Nuclear vorticity and general treatment of vortical, toroidal, and compression modes

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Motivation

Nuclei demonstrate both

- **irrotational flow (most of electric GR)** $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) = 0$
- **vortical flow (toroidal GR, s-p excitations)** $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \neq 0$

Vorticity $\vec{w}(\vec{r})$ is a fundamental quantity:

- **does not contribute to the continuity equation,** $\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$
- **represents an independent part of charge-current distribution beyond the continuity equation.**

Vorticity is related to the exotic modes:

- **toroidal E1 mode (TM) ,**
- **compression E1 mode (CM),**

which are now of a keen interest .

Theoretical studies:

Many publications on toroidal and compressional (ISGDR) modes and manifestations of vorticity:

V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).
M.N. Harakeh et al, PRL 38, 676 (1977).
S.F. Semenko, SJNP 34 356 (1981).
J. Heisenberg, Adv. Nucl. Phys. 12, 61 (1981).
S. Stringari, PLB 108, 232 (1982).
E. Wust et al, NPA 406, 285 (1983).
E.E. Serr, T.S. Dumitrescu, T.Suzuki, NPA 404 359 (1983).
D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).
E.B. Balbutsev and I.N. Mikhailov, JPG 14, 545 (1988).
S.I. Bastrukov, S. Misicu, A. Sushkov, NPA 562, 191 (1993).
I. Hamamoto, H.Sagawa, X.Z. Zang, PRC 53 765 (1996).
E.C.Caparelli, E.J.V.de Passos, JPG 25, 537 (1999).
N.Ryezayeva et al, PRL 89, 272502 (2002).
G.Colo, N.Van Giai, P.Bortignon, M.R.Quaglia, PLB 485, 362 (2000).
D. Vretenar, N. Paar, P. Ring,T. Nikshich, PRC 65, 021301(R) (2002).
V.Yu. Ponomarev, A.Richter, A.Shevchenko, S.Volz, J.Wambach, PRL 89, 272502 (2002).
J. Kvasil, N. Lo Iudice, Ch. Stoyanov, P. Alexa, JPG 29, 753 (2003).
A. Richter, NPA 731, 59 (2004).
……….
N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007).

Recent review

Multipole operators of the modes:

Toroidal mode E1(T=0):

V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).

$$\hat{M}_{tor}(E1\mu) = \frac{1}{20c}\int d\vec{r}\,\hat{\vec{j}}_{nuc}(\vec{r})\cdot[\vec{\nabla}\times(\vec{r}\times\vec{\nabla})\underbrace{(r^3 - \frac{5}{3}r<r^2>_0)}_{\text{cmc}}Y_{1\mu}]$$

$$= -\frac{i}{2\sqrt{3}c}\int d\vec{r}\,\hat{\vec{j}}_{nuc}(\vec{r})\cdot[\frac{\sqrt{2}}{5}r^2\vec{Y}^2_{1\mu} + (r^2 - \underbrace{<r^2>_0}_{\text{cmc}})\vec{Y}^0_{1\mu}]$$

Compression mode E1(T=0):

S.F. Semenko, SJNP 34 356 (1981).

$$\hat{M}'_{com}(E1\mu) = \frac{1}{10}\int d\vec{r}\,\hat{\rho}(\vec{r})[r^3 - \frac{5}{3}r<r^2>_0]Y_{1\mu}$$

The TM and CM operators are related.

Vortical mode E1(T=0): **NO yet OPERATOR**

multipole vortical operator introduced in (discussed later) :

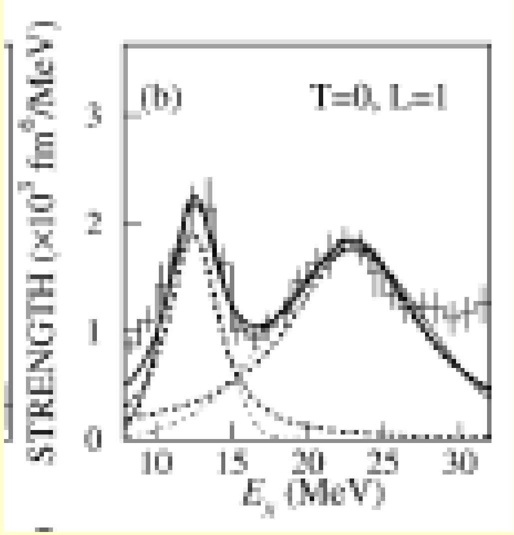
J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC 84, 034303, 2011

Observation of ISGDR : CM and perhaps TM:

(α,α')

D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)



(γ,γ')

N.Ryezayeva et al, PRL 89, 272502 (2002).

(e,e')

J.Heisenberg at al. PRC 25, 5 (1982)

Open problems

different conclusions
on CM vorticity

- definition of nuclear vorticity (HD vs Wambach),

- IS (T=0) and IV(T=1) branches of the modes,

- role of magnetization (spin) nuclear current,

- there is no the VM operator, VM vs TM/CM,

J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034403, 2011

We showed that the VM operator may be derived and related to TM and CM operators:

$$\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)$$

This relation allows to understand better the connection between VM, TM, and CM

Two definitions of vorticity in nuclear theory

definition 1 - from hydrodynamics (HD) :

$\vec{\nabla}\times\vec{v}(\vec{r})=0$ - nonvortical (without whirls)

$\vec{\nabla}\times\vec{v}(\vec{r})\neq 0$ - vortical (with whirls)

excited state $|i>$ is vortical if $\vec{\nabla}\times\vec{v}_i(\vec{r})\neq 0$

excited state $|i>$ is not vortical if $\vec{\nabla}\times\vec{v}_i(\vec{r})=0$

$$\vec{v}_i(\vec{r})=\frac{\delta\vec{j}^{(i)}_{nuc}(\vec{r})}{\rho_0(\vec{r})} \qquad \delta\vec{j}_i(\vec{r})=<i|\hat{\vec{j}}_{nuc}(\vec{r})|gs> \qquad \rho_0(\vec{r})=<gs|\hat{\rho}(\vec{r})|gs>$$

$$\hat{\vec{j}}_{nuc}(\vec{r})=\hat{\vec{j}}_{con}(\vec{r})+\hat{\vec{j}}_{mag}(\vec{r})=\frac{e\hbar}{m}\sum_{q=n,p}(\hat{\vec{j}}^{q}_{com}(\vec{r})+\hat{\vec{j}}^{q}_{mag}(\vec{r}))$$

usually is neglected in the HD papers

$$\hat{\vec{j}}^{q}_{con}(\vec{r})=-ie^{q}_{eff}\sum_{k\ni q}\left(\delta(\vec{r}-\vec{r}_k)\vec{\nabla}_k-\overleftarrow{\nabla}_k\delta(\vec{r}-\vec{r}_k)\right)$$

$$\hat{\vec{j}}^{q}_{mag}(\vec{r})=\frac{g_s}{2}\sum_{k\ni q}\vec{\nabla}_k\times\hat{\vec{s}}_{qk}\delta(\vec{r}-\vec{r}_k)$$

$$<i|\hat{M}(E\lambda\mu)|0>=\int d\vec{r}\,(\vec{\nabla}\times\delta\vec{j}^{(i)}_{nuc})\,[....]$$

$$<i|\hat{M}_{vor}(E\lambda\mu)|0>=\int d\vec{r}\,\rho_0(\vec{r})\,(\vec{\nabla}\times\vec{v}_i(\vec{r}))\,[....]$$

from the point of view of HD comp. modes are irrotational (not vortical):

$$\vec{v}_{com}(E1\mu)\propto\vec{\nabla}[r^3-\frac{5}{3}r<r^2>_0]Y_{1\mu} \qquad \vec{\nabla}\times\vec{v}_{com}(\vec{r})=0$$

