19th Nuclear Physics Workshop "Marie & Pierre Curie" Kazimierz 2012

Energy density functional description of nuclear low-lying spectrum and shape transition

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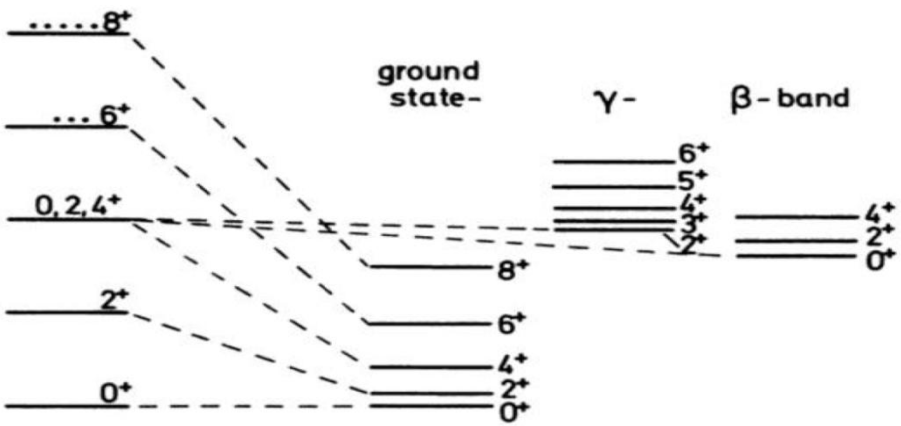
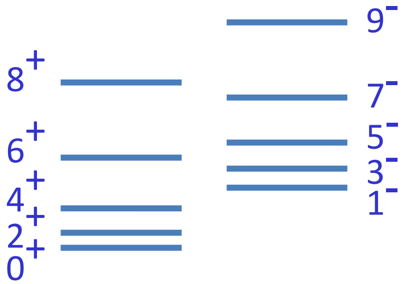
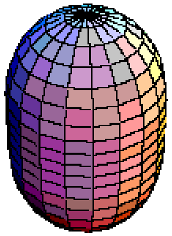
Outline

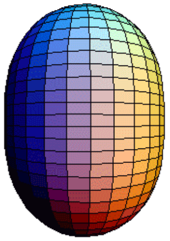
Nuclear Low-lying Spectrum

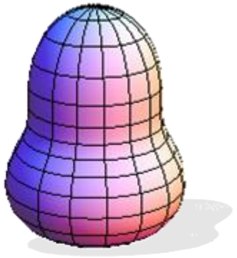
- **Nuclear low-lying spectroscopy can reveal rich structure information of atomic nuclei**
  - **Shape and shape transition**





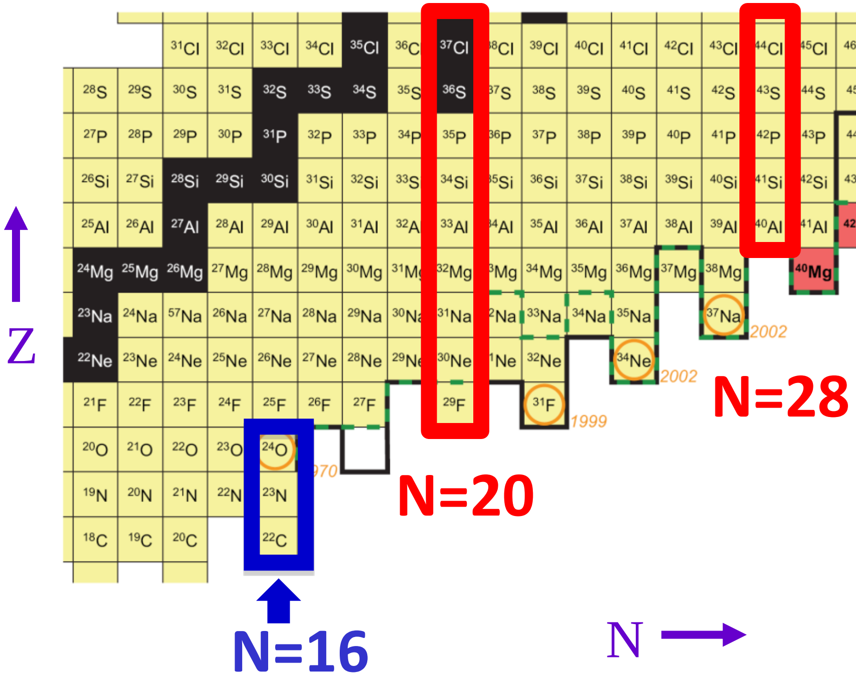




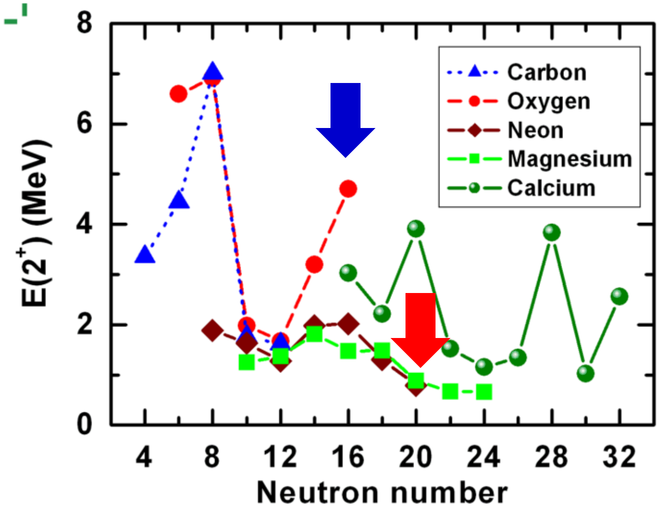


Nuclear Low-lying Spectrum

- **Nuclear low-lying spectroscopy can reveal rich structure information of atomic nuclei**
  - **Shape and shape transition**
  - **Evolution of the shell structure**



T. Baumann
Nature06213(2007)



Nuclear Low-lying Spectrum

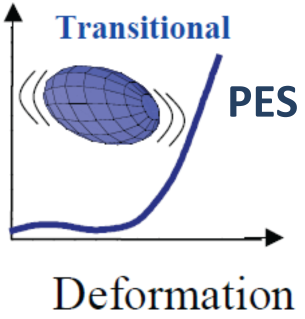
- **CEDF: nuclear structure over almost the whole nuclide chart**

Ring96, Vretenar2005, Meng2006

  - **Spin-orbit splitting**
  - **Scalar and vector fields: nuclear saturation properties**
  - **Origin of the pseudo-spin symmetry**
  - **Spin symmetry in anti-nucleon spectrum**
  - **……**

- **Spectrum: beyond the mean-field approximation**

  - **Restoration of broken symmetry, e.g. rotational**
  - **Mixing of different shape configurations**



AMP+GCM: Niksic2006, Yao2010

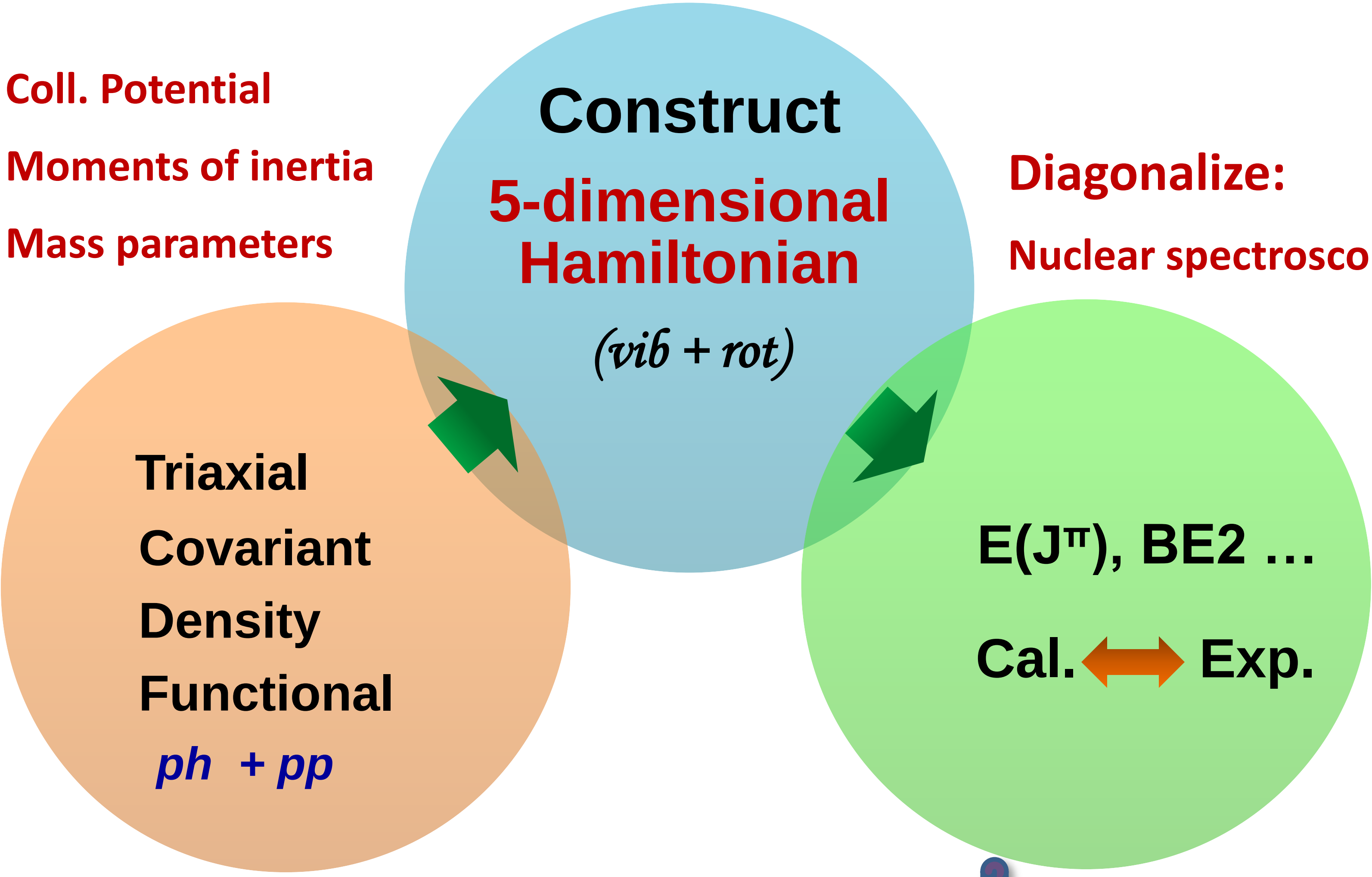
5D Collective Hamiltonian based on CEDF

Theoretical Framework

Coll. Potential

Moments of inertia

Mass parameters



Diagonalize:

