Symmetry energy in bulk matter from neutron skin thickness in finite nuclei

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M. Centelles, X. Roca-Maza, X. Viñas and M. Warda,
Phys. Rev. Lett. **102 122502 (2009)**
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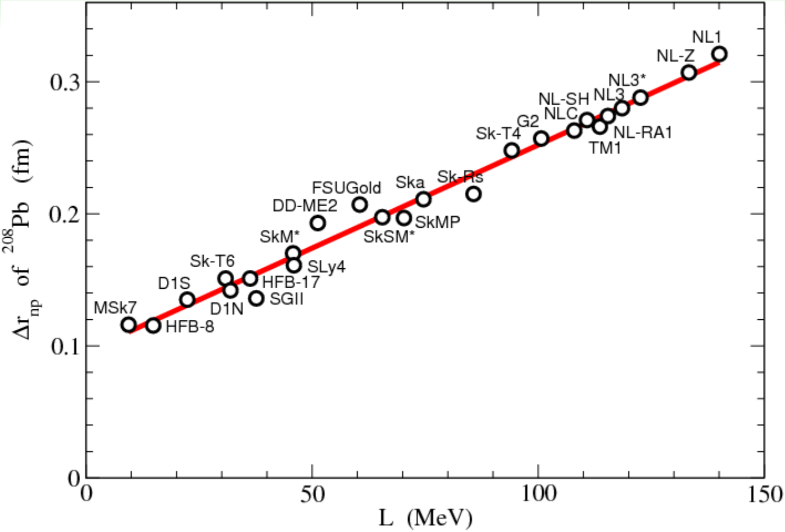
Phys. Rev. C81 054309 (2010)
X. Roca-Maza, M. Centelles, X. Viñas and M. Warda,
Phys. Rev. Lett. 106 252501 (2011)

Why is important the nuclear symmetry energy ?

The nuclear symmetry energy is a fundamental quantity in Nuclear Physics and Astrophysics because it governs, at the same time, important properties of very small entities like the atomic nucleus ($R \sim 10^{-15}$ m) and very large objects as neutron stars ($R \sim 10^{4}$ m)

- Nuclear Physics: Neutron skin thickness in finite nuclei, structure of neutron rich nuclei, Heavy-Ion collisions, Giant Resonances....
- High-Energy Physics: Test of the Standard Model through atomic parity non-conservation observables.
- Astrophysics: Supernova explosion, Neutron emission and cooling of protoneutron stars, Mass-Radius relations in neutron stars, Composition of the crust of neutron stars...



$$\Delta r_{np} = \langle r^2 \rangle_n^{1/2} - \langle r^2 \rangle_p^{1/2}$$

Equation of State in asymmetric matter

$$e(\rho,\delta) = e(\rho,0) + c_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) \qquad \left(\delta = \frac{\rho_n - \rho_p}{\rho}\right)$$

Around the saturation density we can write

$$e(\rho,0) \simeq a_v + \frac{1}{2}K_v\epsilon^2 \quad and \quad c_{sym}(\rho) \simeq J - L\epsilon + \frac{1}{2}K_{sym}\epsilon^2 \quad \left(\epsilon = \frac{\rho_0 - \rho}{3\rho_0}\right)$$

$$\rho_0 \approx 0.16 fm^{-3}, \quad a_v \approx -16 MeV, \quad K_v \approx 230 MeV, \quad J \approx 32 MeV$$

However, the values of

$$L = 3\rho\partial c_{sym}(\rho)/\partial\rho|_{\rho_0} \qquad and \qquad K_{sym} = 9\rho^2\partial^2 c_{sym}(\rho)/\partial\rho^2|_{\rho_0}$$

which govern the density dependence of c_{sym} near ρ_0 are less certain and predictions vary largely among nuclear theories.

Symmetry energy and neutron skin thickness in the Liquid Drop Model

- Symmetry Energy

$$a_{sym}(A) = \frac{J}{1 + x_A}, \quad x_A = \frac{9J}{4Q} A^{-1/3}$$

$$E_{sym}(A) = a_{sym}(A)(I + x_A I_C)^2 A$$

where

$$I = (N - Z)/A, \quad I_{\mathrm{C}} = e^2 Z/(20JR), \quad R = r_0 A^{1/3}$$

.

