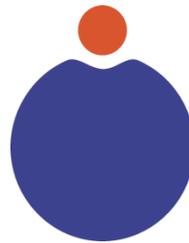


# Skyrme-HFB description of shape phase transitions in even-even SHN

A. Staszczak, L. Próchniak

*Natura non facit saltus*



19<sup>th</sup> Nuclear Physics Workshop  
"Marie & Pierre Curie"  
Kazimierz 2012



# The Segre Chart of the SHN

proton number

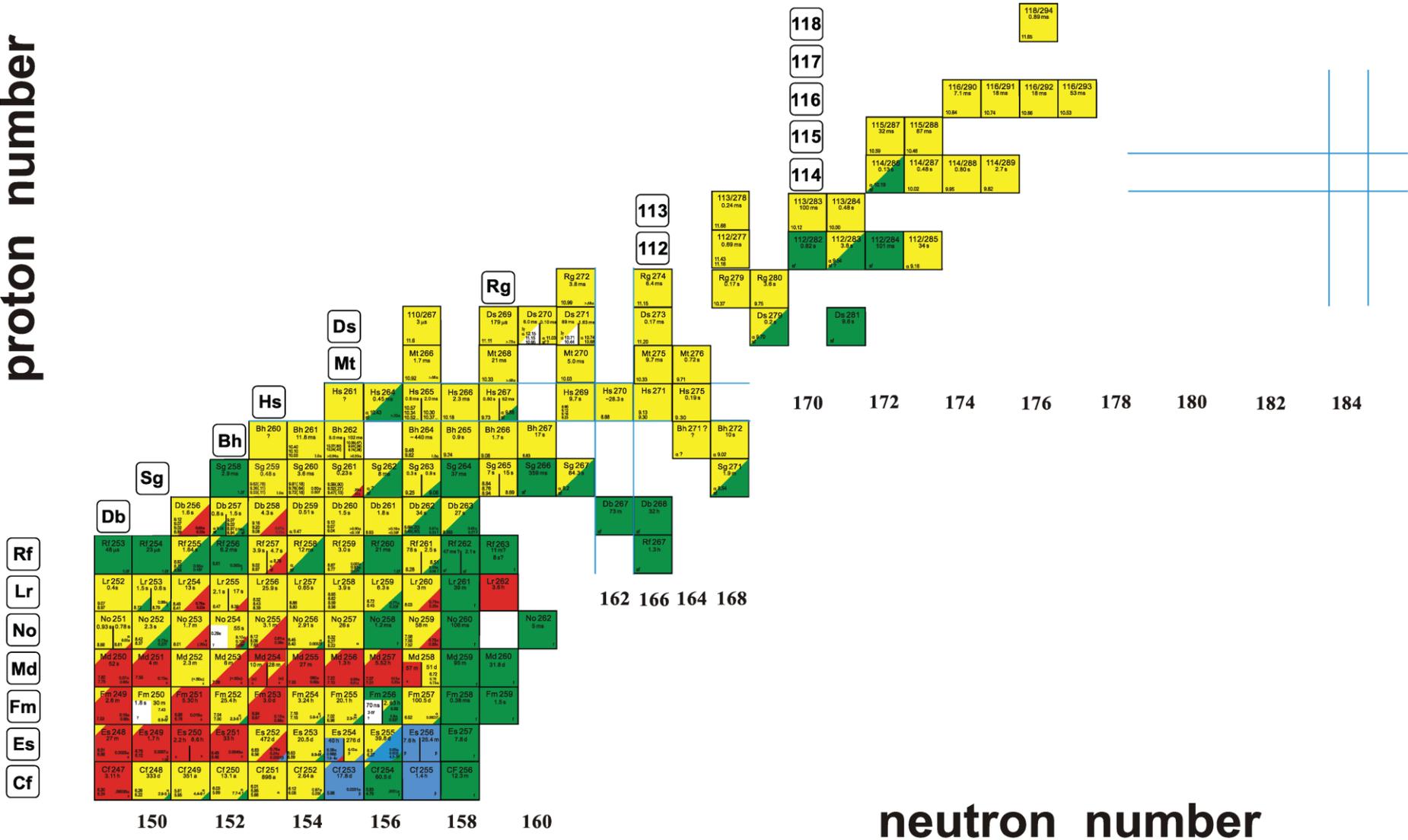


Fig. J. Dvořák (2007)

# The Segre Chart of the SHN

proton number

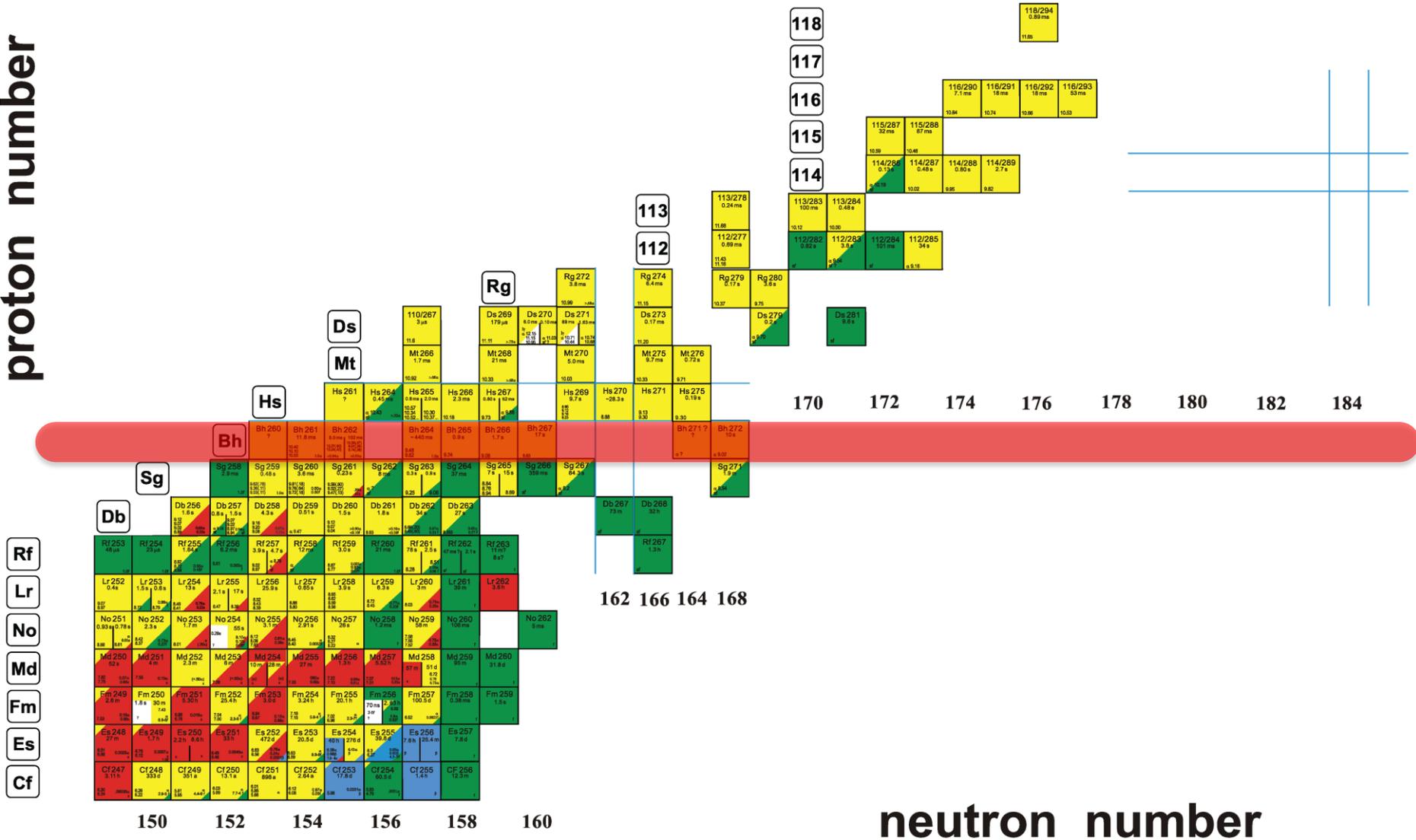
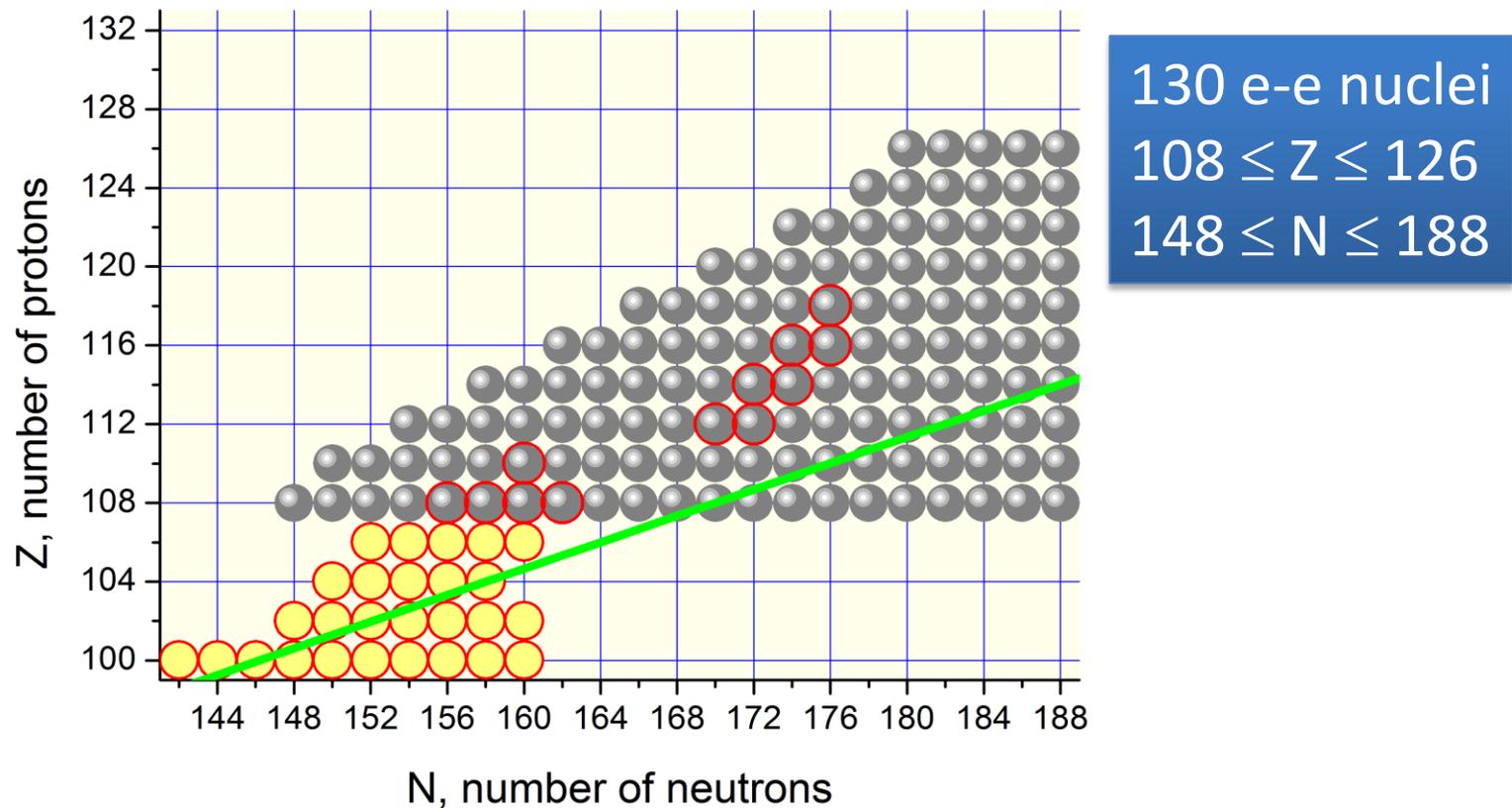


Fig. J. Dvořák (2007)

# Model

The ground states properties of even-even super heavy nuclei (SHN), with  $108 \leq Z \leq 126$  and  $148 \leq N \leq 188$ , were studied within Hartree-Fock-Bogoliubov (HFB) model with a zero-range Skyrme effective interaction.



# Model

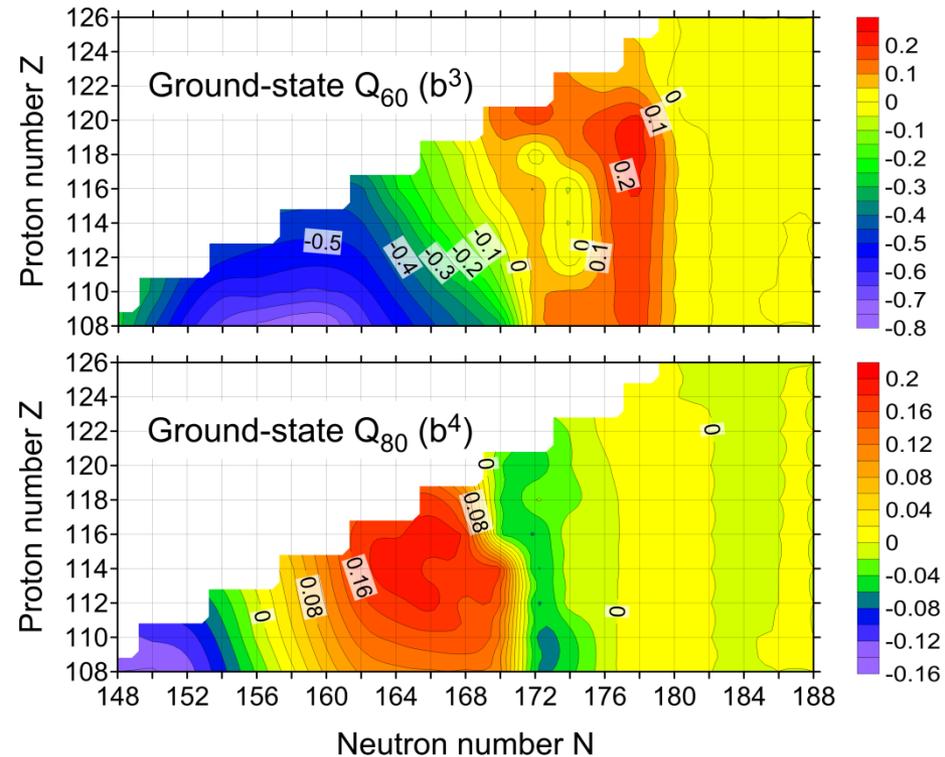
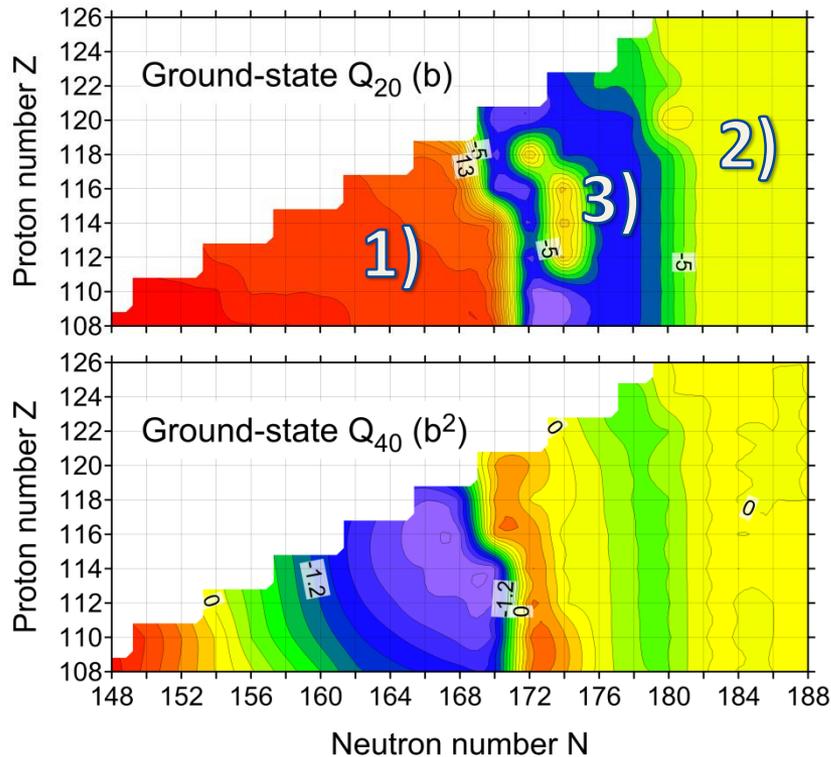
The **symmetry unrestricted code HFODD** [1] and **an augmented Lagrangian method** [2] were used to solve constrained HFB equations with **SkM\* Skyrme force** [3] in the particle-hole channel and **a density dependent mixed pairing** [4] interaction in the particle-particle channel.