definition 2 : D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

$$\vec{j}_{nuc}(\vec{r}) = \rho(\vec{r})\,\vec{v}(\vec{r}) \Rightarrow \vec{\nabla}\times\vec{j}_{nuc}(\vec{r}) = \rho(\vec{r})\,\vec{\nabla}\times\vec{v}(\vec{r}) + \left(\vec{\nabla}\rho(\vec{r})\right)\times\vec{v}(\vec{r})$$

$$\Rightarrow \left\langle f\,|\,\rho(\vec{r})\,\vec{\nabla}\times\vec{v}(\vec{r})\,|\,i\right\rangle \approx \left\langle f\,|\,\vec{\nabla}\times\vec{j}_{nuc}(\vec{r})\,|\,i\right\rangle - \left\langle f\,|\left(\vec{\nabla}\rho(\vec{r})\right)\times\vec{v}\,|\,i\right\rangle$$

$$\approx \left\langle f\,|\,\vec{\nabla}\times\vec{j}_{nuc}(\vec{r})\,|\,i\right\rangle - \frac{ikc}{\lambda}\left\langle f\,|\left(\vec{\nabla}\rho(\vec{r})\right)\times\vec{r}\,|\,i\right\rangle$$

0 ← if transition is not vortical

$S_{fi}(\vec{r})$

using the decomposition into multipoles:

$$\delta\vec{j}_{(fi)}(\vec{r}) = \left\langle j_f m_f\,|\,\hat{\vec{j}}_{nuc}(\vec{r})\,|\,j_i m_i\right\rangle = \sum_{\substack{\lambda\mu\\ l=\lambda\pm1}} \frac{(j_i m_i \lambda\mu\,|\,j_f m_f)}{\sqrt{2j_f+1}}\, j^{(fi)}_{\lambda l}(r)\,\vec{Y}_{\lambda l\mu} =$$

$$= \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu\,|\,j_f m_f)}{\sqrt{2j_f+1}} [j^{(fi)}_{\lambda\lambda-1}(r)\vec{Y}_{\lambda\lambda-1\mu} + j^{(fi)}_{\lambda\lambda+1}(r)\vec{Y}_{\lambda\lambda+1\mu}]$$

$$\delta\rho_{(fi)}(\vec{r}) = \left\langle j_f m_f\,|\,\hat{\rho}_{nuc}(\vec{r})\,|\,j_i m_i\right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu\,|\,j_f m_f)}{\sqrt{2j_f+1}}\, \rho^{(fi)}_{\lambda}(r)\,Y_{\lambda\mu}$$

and continuity equation:

$$\dot{\rho}(\vec{r}) + \vec{\nabla}\cdot\vec{j}_{nuc}(\vec{r}) = 0$$

we have

$$\left\langle f\,|\,\rho(\vec{r})\nabla\times\vec{v}(\vec{r})\,|\,i\right\rangle \approx \left\langle f\,|\,\vec{\nabla}\times\vec{j}_{nuc}(\vec{r})\,|\,i\right\rangle - S_{fi}(\vec{r}) =$$

$$= \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu\,|\,j_f m_f)}{\sqrt{2j_f+1}} \sqrt{\frac{2\lambda+1}{\lambda}} \left[\left(\frac{d}{dr} + \frac{\lambda+2}{r}\right) j^{(fi)}_{\lambda\lambda+1}(r)\right] \vec{Y}_{\lambda\lambda\mu}$$

$w^{(fi)}_{\lambda\lambda}(r)$

current component $j^{(fi)}_{\lambda\lambda+1}(r)$ is responsible for vortical behavior

in the papers of J.W. vorticity strength

$$\nu^{(fi)}_{\lambda} \equiv \int_0^\infty r^{\lambda+4}\, w^{(fi)}_{\lambda\lambda}(r)\, dr$$

is introduced as a mesure of the vorticity

Difference between HD and Wambach vorticity.

Wambach vorticity

The mode is vortical if it is unconstrained by CE and involves $j^{(fi)}_{\lambda\lambda+1}(r)$

$$\hat{M}'_{com}(E1\mu) = \frac{1}{10}\int d\vec{r}\,\hat{\rho}(\vec{r})[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}$$

$$\hat{M}_{com}(E1\mu) = -\frac{i}{2c\sqrt{3}}\int d\vec{r}\;\;\hat{\vec{j}}_{nuc}(\vec{r})\;[r^2 \frac{2\sqrt{2}}{5}\vec{Y}_{12\mu} + (r^2 - < r^2 >_0)\vec{Y}_{10\mu}]$$

CM involves $\vec{Y}_{12\mu}$ **and so** $j_{12}(r)$**. Hence CM is vortical despite its gradient flow according to HD !?**

$$\vec{v}_{com} \propto \vec{\nabla}(r^3 Y_{\lambda\mu})$$

The reason of contradiction:
The Wambach vorticity $w_{\lambda\lambda}(r) \propto j_{\lambda\lambda+1}(r)$ **was introduced mainly as a quantity fully unconstrained by the CE rather than the purely vortical value in the HD sense.**

Derivation of the vorticity multipole operator (J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC 84, 034403 (2011))

Starting point – standard electric multipole operator:

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\quad j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})\,]$$

and the substitution (see the idea of J.W.)

$$\left\langle f\,|\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})|\,i\right\rangle \Rightarrow \underbrace{\left\langle f\,|\,\rho(\vec{r})\,\vec{\nabla}\times\vec{v}(\vec{r})\,|\,i\right\rangle}_{\text{truly vortical}} \approx \left\langle f\,|\,\vec{\nabla}\times\vec{j}_{nuc}(\vec{r})\,|\,i\right\rangle - \left\langle f\,|\left(\vec{\nabla}\rho(\vec{r})\right)\times\vec{v}\,|\,i\right\rangle$$

$$\approx \left\langle f\,|\,\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})\,|\,i\right\rangle - i\frac{kc}{\lambda}\left\langle f\,|\,\vec{\nabla}\hat{\rho}(\vec{r})\times\hat{\vec{r}}\,|\,i\right\rangle$$

$$= \left\langle f\,|\,\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})\,|\,i\right\rangle - S_{fi}(\vec{r})$$

$$\left\langle f\,|\,\hat{M}_{vor}(Ek\lambda\mu)\,|\,i\right\rangle = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d^3r\quad j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot\left[\vec{\nabla}\times\left\langle f\,|\,\hat{\vec{j}}_{nuc}(\vec{r})\,|\,i\right\rangle - S_{fi}(\hat{r})\right]$$

$\Rightarrow$

$$\hat{M}_{vor}(Ek\lambda\mu) = \hat{M}(Ek\lambda\mu) - \hat{M}_S(Ek\lambda\mu) =$$

$$= \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d^3r\quad j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot\left[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - \frac{ikc}{\lambda}\left(\vec{\nabla}\rho(\vec{r})\times\vec{r}\right)\right]$$

Nonzero value of $\left\langle f\,|\,\hat{M}_{vor}(Ek\lambda\mu)\,|\,i\right\rangle$ is the indication of the vorticity of given transitioin

In the paper J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC 84, 034403 (2011) **long-wave decomposition was used to show that the second order term gives the connection between toroidal, compression and vortical transition operators:**

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}[1 - \frac{(kr)^2}{2(2\lambda+3)} + \ldots]$$

$$\hat{M}(E\lambda\mu,k) = \hat{M}(E\lambda\mu) + k\,\hat{M}_{tor}(E\lambda\mu) + \ldots$$

