STATISTICS OF THE SINGLE-PARTICLE LEVELS

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Plan of talk

1. INTRODUCTION. Order and Chaos: Quantum-classical correspondence

2. NEAREST NEIGHBOR SPACING DISTRIBUTIONS

3. NUMERICAL RESULTS

3a. QUANTUM SPECTRA AND LEVEL DENSITIES

3b LEVEL-SPACING DISTRIBUTIONS

3c SHELL EFFECTS

4. CONCLUSIONS

1) LEVEL STATISTICS AND RMT, Brody et al. (1981),
REGULAR AND CHAOTIC BEHAVIOR $\Longleftrightarrow$
Poisson and Wigner (GOE) level-spacing distributions,
ONE-BODY motion, INTEGRABILITY

2) WALL FORMULA, Swiatecki et al. (1977,78),
ORDER VS CHAOS, POINCARE SECTIONS AND
LYAPUNOV EXPONENTS, Swiatecki, Blocki, Skalski
et al. (1995-1999)

3) QUANTUM AND CLASSICAL EXCITATION ENERGY,
Swiatecki, Blocki, Skalski, Magierski et al. (1995-2007);
Blocki, Yatsyshyn, Magner (2010,2011)
FOR 10-20 PERIODS OF OSCILLATIONS

4) NEAREST NEIGHBOR SPACING DISTRIBUTIONS (NND)
AND SHELL EFFECTS, INTEGRABILITY?

2. NEAREST NEIGHBOR SPACING DISTRIBUTIONS (NND)

$$\mathcal{P}(S) = g(S)\,\exp\left(-\int_0^S g(x)\,\mathrm{d}x\right)/\aleph\,, \qquad \textbf{Wigner, 1967}$$

$$\int \mathrm{d}x \mathcal{P}(x) = \int \mathrm{d}x x \mathcal{P}(x) = 1 \qquad \Rightarrow \qquad \aleph$$

$$\mathcal{P}(S) = \exp\left(-S/D\right) \iff \text{Poisson for } g(S) = 1/D$$

$$\mathcal{P}(S) = \left(\pi S/2D\right)\,\exp\left(-\pi S^2/4D^2\right) \iff \text{Wigner, } g(S) \propto S$$

$$P(S) = \exp\left[(q-1)S\right]\Big[(1-q)^2 \mathrm{erf}(\sqrt{\pi q S/2}) +$$

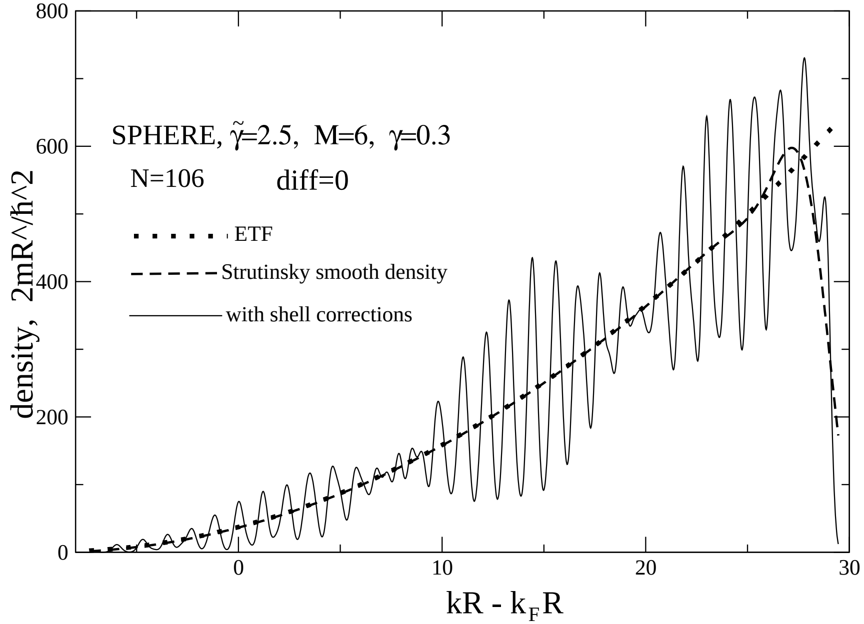
$$\left(2q(1-q) + \pi q^3 S\right)\exp\left(-\pi q^2 S^2/4\right), \text{ Berry, Robnik, 1984}$$

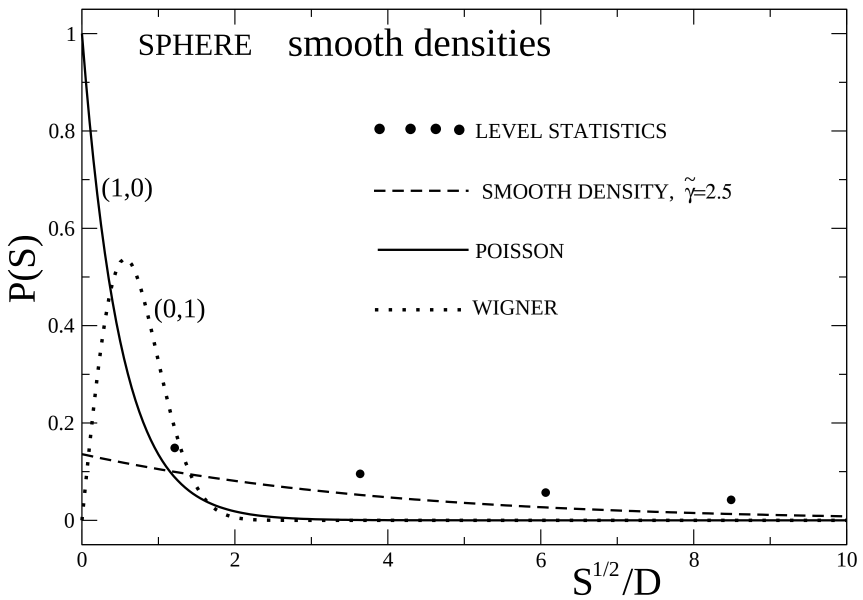
$$g(S) = \left[\mathcal{A} + \mathcal{B}S/D\right]/D, \quad \text{P.: } \mathcal{A} = 1, \mathcal{B} = 0; \quad \text{W.: } \mathcal{A} = 0, \mathcal{B} = 1:$$