Nuclear spectroscopy

Libert, Girod & Delaroche, PRC**60, 054301 (99)**
Prochniak & Rohozinski, JPG36, 123101 (09)
Niksic, Li, Vretenar, Prochniak, Meng & Ring, PRC**79, 034303 (09)**

Collective Hamiltonian

The five-dimensional Hamiltonian: [Bohr1952]

$$\hat{H}(\beta,\gamma,\Omega) = \hat{T}_{\mathrm{rot}}(\beta,\gamma,\Omega) + \hat{T}_{\mathrm{vib}}(\beta,\gamma) + V_{\mathrm{coll}}(\beta,\gamma). \quad (1)$$

- rotational kinetic energy:

$$\hat{T}_{\mathrm{rot}} = \frac{1}{2}\sum_{k=1}^{3} \hat{J}_k^2/\mathcal{I}_k, \quad (2)$$

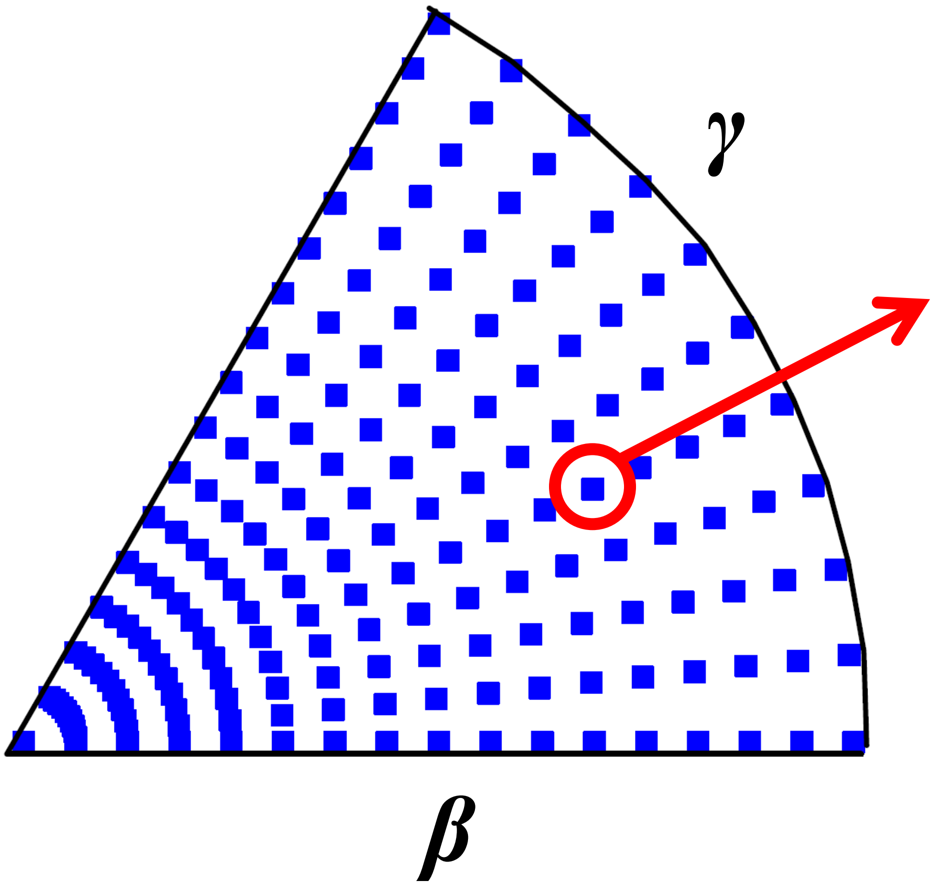
- vibrational kinetic energy:

$$\begin{aligned}\hat{T}_{\mathrm{vib}} \quad = \quad & -\frac{\hbar^2}{2\sqrt{wr}}\left\{\frac{1}{\beta^4}\left[\partial_\beta\left(\beta^4\sqrt{\frac{r}{w}}B_{\gamma\gamma}\partial_\beta\right) - \partial_\beta\left(\beta^3\sqrt{\frac{r}{w}}B_{\beta\gamma}\partial_\gamma\right)\right]\right. \\ & \left. + \frac{1}{\beta\sin 3\gamma}\left[-\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\partial_\beta\right) + \frac{1}{\beta}\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\partial_\gamma\right)\right]\right\}.\end{aligned} \quad (3)$$

- V_{coll} is the collective potential.

7 collective parameters (β,γ): $\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3$, $B_{\beta\beta}, B_{\beta\gamma}, B_{\gamma\gamma}$, and V_{coll}

Collective Hamiltonian



$\mathcal{I}_1(\beta,\gamma)$ $\mathcal{I}_2(\beta,\gamma)$ $\mathcal{I}_3(\beta,\gamma)$

$B_{\beta\beta}(\beta,\gamma)$ $B_{\beta\gamma}(\beta,\gamma)$ $B_{\gamma\gamma}(\beta,\gamma)$

$V_{\text{coll}}(\beta,\gamma)$

How to determine these 7 collective parameters in a microscopic way?

Covariant Energy Density Functional (CEDF)

The CEDF (*contact interaction* or *Point-Coupling*):

Burvenich02, Niksic08, Zhao10

$$
\begin{aligned}
E_{\mathrm{RMF}} &= \int d\boldsymbol{r}\; \mathcal{E}_{\mathrm{RMF}}(\boldsymbol{r}) \\
&= \sum_k \int d\boldsymbol{r}\; v_k^2\, \bar{\psi}_k(\boldsymbol{r}) \left(-i\boldsymbol{\gamma}\boldsymbol{\nabla} + m\right) \psi_k(\boldsymbol{r}) \\
&+ \int d\boldsymbol{r} \left(\frac{\alpha_S}{2}\rho_S^2 + \frac{\beta_S}{3}\rho_S^3 + \frac{\gamma_S}{4}\rho_S^4 + \frac{\delta_S}{2}\rho_S \triangle \rho_S + \frac{\alpha_V}{2} j_\mu j^\mu + \frac{\gamma_V}{4}(j_\mu j^\mu)^2 + \frac{\delta_V}{2} j_\mu \triangle j^\mu \right. \\
&+ \left. \frac{\alpha_{TV}}{2} j_{TV}^\mu (j_{TV})_\mu + \frac{\delta_{TV}}{2} j_{TV}^\mu \triangle (j_{TV})_\mu + \frac{\alpha_{TS}}{2}\rho_{TS}^2 + \frac{\delta_{TS}}{2}\rho_{TS}\triangle\rho_{TS} + \frac{e}{2}\rho_p A^0 \right),
\end{aligned}
$$

Collective parameters: Cranking approximation

$$
\mathcal{I}_k = \sum_{i,j} \frac{(u_i v_j - v_i u_j)^2}{E_i + E_j} |\langle i|\hat{J}_i|j\rangle|^2
$$

$$
V_{\mathrm{coll}} = E_{\mathrm{tot}} - \Delta V_{\mathrm{vib}} - \Delta V_{\mathrm{rot}},
$$

$$
B_{\mu\nu} = \frac{1}{2}\left[\mathcal{M}_{(1)}^{-1}\mathcal{M}_{(3)}\mathcal{M}_{(1)}^{-1}\right]_{\mu\nu},
$$

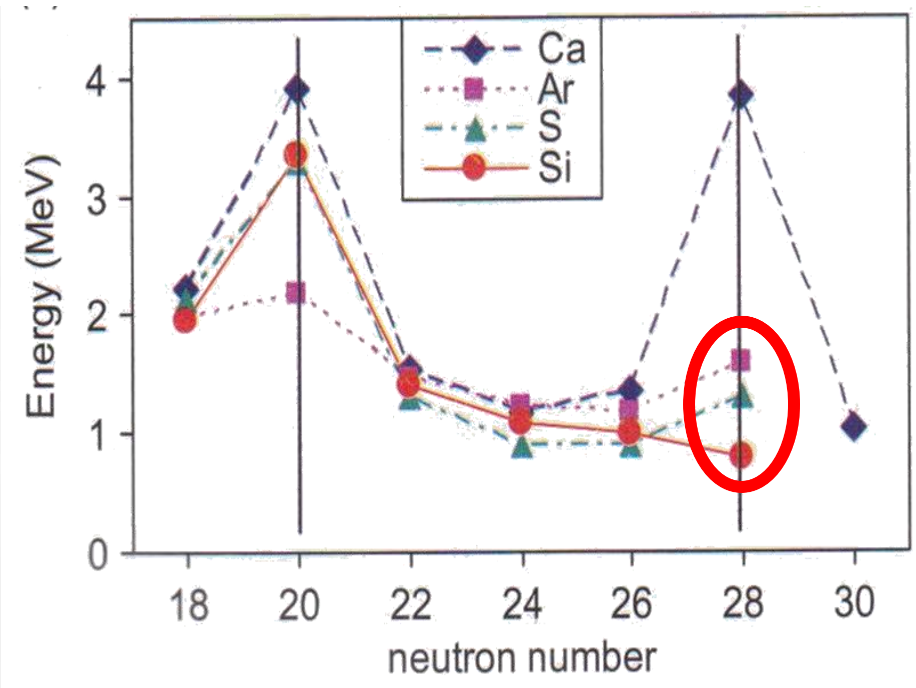
$$
\mathcal{M}_{(n),\mu\nu} = \sum_{i,j} \frac{(u_i v_j + v_i u_j)^2}{(E_i + E_j)^n} \langle i|\hat{Q}_{2\mu}|j\rangle\langle j|\hat{Q}_{2\nu}|i\rangle
$$