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})\,r^\lambda Y_{\lambda\mu}$$

$$\hat{M}_{tor}(E\lambda\mu) = -\frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\ \ \hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} + \sqrt{\frac{\lambda}{\lambda+1}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu})$$

$$\hat{M}_S(E\lambda\mu,k) = \hat{M}(E\lambda\mu) - k\,\hat{M}_{com}(E\lambda\mu) + \cdots$$

$$\hat{M}_{com}(E\lambda\mu) = \frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\ \ \hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} - \sqrt{\frac{\lambda+1}{\lambda}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu}) = -k\,\hat{M}'_{com}(E\lambda\mu)$$

$$\hat{M}'_{com}(E\lambda\mu) = \frac{1}{2(2\lambda+3)}\int d\vec{r}\,\hat{\rho}(\vec{r})\,r^{\lambda+2}Y_{\lambda\mu}$$

$$\hat{M}_{vor}(E\lambda\mu,k) = \hat{M}(E\lambda\mu,k) - \hat{M}_S(E\lambda\mu,k) = k\left[\hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)\right]$$

$$\hat{M}_{vor}(E\lambda\mu) = -\frac{i}{c(2\lambda+3)}\sqrt{\frac{\lambda+1}{2\lambda+1}}\int d\vec{r}\ \ \hat{\vec{j}}_{nuc}(\vec{r})\,r^{\lambda+1}\vec{Y}_{\lambda\lambda+1\mu}$$

$$\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)$$

vorticity, toroidal and compressional multipole operators - survey

$$\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)$$

toroidal multipole operator

$$\hat{M}_{tor}(E\lambda\mu) = -\frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\;\; \hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} + \sqrt{\frac{\lambda}{\lambda+1}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu})$$

$$= -\frac{1}{2c}\sqrt{\frac{\lambda}{\lambda+1}}\frac{1}{2\lambda+3}\int d\vec{r}\;\; r^{\lambda+2}\vec{Y}_{\lambda\lambda\mu}[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

compressional multipole operator

$$\hat{M}_{com}(E\lambda\mu) = \frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\;\; \hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} - \sqrt{\frac{\lambda+1}{\lambda}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu})$$

$$= \frac{i}{2c}\frac{\lambda}{2\lambda+3}\int d\vec{r}\;\; r^{\lambda+2}Y_{\lambda\mu}[\vec{\nabla}\cdot\hat{\vec{j}}_{nuc}(\vec{r})]$$

vorticity multipole operator

$$\hat{M}_{vor}(E\lambda\mu) = -\frac{i}{c(2\lambda+3)}\sqrt{\frac{\lambda+1}{2\lambda+1}}\int d\vec{r}\;\; r^{\lambda+1}\vec{Y}_{\lambda\lambda+1\mu}\hat{\vec{j}}_{nuc}(\vec{r})$$

In all formulas for electric Eλ, compressional electric, toroidal electric, vorticity multipole operators nuclear current $\hat{\vec{j}}_{nuc}(\vec{r})$ is involved. This current consists from convection and magnetization (spin) parts

$$\hat{\vec{j}}_{nuc}(\vec{r}) = \hat{\vec{j}}_{con}(\vec{r}) + \hat{\vec{j}}_{mag}(\vec{r}) = \frac{e\hbar}{m}\sum_{q=n,p}(\hat{\vec{j}}^{(q)}_{com}(\vec{r}) + \hat{\vec{j}}^{(q)}_{mag}(\vec{r})) \qquad q = n,\ p$$

$$\hat{\vec{j}}^{(q)}_{con}(\vec{r}) = -ie^{(q)}_{eff}\sum_{k\ni q}\left(\delta(\vec{r}-\vec{r}_k)\vec{\nabla}_k - \overleftarrow{\nabla}_k\delta(\vec{r}-\vec{r}_k)\right) \quad \Bigg| \quad \hat{\vec{j}}^{(q)}_{mag}(\vec{r}) = \frac{g^{(q)}_s}{2}\sum_{k\ni q}\vec{\nabla}_k \times \hat{\vec{s}}_{qk}\delta(\vec{r}-\vec{r}_k)$$

experimentally el.mg., T=0 and T=1 channels can be recognized:

R.Alacron et al., PRC40, R1097 (1989)

el.mg. $e^{(n)}_{eff} = 0, \quad e^{(p)}_{eff} = 1, \quad g^{(n)}_s(el.mg.) = -3.82\varsigma, \quad g^{(p)}_s(el.mg.) = 5.58\varsigma, \quad \varsigma \approx 0.7$

(e,e'), γ - abs

T=0 $e^{(n)}_{eff} = e^{(p)}_{eff} = 1, \quad g^{(n,p)}_s(T=0) = \frac{1}{2}(g^{(n)}_s + g^{(p)}_s) = 0.88\varsigma, \qquad \varsigma \approx 0.7$

(α,α'), (p,p'), …

T=1 $e^{(n)}_{eff} = -e^{(p)}_{eff} = 1, \quad g^{(n,p)}_s(T=1) = \frac{1}{2}(g^{(n)}_s - g^{(p)}_s) = -4.70\varsigma, \qquad \varsigma = 0.7$

(p,p') ??

$$|g^{(n,p)}_s(T=0)| << |g^{(n,p)}_s(T=1)|$$

numerical results were obtained using the fully self-consistent Skyrme Separable RPA (SRPA approach)

SRPA : →

electric GR — Phys. Rev. C: 66, 044307 (2002); 74, 064306 (2006); 78, 044313 (2008); IJMP(E): 16, 624 (2007); 17, 89 (2008); 18, 975 (2009).

magnetic spin-flip GR — PRC, 80, 031302(R) (2009); JPG, 37, 064034 (2010); IJMP (E), 19, n.4, 558 (2010).

vorticity — Phys Rev. C 84, 034303 (2010)

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basic idea of the SRPA approach:

We start with the general energy functional

$$E = < HFB \,|\, \hat{H} \,|\, HFB > = \int \mathcal{H}(J_\alpha(\vec{r}))\, d^3r$$

with $J_\alpha(\vec{r}) = < HFB \,|\, \hat{J}_\alpha(\vec{r}) \,|\, HFB >$ — some $\hat{J}_\alpha(\vec{r})$ are time-even and some are time-odd

nucleus is excited by external s.p. fields $(\hat{Q}_k, \hat{P}_k) \quad k = 1, \cdots, K$:

$$\hat{Q}^+_{\tau k} = \hat{Q}_{\tau k} \quad ; \quad T\hat{Q}_{\tau k}T^{-1} = \hat{Q}_{\tau k} \quad ; \quad [\hat{H}, \hat{Q}_{\tau k}] = -i\hat{P}_{\tau k}$$

$$\hat{P}^+_{\tau k} = \hat{P}_{\tau k} \quad ; \quad T\hat{P}_{\tau k}T^{-1} = -\hat{P}_{\tau k} \quad ; \quad [\hat{H}, \hat{P}_{\tau k}] = -i\hat{Q}_{\tau k}$$

$$\hat{Q}_k = \sum_i r_i^l Y_{\lambda\mu}(i)$$ for electric type excitation

$$\hat{P}_k = \sum_i r_i^{l'} [\vec{\sigma} \otimes Y_l]_{\lambda\mu}$$ for magnetic type excitation