To truncate the quasiparticle space of HFB, we adapted the quasiparticle cut-off value of 60 MeV in the equivalent energy spectrum. The pairing strengths were adjusted to reproduce the neutron and proton pairing gaps in  $^{252}\text{Fm}$  [5]; the resulting values are  $V_{n0} = -268.9 \text{ MeV fm}^3$  and  $V_{p0} = -332.5 \text{ MeV fm}^3$ .

The stretched harmonic oscillator basis of HFODD was composed of states having not more than  $N_0 = 26$  quanta in either of the Cartesian directions, and not more than 1140 states in total.

- 
- [1] J. Dobaczewski and J. Dudek, *Comput. Phys. Commun.* **102**, 166 (1997); **102**, 183 (1997); **131**, 164 (2000); J. Dobaczewski and P. Olbratowski, **158**, 158 (2004); **167**, 214 (2005); J. Dobaczewski *et al.*, **180**, 2361 (2009); “*HFODD (v2.40h) User’s Guide*”, (2009), arXiv:0909.3626; N. Schunck *et al.*, **183**, 166 (2012).
  - [2] A. Staszczak, M. Stoitsov, A. Baran, and W. Nazarewicz, *Eur. J. Phys. A* **46**, 85 (2010).
  - [3] J. Bartel *et al.*, *Nucl. Phys. A* **386**, 79 (1982).
  - [4] J. Dobaczewski, W. Nazarewicz, and M. V. Stoitsov, *Eur. J. Phys. A* **15**, 21 (2002).
  - [5] A. Staszczak, A. Baran, J. Dobaczewski, and W. Nazarewicz, *Phys. Rev. C* **80**, 014309 (2009).

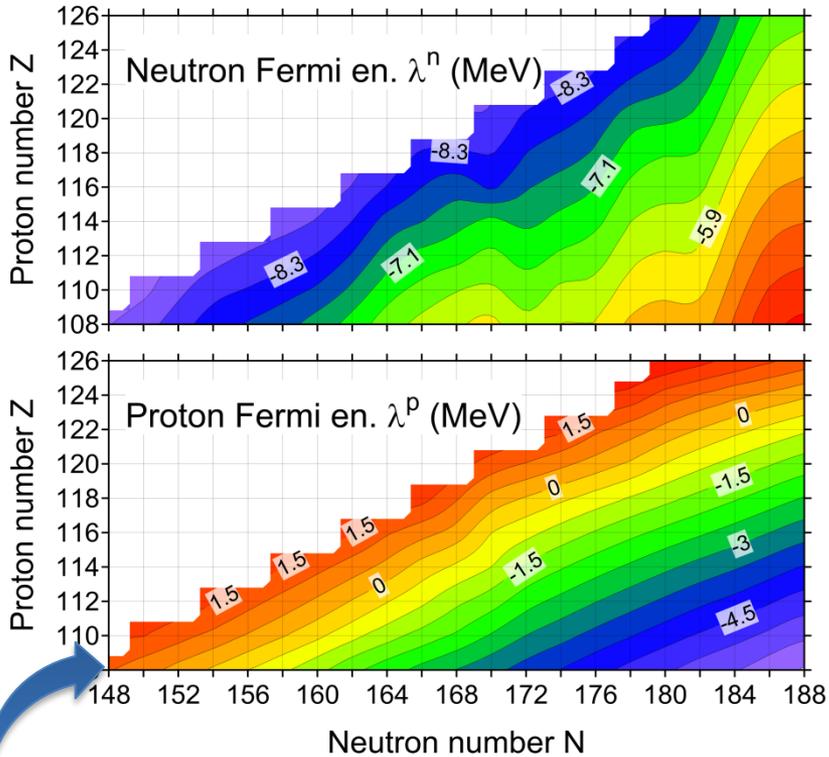
# Ground state deformations



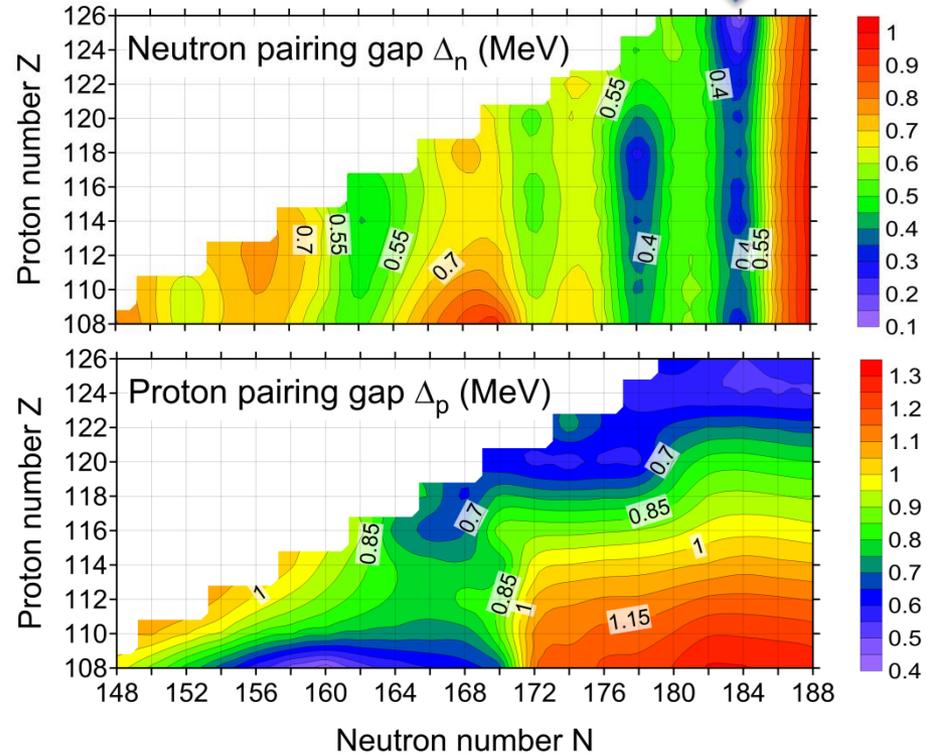
The e-e SHN form three regions:

- 1) a prolate-deformed (for  $N < 172$ ),
- 2) spherical ( $N > 180$ ),
- 3) the transitional region (between the former two).

# Ground state pairing properties of e-e SHN

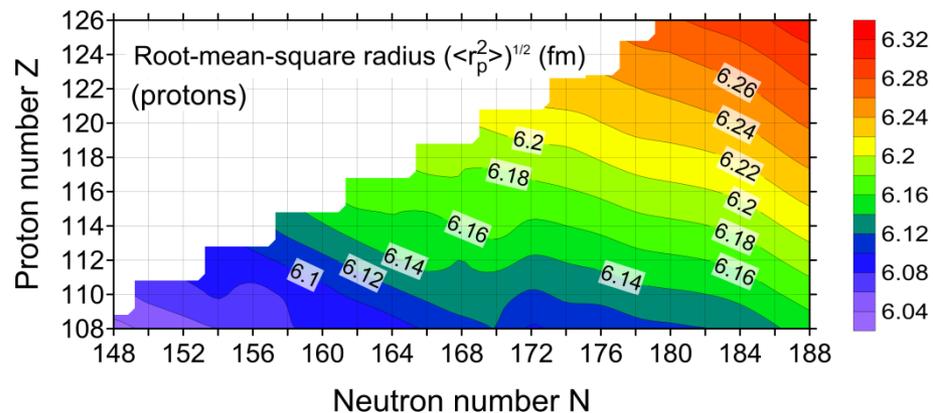
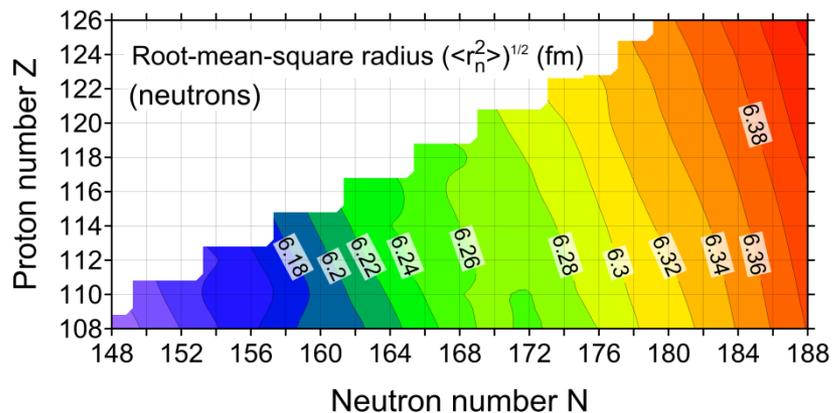
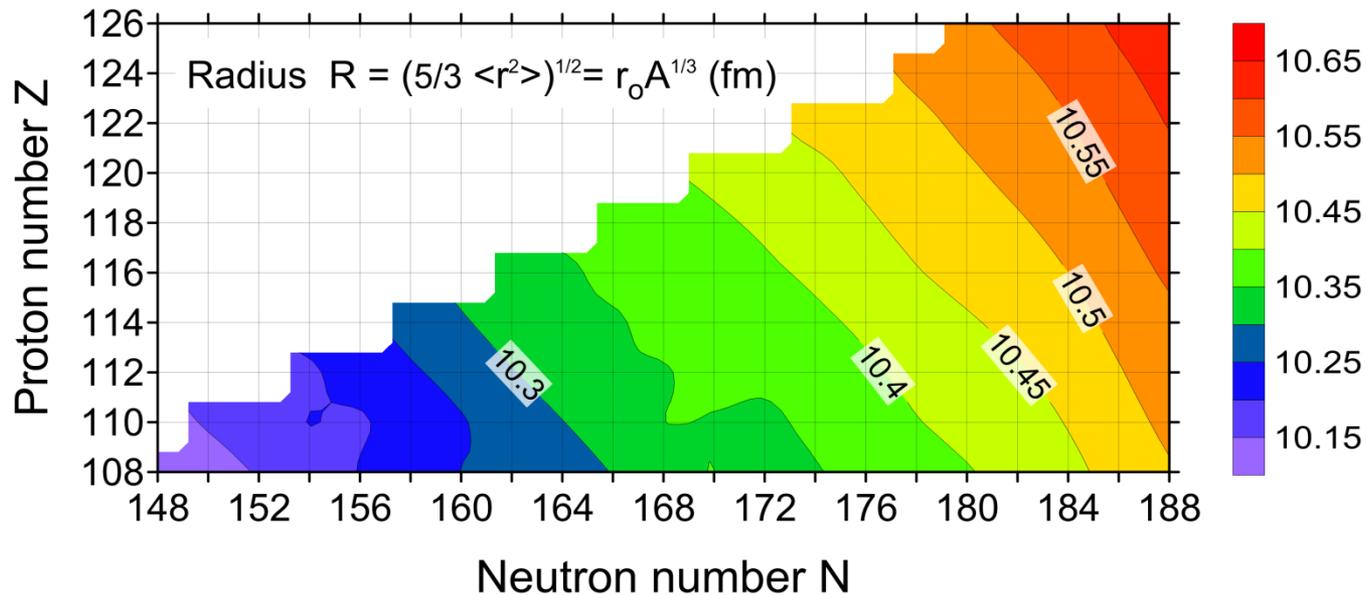


$\Delta_n \approx 0$  MeV for N = 184



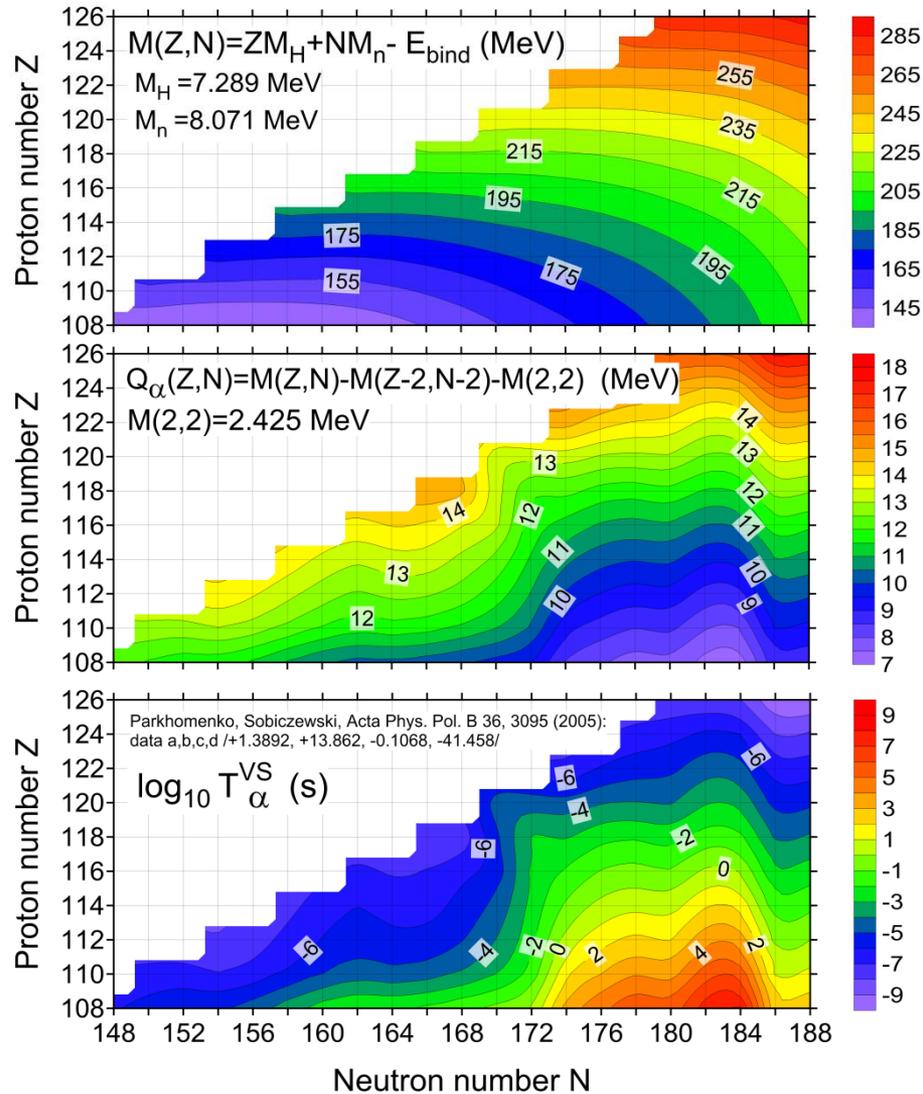
Proton drip line:  
Fermi energy  $\lambda^p \leq 2$  MeV.

