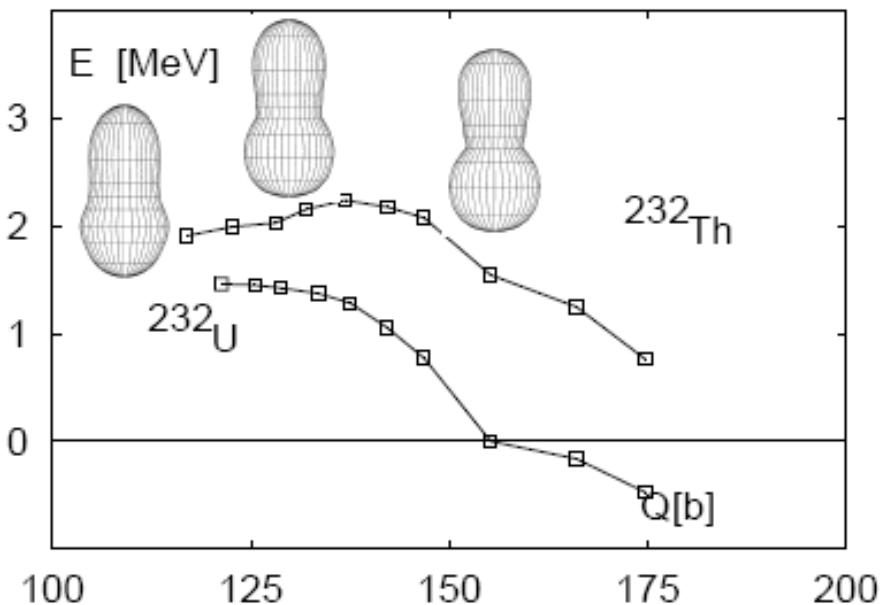
$$-0.15 < \beta_6 < 0.15$$



# III minima – type: B