Using TDHFB with the linear response theory we obtain : $\hat{H} = \hat{h}_{HFB} + \hat{V}_{res}$

$$\hat{h}_{HFB} = \int d^3r \sum_{\alpha_+} [\frac{\partial E}{\partial J_{\alpha_+}(\vec{r})}] \hat{J}_{\alpha_+}(\vec{r})$$

$$\hat{V}_{res}^{(sep)} = \frac{1}{2} \sum_{k,k'=1}^{K} \left\{ \kappa_{kk'} \hat{X}_k \hat{X}_{k'} + \eta_{kk'} \hat{Y}_k \hat{Y}_{k'} \right\}$$

$$\hat{X}_k = i \int d^3r \int d^3r' \sum_{\alpha_+ \alpha'_+} [\frac{\partial^2 E}{\partial J_{\alpha_+}(\vec{r}') \partial J_{\alpha'_+}(\vec{r})}] <|[\hat{P}_k, \hat{J}_{\alpha_+}(\vec{r})]|> \hat{J}_{\alpha'_+}(\vec{r}')$$

$$\hat{Y}_k = i \int d^3r \int d^3r' \sum_{\alpha_- \alpha'_-} [\frac{\partial^2 E}{\partial J_{\alpha_-}(\vec{r}') \partial J_{\alpha'_-}(\vec{r})}] <|[\hat{Q}_k, \hat{J}_{\alpha_-}(\vec{r})]|> \hat{J}_{\alpha'_-}(\vec{r}')$$

$$T \hat{J}_{\alpha_+} T^{-1} = \hat{J}_{\alpha_+}, \; T \hat{J}_{\alpha_-} T^{-1} = -\hat{J}_{\alpha_-} \quad \Rightarrow \quad T \hat{X}_k T^{-1} = \hat{X}_k, \; T \hat{Y}_k T^{-1} = -\hat{Y}_k$$

where strength constant matrixes are

$$\kappa_{kk'}^{-1} = \int d^3r \int d^3r' \sum_{\alpha_+ \alpha'_+} <|[\hat{P}_k, \hat{J}_{\alpha_+}(\vec{r})]|> [\frac{\partial^2 E}{\partial J_{\alpha'_+}(\vec{r}') \partial J_{\alpha_+}(\vec{r})}] <|[\hat{P}_{k'}, \hat{J}_{\alpha'_+}(\vec{r}')]|>$$

$$\eta_{kk'}^{-1} = \int d^3r \int d^3r' \sum_{\alpha_- \alpha'_-} <|[\hat{Q}_k, \hat{J}_{\alpha_-}(\vec{r})]|> [\frac{\partial^2 E}{\partial J_{\alpha'_-}(\vec{r}') \partial J_{\alpha_-}(\vec{r})}] <|[\hat{Q}_{k'}, \hat{J}_{\alpha'_-}(\vec{r}')]|>$$

RPA equations:

$$[H, O_\nu^+] = \omega_\nu O_\nu^+ \qquad [H, O_\nu] = -\omega_\nu O_\nu \qquad [O_\nu, O_{\nu'}^+] = \delta_{\nu\nu'}$$

gives energies, forward and backward amplitudes of phonon operator

$$O_\nu^+ = \sum_{ij} \left\{ \psi_{ij}^{(\nu)} b_{ij}^+ - \varphi_{ij}^{(\nu)} b_{ij} \right\} \Longleftrightarrow \omega_\nu$$

RPA equations with the separable residual interactions can be transferred into the homogeneous system of algebraic equations. Dimension of the matrix of this system is given by the number of s.p. operators $\hat{X}_k$ and $\hat{Y}_k$ in the residual interaction. Detailed description of our SRPA method can be found in the papers:

W.Kleinig, V.O.Nesterenko, J.Kvasil, P.-G.Reinhard, P.Vesely, PRC78, 044315 (2008)

V.O.Nesterenko, W.Kleinig, J.Kvasil, P.Vesely, P.-G.Reinhard, PRC74, 064306 (2006)

Knowing the structure of phonons we can calculate el.mg. reduced probability from the RPA ground state $|RPA>$ to one-phonon state $O_\nu^+ |RPA>$ with the energy ω_ν

$$B(Z\lambda\mu, |RPA> \to |\nu>) = |< RPA|[O_\nu, M_{Z\lambda\mu}]|RPA>$$

$Z = el., mg.$ | $\lambda -$ transition multipolarity | $M_{Z\lambda\mu} -$ transition multipole operator

Then the energy weighted strength function is:

$$S_L(Z\lambda\mu\,;E) = \sum_\nu B(Z\lambda\mu\,;|RPA> \to |\nu>)\,\omega_\nu^L\,\delta(E-\omega_\nu)$$

$$\approx \sum_\nu B(Z\lambda\mu\,;|RPA> \to |\nu>)\,\omega_\nu^L\,\xi(E-\omega_\nu)$$

This quantity can be determined even without the solving the RPA equations for each individual phonon state $|\nu>= O_\nu^+\,|RPA>$ using the Cauchy theorem and the substitution

$$\delta(E-\omega_\nu) \;\to\; \xi(E-\omega_\nu) = \frac{1}{2\pi}\frac{1}{(E-\omega_\nu)^2 + (\Delta/2)^2}$$

The SRPA method was used for the Skyrme interaction with its densities and currents:

standard density $\hat{\rho}(\vec{r})$

kinetic energy density $\hat{\tau}(\vec{r})$

spin-orbital current $\hat{\vec{J}}(\vec{r})$

standard current $\hat{\vec{j}}(\vec{r})$

spin-current $\hat{\vec{s}}(\vec{r})$

kinetic energy current $\hat{\vec{T}}(\vec{r})$

pairing density $\hat{\chi}(\vec{r})$

Further the SRPA method is used for the SLy6 parametrization and ^{208}Pb

$\Delta = 1\ MeV$

We use the Skyrme energy density for the energy functional - see e.g. J.Dobaczewski, J.Dudek, Phys.Rev. C52, 1827 (1995):

$$E(\rho,\tau,\vec{s},\vec{j},\vec{\mathfrak{I}},\vec{T},\chi)=\int \mathcal{H}(\vec{r})\,d^3r$$

with

$$\mathcal{H}(\vec{r})=\mathcal{H}_{kin}(\vec{r})+\mathcal{H}_{Sk}(\vec{r})+\mathcal{H}_{pair}(\vec{r})+\mathcal{H}_{Coul}(\vec{r})$$

$$\mathcal{H}_{kin}(\vec{r})=\frac{\hbar^2}{2m}\tau(\vec{r})$$

$$\mathcal{H}_{Coul}(\vec{r})=\frac{e^2}{2}\int d^3r'\,\rho_p(\vec{r})\frac{1}{|\vec{r}-\vec{r}'|}\rho_p(\vec{r}')-\frac{3}{4}e^2\left(\frac{3}{\pi}\right)^{1/3}[\rho_p(\vec{r})]^{4/3}$$

$$\mathcal{H}_{Sk}(\vec{r})=\sum_{t=0,1}\mathcal{H}_t^{(even)}+\sum_{t=0,1}\mathcal{H}_t^{(odd)}$$

$$\mathcal{H}_{pair}(\vec{r})=\frac{1}{4}\sum_{t=n,p}\chi_t^2V_t^{(0)}[1-(\frac{\rho}{\rho_{nm}})^{\gamma}]$$

$$\mathcal{H}_t^{(even)}(\vec{r})=C_t^{(\rho)}\rho_t^2+C_t^{(\Delta\rho)}\rho_t(\Delta\rho)+C_t^{(\tau)}\rho_t\tau_t+C_t^{(\mathfrak{I})}\vec{\mathfrak{I}}_t^2+C_t^{(\Delta\mathfrak{I})}\rho_t(\vec{\nabla}\vec{\mathfrak{I}}_t)$$

$$\mathcal{H}_t^{(odd)}(\vec{r})=C_t^{(s)}\vec{s}_t^2+C_t^{(\Delta s)}\vec{s}_t(\Delta\vec{s})+C_t^{(T)}\vec{s}_t\vec{T}_t+C_t^{(j)}\vec{j}_t^2+C_t^{(\Delta j)}\vec{s}_t(\vec{\nabla}\times\vec{j}_t)$$

$$C_t^{(\rho)}(\rho),C_t^{(s)}(\rho),C_t^{(\Delta\rho)},C_t^{(\tau)},C_t^{(\mathfrak{I})},$$
$$C_t^{(\Delta\mathfrak{I})},C_t^{(\Delta s)},C_t^{(T)},C_t^{(j)},C_t^{(\Delta j)},V_t^{(0)},\gamma$$