- Neutron skin thickness

$$S = \sqrt{3/5}\left[t - e^2 Z/(70J) + \frac{5}{2R}(b_n^2 - b_p^2)\right]$$

where

$$t = \frac{3r_0}{2}\,\frac{J/Q}{1 + x_A}\,(I - I_{\mathrm{C}}) = \frac{2r_0}{3J}\,[J - a_{sym}(A)]\,A^{1/3}\,(I - I_{\mathrm{C}})$$

M. Centelles, M. Del Estal and X. Viñas, Nucl. Phys. **A635**, 193 (1998)

The $c_{sym}(\rho)$-$a_{sym}(A)$ correlation

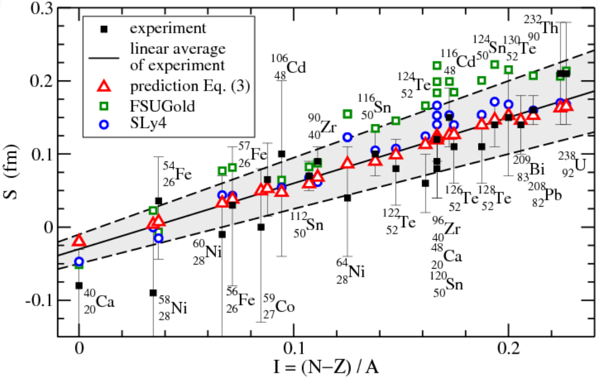
- There is a genuine relation between the symmetry energy coefficients of the EOS and of nuclei: $c_{sym}(\rho)$ equals $a_{sym}(A)$ of heavy nuclei like ^{208}Pb at a density $\rho = 0.1 \pm 0.01\,\mathrm{fm}^{-3}$ practically independent of the mean field model used to compute them.
- A similar situation occurs down to medium mass numbers, at lower densities.
- We find that this density can be very well simulated by

$$\rho \approx \rho_A = \rho_0 - \rho_0/(1 + cA^{1/3})\,,$$

  where c is fixed by the condition $\rho_{208} = 0.1\,\mathrm{fm}^{-3}$.
- Using the equality $c_{sym}(\rho) = a_{sym}(A)$ and the LDM , the neutron skin thickness can be finally written as:

$$S = \sqrt{\frac{3}{5}}\frac{2r_0}{3}\frac{L}{J}\left(1 - \epsilon\frac{K_{sym}}{2L}\right)\epsilon A^{1/3}(I - I_{\mathrm{C}})$$

- See also Lie-Wen Chen Phys. Rev. **C83**, 044308 (2011)

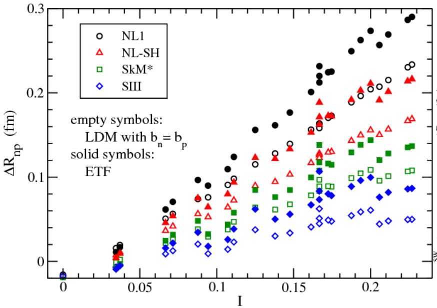


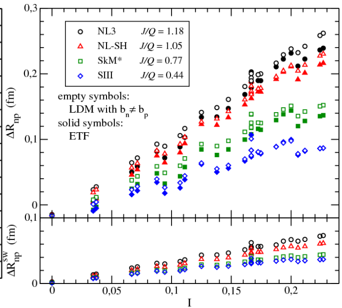
$$S = (0.9 \pm 0.15) I + (-0.03 \pm 0.02) \text{ fm}$$

A. Trzcińska et al, Phys. Rev. Lett. **87**, 082501 (2001)

Assuming $c(\rho) = 31.6(\rho/\rho_0)^\gamma$ with ρ_0=0.16 fm^{-3} we predict ($b_n = b_p$): $L = 75 \pm 25$ MeV

Influence of the surface width ($b_n \neq b_p$)



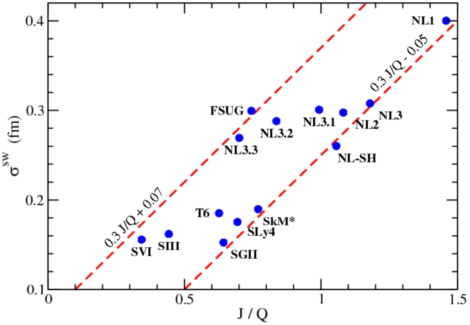


$$\sqrt{\frac{3}{5}}\frac{5}{2R}(b_n^2 - b_p^2) = 0.31I(NL3) - 0.15I(SGII)$$

b_n and b_p are obtained semiclassically at ETF level

M.Centelles et al. NPA **635**, 193 (1998).