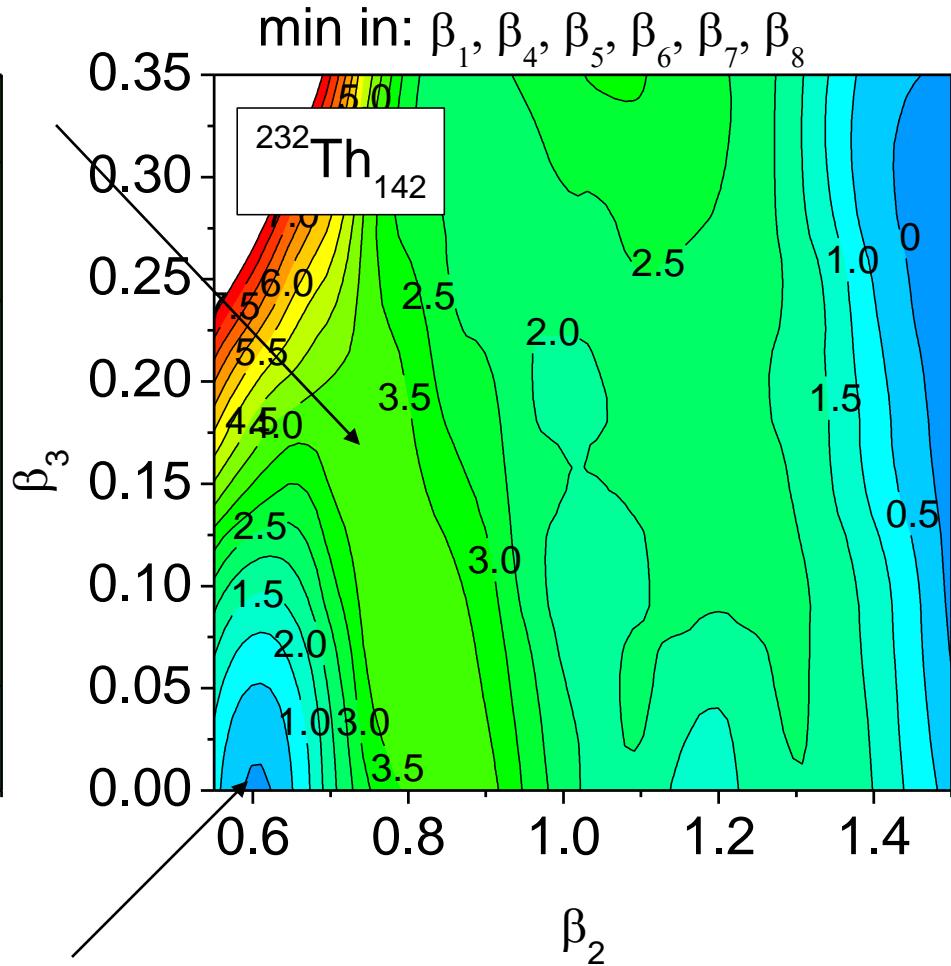
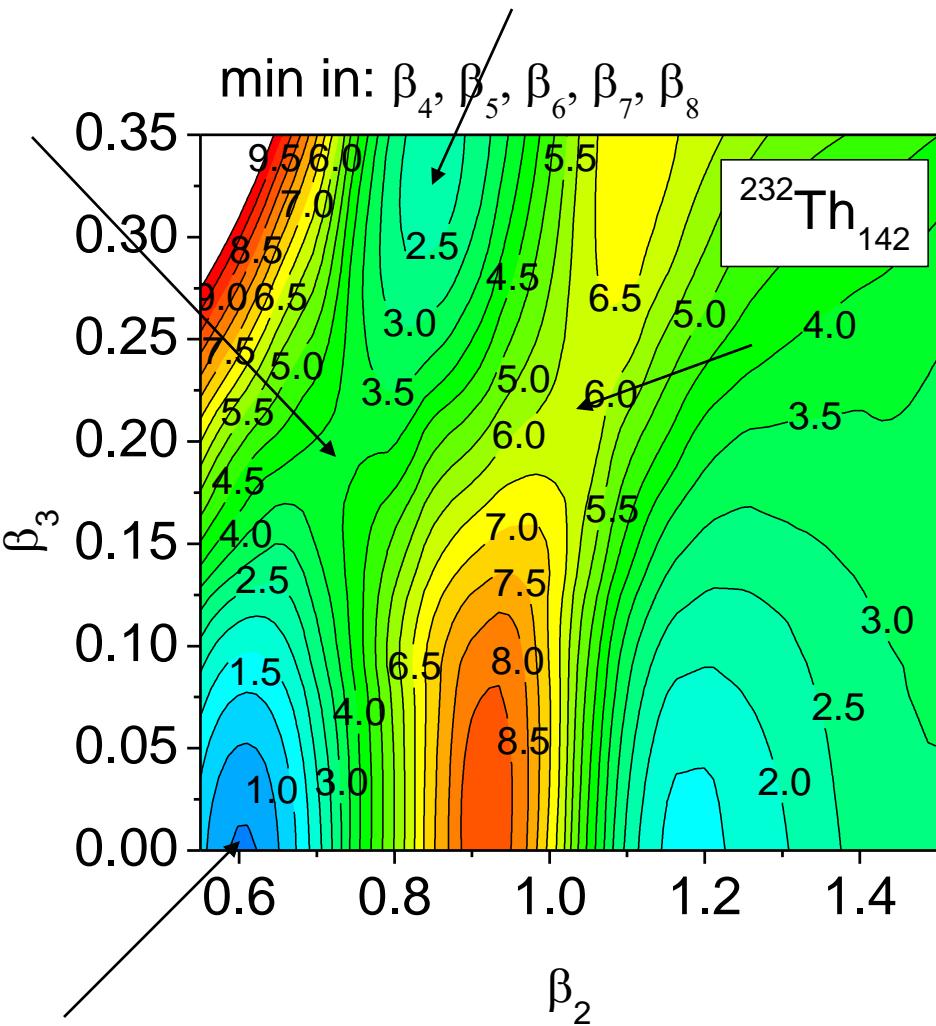
# III minima – type: B



