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Mass and lifetime of unstable nucleus in covariant density functional theory

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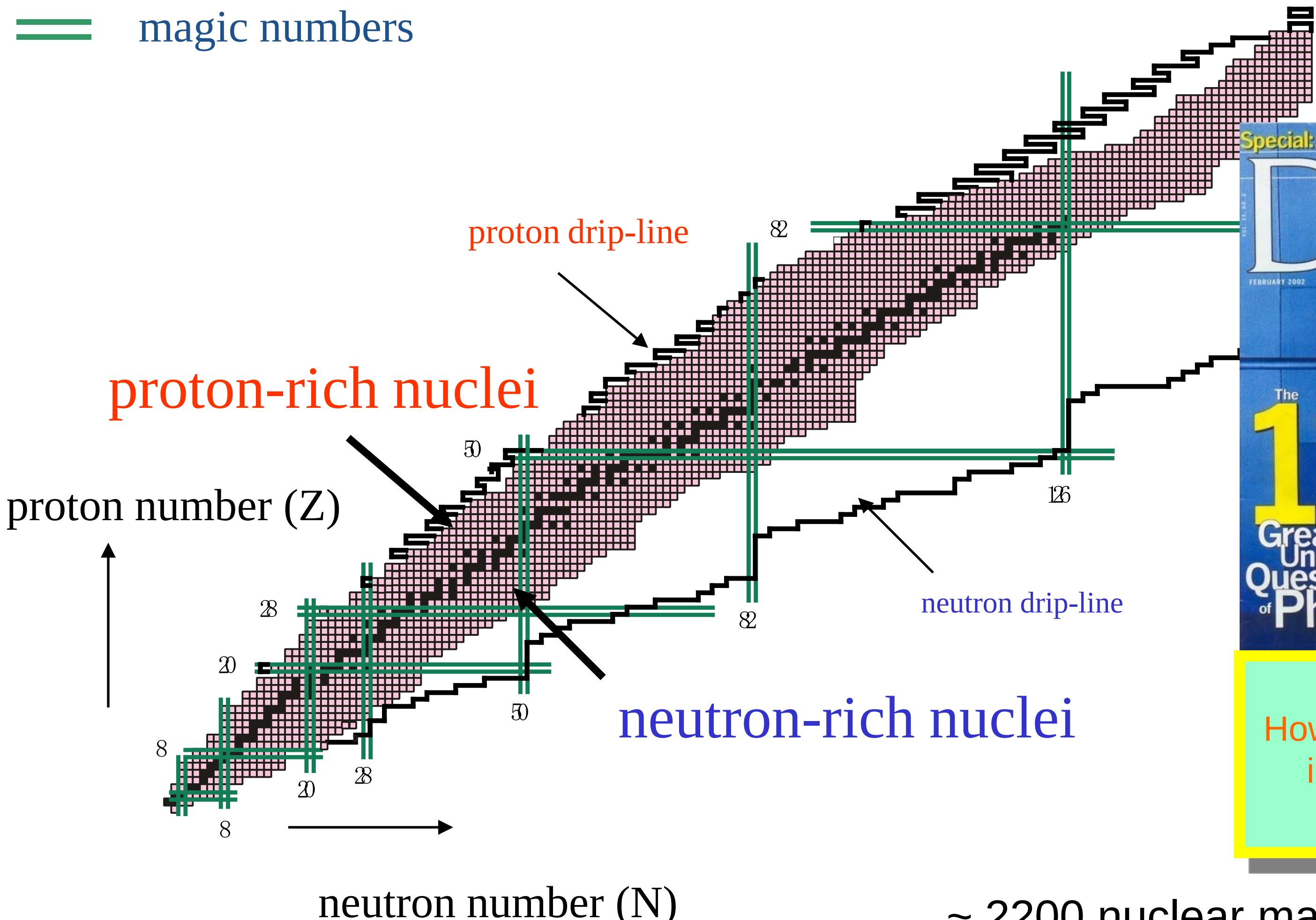
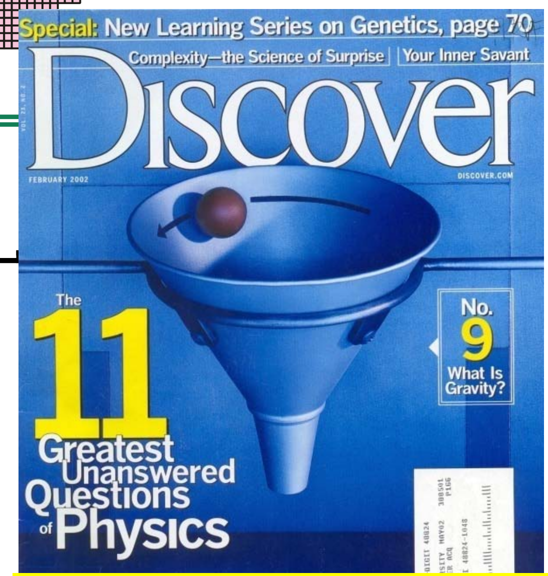
Who did the work ?

Existence Limit of nucleus

■	stable nuclei	~300 nuclei
□	unstable nuclei observed so far	~2700 nuclei
—	drip-lines (limit of existence)（theoretical predictions)	~8000 nuclei
═	magic numbers	





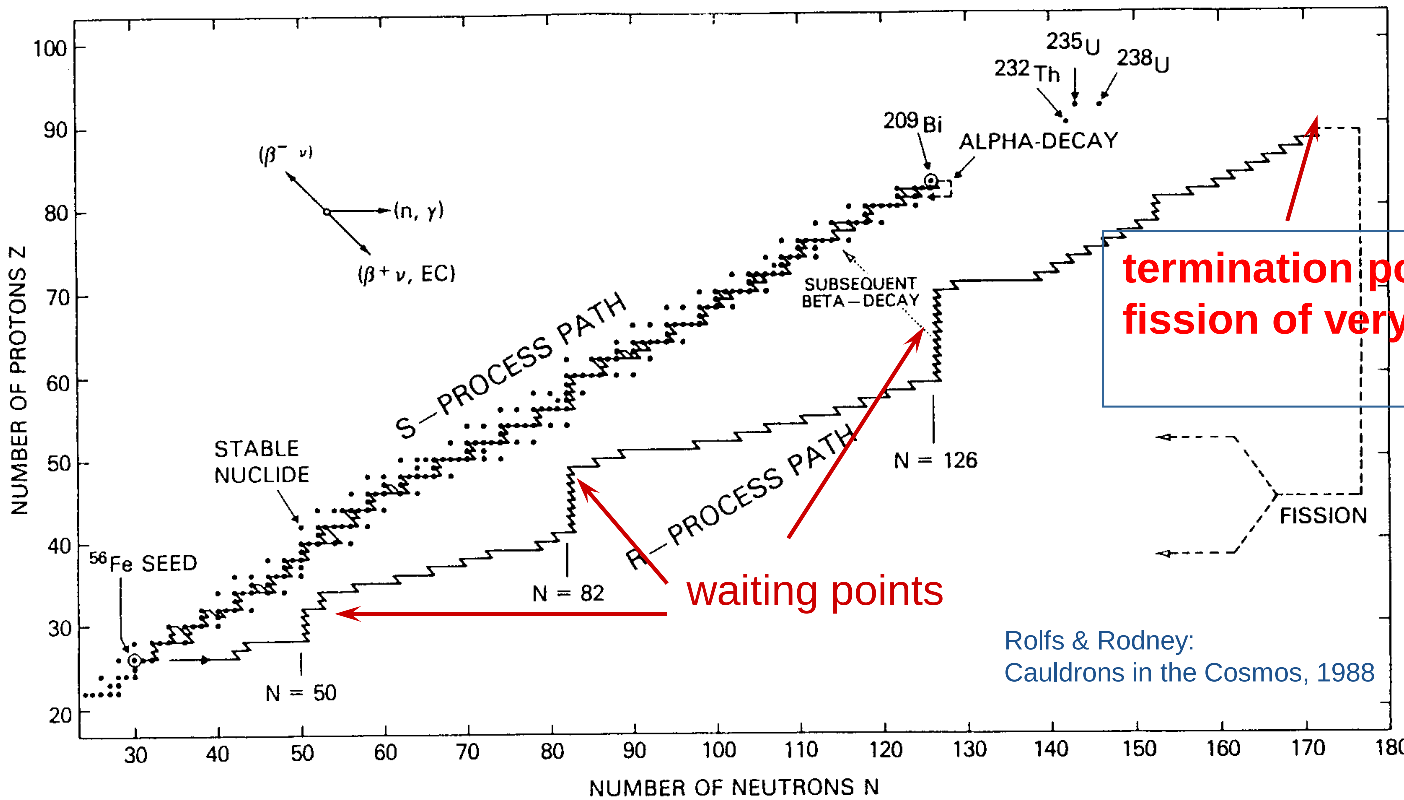
Question 3
How were the elements from iron to uranium made ?

~ 2200 nuclear masses were measured

Astrophysical environment for r-process

r-process (r = rapid neutron capture process)

unstable nucleus reacts before capturing decay ⇔ **n capture time: $\tau_n \ll \tau_\beta$**



typical lifetimes for unstable nuclei far from the valley of β stability: $10^{-4} - 10^{-2}$ s

requiring n capture time: $\tau_n \sim 10^{-4}$ s ⇔ $n_n \sim 10^{20}$ n/cm^3

explosive scenarios needed to account for such high neutron fluxes

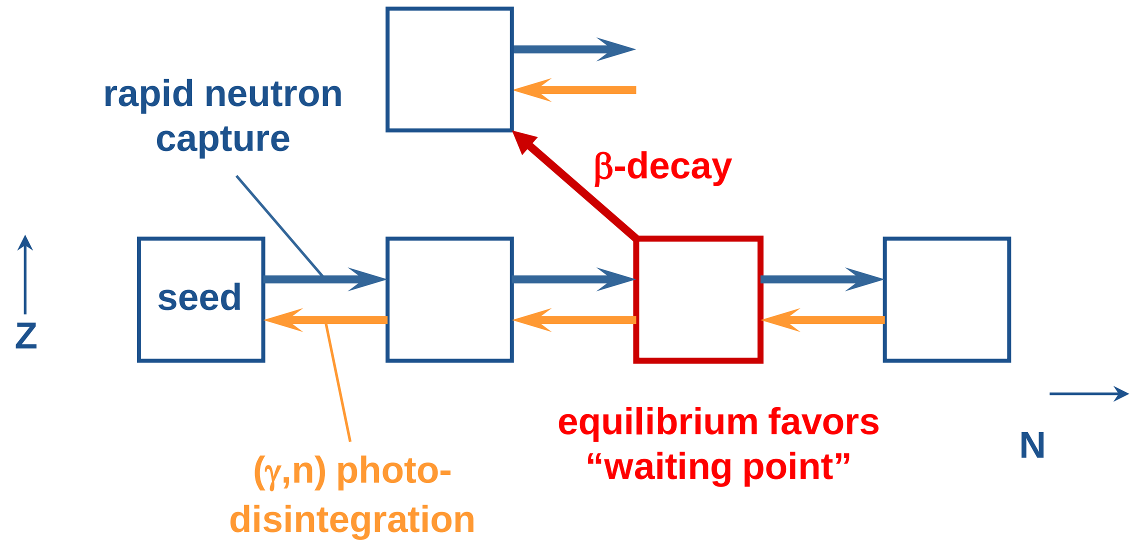
Nuclear physics input in the r-process

Quantity		Effect
S_n	neutron separation energy	path
$T_{1/2}$	b-decay half-lives	▪ abundance pattern ▪ timescale
P_n	b-delayed n-emission branchings	▪ final abundance pattern smooth r-abundance
G	Nuclear Partition function	▪ abundance pattern (weakly)
fission (branchings and products)		▪ endpoint ▪ abundance pattern? ▪ degree of fission cycling
N_A<sv>	neutron capture rates	▪ final abundance pattern during freezeout ? ▪ conditions for waiting point approximation
Isomeric states…		▪ Branch of the r-process path ▪ final abundance pattern ▪ timescale

Classical r-process calculation

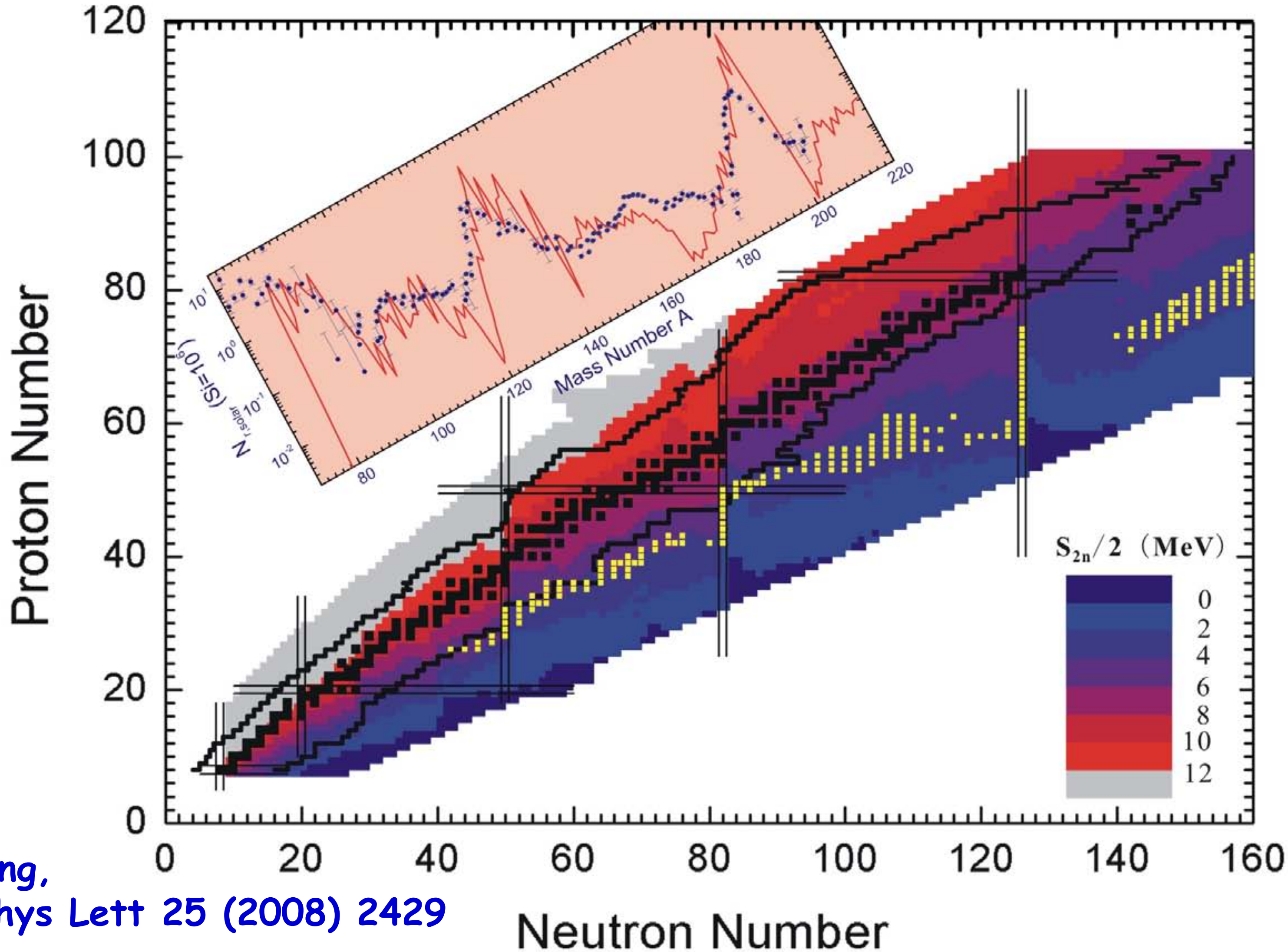
Assume:

- **(n,γ) ↔ (γ,n) equilibrium within isotopic chain, and**
- **elemental distribution of neighboring z-chain is determined by the β-decays**
- **neglect the effect of fission**
- **constant T_9, multi r-process components with $n_n=10^{20-27}$.**



The nucleus with maximum abundance in each isotopic chain has smaller neutron capture rate and must wait for the longer time to continue via β-decay

nuclear inputs: S_n(RMF), $T_{1/2}$(β-decay), P_{1n}, P_{2n}, P_{3n} (FRDM),
astrophysical parameters: T_9=1.5, n_n=10^{20-28}, ω, τ (least-square fit),



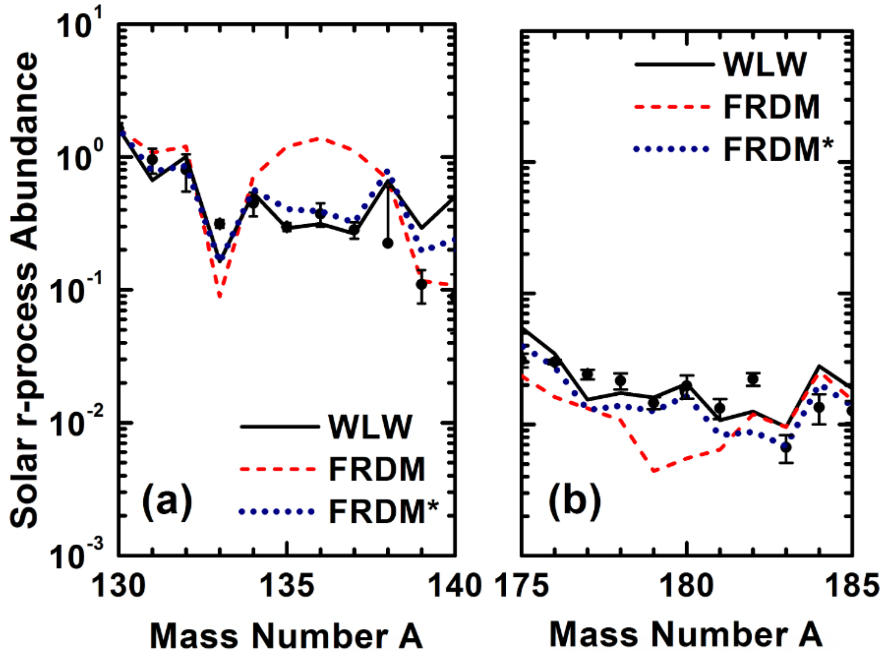
Sun & Meng, Chinese Phys Lett 25 (2008) 2429

Sun et al., PRC 78 025806 (2009)

Niu et al., PRC 80 065806 (2009)

Constraints of nuclear mass model by Solar abundance

- Solar abundance

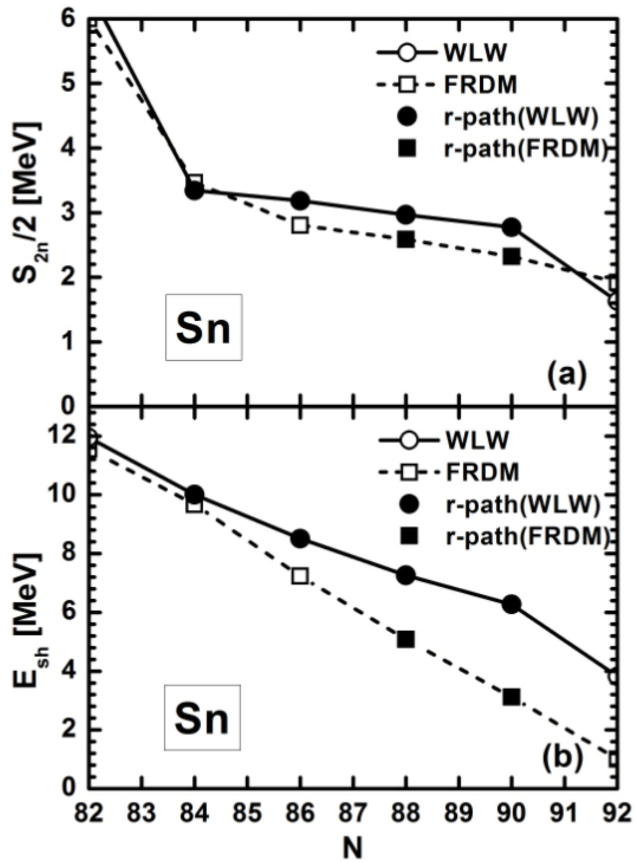


- ➢ Single Sn isotopic chain influences the abundance for A around 135
- ➢ Sm and Eu isotopic chains influences the abundance for A around 180

*FRDM: ADNDT **59** 185*
*Wang: PRC **81** 044322*

Isospin for S-O & E_sym + mirror nuclei

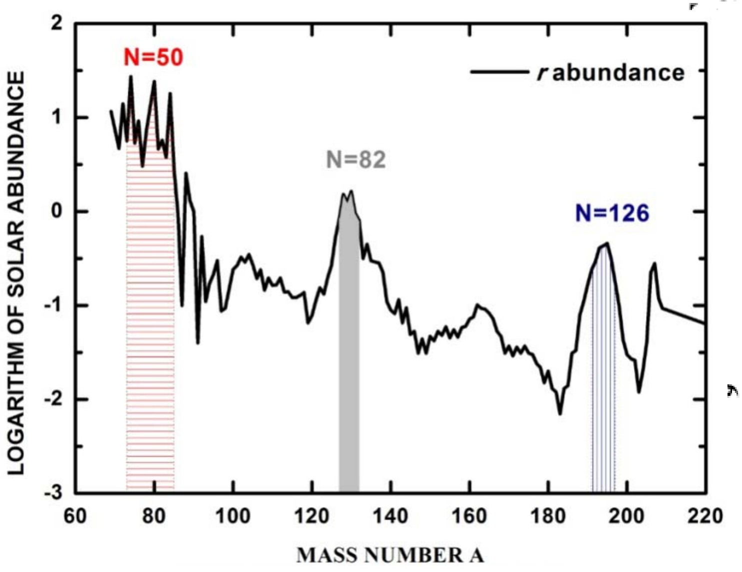
- Abundance difference roots in nuclear structure
- separation energy, waiting point, and shell correction for FRDM & WLW

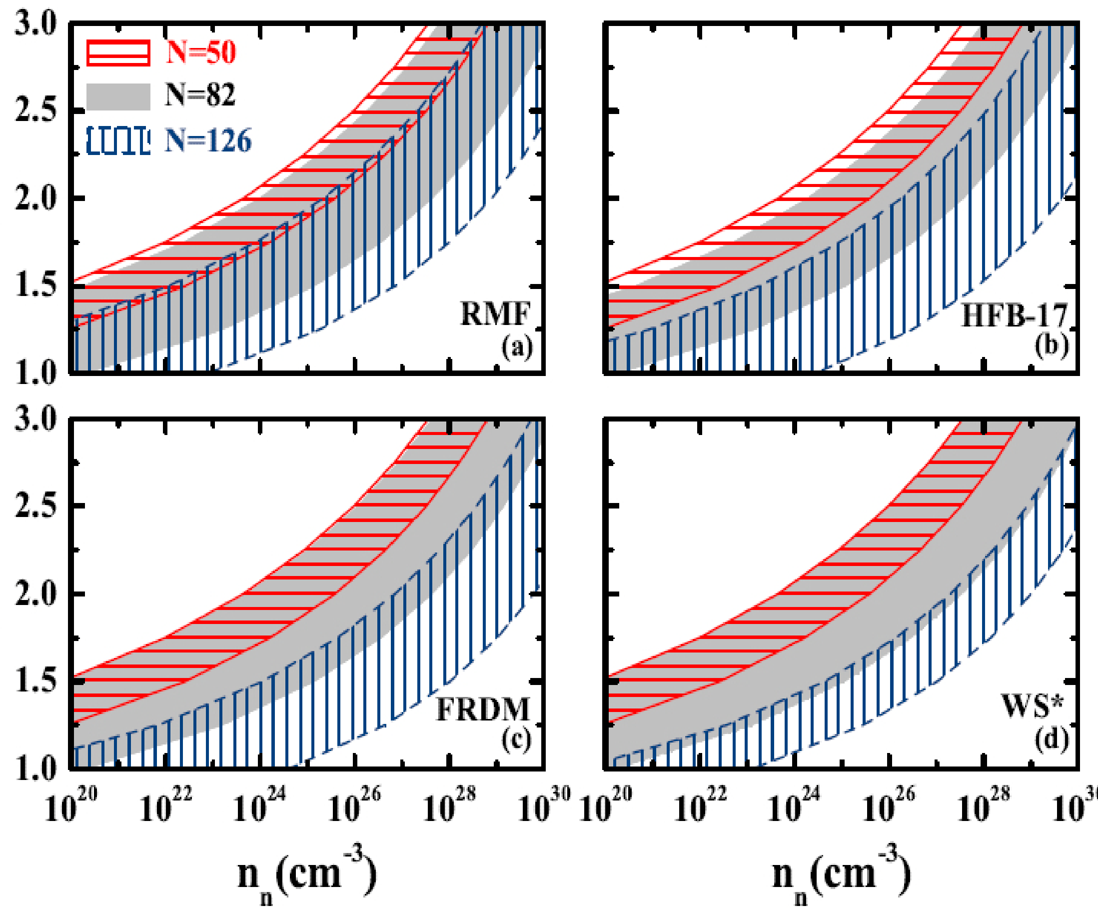


Z Li et al, Acta Phys. Sin. **61** 072601

Constraints of astrophysical condition by Solar abundance

Corresponding T_9-n_n correlation in reproducing the N=50, 82, and 126 abundance peak





XD Xu et al, arXiv:1208.2341[nucl-th], 2012

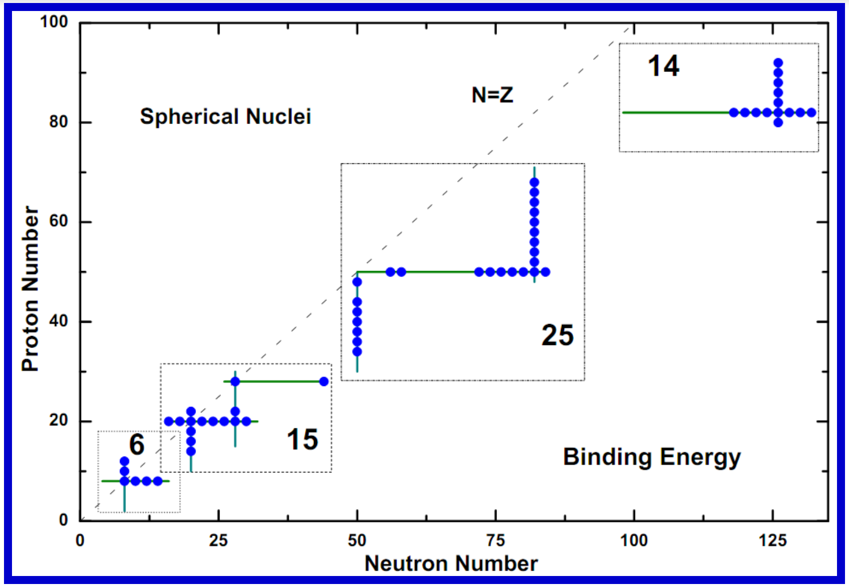
Different astrophysical conditions for producing nuclei with neutron number N = 50 and those with N = 126 !

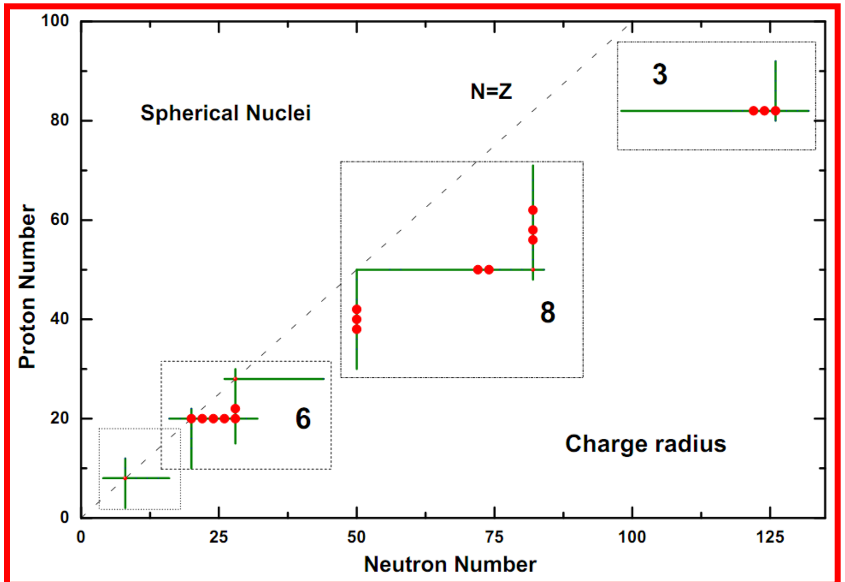
CDFT with non-linear point coupling interaction

Lagrangian density

$$
\begin{aligned}
L = {} & \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi \\
& -\frac{1}{2}\alpha_S(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) - \frac{1}{2}\alpha_{TV}(\bar{\psi}\vec{\tau}\gamma_\mu\psi)(\bar{\psi}\vec{\tau}\gamma^\mu\psi) \\
& -\frac{1}{3}\beta_S(\bar{\psi}\psi)^3 - \frac{1}{4}\gamma_S(\bar{\psi}\psi)^4 - \frac{1}{4}\gamma_V[(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)]^2 \\
& -\frac{1}{2}\delta_S\partial_\nu(\bar{\psi}\psi)\partial^\nu(\bar{\psi}\psi) - \frac{1}{2}\delta_V\partial_\nu(\bar{\psi}\gamma_\mu\psi)\partial^\nu(\bar{\psi}\gamma^\mu\psi) - \frac{1}{2}\delta_{TV}\partial_\nu(\bar{\psi}\vec{\tau}\gamma_\mu\psi)\partial^\nu(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\
& -e\frac{1-\tau_3}{2}\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}
\end{aligned}
$$

Parameterizations: PC-PK1





Coupl.	Cons.	PC-PK1	Dimension
α_S	$[10^{-4}]$	-3.96291	MeV^{-2}
β_S	$[10^{-11}]$	8.66530	MeV^{-5}
γ_S	$[10^{-17}]$	-3.80724	MeV^{-8}
δ_S	$[10^{-10}]$	-1.09108	MeV^{-4}
α_V	$[10^{-4}]$	2.69040	MeV^{-2}
γ_V	$[10^{-18}]$	-3.64219	MeV^{-8}
δ_V	$[10^{-10}]$	-4.32619	MeV^{-4}
α_{TV}	$[10^{-5}]$	2.95018	MeV^{-2}
δ_{TV}	$[10^{-10}]$	-4.11112	MeV^{-4}
V_n	$[10^{0}]$	-349.5	$\mathrm{MeV\ fm}^{3}$
V_p	$[10^{0}]$	-330	$\mathrm{MeV\ fm}^{3}$

Zhao, Li, Yao, Meng, PRC 82, 054319 (2010)