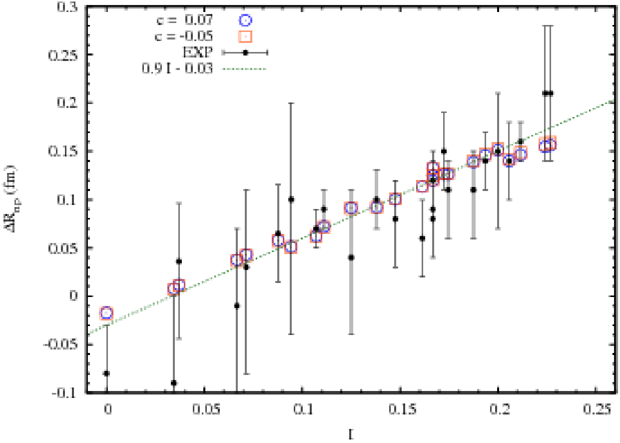
Surface contribution to the neutron skin thickness



$$\sqrt{\frac{3}{5}}\frac{5}{2R}(b_n^2 - b_p^2) = \sigma^{sw} I = (0.3\frac{J}{Q} + c)I$$

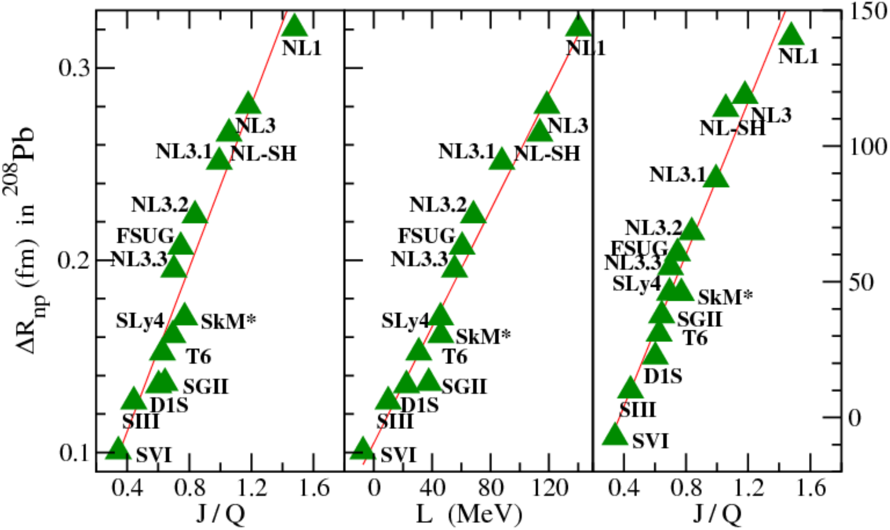
$$c = 0.07fm \quad and \quad c = -0.05$$