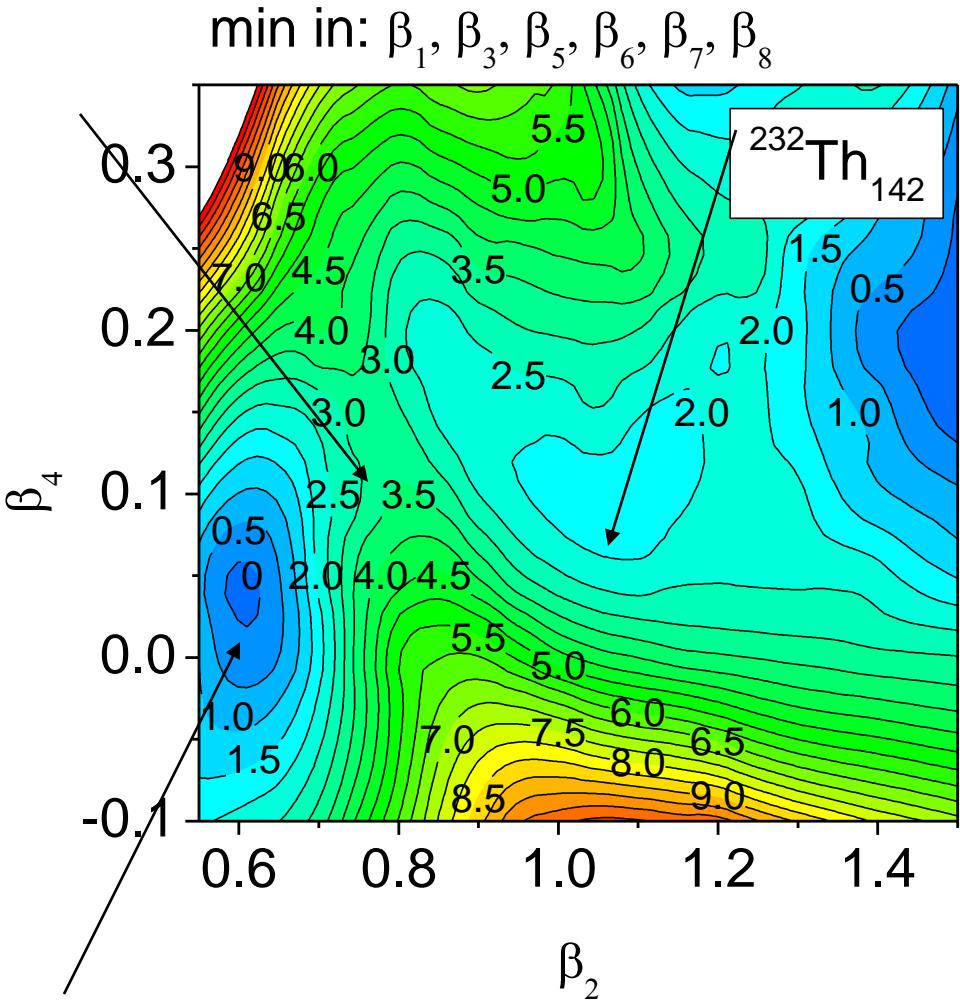
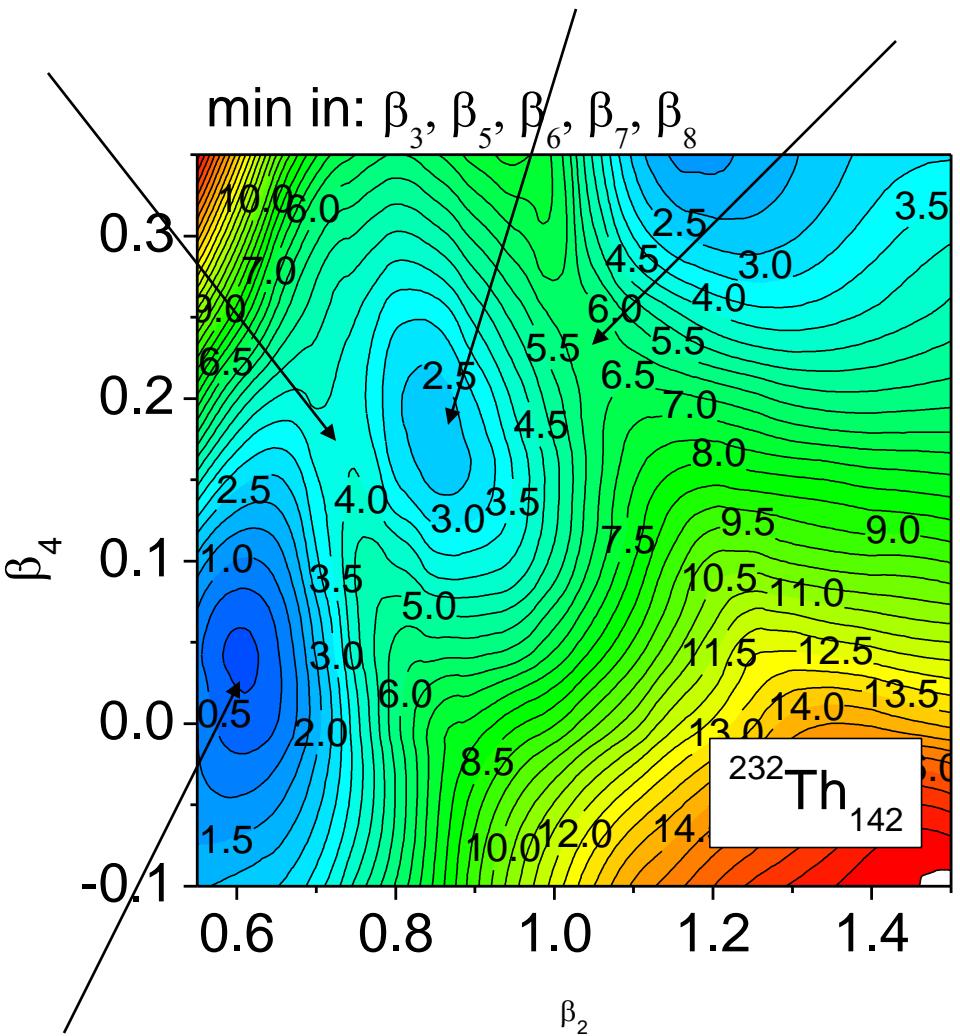
8D paths from the supposed IIIrd minima towards scission. Energies vs. quadrupole moments along such paths are shown

