…Fission is complex…

Collective large-scale amplitude motion

involving many (all) nucleons of the system

Mass re-arrangement in fission as an ideal tool for

probing the interplay between static and dynamical effects

for fundamental and industrial issues

Available theories

Multi-dimensional fission

⇨ Quantum mechanical / Microscopic approaches

- ✗ **Thorough « static » analysis of multi-dimensional potential energy surfaces**
- ✗ **Dynamics within time-dependent HFB treatment (GCM+GOA) in 2 dimensions**

→ today: low-energy only, no evaporation, huge computing time

⇨ Stochastic Langevin (or FPE) transport approach

- ✗ **Classical equation of motion**
- ✗ **Macroscopic ingredients and coupling to evaporation**
- ✗ **Extension to lower E^* proved successful**

→ today: up to 4D and A~100 region tractable

On some limitation of current multi-dimensional Langevin calculations

P.N. Nadtochy [1], K. Mazurek [2] , C. Schmitt [3]

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[2] IFJ PAN, 31-342 Krakow, Poland

[3] Ganil, CEA/DSM-CNRS/IN2P3, 14076 Caen, France

OUTLINE

- 🗁 **Basic principle of the Langevin approach and recent achievement**
- 🗁 **Limitations and consequences**
- 🗁 **« Practical implementation » issue : Cures ?**

The stochastic Langevin approach

❐ Transport theory for the slow dissipative fission process

⇨ **Most relevant collective degrees of freedom in interaction with heat bath**

(analogy with Brownian motion, Kramers, 1940)

❐ Multi-dimensional classical Langevin equation

$$q_i^{(n+1)} = q_i^{(n)} + \frac{1}{2}\mu_{ij}^{(n)}(\vec{q})(p_j^{(n)} + p_j^{(n+1)})\tau,$$

$$p_i^{(n+1)} = p_i^{(n)} - \tau\left[\frac{1}{2}p_j^{(n)}p_k^{(n)}\left(\frac{\partial\mu_{jk}(\vec{q})}{\partial q_i}\right)^{(n)} - K_i^{(n)}(\vec{q}) - \gamma_{ij}^{(n)}(\vec{q})\mu_{jk}^{(n)}(\vec{q})p_k^{(n)}\right] + \theta_{ij}^{(n)}\xi_j^{(n)}\sqrt{\tau},$$

- Inertia Tensor: $\|\mu_{ij}(\vec{q})\| = \|m_{ij}(\vec{q})\|^{-1}$
- Friction Tensor: $\gamma_{ij}(\vec{q})$
- Conservative Force: $K_i(\vec{q}) = -\frac{\partial F(\vec{q})}{\partial q_i}$, where $F(\vec{q})$ - free energy
- Random Force: $\theta_{ij}\xi_j$, where θ_{ij} is the random force amplitude

$$\langle \xi_i^{(n)} \rangle = 0,$$

$$\langle \xi_i^{(n_1)} \xi_j^{(n_2)} \rangle = 2\delta_{ij}\delta_{n_1 n_2},$$

$$D_{ij} = \theta_{ik}\theta_{kj} = T\gamma_{ij}$$

- Track time evolution by Monte-Carlo sampling of single trajectories
- Solvable exactly in *n*D
- « Easy » coupling with evaporation

The multi-dimensional model of Omsk (1)

G.D.Adeev, A.V.Karpov, P.N.Nadtochy, and D.V.Vanin, Phys. Part. Nucl. 36 (2005) 378

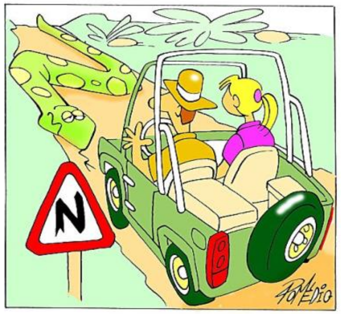
- **Sophisticated three-dimensional Langevin code**
  - ⇨ **Successful description of various observables over wide range of systems (A_{CN}, E*, L) for fission at high temperature**
  - ⇨ **Recent extension to 4D (3 deformation coordinates + *K*-rotational mode)**

- **Experiment has a lot to do with much of the (early and present) progress !**

- **Cautious survey of the theoretical results is mandatory**

  at the risk of mis-interpretation of underlying physics

NB: Present study performed with the 3D model



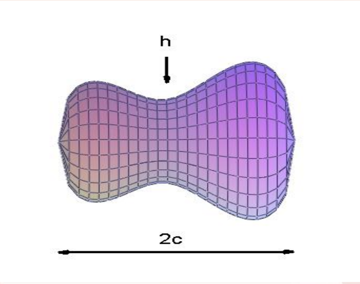
The multi-dimensional model of Omsk (2)

❒ Shape parameterization (q_1, q_2, q_3) based on (c, h, α) Funny Hills coordinates

$$q_1 = c$$

$$q_2 = \frac{h+3/2}{\frac{5}{2c^3}+\frac{1-c}{4}+3/2}$$

$$q_3 = \begin{cases} \alpha/(A_s+B), & B \geq 0 \\ \alpha/A_s, & B < 0 \end{cases}$$



- c - the elongation of the nucleus
- h - constriction coordinate
- α - mass-asymmetry parameter related to the ratio of the masses of nascent fragments

M.Brack et al., Rev. Mod. Phys. 44 (1972) 320

❒ Driving potential : FREE energy

$$F(\mathbf{q}, I, K, T) = V(\mathbf{q}, I, K) - a(\mathbf{q})T^2$$

- $a(\mathbf{q})$ - level density parameter **(various options)**
- $T = \sqrt{E_{\text{int}}/a(\mathbf{q})}$ - temperature of the nucleus
- The potential energy of the nucleus was calculated with Sierk's parameters[a] **(FRLDM)**

Ignatyuk et al.,
Yad. Fiz. 21 (1975) 1185

Sierk et al.,
Phys. Rev. C 33 (1986) 2039

❒ Transport coefficients

Inertia M_{ij} : Hydrod

WW irrotati

Davies et al.,
PRC 13(1976) 2385

with wall contribution scaled with k_s

astic force : Fluctuation-

em $D_{ij} = \gamma_{ij}$ T

locki et al., Ann.Phys.
(N.Y.) 113(1978) 330
Nix and Sierk, Dubna Report (1976)

- ▸ **Deformation-dependence accounted for**
- ▸ **Single free parameter i.e. k_s**

The multi-dimensional model of Omsk (3)

❐ **Initial conditions**

- Spherical and thermalized CN
- Spin $d\sigma_l/dl$ for fusion from Fröbrich and Gontchar, Phys. Rep. 292 (1998) 131 (≈ surface friction model)

❐ **Scission criterion**

$$R_N = 0.3 R_0,$$

R_N - neck thickness and R_0 - radius at ground state

Compatible with rupture condition (instability with respect to neck variation, stochastic rupture, Coulomb repulsion ≈ nuclear attraction)

❐ **Dynamics coupled with evaporation within Monte Carlo method**

- Hauser-Feschbach theory for $\Gamma_{n,p,\alpha}$

  statistical code LILITA, J. Gomez del Campo et al., ORNL TM-7295 (1981)