Fit and results

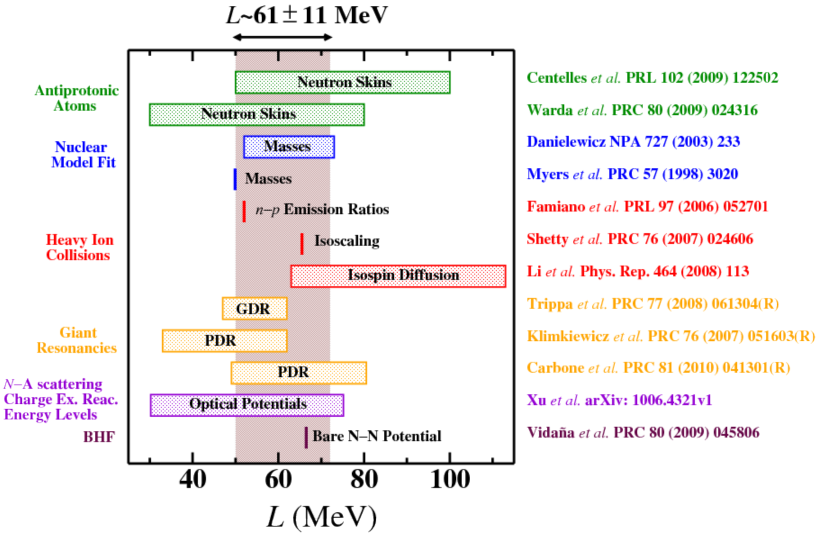


$$\frac{J}{Q} = 0.6 - 0.9 \qquad L = 31 - 81 MeV$$

Neutron skin thickness



Constraints on the slope of the symmetry energy



What can we learn from parity-violating electron scattering ?

- See C.J. Horowitz et al. Phys. Rev. **C63**, 025501 (2001); Shufang Ban et al arXiv:1010.3246 [nucl-th]

- A_{LR} is the parity-violating asymmetry

- $A_{LR} \equiv \dfrac{\frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega}}{\frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega}}$

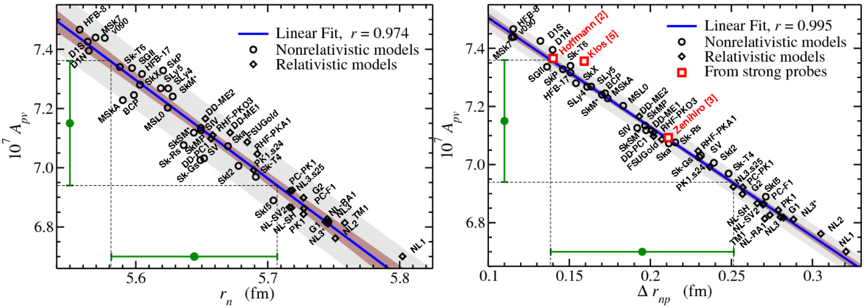
- $V_{\pm}(r) = V_{\mathrm{Coulomb}}(r) \pm V_{\mathrm{weak}}(r)$

- $V_{\mathrm{weak}}(r) = \frac{G_F}{2^{3/2}}\left[(1-4\sin^2\theta_W)Z\rho_p(r) - N\rho_n(r)\right]$

- $A_{LR}^{\mathrm{PWBA}} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{F_n(q)-F_p(q)}{F_p(q)}\right]$

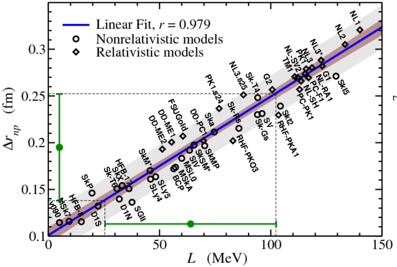
- PREX experiment $E \sim 1.05$ GeV and $\theta \sim 5^\circ$

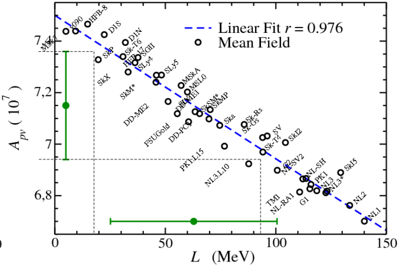
From parity-violating electron scattering