# Geometric sizes



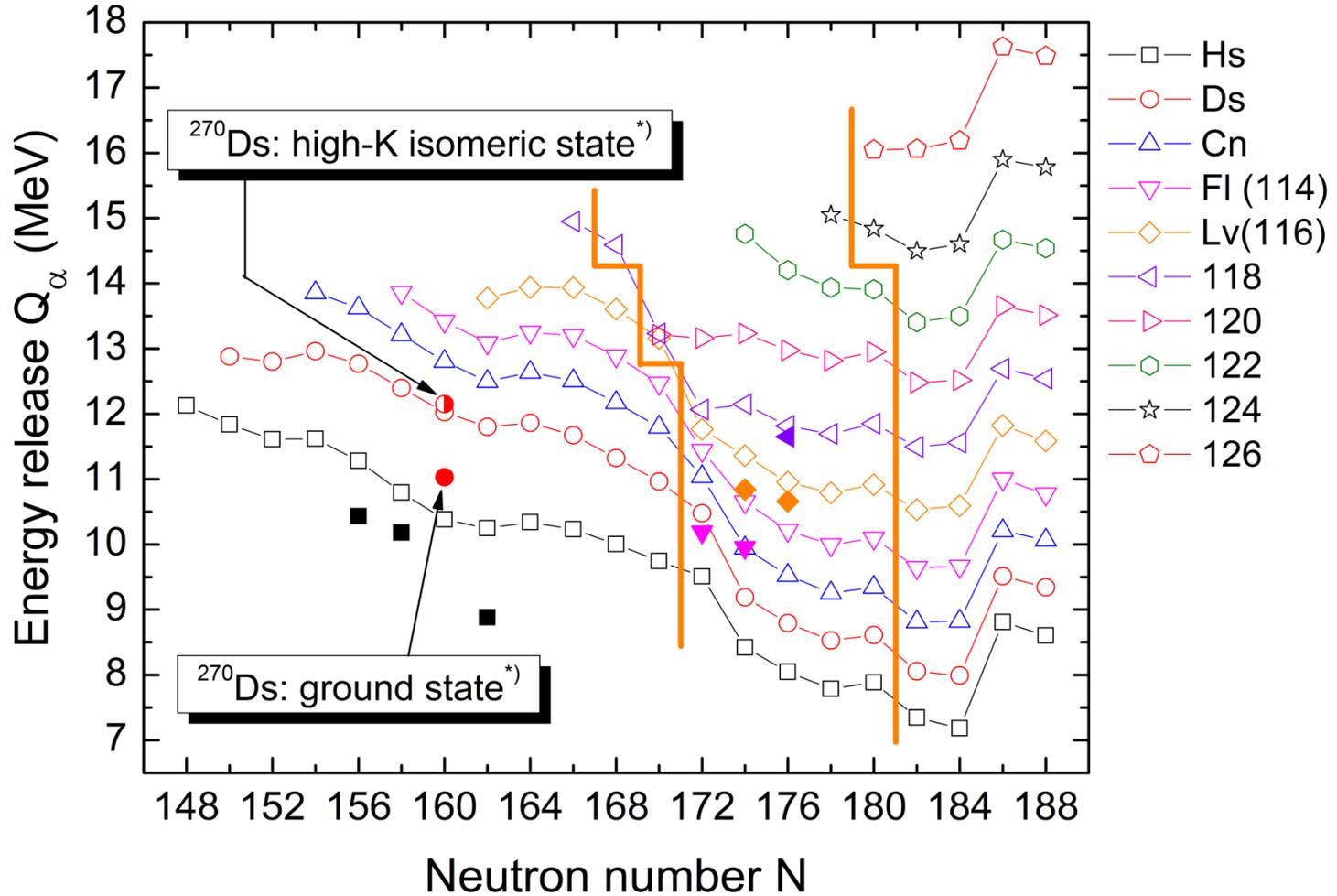


# Alpha emission



# $Q_\alpha$ - values

<sup>\*)</sup>S. Hofmann, *et al.*, Eur. Phys. J. A **10**, 5 (2001)

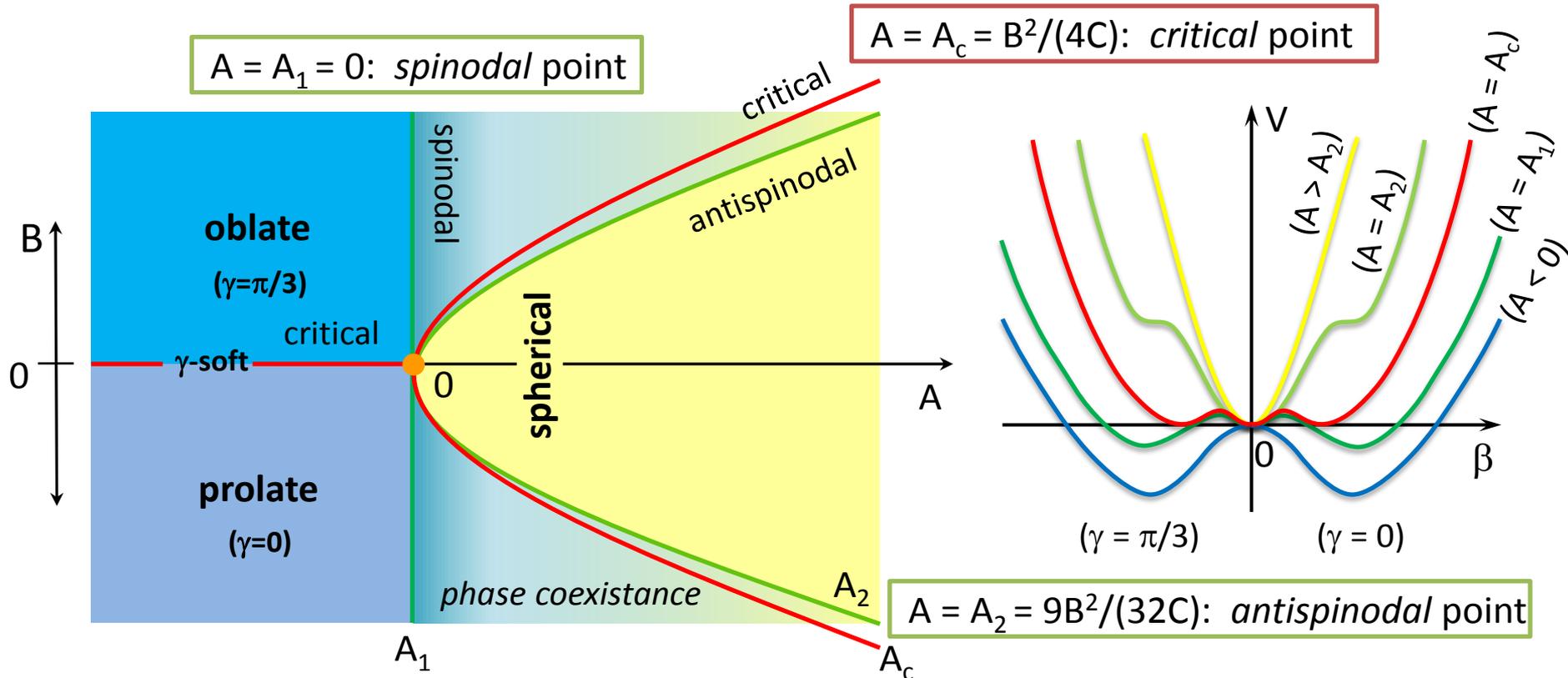


# Geometric collective model (GCM) - A. Bohr (1952)

$$V(\beta, \gamma) = A\beta^2 + B\beta^3 \cos 3\gamma + C\beta^4, \quad (C > 0)$$

$A = A_1 = 0$ : spinodal point

$A = A_c = B^2/(4C)$ : critical point



$A = A_2 = 9B^2/(32C)$ : antispinodal point

$A = A_c$  ( $B \neq 0$ ) and  $A < 0$  ( $B = 0$ ): first-order phase transition lines

$A = B = 0$ : second-order phase transition point (triple-point)

# Interacting boson approximation (IBA-1) – Arima, Iachello

$$U(6) \supset U(5) \supset O(5) \supset O(3)$$

$$U(6) \supset SU(3) \supset O(3)$$

$$U(6) \supset O(6) \supset O(5) \supset O(3)$$

$$U(6) \supset \overline{SU(3)} \supset O(3)$$

$$U(6) \supset \overline{O(6)} \supset O(5) \supset O(3)$$

## Critical-point solutions:

$$V(\beta, \gamma) = A\beta^2 + B\beta^3 \cos 3\gamma + C\beta^4$$

$$V(\beta, \gamma) \approx V_1(\beta) + V_2(\gamma)$$

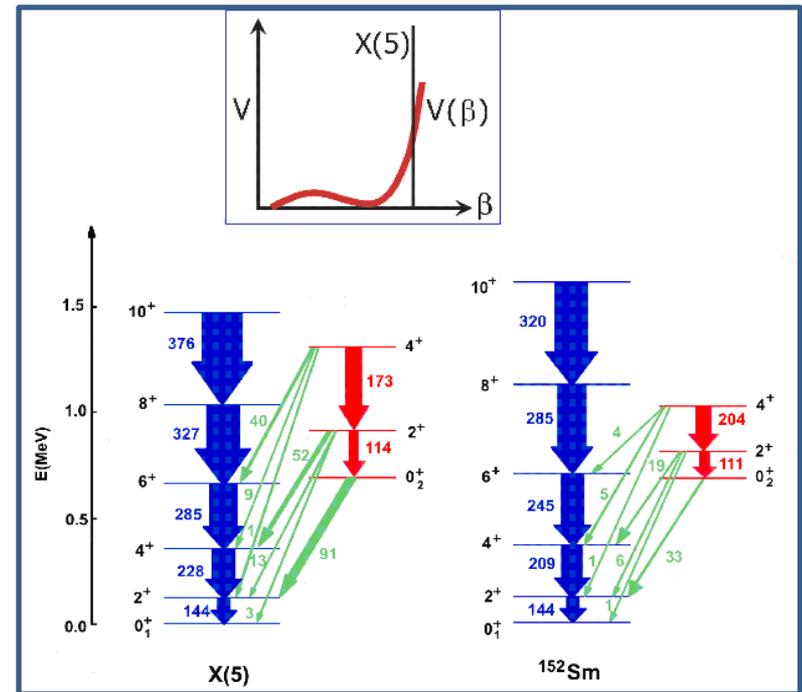
$$X(5): \quad V_1 = V_{well}(\beta), \quad V_2 = c(\gamma - \gamma_0)^2, \quad (c > 0)$$

$$E(5): \quad V_1 = V_{well}(\beta), \quad V_2 \equiv 0$$

F. Iachello, PRL **85**, 3580 (2000);  
**87**, 052502 (2001).

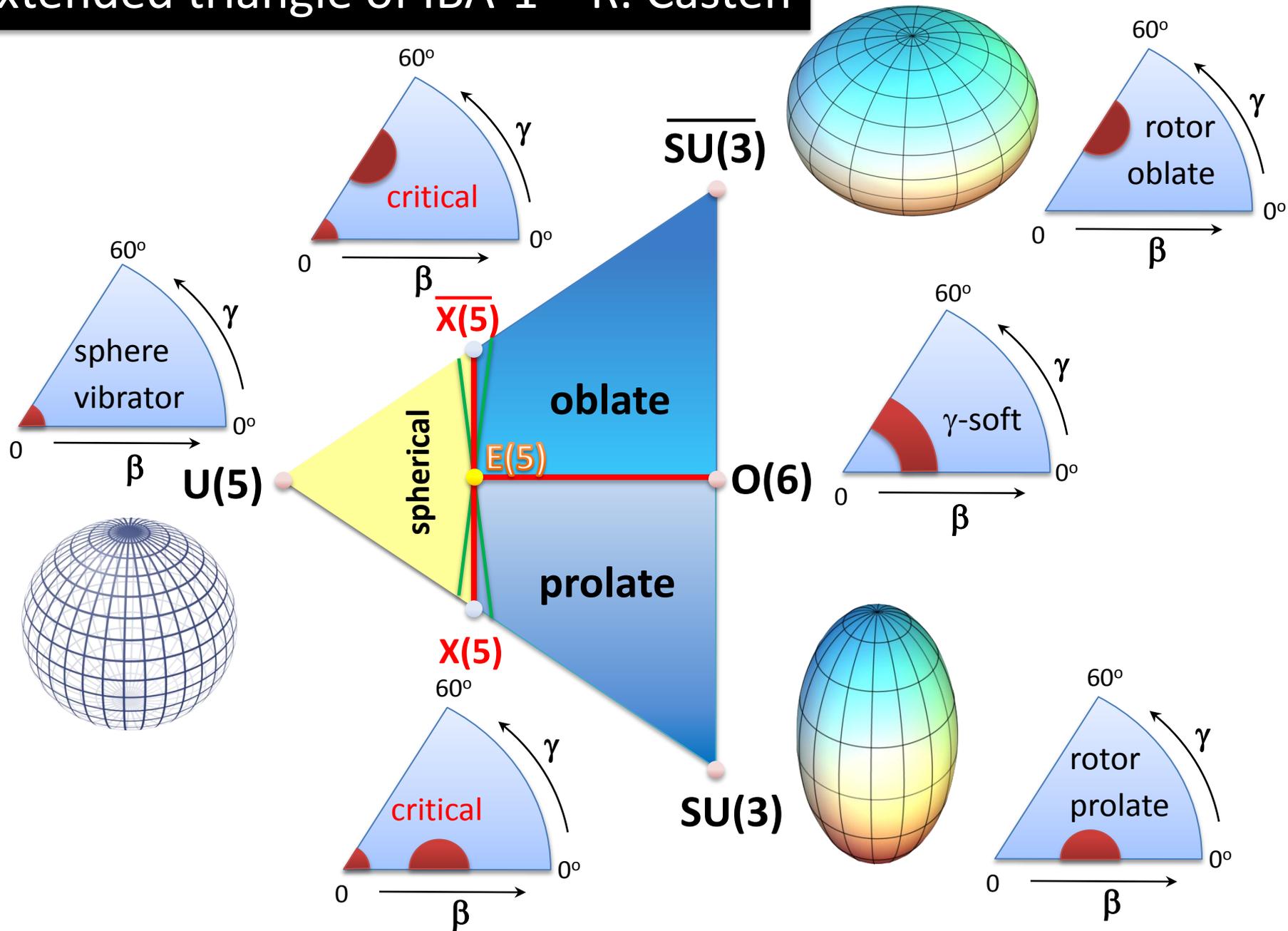
## Dynamical symmetries:

$U(5)$  (vibrational)  
 $SU(3), \overline{SU(3)}$  (rotational)  
 $O(6), \overline{O(6)}$  ( $\gamma$ -soft)

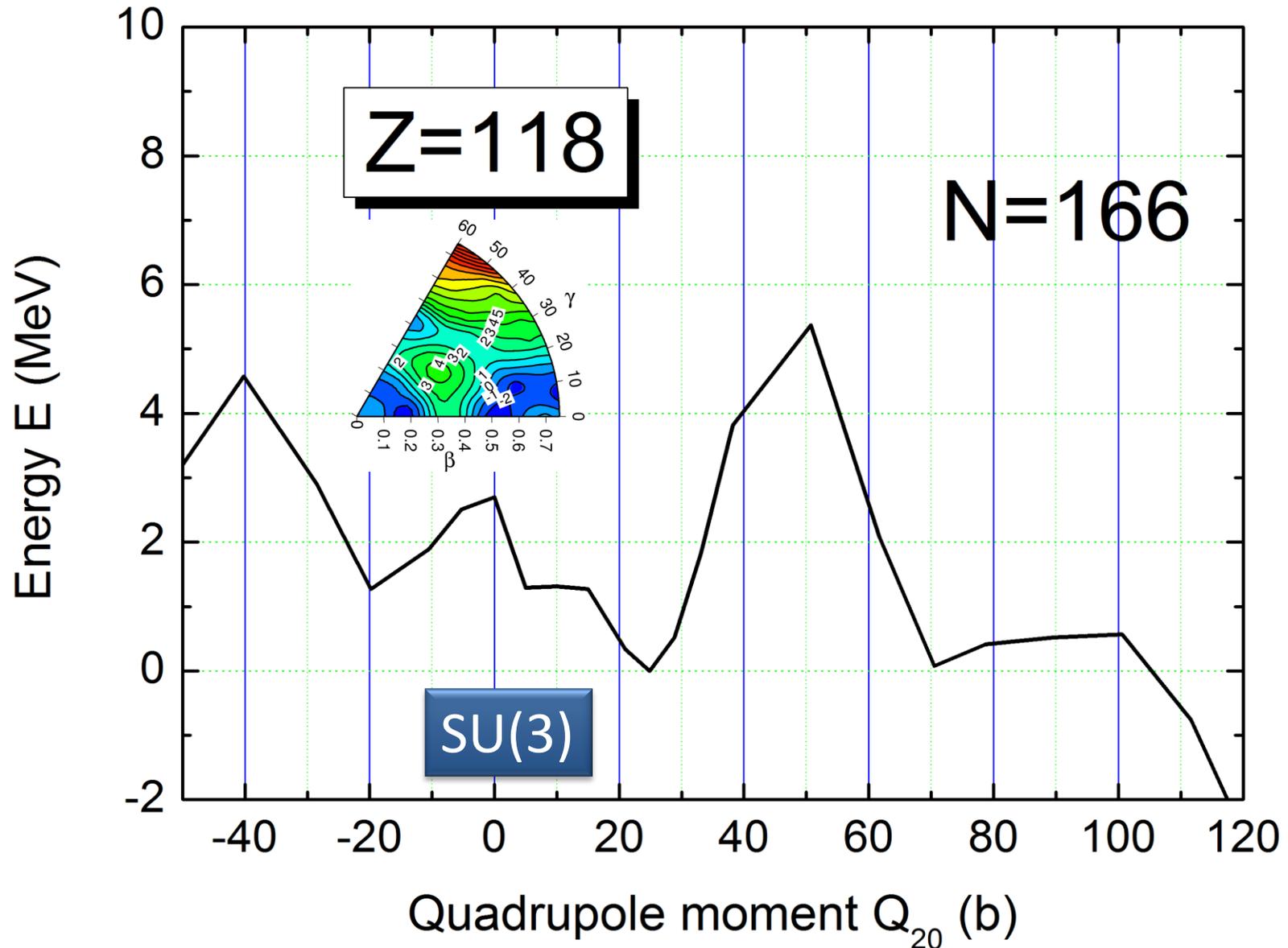


(Fig. Casten)

