Nuclear alternating-parity bands and transition rates in a model of coherent quadrupole-octupole motion

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Hamilton operator

$$H_{qo} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + U(\beta_2,\beta_3,I)$$

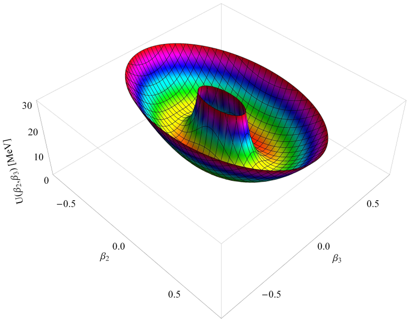
$$U(\beta_2,\beta_3,I) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{X(I)}{d_2\beta_2^2 + d_3\beta_3^2}$$

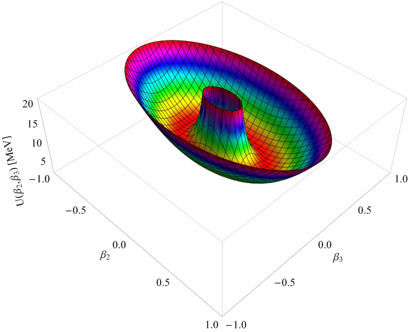
$$X(I) = [d_0 + I(I+1)]/2$$

- β_2 and β_3 axial deformation variables
- Mass parameters B_2 and B_3
- Stiffness parameters C_2 and C_3
- Moment of inertia parameters d_2 and d_3

[N. M. et al, Phys. Rev. C **73**, 044315 (2006); **76**, 034324 (2007)]

Potential Shapes

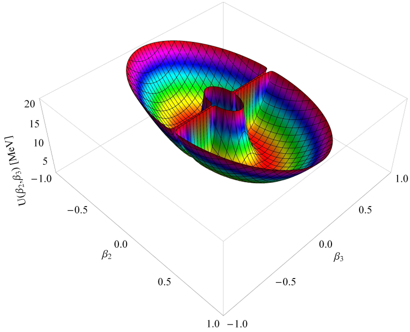


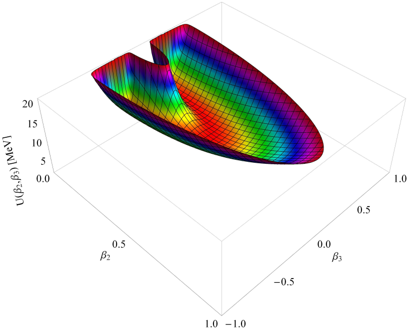


general form

$d_2/C_2 = d_3/C_3$

Prolate quadrupole deformation $\beta_2 > 0$





boundary condition

model space $\beta_2 > 0$

Coherent quadrupole-octupole mode

$$\beta_2 = \sqrt{d/d_2}\eta \cos\phi \ , \quad \beta_3 = \sqrt{d/d_3}\eta \sin\phi \ , \quad d = (d_2 + d_3)/2$$

$$U_I(\eta) = \frac{1}{2}C\eta^2 + \frac{X(I)}{d\eta^2}$$

$$\omega = \sqrt{C_2/B_2} = \sqrt{C_3/B_3} \equiv \sqrt{C/B}$$

$$\frac{\partial^2}{\partial\eta^2}\psi(\eta) + \frac{1}{\eta}\frac{\partial}{\partial\eta}\psi(\eta) + \frac{2B}{\hbar^2}\left[E - \frac{\hbar^2}{2B}\frac{k^2}{\eta^2} - U_I(\eta)\right]\psi(\eta) = 0$$

$$\frac{\partial^2}{\partial\phi^2}\varphi(\phi) + k^2\varphi(\phi) = 0$$

Analytic solution of the Schrödinger equation

Energy spectrum:

$$E_{n,k}(I,\pi) = \hbar\omega\left[2n + 1 + \sqrt{k^2 + bX(I)}\right], \quad b = \frac{2B}{\hbar^2 d}, \quad n = 0, 1, 2, \ldots$$