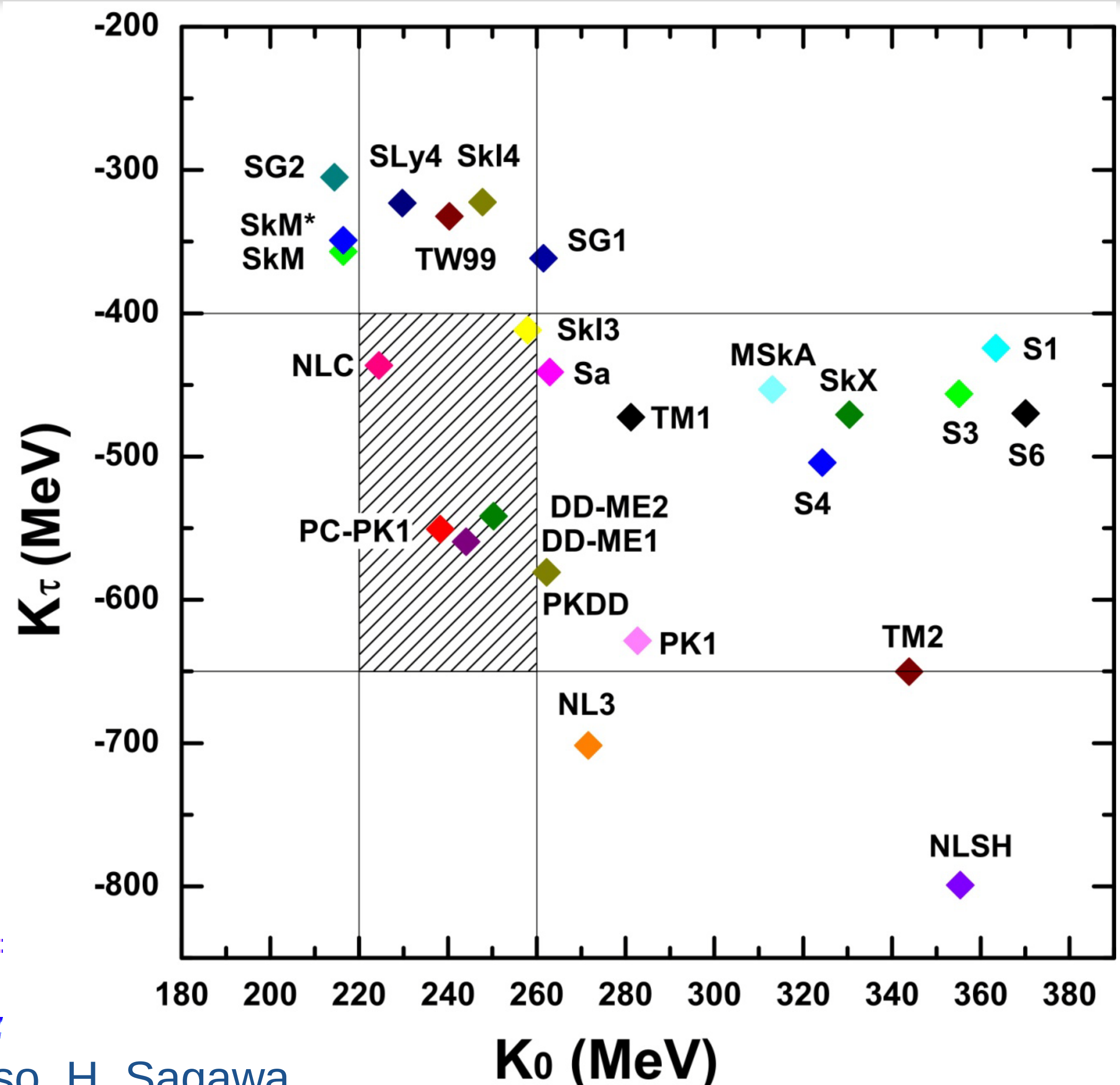
Nuclear matter properties

Saturation properties:

Model	ρ_0 (fm^{-3})	E/A (MeV)	M_D^*/M	M_L^*/M	E_{sym} (Mev)	L (MeV)	K_{sym} (MeV)	K_0 (MeV)	K_{asy} (MeV)
Empirical	0.166 ±0.018	−16 ±1	0.55 − 0.60	0.8 ±0.1	∼ 32	88 ±25		240 ±20	−550 ±100
NL3	0.148	-16.25	0.59	0.65	37.4	119	101	272	-611
PK1	0.148	-16.27	0.61	0.66	37.6	116	55	283	-640
TW99	0.153	-16.25	0.55	0.62	32.8	55	-125	240	-457
DD-ME1	0.152	-16.2	0.58	0.64	33.1	56	-101	245	-435
PKDD	0.15	-16.27	0.57	0.63	36.8	90	-81	262	-622
PC-LA	0.148	-16.13	0.58	0.64	37.2	108	-61	264	-711
PC-F1	0.151	-16.17	0.61	0.67	37.8	117	74	255	-628
PC-PK1	0.153	-16.12	0.59	0.65	35.6	113	95	238	-582
DD-PC1	0.152	-16.06	0.58	0.64	33	70	-108	230	-529

EoS for Nuclear matter

$K_\tau = K_{\tau,v} + K_{\tau,s} A^{-1/3}$



We know K_A from E_{GMR}:

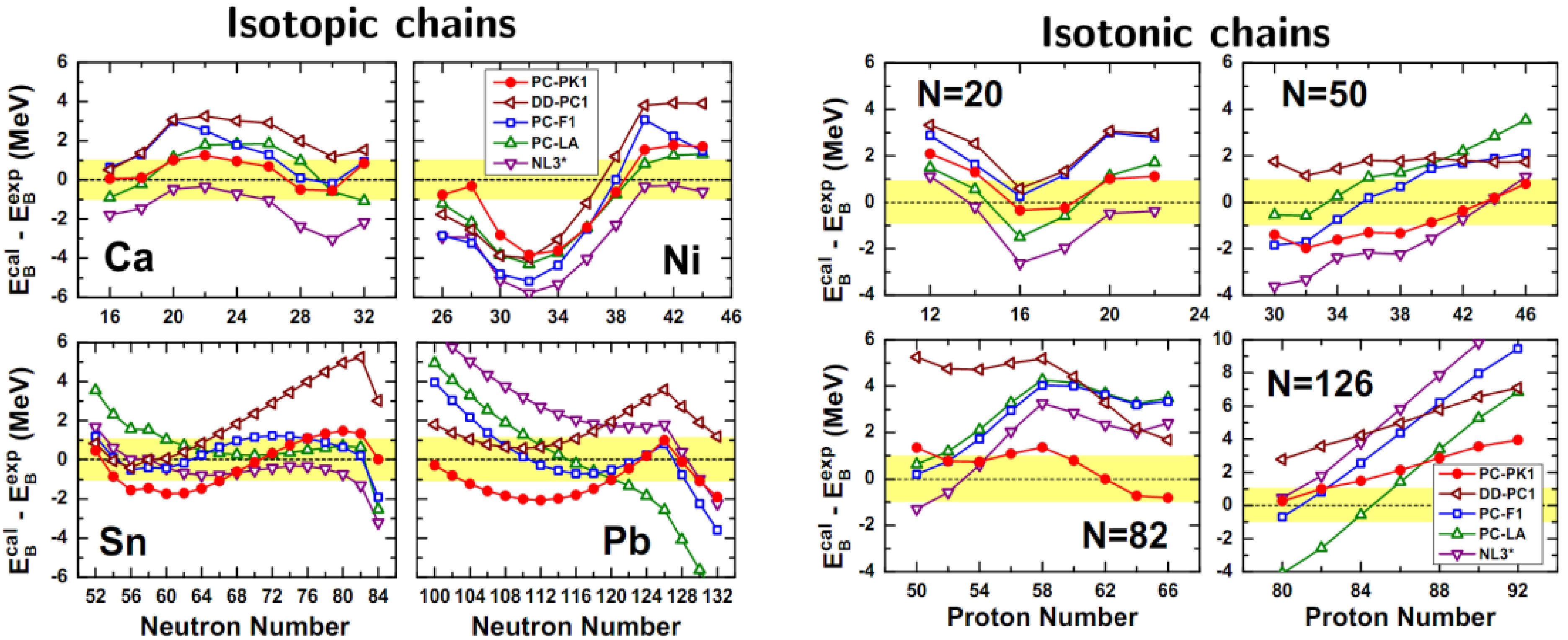
$$E_{GMR} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}$$

In an approximate way, K_A may be expressed as:

$$K_A \sim K_\infty\,(1 + cA^{-1/3}) + K_\tau\,((N-Z)/A)^2 + K_{Coul}\,Z$$

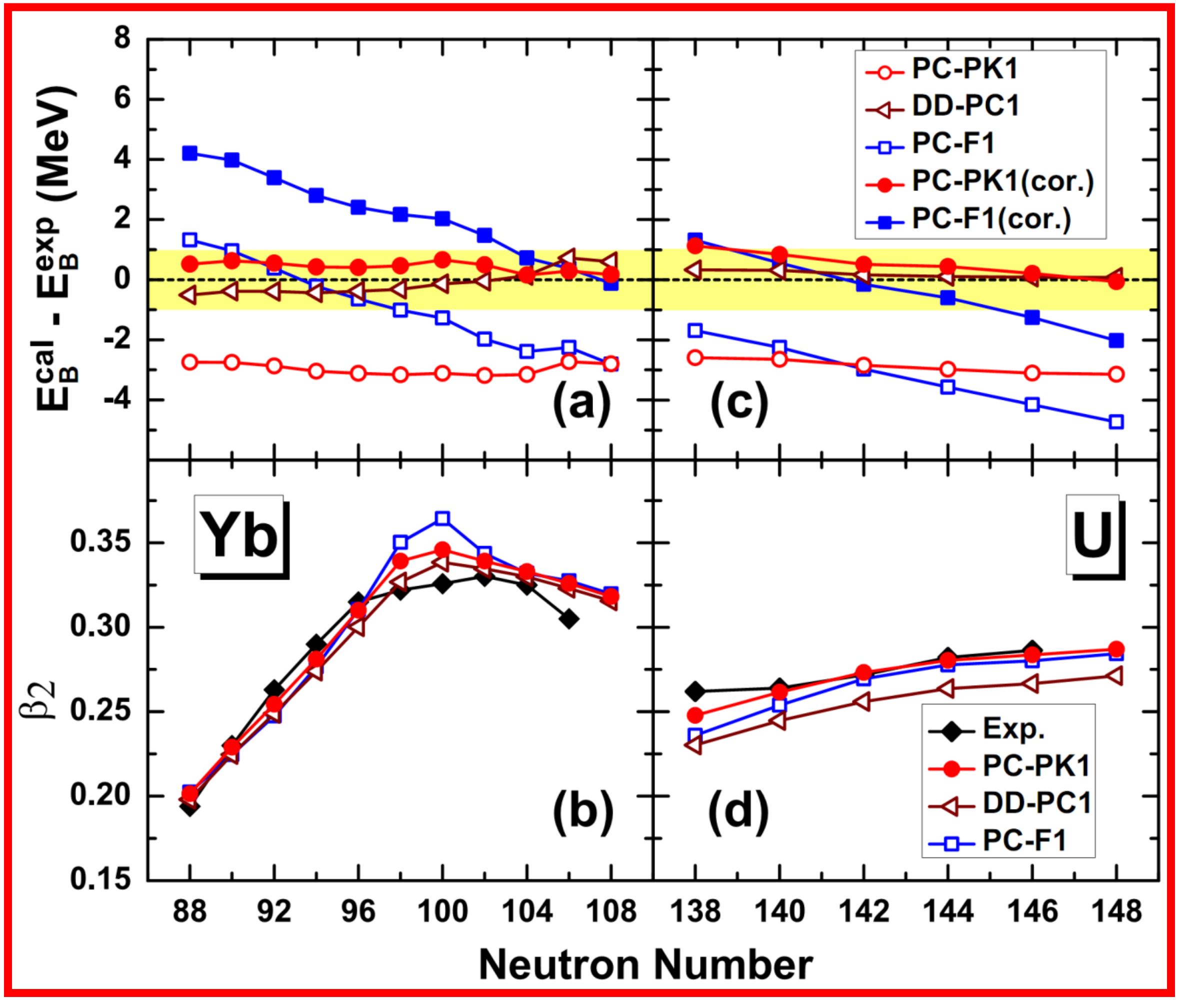
Data from from Umesh Garg, also H. Sagawa *et al., Phys. Rev. C* ***76, 034327 (2007)***

Spherical nuclei



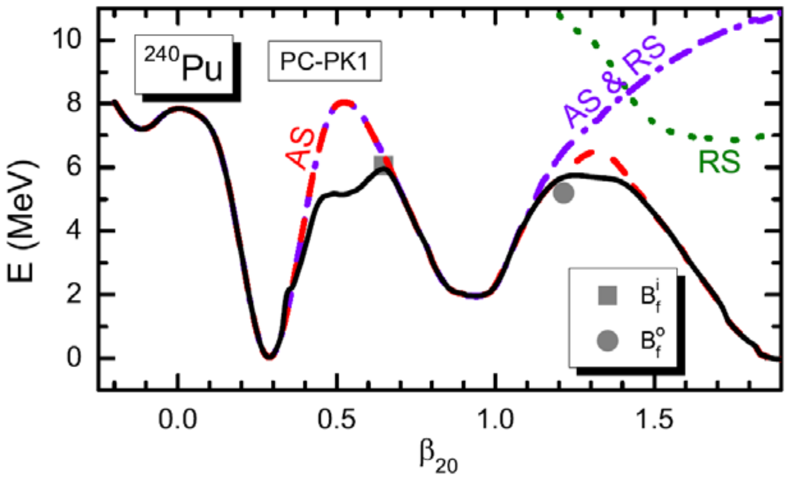
Zhao, Li, Yao, Meng, PRC 82, 054319 (2010)

Deformed nuclei



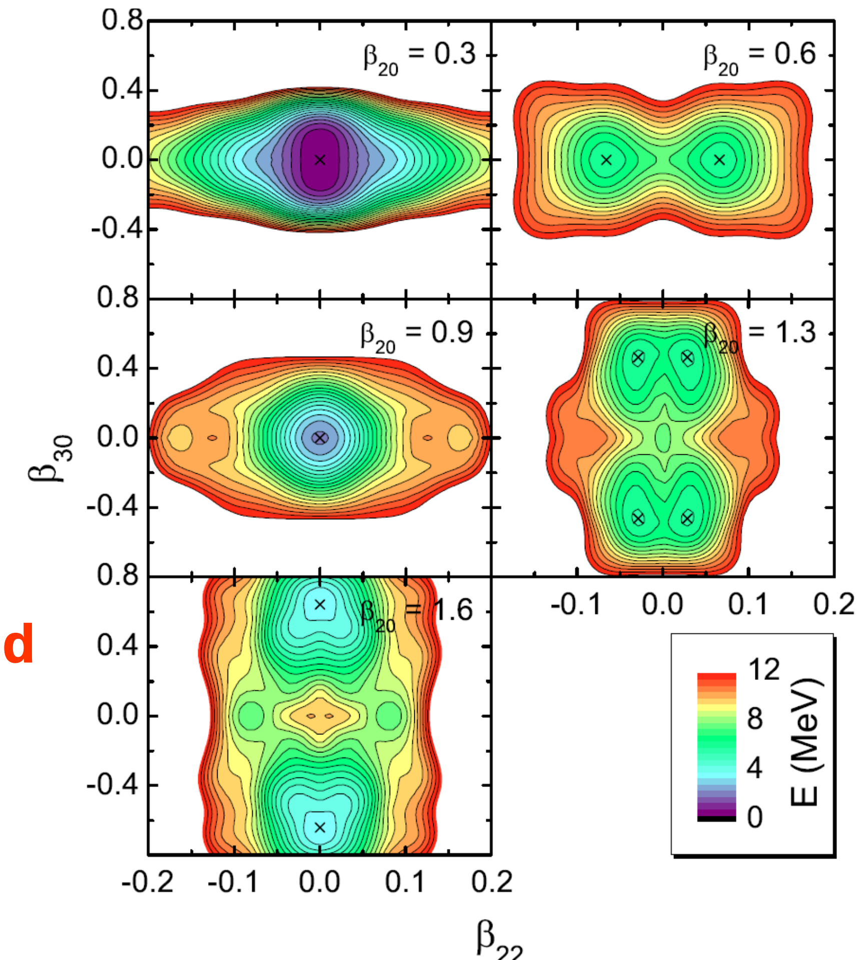
Zhao, Li, Yao, Meng, PRC 82, 054319 (2010)

