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Dynamical Coupling of Rotation with the Pairing Field in Heavy Nuclei

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Our previous papers related to the subject:

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Outline

1. **Summary of the theoretical model:**

Method	**macroscopic-microscopic**
Single particle potential	**Yukawa-folded**
Shape parametrization	**Modified-Funny-Hills**
Macroscopic energy	**Lublin-Strasbourg-Drop**
Microscopic corrections	**Strutinsky, BCS**
Moments of inertia	**cranking**
Dynamical effects	**coupling with pairing field**

2. Results

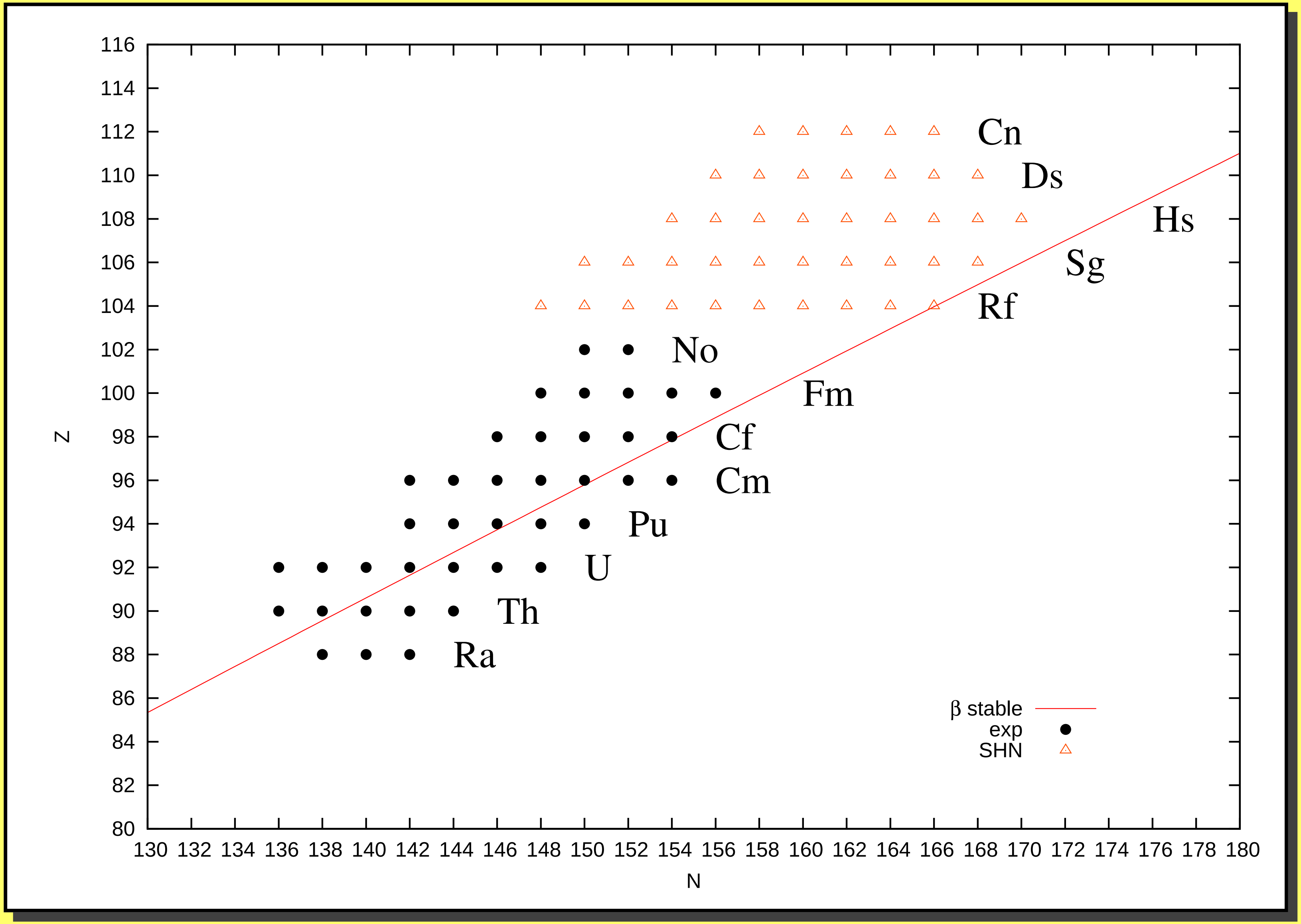
Obtained observables for heavy and superheavy nuclei:

- **equilibrium deformations ,**
- **deformation energies ,**
- **single-particle levels ,**
- **moments of inertia ,**
- **strength of the pairing force ,**
- **rotational energies ,**
- **ground state masses .**

3. Calculation scheme:

I	**Single particle levels with Yukawa folded mean field**
II	**Potential energy surface E with a standard pairing strength**
III	**Equilibrium deformation by minimization of this energy**
IV	**Cranking moments of inertia in equilibrium points**
V	**Rotational energies**
VI	**If the 2^+ rotational energy does not agree with the data we change the pairing strength and go back to point II**
VII	**If the lowest rotational energies agree with the data we evaluate the masses and include dynamical effects to obtain higher members of the rotational band**

Set of nuclei under consideration



Nuclear binding energy

Single-particle energies e_ν are obtained by diagonalization of the Yukawa-folded mean-field Hamiltonian for a given nucleus $Z + N = A$.

The total binding energy E consists of the macroscopic part E_{macr} and microscopic corrections E_{shell} and E_{pair}:

$$E = E_{\text{macr}} + E_{\text{corr}} = E_{\text{LSD}} + E_{\text{shell}} + E_{\text{pair}}\,,$$

where $E_{\text{corr}} = E^n_{\text{corr}} + E^p_{\text{corr}}$ and

$$E_{\text{shell}} = \sum_\nu 2e_\nu - \tilde{E}\,,$$

$$E_{\text{pair}} = E_{\text{BCS}} - \sum_\nu 2e_\nu + < E_{\text{pair}} >\,,$$

and

$$E_{\text{BCS}} = \sum_{k>0} 2e_k v_k^2 - \frac{\Delta^2}{G} - G \sum_{k>0} v_k^4\,.$$

Yukawa-folded potential

The Yukawa-folded mean-field potential consists of the central, spin-orbit and Coulomb terms:

$$V^{\mathbf{YF}} = V_{\mathbf{c}} + V_{\mathbf{so}} + V_{\mathbf{Coul}} \,.$$

The single-particle central potential is given the following folding integral:

$$V_{\mathbf{c}}\left(\vec{r}_1\right) = \int\limits_V d^3 r_2 \, V(r_{12}) \frac{\rho\left(\vec{r}_2\right)}{\rho_0} \,,$$

where $V(r_{12})$ is the Yukawa two-body nucleon-nucleon interaction $V(r_{12})$

$$V(r_{12}) = -\frac{V_0^q}{4\pi\lambda^3} \frac{e^{-|\vec{r}_1-\vec{r}_2|/\lambda}}{|\vec{r}_1-\vec{r}_2|/\lambda} \,,$$

with $r_{12} = |\vec{r}_1 - \vec{r}_2|$ and $q = \{n, p\}$.

Modified Funny Hill shape parametrization

An essential ingredient of any macroscopic–microscopic approach consists, in the use of a parametrization of the relevant nuclear shapes that contains only a rather small number of deformation parameters.

The so-called Modified Funny-Hills (MFH) shape parametrization contains four deformation parameters only:

c-elongation, h-neck, α-mass asymmetry, η-nonaxiality

and it is defined as following:

$$\varrho_s^2(u) \sim (1-u^2)\left(1 - B e^{-a(u-\alpha')^2}\right)(1 - g\alpha' u)\,\frac{1-\eta^2}{1+\eta^2+2\eta\cos(2\phi)}\,,$$

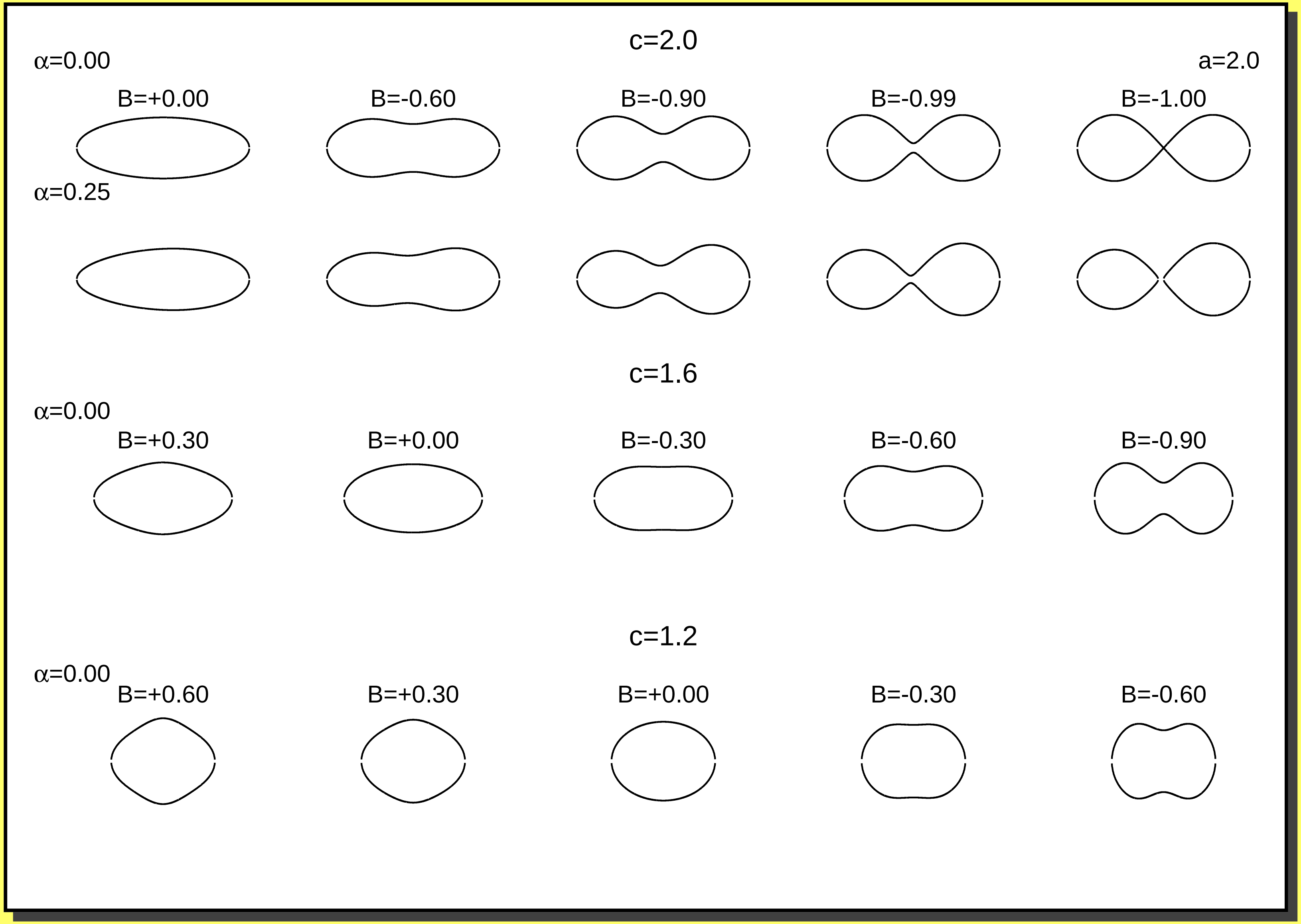
where $\varrho_s(u)$ is the distance of the surface to the z-axis as function of $u = \frac{z - z_{sh}}{z_0}$ with $z_0 = cR_0$ and z_{sh} ensuring the center of mass at $z = 0$.