interaction parameters

$$C_t^{(j)}=-C_t^{(\tau)}$$
$$C_t^{(T)}=-C_t^{(\mathfrak{I})}$$
$$C_t^{(\Delta j)}=C_t^{(\Delta\mathfrak{I})}$$

gauge invariance

The dependence of the energy density $\mathcal{H}(\vec{r})$ on $\vec{r}$ goes through the following densities and currents:

density

$$\hat{\rho}(\vec{r}) = \sum_{\tau} \sum_{ij\in\tau} \psi_i^+(\vec{r})\, \psi_j(\vec{r})\, a_i^+ a_j$$

kinetic energy density

$$\hat{\tau}(\vec{r}) = \sum_{\tau} \sum_{ij\in\tau} \left(\vec{\nabla}\psi_i(\vec{r})\right)^+ \left(\vec{\nabla}\psi(\vec{r})\right) a_i^+ a_j$$

spin-orbit current

$$\hat{\vec{\mathfrak{I}}}(\vec{r}) = -\frac{i}{2} \sum_{\tau} \sum_{ij\in\tau} \left\{ \psi_i^+(\vec{r}) \left(\vec{\nabla}\times\psi_j(\vec{r})\right) + \left(\vec{\nabla}\times\psi_i(\vec{r})\right)^+ \psi_j(\vec{r}) \right\} a_i^+ a_j$$

current

$$\hat{\vec{j}}(\vec{r}) = \frac{i}{2} \sum_{\tau} \sum_{ij} \left\{ \left(\vec{\nabla}\psi_i(\vec{r})\right)^+ \psi_j(\vec{r}) - \psi_i^+(\vec{r}) \left(\vec{\nabla}\psi_j(\vec{r})\right) \right\} a_i^+ a_j$$

spin-current

$$\hat{\vec{s}}(\vec{r}) = \sum_{\tau} \sum_{ij\in\tau} \psi_i^+(\vec{r})\, \vec{\sigma}\, \psi_j(\vec{r})\, a_i^+ a_j$$

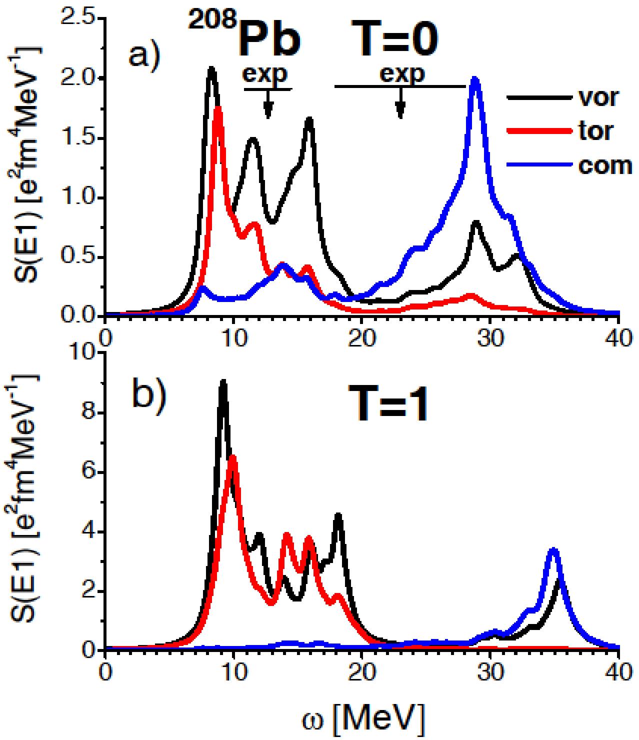
pairing density

$$\hat{\chi}_\tau(\vec{r}) = \sum_{i\in\tau} \psi_i^+(\vec{r})\, \psi_i(r)\, (a_i^+ a_{\bar{i}}^+ + a_{\bar{i}}\, a_i)$$

kinetic energy – spin current

$$\hat{\vec{T}}(\vec{r}) = \sum_{\tau} \sum_{ij\in\tau} \left(\vec{\nabla}\psi_i(\vec{r})\right)^+ \vec{\sigma} \left(\vec{\nabla}\psi_j(\vec{r})\right) a_i^+ a_j$$

Comparison of VM, TM, and CM



First direct comparison of VM, TM and CM!

- Broad low-energy (LE) and high-energy (HE) bumps for VM, TM, and CM.
- LE strength is dominated by VM and TM
- HE strength is dominated by VM and CM

- General agreement for TM and CM with previous studies.
- Poor agreement with exper. of Ichida (like in previous studies).

Uchida et al., 2003:

$E_1 = 12.7\,\text{MeV}, \quad \Gamma_1 = 3.5\,\text{MeV}$

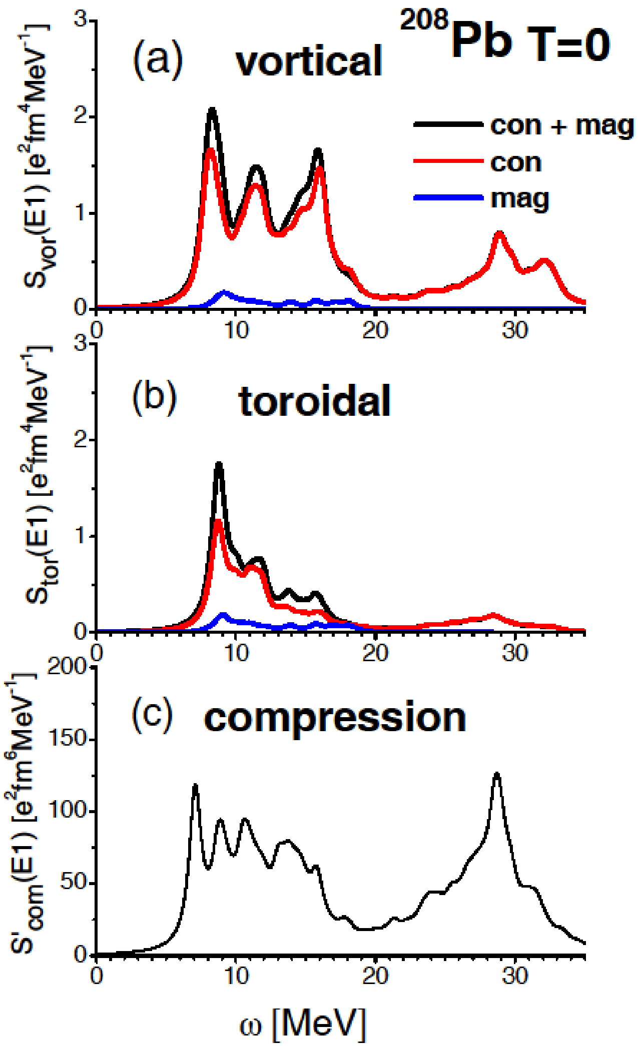
$E_2 = 23.0\,\text{MeV}, \quad \Gamma_2 = 10.3\,\text{MeV}$

- Purely vortical VM does not coincide with partly vortical TM, especially at HE.