Rotation-vibration wave function:

$$\Psi^{\pi}_{nkIM0}(\eta,\phi) = \sqrt{\frac{2I+1}{8\pi^2}} D^{I}_{M\,0}(\theta)\Phi^{\pi}_{n,k,I}(\eta,\phi)$$

$$\Phi^{\pi}_{n,k,I}(\eta,\phi) = \psi^{I}_{nk}(\eta)\varphi^{\pi}_{k}(\phi)$$

Quadrupole-octupole vibration function

$$\Phi^{\pi}_{n,k,I}(\eta,\phi) = \psi^{I}_{nk}(\eta)\varphi^{\pi}_{k}(\phi)$$

$$\psi^{I}_{nk}(\eta) = \sqrt{\frac{2c\Gamma(n+1)}{\Gamma(n+2s+1)}}\,e^{-c\eta^2/2}c^{s}\eta^{2s}L^{2s}_{n}(c\eta^2)$$

$$c = \sqrt{BC}/\hbar, \quad s = \sqrt{k^2 + bX(I)}/2$$

$$\beta_2 > 0 \quad \Rightarrow \quad \varphi(-\pi/2) = \varphi(\pi/2) = 0$$

$$\varphi^{+}_{\mathbf{k}}(\phi) \quad = \quad \sqrt{\mathbf{2}/\boldsymbol{\pi}}\mathbf{cos}(\mathbf{k}\phi)\ , \quad \mathbf{k} = \mathbf{1}, \mathbf{3}, \mathbf{5}, \ldots \ \rightarrow \ \boldsymbol{\pi} = (+), I = \text{even}$$

$$\varphi^{-}_{\mathbf{k}}(\phi) \quad = \quad \sqrt{\mathbf{2}/\boldsymbol{\pi}}\mathbf{sin}(\mathbf{k}\phi)\ , \quad \mathbf{k} = \mathbf{2}, \mathbf{4}, \mathbf{6}, \ldots \ \rightarrow \ \boldsymbol{\pi} = (-), I = \text{odd}$$

Structure of the spectrum

Yrast alternating-parity sequence: $n = 0,\ k_n \to k_0^{(+)},\ k_0^{(-)}$
$\Rightarrow$ unites the ground-state band $(0_1^+, 2_1^+, 4_1^+, 6_1^+, \ldots)$
with the first negative-parity band $(1_1^-, 3_1^-, 5_1^-, \ldots)$

First non-yrast sequence: $n = 1,\ k_n \to k_1^{(+)},\ k_1^{(-)}$
$\Rightarrow$ unites the first β-band $(0_2^+, 2_2^+, 4_2^+, \ldots)$ with the
second negative-parity band $(1_2^-, 3_2^-, 5_2^-, \ldots)$

Second non-yrast sequence: $n = 2,\ k_n \to k_2^{(+)},\ k_2^{(-)}$
$\Rightarrow$ unites the second β-band $(0_3^+, 2_3^+, 4_3^+, \ldots)$ with
the third negative-parity band $(1_3^-, 3_3^-, 5_3^-, \ldots)$

and so on ...

Reduced transition probabilities

$$B(E\lambda; n_i k_i I_i \rightarrow n_f k_f I_f)$$
$$= \frac{1}{2I_i+1} \sum_{M_i M_f \mu} \left| \left\langle \Psi^{\pi_f}_{n_f k_f I_f M_f 0}(\eta, \phi) | \mathcal{M}_\mu(E\lambda) | \Psi^{\pi_i}_{n_i k_i I_i M_i 0}(\eta, \phi) \right\rangle \right|^2$$

$$\mathcal{M}_\mu(E1) = \sqrt{\frac{3}{4\pi}} \hat{Q}_{10} D^1_{0\mu}, \quad \mu = 0, \pm 1,$$
$$\mathcal{M}_\mu(E\lambda) = \sqrt{\frac{2\lambda+1}{16\pi}} \hat{Q}_{\lambda 0} D^\lambda_{0\mu}, \quad \lambda = 2, 3, \quad \mu = 0, \pm 1, ..., \pm\lambda$$

