# Collective properties of stable even-even Cd isotopes

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# Motivation

1. Vibrations, rotations or something more general?

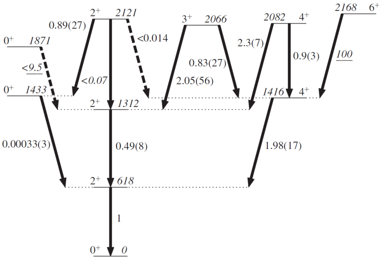
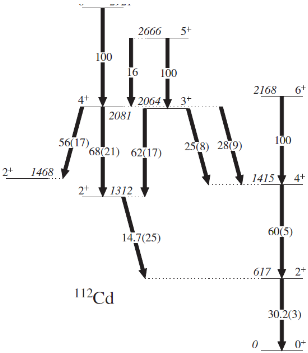



**Figure 4.** $B(E2)$ values, relative to the $B(E2; 2^+_1 \rightarrow 0^+_1)$ value, observed for $^{112}$Cd. The uncertainties for the last digit are listed in parentheses. Dashed transitions indicate an unobserved transition. Underlined numbers indicate a $B(E2)$ value relative to the strongest transition (defined as 100 units) from the level. (The upper limit for the $E2$, 2121 → 1312 transition is dictated by the $\delta(E2/M1)$ value for the 2121 → 1312 transition.)




*On the robustness of surface vibrational modes: case studies in the Cd region*
P.E. Garrett and J. L. Wood, J.Phys.G: Nucl.Part.Phys. **37** 064028, 2010

# Motivation cont.

2. Treatment of pairing and particle number conservation, going beyond BCS?

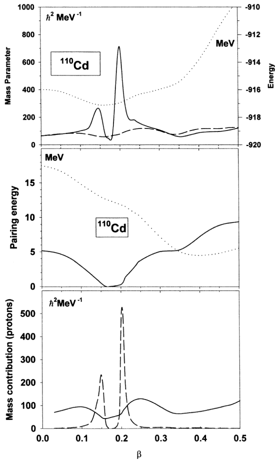



Fig. 1. In the upper part, mass parameters $M^P(\beta)$(solid curve)

*Mass parameters for large amplitude collective motion: A perturbative microscopic approach*
E.Kh. Yuldashbaeva, J. Libert, P. Quentin, M. Girod, Phys. Lett. B **461** 1 (1999).

# The Bohr Hamiltonian

Hamiltonian

$$H_{\mathrm{GBH}}(\beta,\gamma,\Omega) = T_{\mathrm{vib}} + T_{\mathrm{rot}} + V$$

$$\begin{aligned} T_{\mathrm{vib}}(\beta,\gamma) &= -\frac{1}{2\sqrt{wr}}\left\{\frac{1}{\beta^4}\left[\partial_\beta\left(\beta^4\sqrt{\frac{r}{w}}B_{\gamma\gamma}\right)\partial_\beta - \partial_\beta\left(\beta^3\sqrt{\frac{r}{w}}B_{\beta\gamma}\right)\partial_\gamma\right]+\right.\\ &\left.+\frac{1}{\beta\sin 3\gamma}\left[-\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\right)\partial_\beta + \frac{1}{\beta}\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\right)\partial_\gamma\right]\right\}\\ T_{\mathrm{rot}}(\beta,\gamma,\Omega) &= \frac{1}{2}\sum_{k=1}^{3} I_k^2(\Omega)/J_k; \quad J_k(\beta,\gamma) = 4B_k(\beta,\gamma)\beta^2\sin^2(\gamma - 2\pi k/3)\\ & w = B_{\beta\beta}B_{\gamma\gamma} - B_{\beta\gamma}^2; \;\; r = B_xB_yB_z \end{aligned}$$

Collective variables $\beta$, $\gamma$, Euler angles $\Omega$

$$\begin{aligned} \beta\cos\gamma &= Dq_0, \qquad q_0 = \langle Q_0\rangle = \langle \textstyle\sum_i^A 3z_i^2 - r_i^2\rangle\\ \beta\sin\gamma &= \sqrt{3}Dq_2, \qquad q_2 = \langle Q_2\rangle = \langle \textstyle\sum_i^A x_i^2 - y_i^2\rangle \end{aligned}$$

$$D = \sqrt{\pi/5}/A\overline{r^2}, \quad \overline{r^2} = \tfrac{3}{5}(r_0A^{1/3})^2, \quad r_0 = 1.2 \text{ fm}$$

## Microscopic formulas from the ATDHFB theory

Simplest (most approximate) version
Vibrational mass parameters for constrained HFB calculations

$$B_{kj} = \frac{\hbar^2}{2}\left(M_{(1)}^{-1} M_{(3)} M_{(1)}^{-1}\right)_{kj}$$

$$M_{(n),kj} = \sum_{\mu,\nu} \frac{\langle\mu|Q_k|\nu\rangle\langle\nu|Q_j|\mu\rangle}{(E_\mu + E_\nu)^n}(u_\mu v_\nu + u_\nu v_\mu)^2$$