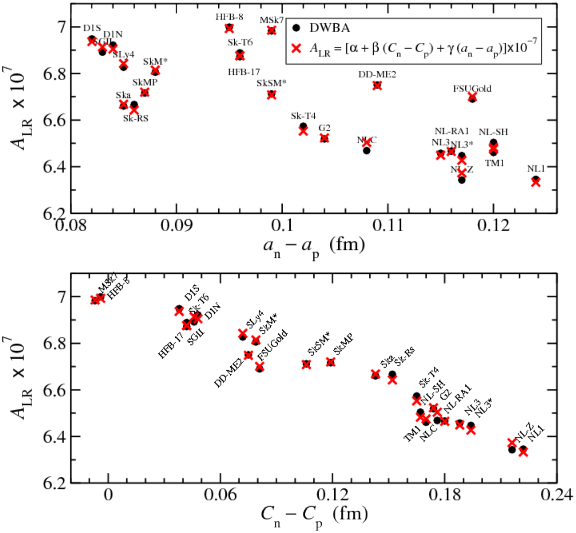
with E=1.06 GeV and $\theta = 5^\circ$

From parity-violating electron scattering





From parity-violating electron scattering

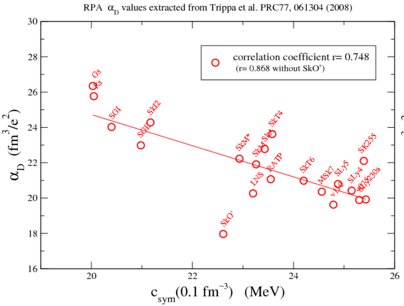


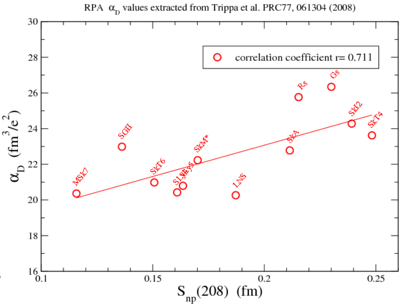
with E=1 GeV and $\theta = 5^\circ$

Dipole polarizability and symmetry energy

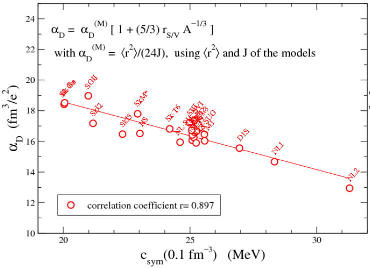
The hydrodynamical model of Lipparini and Stringari (Phys. Rep. **175**, 103 (1989) (see also W.Satula et al, Phys. Rev. **C74**, 011301 (2009)) suggest a relation between the dipole polarizability and the bulk and surface contributions to the symmetry energy:

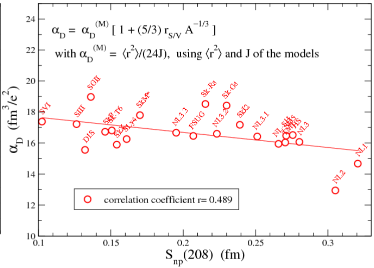
$$\alpha_{hydro} = \frac{A < r^2 >}{24J}\left(1 + \frac{5}{3}\frac{J - a_{sym}}{J}A^{-1/3}\right); \quad E_{-1} = \sqrt{\frac{\hbar^2 A(1+\kappa)}{4m\alpha(D)}}$$

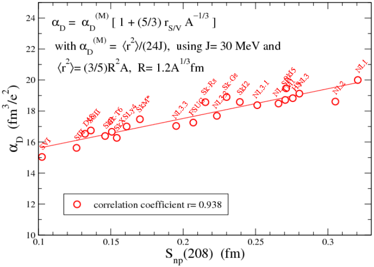




Dipole polarizability and symmetry energy





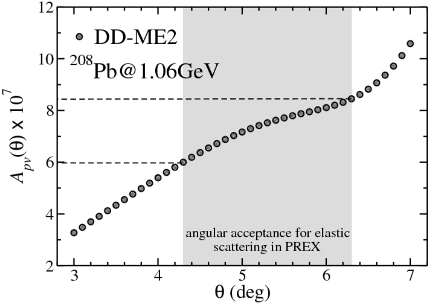


Summary and Conclusions

- We have described a generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation.
- We take advantage of this relation to explore constraints on $c_{sym}(\rho)$ from neutron skins measured in antiprotonic atoms. These constraints points towards a soft symmetry energy.
- We discuss the L values constrained by neutron skins in comparison with most recent observations from reactions and giant resonances.
- We learn that in spite of present error bars in the data of antiprotonic atoms, the size of the final uncertainties in L is comparable to the other analyses.