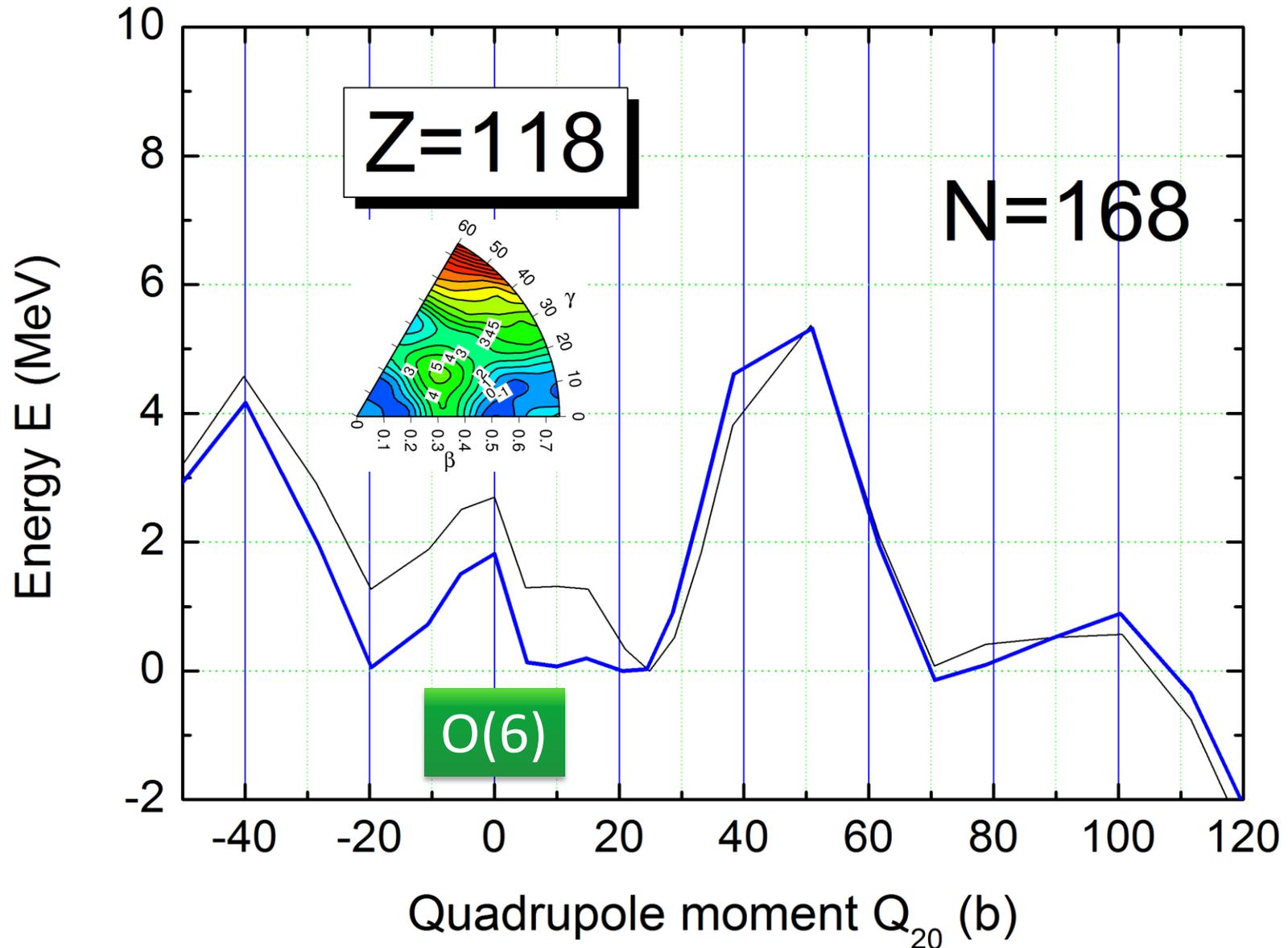
# Extended triangle of IBA-1 – R. Casten