Multipole operators $\lambda = 1, 2, 3$

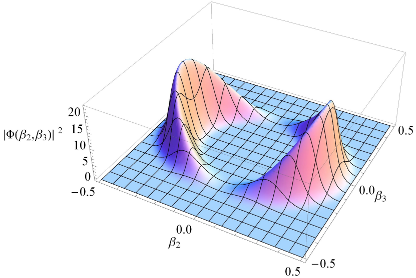
$$\hat{Q}_{10} = M_1 \beta_2 \beta_3 = M_1 p q \eta^2 \cos\phi \sin\phi$$
$$\hat{Q}_{20} = M_2 \beta_2 = M_2 p \eta \cos\phi$$
$$\hat{Q}_{30} = M_3 \beta_3 = M_3 q \eta \sin\phi$$

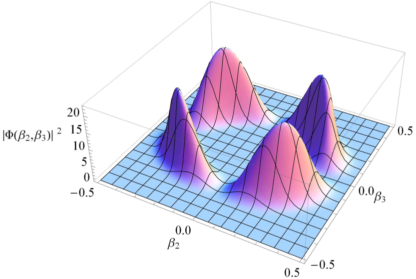
$$M_\lambda = \frac{3}{\sqrt{(2\lambda+1)\pi}} Z e R_0^\lambda, \qquad \lambda = 2, 3$$
$$M_1 = \frac{9AZe^3}{56\sqrt{35}\pi}\left(\frac{1}{J} + \frac{15}{8QA^{\frac{1}{3}}}\right)$$

J- surface, Q- volume symmetry-energy parameters

$\hat{Q}_{\lambda 0}$ connect states with fixed β_2 and β_3 deformations

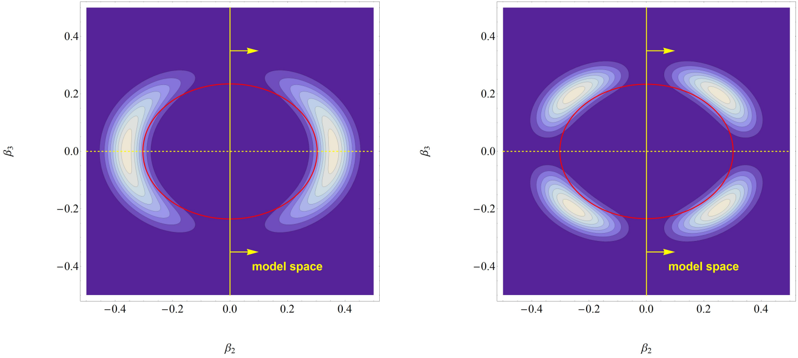
Complex-shape density distribution $|\Phi^{\pi}_{n,k,I}(\beta_2,\beta_3)|^2$