The other parameters are:

$$a = 3 - B,\quad g = e^{-4B^2},\quad B = \tfrac{1}{2}(c - \tfrac{1}{c})^2 + 1 - e^{-h},\quad \alpha' = \alpha\, e^{\frac{1}{2}(c-\frac{1}{c})^2}\,.$$

The MFH shapes are very close to the optimal in the LD energy forms.

Funny Hill shapes for different values of c,B,α



Lublin Strasbourg Drop - LSD

$$M_{\rm th} = ZM_{\rm H} + NM_n - 0.00001433Z^{2.39} + E_{\rm LSD} + E_{\rm corr}$$

$$\begin{aligned} E_{\rm LSD} = & -b_{vol}(1-\kappa_{vol}I^2)A \\ & +b_{\rm surf}(1-\kappa_{surf}I^2)A^{2/3} \\ & +b_{\rm cur}(1-\kappa_{cur}I^2)A^{1/3} \\ & +\frac{3}{5}e^2\frac{Z^2}{r_0^{ch}A^{1/3}} - C_4\frac{Z^2}{A} - 10\cdot\exp\left(-42|I|/10\right) \end{aligned}$$

$b_{\rm vol}$ = 15.4920 MeV,

$\kappa_{\rm vol}$ = 1.8601

$b_{\rm surf}$ = 16.9707 MeV,

$\kappa_{\rm surf}$ = 2.2938

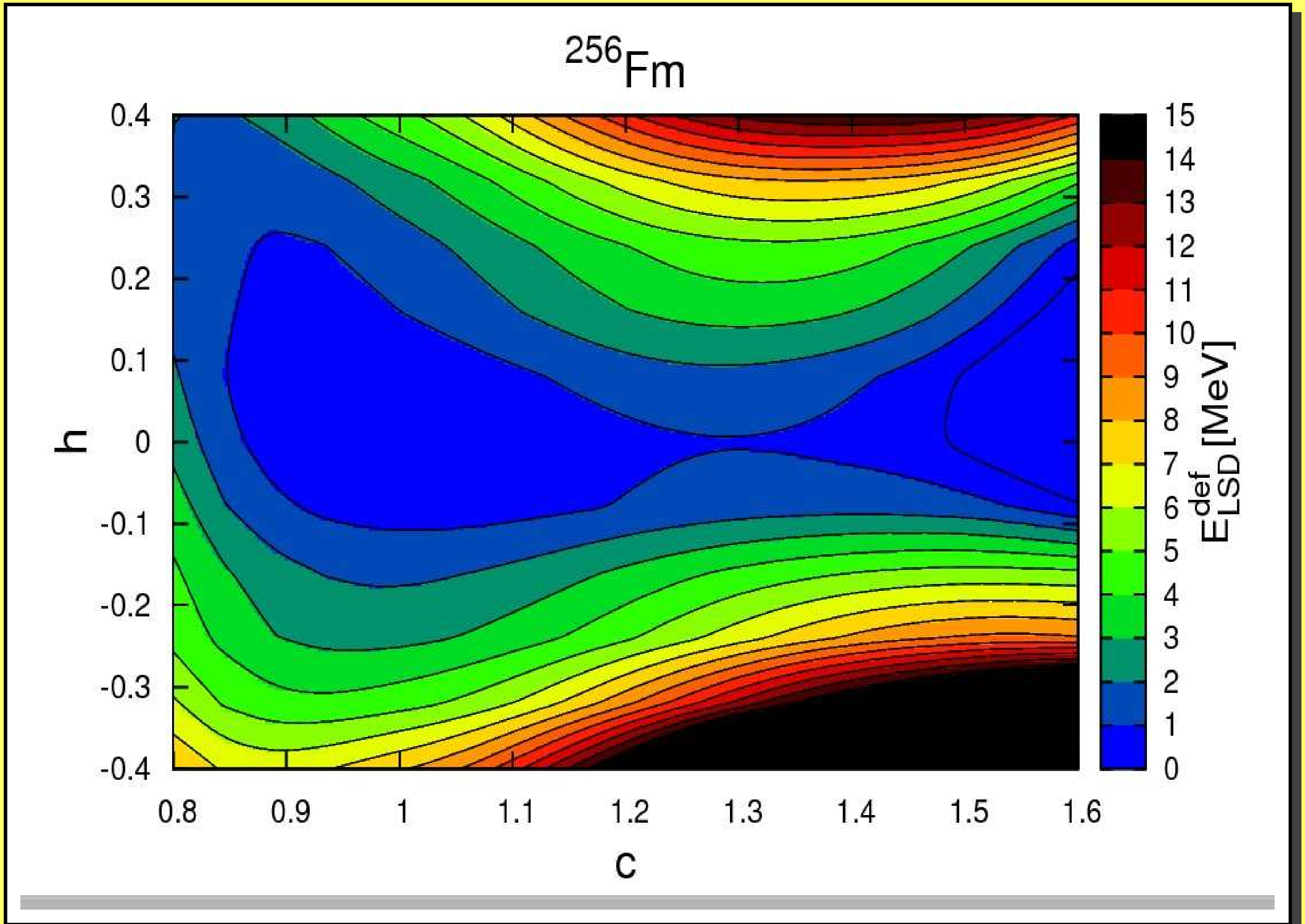
$b_{\rm cur}$ = 3.8602 MeV

$\kappa_{\rm cur}$ = -2.3764

$r_0^{\rm ch}$ = 1.21725 fm

C_4 = 0.91810 MeV and $I = (N-Z)/A$

Macroscopic LSD energy $E_{\rm LSD}^{\rm def} = E_{\rm LSD}(c,h) - E_{\rm LSD}(1,0)$



Strutinsky shell correction method:

$$E_{\text{shell}} = \sum_{\nu} 2e_{\nu} - \widetilde{E}$$

$$\widetilde{E} = 2 \int_{-\infty}^{\lambda} e\tilde{\rho}(e)de$$

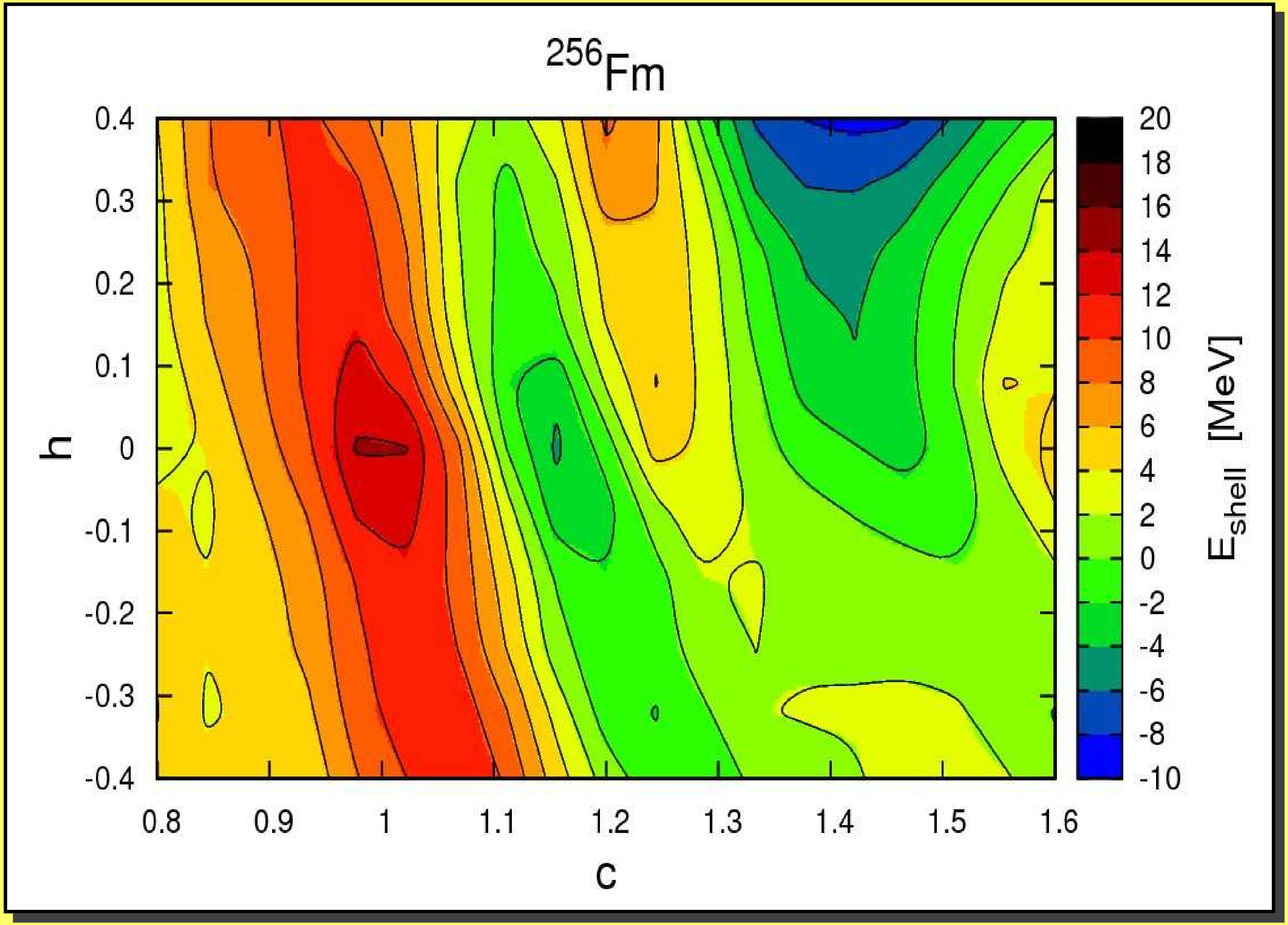
$$\tilde{\rho}(e) = \frac{1}{\gamma} \int_{-\infty}^{+\infty} \rho(e')\, e^{-(\frac{e-e'}{\gamma})^2} j_6(\frac{e-e'}{\gamma})\, de'$$