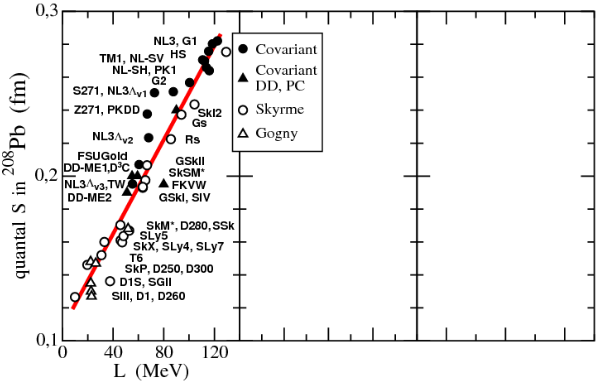
- We have investigated parity-violating electron scattering in nuclear models constrained by available experimental data to extract the neutron radius and skin of ^{208}Pb without specific assumptions on the shape of the nucleon densities.
- We have demonstrated a linear correlation, universal in mean field framework, between A_{pv} and Δr_{np} that has very small scatter.
- It is predicted that a 1% measurement of A_{pv} would allow to constrain the slope L of the symmetry energy to near a novel 10 MeV level.
- We have found a simple parametrization of the parity-violating asymmetry for electron scattering in terms of the parameters $C_n - C_p$ and $a_n - a_p$ of the equivalent 2pF distributions.

From parity-violating electron scattering



- The generic relation between the symmetry The generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation plausibly encompasses other prime correlations of nuclear observables with the density content of the symmetry energy as e.g. the constrains of $c_{sym}(0.1)$ from the GDR of ^{208}Pb (L. Trippa et al. Phys. Rev. **C77**, 061304(R) (2008)).
- The properties of $c_{sym}(\rho)$ derived from terrestrial nuclei also have intimate connections to astrophysics. As an example, we can estimate the transition density ρ_t between the crust and the core of a neutron star as $\rho_t/\rho_0 \sim 2/3 + (2/3)^\gamma K_{sym}/2K_v$ (J. M. Lattimer, M. Prakash, Phys. Rep. **442**, 109 (2007)). . The constraints from neutron skins hereby yield $\rho_t \sim 0.095 \pm 0.01$ fm^{-3}. This value would not support the direct URCA process of cooling of a neutron star that requires a higher ρ_t. Our prediction is in consonance with $\rho_t \sim 0.096\,\mathrm{fm}^{-3}$ of the microscopic EOS of Friedman and Pandharipande as well as with $\rho_t \sim 0.09\,\mathrm{fm}^{-3}$ predicted by a recent $\rho_t \sim 0.09\,\mathrm{fm}^{-3}$ predicted by a recent analysis of pygmy dipole resonances in nuclei.

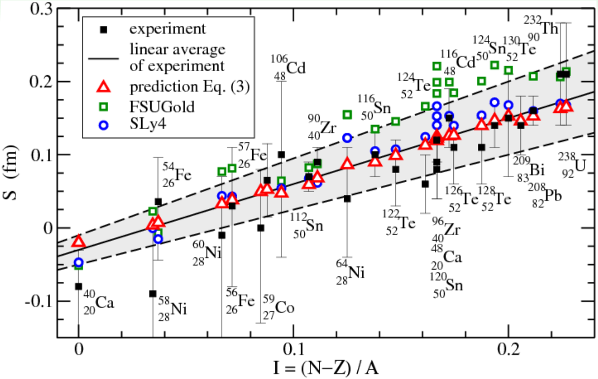
Neutron skin thickness



$$\frac{de(\rho,\delta=1)}{d\rho} = \frac{L}{3\rho_0} - \frac{K+K_{sym}}{3\rho_0}\epsilon \qquad \frac{dc_{sym}(\rho)}{d\rho} = \frac{L}{3\rho_0} - \frac{K_{sym}}{3\rho_0}\epsilon$$

What is experimentally know about neutron skin thickness in nuclei ?

- The neutron skin thickness is defined as $S = R_n - R_p$, where R_n and R_p are the rms of the neutron and proton distributions respectively.
- R_p is known very accurately from elastic electron scattering measurements
- R_n has been obtained with hadronic probes such as:
  a) Proton-nucleus elastic scattering
  b) Inelastic scattering excitation of the giant dipole and spin-dipole resonances
  c) Antiprotonic atoms: Data from antiprotonic X rays and radiochemical analysis of the yields after the antiproton annihilation



$$S = (0.9 \pm 0.15) I + (-0.03 \pm 0.02) \text{ fm}$$

A. Trzcińska et al, Phys. Rev. Lett. **87**, 082501 (2001)

CAN THE NEUTRON SKIN THICKNESS of 26 STABLE NUCLEI, FROM ^{40}Ca TO ^{238}U, ESTIMATED USING ANTIPROTONIC ATOMS DATA CONSTRAINT THE SLOPE AND CURVATURE OF c_{sym} ?