Results and discussion

- **Rapid shape evolution in *N*=28 isotones**
- **Enhanced collectivity in Sn isotopes**
- **Effect of time-odd mean field:**

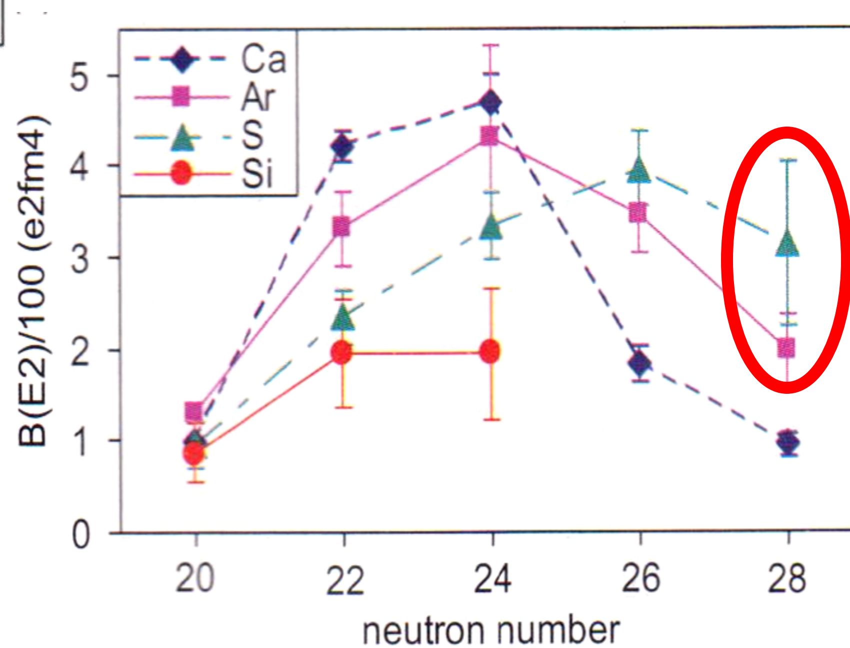
Expansion method for TV inertia parameter

Shape evolution & coexistence in N=28 isotones



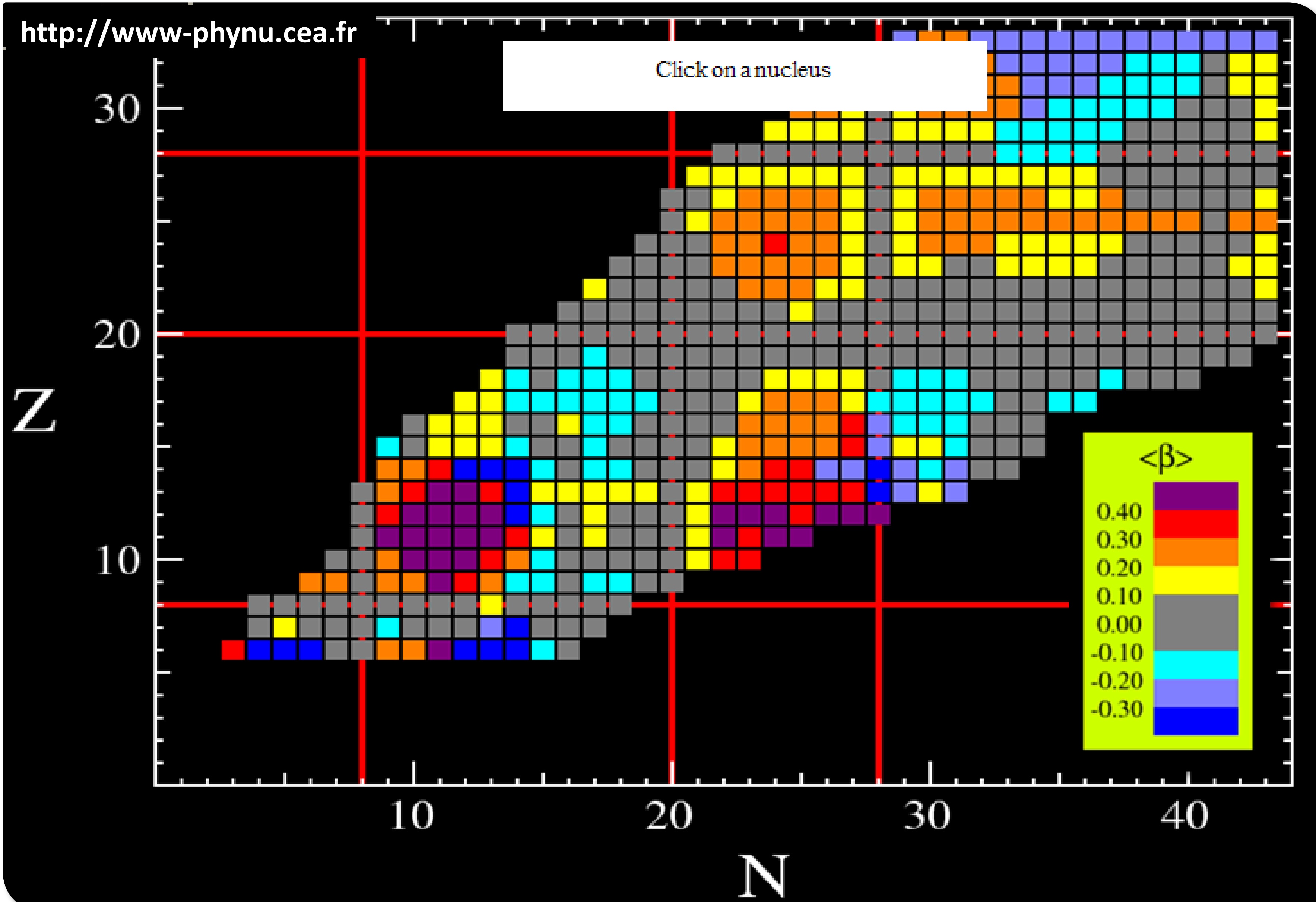
$E(2_1^+)$

B(E2)



http://www.nndc.bnl.gov/

Shape evolution & coexistence in N=28 isotones



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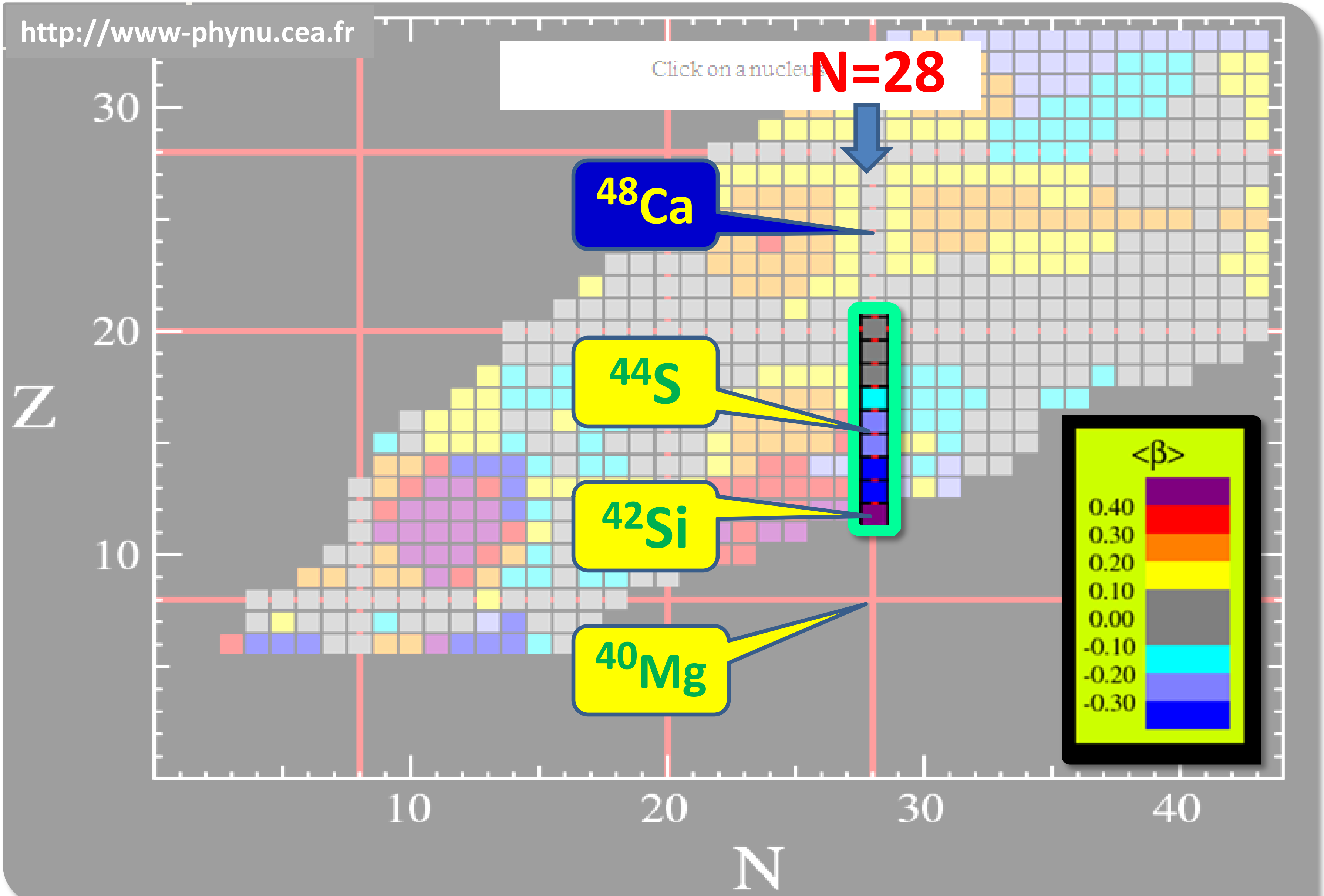