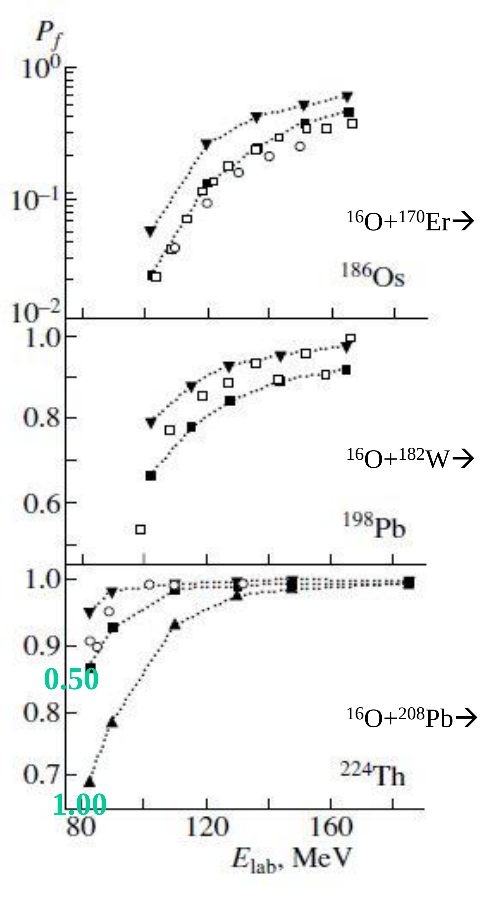
- Conservation laws used at each step

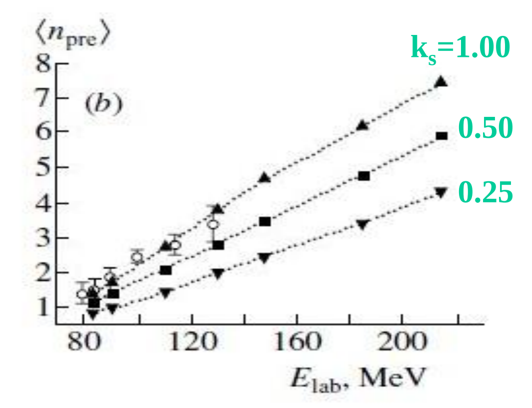
❐ **Trajectory terminates either at scission or with ER formation**

Achievement (1)

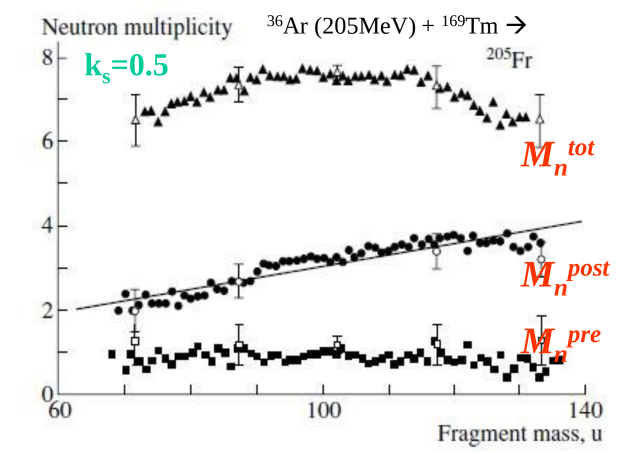
❐ **Survey of various observables over wide (A_{CN}, E*, L) plane to study nuclear viscosity (i.e. extract k_s)**

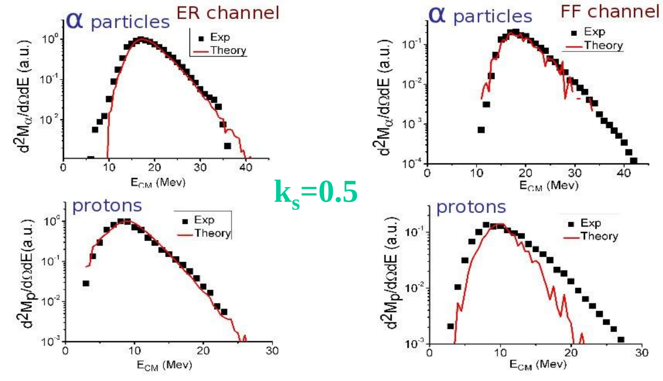
- **Fission/ER cross sections, light particle multiplicities and energies**





P.N. Nadtochy et al.,
Phys. At. Nucl. 66 (2003) 1203





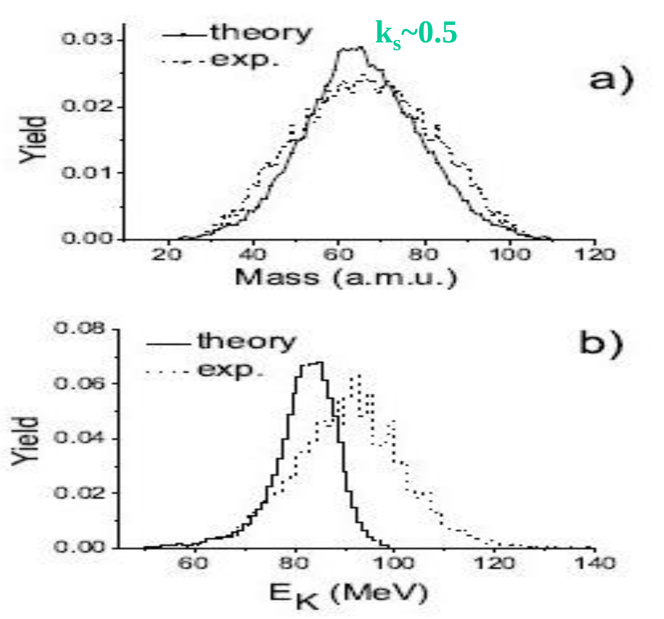
^{32}S (200MeV) + ^{100}Mo → ^{132}Ce*

P.N. Nadtochy et al.,
EPJ Web Conf. 2 (2010) 08003

Achievement (2)

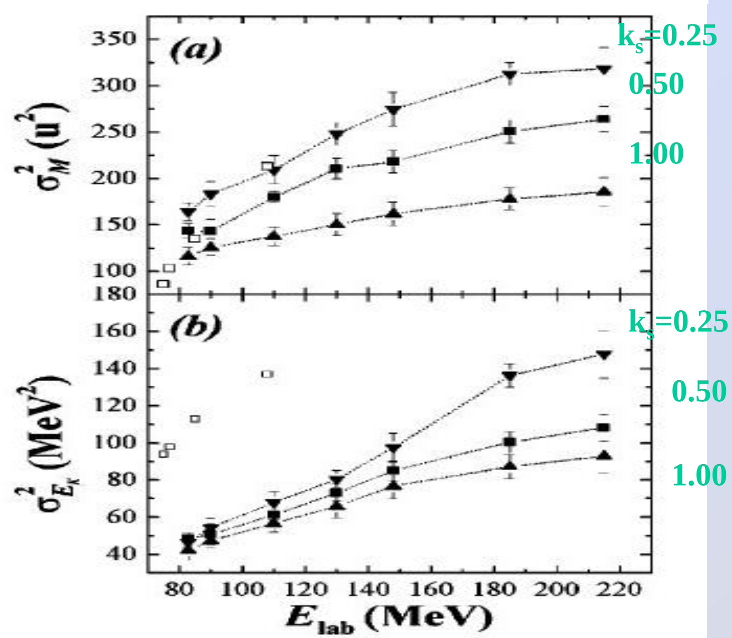
Fission-fragment mass and TKE distributions - Angular anisotropies