^{240}Pu: 3D PES (β_{20}, β_{22}, β_{30}) in MD constraint CDFT



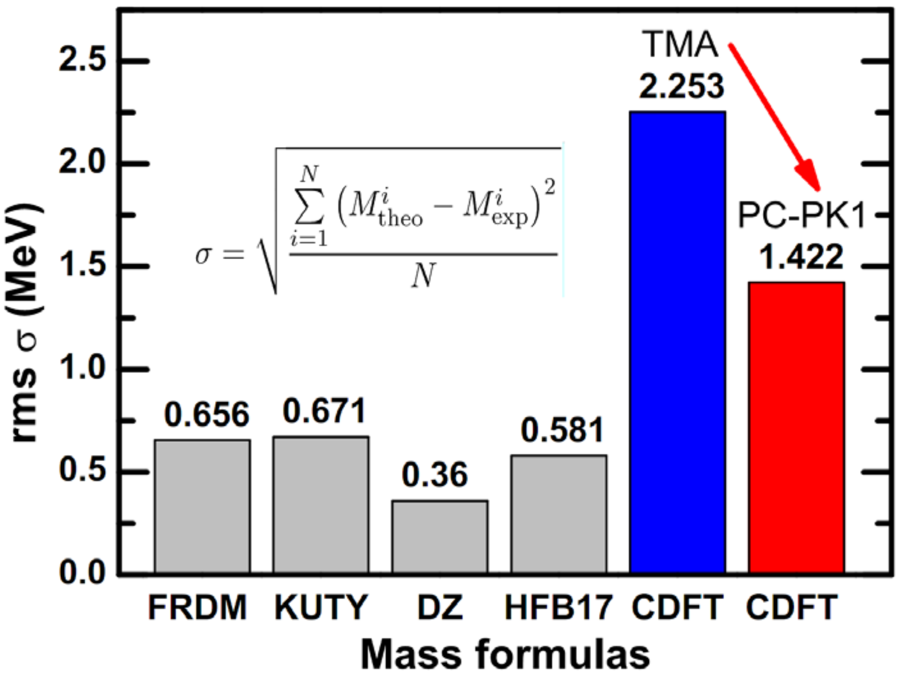
Lu, Zhao, Zhou, PRC85 (2012) 011301R

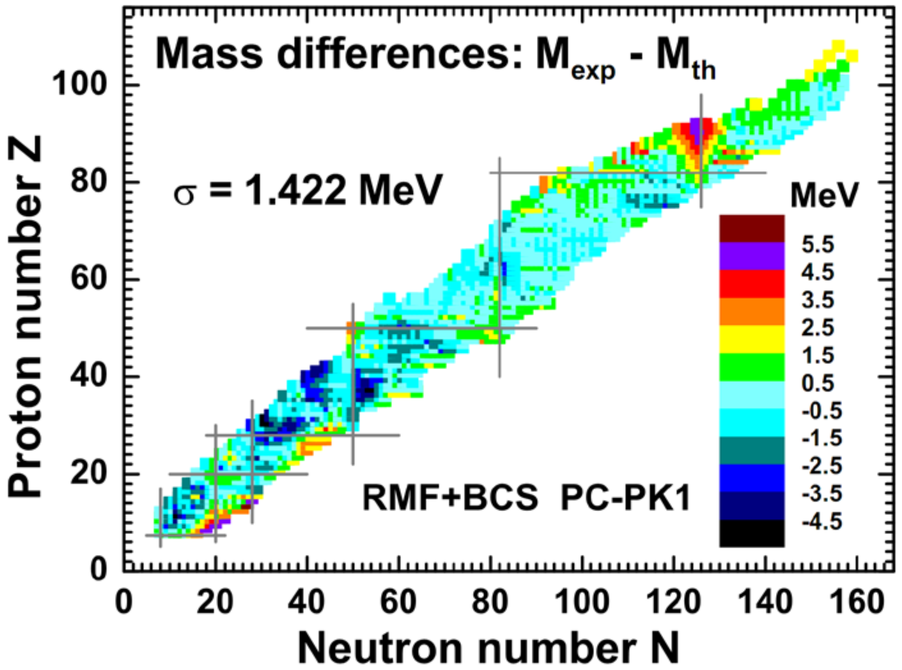
$\beta_{\lambda\mu}$ with even μ are included automatically



- **Axial & reflection symmetric shapes for ground state & isomer, the latter is stiffer**
- **Triaxial shape around the inner barrier**
- **Triaxial & octupole shape around the outer barrier; this is also true for other actinide nuclei**

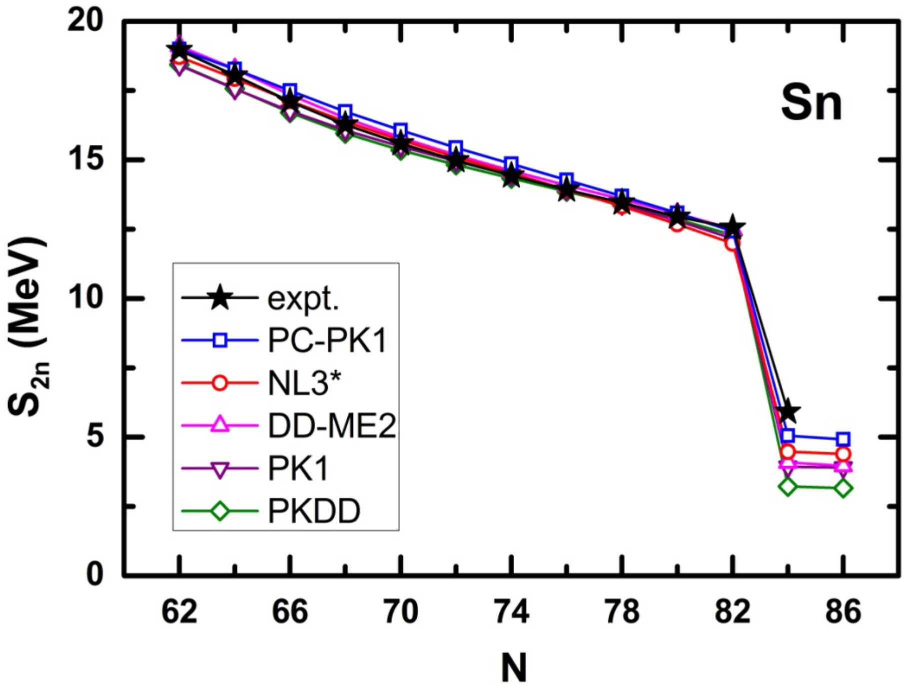
Nuclear Mass

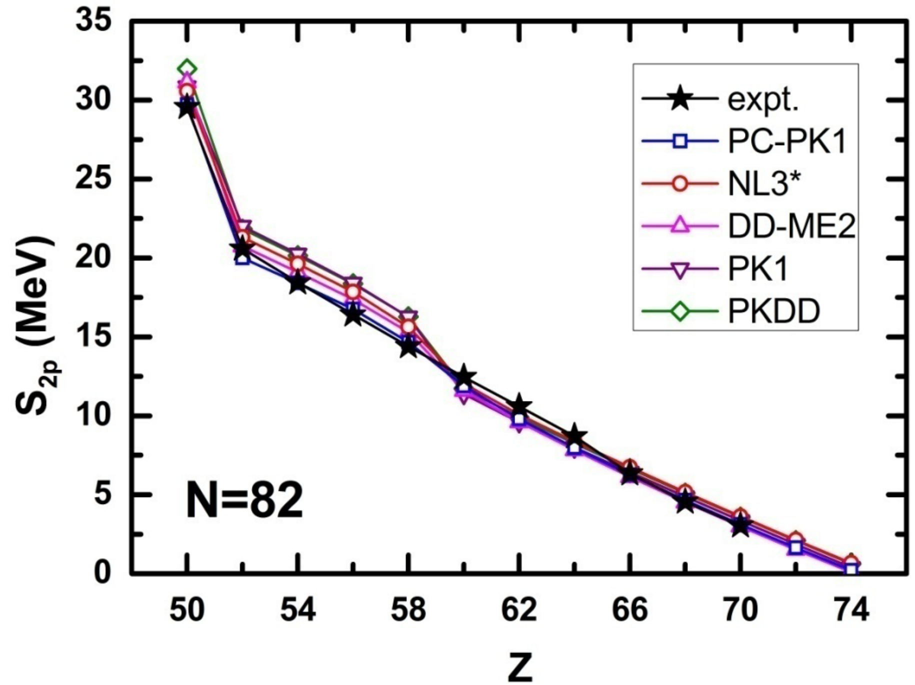




Exp value for 2149 nuclei from Audi et al. NPA2003

Two-neutron & two-proton separation energies



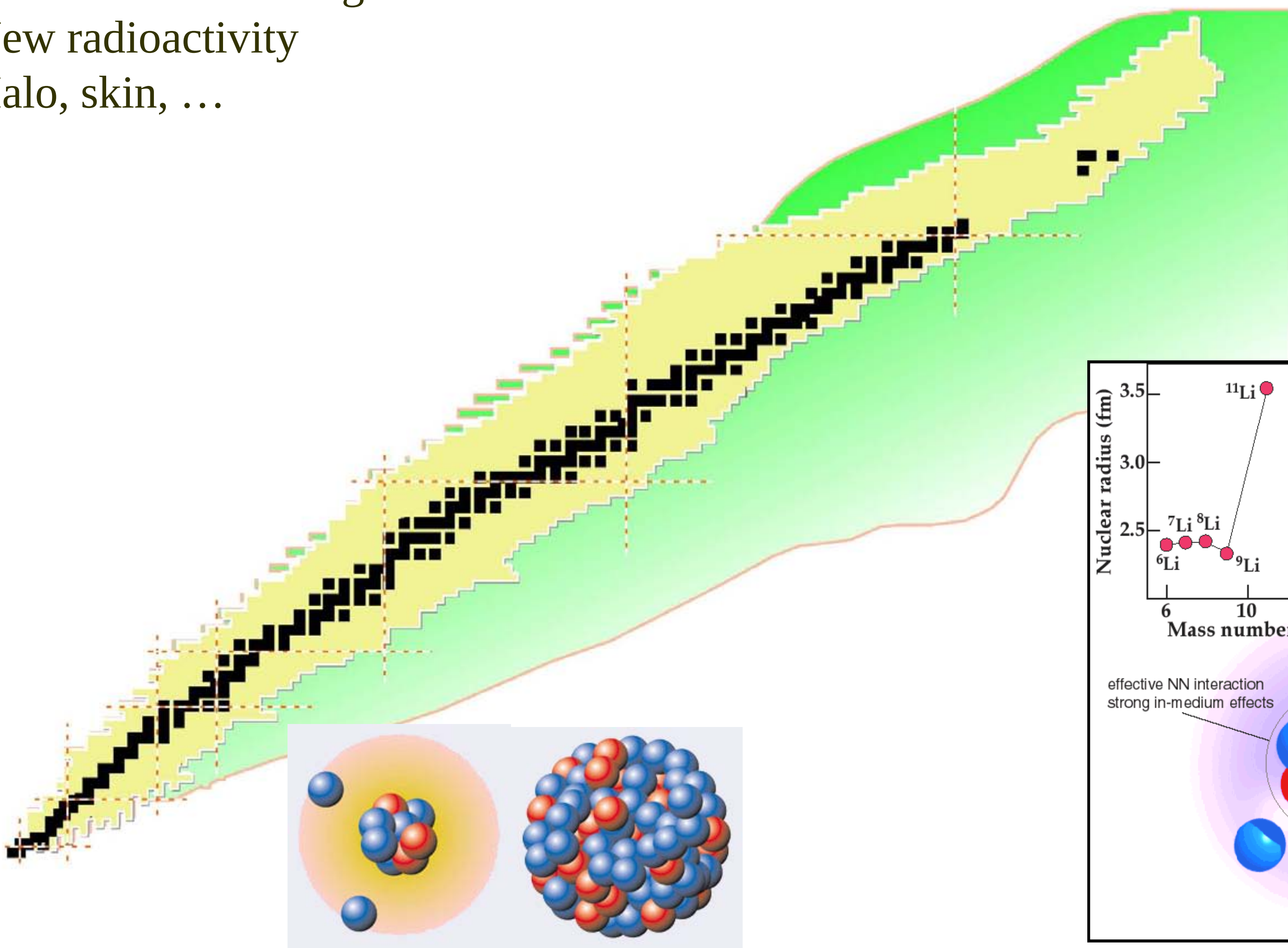


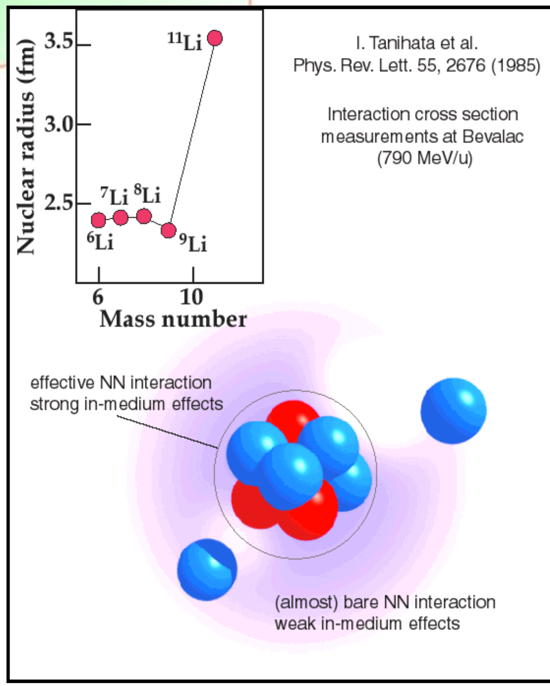
➢ Two-neutron and two-proton separation energies of the Sn isotopes and N=82 isotones.