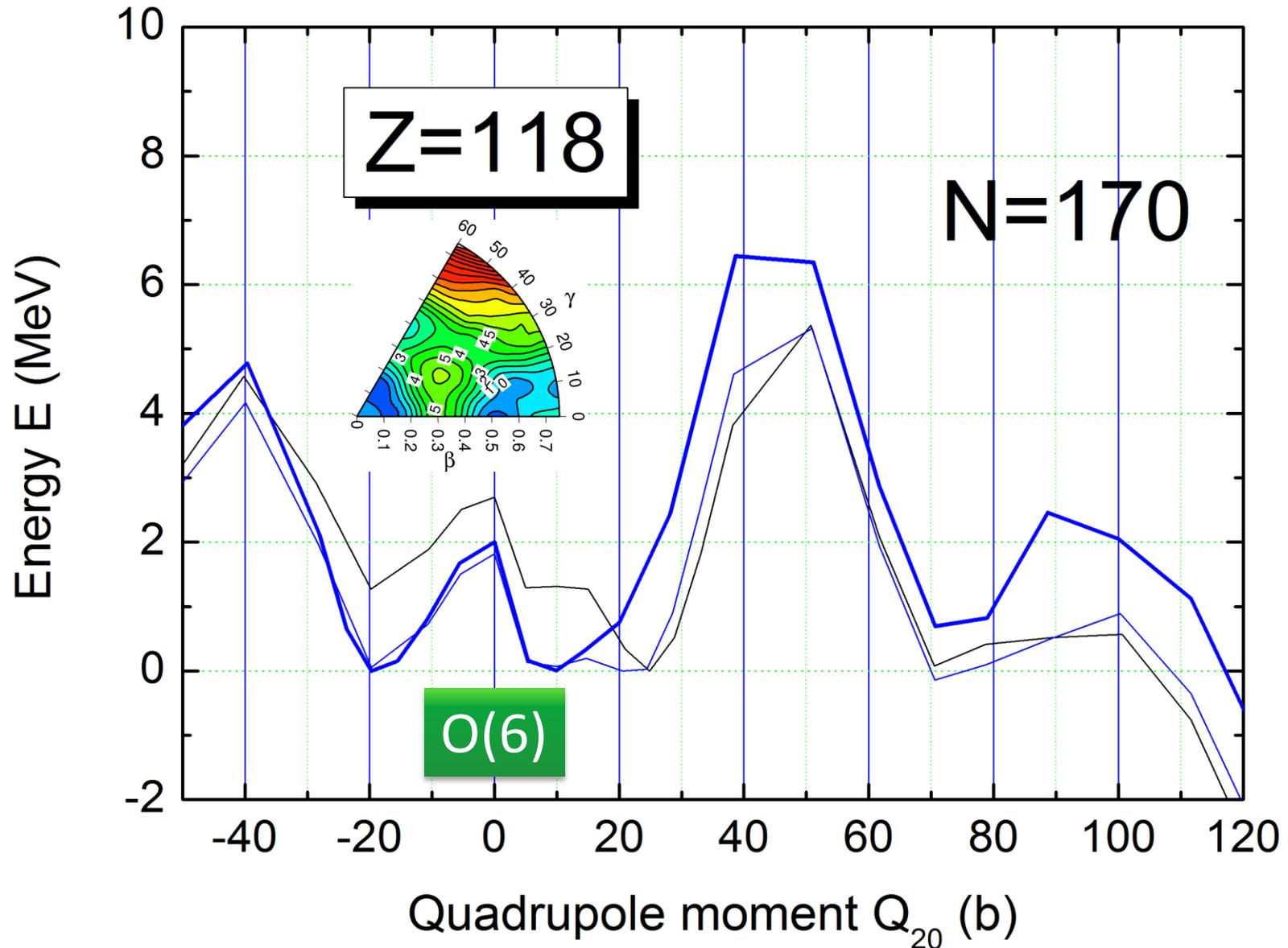
# Nuclear shape phase transitions



# Second order phase transition O(6) – U(5)

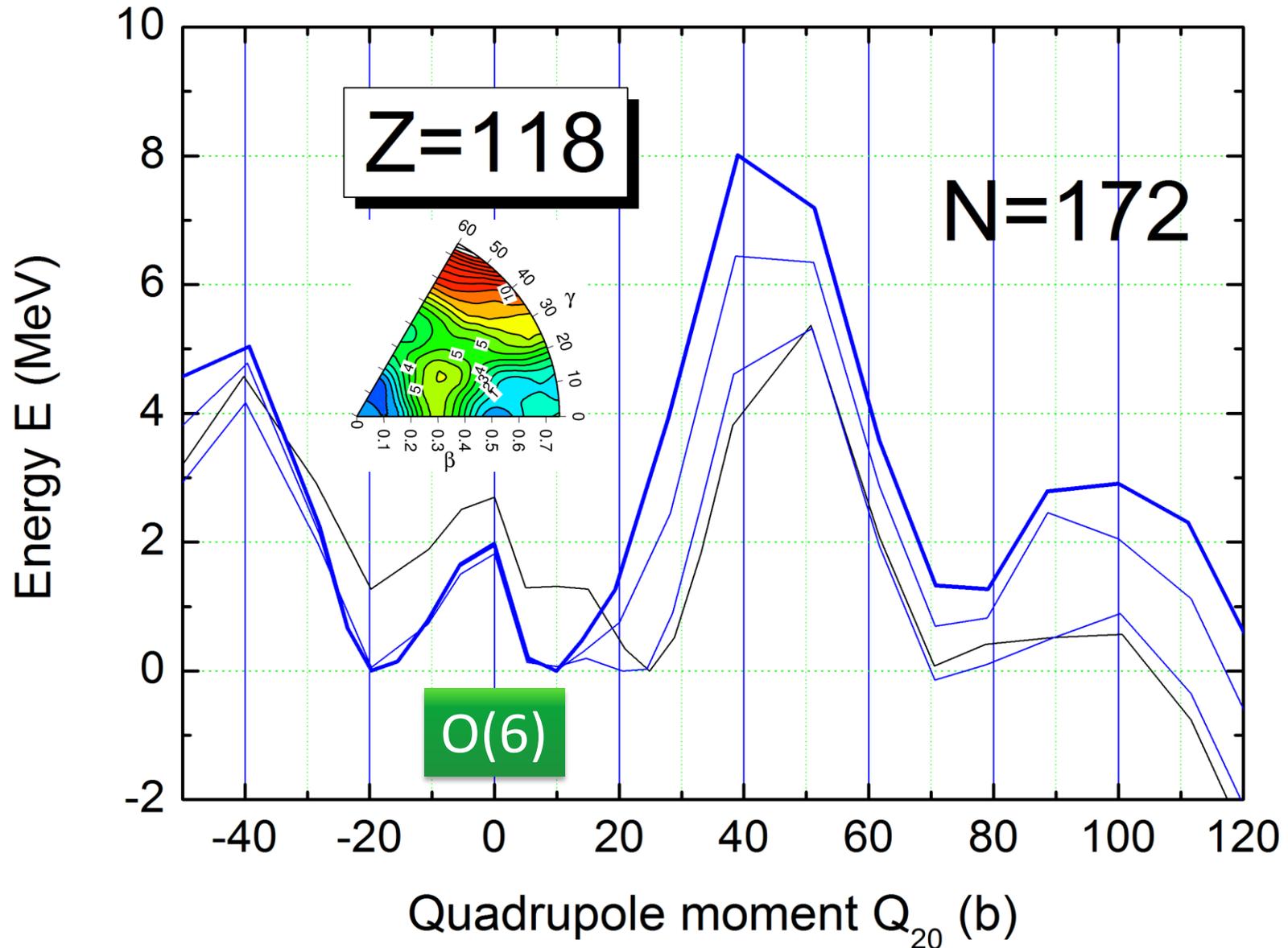


# Second order phase transition O(6) – U(5)

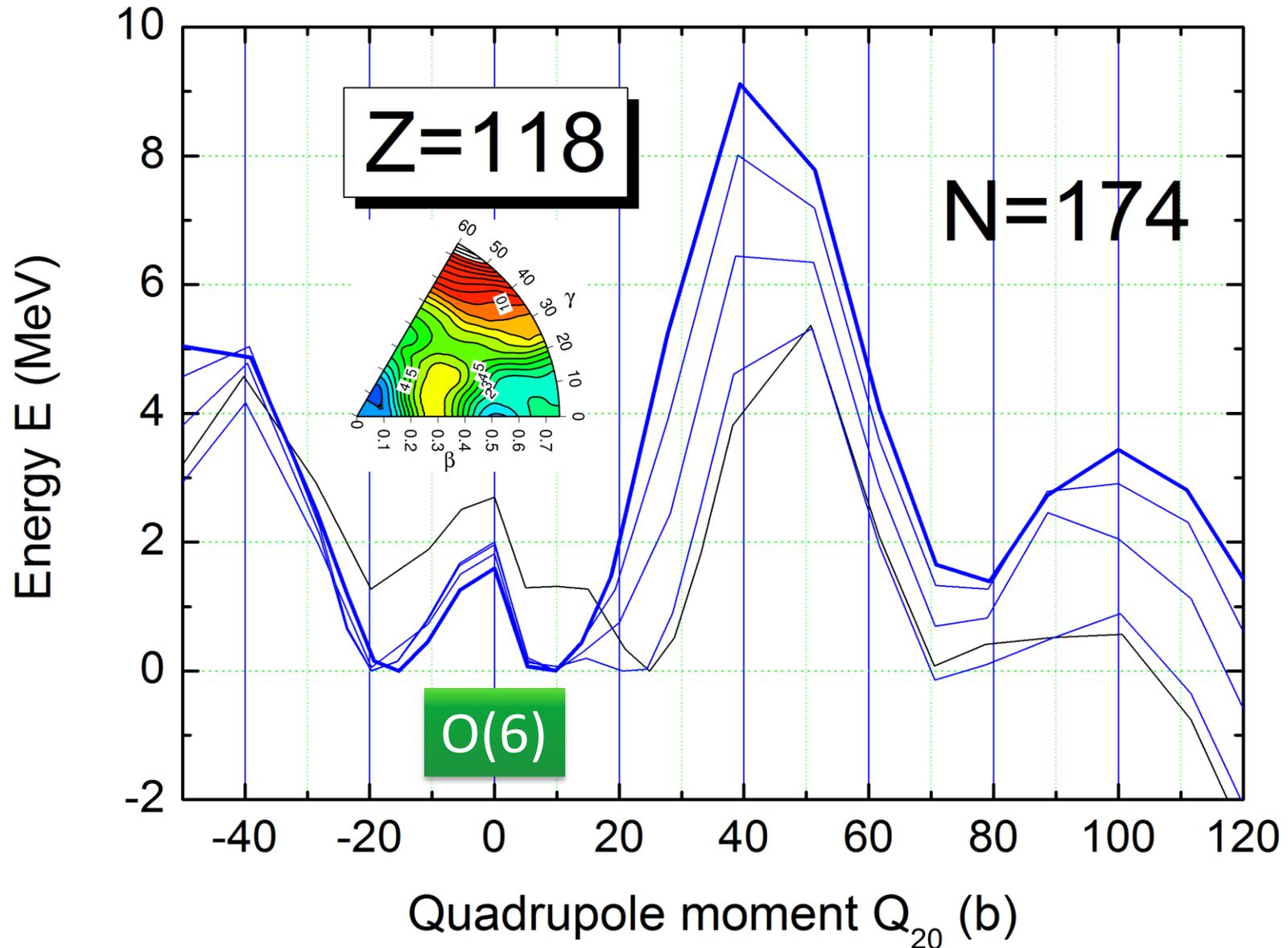




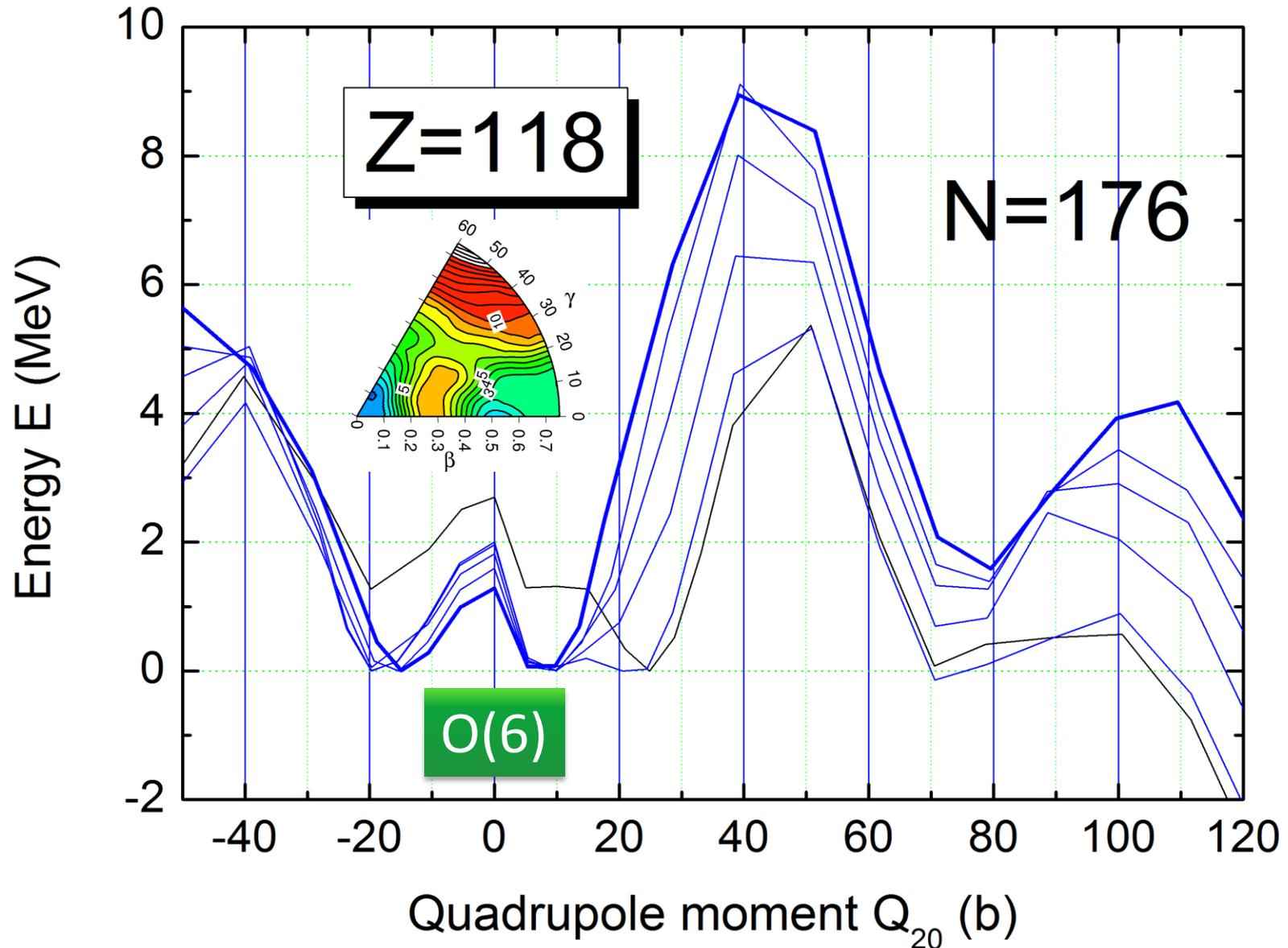
# Second order phase transition O(6) – U(5)



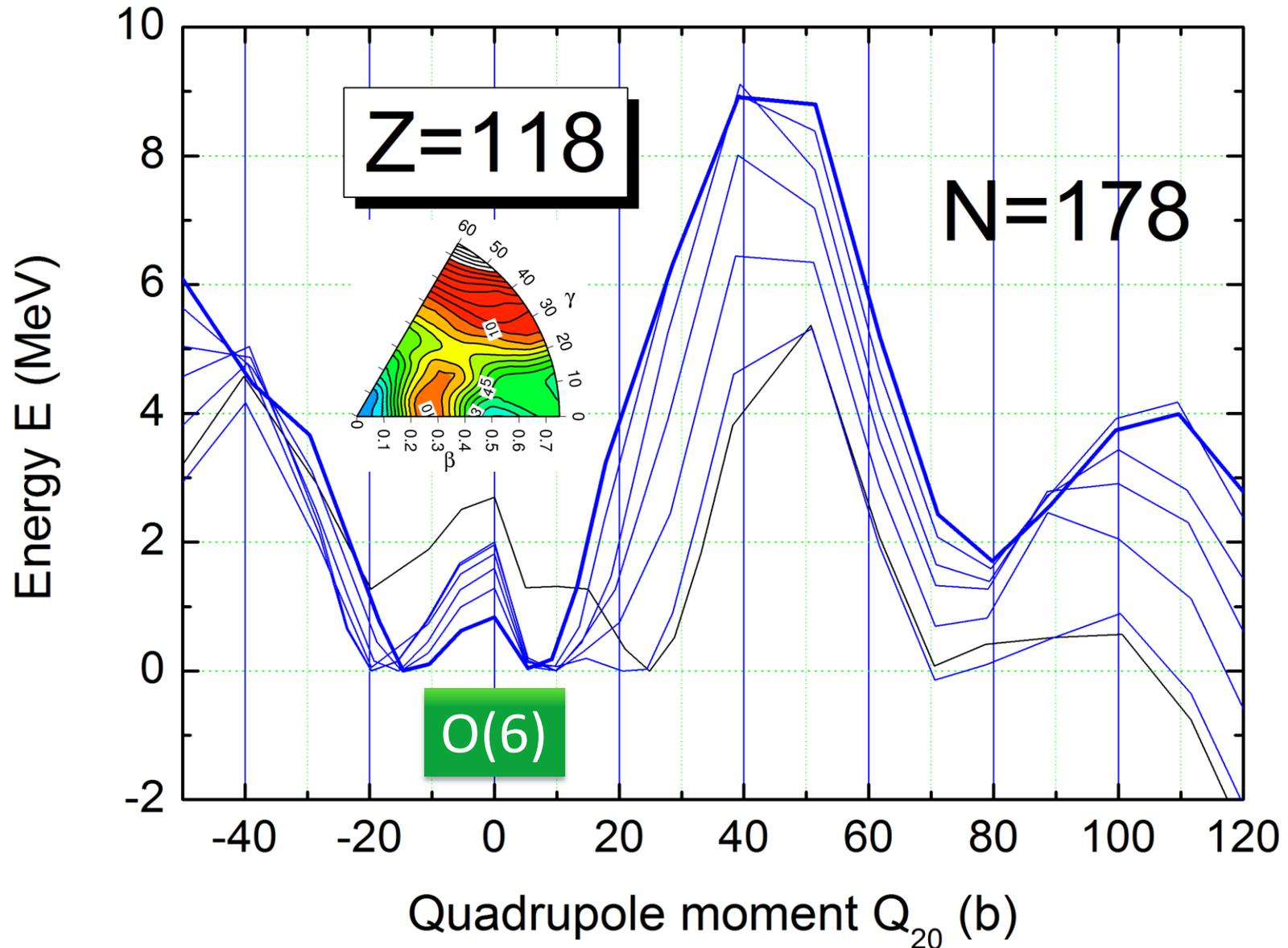
# Second order phase transition O(6) – U(5)



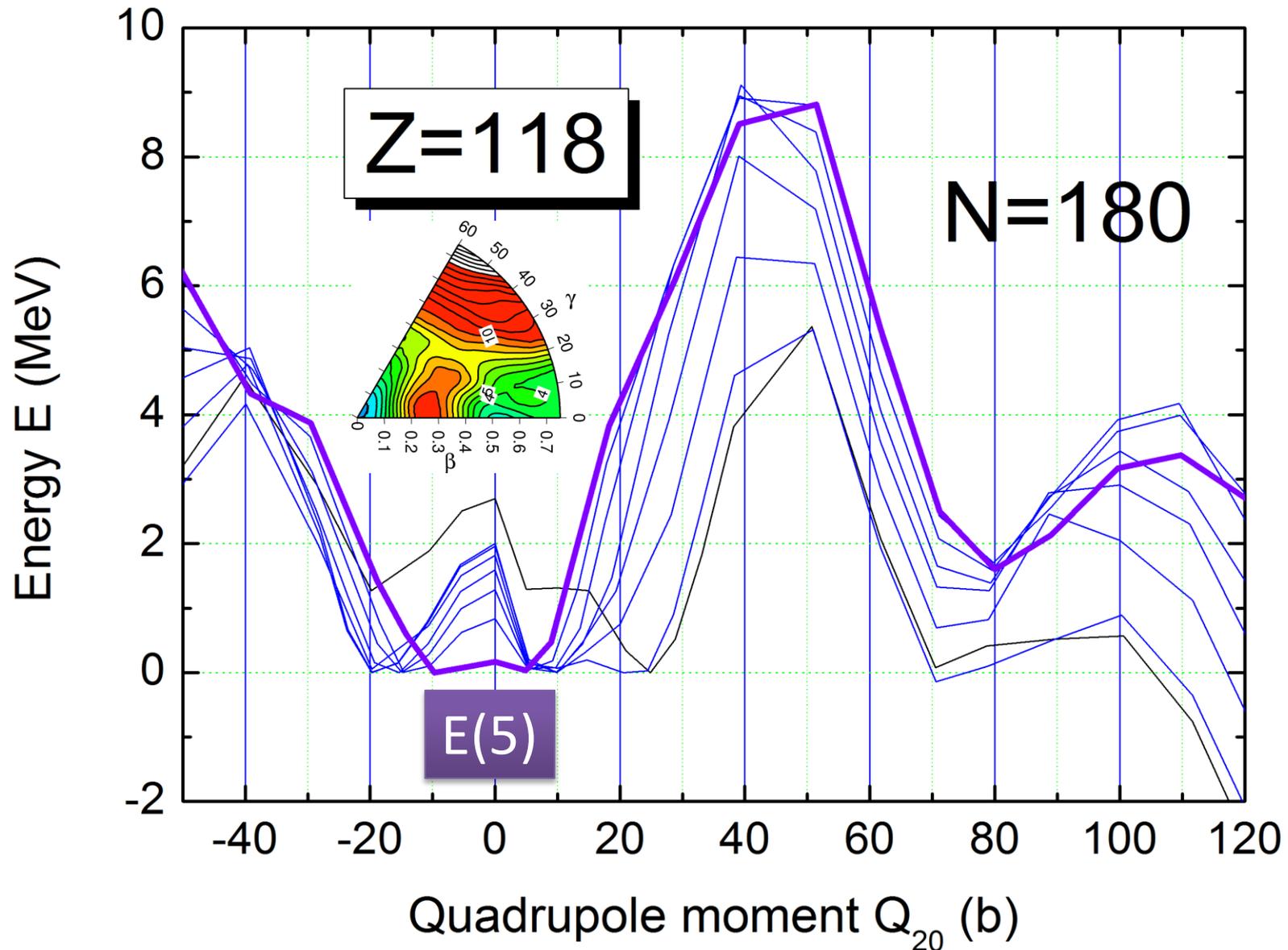
# Second order phase transition O(6) – U(5)



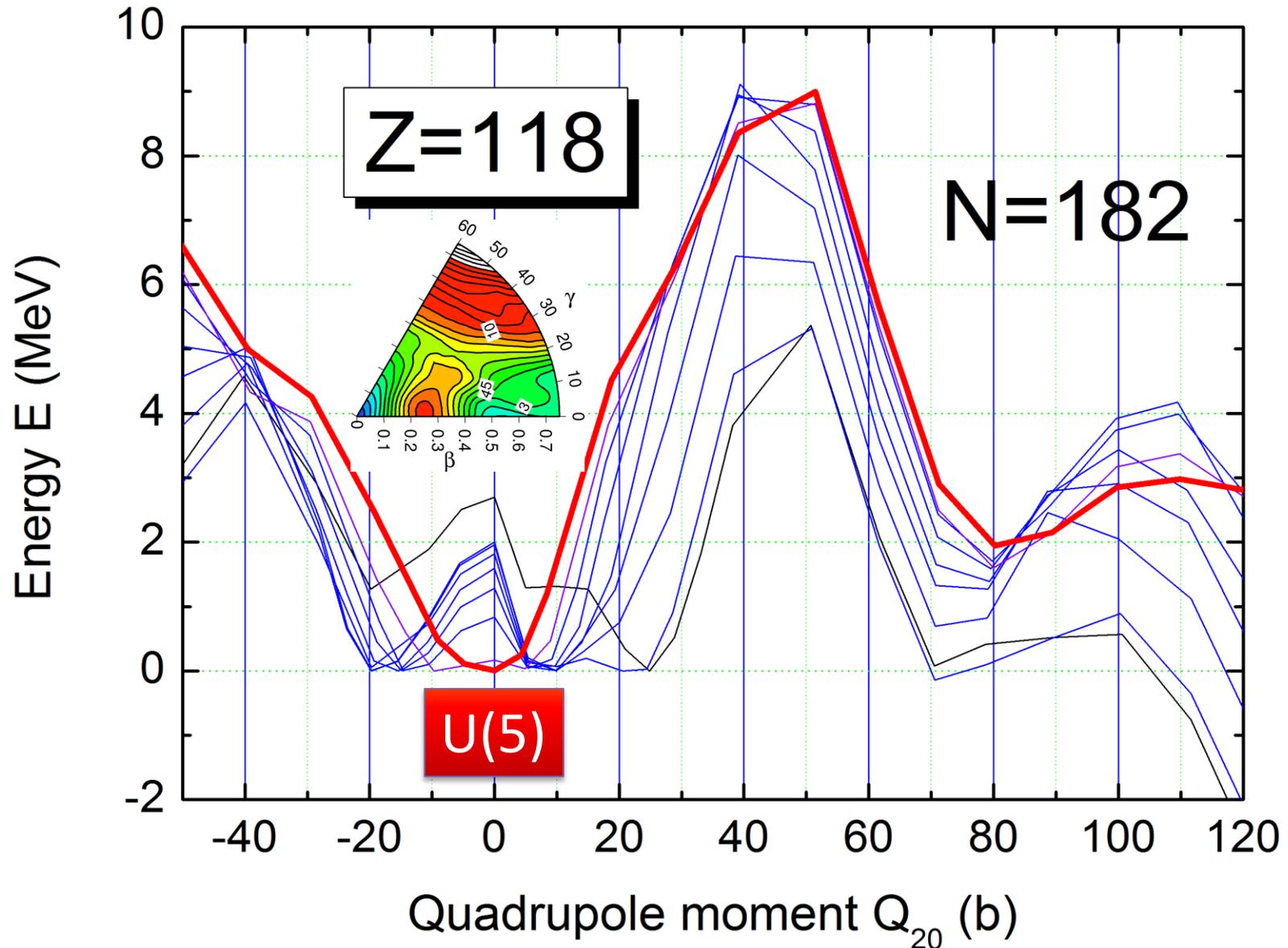
# Second order phase transition O(6) – U(5)



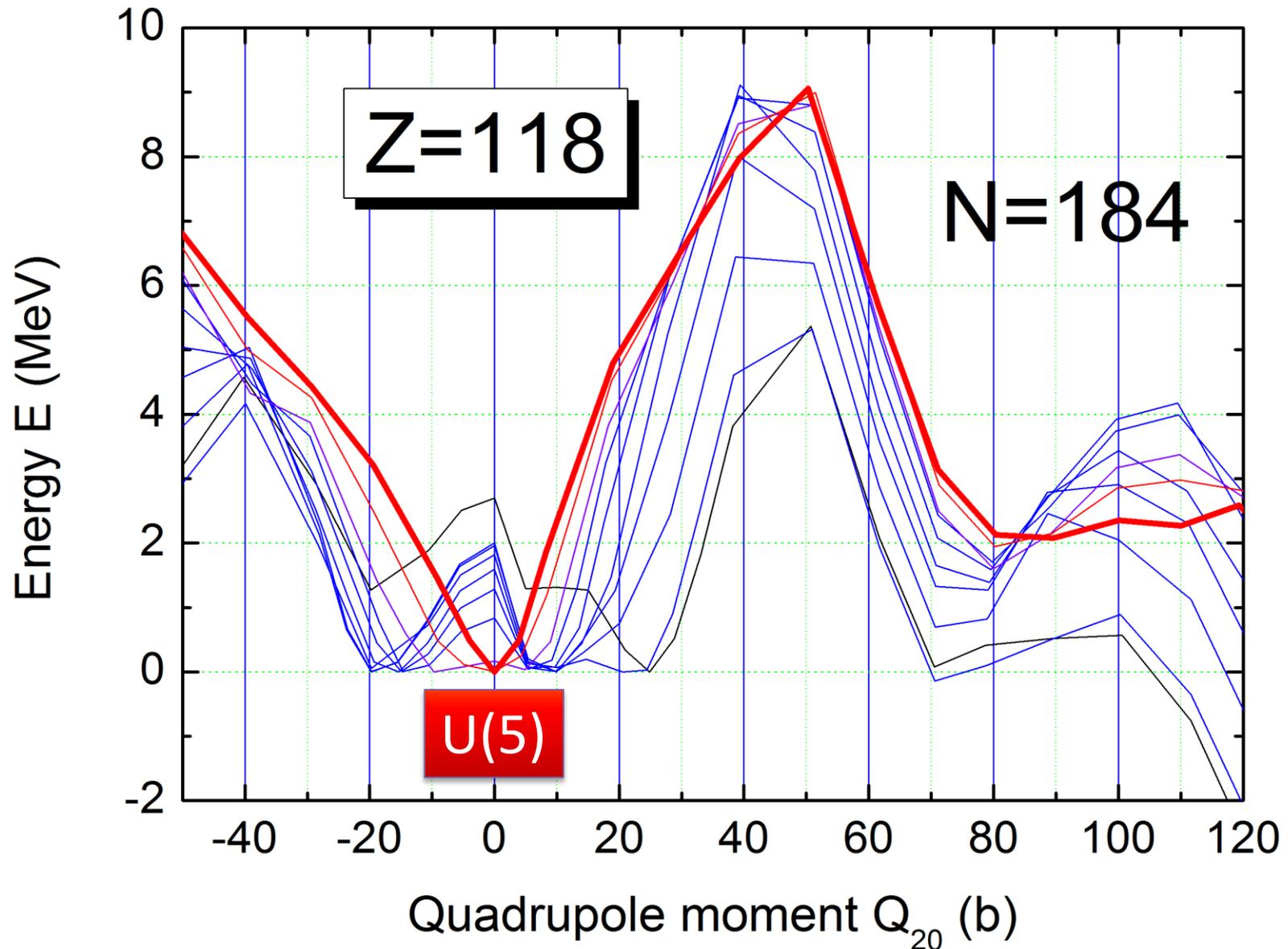
# Critical (triple) point E(5)