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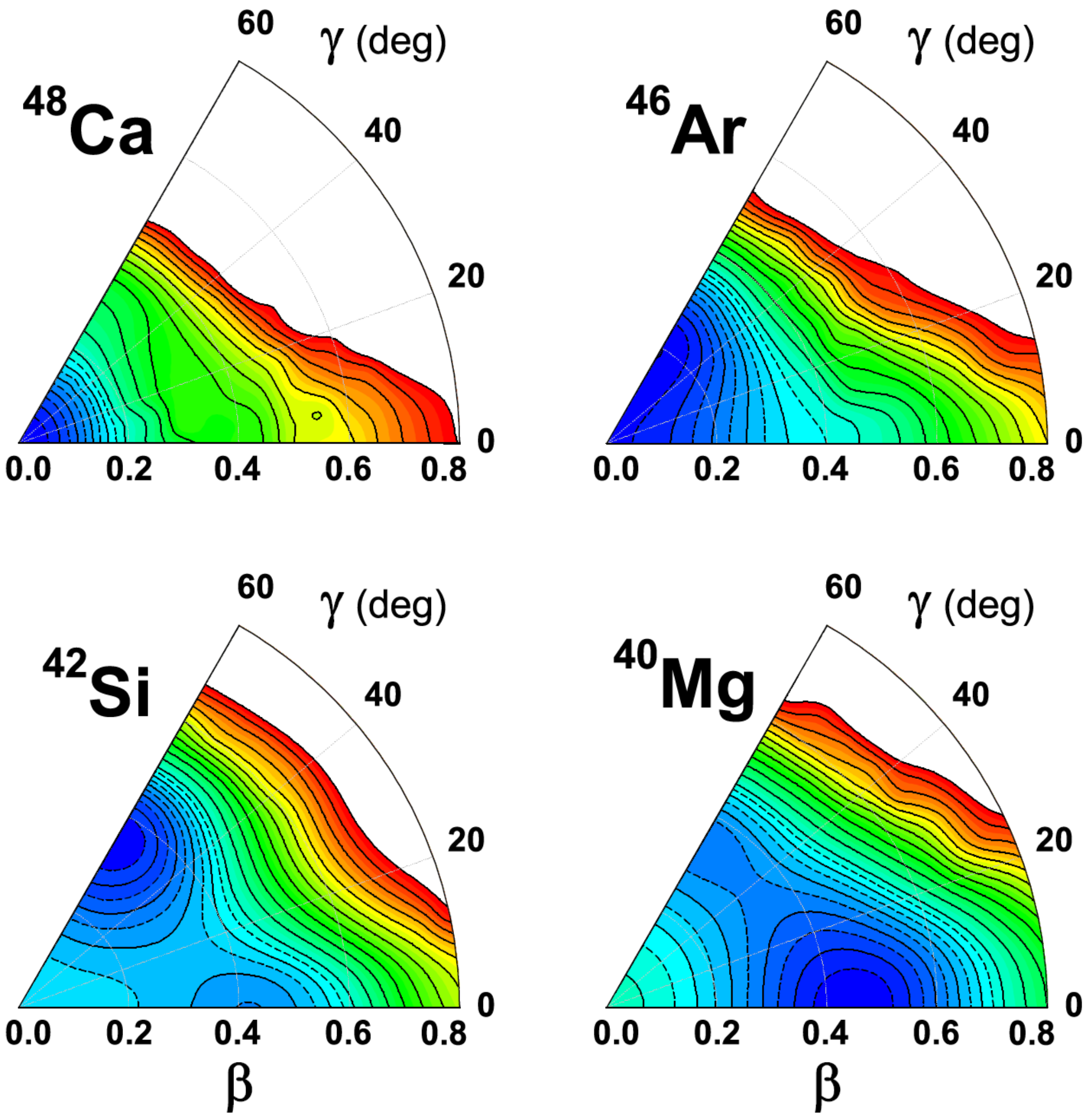
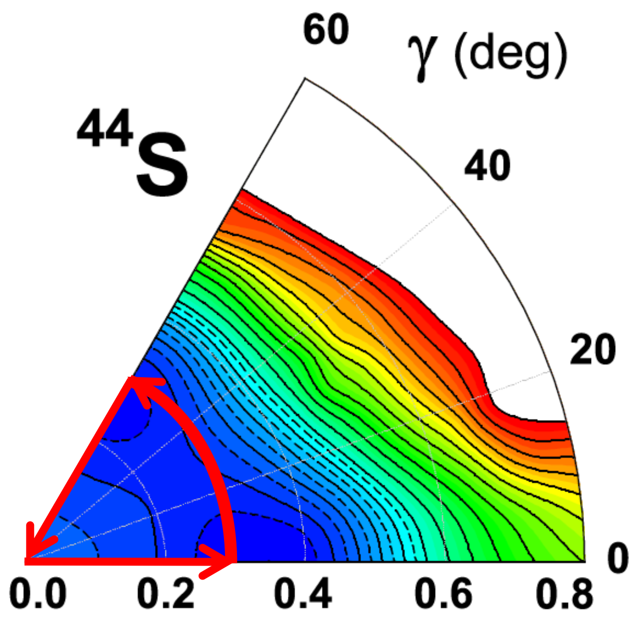
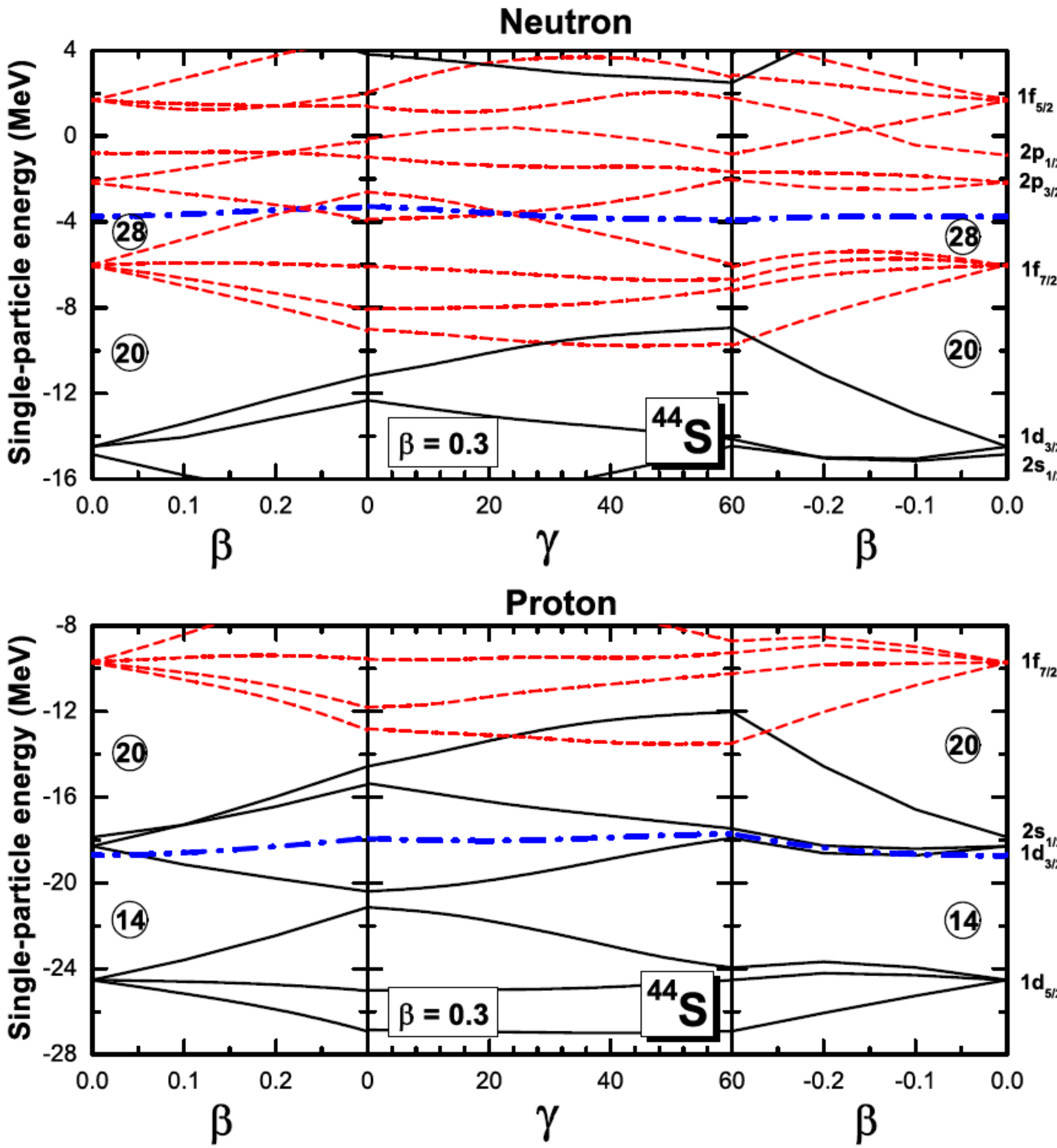


	$\Delta^{sph.}_{N=28}$	β_{min}
^{48}Ca	4.73	0.00
^{46}Ar	4.48	-0.19
^{44}S	3.86	0.34
^{42}Si	3.13	-0.35
^{40}Mg	2.03	0.45