❖ **The two-neutron and two-proton separation energies are well reproduced**

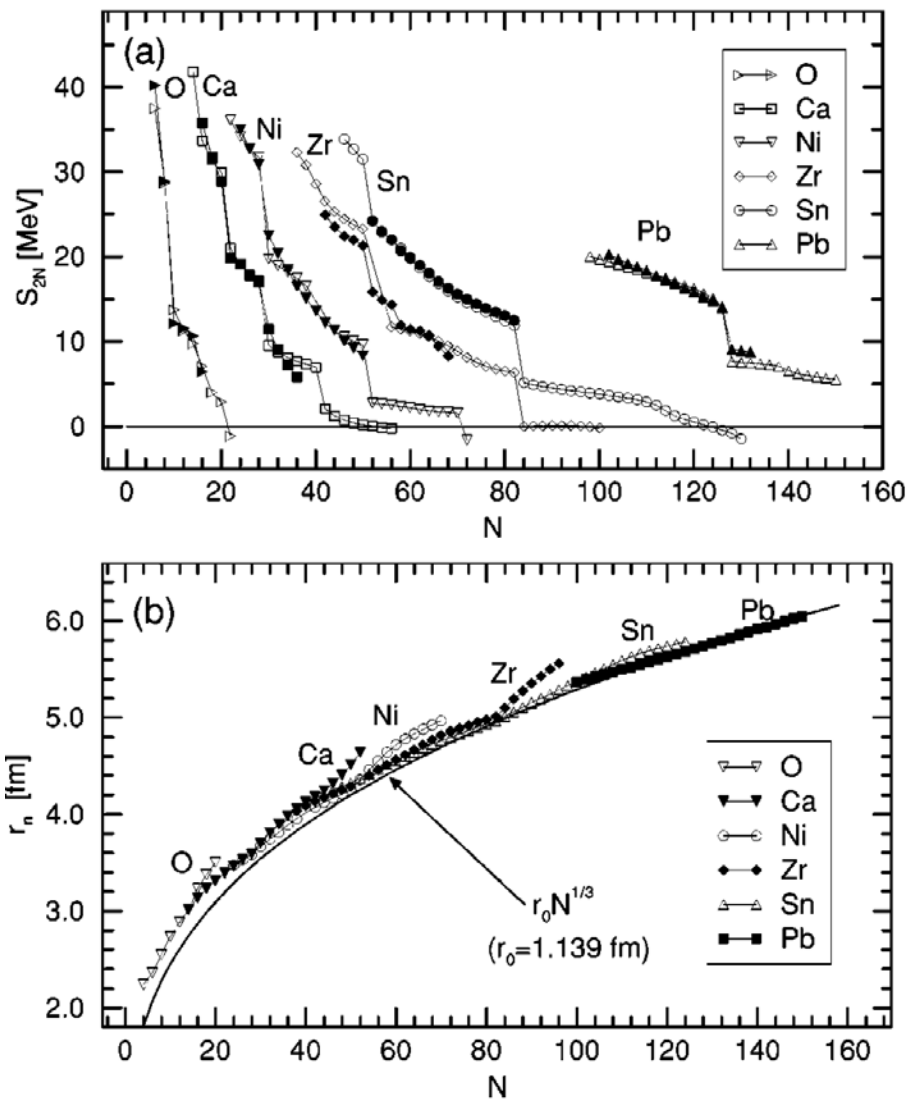
Exotic phenomena in nuclei with extreme N/Z

- Modifications of magic numbers
- New radioactivity
- Halo, skin, …
- …





Description of unstable nucleus and Prediction of giant halo



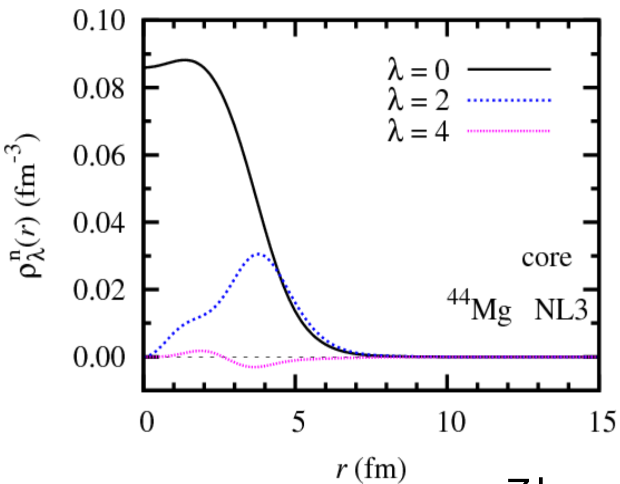
Spherical nucleus:
Meng & Ring, PRL77,3963 (96)
Meng & Ring, PRL80,460 (1998)
Meng, NPA 635, 3-42 (1998)
Meng, Tanihata & Yamaji, PLB 419, 1(1998)
Meng, Toki, Zeng, Zhang & Zhou, PRC65, 041302R

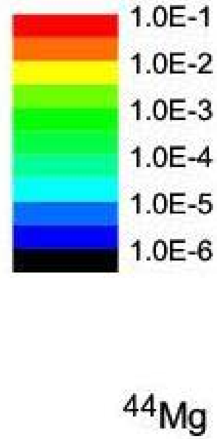
Spherical nucleus but in DDRHFB:
Long, Ring, Meng & Van Giai, PRC81 , 031302

Deformed nucleus:
Zhou, Meng, Ring & Zhao, Phys. Rev. C 82, 011301 (2010)
Li, Meng, Ring, Zhao & Zhou, Phys. Rev. C 85, 024312 (2012)
Chen, Li, Liang & Meng, Phys. Rev. C 85, 067301 (2012)
Li, Meng, Ring, Zhao & Zhou, Chin. Phys. Lett. 29, 042101 (2012).

Reviews:
Meng,Toki, Zhou, Zhang, Long & Geng, PPNP 57. 460 (2006)

Prolate core & oblate halo

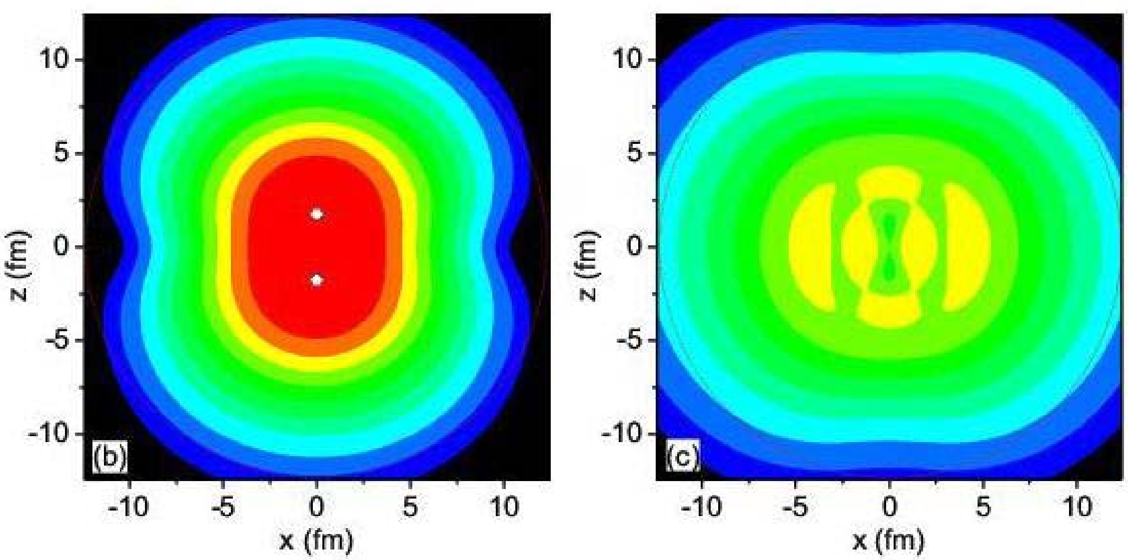


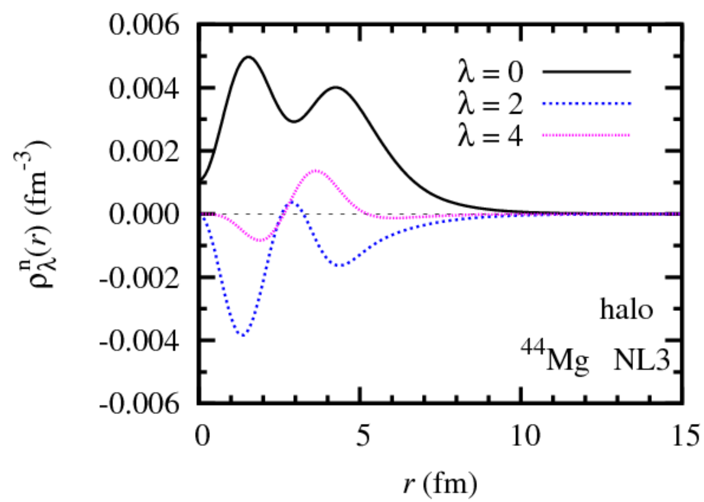


- **Prolate core, but slightly oblate halo with sizable hexadecapole component !**
- **Decoupling of deformation between core & halo**

Zhou, Meng, Ring & Zhao, Phys. Rev. C 82, 011301 (2010)

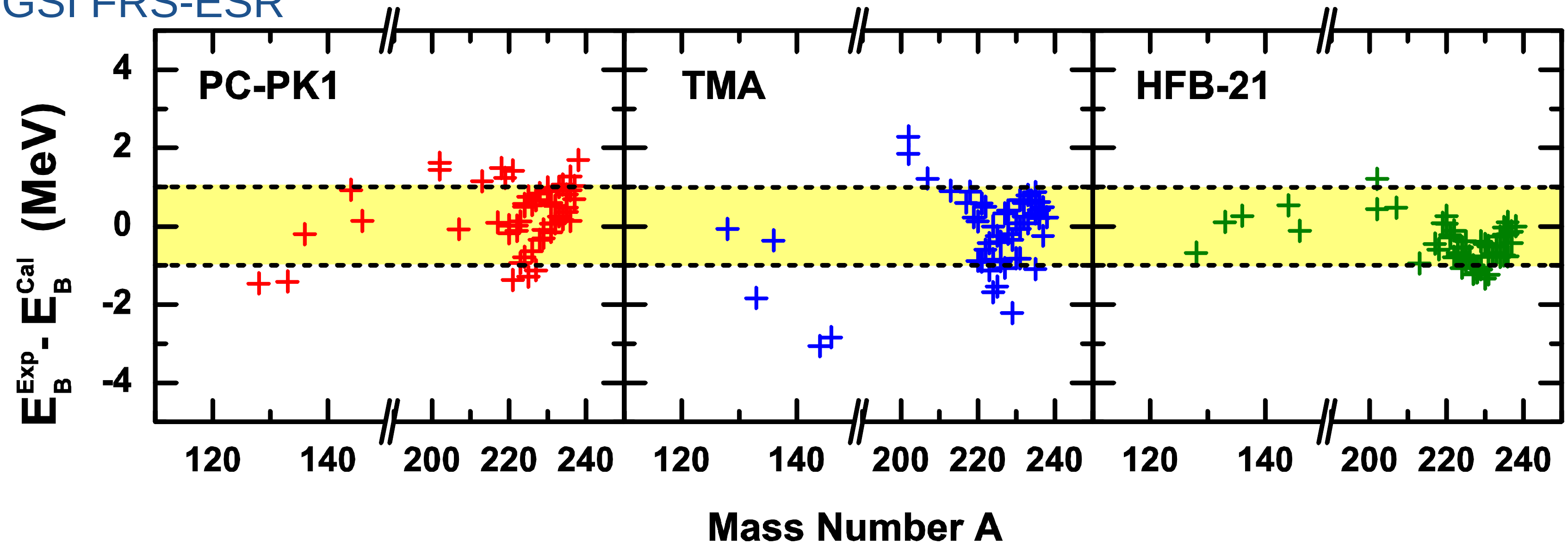
x (fm)