$$\gamma = 1.2\,\hbar\omega_o\ ; \qquad \hbar\omega_o = 41/A^{1/3} MeV$$

$$j_6(u) = \frac{1}{\sqrt{\pi}} e^{-u^2} \left(\frac{35}{16} - \frac{35}{8}u^2 + \frac{7}{4}u^4 - \frac{1}{6}u^6\right)$$

$$\mathcal{N} = 2 \int_{-\infty}^{\lambda} \rho(e)de$$

Shell correction for ^{256}Fm



Pairing correction

$$E_{pair} = E_{\rm BCS} - \sum_k 2e_k + < \mathrm{E_{pair}} >$$

$$E_{\rm BCS} = \sum_{k>0} 2e_k v_k^2 - \frac{\Delta^2}{G} - G \sum_{k>0} v_k^4 \ ,$$

where

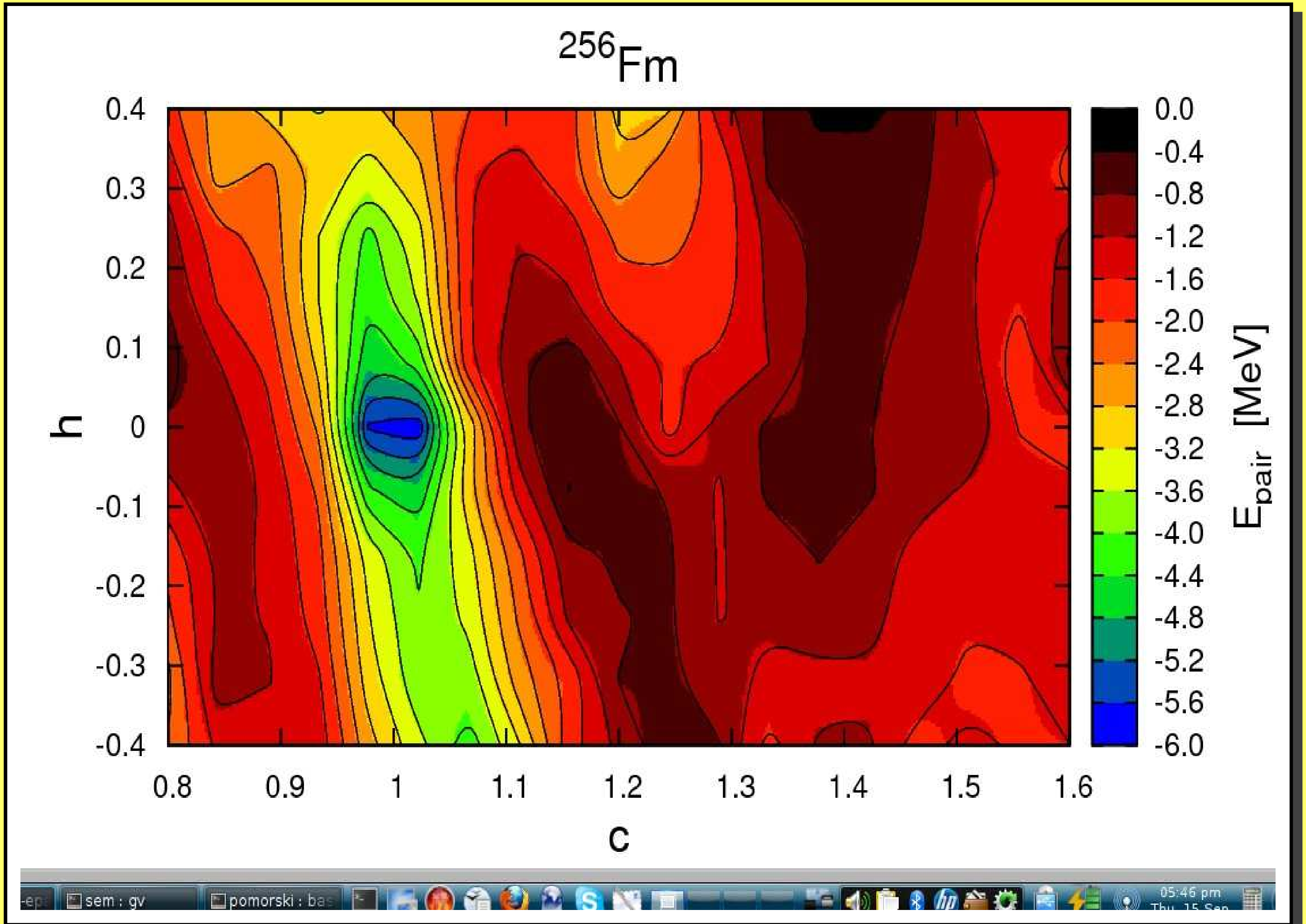
$$e'_k = e_k - G v_k^2 \ , \quad v_k^2 = \frac{1}{2}\left(1 - \frac{e'_k - \lambda}{E_k}\right)$$

and$E_k = \sqrt{\Delta^2 + (e'_k - \lambda)^2}$.

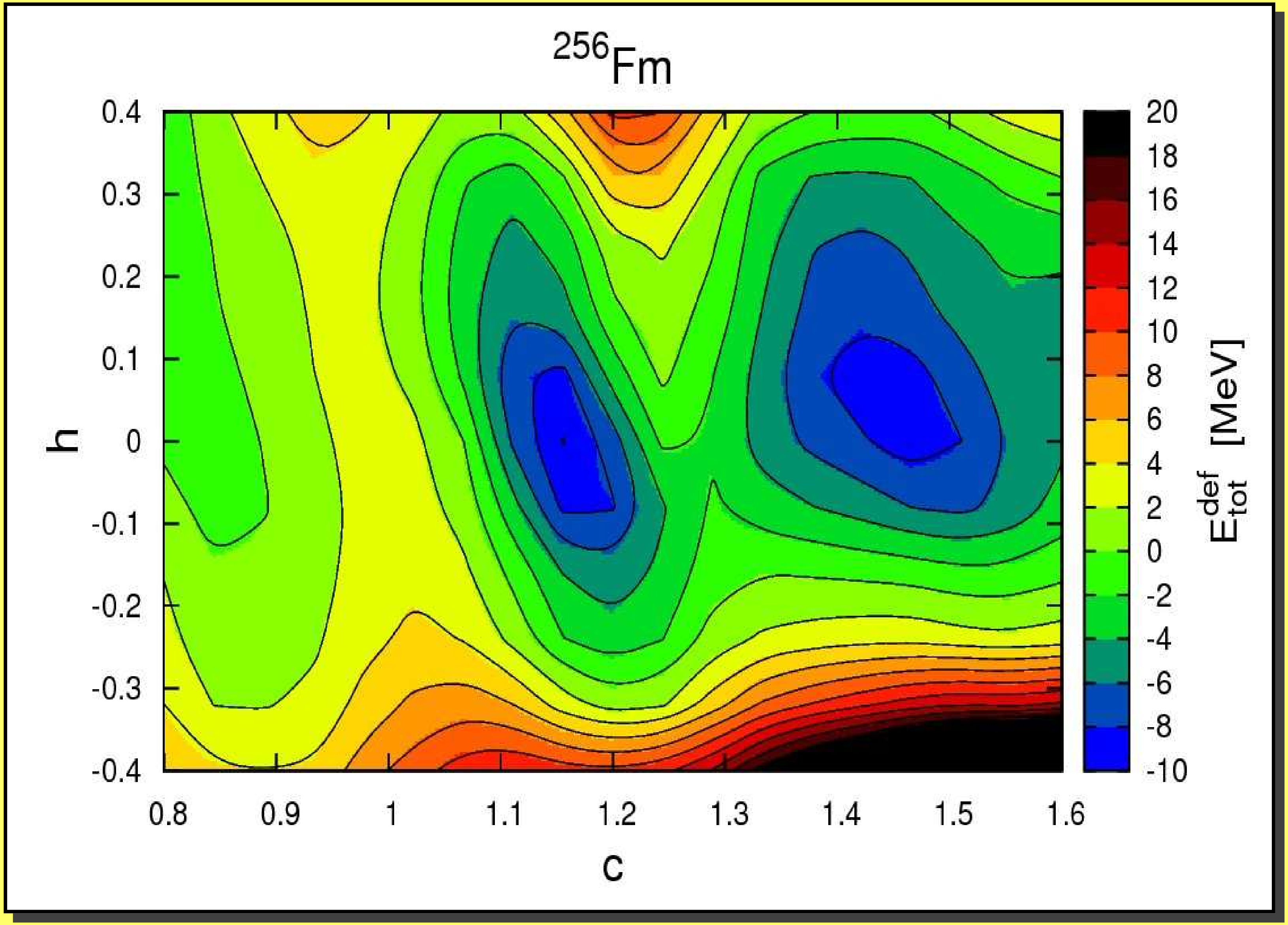
$$N = \sum_{k>0}\left(1 - \frac{e'_k - \lambda}{E_k}\right)$$

$$\frac{2}{G} = \sum_{k>0} \frac{1}{E_k} \ , \text{ with } \quad G = g_o / N^{2/3} \hbar\omega_o$$

Pairing correction for ^{256}Fm



Deformation energy $E_{tot}^{def} = E(c,h) - E(1,0)$



Cranking moment of inertia

Time dependent Schroedinger equation for the rotation around x-axis reads:

$$i\hbar\frac{\partial}{\partial t}\psi'(x'y'z') = (\widehat{H}' - \hbar\omega\hat{j}_{x'})\psi'(x'y'z')\,.$$

The term $\hbar\omega\,\hat{j}_{x'}$ can be treated as perturbation when ω is small

$$E = E^{(0)} + E^{(1)}\omega + E^{(2)}\omega^2 \ldots\,,$$

where $E^{(1)} = 0$ and $E^{(2)} = 0$ gives the energy correction to the non-rotation case:

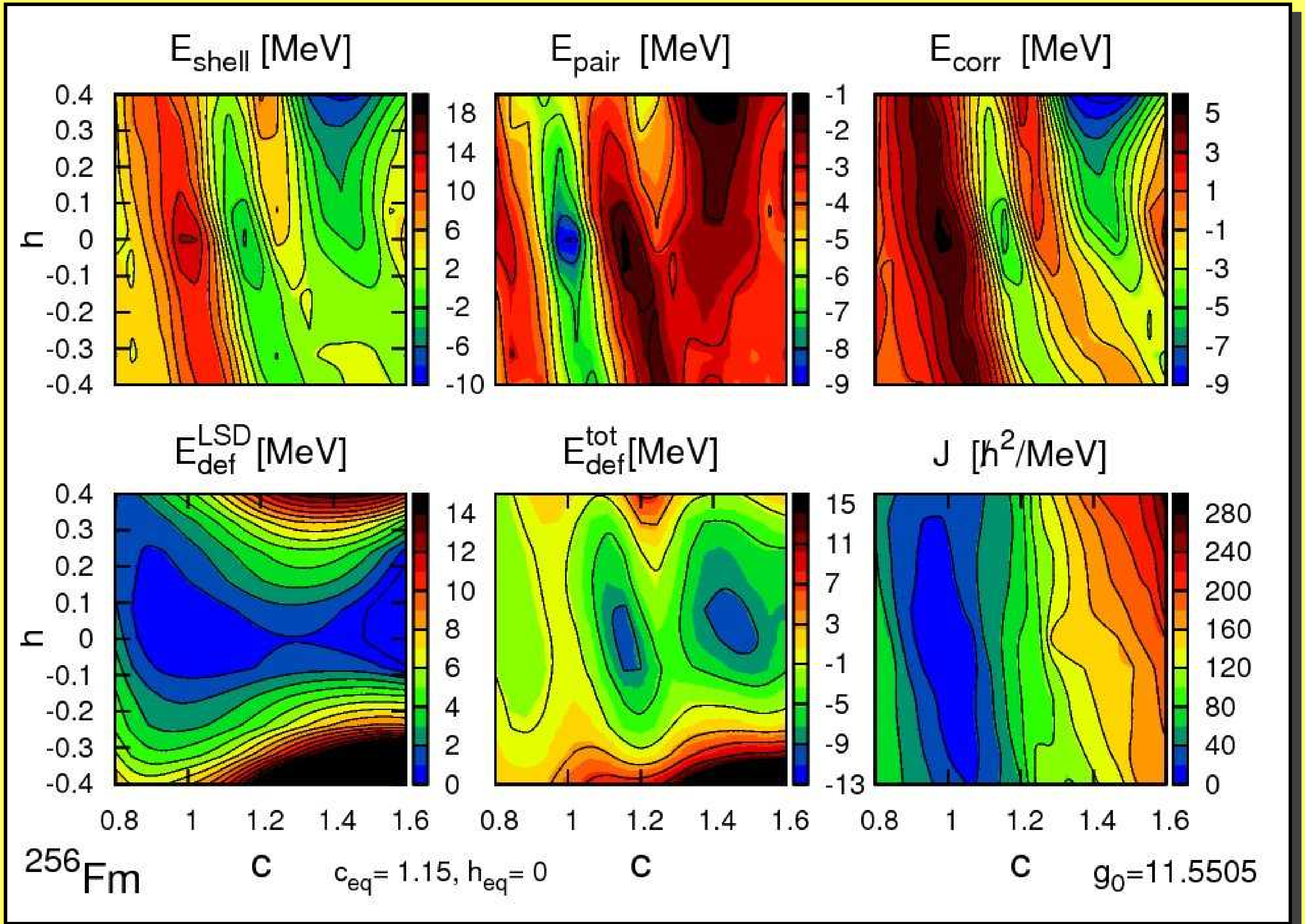
$$E^{(2)} = \hbar^2\omega^2 \sum_{k\neq 0} \frac{|\langle\Phi_k|\hat{j}_{x'}|\Phi_o\rangle|^2}{(E_k - E_o)} \equiv \frac{1}{2}\mathcal{J}_x^{\rm cr}\omega^2\,.$$

In the BCS model the cranking moment of inertia has the following form:

$$\mathcal{J}_x^{\rm cr} = 2\hbar^2 \sum_{\mu\nu} \frac{|\langle\mu|\hat{j}_{x'}|\nu\rangle|^2(u_\nu v_\mu - v_\nu u_\mu)^2}{E_\nu + E_\mu}\,,$$

where E_ν is the quasiparticle energy and u_ν, v_ν are usual occupation factors.

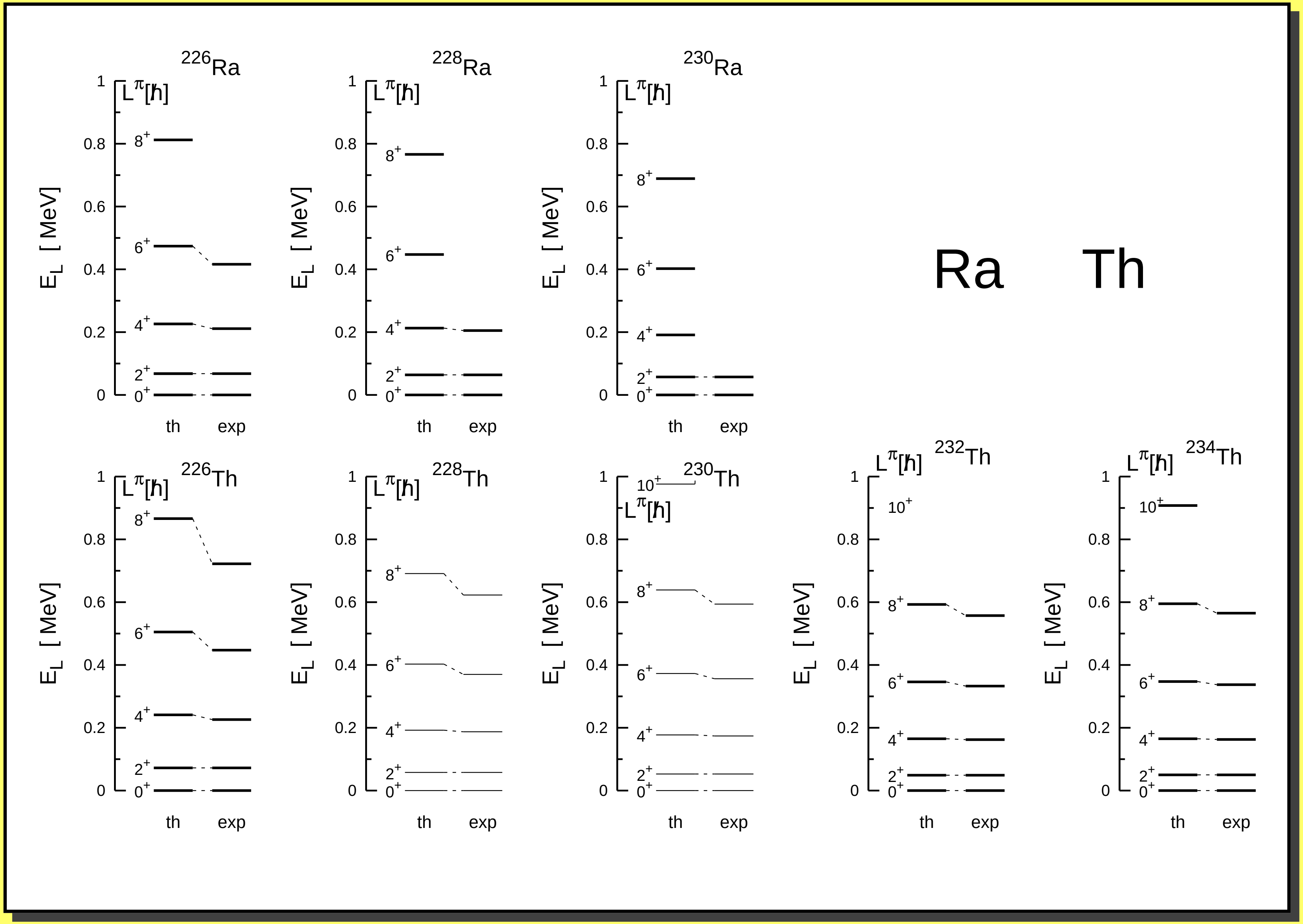
PES and moment of inertia of ^{256}Fm



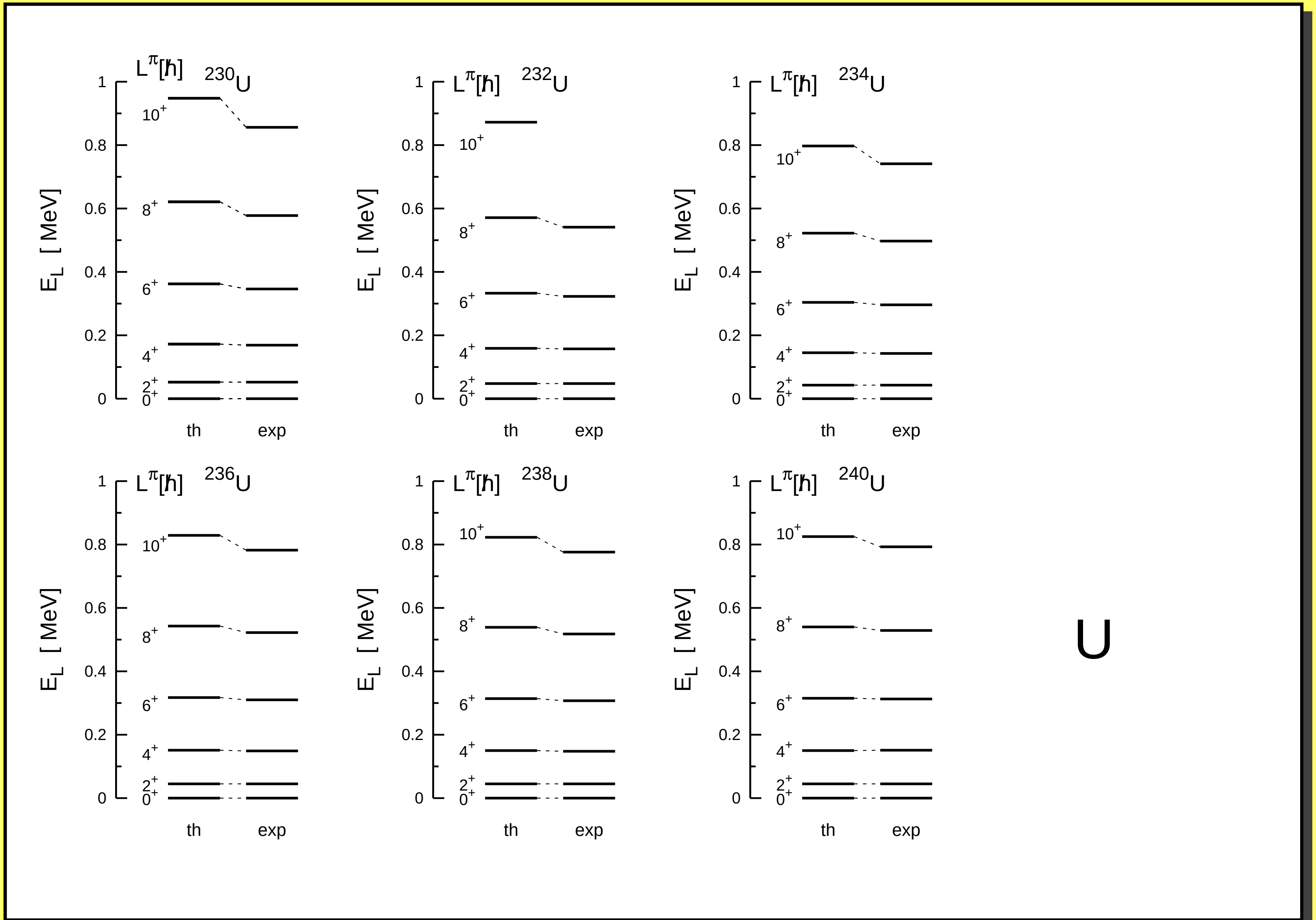
Simple rotational model:

$$\boldsymbol{E_L} = \frac{\boldsymbol{L(L+1)}}{\mathbf{2}\boldsymbol{\mathcal{J}}(\mathbf{def_{eq}},\boldsymbol{\Delta})}$$