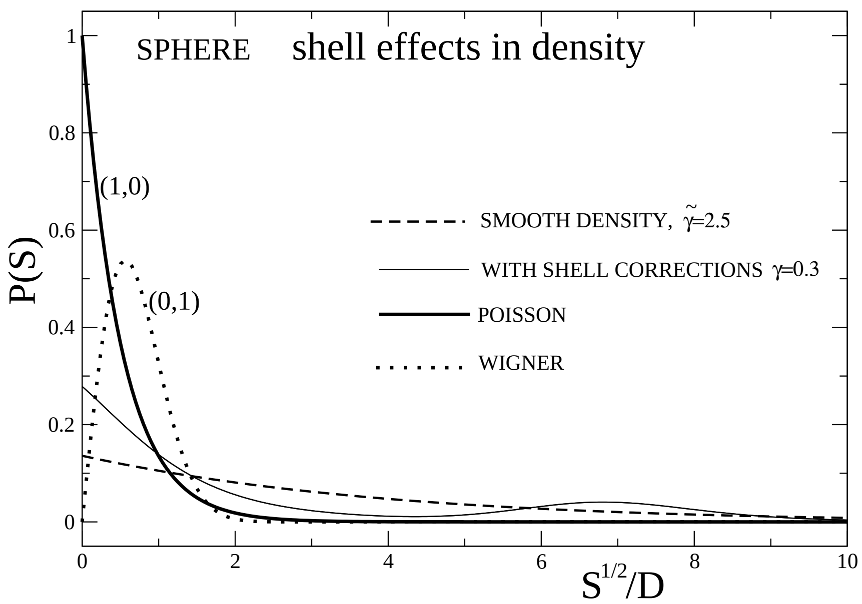
$$\mathcal{P}(S) = \left(1 + \mathcal{B}\xi/\mathcal{A}\right)\,\exp\left(-\mathcal{B}\xi^2/2 - \mathcal{A}\xi\right)/\left[\aleph_{PW}^{(0)} + \mathcal{B}\,\aleph_{PW}^{(1)}/\mathcal{A}\right]$$

$$\xi = S/D\,, \qquad \aleph_{PW}^{(0)} = \sqrt{\pi/2\mathcal{B}}\,\exp\left(\mathcal{A}^2/2\mathcal{B}^2\right)\,\mathrm{erf}\left[\left(\mathcal{A} + \mathcal{B}\xi\right)/\sqrt{2\mathcal{B}}\right]$$

$$\aleph_{PW}^{(1)} = -\left[\exp\left(-\mathcal{B}\xi_{max}^2/2 - \mathcal{A}\xi_{max}\right) + \mathcal{A}\,\aleph_{PW}^{(0)}\right]/\mathcal{B}$$


SPHERE, $\tilde{\gamma}$=2.5, M=6, γ=0.3
N=106 diff=0
ETF
Strutinsky smooth density
with shell corrections
density, 2mR^/h^2
kR - k_FR


SPHERE smooth densities
LEVEL STATISTICS
SMOOTH DENSITY, $\tilde{\gamma}$=2.5
POISSON
WIGNER
(1,0)
(0,1)
P(S)
$S^{1/2}$/D

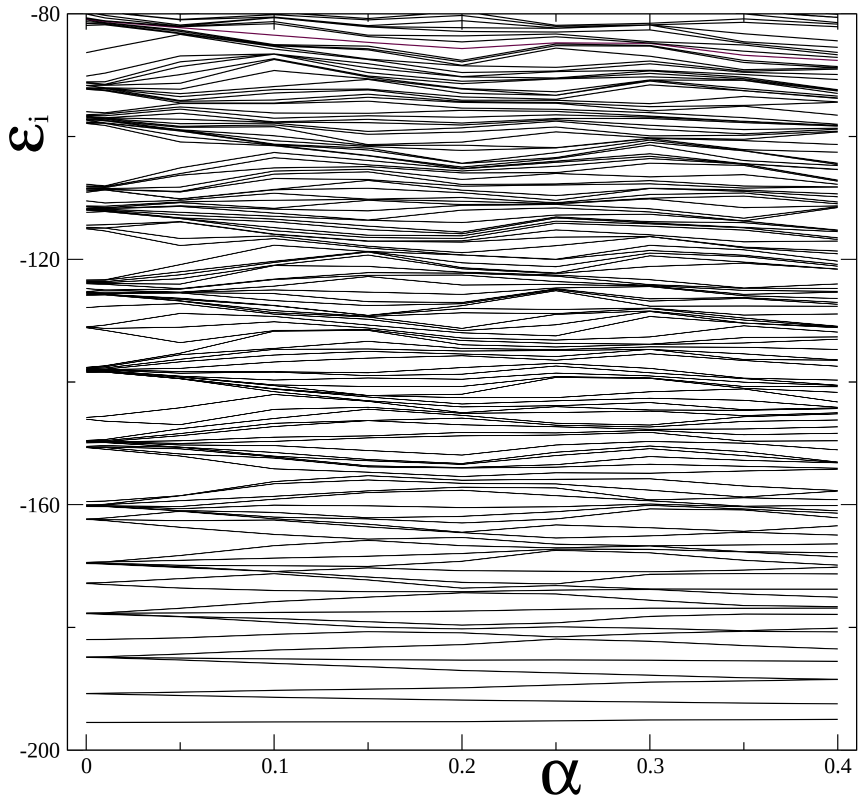

SPHERE shell effects in density
SMOOTH DENSITY, $\tilde{\gamma}$=2.5
WITH SHELL CORRECTIONS γ=0.3
POISSON
WIGNER
(1,0)
(0,1)
P(S)
$S^{1/2}$/D

2.1. QUANTUM EQUILIBRIUM SPECTRUM: P_2, P_5

$$H\phi_i = \varepsilon_i \phi_i, \quad V(r,\theta) = -V_0 \left[1 + \exp\left\{\left[r - R(\theta)\right]/a\right\}\right]^{-1}$$

$$R(\theta) = R_0 \left[1 + \alpha_n P_n(cos\theta) + \alpha_1 P_1(cos\theta)\right]/\lambda, \quad \alpha_n = \alpha\sqrt{(2n+1)/5}$$