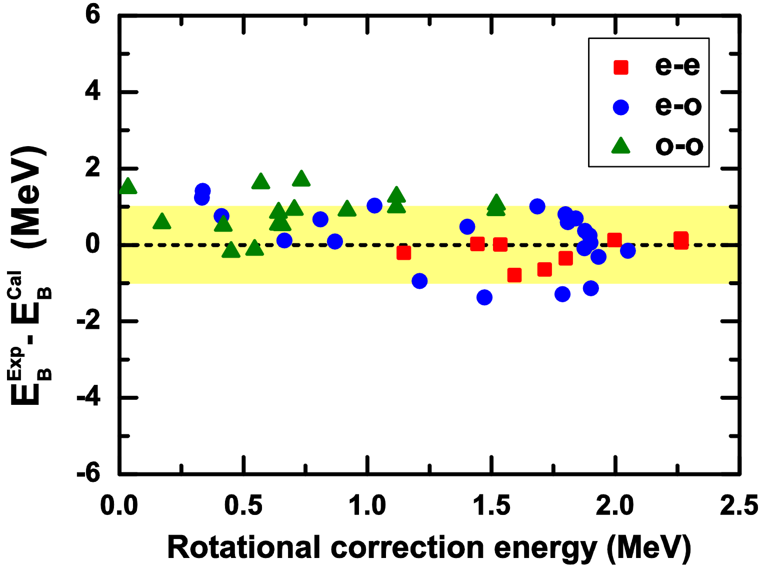


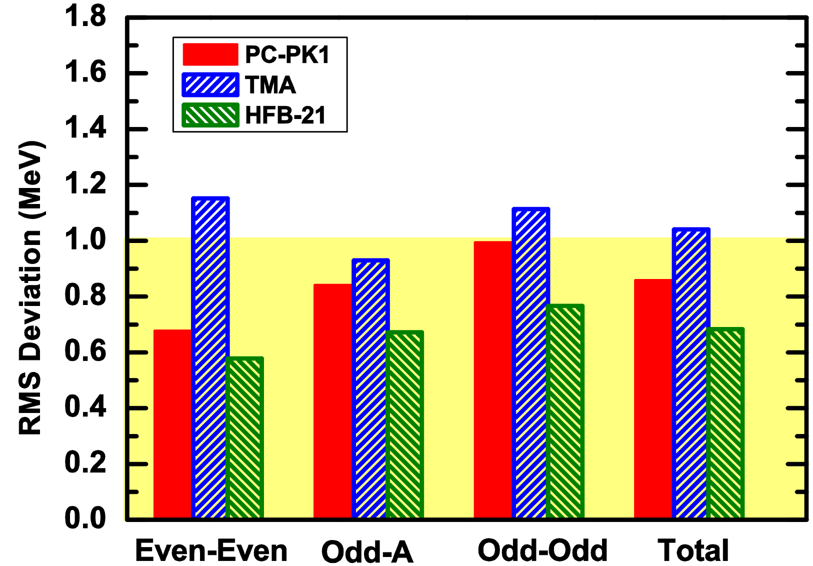


Newly measured masses of heavy neutron-rich nuclei

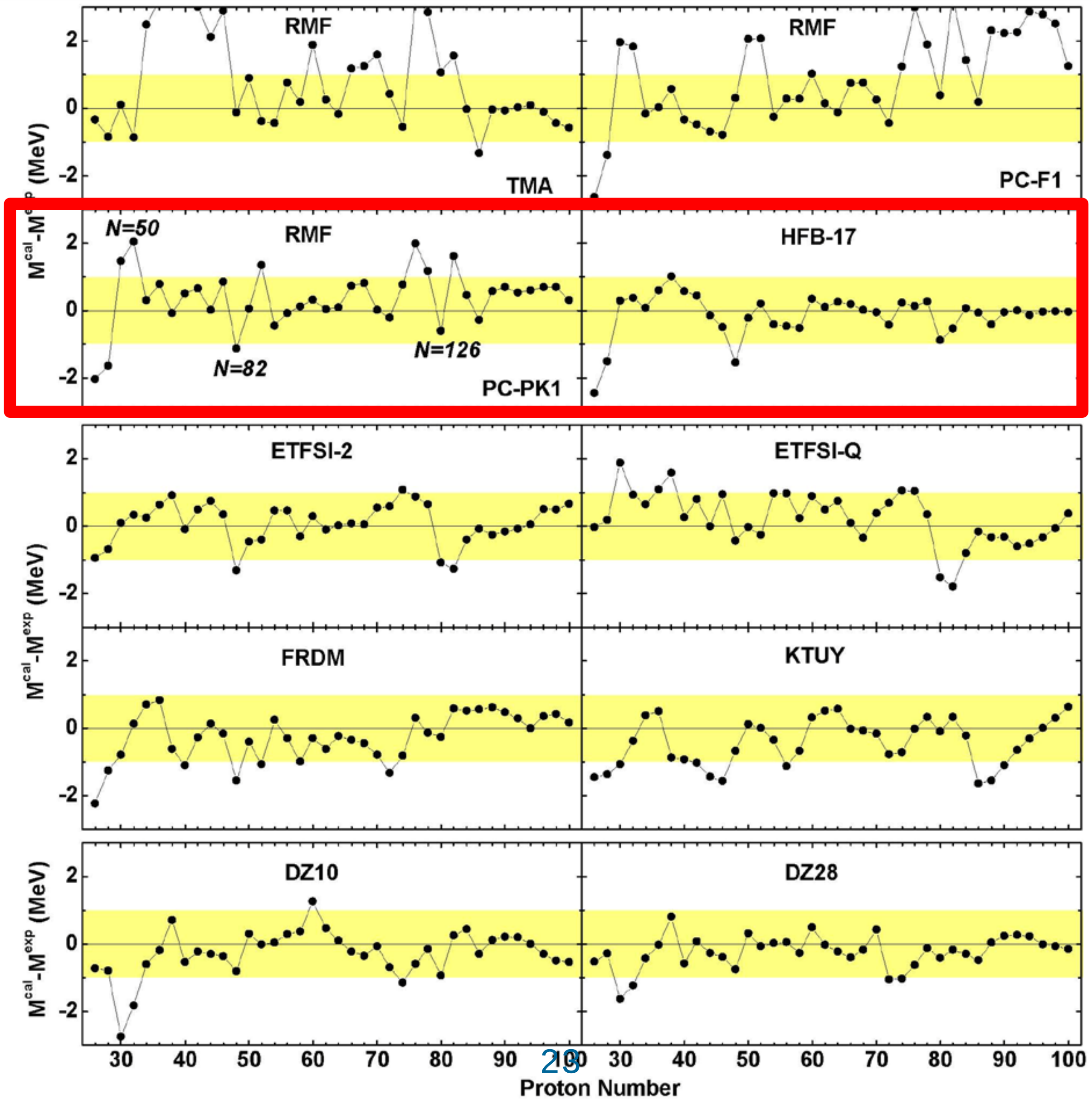
@GSI FRS-ESR







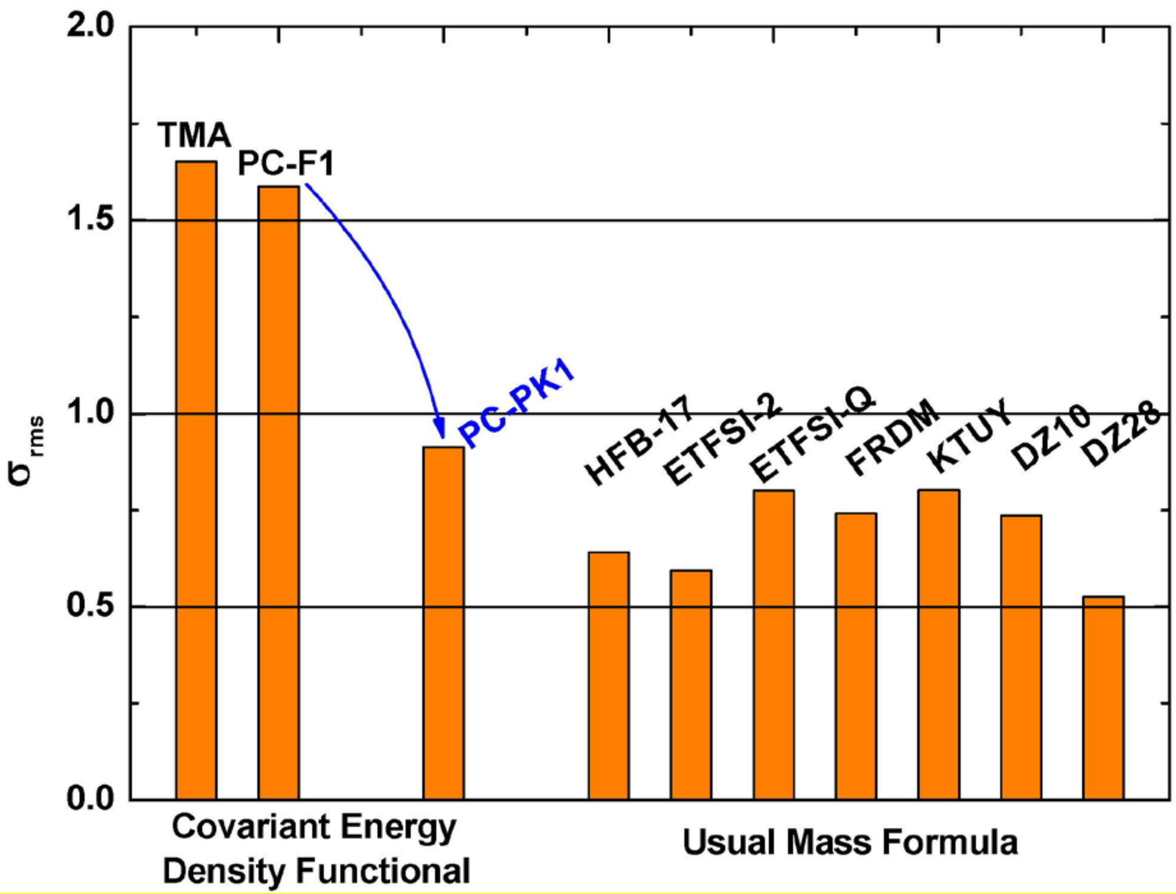
Observed neutron-richest e-e nuclei with 26≤Z≤100



Observed neutron-richest e-e nuclei with 26≤Z≤100

rms mass deviations:

$$\sigma_{rms} = \sqrt{\sum_{i=1}^{N} \frac{(M_i^{cal} - M_i^{exp})^2}{N}}$$



- PC-PK1 improves the description remarkably .
- Similar accuracy for the others

TMA: PTP 113, 785.
PC-F1: PRC 65, 044308
HFB-17: PRL 102, 152503.
ETFSI-2: AIP 529, 287.
ETFSI-Q: PLB 387, 455.
KTUY: PTP 113, 305.
DZ28: PRC 52, R23.

PC-PK1: RRC 82, 054319

Life-time of Neutron Rich Nuclei

APS » Journals » Phys. Rev. Lett. » Volume 106 » Issue 5

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Phys. Rev. Lett. 106, 052502 (2011) [5 pages]

β-Decay Half-Lives of Very Neutron-Rich Kr to Tc Isotopes on the Boundary of the *r*-Process Path: An Indication of Fast *r*-Matter Flow

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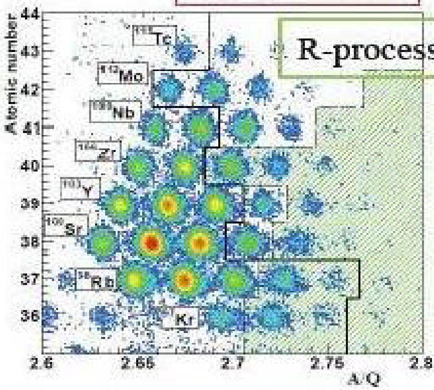
Viewpoint: RNA in cycles

...aphene prêt-à-porter

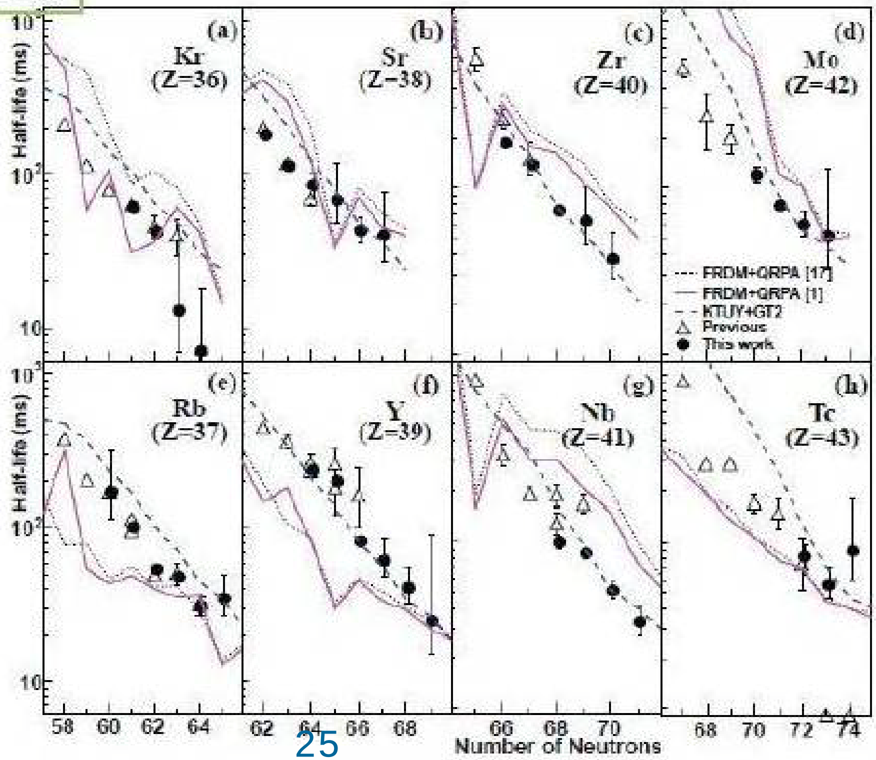
...nic heterogeneity in amorphous

T1/2 unknown

R-process waiting points



S. Nishimura et al., PRL 106 (11) 052502

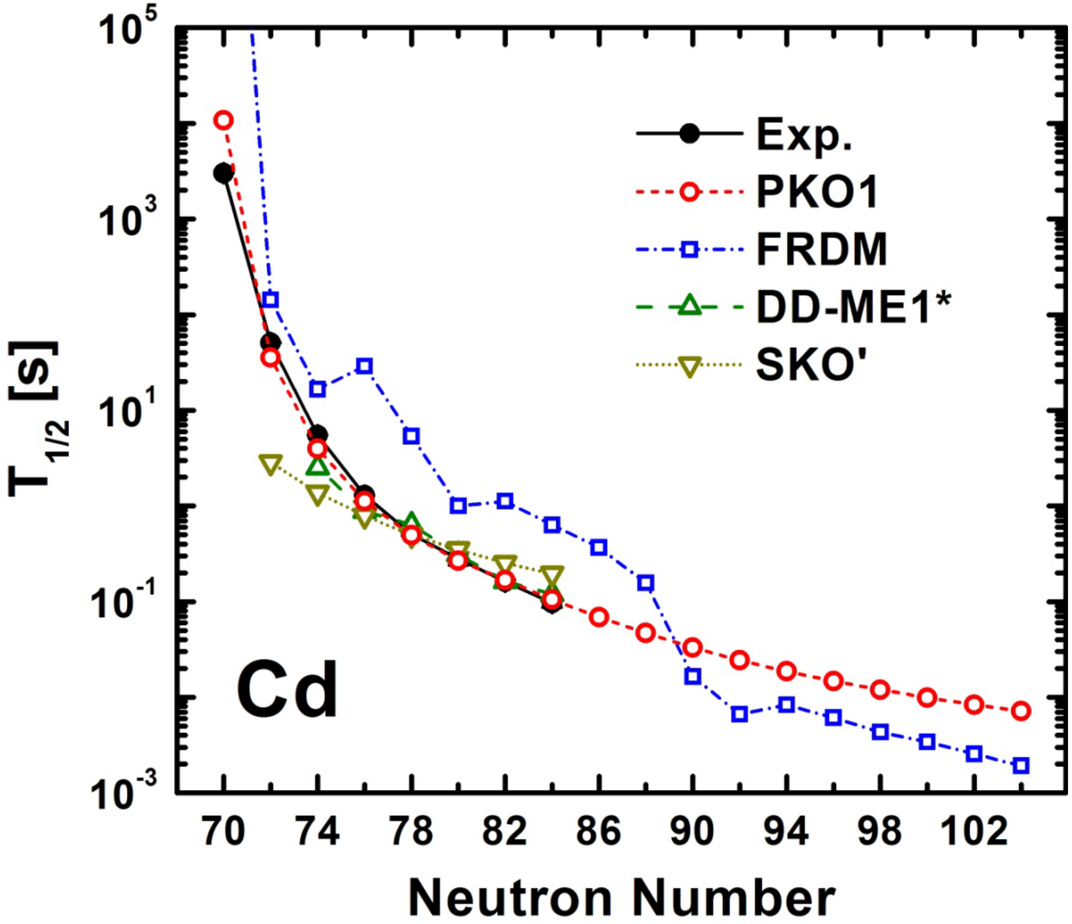


8 hour data acquisition

T1/2 data of 38 isotopes including first data for 18 isotopes

More rapid flow in the rapid neutron-capture process than expected

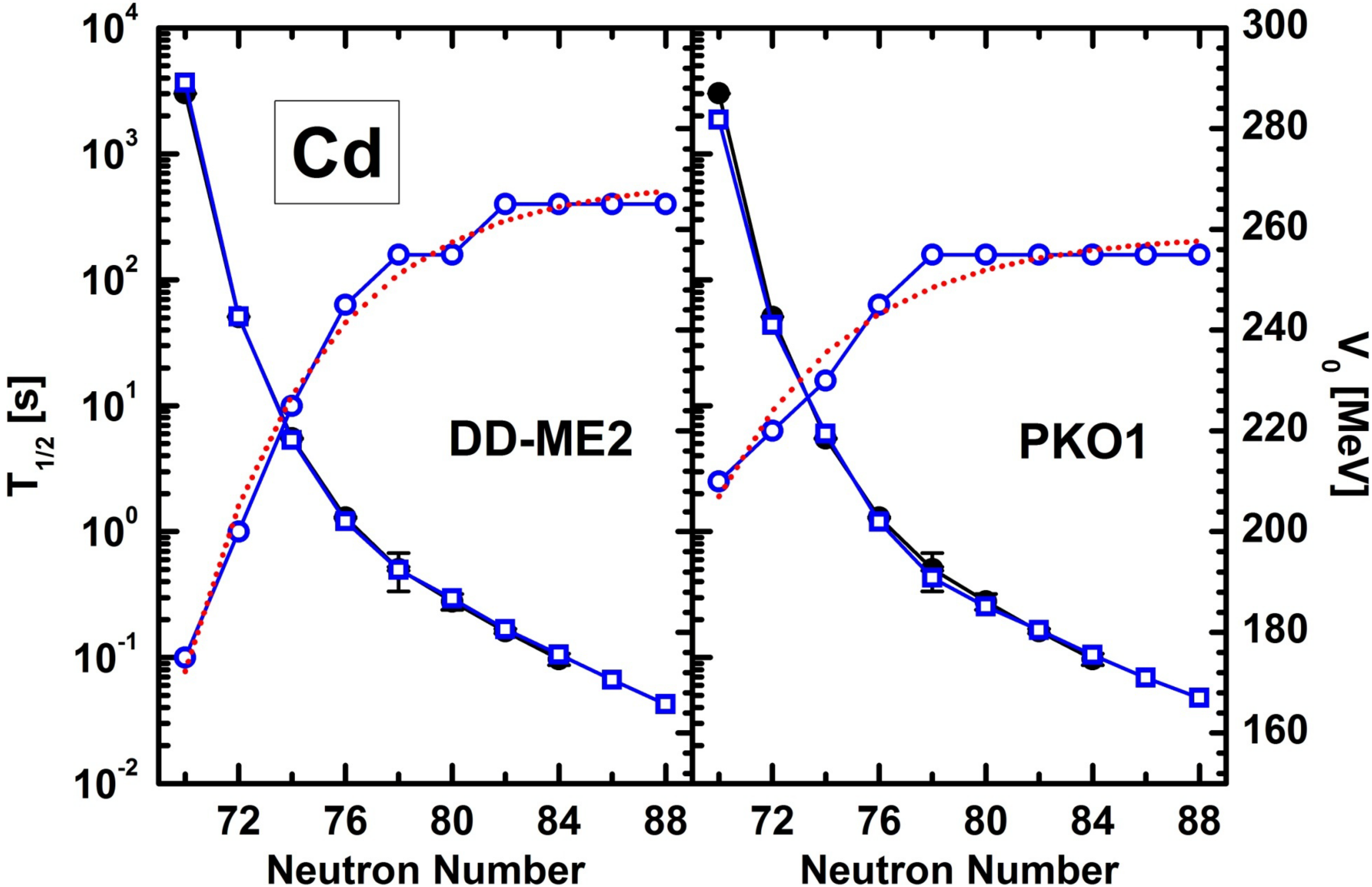
β - decay half - lives of Cd



- RHFB+QRPA: the data is well reproduced, slightly overestimates half - lives of $1^{30,132}$Cd