$I = 2, k = 1$ $\quad (n = 0) \quad$ $I = 1, k = 2$

Complex-shape density distribution $|\Phi^{\pi}_{n,k,I}(\beta_2,\beta_3)|^2$



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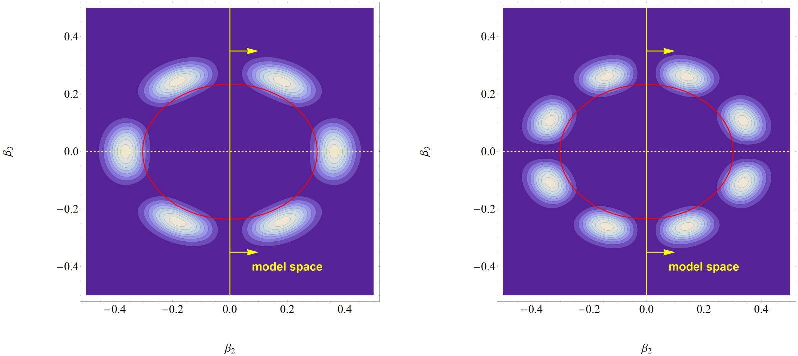
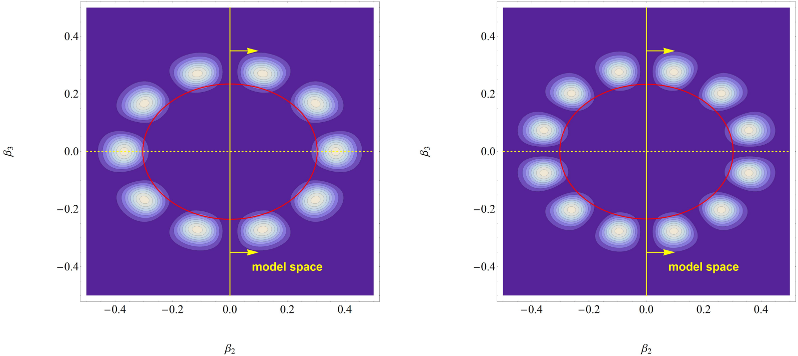


$I = 2, k = 3$ $(n = 0)$ $I = 1, k = 4$

Complex-shape density distribution $|\Phi^{\pi}_{n,k,I}(\beta_2,\beta_3)|^2$



$I = 2, k = 5$ $\quad (n = 0) \quad$ $I = 1, k = 6$

Generalization in the angular variable ϕ

$$\cos\phi \quad \rightarrow \quad \hat{A}_{20}(\phi) \equiv \sum_{k=1}^{\infty} \frac{\cos(k\phi)}{k} = -\frac{1}{2}[\ln 2 + \ln(1-\cos\phi)]$$

$$\sin\phi \quad \rightarrow \quad \hat{A}_{30}(\phi) \equiv \sum_{k=1}^{\infty} \frac{\sin(k\phi)}{k} = \frac{\pi-\phi}{2} + \pi \mathrm{Floor}\left(\frac{\phi}{2\pi}\right)$$

$$\cos\phi\sin\phi \quad \rightarrow \quad \hat{A}_{10}^{(1)}(\phi) \equiv \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\cos(m\phi)}{m}\frac{\sin(n\phi)}{n}$$

$$\hat{Q}_{10}(\eta,\phi) = M_1 pq\eta^2 \hat{A}_{10}(\phi)$$

$$\hat{Q}_{20}(\eta,\phi) = M_2 p\eta \hat{A}_{20}(\phi)$$

$$\hat{Q}_{30}(\eta,\phi) = M_3 q\eta \hat{A}_{30}(\phi)$$

Connection between states with any difference $\Delta k = k_f - k_i$

Model expressions for B(E1), B(E2) and B(E3)

$$B(E\lambda; n_i k_i I_i \rightarrow n_f k_f I_f) = \frac{2\lambda+1}{4\pi(4-3\delta_{\lambda,1})}\langle I_i 0 \lambda 0 | I_f 0\rangle^2 R_\lambda^2(n_i k_i I_i \rightarrow n_f k_f I_f)$$

$$R_\lambda(n_i k_i I_i \rightarrow n_f k_f I_f) = \left\langle \Phi^{\pi_f}_{n_f k_f I_f}(\eta,\phi) | \hat{Q}_{\lambda 0} | \Phi^{\pi_i}_{n_i k_i I_i}(\eta,\phi) \right\rangle$$

$$\begin{aligned} R_1 &= M_1 pq S_2(n_i, I_i; n_f, I_f) I_1^{\pi_i,\pi_f}(k_i, k_f) \\ R_2 &= M_2 p S_1(n_i, I_i; n_f, I_f) I_2^{\pi_i,\pi_f}(k_i, k_f) \\ R_3 &= M_3 q S_1(n_i, I_i; n_f, I_f) I_3^{\pi_i,\pi_f}(k_i, k_f) \end{aligned}$$