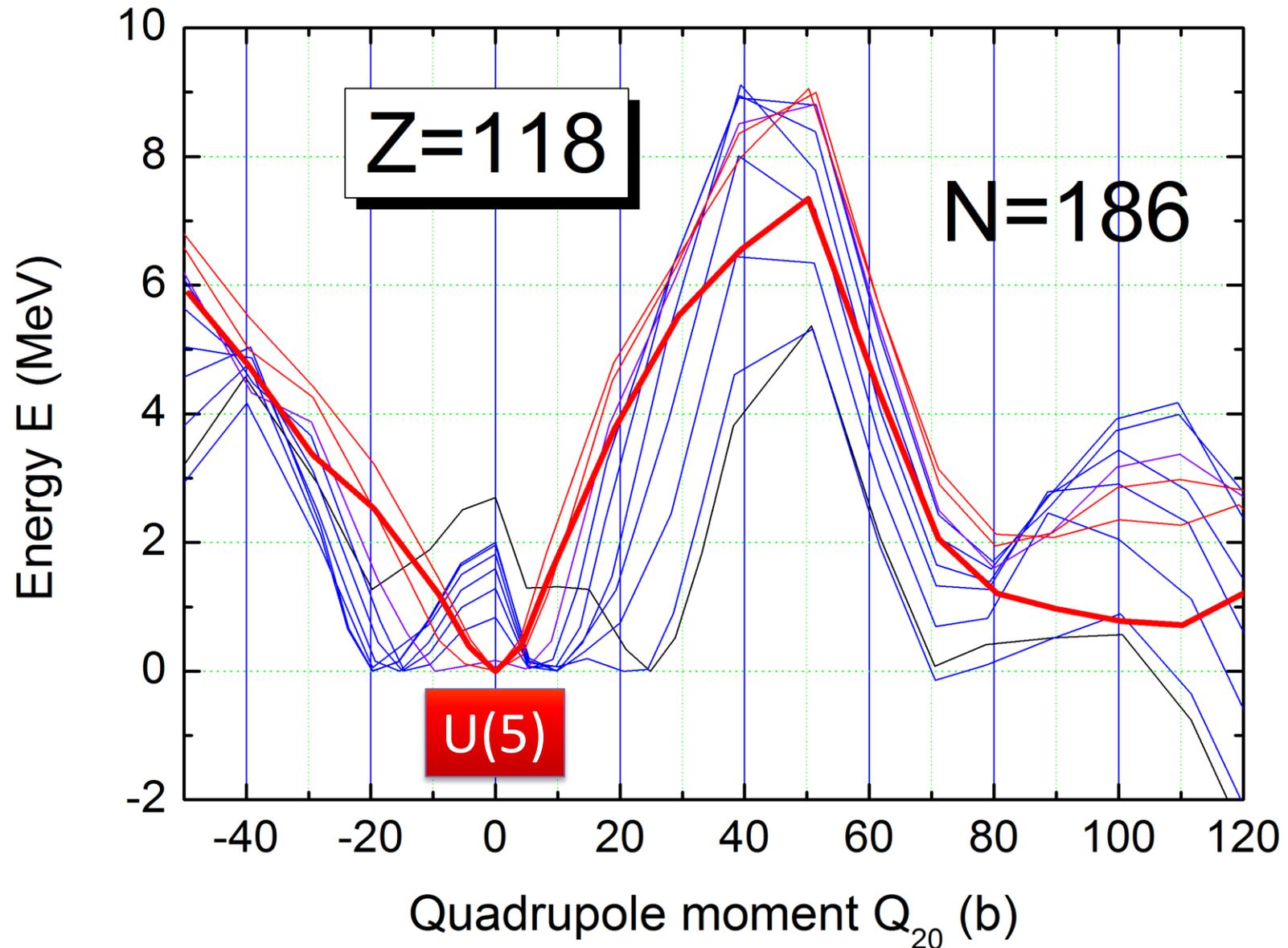
# Second order phase transition O(6) – U(5)



# Second order phase transition O(6) – U(5)

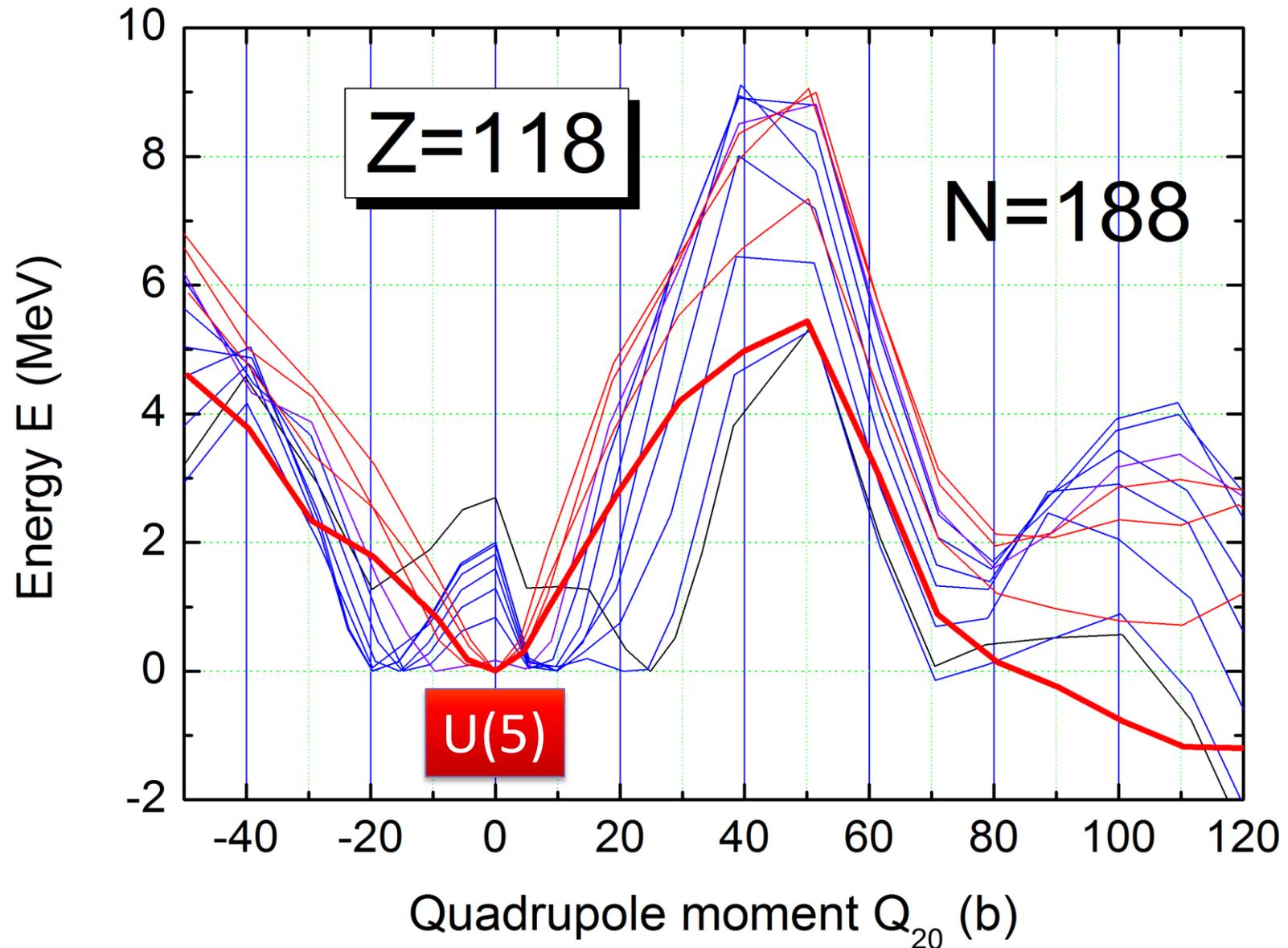


# Second order phase transition O(6) – U(5)

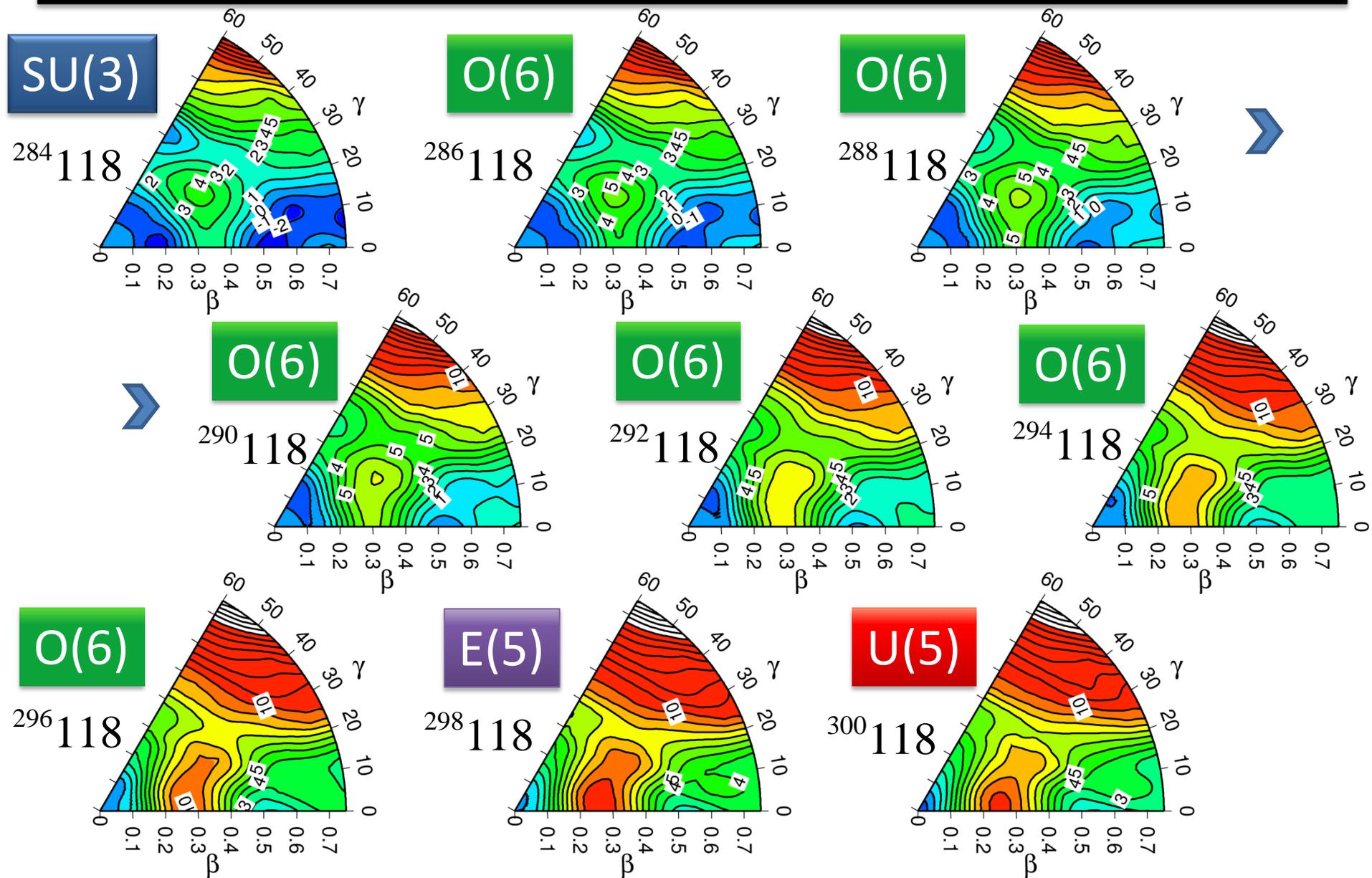




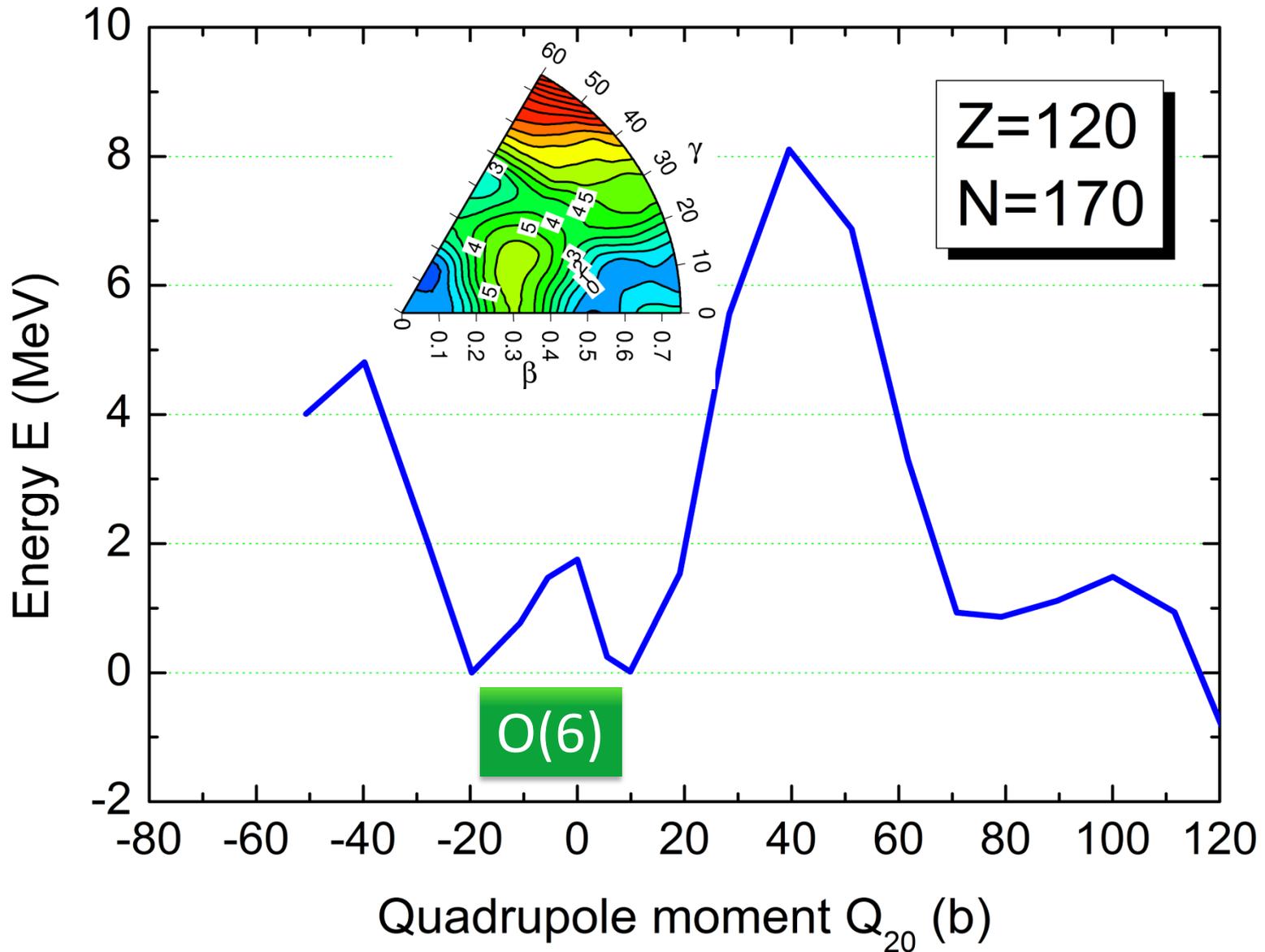
# Second order phase transition O(6) – U(5)