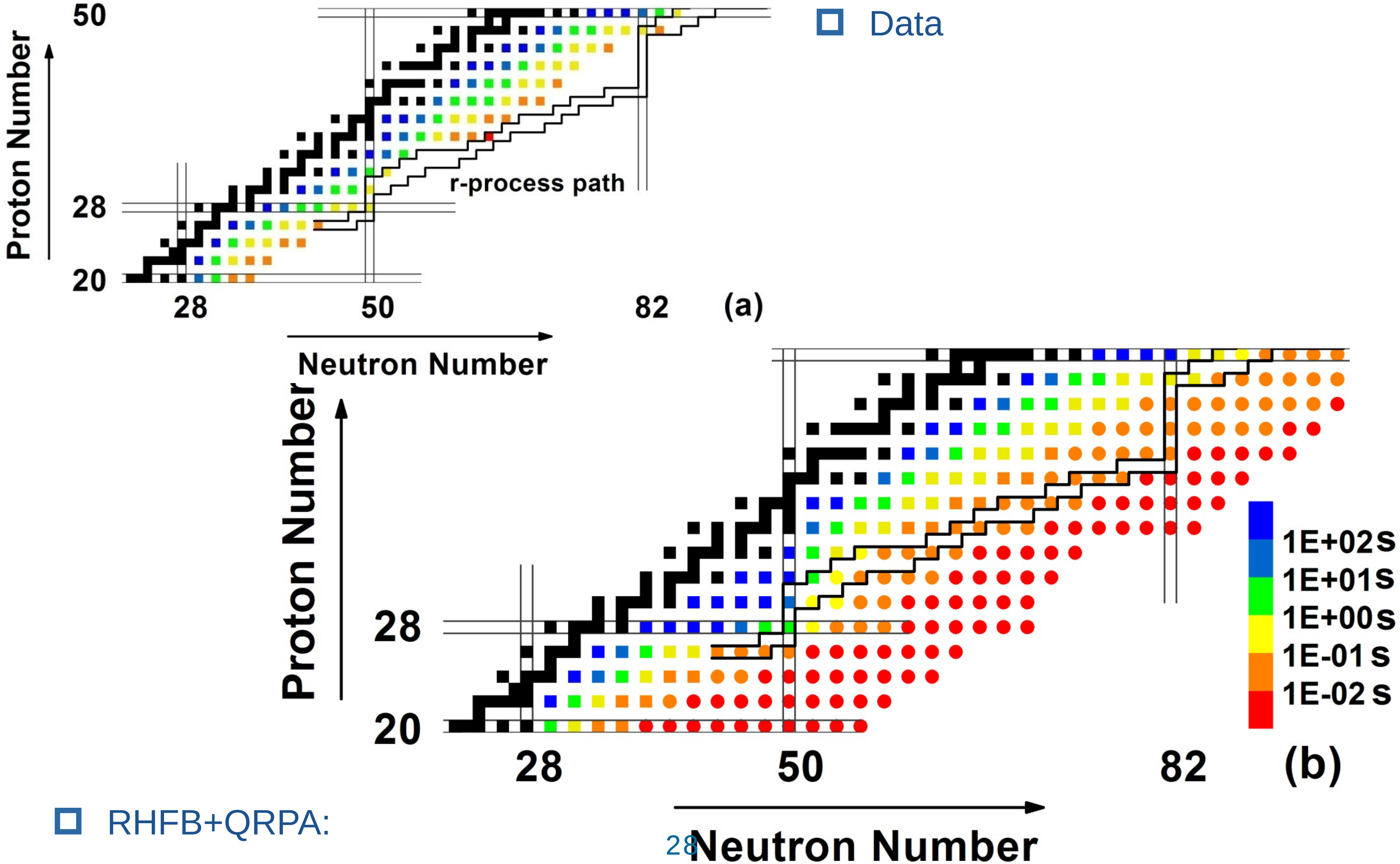
- FRDM+QRPA: overestimates the nuclear half - lives (the pp residual interactions in the T=0 channel are not considered.)

The nuclear β-decay half-lives are sensitive to the strength of T=0 pairing, which significantly reduce the β-decay half-lives.


Cd
DD-ME2
PKO1
$T_{1/2}$ [s]
V_0 [MeV]
Neutron Number
Neutron Number
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
300
280
260
240
220
200
180
160
72
76
80
84
88

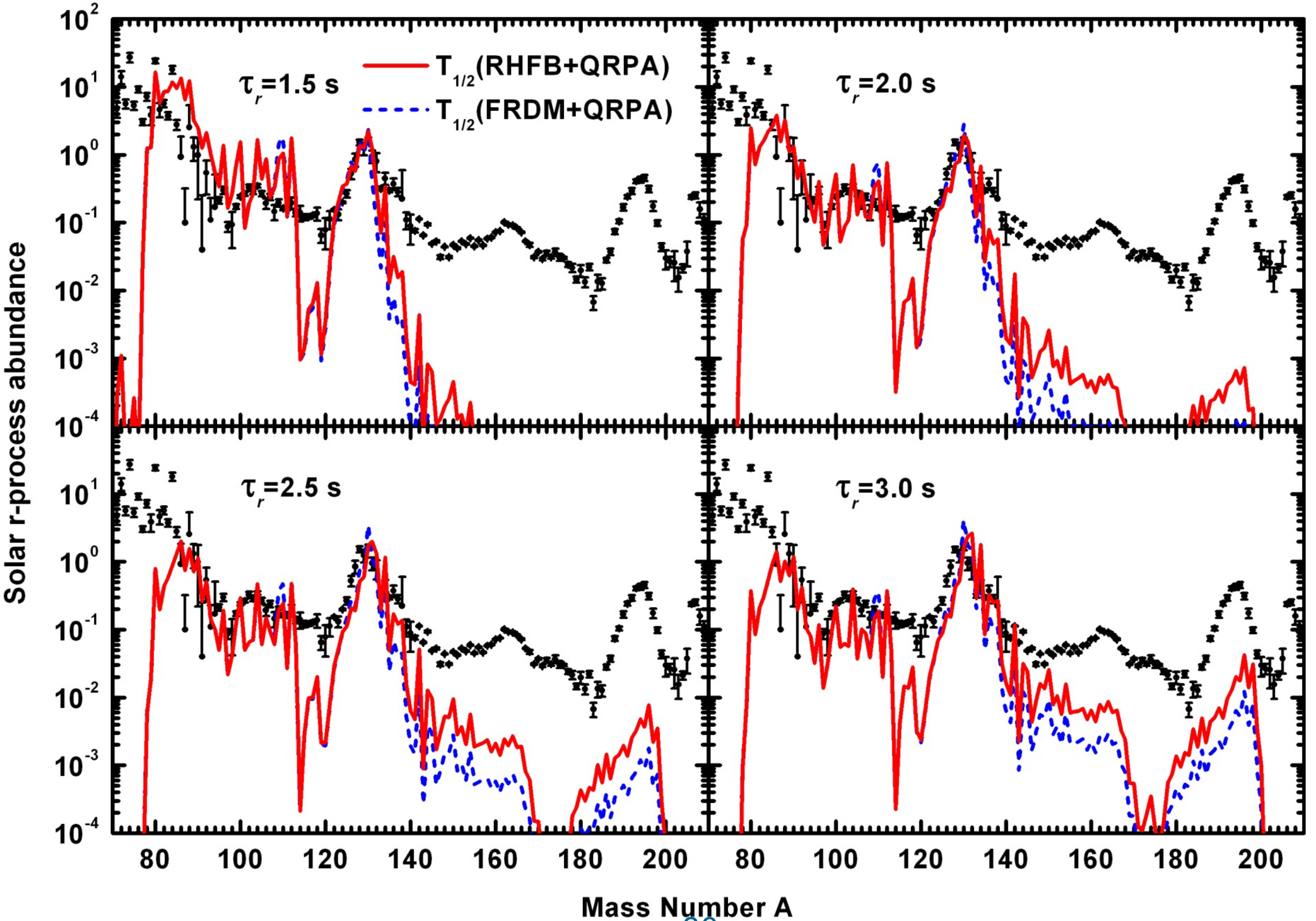
β - decay half - lives of neutron rich nuclei Z=20~50

- Data



- RHFB+QRPA:

Impact on r-process by the β⁻ decay half-lives of Sn, Cd, Pd, Ru, Mo, Zr

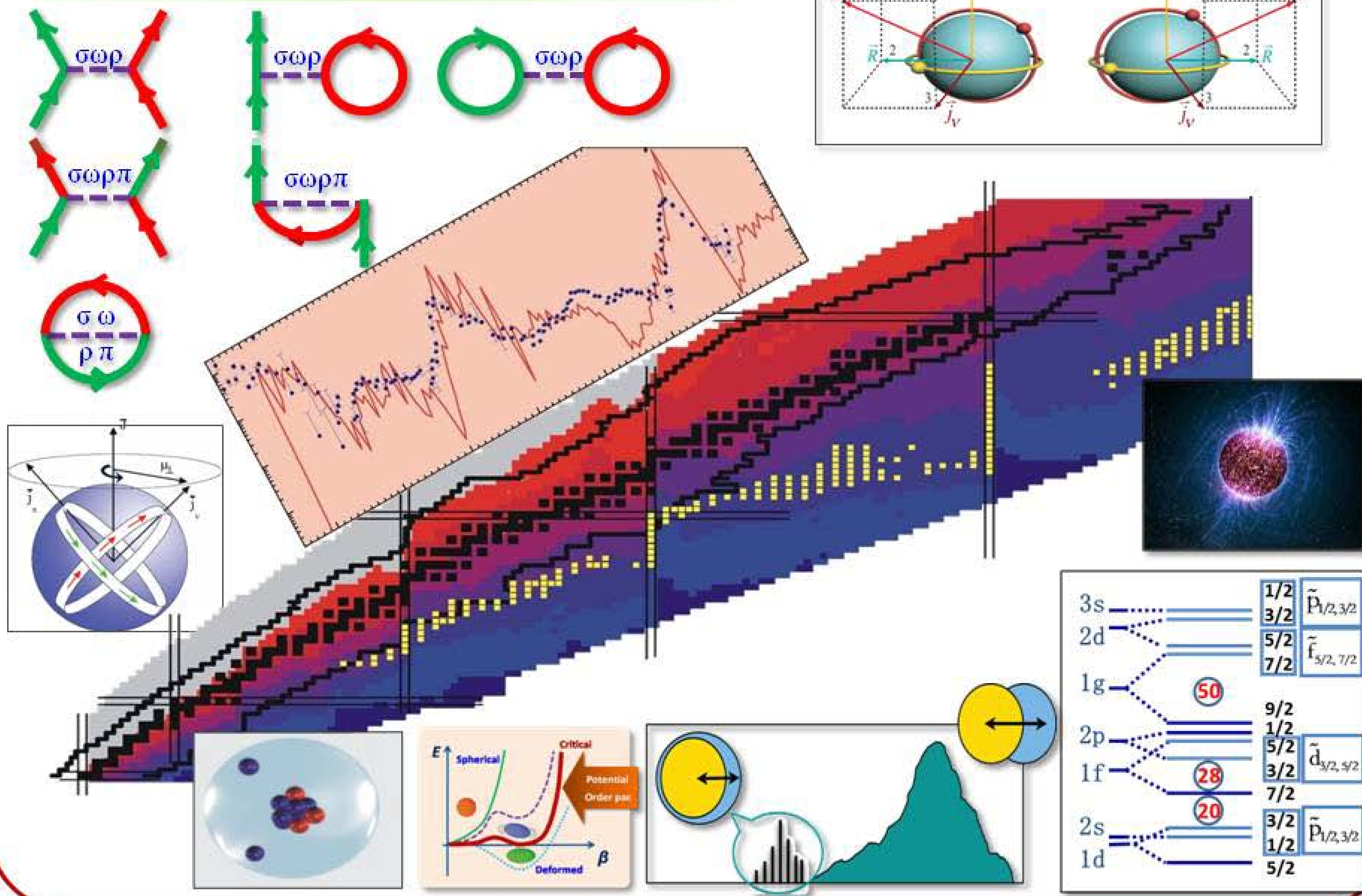


Summary and Perspectives

- Lots of efforts have been devoted for CDFT theory in the last decades
- CDFT theory includes either meson exchange (RH or RHF) or Point Coupling are equally good for nuclear ground properties
- Nuclear dynamic properties can be described well by CDFT of the RHF version or the RH version with improved tensor and pseudo-vector channels
- Newly proposed PC-PK1 provides better description for nuclear mass, nuclear matter EOS, shell structure, magnetic moment, magnetic and antimagnetic rotation, exotic phenomena such as halos, β-decay halflives of neutron rich nuclei
- …

Thank you for your attention!