Vortical, toroidal, and compressional T=0 strength

SLy6

$\Delta = 1\,\mathrm{MeV}$



- dominant contribution of j_{con} to VM and TM

-no j_{mag} contribution to:

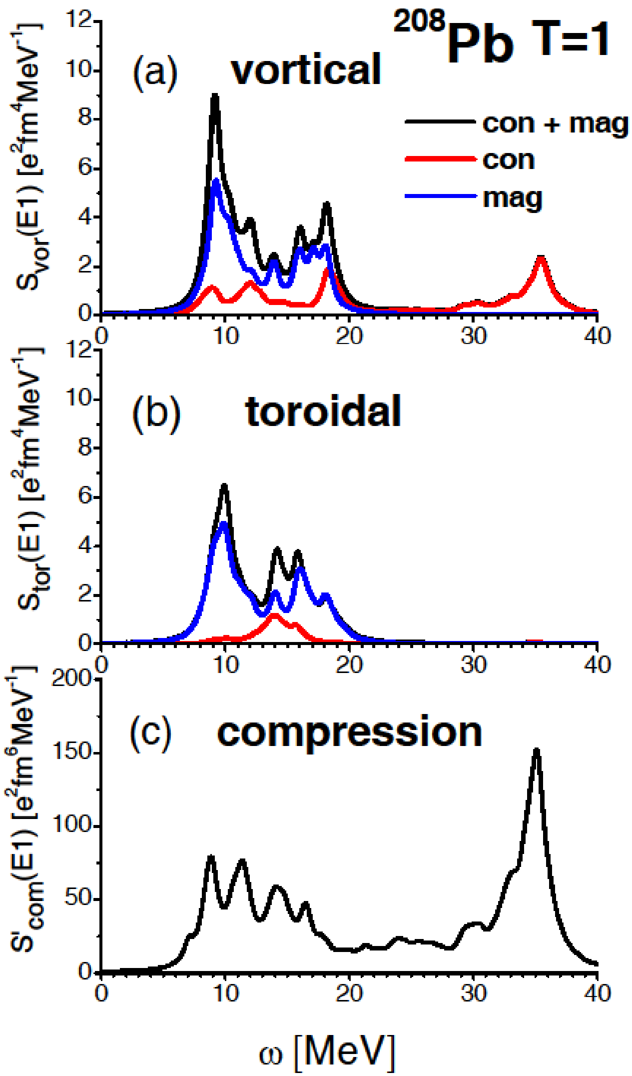
- CM
- HE strength

$$g_s^p = 5.58\varsigma, \quad g_s^n = -3.82\varsigma$$

$$g_s^{T=0} = \frac{1}{2}(g_s^p + g_s^n) = 0.88\varsigma$$

Small T=0 g-factors!

Vortical, toroidal, and compressional T=1 strength



SLy6

$$\Delta = 1\,\text{MeV}$$

VM and TM:

- dominant contribution of j_{mag} !!

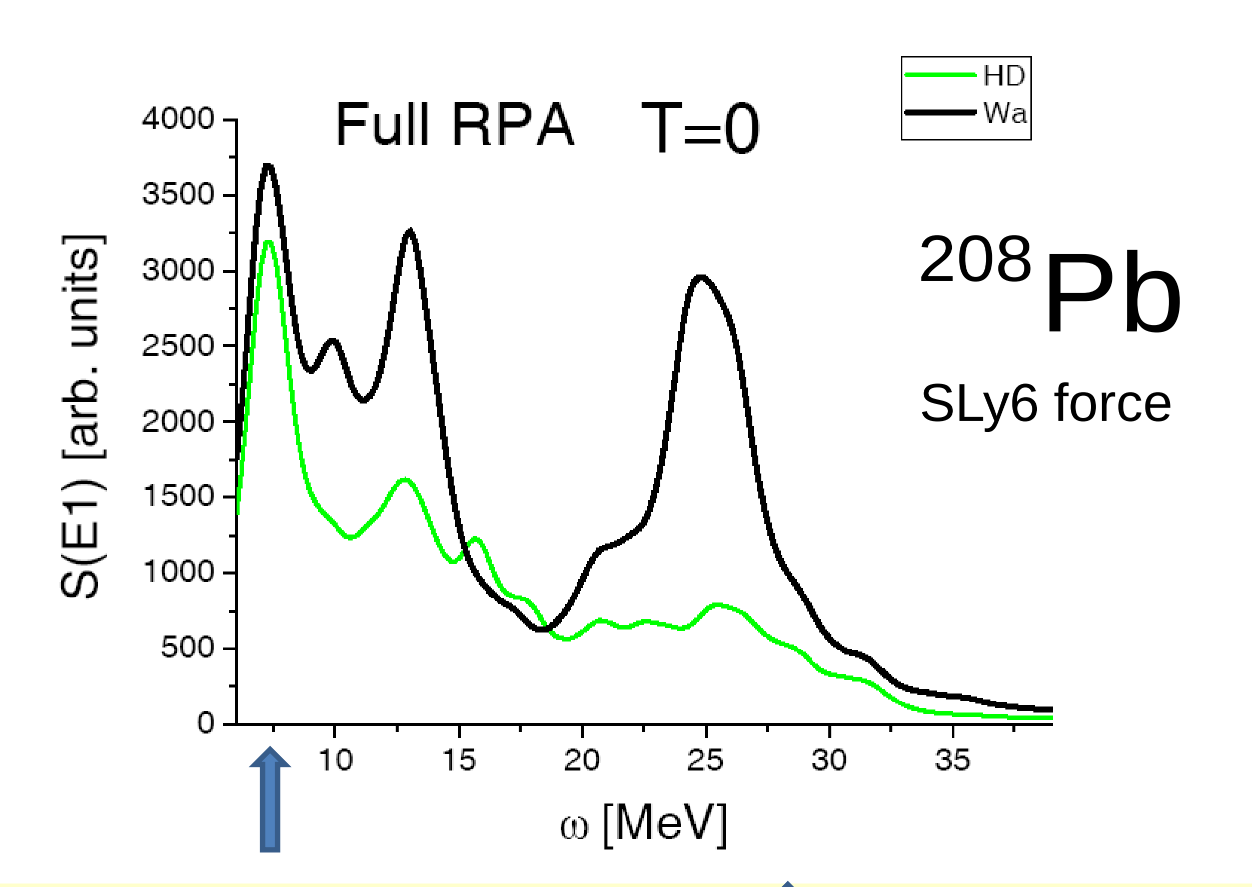
$$g_s^p = 5.58\varsigma, \quad g_s^n = -3.82\varsigma,$$

$$g_s^{T=1} = \frac{1}{2}(g_s^p - g_s^n) = 4.7\varsigma$$

Large T=1 g-factors!

VM and TM in T=1 channel are suitable to see the effect of j_{mag} in electric excitations.

Wambach vs HD vorticity (VM)



W ~ HD ~T
vortical region

HD is modest

W ~ CM
HD is small → irrotational region from HD point of view

Conclusions

-The **(Wambach) vortical operator** **has been derived.**
This allows to treat and compare VM,TM, and CM on the same theor. ground.

-**The difference between Wambach and HD voirticities was discussed.**

- **Both T=0 and T=1 VM,TM, and CM were considered**

- **The dominant role of:**
  - **convection nuclear current in isoscalar VM and TM**
  - **magnetization nuclear current in isovector VM and TM**
  - **E1(T=1) VM and TM: remarkable example of strong** j_{mag} **effect in electric GR.**

Outlook:

- **more comparison of HD and Wambach vorticity**
- **dependence of Skyrme forces and other interactions**
- **going beyond the RPA**
- **proposals for (e,e') and hadron reactions**

Thank you for your attention!