$B_{kj} \longrightarrow B_{\beta\beta}, B_{\beta\gamma}, B_{\gamma\gamma}$
Moments of inertia

$$J_k = 2\hbar^2 \sum_{\mu,\nu} \frac{|\langle\mu|j_k|\nu\rangle|^2}{E_\mu + E_\nu}(u_\mu v_\nu - u_\nu v_\mu)^2$$

Thouless-Valatin correction, factor 1.3

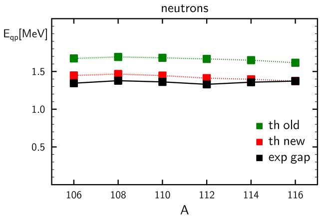
# Microscopic calculations

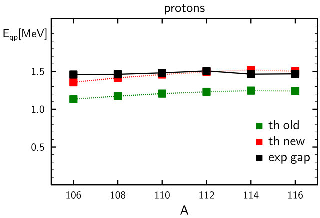
Even-even $^{106-116}$Cd isotopes
The Skyrme interaction, SIII parameters
Seniority (constant $G$) pairing interaction

Two variants of the pairing strength ($g_i = G_i/(11 + N_i)$, $i = n, p$)

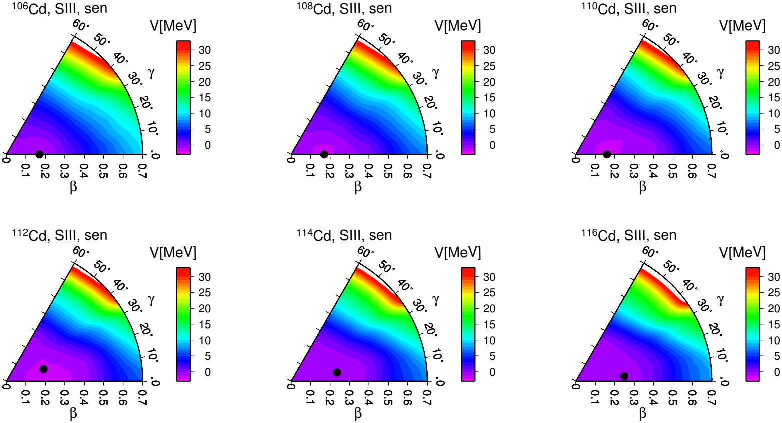
| | $G_n$ | $G_p$ |
|---|---|---|
| old | 17.1 | 16.5 |
| new | 16.1 | 17.5 |

$E_{qp} = \min \sqrt{(e_i - \lambda)^2 + \Delta^2}$
Experimental gap from the 5-point mass formula