P_2



P_5

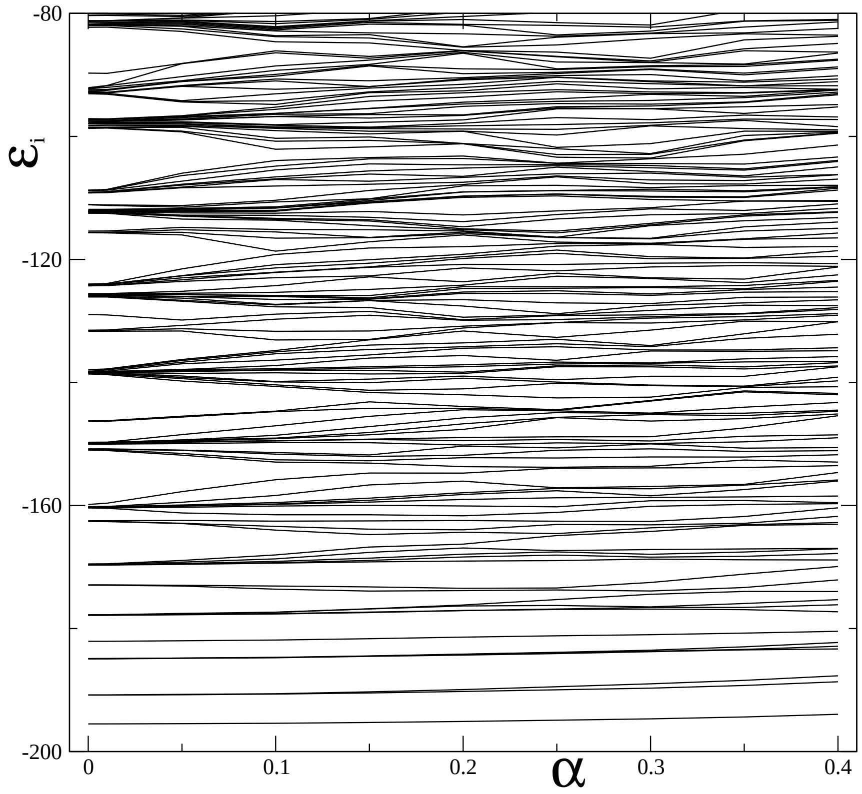


Fig. 1a. The s.p. energy levels ε_i in the WS potential ($V_0 = 200$ MeV, $R_0 = 6.622$ fm, $a = 0.1$ fm) as function of the deformation α for the P_2 (left) and P_5 (right) shapes.

2.1. QUANTUM EQUILIBRIUM SPECTRUM: P_2, P_3

$$H\phi_i = \varepsilon_i\phi_i, \quad V(r,\theta) = -V_0\left[1+\exp\left\{\left[r - R(\theta)\right]/a\right\}\right]^{-1}$$

$$R(\theta) = R_0\left[1+\alpha_n P_n(cos\theta) + \alpha_1 P_1(cos\theta)\right]/\lambda, \quad \alpha_n = \alpha\sqrt{(2n+1)/5}$$

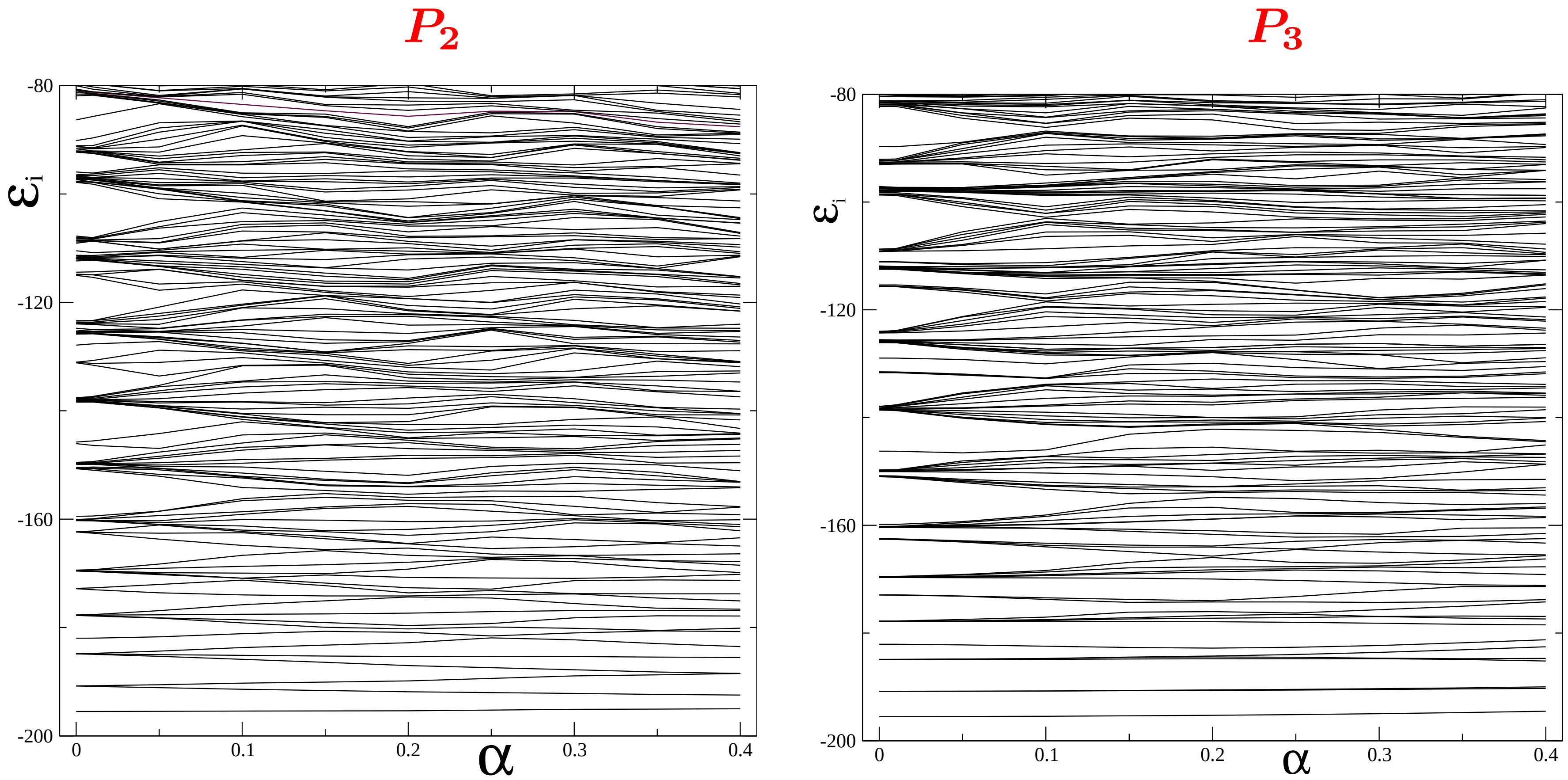


Fig. 1b. The s.p. energy levels ε_i in the WS potential ($V_0 = 200$ MeV, $R_0 = 6.622$ fm, $a = 0.1$ fm) as function of the deformation α for the P_2 (left) and P_3 (right) shapes.