- the barrier vanishes in uranium and must be smaller than 330 keV in  $^{232}\text{Th}$ . The only other nonzero upper limit on the IIIrd barrier of 200 keV we nd in  $^{230}\text{Th}$ .

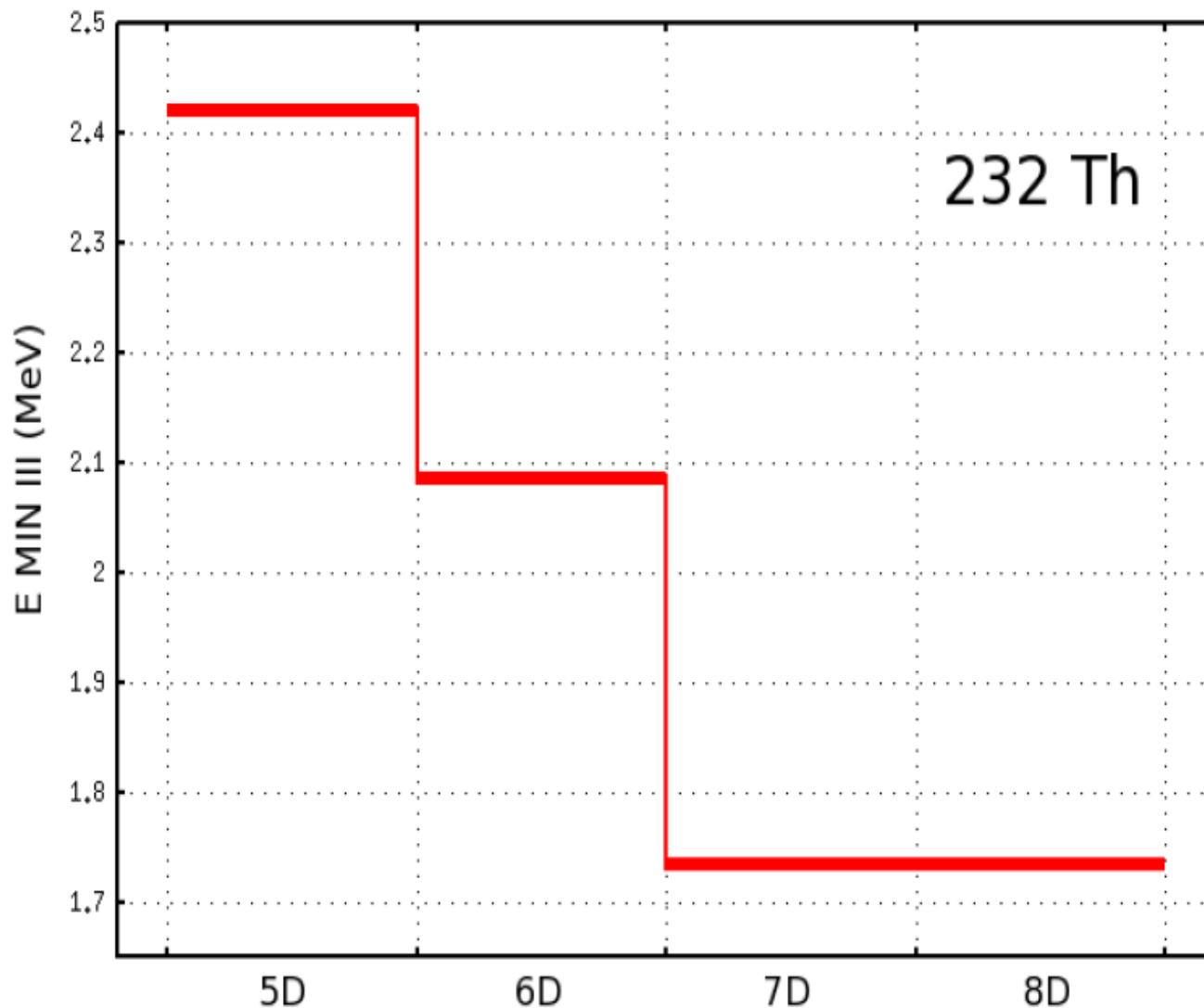
# III saddle from the mesh 5D-8D; 8D mesh (beta1-beta8) – 50803200 points!