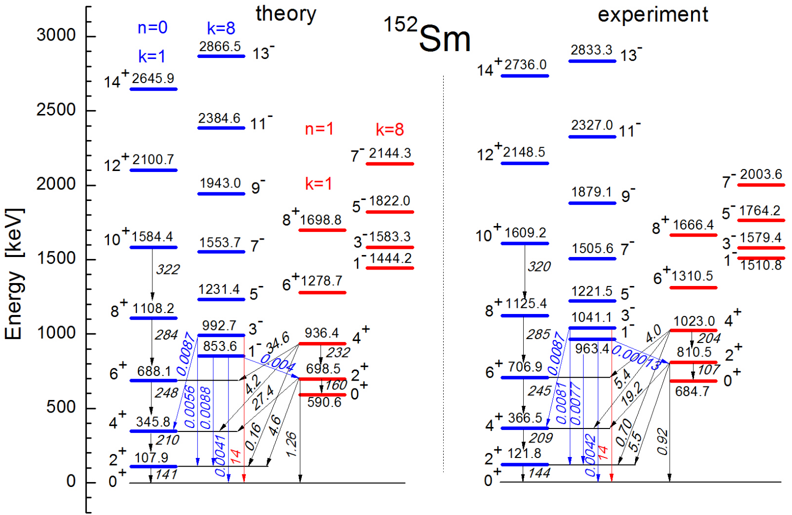
Model expressions for B(E1), B(E2) and B(E3)

$$S_1(n_i, I_i; n_f, I_f) = \int_0^\infty d\eta \psi_{n_f}^{I_f}(\eta)\eta^2\psi_{n_i}^{I_i}(\eta)$$

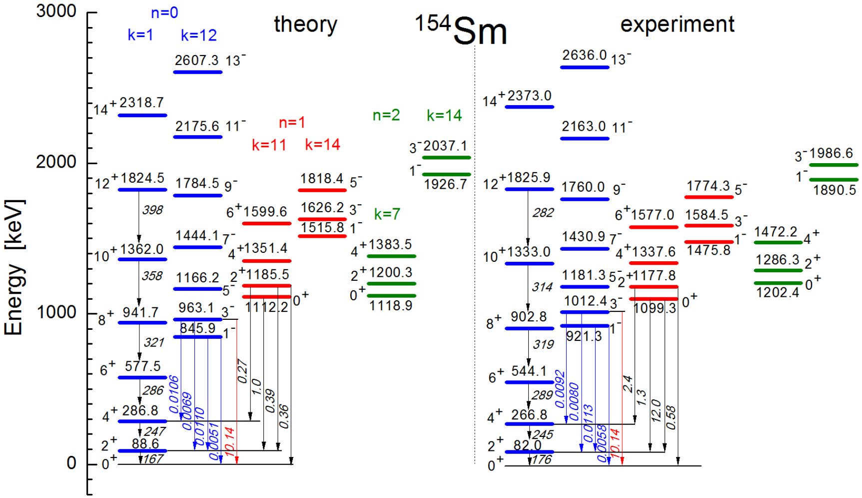
$$S_2(n_i, I_i; n_f, I_f) = \int_0^\infty d\eta \psi_{n_f}^{I_f}(\eta)\eta^3\psi_{n_i}^{I_i}(\eta)$$

$$I_\lambda^{\pi_i,\pi_f}(k_i, k_f) = \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A_{\lambda 0}(\phi)\varphi_{k_f}^{\pi_f}(\phi)\varphi_{k_i}^{\pi_i}(\phi)d\phi, \quad \lambda = 1, 2, 3$$