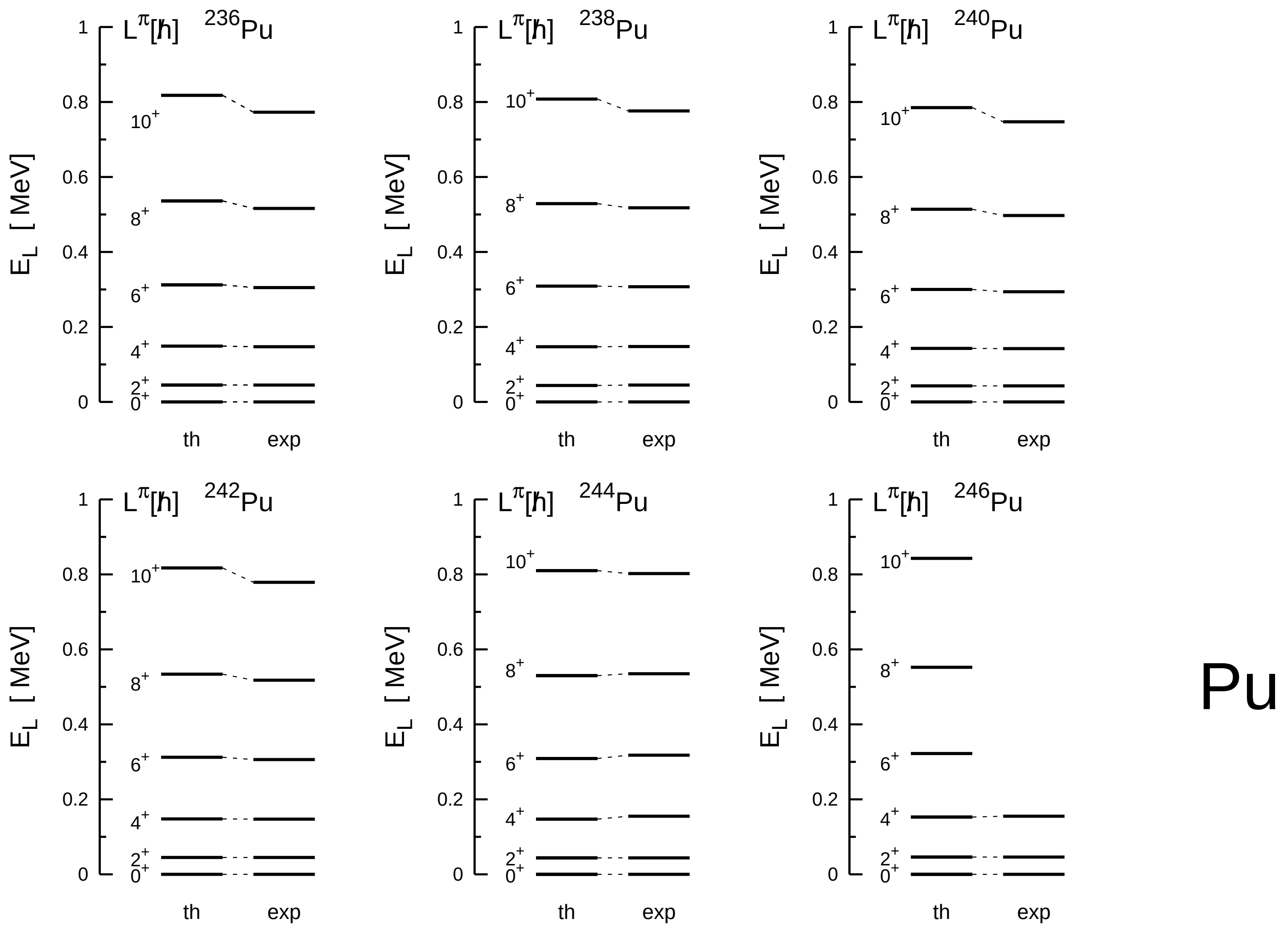
Rotational energies E_{L+} for Ra and Th.



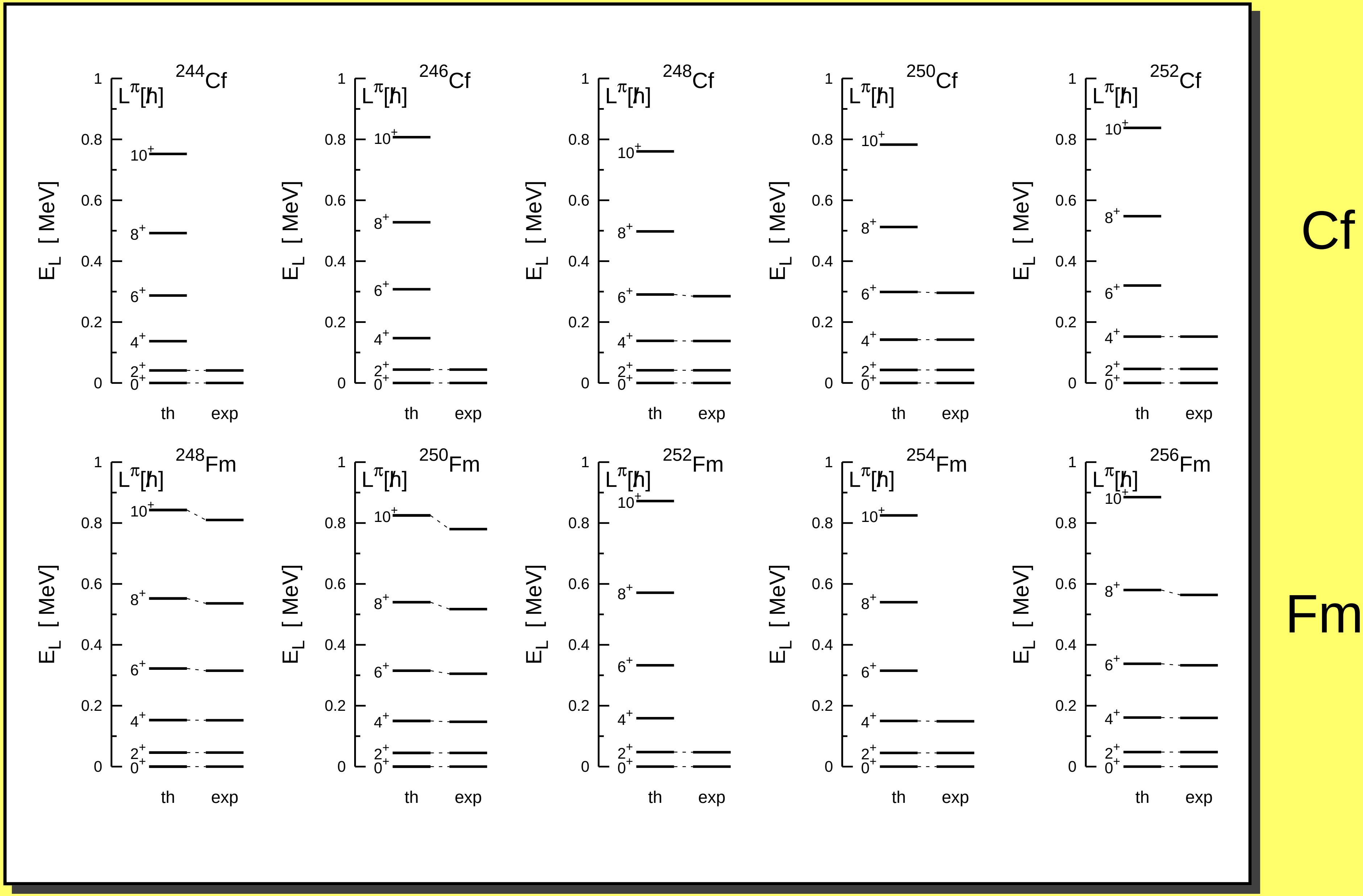
Rotational energies E_{L^+} for U isotopes.



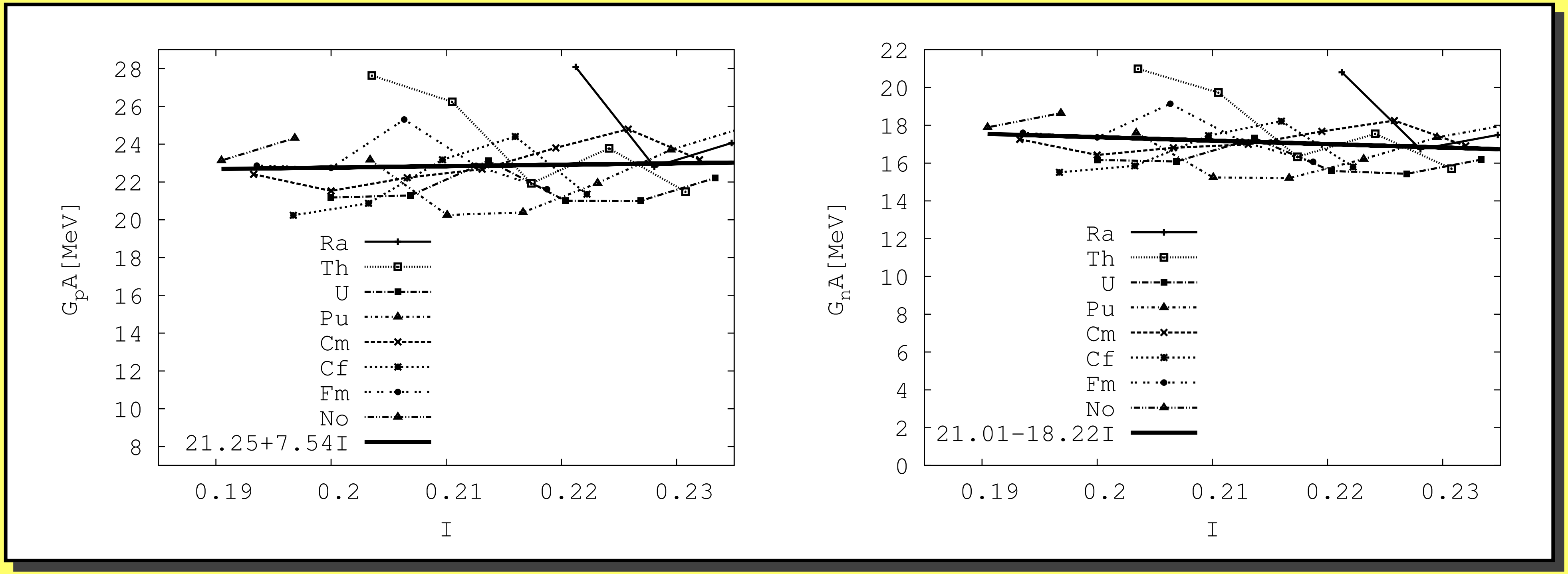
Rotational energies E_{L^+} for Pu isotopes.