# Second order phase transition $O(6) - U(5)$

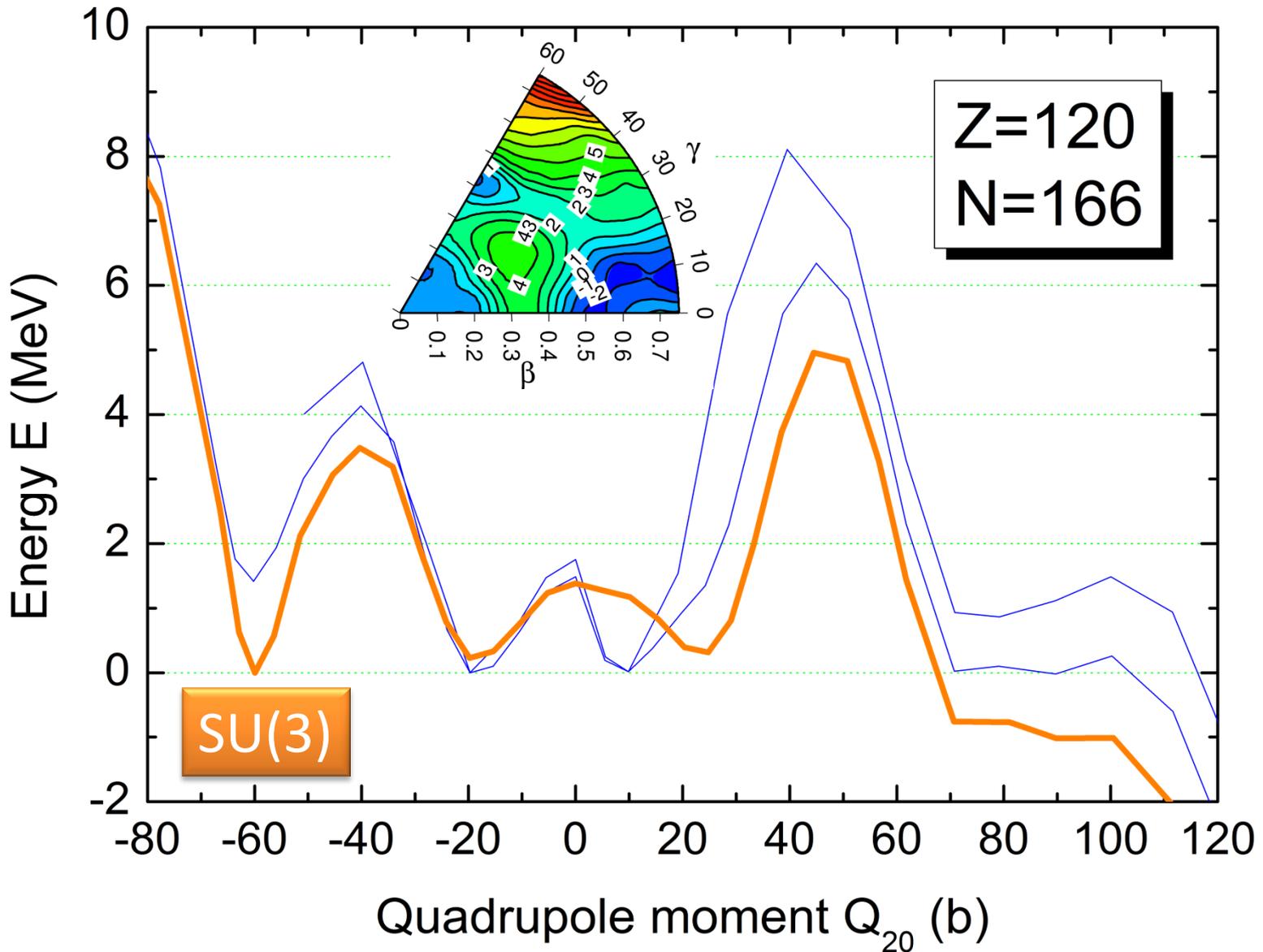


# Superdeformed oblate (SDO) SHN?



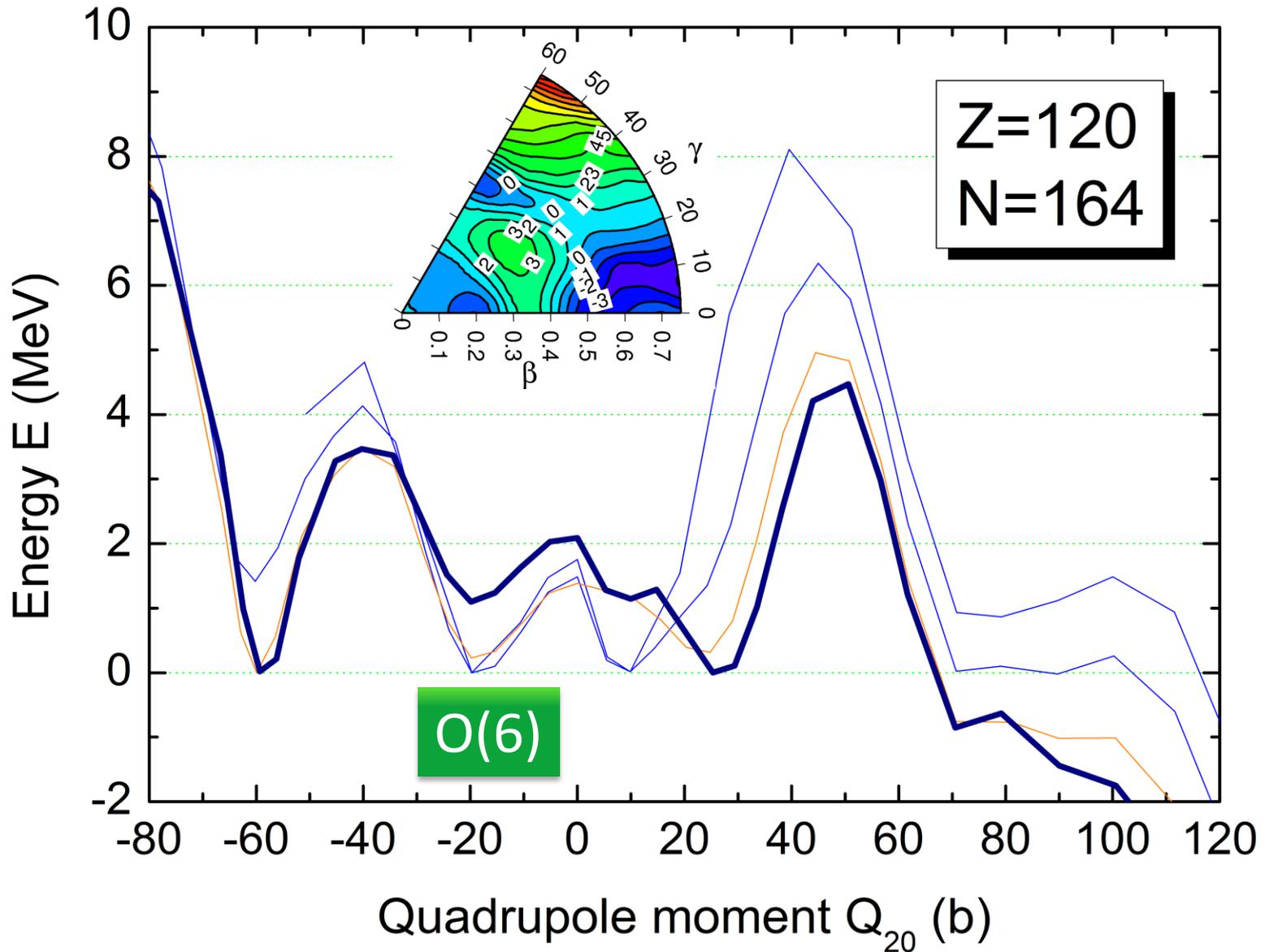


# Superdeformed oblate (SDO\*) SHN!

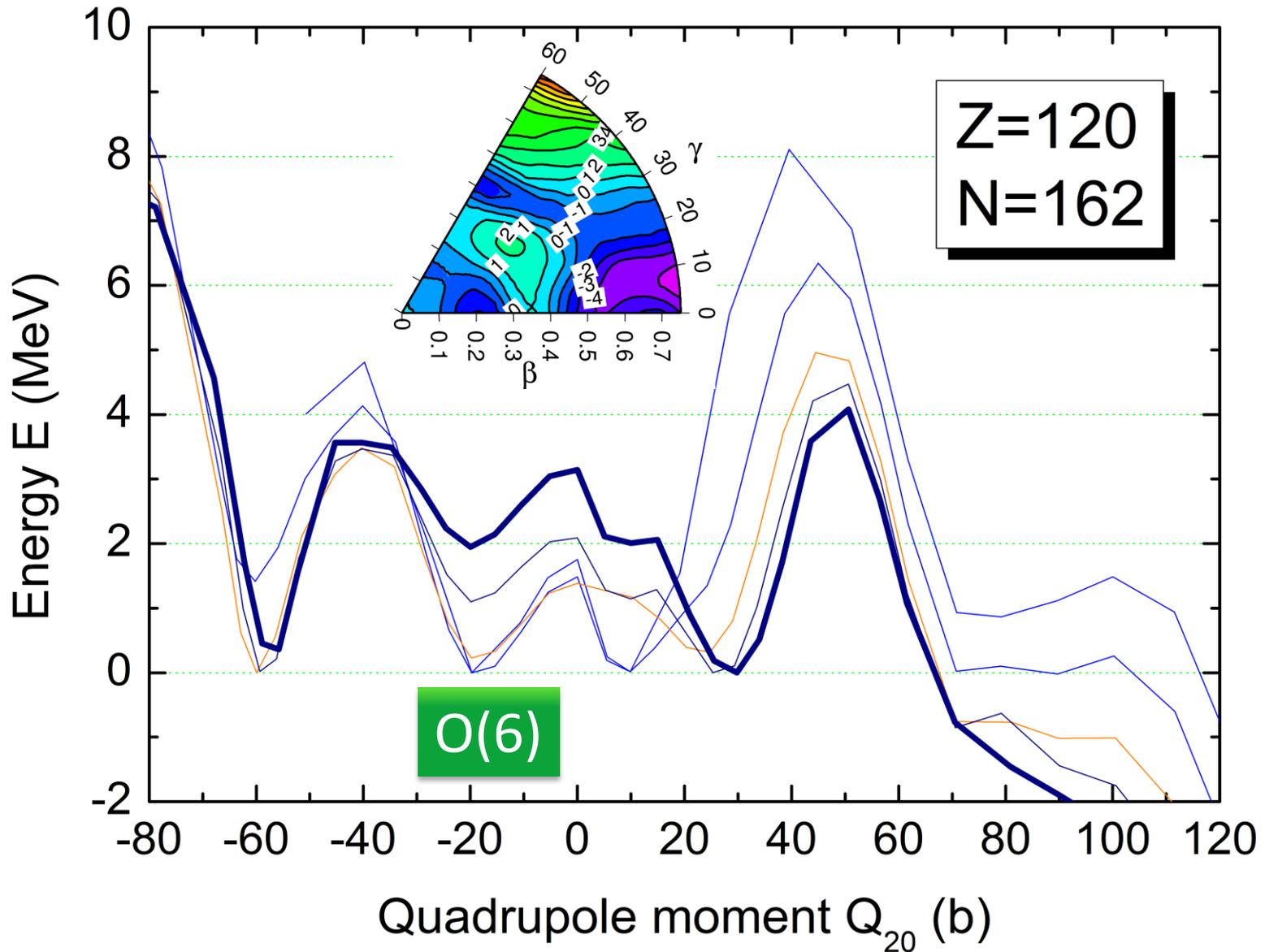


\*P. Jachimowicz, M. Kowal, J. Skalski, Phys. Rev. C **83**, 054302 (2011)  
L. Próchniak, A. S., ZAKOPANE 2012