Shape evolution & coexistence in *N*=28 isotones

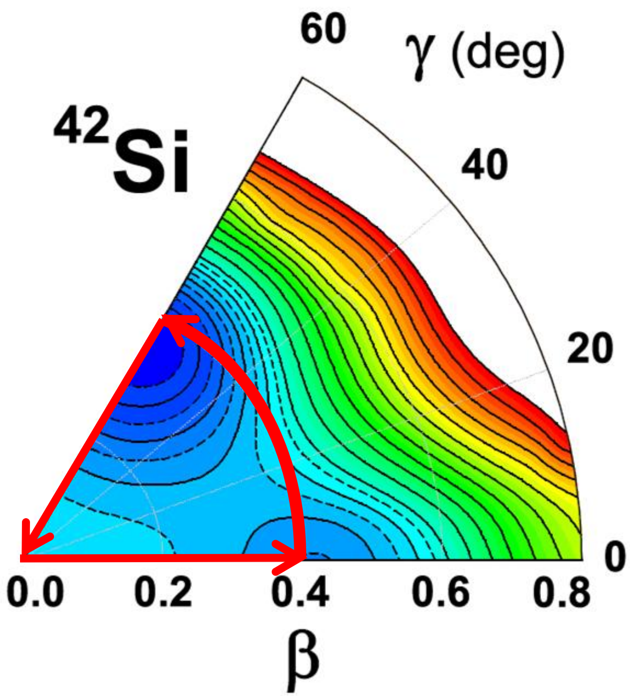


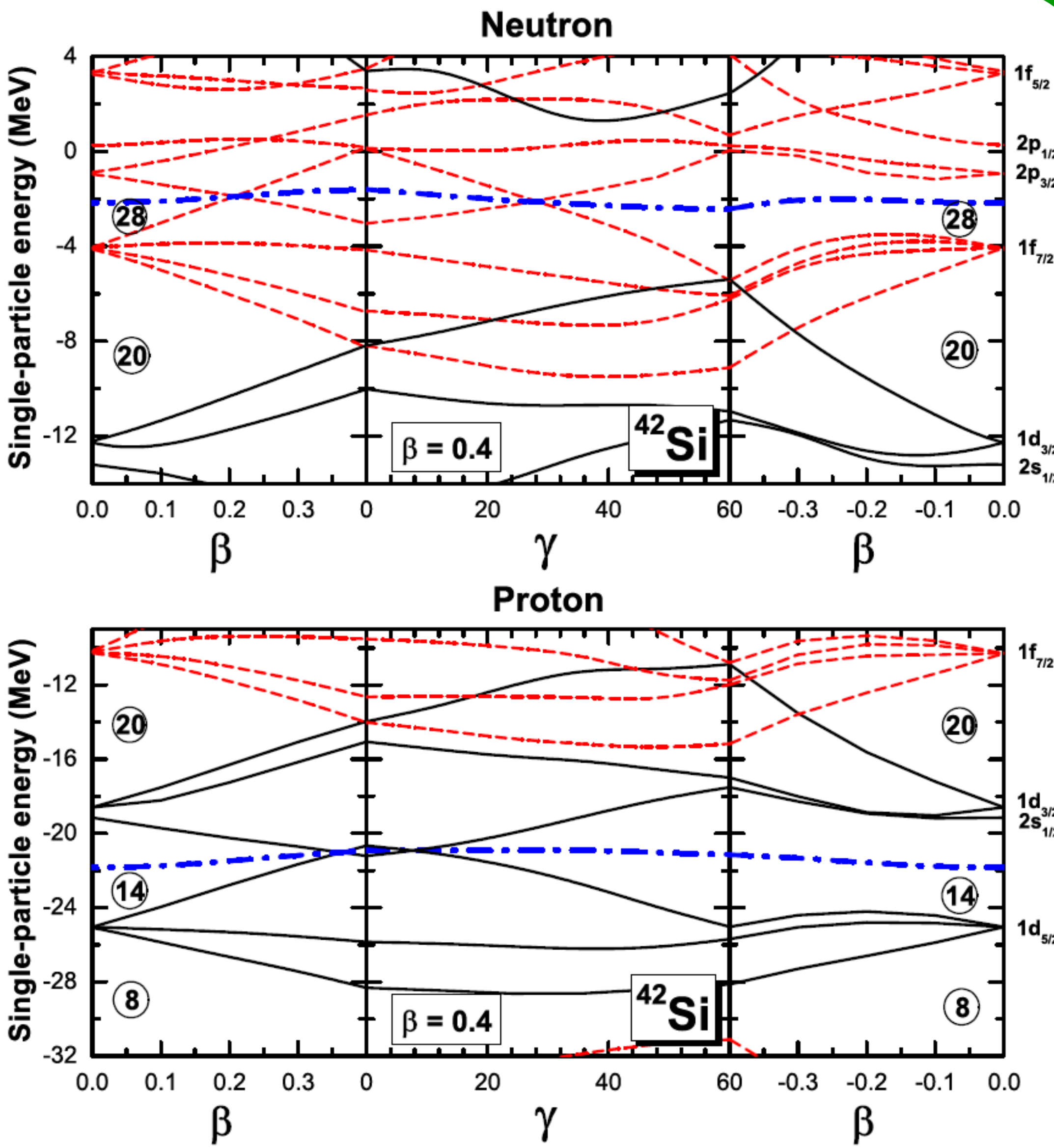


Oblate: **Δn = 3.91 MeV**

Prolate: **Δp = 5.05 MeV**

Shape evolution & coexistence in N=28 isotones

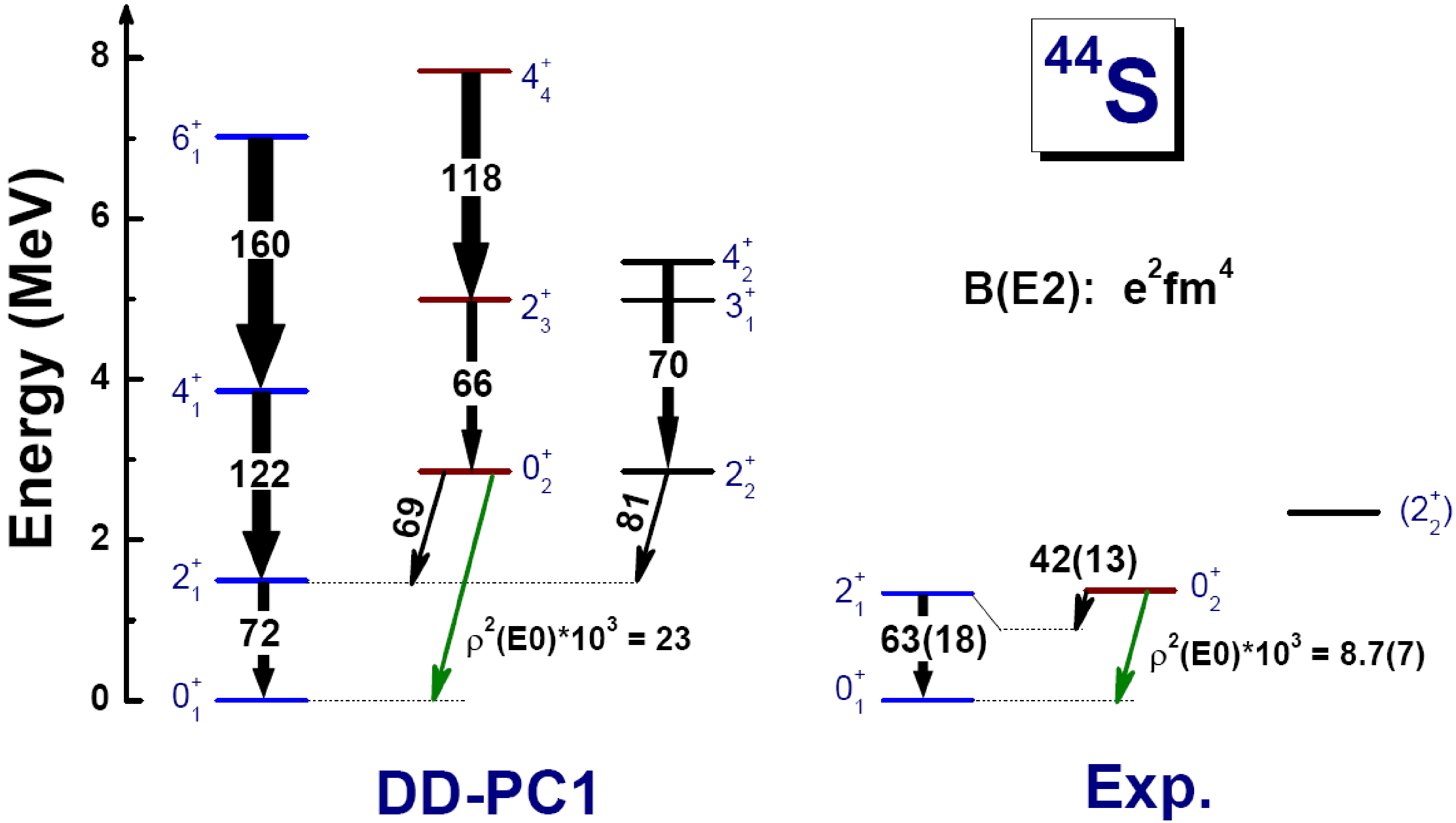