^{32}S (200MeV)+ ^{100}Mo → $^{132}Ce^*$



P.N. Nadtochy et al., EPJ Web Conf. 2 (2010) 08003

^{16}O + ^{208}Pb → $^{224}Th^*$



P.N. Nadtochy et al., Phys. Rev. C 65 (2002) 064615

- **k_s ~ (0.25-0.5) suited for whole data set on $\sigma_{ER,fiss}$, M_n^{pre}, σ_M^2 , $W(\vartheta)$**
- **σ_{TKE} un-explained → too poor variety of scission configurations** (elongation/neck)
- **Three dimensions crucial for consistent description of experiment**

Too strong wall-and-window dissipation (related to theory of chaos)

Nix and Sierk, Proceedings JINR,1987, p.453
Pal et al., Phys. Rev. C 54 (1996) 1333)

Consequence of “practical” limitations?

❒ **Theoretical TKE distribution too narrow due to poor variety of scission shapes**

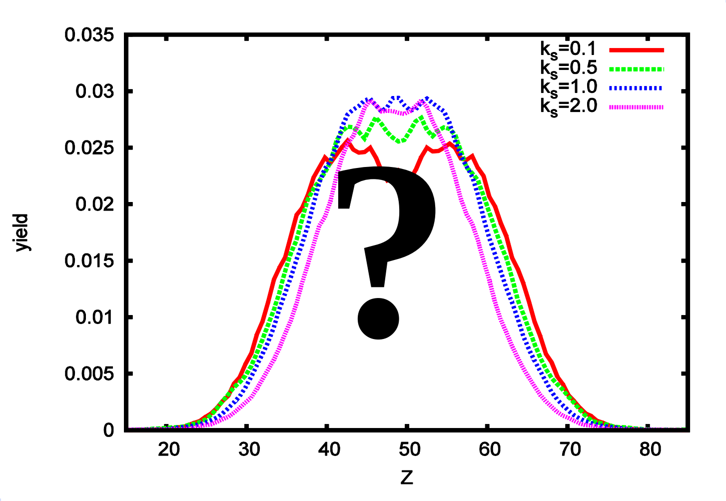
❒ **<u>More generally</u>: «Practical» implementation of theory often implies simplificatio**

or approximations due to space/computing/mathematical limitation

→ Need to ensure that no hazardous consequence as for interpretation of experiment

❒ **Meticulous survey of all calculated observables over a very wide range of the model ingredients, and with large statistics (a few 10^5 trajectories)**

E.g: $0.1 < k_s < 2$



Asymmetric fission peaks with:

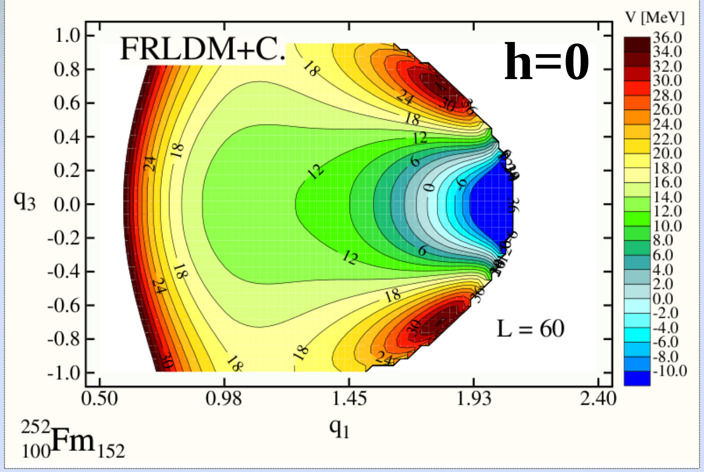
Classical equation

Macroscopic ingredients (PES, transport coeff

High E*

Model ingredients and dynamics

❒ **Potential energy surface** → Single fission valley to symmetry

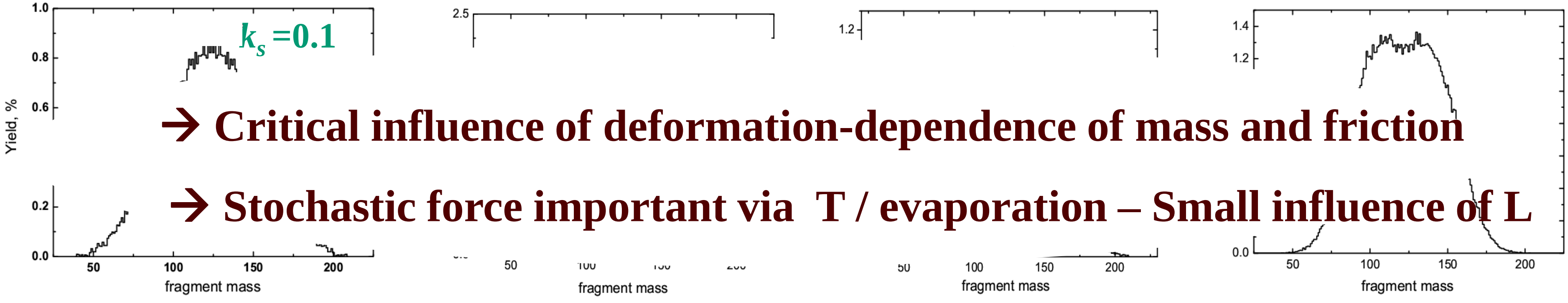


Asym fission dominates at LOW viscosity (ks ≤0.2):

- **deformation dependent mass AND friction**
- **substantial fluctuations by stochastic force**
- **stiffness/softness of the PES**

SUBTLE INTERPLAY BETWEEN INGREDIENTS

❒ **Transport coefficients**

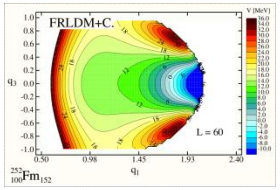


→ **Critical influence of deformation-dependence of mass and friction**

→ **Stochastic force important via T / evaporation – Small influence of L**

Mass and Friction cst, no evaporation	**Overdamped, F cst, no evaporation**	**Overdamped, F =f(def) no evaporation**	**M=f(def), F =f(def), evaporation**