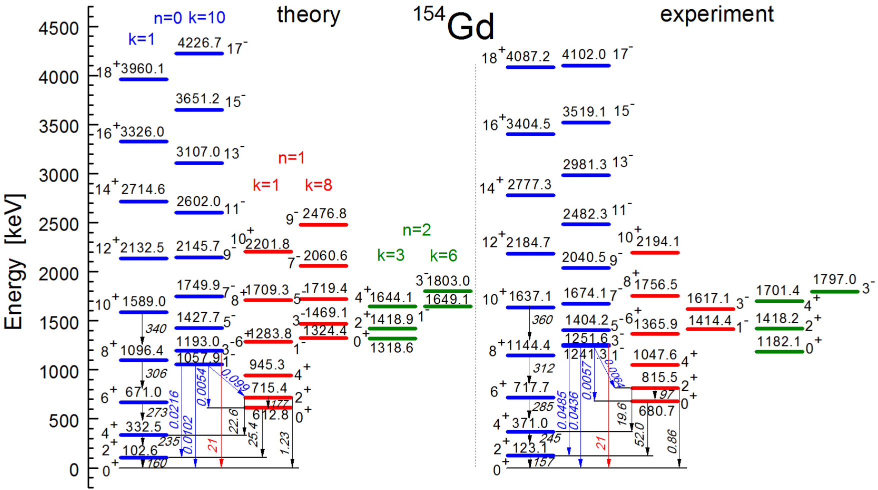
Energies and transition probabilities in ^{152}Sm



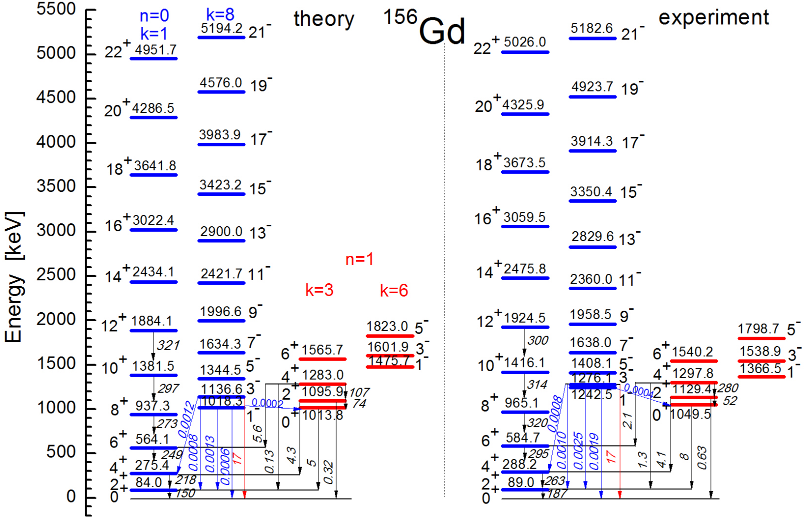
Energies and transition probabilities in ^{154}Sm



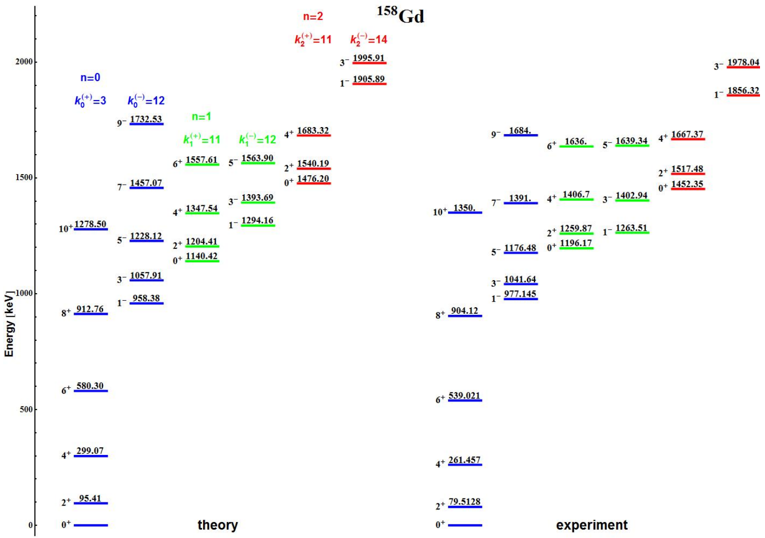
Energies and transition probabilities in ^{154}Gd