Shape evolution & coexistence in *N*=28 isotones

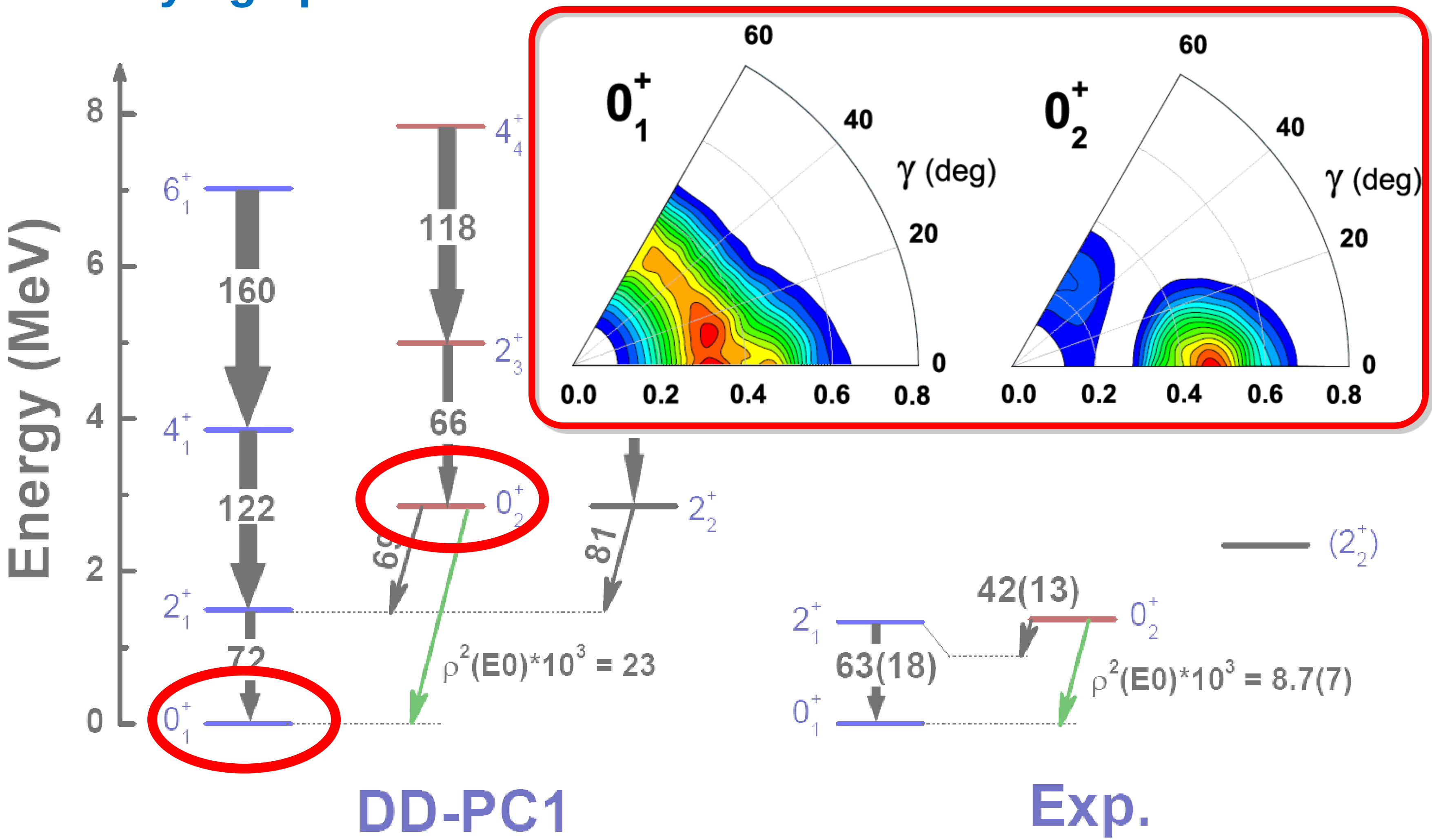
➢ **Low-lying spectrum**

C. Force et al., PRL105



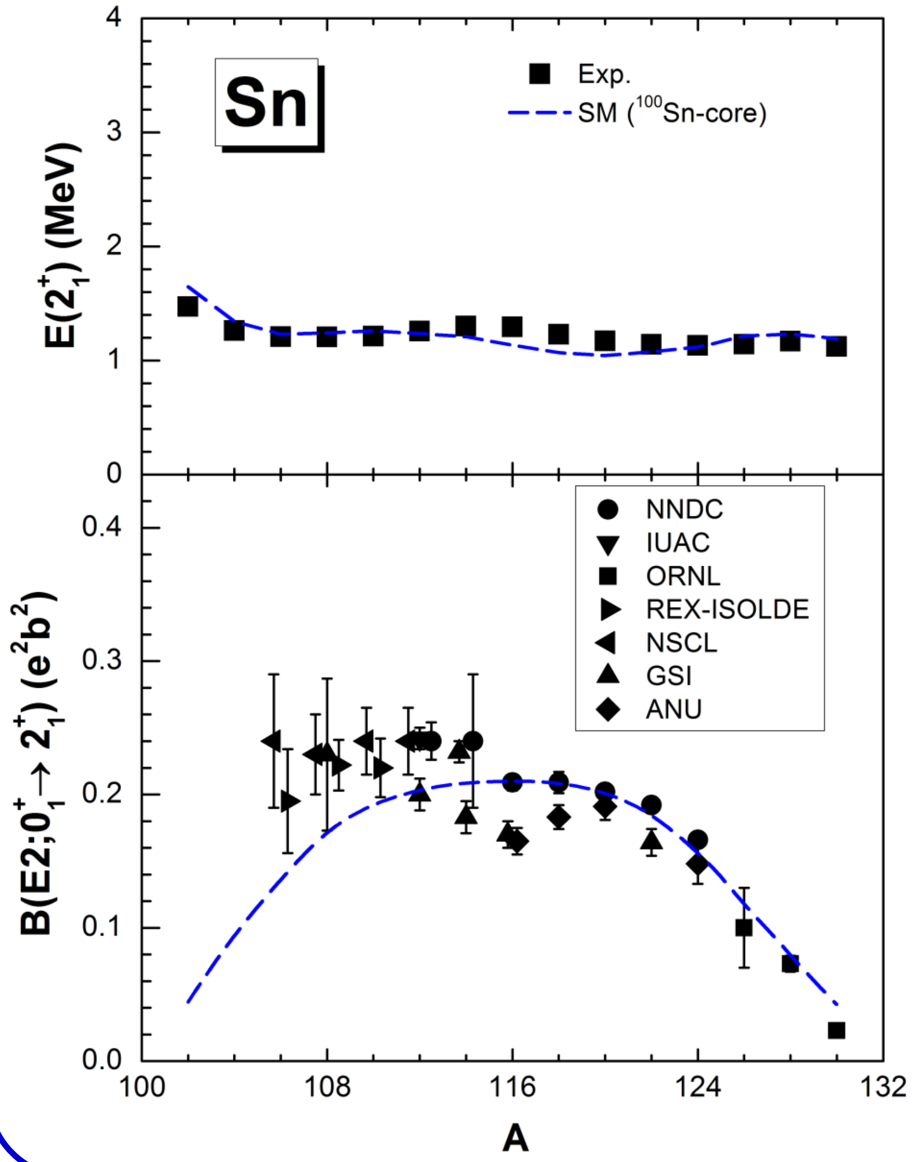
Shape evolution & coexistence in *N*=28 isotones

➢ Low-lying spectrum



Li, Yao, Vretenar, Niksic, Chen & Meng, PRC84, 054304 (2011)