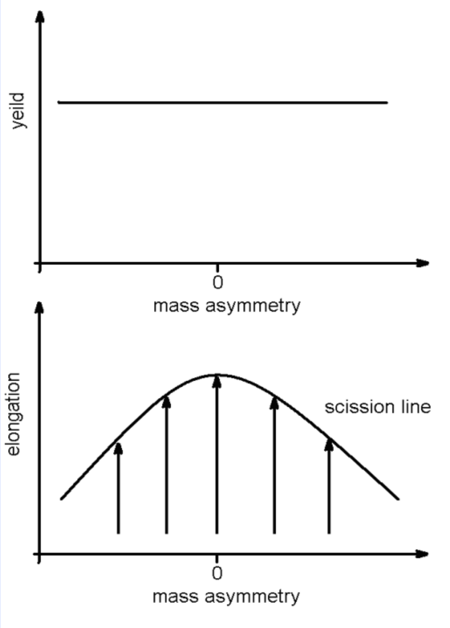
Shape parameterization,scission line and dynamics



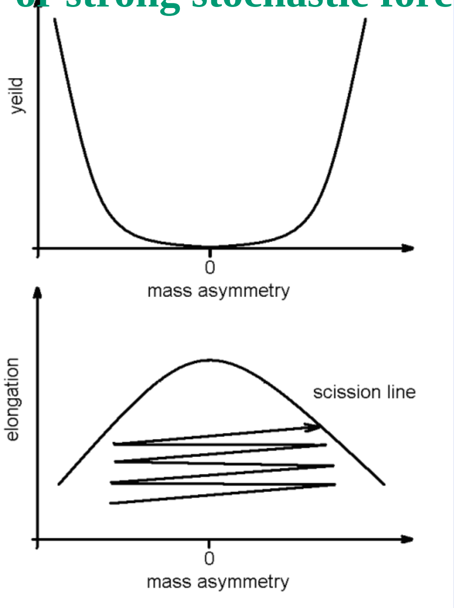
- **Curved scission line (« hyper-space» in *n*D)**

  → influence on the mass distribution

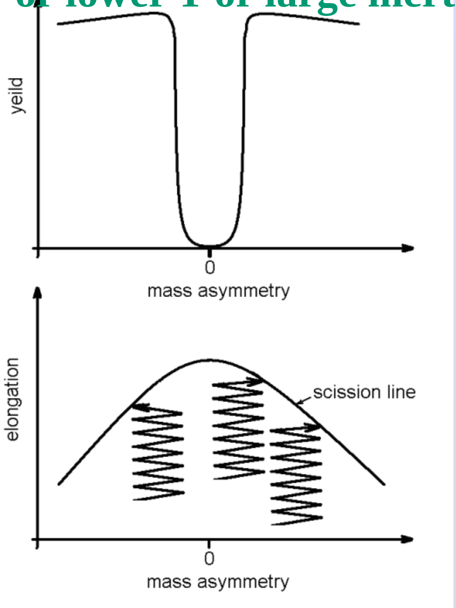
Cst flux of particles



Cst flux + weak friction or strong stochastic force



Cst flux + larger friction or lower T or large inertia



- **Tricky <u>puzzle</u> between magnitude and deformation-dependence of mass, friction, stochastic force, PES topolography combined to curved scission line**

⇨ much too much asym splitting !!

Practical implementation of theory …

- Shape parameterization $\{q_i\}$ + scission criterion

  → **define the « scission hyper-surface »**

- Scission should have a specific geometry to avoid pratical bias

  → **line in 2D, plane in 3D, etc**

- At day, no existing prescription that can guarantee being free of any such limitation ($\forall$ *n*D)

… calls for careful analysis of result

Do we master the conditions under which implementation may cause trouble?

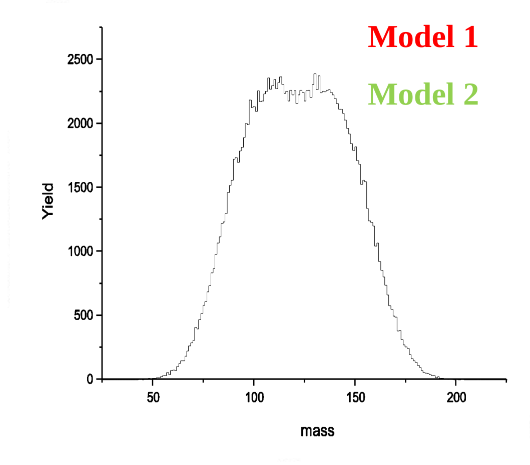
Hazardeous… as the appearance of « un-physical » results is due to interplay involving puzzling dynamics and practical limitations

Consequences

❒ Modelling of washing out of shell effects with excitation energy

Model 1: dynamics on macro-micro PES including δU=f(T) and with some k_s^1

M purely macro PES with some other k_s^2



E.g: Fission at 1 < T < 2MeV

Persistence of shell effects ?

«Mere» dynamical effect @ moderate viscosity ?

❒ Viscous nature of nuclear matter

σ_M^2, <TKE>, σ_{TKE}^2 commonly used to infer the magnitude of dissipation

σ_M^2, <TKE>, σ_{TKE}^2 = f (saddle descent, scission configuration)

⇨ Uncontrolled practical bias ⇨ Mis-interpretation on underlying viscosity

Conclusion ?

- ❐ Dynamical multi-dimensional Langevin approach as a powerful tool for understanding fission dynamics

- ❐ Practical implementation implies restriction on the modeling as compared to the richness of the available phase-space

  ➪ **often neglected or forgotten**

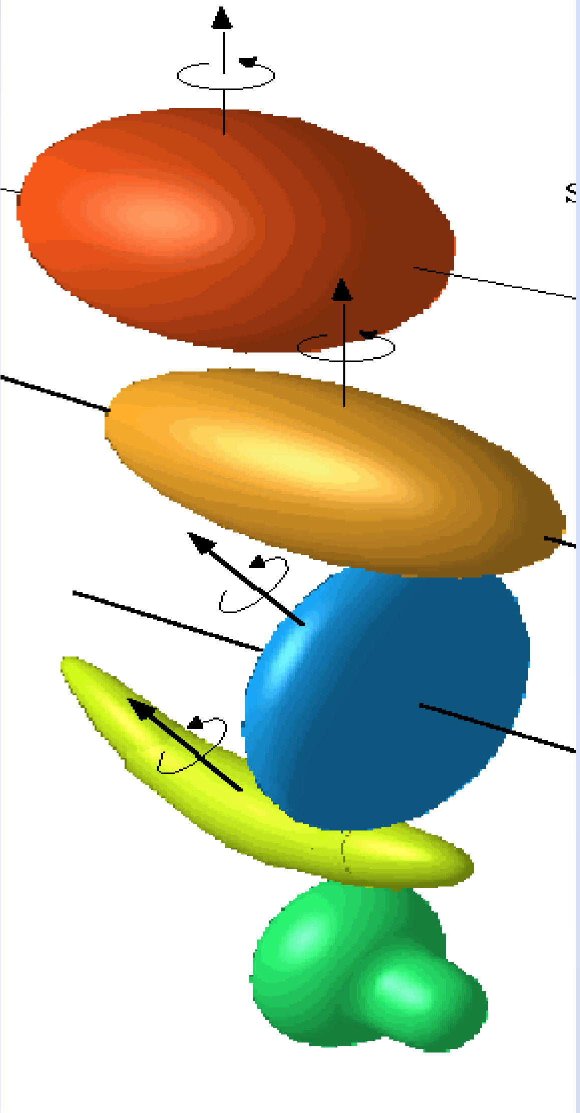
  ➪ **due to intricate effects, potentially un-controlled, if not un-physical, result**

Wise analysis of prediction due to still un-perfect modeling of shape and scission configurations : any input is welcome !