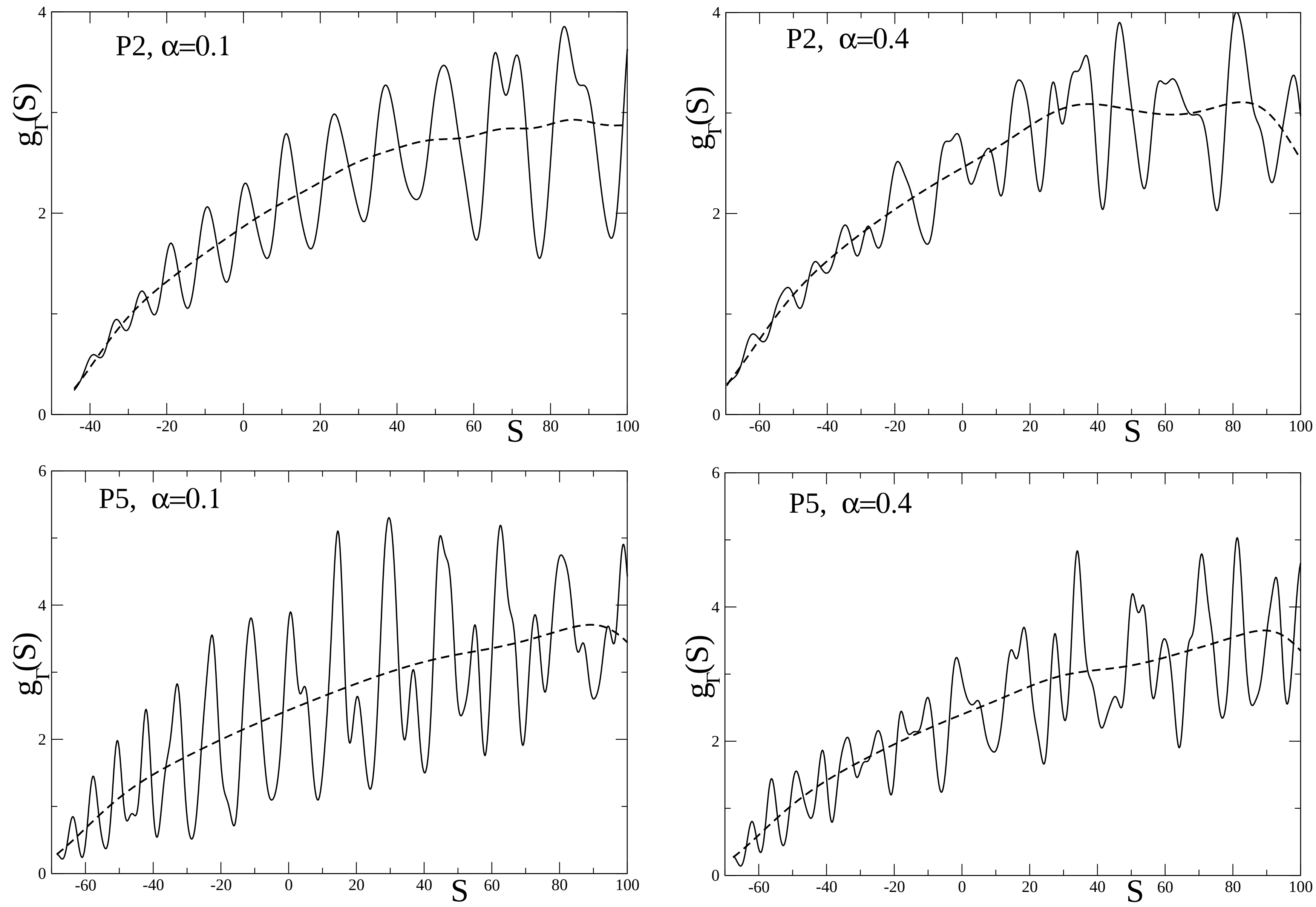


Fig. 2. **Level densities $g_\Gamma(S)$ as function of the energy S; dashed is smooth density $\tilde{g}(S)$; solid is the total density $g_\Gamma(S)$ ($\Gamma = 3$ MeV).**

2.3. QUANTUM EQUILIBRIUM SPECTRUM $m = 0$

$$H\phi_i = \varepsilon_i \phi_i, \quad V(r,\theta) = -V_0 \left[1 + \exp\left\{\left[r - R(\theta)\right]/a\right\}\right]^{-1}$$

$$R(\theta) = R_0 \left[1 + \alpha_n P_n(cos\theta) + \alpha_1 P_1(cos\theta)\right]/\lambda, \quad \alpha_n = \alpha\sqrt{(2n+1)/5}$$