The dependence of the energy density $\mathcal{H}(\vec{r})$ on $\vec{r}$ goes through the following densities and currents:

density

$$\hat{\rho}(\vec{r}) = \sum_{\tau}\sum_{ij\in\tau} \psi_i^+(\vec{r})\,\psi_j(\vec{r})\,a_i^+ a_j$$

kinetic energy density

$$\hat{\tau}(\vec{r}) = \sum_{\tau}\sum_{ij\in\tau} \left(\vec{\nabla}\psi_i(\vec{r})\right)^+ \left(\vec{\nabla}\psi(\vec{r})\right) a_i^+ a_j$$

spin-orbit current

$$\hat{\vec{\mathfrak{J}}}(\vec{r}) = -\frac{i}{2}\sum_{\tau}\sum_{ij\in\tau} \left\{ \psi_i^+(\vec{r})\left(\vec{\nabla}\times\psi_j(\vec{r})\right) + \left(\vec{\nabla}\times\psi_i(\vec{r})\right)^+ \psi_j(\vec{r}) \right\} a_i^+ a_j$$

current

$$\hat{\vec{j}}(\vec{r}) = \frac{i}{2}\sum_{\tau}\sum_{ij} \left\{ \left(\vec{\nabla}\psi_i(\vec{r})\right)^+ \psi_j(\vec{r}) - \psi_i^+(\vec{r})\left(\vec{\nabla}\psi_j(\vec{r})\right) \right\} a_i^+ a_j$$

spin-current

$$\hat{\vec{s}}(\vec{r}) = \sum_{\tau}\sum_{ij\in\tau} \psi_i^+(\vec{r})\,\vec{\sigma}\,\psi_j(\vec{r})\,a_i^+ a_j$$

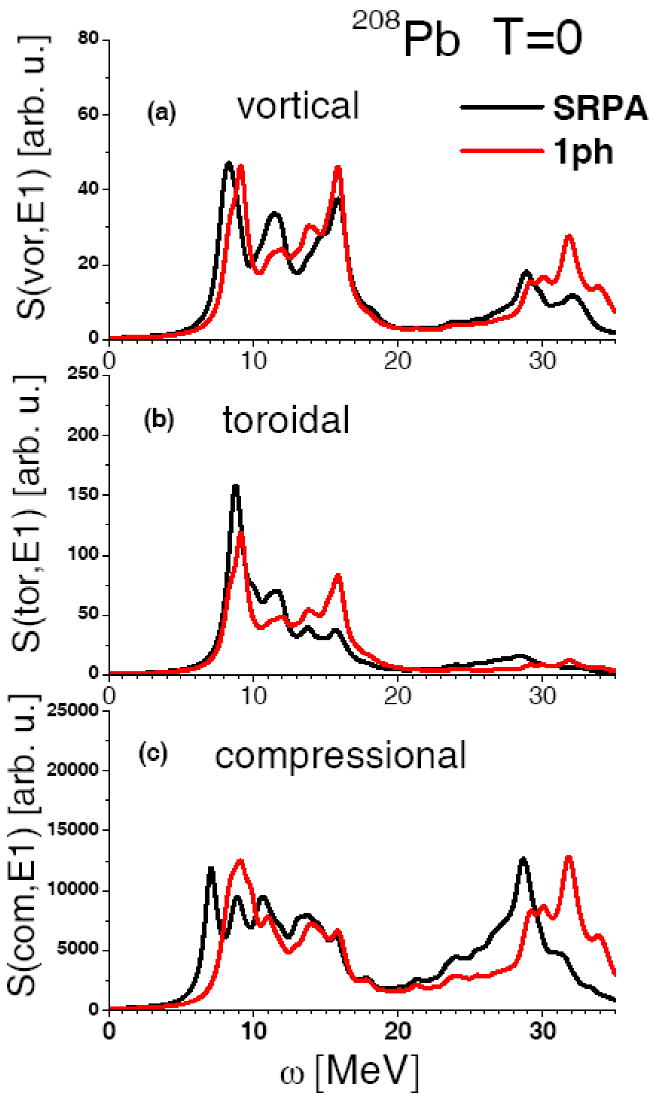
pairing density

$$\hat{\chi}_\tau(\vec{r}) = $$
$$= \sum_{i\in\tau} \psi_i^+(\vec{r})\,\psi_i(r)\,(a_i^+ a_{\bar{i}}^+ + a_{\bar{i}}\, a_i\,)$$

kinetic energy – spin current

$$\hat{\vec{T}}(\vec{r}) = \sum_{\tau}\sum_{ij\in\tau} \left(\vec{\nabla}\psi_i(\vec{r})\right)^+ \vec{\sigma}\left(\vec{\nabla}\psi_j(\vec{r})\right) a_i^+ a_j$$

SRPA vs 1ph strength



- Collective down-shifts:
  - ~ 1-3 MeV for LE bump
  - ~ 4 MeV for HE bump

- Quite collective RPA states:

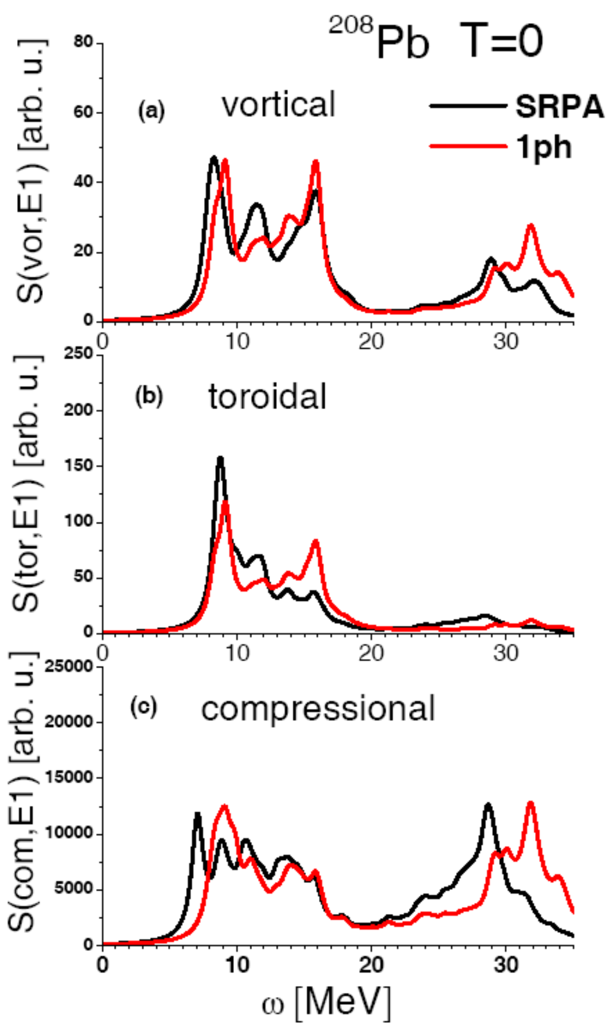
  state at 8.3 MeV:

- In LE bump the structure of VM, TM, and CM responses is mainly of 1ph origin

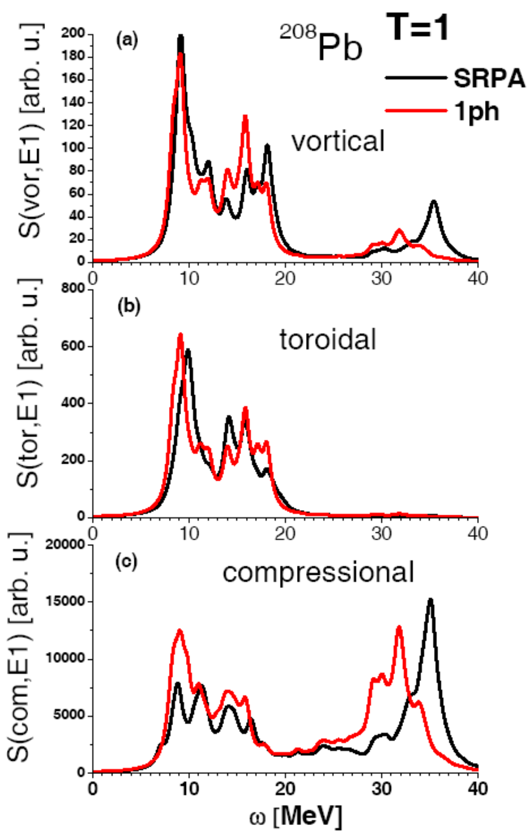
The 1ph origin of the vorticity ?