- ❐ Other difficult aspect: find the true saddle ridges and fission valleys out of a multi-dimensional space

  ➪ **Need of a discriminant observable**

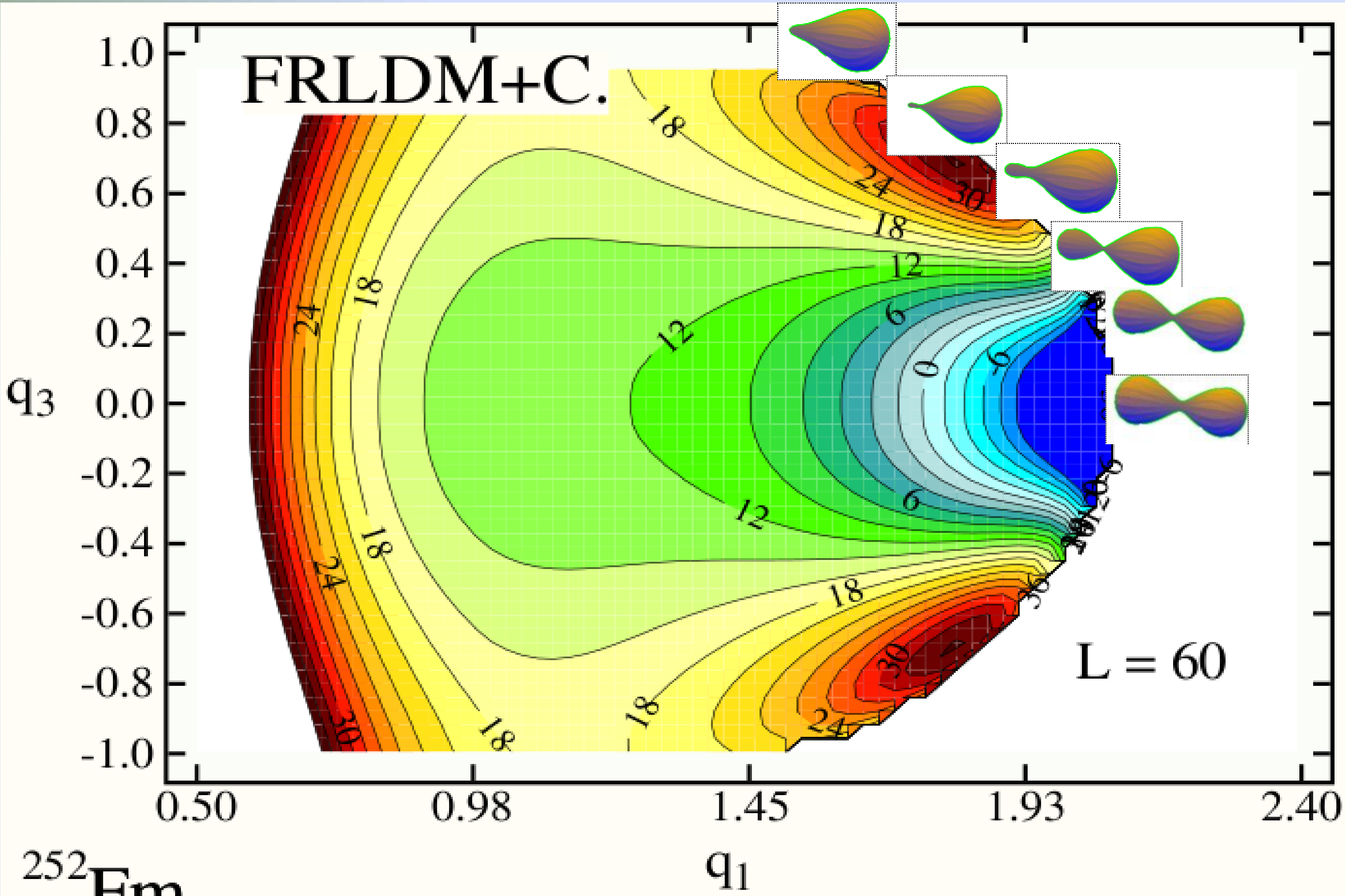
N. Dubray and D. Regnier,
Comp.Phys. Com.
183 (2012) 2035



Thank you

for your attention !

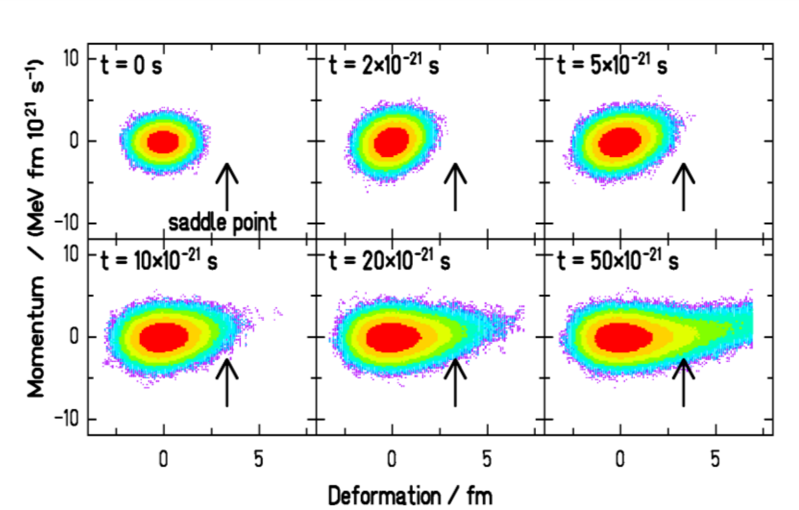
EXTRAS



$^{252}_{100}\mathrm{Fm}_{152}$

by Kasia Mazurek

The stochastic Langevin vs. FP approach



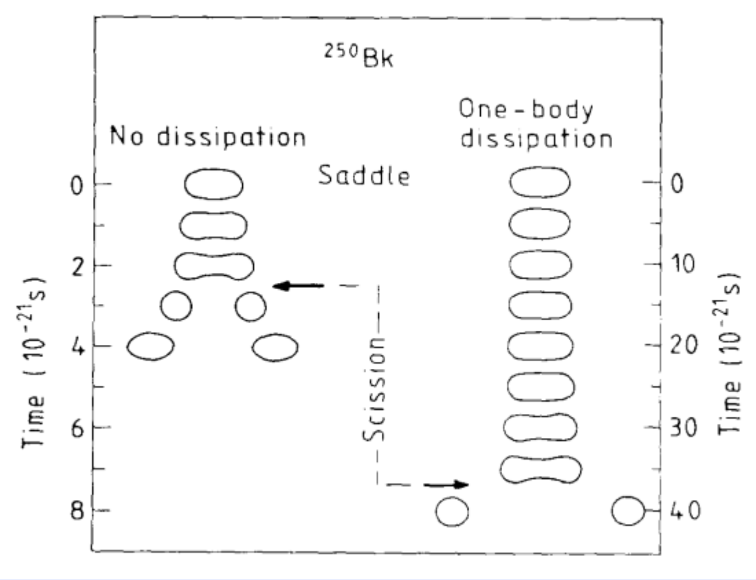
^{248}Cm, T=3MeV, $\beta=5 \cdot 10^{21}$s^{-1}

K.-H.Schmidt , Ch. Schmitt, et al.