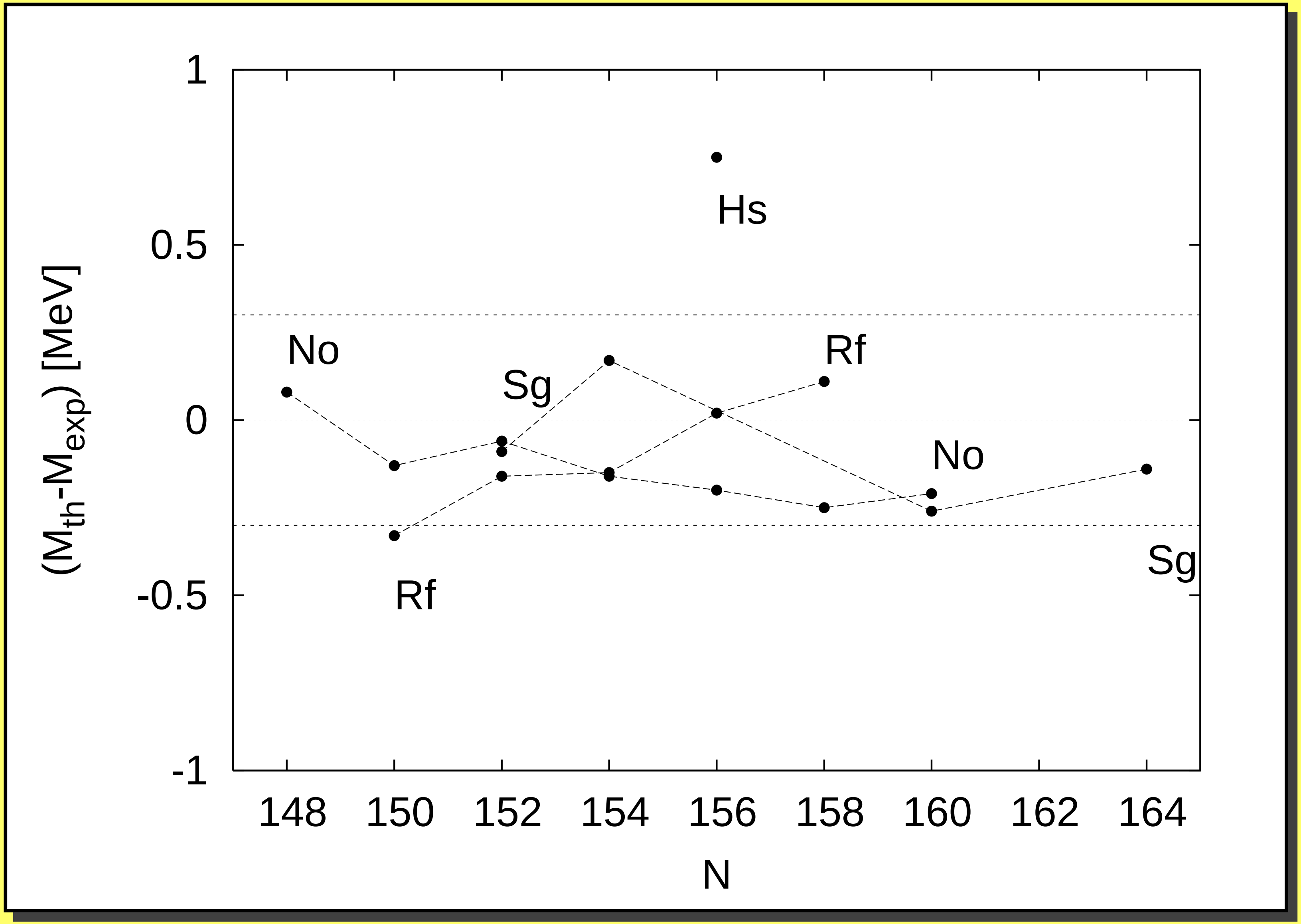
Rotational energies E_{L+} for Cf and Fm.



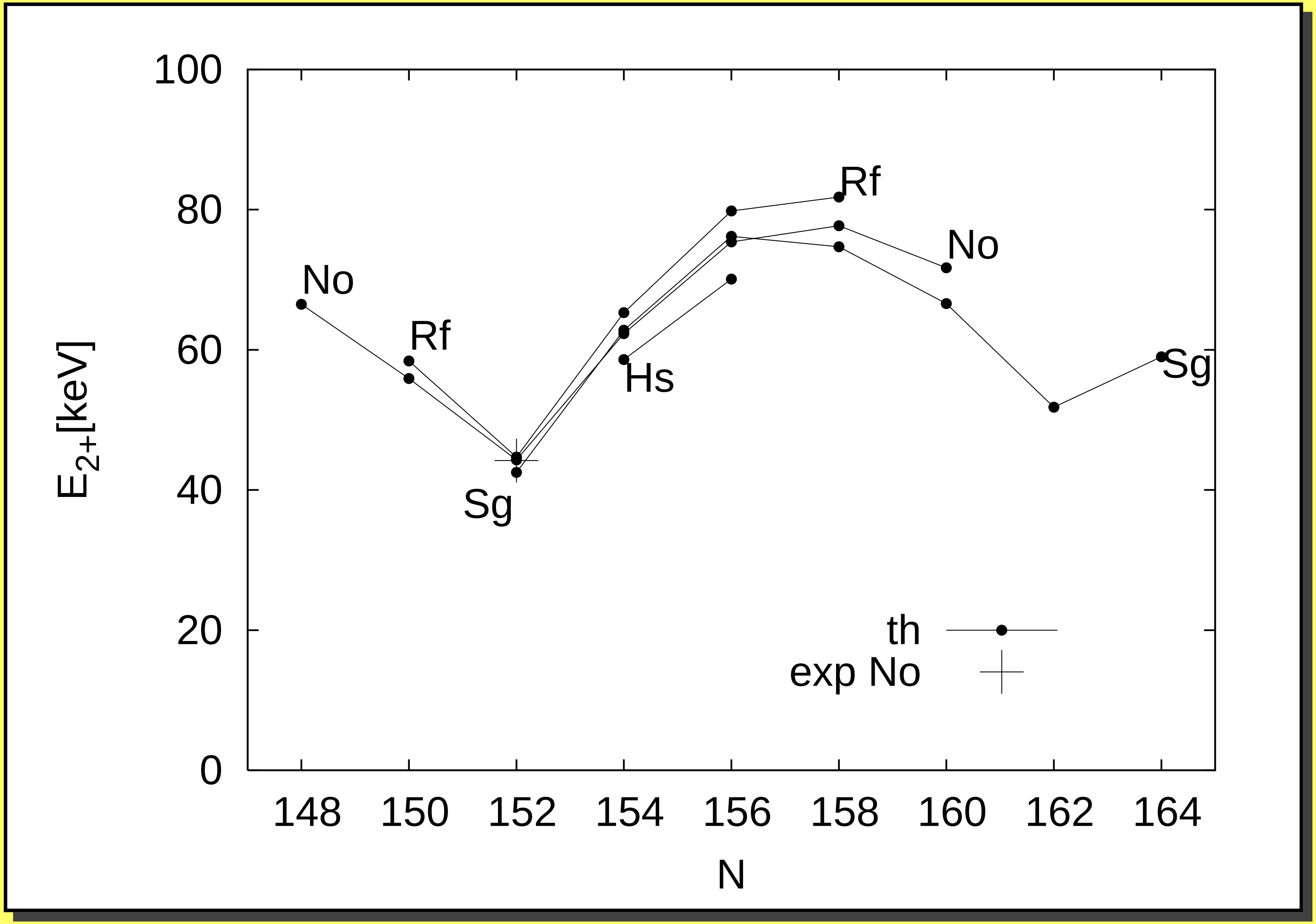
Pairing strength adjusted to energy of 2^+ levels