Some technical details

- The surface stiffness coeficient Q and the surface widths b_n and b_p are obtained from self-consistent calculations of the neutron and proton density profiles in asymmetric semi-infinite nuclear matter.
- To this end one has to minimize the total energy per unit area with the constraint of conservation of the number of protons and neutrons with respect to arbitrary variations of the densities.

$$\frac{E_{\rm const}}{S} = \int_{-\infty}^{\infty} \left[\varepsilon(z) - \mu_n\rho_n(z) - \mu_p\rho_p(z)\right] dz,$$

  where $\varepsilon(z)$ is the nuclear energy density functional.
- In the non-relativistic framework the densities ρ_n and ρ_p obey the coupled local Euler-Lagrange equations:

$$\frac{\delta\varepsilon(z)}{\delta\rho_n} - \mu_n = 0, \qquad \frac{\delta\varepsilon(z)}{\delta\rho_p} - \mu_p = 0.$$

  The relative neutron excess $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is a function of the z-coordinate. When $z \to -\infty$, the densities ρ_n and ρ_p approach the values of asymmetric uniform nuclear matter in equilibrium with a bulk neutron excess δ_0.

- From the calculated density profiles one computes:

$$z_{oq} = \frac{\int_{-\infty}^{\infty} z\rho_q'(z)dz}{\int_{-\infty}^{\infty} \rho_q'(z)dz},$$

$$b_q^2 = \frac{\int_{-\infty}^{\infty} (z - z_{0q})^2 \rho_q'(z)dz}{\int_{-\infty}^{\infty} \rho_q'(z)dz}.$$

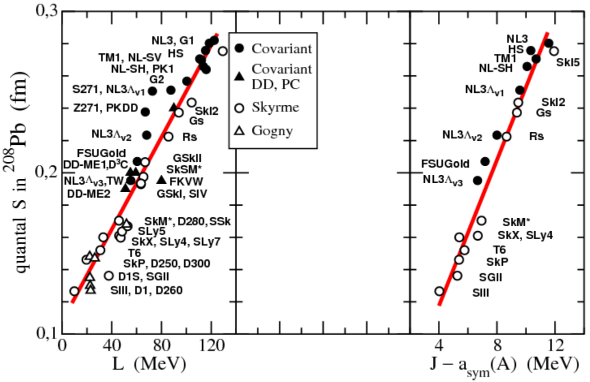
- From the relation

$$t = z_{0n} - z_{0p} = \frac{3r_0}{2}\frac{J}{Q}\delta_0,$$

  one can evaluate Q from the slope of t at $\delta_0 = 0$.
- The distance t and the surface widths b_n and b_p in finite nuclei with neutron excess $I = (N_Z)/A$ are obtained using δ_0 given by:

$$\delta_0 = \frac{I + \frac{3}{8}\frac{c_1}{Q}\frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4}\frac{J}{Q}A^{-1/3}}.$$

Neutron skin thickness



$$\frac{de(\rho, \delta = 1)}{d\rho} = \frac{L}{3\rho_0} - \frac{K + K_{sym}}{3\rho_0}\epsilon \qquad \frac{dc_{sym}(\rho)}{d\rho} = \frac{L}{3\rho_0} - \frac{K_{sym}}{3\rho_0}\epsilon$$

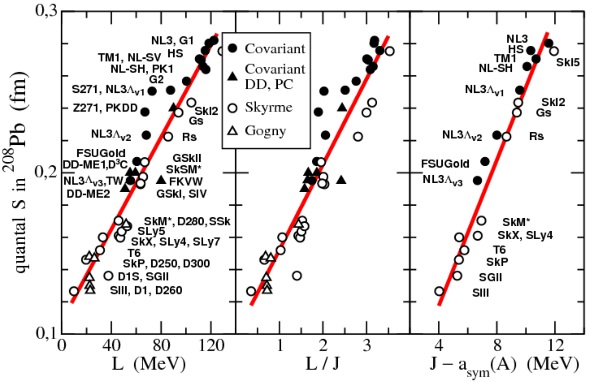
Table: Value of $a_{sym}(A)$ and density ρ that exactly fulfils $c_{sym}(\rho) = a_{sym}(A)$ for $A = 208, 116, 40$, in various nuclear models. J and a_{sym} are in MeV and ρ is in fm^{-3}.