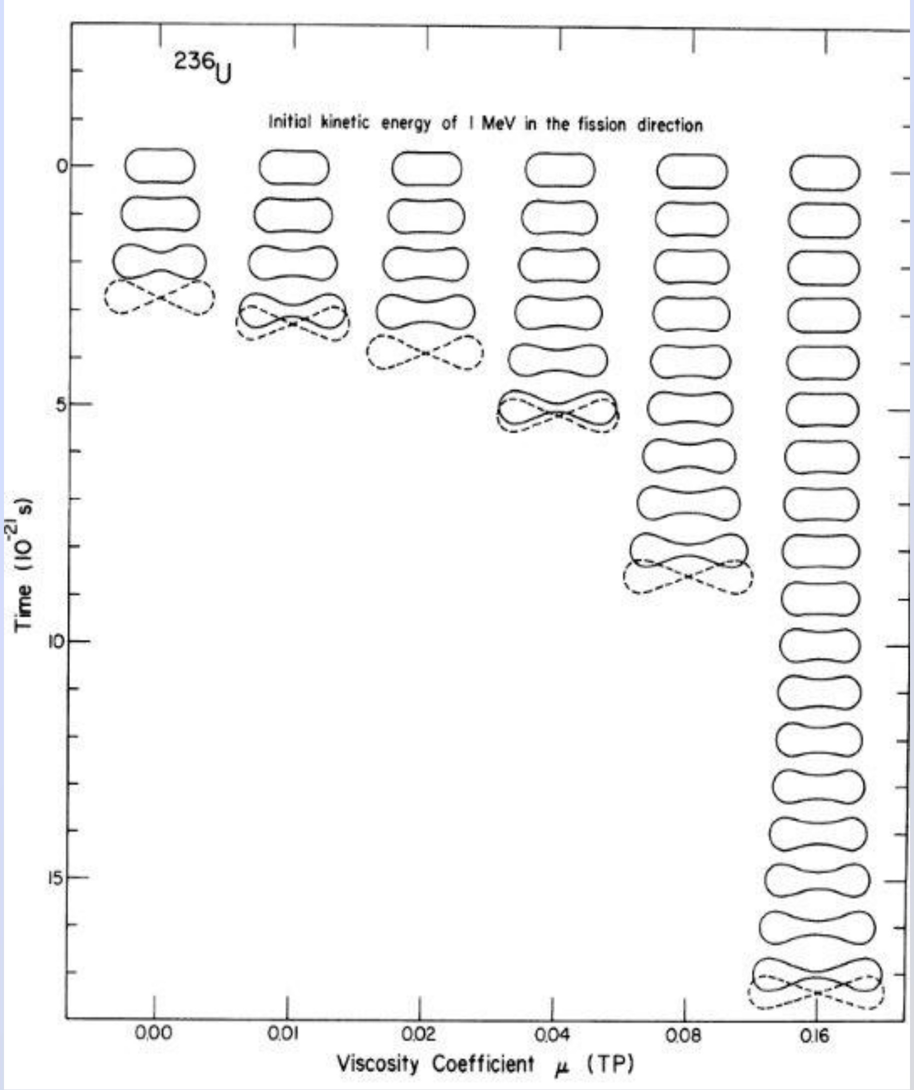
Y. Abe, S. Ayik, P.G. Reinhard, E. Suraud, Phys. Rep. 275 (1996) 49

- Track time evolution by Monte-Carlo sampling of single trajectories → $P(q_i,p_i;t)$ as with FPE
- Solvable exactly in nD
- « Easy » coupling with evaporation

Viscosity and shape evolution



D. Hilscher, Ann. Phys. Fr. 17 (1992) 471



Complex interplay between transport coefficients and their dependence on deformation

Adeev et al., Phys. Part. Nucl. 36 (2005) 378 and therein

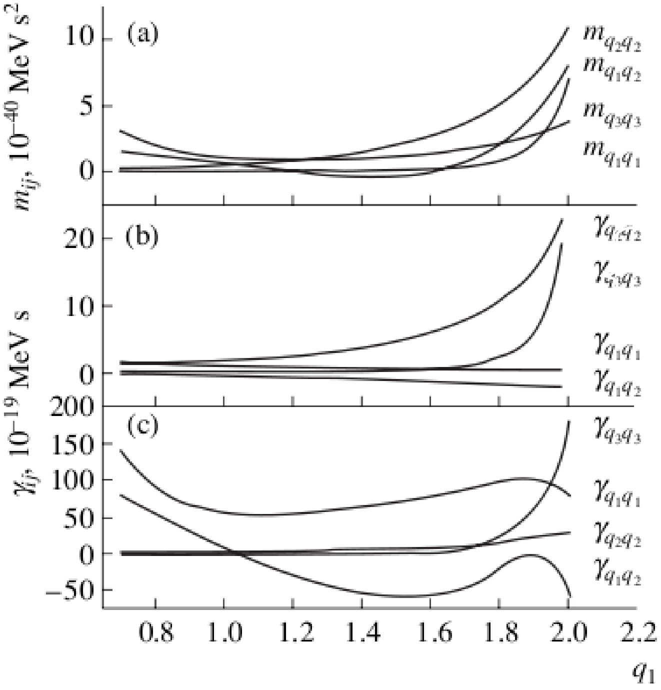
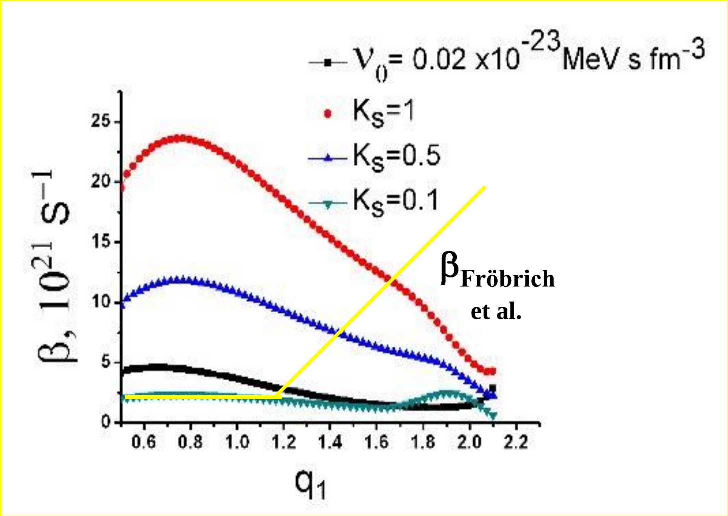
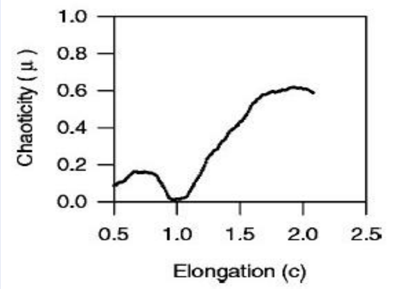
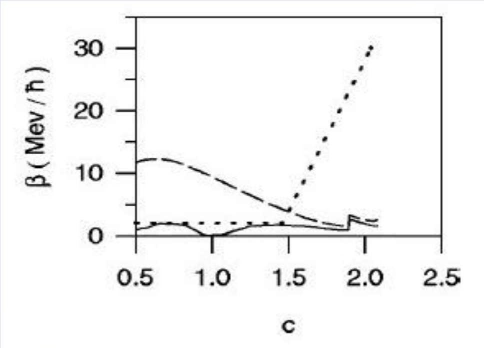