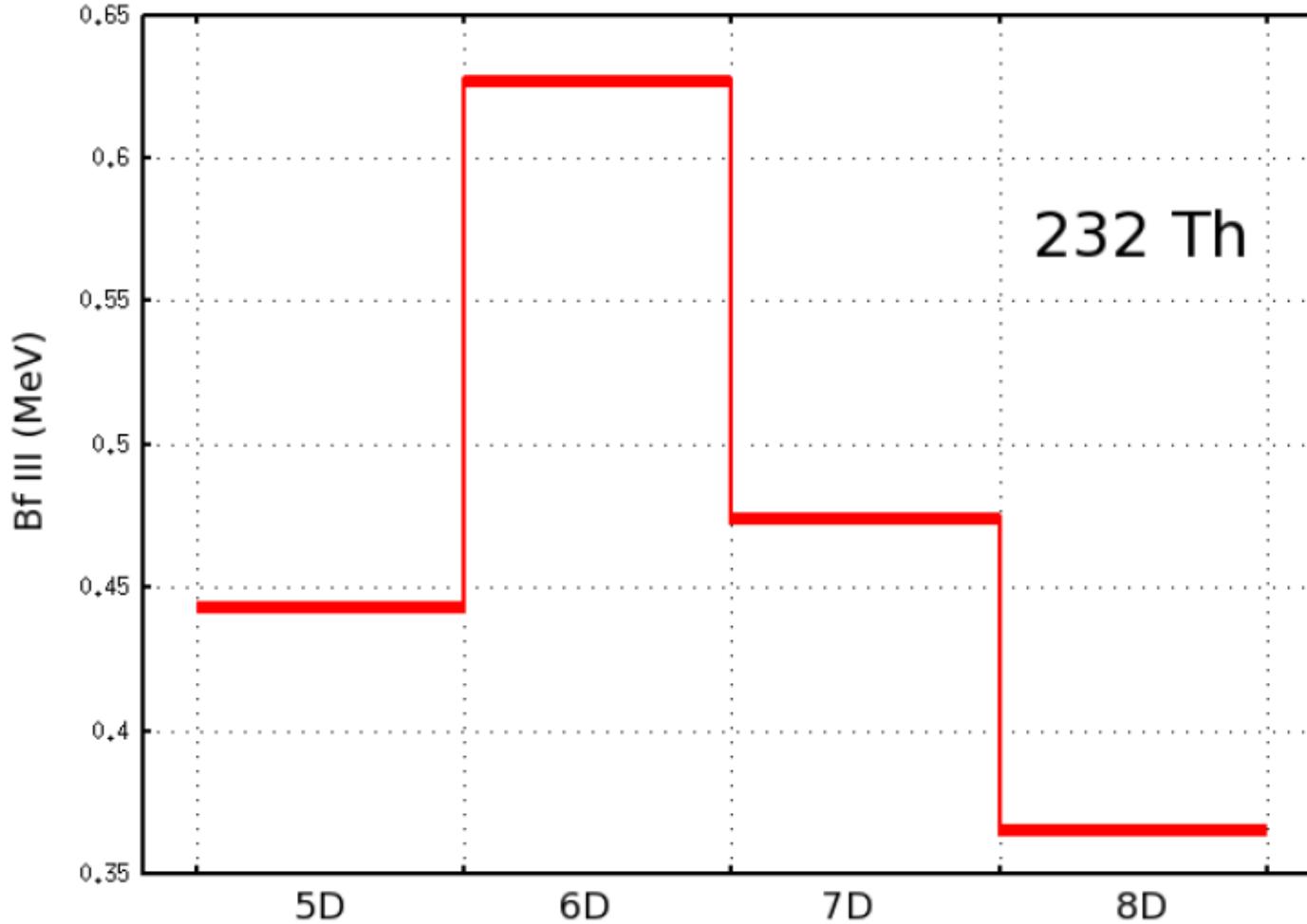
# III saddle from the mesh 5D-8D; 8D mesh (beta1-beta8) – 50803200 points!



# IIIrd minimum from the 5D-8D mesh; beta1-beta5(6,7,8)



IIIrd saddle from the mesh 5D-8D;  
8D mesh (beta1-beta8) – **50803200 points!**



# Status of third minimum in actinides:

Shallow minima  
(0.5 MeV or less )

?

Deep minima  
(3 - 4 MeV)

Theory:

self-consistent  
models

=

mac-mic model  
P. Moller et, al.

?

mac-mic model  
S. Ćwiok et, al.

||

||

Experiment:

Blons et, al.  
(231,232,233Th)

?

— — /

Debrecen-Munich  
(232,234,236U)

# III minima in actinides – How deep? Do they exist?

- - At present no predictions of deep IIIrd minima;
- - In  $^{232,230}\text{Th}$  shallow minima – experiment & theory consistent;
- Uranium nuclei: predictions conflicting experimental results.
- Other data interpretation?
- Theory change?
- Possibilities for theory:
  - Rotation (?) Temperature?
  - Importance of beyond-mean-field effects?