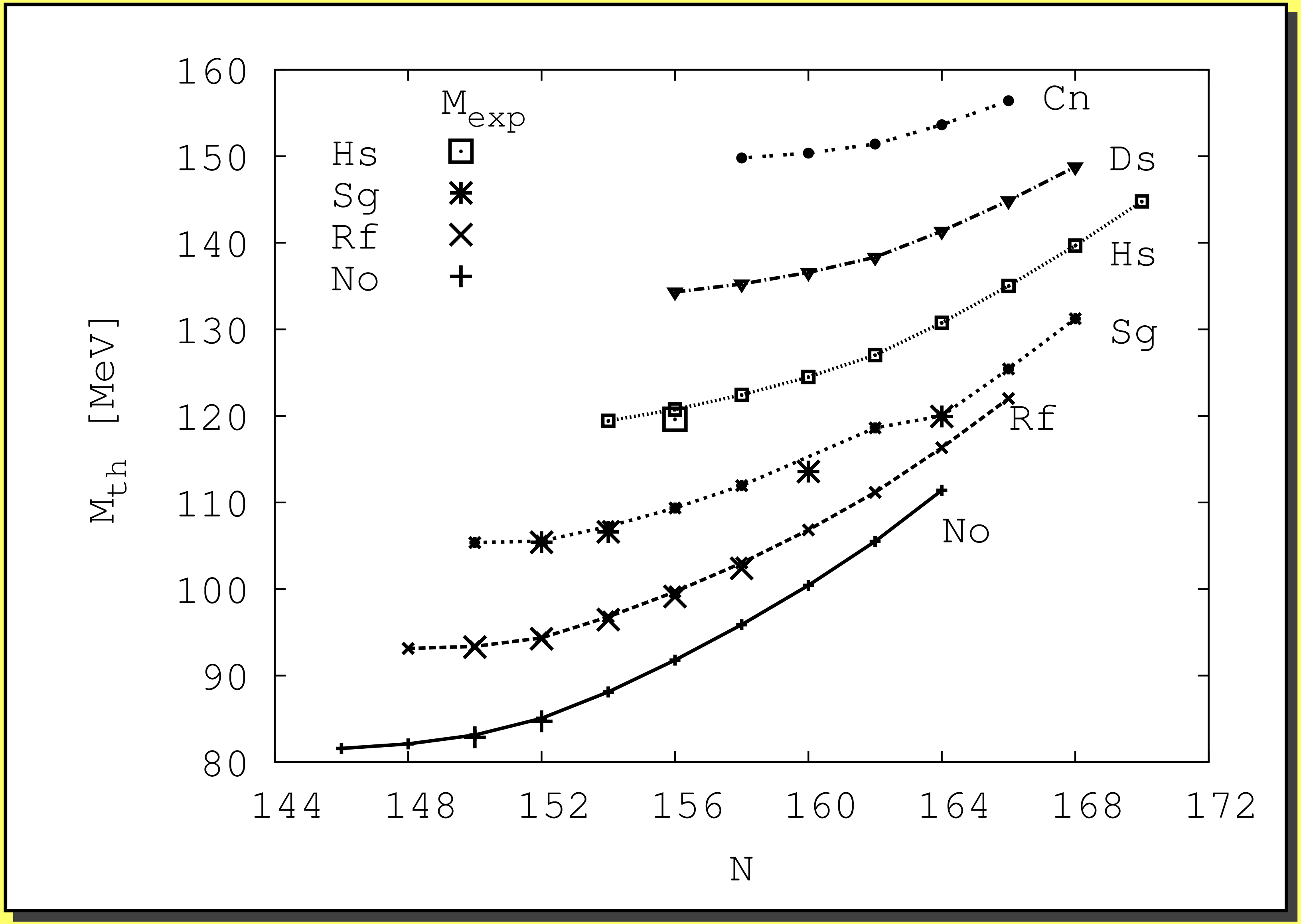
Mass discrepancies



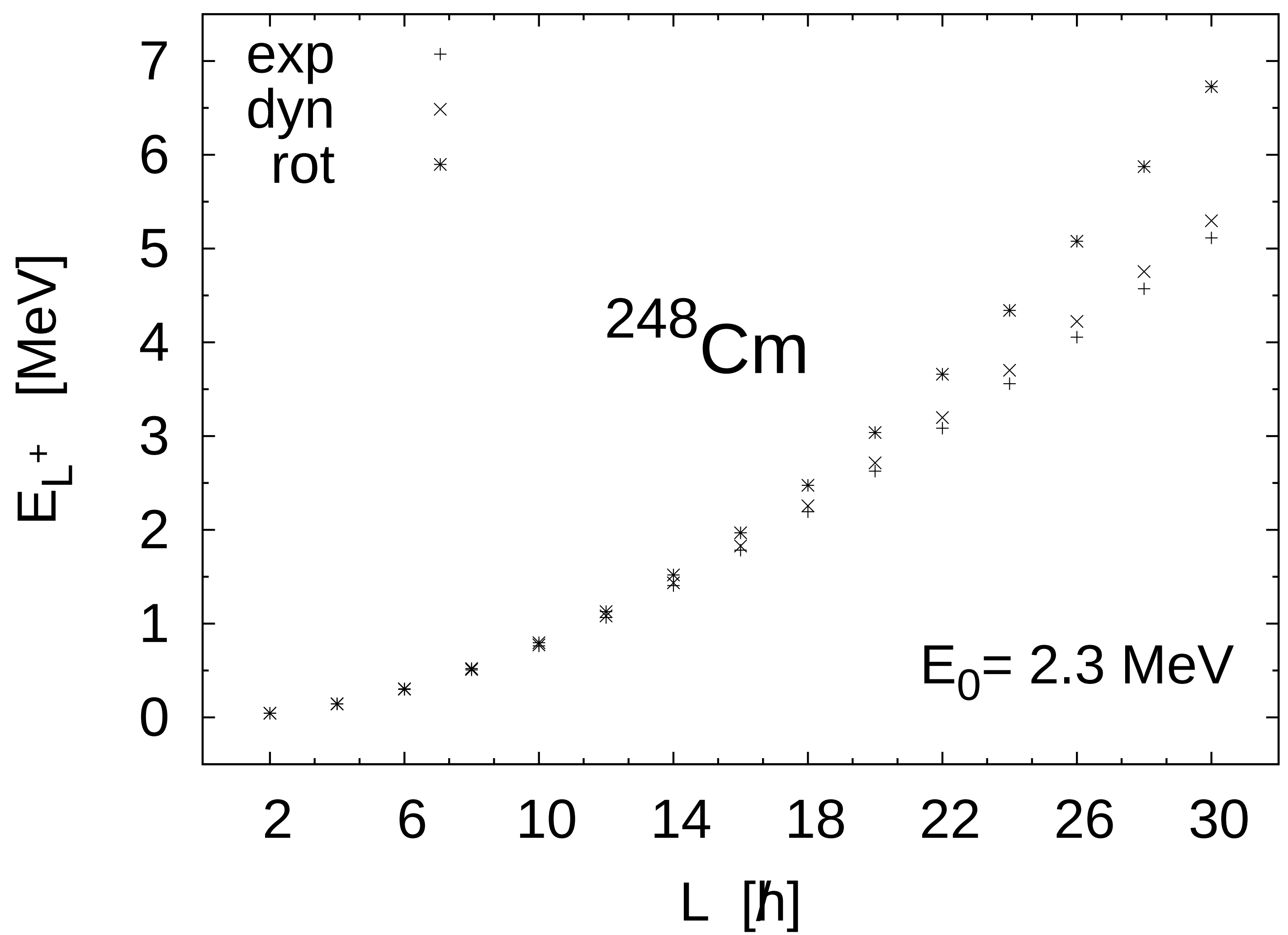
Forseen rotational energies E_{2+}



Forseen masses for superheavies nuclei



Rotational energies at high spins



Dynamical coupling of rotation with pairing field

It is well known that a rapid rotation influences the pairing correlations. We therefore propose here a simple model which allows to explain this mechanism and obtain some quantitative estimates of high-spin rotational states.

In a standard calculation the ground-state pairing gaps for protons (Δ_0^p) and neutrons (Δ_0^n) are determined from the BCS equation, i.e. looking for the minimum with respect of Δ^q of the BCS energy of a non-rotating nucleus but not of the sum of BCS and rotational energies.

One can show that in absence of pairing correlations the cranking moment of inertia at $\Delta = 0$ is approximately equal to the rigid-body moment of inertia ($\mathcal{J}_{\rm rig}$) and it decreases with growing Δ. Its Δ dependence can be approximated by

$$\mathcal{J}(\Delta) = \frac{\mathcal{J}_{\rm rig}}{1 + a(\Delta/\Delta_0)^2} \; .$$

Here Δ/Δ_0 corresponds to the line $\Delta^p/\Delta_0^p = \Delta^n/\Delta_0^n$ on the (Δ^p, Δ^n) plane, where Δ_0^q is the ground state pairing gap for protons (p) or neutrons (n) and $a = \mathcal{J}_{\rm rig}/\mathcal{J}_0 - 1$ with $\mathcal{J}_0$ being the ground-state moment of inertia.

Such a dependence of the moment of inertia reflects in the rotational energy which being inversely proportional to $\mathcal{J}(\Delta)$ grows with Δ and shifts the minimum of the sum BCS and rotational energies

$$E^{\mathrm{R}}_{\mathrm{BCS}}(\Delta; L) = E_{\mathrm{BCS}}(\Delta) + \frac{\hbar^2 L(L+1)}{2\mathcal{J}(\Delta)}$$

towards smaller Δ with respect to the BCS ground state energy $E_{\mathrm{BCS}}(\Delta_0)$.

The BCS energy dependence on Δ one can approximate by a cubic formula

$$E_{\mathrm{BCS}}(\Delta) = E_0 \left[-3 \left(\frac{\Delta}{\Delta_0} \right)^2 + 2 \left(\frac{\Delta}{\Delta_0} \right)^3 \right] ,$$

where $E_0 \approx 2.3$ MeV[a] is the average pairing correlation energy.