P_2 P_5

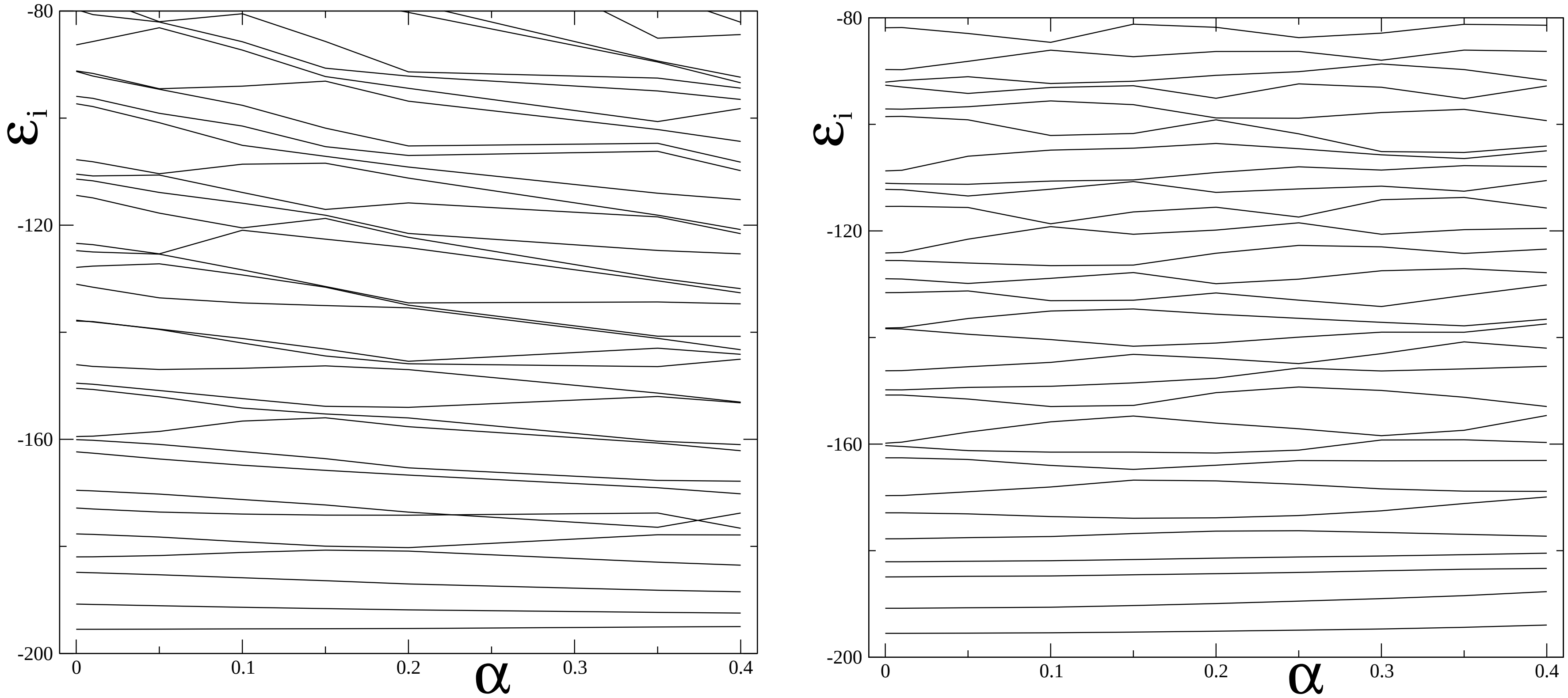


Fig. 3. The s.p. energy levels ε_i with $m = 0$ in the WS potential ($V_0 = 200$ MeV, $R_0 = 6.622$ fm, $a = 0.1$ fm) as function of the deformation α for the P_2 (left) and P_5 (right) shapes.

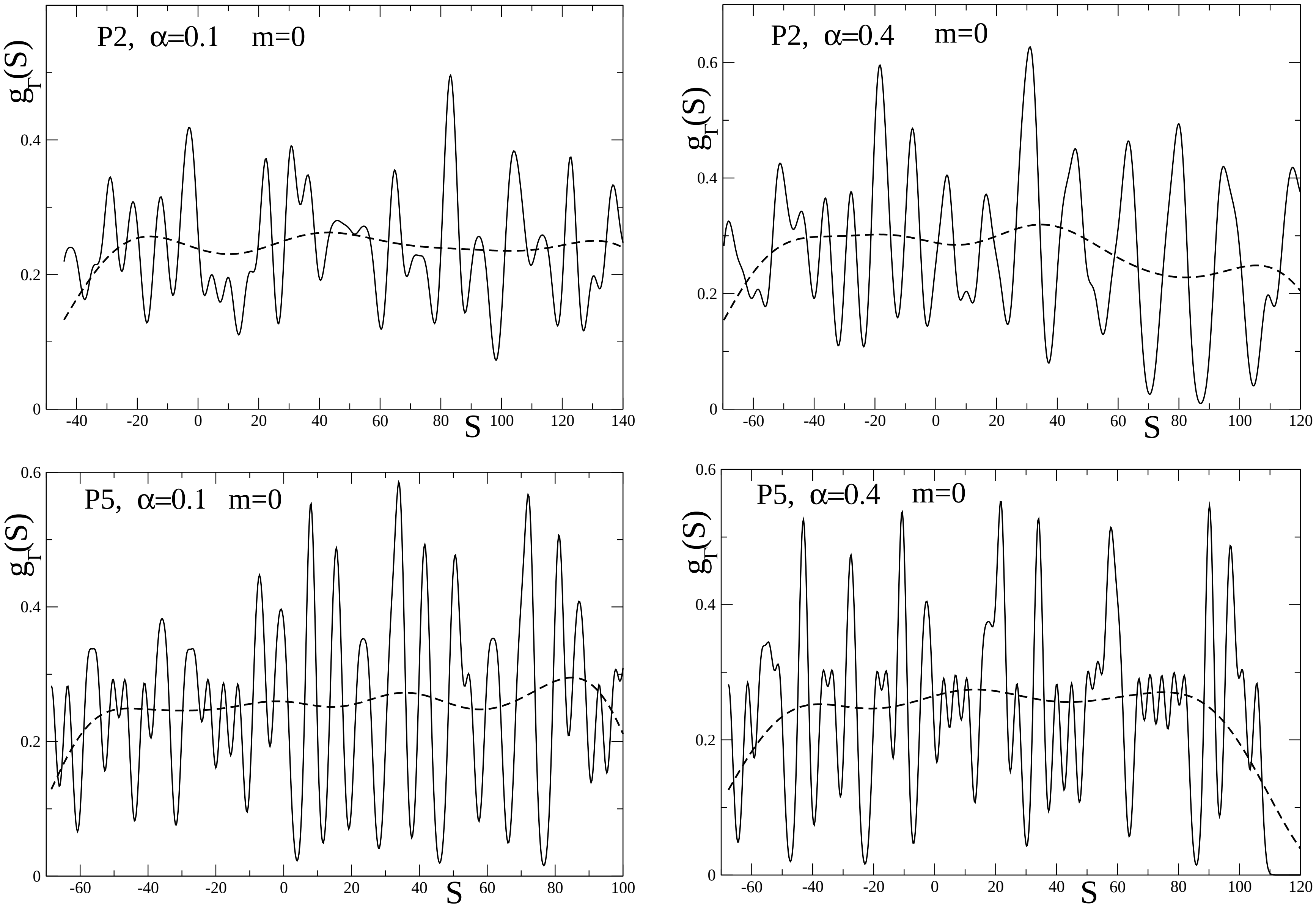


Fig. 4. Level densities $g_\Gamma(S)$ with $m = 0$ as function of the energy S; dashed is smooth density $\tilde{g}(S)$ solid is the total density $g_\Gamma(S)$.