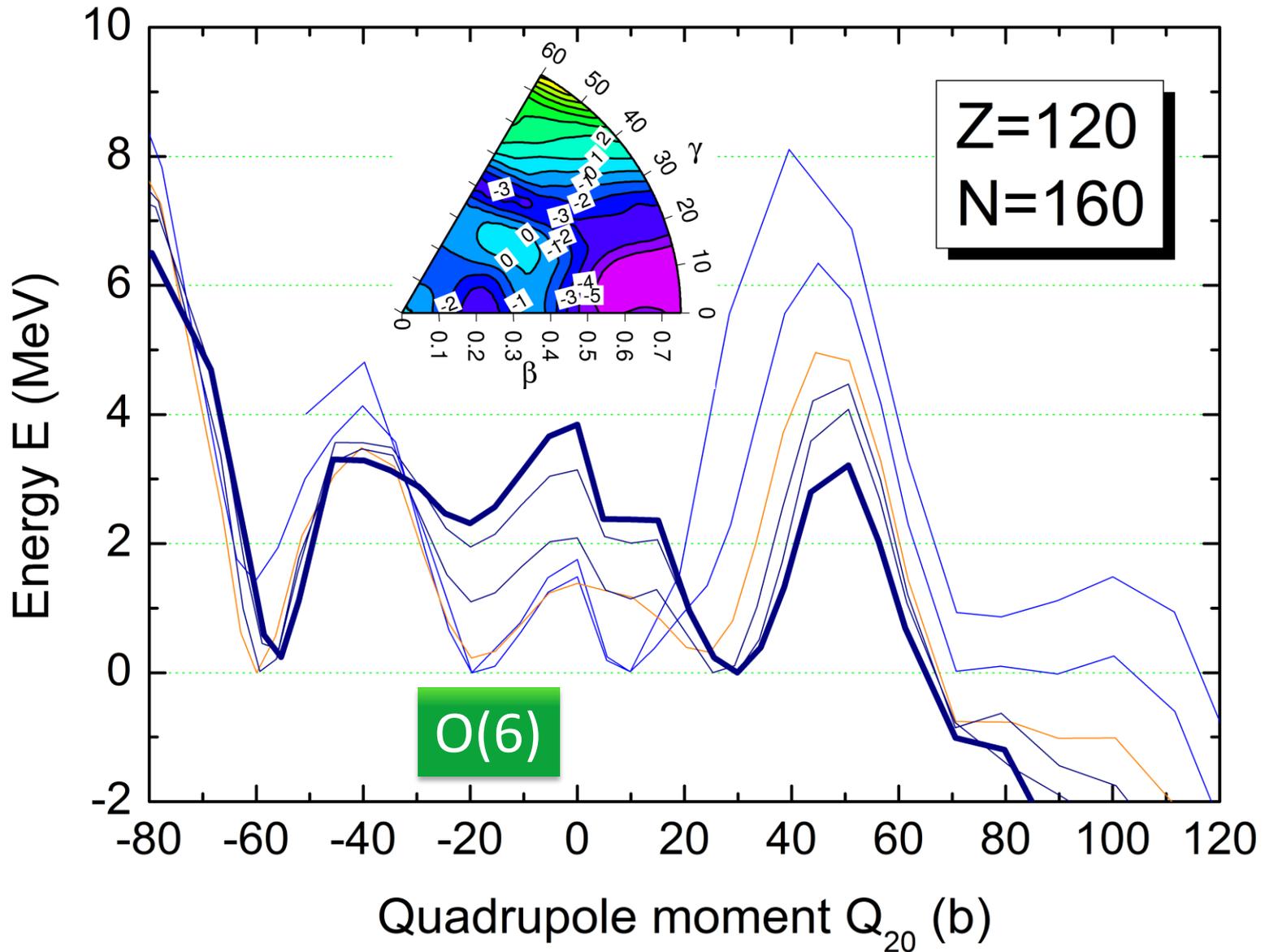
# Superdeformed oblate (SDO) SHN



# Superdeformed oblate (SDO) SHN

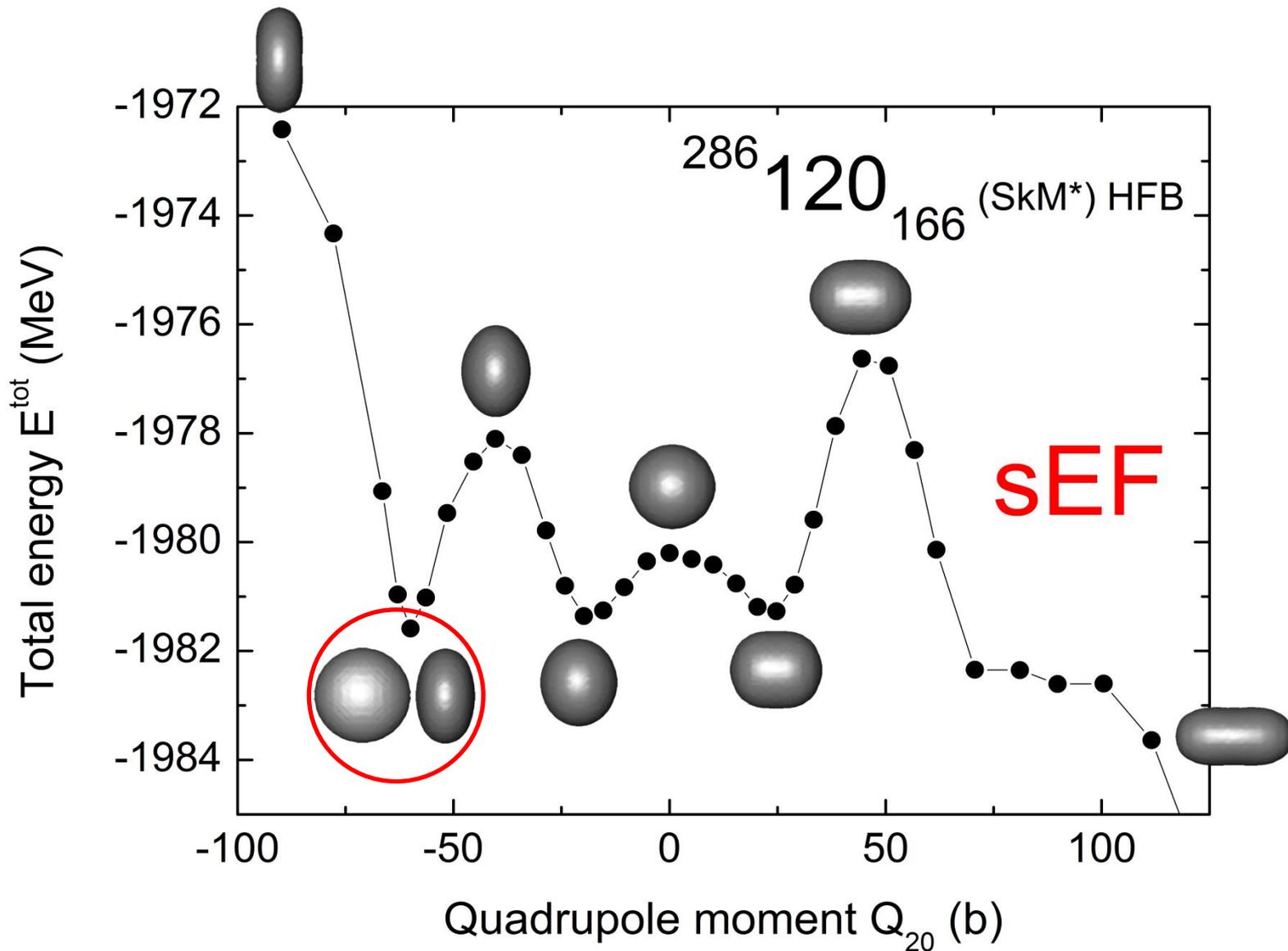


# Superdeformed oblate (SDO) SHN



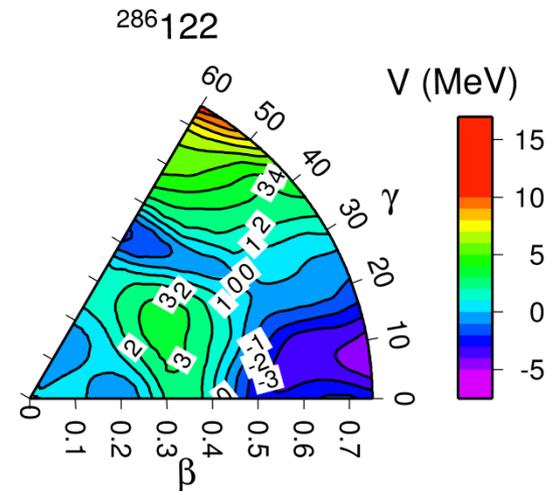
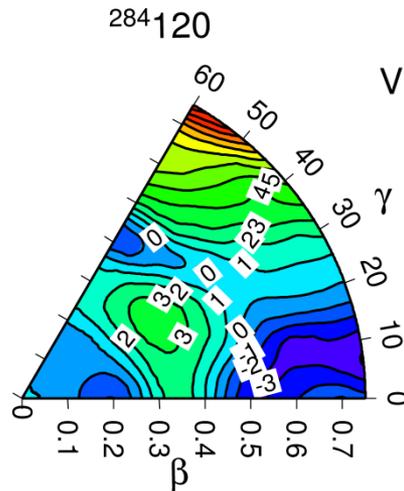
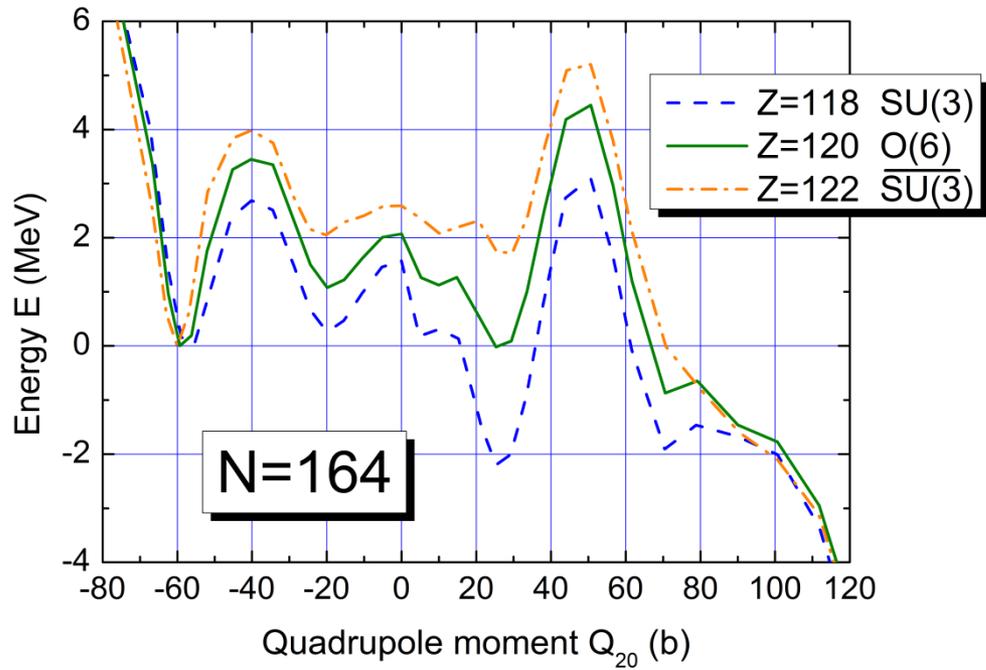


# Superdeformed oblate (SDO) SHN

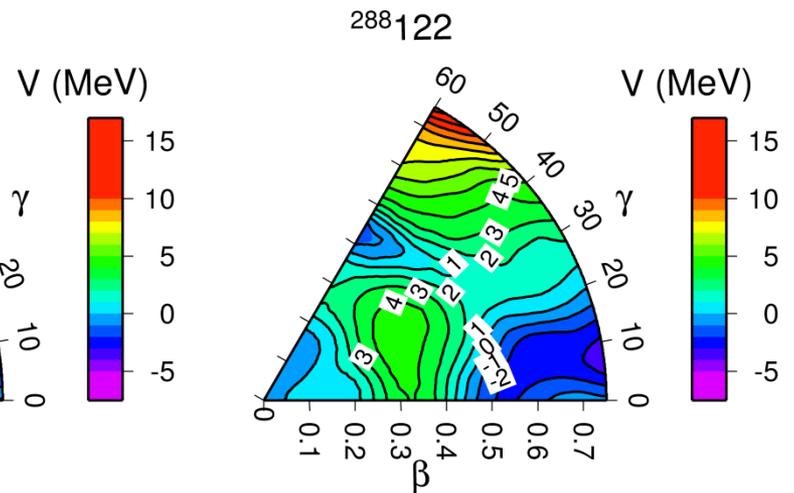
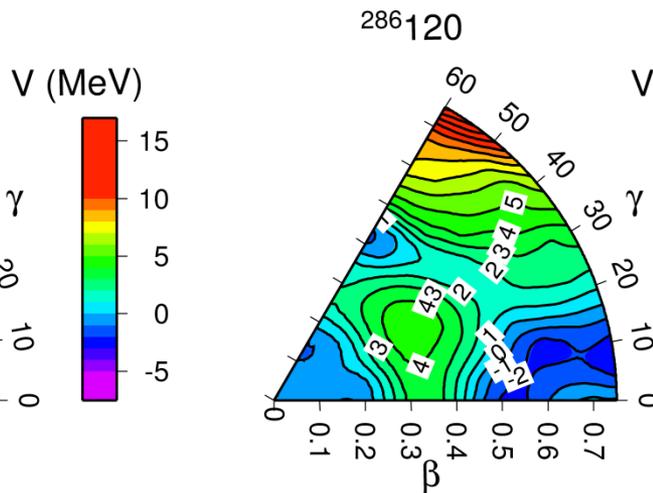
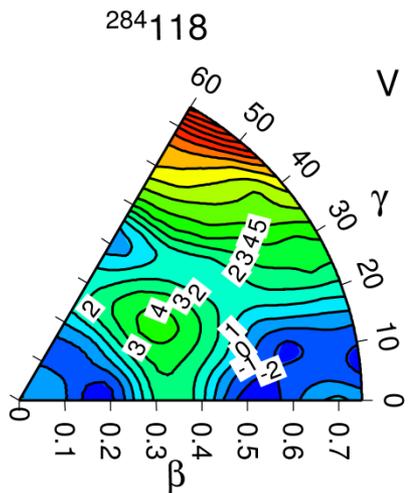
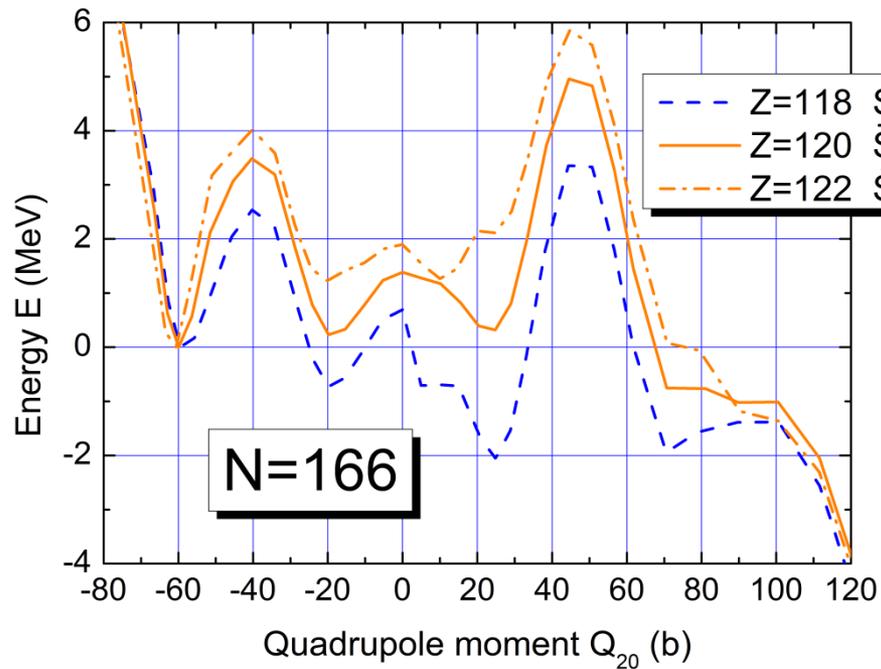


# Prolate-oblate phase transition SU(3) - O(6) - $\overline{\text{SU}}(3)$

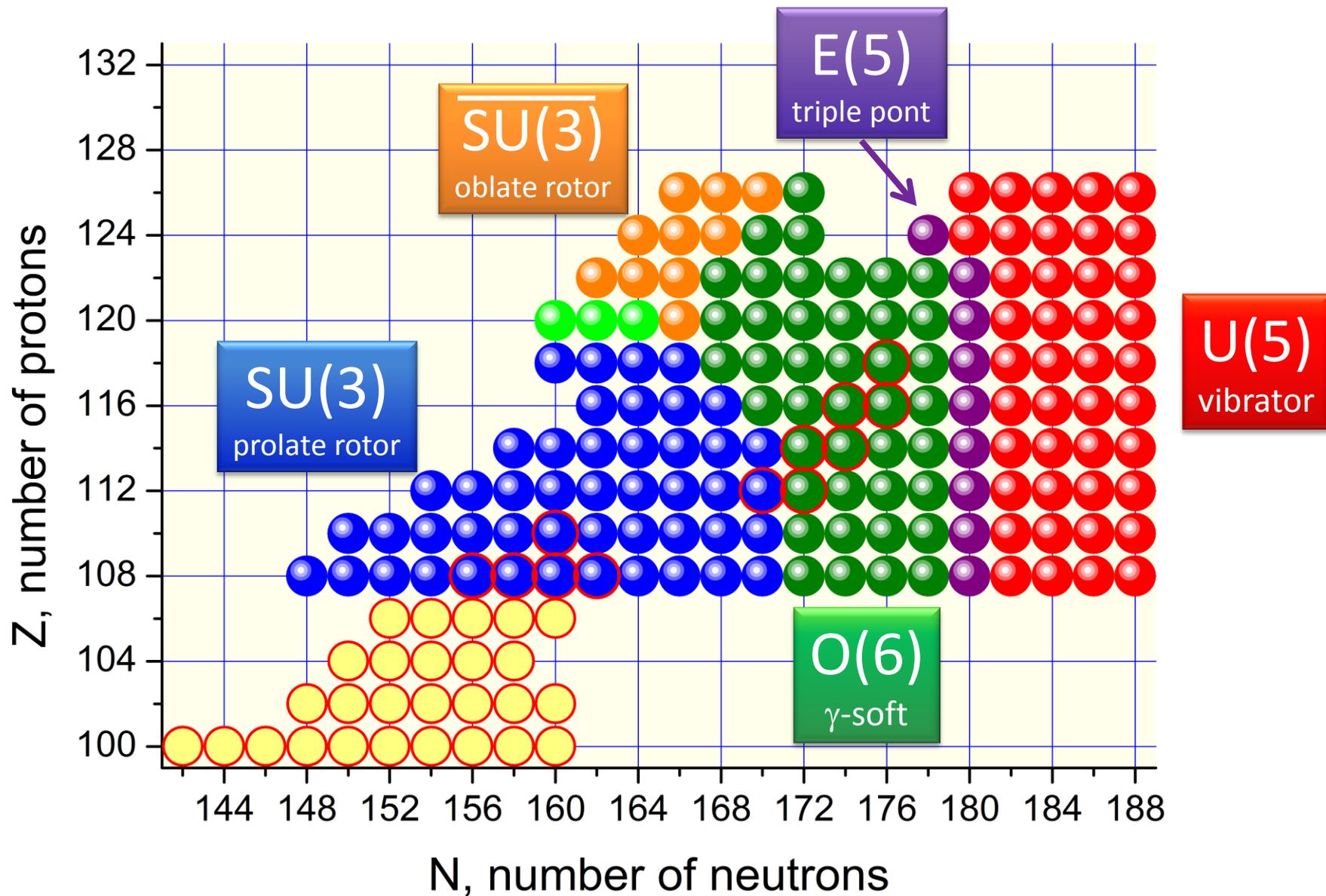
J. Jolie, R. Casten, P. von Brentano, V. Werner,  
PRL **87**, 162501 (2001).



# Prolate-oblate phase transition SU(3) - O(6) - $\overline{\text{SU}}(3)$



# *Natura non facit saltus*



# Conclusions

- ✓ The e-e SHN form three regions: **the prolate-deformed SU(3)** (for  $N < 172$ ), **spherical U(5)** (for  $N > 180$ ), and transitional region ( **$\gamma$ -soft**) **O(6)** between the former two.
- ✓ On the border between the O(6) and U(5) regions (for  $N = 180$ ) nuclei exhibit a rather flat potential bottom and acquire **the triple-point solutions - E(5)**.
- ✓ The existence of **superdeformed oblate (SDO) nuclei -  $\overline{\text{SU(3)}}$**  for  $N \leq 166$  and  $Z \geq 120$  was validated.
- ✓ The heaviest even-even nuclei produced by  $^{48}\text{Ca}$  induced reactions on actinide targets fall into the class of O(6)  $\gamma$ -soft nuclei.

Thank you!

## Ehrenfest classification, 1933:

The phase transition is of the  $k$ -th order if the  $k$ -th derivative of the thermodynamic free energy with respect to some thermodynamic variable changes discontinuously at the critical point.