Fig. 5. Transport coefficients as functions of the coordinate q_1 ($h = \alpha = 0$): (a) the inertia tensor; (b) the friction tensor under the assumption of the two-body mechanism of nuclear viscosity $\nu_0 = 2 \times 10^{-23}$ MeV s fm^{-3}, and (c) the friction tensor under the one-body mechanism of viscosity with $k_s = 0.25$. The calculations are performed for the ^{224}Th nucleus.

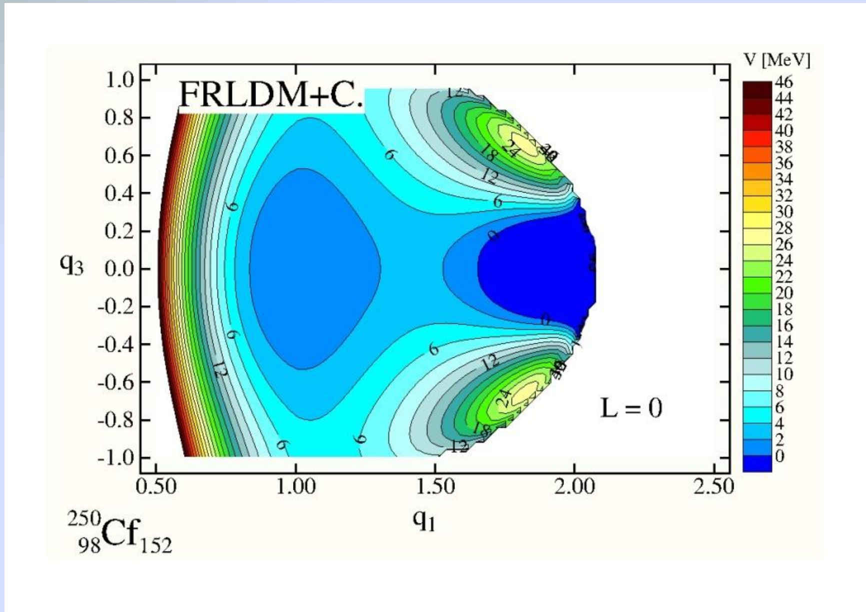


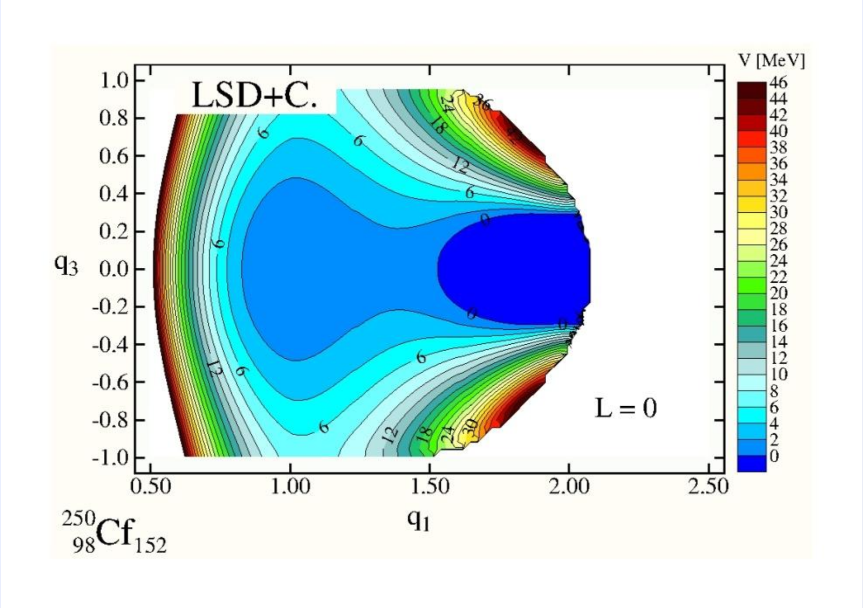
« Chaocity » in nuclei ?