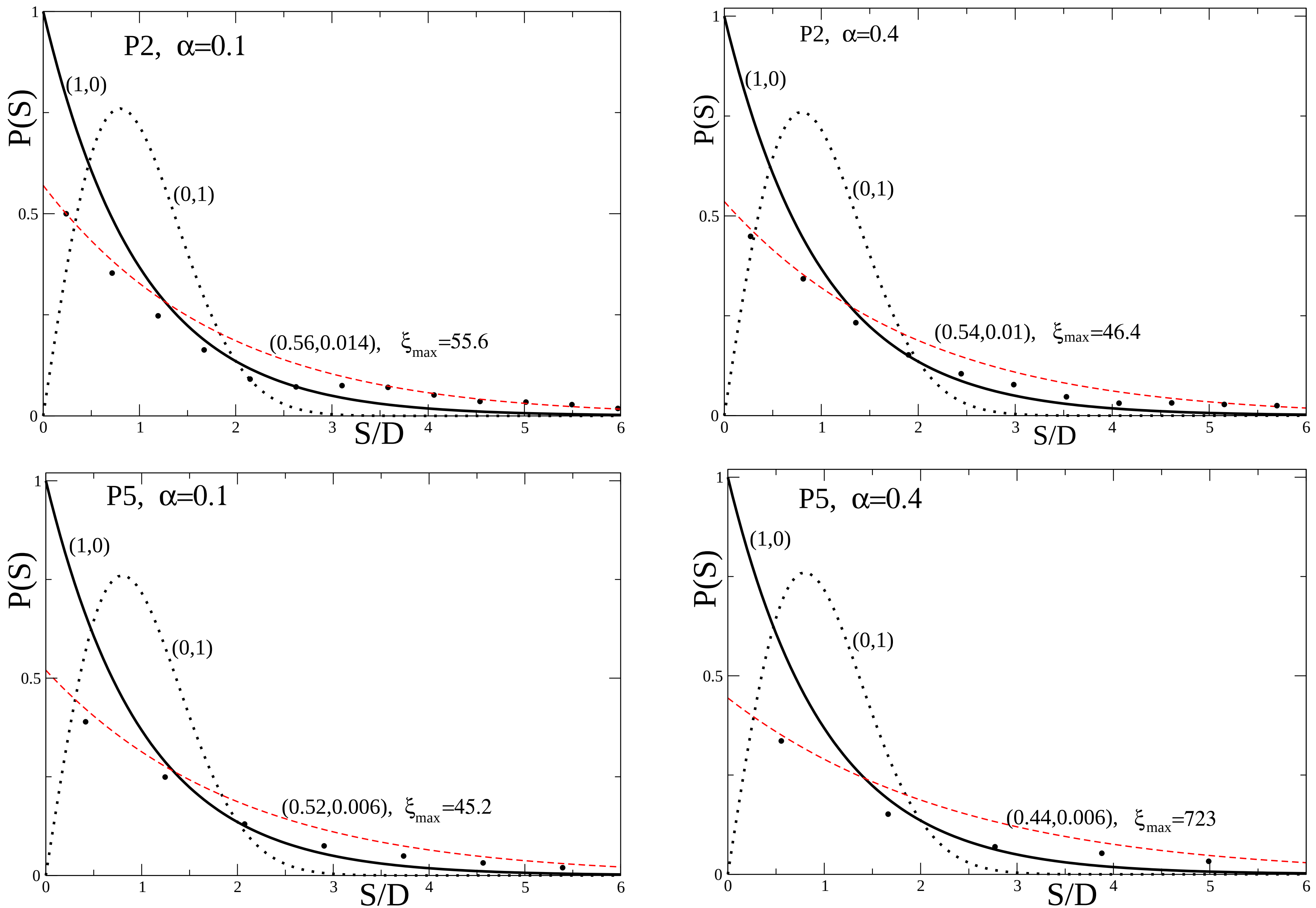


Fig. 5. NND $P(S)$ (heavy dots) vs. the energies S; solid is Poisson (1,0), $\mathcal{A} = 1$, $\mathcal{B} = 0$), frequent dots is Wigner (0,1) and dashed is for the linear level density.

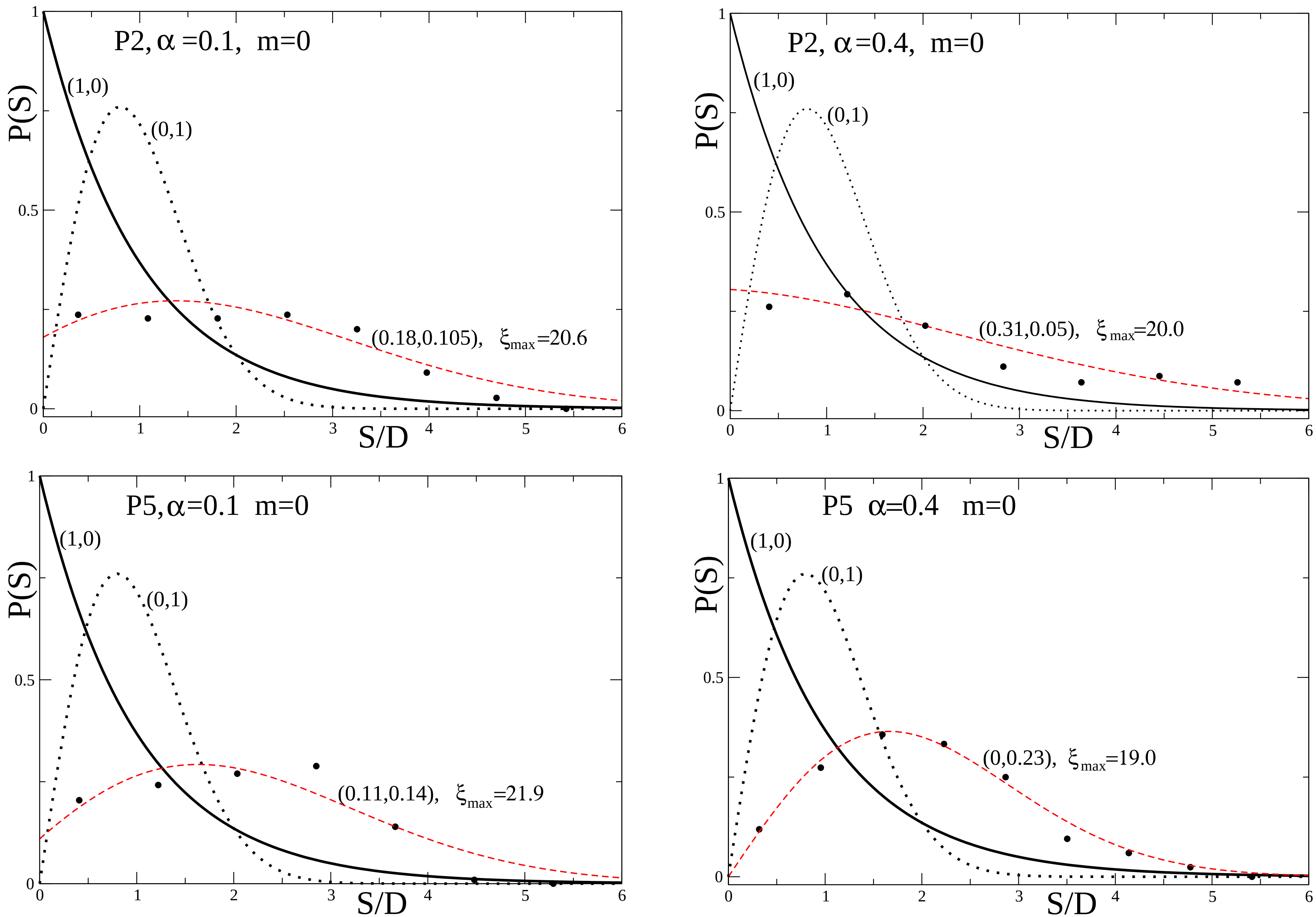


Fig. 6. NND $P(S)$ for levels with $m = 0$ (heavy dots) vs. the energy S; solid is Poisson (1,0), $\mathcal{A} = 1$, $\mathcal{B} = 0$, frequent dots is Wigner (0,1) and dashed is for the linear density.