原子核协变密度泛函理论



Nuclear matter properties

Saturation point: $p(\rho_0) = 0$

- Symmetry energy

$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2$$

$$E_{sym}(\rho) = \frac{1}{2}\left(\frac{\partial^2(\varepsilon/\rho)}{\partial t^2}\right)_{t=0}, t = \frac{\rho_n - \rho_p}{\rho_n}$$

$$L = 3\rho_0\left(\frac{\partial E_{sym}(\rho)}{\partial \rho}\right)_{\rho_0}, K_{sym} = 9\rho_0^2\left(\frac{\partial^2 E_{sym}(\rho)}{\partial \rho^2}\right)_{\rho_0}$$

- Compressibility

$$K(\alpha) \approx K_0 + K_{asy}\alpha^2, \alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$$

$$K_0 = 9\rho_0^2\left[\frac{\partial^2(\varepsilon/\rho)}{\partial \rho^2}\right]_{\rho_0}$$

$$K_{asy} = K_{sym} - 6L$$

- Effective masses

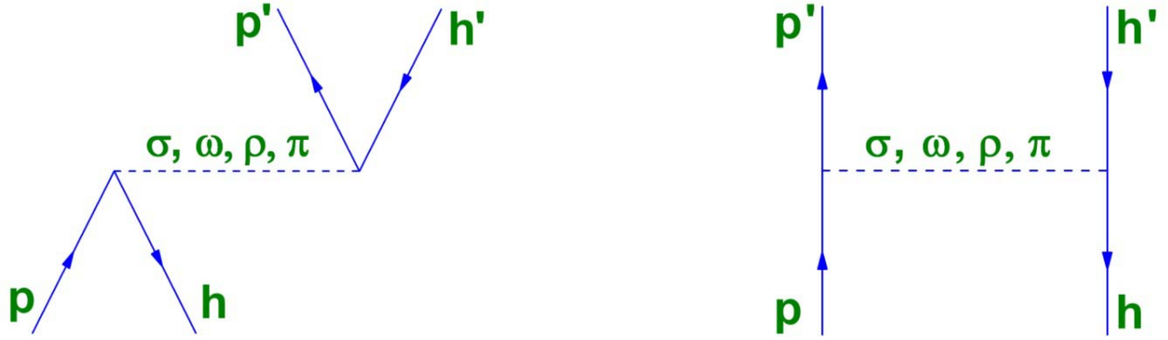
$$M_D^* = M^* = M + S$$

$$M_L^* = \sqrt{(M_D^*)^2 + k_F^2}$$

Merit of RPA based on DDRHF

- Relativistic Hartree + RPA (not self-consistent)
  De Conti:1998,2000, Vretenar:2003, Ma:2004, Paar:2004,2008, Niksic:2005
  - pion is added by hand
  - a parameter to fit E_{GTR}

- Relativistic Hartree-Fock + RPA (fully self-consistent)
  Liang, Giai, Meng, PRL 101, 122502 (2008)
  - both direct and exchange terms are kept
  - pion is naturally included
  - no readjustment of particle-hole residual interaction

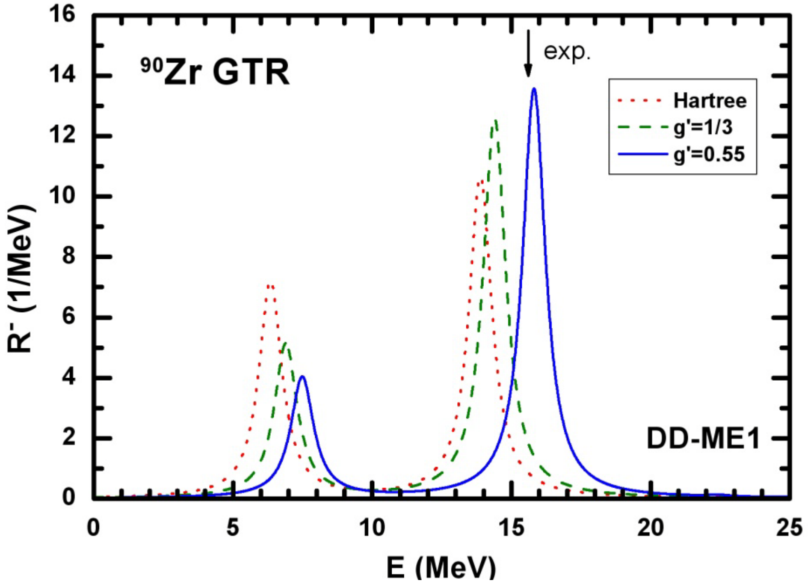


direct term   exchange term

Fock-terms : Physical mechanisms of GTR

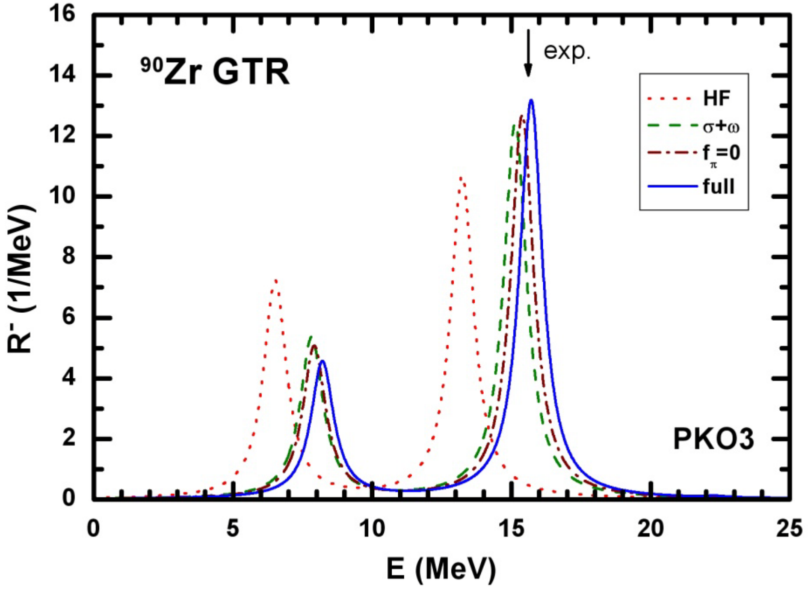
RH + RPA

- ✧ No contribution from isoscalar mesons (σ,ω), because exchange terms are missing.
- ✧ π-meson is dominant in this resonance.
- ✧ g' has to be refitted to reproduce the experimental data.



RHF + RPA

- ✧ Isoscalar mesons (σ,ω) play an essential role via the exchange terms.
- ✧ While, π-meson plays a minor role.
- ✧ g' = 1/3 is kept for self-consistency.



Liang, Giai, Meng, PRL 101, 122502 (2008)

Relativistic Hartree-Fock-Bogoliubov theory

- Effective Lagrangian density

Long, Ring, Giai, Meng, PRC 81, 024308 (2010)
Long, Ring, Meng, Giai, Bertulani, PRC 81, 031302(R) (2010)

$$\mathscr{L} = \bar{\psi}\left[i\gamma^\mu\partial_\mu - M - g_\sigma\sigma - \gamma^\mu\left(g_\omega\omega_\mu + g_\rho\vec{\tau}\cdot\vec{\rho}_\mu + e\frac{1-\tau_3}{2}A_\mu\right) - \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu\partial_\mu\vec{\pi}\cdot\vec{\tau}\right]\psi$$
$$+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu$$
$$+\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}\cdot\vec{\pi} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

- Relativistic Hartree-Fock-Bogoliubov equations Kucharek1991ZPA, Long2010PRC

$$\int d\mathbf{r}'\begin{pmatrix} h(\mathbf{r},\mathbf{r}')-\lambda & \Delta(\mathbf{r},\mathbf{r}') \\ \Delta(\mathbf{r},\mathbf{r}') & -h(\mathbf{r},\mathbf{r}')+\lambda \end{pmatrix}\begin{pmatrix} f_U(\mathbf{r}') \\ f_V(\mathbf{r}') \end{pmatrix} = E\begin{pmatrix} f_U(\mathbf{r}') \\ f_V(\mathbf{r}') \end{pmatrix},$$

where f_U and f_V are the quasiparticle spinors, λ is the chemical potential.

- The average field $h(\mathbf{r},\mathbf{r}')$ and the pairing potential $\Delta(\mathbf{r},\mathbf{r}')$ are given by

$$h(\mathbf{r},\mathbf{r}') = h^{\rm kin}(\mathbf{r},\mathbf{r}') + h^{\rm D}(\mathbf{r},\mathbf{r}') + h^{\rm E}(\mathbf{r},\mathbf{r}'), \quad \Delta_\alpha(\mathbf{r},\mathbf{r}') = -\frac{1}{2}\sum_\beta V_{\alpha\beta}^{pp}(\mathbf{r},\mathbf{r}')\kappa_\beta(\mathbf{r},\mathbf{r}').$$

- $h^{\rm kin}$, $h^{\rm D}$, $h^{\rm E}$: derived from $\mathscr{L}$ with sets PKO1 and DD-ME2 . Long2006PLB, Lalazissis2005PRC
- V^{pp}: phenomenological Gogny force with the set D1S. Berger1991CPC

Role of pion and Exchange term: DDRHFB

- **RHFB equation**

$$\int d\mathbf{r}' \begin{pmatrix} h(\mathbf{r},\mathbf{r}') - \lambda & \Delta(\mathbf{r},\mathbf{r}') \\ \Delta(\mathbf{r},\mathbf{r}') & -h(\mathbf{r},\mathbf{r}') + \lambda \end{pmatrix} \begin{pmatrix} \psi_U(\mathbf{r}') \\ \psi_V(\mathbf{r}') \end{pmatrix} = E \begin{pmatrix} \psi_U(\mathbf{r}) \\ \psi_V(\mathbf{r}) \end{pmatrix}$$

- **Single particle Hamiltonian:** $h = h^{\rm kin} + h^{\rm D} + h^{\rm E}$

Kinetic energy: $h^{\rm kin}(\mathbf{r},\mathbf{r}') = [\alpha \cdot \mathbf{p} + \beta M]\,\delta(\mathbf{r},\mathbf{r}'),$

Local potentials: $h^{\rm D}(\mathbf{r},\mathbf{r}') = [\Sigma_T(\mathbf{r})\gamma_5 + \Sigma_0(\mathbf{r}) + \beta\Sigma_S(\mathbf{r})]\,\delta(\mathbf{r},\mathbf{r}'),$

Non-local Potentials: $h^{\rm E}(\mathbf{r},\mathbf{r}') = \begin{pmatrix} Y_G(\mathbf{r},\mathbf{r}') & Y_F(\mathbf{r},\mathbf{r}') \\ X_G(\mathbf{r},\mathbf{r}') & X_F(\mathbf{r},\mathbf{r}') \end{pmatrix}$

- **Pairing Force: Gogny D1S**

$$V(\mathbf{r},\mathbf{r}') = \sum_{i=1,2} e^{((r-r')/\mu_i)^2}(W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau)$$

- **Dirac Woods-Saxon Basis** **S.-G. Zhou (2003)**

➔ **To Solve the integro-differential RHFB equation**

Quasiparticle random phase approximation

- QRPA equations: Ring1995Springer

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X^{\nu} \\ Y^{\nu} \end{pmatrix} = \omega_{\nu} \begin{pmatrix} X^{\nu} \\ Y^{\nu} \end{pmatrix}$$

where ω_{ν} is the excitation energy, X_{ν} and Y_{ν} denote the 2qp amplitudes. The QRPA matrices A and B read:

$$A_{kk'll'} = (E_k + E_{k'})\delta_{kl}\delta_{k'l'} + \frac{\delta^2 E}{\delta R^*_{kk'} \delta R_{ll'}}, \qquad B_{kk'll'} = \frac{\delta^2 E}{\delta R^*_{kk'} \delta R^*_{ll'}}.$$

- In the canonical basis, the matrices A and B for the charge-exchange channel read:

$$\begin{aligned} A_{pnp'n'} &= H^{11}_{pp'}\delta_{nn'} + H^{11}_{nn'}\delta_{pp'} \\ &\quad + V^{ph}_{pnp'n'}(u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}) \\ &\quad + V^{pp}_{pnp'n'}(u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}), \\ B_{pnp'n'} &= V^{ph}_{pnp'n'}(u_p v_n v_{p'} u_{n'} + v_p u_n u_{p'} v_{n'}) \\ &\quad - V^{pp}_{pnp'n'}(u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}). \end{aligned}$$

RPA QRPA

$$E \rightarrow H^{11}$$
$$u,v = 0,1 \rightarrow u,v = [0,1]$$
$$V^{pp} = 0 \rightarrow V^{pp} \neq 0$$

where

$$H^{11}_{kl} = (u_k u_l - v_k v_l) h_{kl} - (u_k v_l - v_k u_l)\Delta_{kl}.$$

Particle-hole residual interactions

- Particle-hole residual interactions:

$$V^{ph}_{pnp'n'} = \int\int d\mathbf{r}_1 d\mathbf{r}_2 f_p^+(\mathbf{r}_1) f_{n'}^+(\mathbf{r}_2) \sum_{\phi_i} V_{\phi_i}(1,2)\left[f_{p'}(\mathbf{r}_2) f_n(\mathbf{r}_1) - f_n(\mathbf{r}_2) f_{p'}(\mathbf{r}_1)\right],$$

  - ⋆ σ-meson: $V_\sigma(1,2) = -[g_\sigma\gamma_0]_1[g_\sigma\gamma_0]_2 D_\sigma(1,2)$,
  - ⋆ ω-meson: $V_\omega(1,2) = [g_\omega\gamma_0\gamma^\mu]_1[g_\omega\gamma_0\gamma_\mu]_2 D_\omega(1,2)$,
  - ⋆ ρ-meson: $V_\rho(1,2) = [g_\rho\gamma_0\gamma^\mu\vec{\tau}]_1 \cdot [g_\rho\gamma_0\gamma_\mu\vec{\tau}]_2 D_\rho(1,2)$,
  - ⋆ π-meson: $V_\pi(1,2) = -[\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\gamma^k\partial_k]_1 \cdot [\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\gamma^l\partial_l]_2 D_\pi(1,2)$,
  - ⋆ zero-range counter-term of π-meson:

$$V_{\pi\delta}(1,2) = g'[\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\gamma]_1 \cdot [\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\gamma]_2 \delta(\mathbf{r}_1 - \mathbf{r}_2).$$

Parameter set PKO1

- π-meson is included naturally.
- $g' = 1/3$ is maintained for the sake of self-consistency.

Particle-particle residual interactions

- Particle-particle residual interactions:

$$V^{pp}_{pnp'n'} = \int\int d\mathbf{r}_1 d\mathbf{r}_2 f_p^{+}(\mathbf{r}_1) f_n^{+}(\mathbf{r}_2) \sum_{T=1,0} V_T(1,2) \left[f_{n'}(\mathbf{r}_2) f_{p'}(\mathbf{r}_1) - f_{p'}(\mathbf{r}_2) f_{n'}(\mathbf{r}_1) \right],$$

⋆ $T = 1$ channel: Gogny force

$$V_{T=1}(1,2) = \sum_{i=1,2} e^{-r_{12}^2/\mu_i^2} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau),$$

with the set D1S, the same as those in RHFB calculations. Berger1991CPC

⋆ $T = 0$ channel:

$$V_{T=0}(1,2) = -V_0 \sum_{j=1}^{2} g_j e^{-r_{12}^2/\mu_j^2} \hat{\prod}_{S=1,T=0},$$

where $\mu_1 = 1.2$ fm, $\mu_2 = 0.7$ fm, $g_1 = 1$, $g_2 = -2$, V_0 is determined by fitting to the measured half-lives of nuclei. Engel1999PRC

Nuclear β-decay half lives

- The nuclear β-decay half-life in the allowed Gamow-Teller approximation reads as follows:

$$T_{1/2} = \frac{\ln 2}{\lambda_\beta} = \frac{D}{g_A^2 \sum_m \left| \sum_{pn} \langle 1_m^+ | \sigma\tau | 0^+ \rangle \right|^2 f(Z, A, E_m)},$$

where $D = \frac{\hbar^7 2\pi^3 \ln 2}{g^2 m_e^5 c^4} = 6163.4$ s, $g_A = 1$. The transition probability $\langle 1_m^+ | \sigma\tau | 0^+ \rangle$ can be directly taken from the QRPA calculations.

★ The integrated $(e, \bar{\nu}_e)$ phase volume $f(Z,A,E_m)$:

$$f(Z, A, E_m) = \frac{1}{m_e^5} \int_{m_e}^{E_m} p_e E_e (E_m - E_e)^2 F(Z, A, E_m) dE_e,$$

★ The maximum value of β-decay energy E_m:

$$E_m = E_i - E_f = (m_n - m_p) - E_{QRPA} = \Delta_{np} - E_{QRPA}.$$

Due to $E_m > m_e$, the sum on m runs over all final states with E_{QRPA} smaller than $\Delta_{nH} = \Delta_{np} - m_e = 0.782$ MeV.