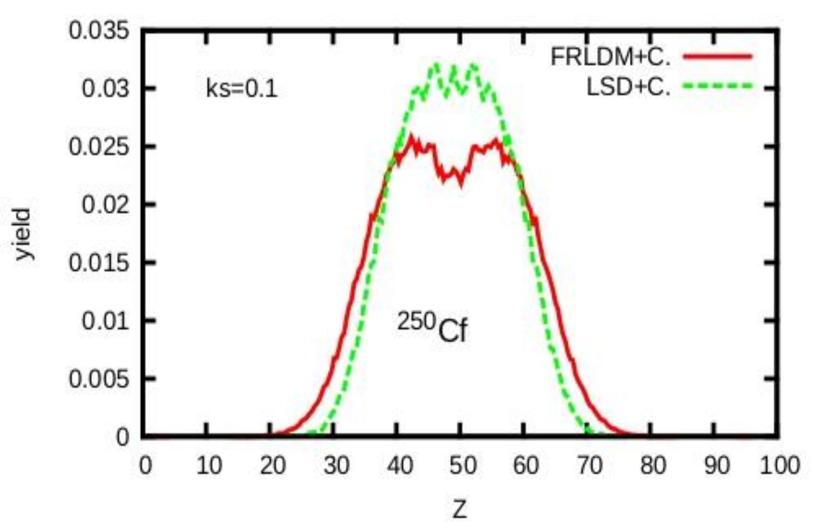




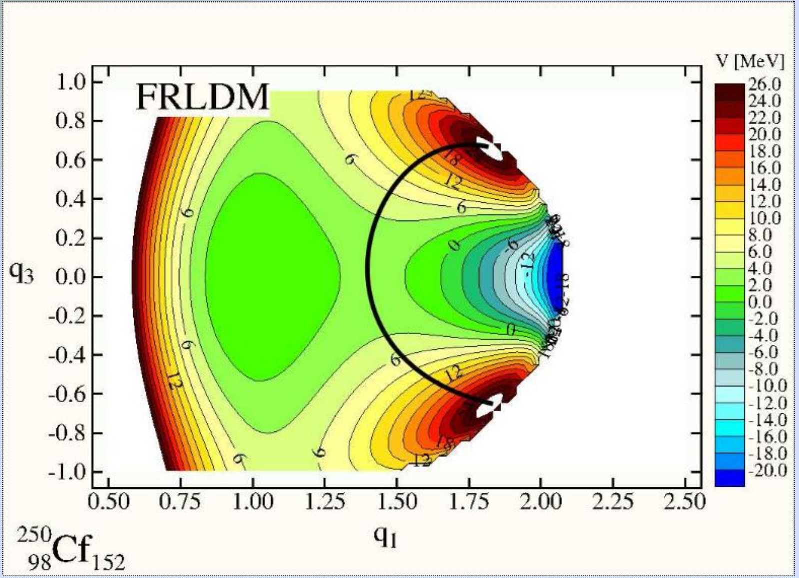
Influence of the PES prescription







Influence of scission condition



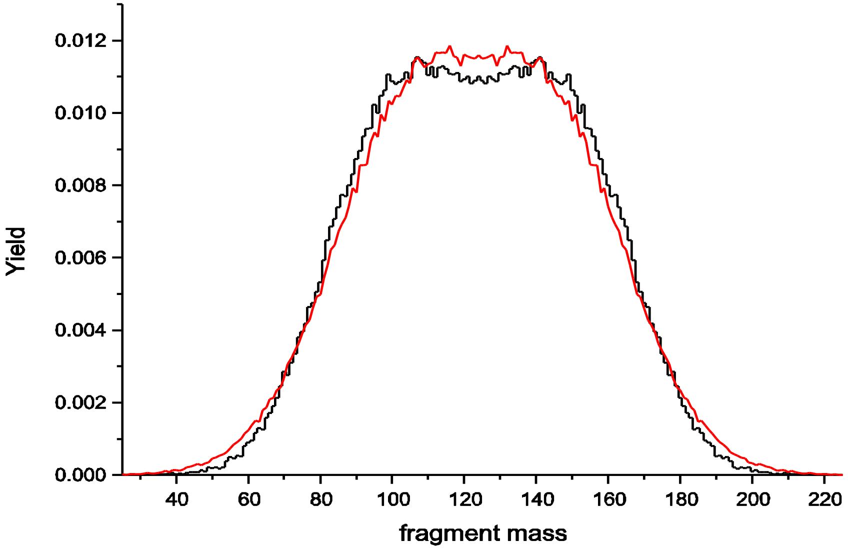
Scission criterion should define

a line instead of a curve (2D),
a planar surface (3D) , etc,

in the corresponding collective coordinates space

Scission at Rneck = 0.3R0

Scission at c=1.98



Potential discrepant interpretation of data ?

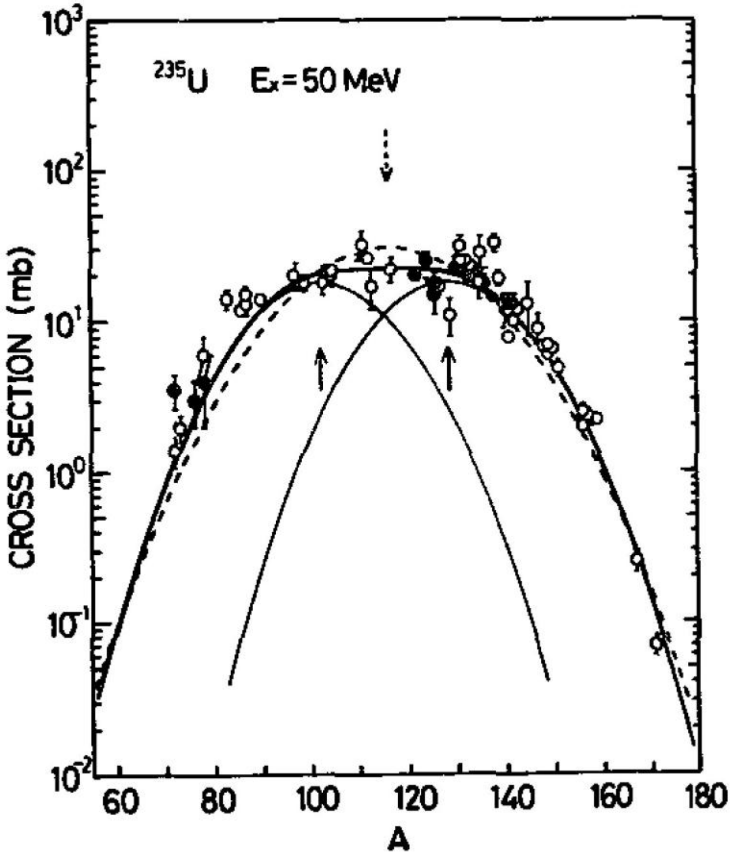
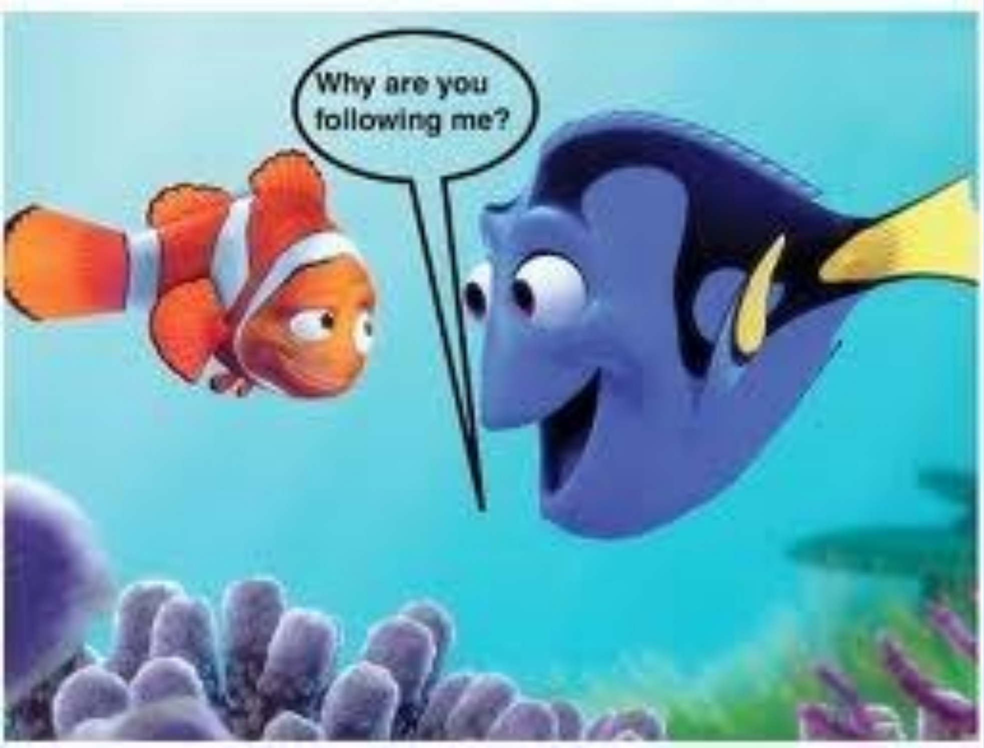


Fig. 8. Mass distribution for the fission of ^{235}U with 90 MeV ^{12}C (50 MeV excitation). Open circles represent the total chain yields deduced from cumulative yields whereas solid circles give those calculated with independent yields. Fitted curves are also shown for a single gaussian (dashed line) and a double gaussian (solid line).

Duh et al., Nucl. Phys. A 550 (1992) 281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