Enhanced collectivity in neutron-deficient Sn isotope



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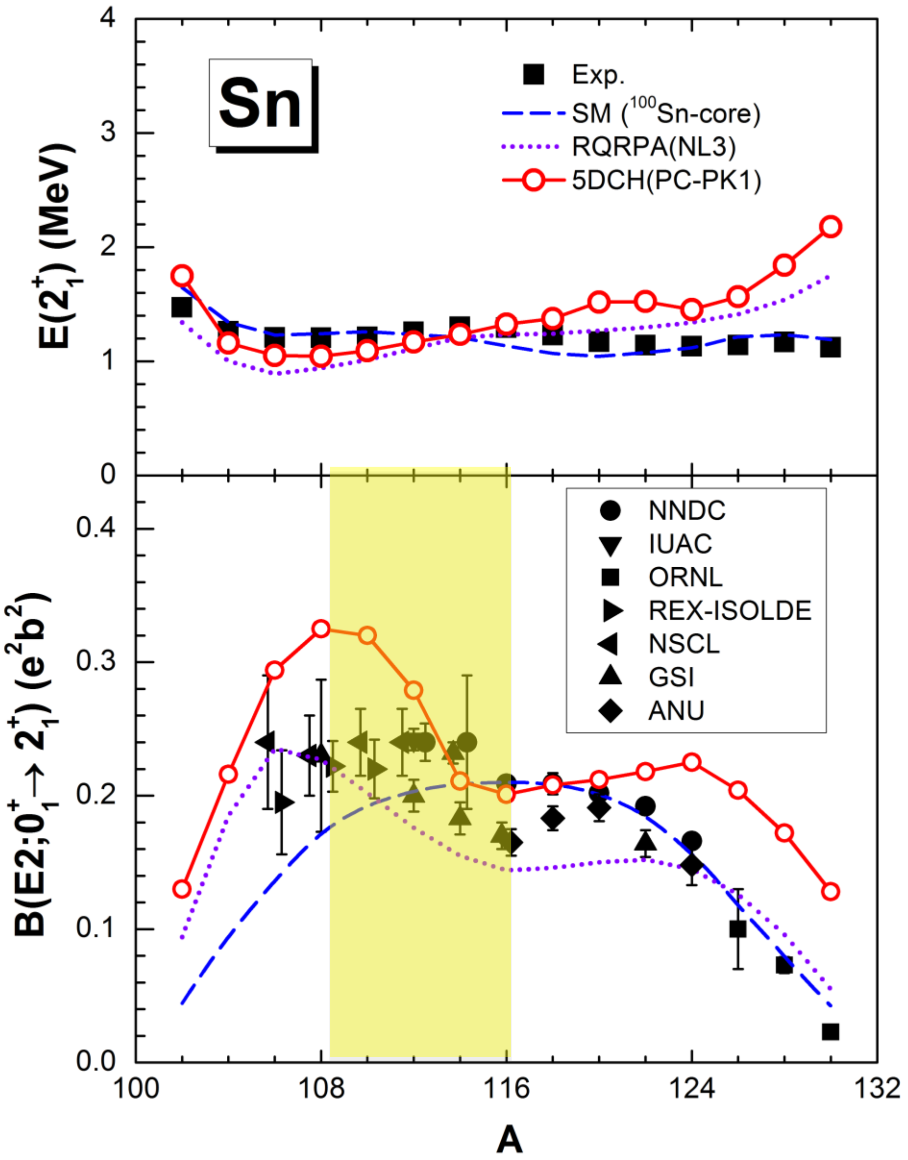
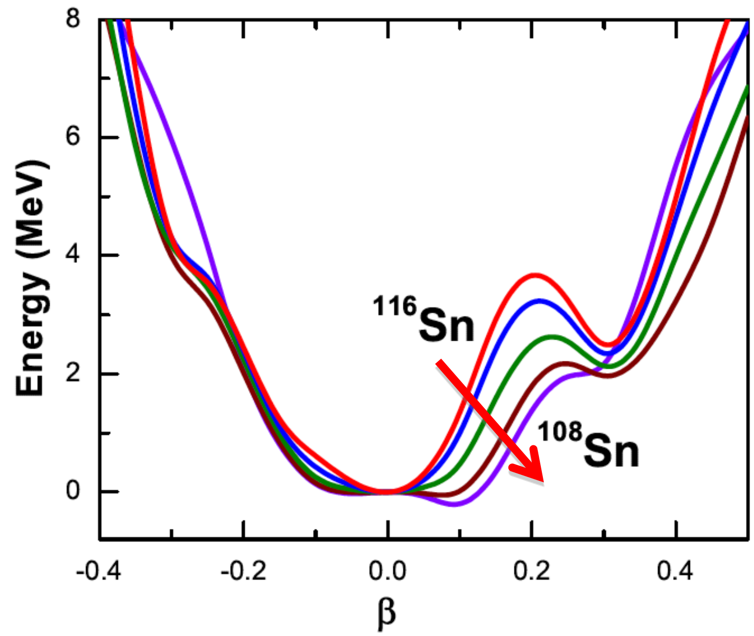
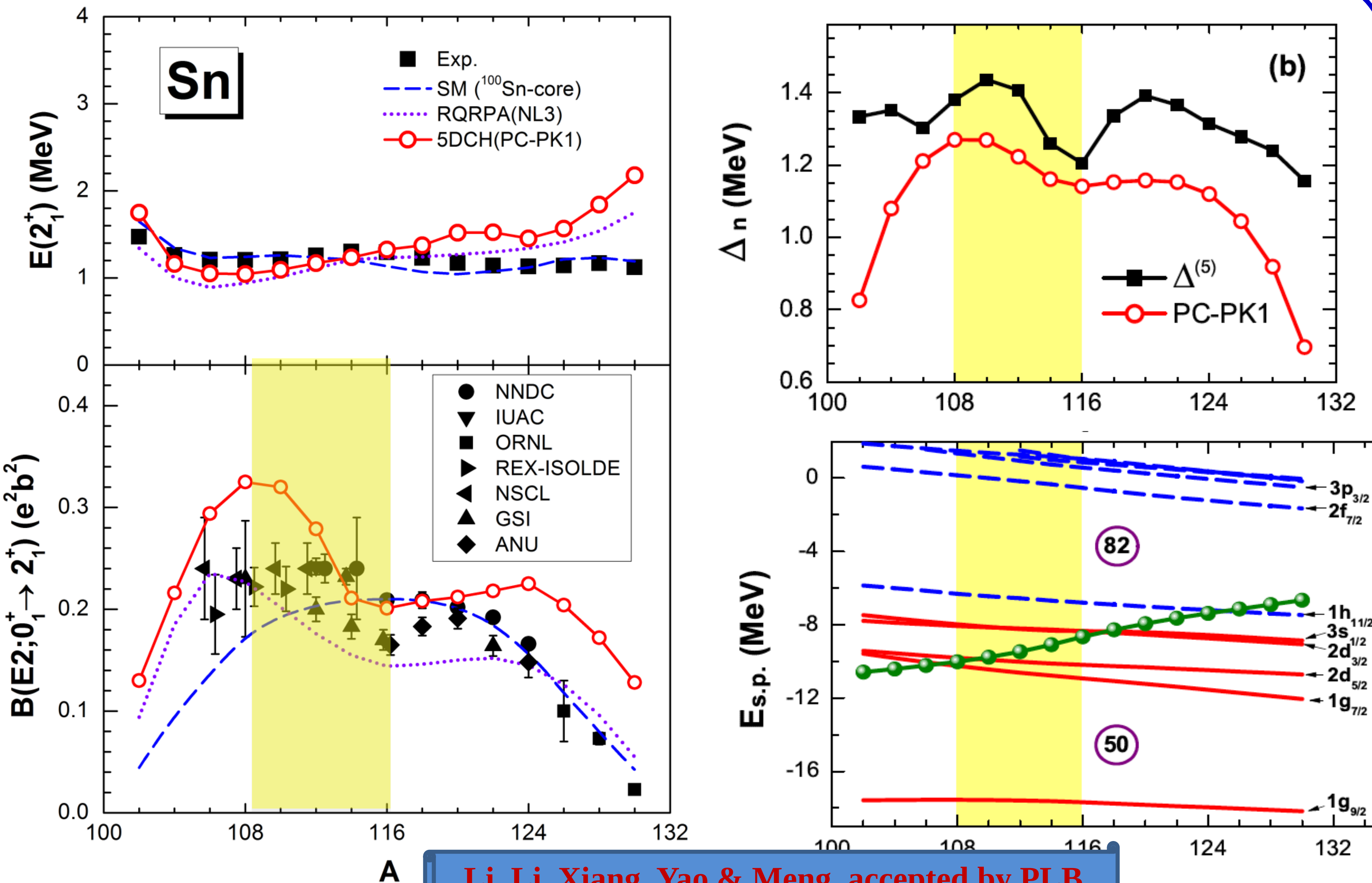




Enhanced collectivity in neutron-deficient Sn isotope



Li, Li, Xiang, Yao & Meng, accepted by PLB

Effect of time-odd mean-field

➢ Expansion method for TV inertia parameter

$$M_{\mu\mu'}(\mathbf{q}) = \begin{pmatrix} P^* & -P \end{pmatrix}_\mu \begin{pmatrix} A & -B \\ -B^* & A^* \end{pmatrix}^{-1} \begin{pmatrix} P \\ -P^* \end{pmatrix}_{\mu'}$$

P. Ring and P. Schuck, *"The Nuclear Many-Body Problem"*

$$A_{php'h'} = (\epsilon_p - \epsilon_h)\delta_{pp'}\delta_{hh'} + V_{phh'p'}, \quad B_{php'h'} = V_{php'h'}.$$

$$\mathcal{M}^{-1} = \begin{pmatrix} A & -B \\ -B^* & A^* \end{pmatrix} = \mathcal{M}_0^{-1} + \mathcal{V}$$

$$V_{adbc} = -\alpha_V \int [\psi_a^\dagger \boldsymbol{\alpha} \psi_b][\psi_d^\dagger \boldsymbol{\alpha} \psi_c] d^3 r$$

Effect of time-odd mean-field

- Expansion method for TV inertia parameter

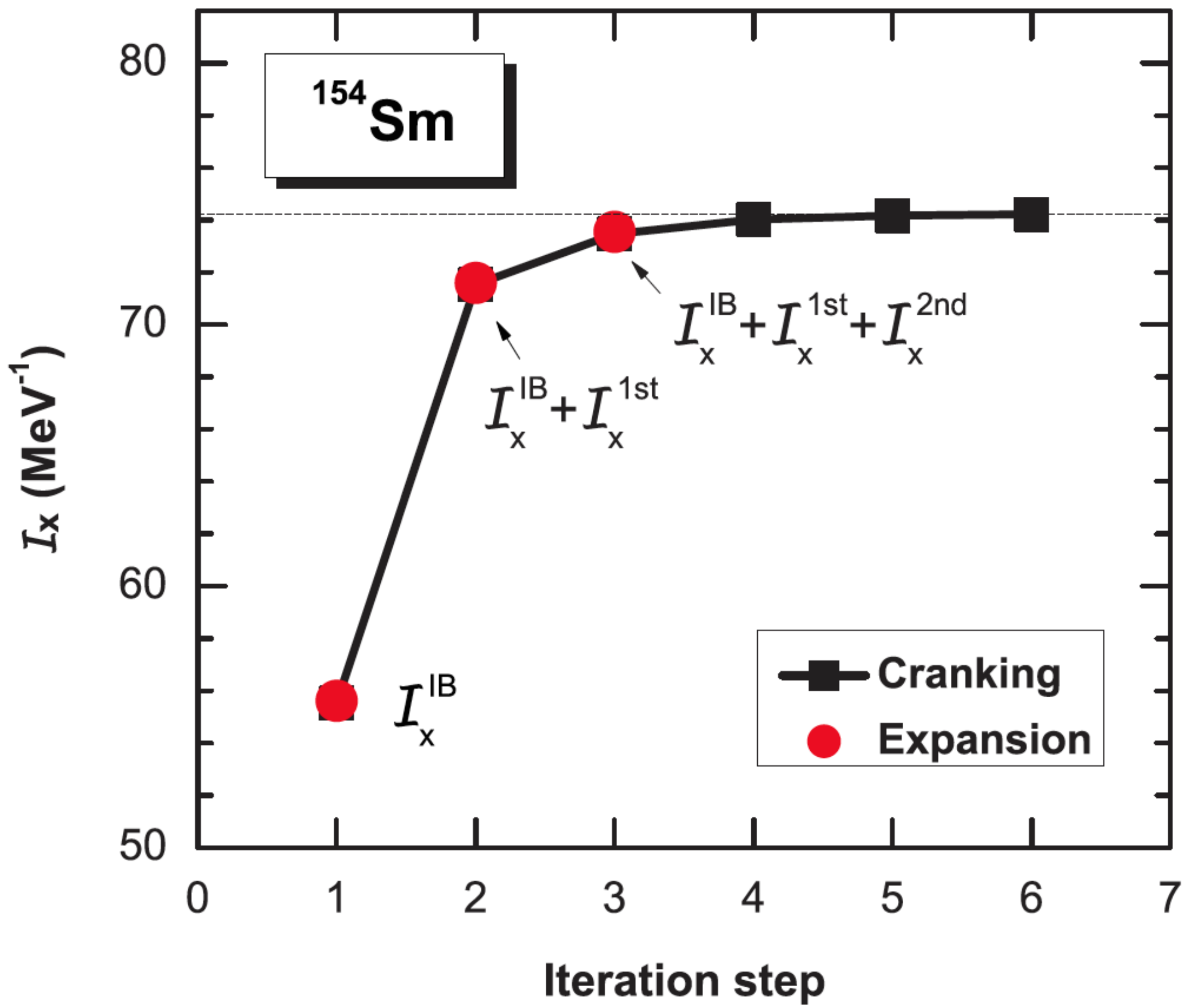
$$\mathcal{M} = [\mathcal{M}_0^{-1} + \mathcal{V}]^{-1} = \mathcal{M}_0[\mathbb{1} + \mathcal{V}\mathcal{M}_0]^{-1}.$$