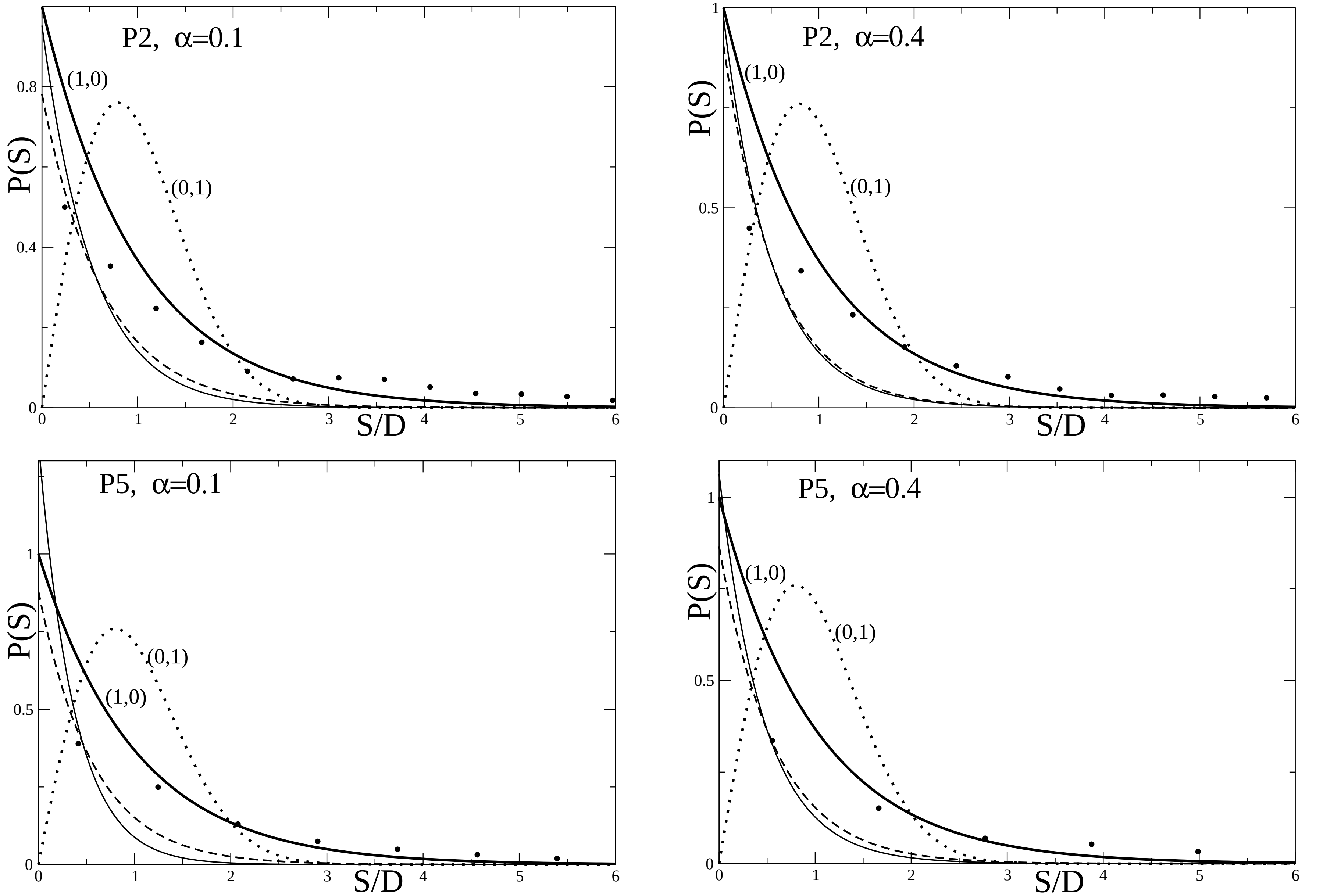


Fig. 7. The general Wigner $P(S)$ vs. energies S; thick dashed is for smooth density $\tilde{g}(S)$ and thin solid is for the total density $g_\Gamma(S)$.