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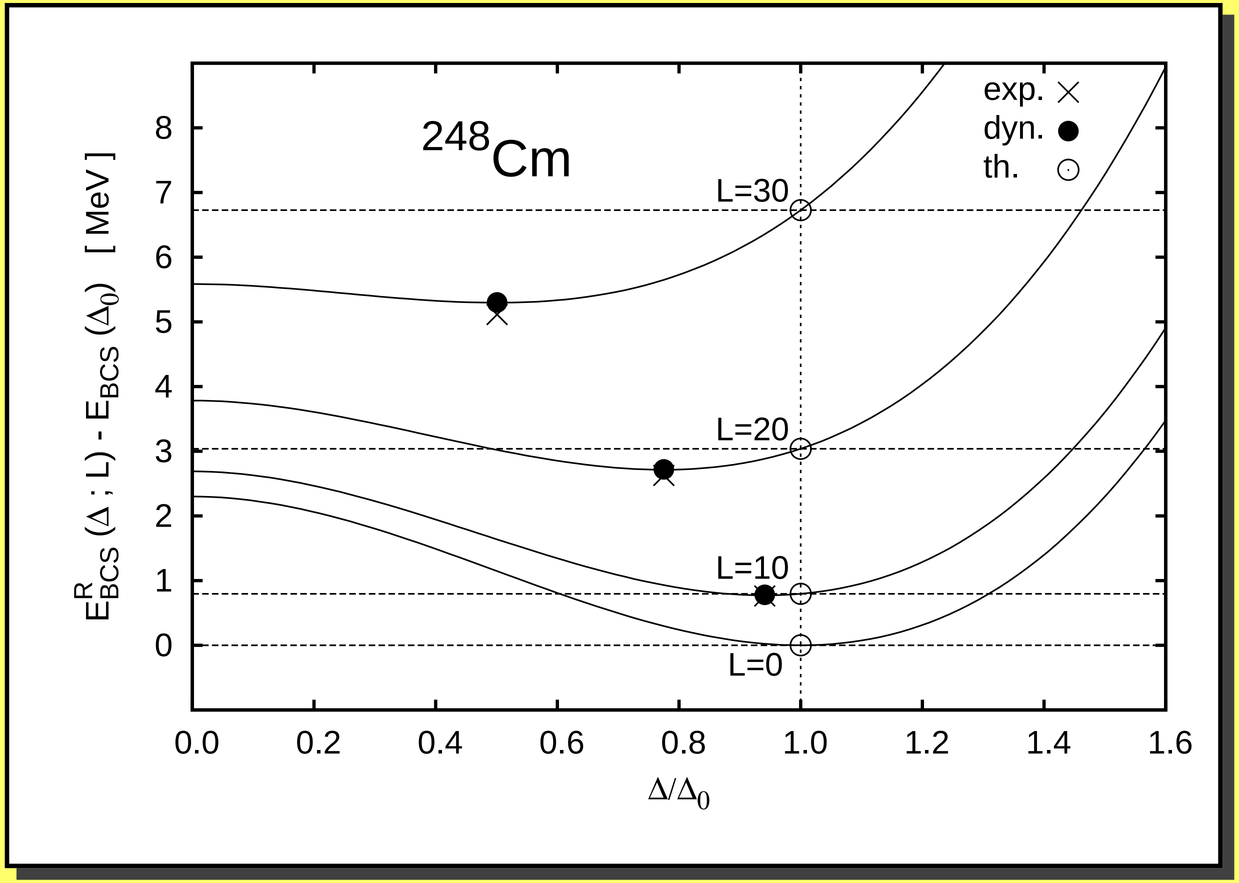
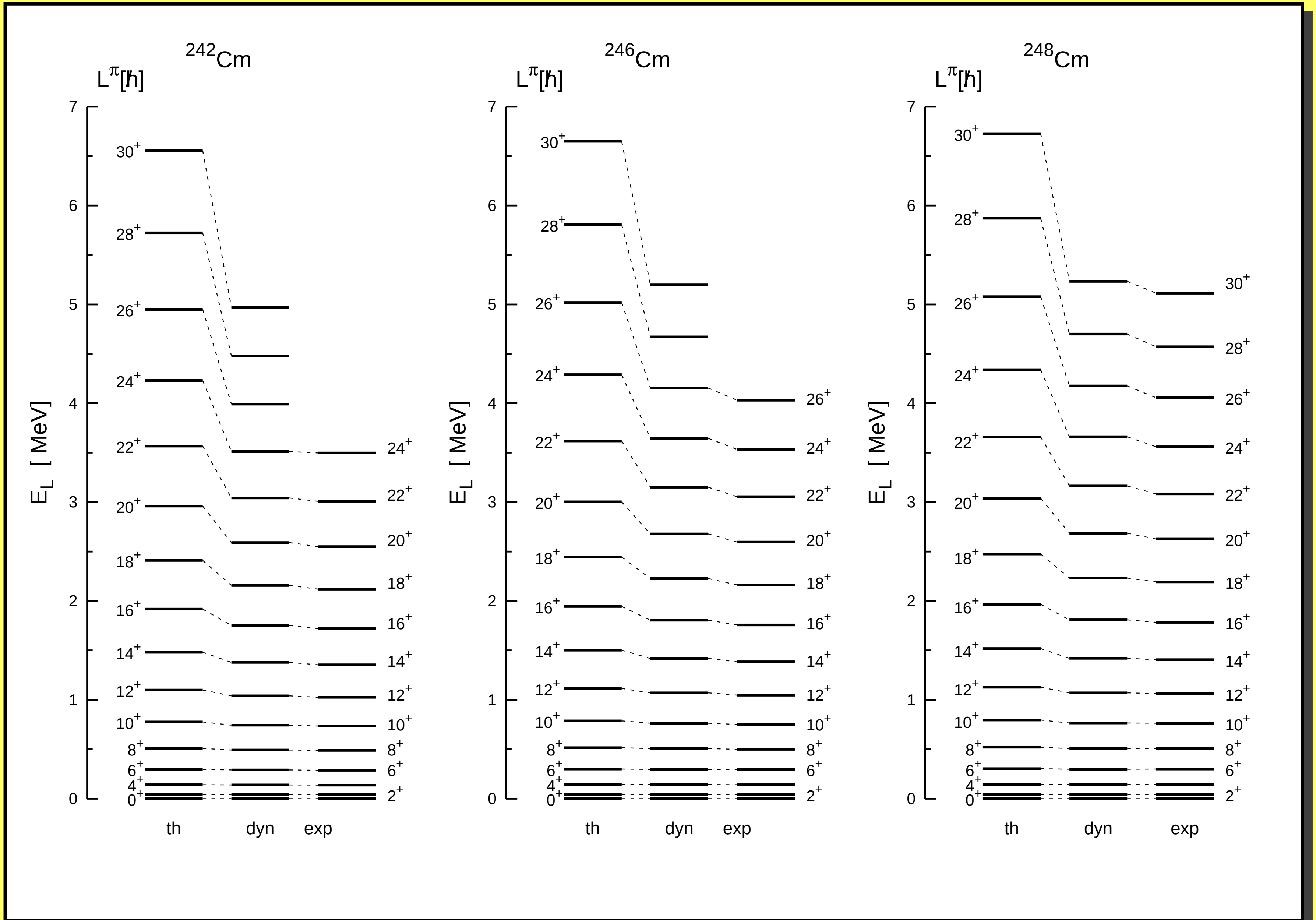
[a] S.G. Nilsson, C.F. Tsang, A. Sobiczewski, Z. Szymański, S. Wycech, C. Gustafson, I.L. Lamm, P. Möller, B. Nilsson, Nucl. Phys. **A131**, 1 (1969).

Dependence of the $E^{\mathrm{R}}_{\mathrm{BCS}}$ energy on Δ

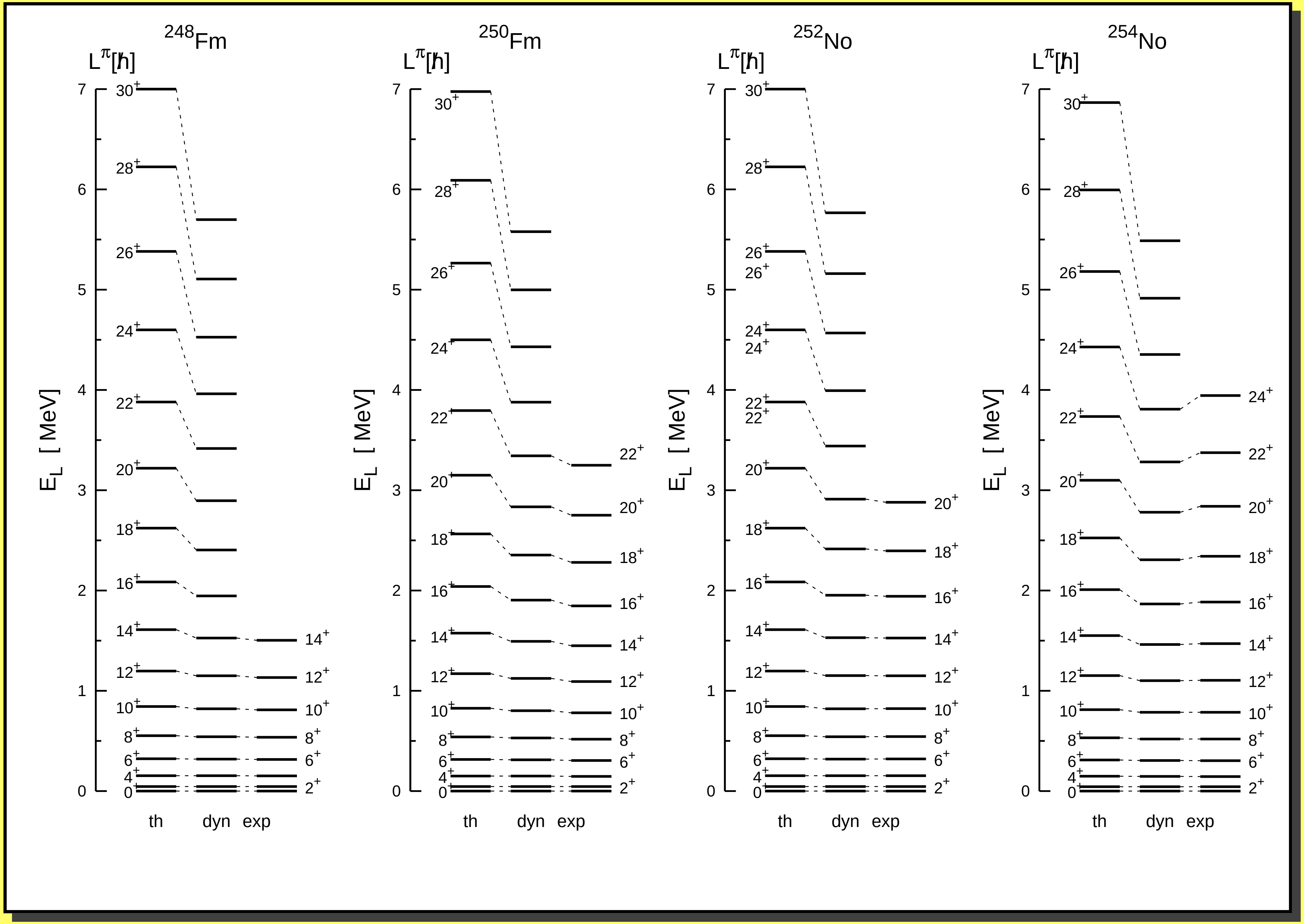


It is seen in the figure that the energy minima (black points) corresponding to $\boldsymbol{L = 10}$, $\boldsymbol{L = 20}$ and $\boldsymbol{L = 30}$ are significantly shifted down in comparison with the pure rotational-model estimates (open circles). This dynamical coupling of rotation with the pairing field brings the theoretical estimates to the experimental data (crosses).

Dynamical estimates of rotational bands for Cm



Dynamical estimates of rotational bands for Fm and No



Conclusions

(i) The Yukawa-folded mean field potential describes well the shell structure of heavy nuclei.

(ii) The Strutinsky shell correction and BCS pairing energy with pairing strength adjusted to the position of L=2 rotational states give the proper equilibrium deformations and masses of heavy nuclei.

(iii) Taking into account the dynamical coupling with the pairing field with rotation brings theoretical estimates of the high spin rotational levels toward experimental data.

(iv) More data would be required for finding the isotopic dependence of pairing strength.

(v) More deformation than two parameters should be included for finding the equilibrium deformations.

(vi) Minimization of the total microscopic energy (BCS plus rotational) over $\boldsymbol{\Delta^p}$, $\boldsymbol{\Delta^n}$ will be performed in future calculations.

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