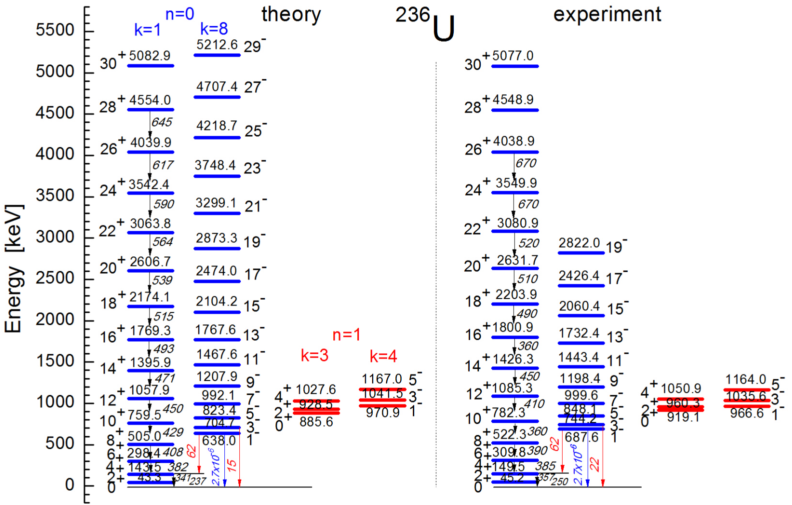
Energies and transition probabilities in ^{156}Gd



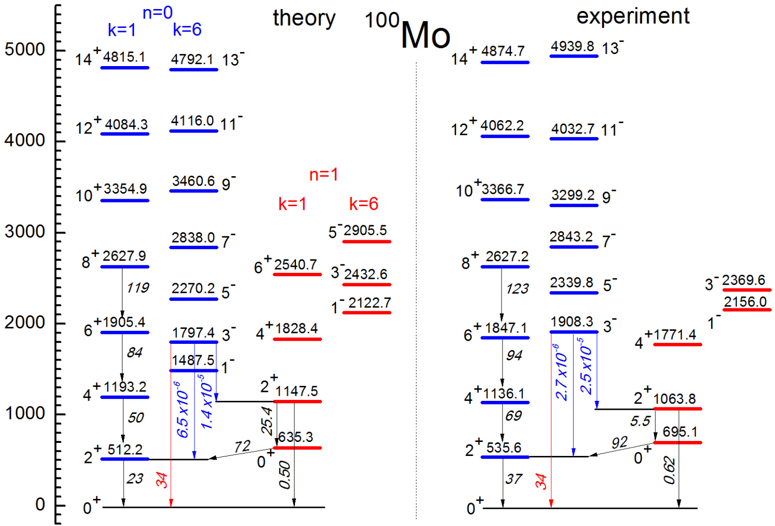
Alternating-parity spectrum in ^{158}Gd



Energies and transition probabilities in ^{236}U



Energies and transition probabilities in ^{100}Mo



SUMMARY

Results

- CQOM formalism for non-yrast alternating parity levels and E1-E3 transition probabilities
- Consideration of complex-shape density distribution $\Rightarrow$ generalized angular part of E1-E3 transition operators
- Description of octupole spectra and the attendant intraband and interband B(E1)-B(E2) transition probabilities in yrast and non-yrast bands in rare-earth, actinide and Mo nuclei

Perspectives

- Extension to odd-mass nuclei
- Extension beyond the limits of the coherent-mode assumption