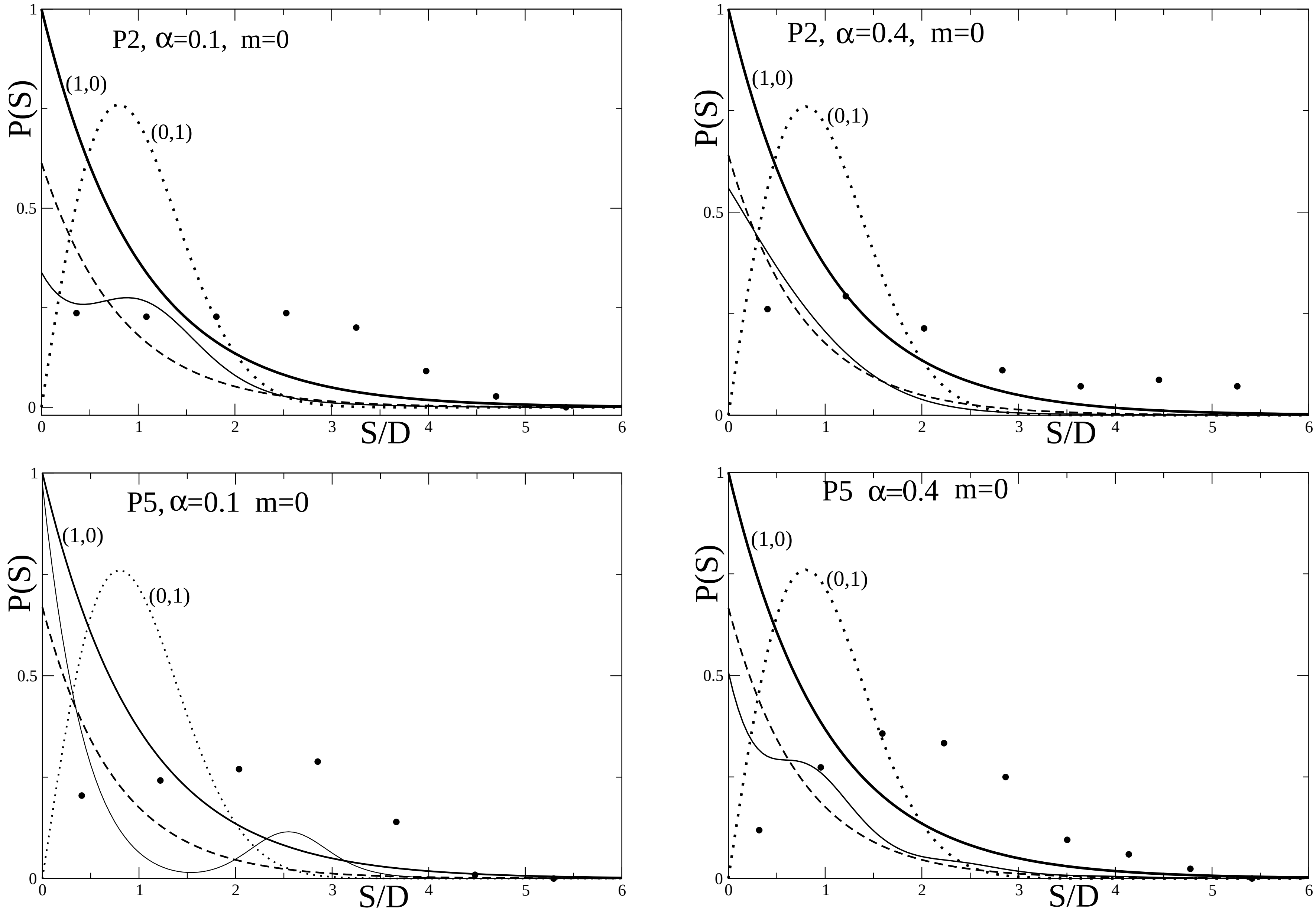


Fig. 8. The general Wigner $P(S)$ with the $m = 0$ vs. energies S; thick dashed is for smooth density $\tilde{g}(S)$ and thin solid is for the total density $g_{\Gamma}(S)$.

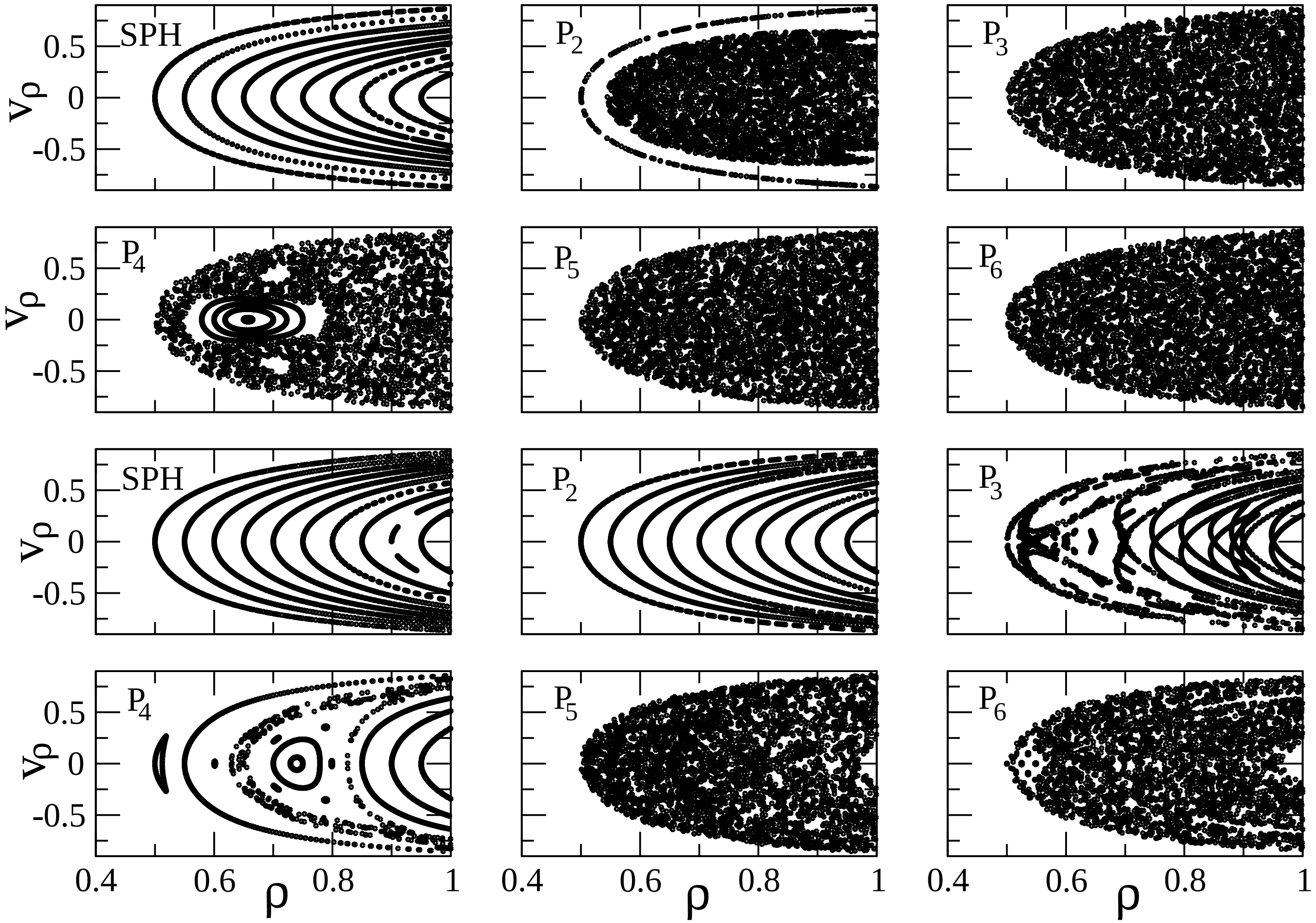


Fig.9. **Poincare sections for six shapes at the large deformation $\alpha = 0.4$ (two upper rows) and small deformation $\alpha = 0.05$ (two lower rows) for the projections of the angular momentum on the symmetry axis $m = 0.5$ with respect to the maximal value.**

CONCLUSIONS

- **NND $P(S)$ for full s.p. spectra and levels with $m = 0$ in a deformed WS potential were analysed in terms of the Poisson and Wigner distributions for several deformations and their multipolarities.**
- **We found the significant deflections of all distributions with a fixed value of the angular momentum projection of the particle, more closely to the Wigner distribution, in contrast to those of the full spectra with Poisson-like behavior though all desired potentials are non-integrable in the symmetry-axis plane.**
- **Notable shell effects are observed in the level distributions for all deformations and their multipoilarities, besides of a small region near the spherical shape.**
- **As perspectives, it would be worth to apply to the collective dynamics for analysis of the order-chaos transitions.**

THANKS!