		$A = 208$		$A = 116$		$A = 40$	
Model	J	a_{sym}	ρ	a_{sym}	ρ	a_{sym}	ρ
NL3	37.4	25.8	0.103	24.2	0.096	21.1	0.083
NL-SH	36.1	25.8	0.105	24.6	0.099	21.3	0.086
FSUGold	32.6	25.4	0.098	24.2	0.090	21.9	0.075
TF-MS	32.6	24.2	0.093	22.9	0.085	20.3	0.068
SLy4	32.0	25.3	0.100	24.2	0.091	22.0	0.075
SkX	31.1	25.7	0.102	24.8	0.096	22.8	0.082
SkM*	30.0	23.2	0.101	22.0	0.093	19.9	0.078
SIII	28.2	24.1	0.093	23.4	0.088	21.8	0.077
SGII	26.8	21.6	0.104	20.7	0.096	18.9	0.082

$$\rho \approx \rho_A = \rho_0 - \rho_0/(1 + cA^{1/3}),$$

with c fixed by the condition $\rho_{208} = 0.1\,\mathrm{fm}^{-3}$.

Neutron skin thickness



$$\frac{de(\rho,\delta=1)}{d\rho} = \frac{L}{3\rho_0} - \frac{K+K_{sym}}{3\rho_0}\epsilon \qquad \frac{dc_{sym}(\rho)}{d\rho} = \frac{L}{3\rho_0} - \frac{K_{sym}}{3\rho_0}\epsilon$$

Fitting procedure and results

- We optimeze

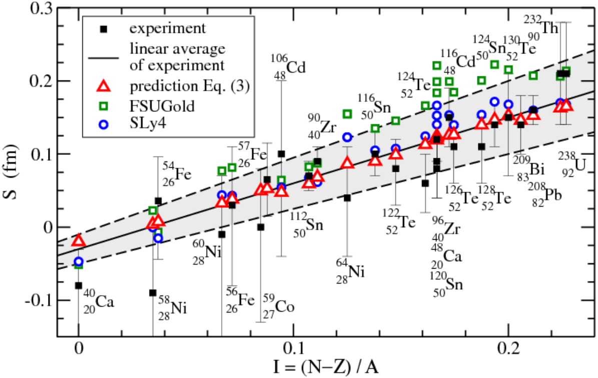
$$S = \sqrt{\frac{3}{5}}\frac{2r_0}{3}\frac{L}{J}\left(1-\epsilon\frac{K_{sym}}{2L}\right)\epsilon A^{1/3}(I-I_{\mathrm{C}})$$

  using

$$c_{sym} = 31.6(\frac{\rho}{\rho_0})^{\gamma} MeV, \quad \epsilon = \frac{1}{3(1+cA^{1/3})}, \quad \rho_0 = 0.16 fm^{-3}$$

  and taking as experimental baseline the neutron skins measured in 26 antiprotonic atoms.

- We predict $(b_n = b_p)$: $L = 75 \pm 25$ MeV



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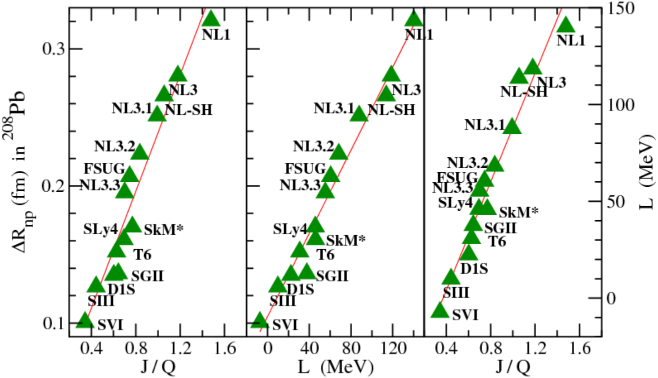
Neutron skin thickness



$$S = \sqrt{3/5}\left[t - e^2 Z/(70J) + \frac{5}{2R}(b_n^2 - b_P^2)\right]$$

$$t = \frac{3r_0}{2}\,\frac{J/Q}{1+x_A}\,(I - I_{\mathrm{C}})$$