$$\mathcal{M} = \mathcal{M}_0 - \mathcal{M}_0\mathcal{V}\mathcal{M}_0 + \mathcal{M}_0\mathcal{V}\mathcal{M}_0\mathcal{V}\mathcal{M}_0 + \cdots.$$

$$M^0_{\mu\mu'}(\mathbf{q}) = \frac{1}{\hbar^2}(P^* \quad -P)_\mu \mathcal{M}_0 \begin{pmatrix} P \\ -P^* \end{pmatrix}_{\mu'} = \frac{2}{\hbar^2}\sum_{ph}\frac{|\hat{P}_{ph}|^2}{\epsilon_p - \epsilon_h}.$$

Effect of time-odd mean-field

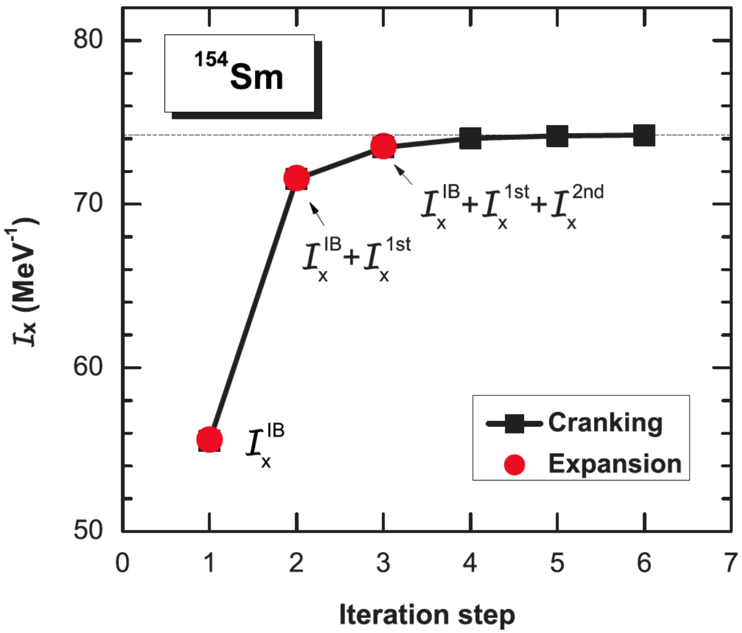
➢ Numerical check

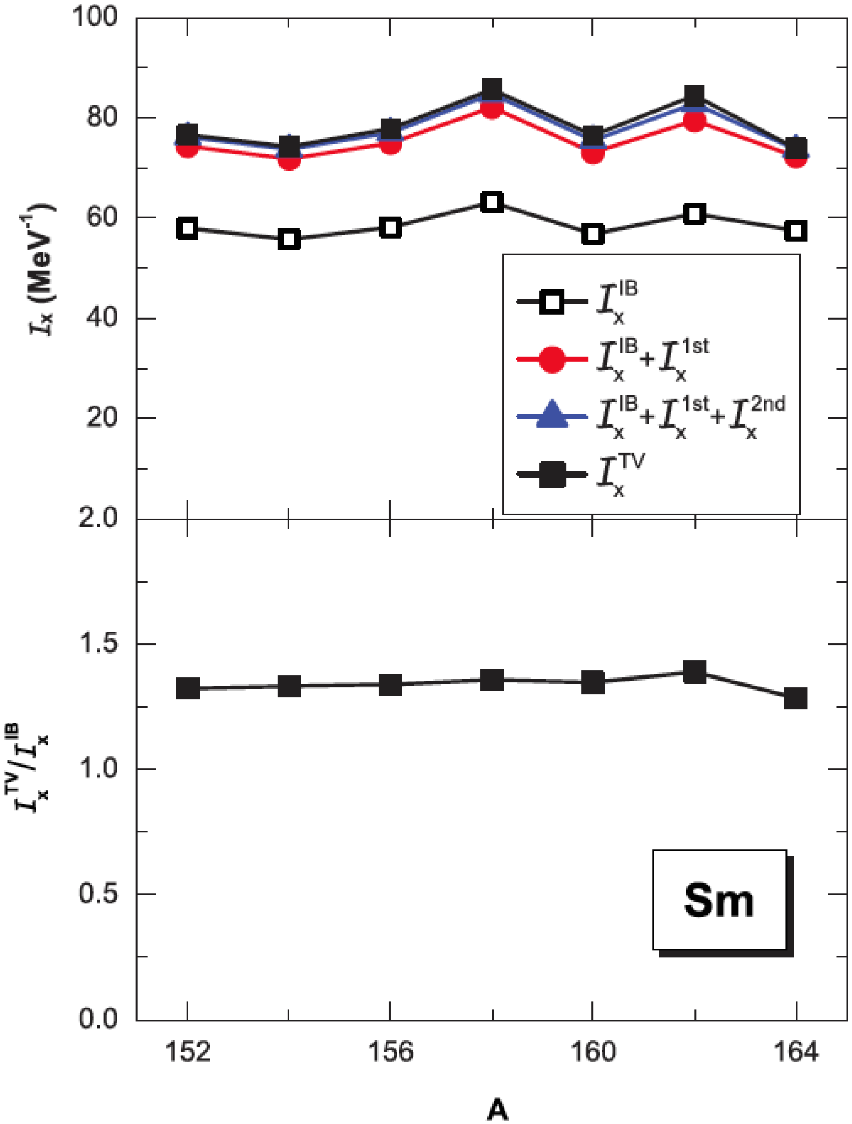


Li, Niksic, Ring, Vretenar, Yao & Meng, PRC86, 034334 (2012)

Effect of time-odd mean-field

- **The ATDHF mass tensor**





The ratio ranges ~1.3- ~1.4

Li, Niksic, Ring, Vretenar, Yao & Meng, PRC86, 034334 (2012)

Summary

- Microscopic Collective Hamiltonian based on CEDF has been developed
- Application to the interesting topics
  - Rapid shape evolution in N=28 isotones
  - Enhanced collectivity in Sn isotopes
- Expansion method for Thouless-Valatin inertia parameter was introduced

Thank You !

J. Meng & JCNP group

D. Vretenar & T. Niksic

P. Ring

L. Prochniak

J. M. Yao

Shape fluctuation

$$\Delta\beta = \sqrt{\langle\beta^4\rangle - \langle\beta^2\rangle^2}/2\langle\beta\rangle,$$

$$\Delta\gamma = \sqrt{\frac{\langle\beta^6 \cos^2 3\gamma\rangle}{\langle\beta^6\rangle} - \frac{\langle\beta^3 \cos 3\gamma\rangle^2}{\langle\beta^4\rangle\langle\beta^2\rangle}} \Bigg/ 3 \sin(3\langle\gamma\rangle),$$

where the average values of β and γ,

$$\langle\beta\rangle = \sqrt{\langle\beta^2\rangle},$$

$$\langle\gamma\rangle = \arccos(\langle\beta^3 \cos 3\gamma\rangle/\sqrt{\langle\beta^4\rangle\langle\beta^2\rangle})/3,$$

Observables

Observables and expected values

For $E_{I\alpha}$, the wave function:

$$\Psi_{\alpha}^{IM}(\beta,\gamma,\Omega)=\sum_{K\in\Delta I}\psi_{\alpha K}^{I}(\beta,\gamma)\Phi_{MK}^{I}(\Omega).$$

The reduced $E2$ transition

$$B(\mathrm{E2};\ \alpha I\rightarrow\alpha' I')=\frac{1}{2I+1}|\langle\alpha' I'||\hat{\mathcal{M}}(E2)||\alpha I\rangle|^2$$

$$\hat{\mathcal{M}}(\mathrm{E2};\mu)=\sqrt{\frac{5}{16\pi}}\left[D_{\mu 0}^{2}Q_0(\beta,\gamma)+\frac{1}{\sqrt{2}}(D_{\mu 2}^{2}+D_{\mu -2}^{2})Q_2(\beta,\gamma)\right]$$

Spectroscopic quadrupole moment

$$Q_{\mathrm{spec},\alpha I}=\frac{1}{\sqrt{2I+1}}C_{II20}^{II}\langle\alpha I||\hat{\mathcal{M}}(E2)||\alpha I\rangle$$

The average value of β & γ

$$\langle\beta\rangle_{I\alpha}=\sqrt{\langle\beta^2\rangle_{I\alpha}},$$

$$\langle\gamma\rangle_{I\alpha}=\frac{1}{3}\arccos\frac{\langle\beta^3\cos 3\gamma\rangle_{I\alpha}}{\sqrt{\langle\beta^2\rangle_{I\alpha}\langle\beta^4\rangle_{I\alpha}}};$$

Distribution of K component

$$N_K=6\int_0^{\pi/3}\int_0^{\infty}|\psi_{\alpha,K}^{I}(\beta,\gamma)|^2\beta^4|\sin 3\gamma|d\beta d\gamma.$$