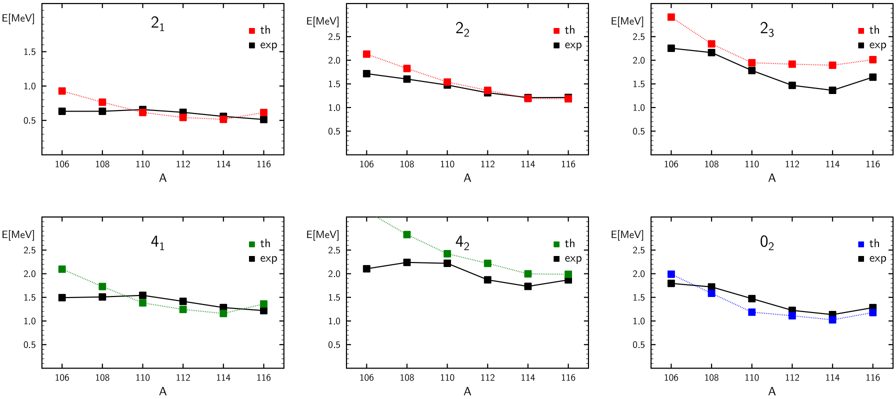



# $^{106-116}$Cd, Potential energy

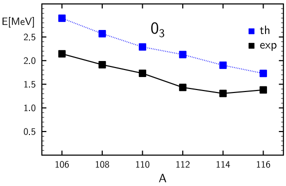


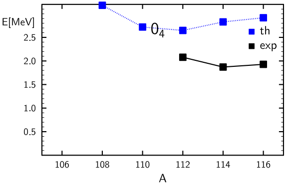

$\beta = 0.7 \rightarrow q_0 \approx 19.3$ b $(A = 110)$

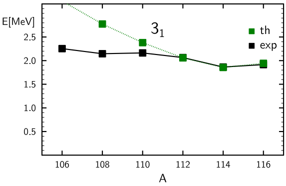
# $^{106-116}$Cd, energy levels

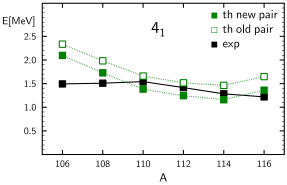


# $^{106-116}$Cd, energy levels, cont



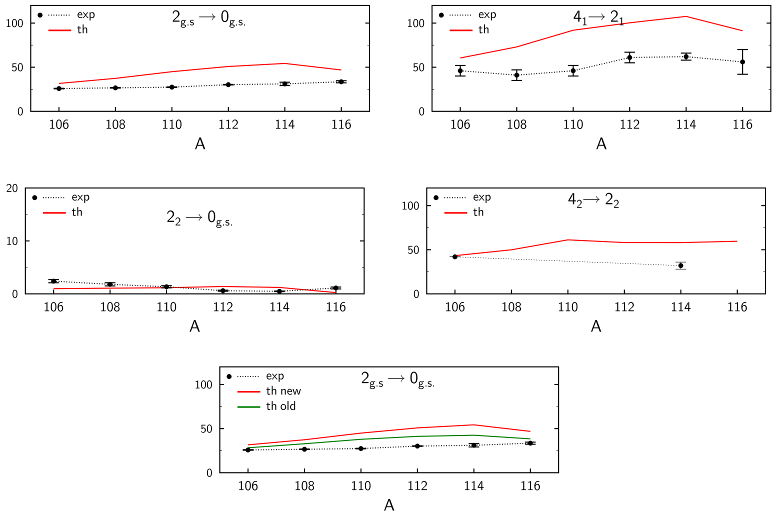



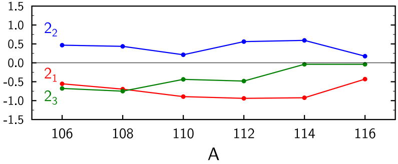

Two variants of pairing

# $^{106-116}$Cd, B(E2) transition probabilities



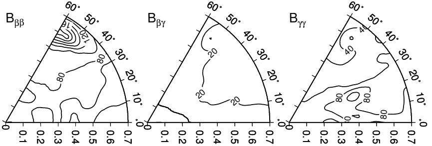
# Diagonal matrix elements of the $Q^{\mathrm{el}}$ operator

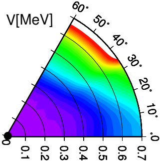
$\langle i \| Q^{\mathrm{el}} \| i \rangle$ [eb] for $2_{1,2,3}$ levels




Test case

micr




SKE
$B_{\beta\beta} = B_{\gamma\gamma} = B_k = B$,
$B_{\beta\gamma} = 0$

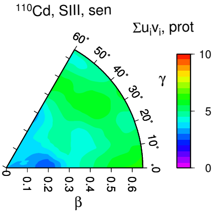
| | micr | SKE |
|---|---|---|
| $2_1$ | -0.480 | -0.514 |
| $2_2$ | 0.213 | 0.259 |
| $2_3$ | -0.220 | -0.591 |

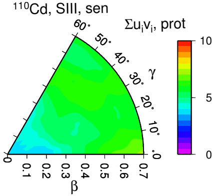
## Higher Tamm-Dancoff Approximation

1. Mean-field (HF+BCS) calculations $\longrightarrow$ one particle basis
2. Many-particle basis built from $m$ particle-hole states (for protons and neutrons)
3. Diagonalization of the $\delta$ residual interaction in the many-particle basis

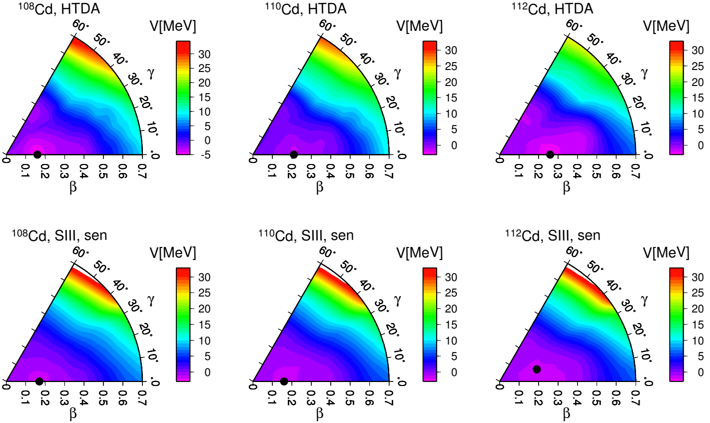
States with a good particle number
Density matrix $\rho_{ii} = \langle \Phi | a_i^+ a_i | \Phi \rangle \longrightarrow v_i^2$

Sum $\sum_i u_i v_i$ for the HF+BCS solution



# HTDA, potential energy



# HTDA, mass parameters

How to get mass parameters in the HTDA?

1. More fundamental theory (TD HTDA)
2. Analogy with ATDHFB, density matrix $\rho_{ii} \rightarrow v_i^2$, what about quasiparticle energies

$$u_k v_k = \Delta_k / 2E_k$$

$$\Delta_k = -\sum_j u_j v_j \langle k\bar{k}|V_{\text{res}}|j\bar{j}\rangle$$

3. Schematic estimations

# Conclusions

- The Bohr Hamiltonian is flexible enough to describe quadrupole dynamics even in the region near to closed shell nuclei
- The Cd region is an interesting field for testing extensions of the present approach (to explain e.g. low lying $0^+$ levels, improper $A$ dependence of some energy levels)