D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

SRPA vs 1ph strength



- **Collective up-shifts:**
  ~ 1-2 MeV for LE bump
  ~ 4 MeV for HE bump



- **In LE bump the structure of VM, TM, and CM responses is mainly of 1ph origin**

The 1ph origin of the vorticity ?

D.G.Raventhall, J.Wambach,
NPA 475, 468 (1987).

How to introduce the vorticity for nuclei?

In HD the vorticity is defined as

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \neq 0$$

Nuclear quantum theory deals with the nuclear current $\vec{j}_{nuc}(\vec{r})$

The vorticity cannot be measured by the current curl

$$\vec{\nabla} \times \vec{j}_{nuc}(\vec{r})$$

because

$$\vec{v}(\vec{r}) = \frac{\delta \vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$

$$\vec{\nabla} \times \vec{v}(\vec{r}) = \frac{\vec{\nabla} \times \delta \vec{j}_{nuc}(\vec{r}) - \left(\vec{\nabla} \rho_0(\vec{r})\right) \times \hat{\vec{v}}(\vec{r})}{\rho_0(\vec{r})}$$

$$\hat{M}(E\lambda\mu) \sim \int d\vec{r}\, (\vec{\nabla} \times \hat{\vec{j}}_{nuc})\, [....]$$

Then, how to introduce the nuclear vorticity?

HD vortical operator

The HD vortical operator (matrix element) may be also constructed:

$$\hat{M}(Ek\lambda\mu) = i\,\frac{(2\lambda+1)!!}{ck^{\lambda+1}(\lambda+1)}\sqrt{\lambda(\lambda+1)}\int d\vec{r}\;\; j_{\lambda}(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})$$

Wambach: $\Rightarrow$

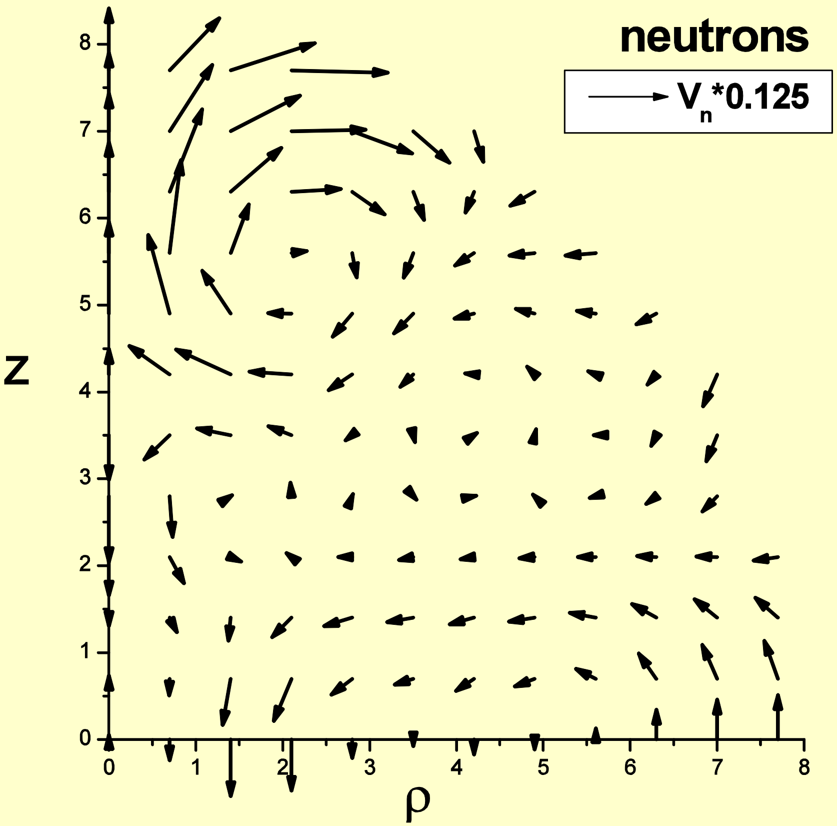
$$\hat{\rho}(\vec{r})\vec{\nabla}\times\hat{\vec{v}}(\vec{r}) = \vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - i\,\frac{kc}{\lambda}\vec{\nabla}\hat{\rho}(\vec{r})\times\hat{\vec{r}}$$

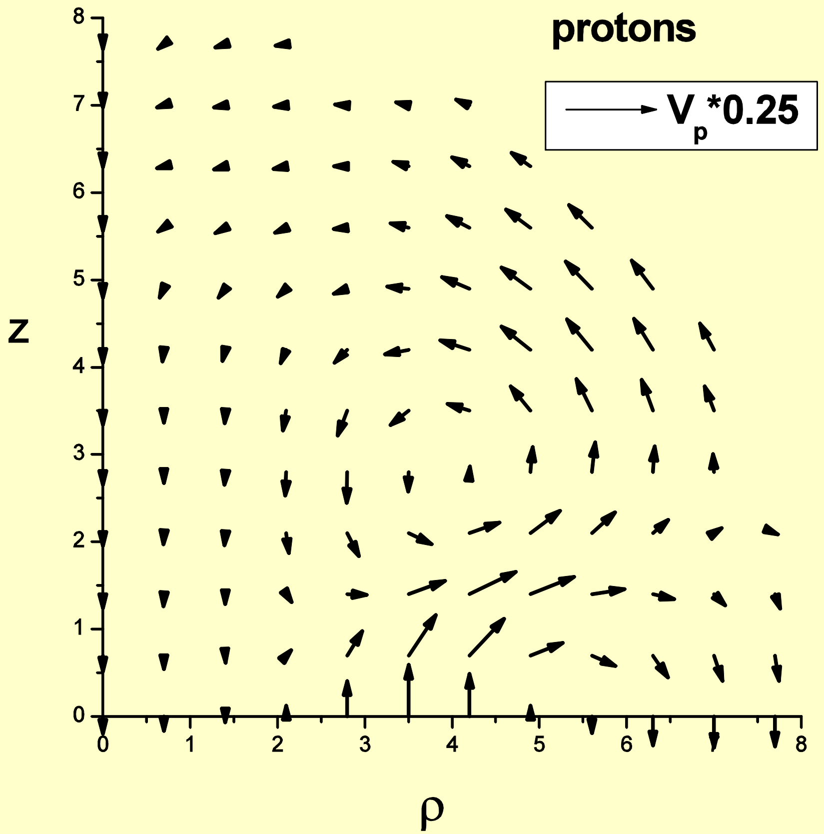
HD $\Rightarrow$

$$\rho_0(\vec{r})\vec{\nabla}\times\hat{\vec{v}}(\vec{r}) = \vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - \frac{1}{\rho_0(\vec{r})}\vec{\nabla}\rho_0(\vec{r})\times\hat{\vec{j}}_{nuc}(\vec{r})$$

$$\vec{v}_{\nu}(\vec{r}) = \frac{\delta\vec{j}_{\nu}(\vec{r})}{\rho_0(\vec{r})} \longrightarrow \quad \rho_0(\vec{r})\vec{\nabla}\times\vec{v}(\vec{r}) = \vec{\nabla}\times\delta\vec{j}_{\nu}(\vec{r}) - \frac{1}{\rho_0(\vec{r})}\vec{\nabla}\rho_0(\vec{r})\times\delta\vec{j}_{\nu}(\vec{r})$$

Velocity fields for T=0 vortical state at 8.3 MeV





s-p manifestation of vorticity

nn[2g7/2 – 2f5/2] 36%

pp[1i13/2-1h11/2] 18%

Wambach does not use the vortical operator
though it would be useful for comparison of VM, TM, and CM.

We will derive the vortical operator and relate it with TM and CM ones:

$$\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)$$

where for $\lambda = 1$ we have

$$\hat{M}_{tor}(E1\mu) = -\frac{i}{2c\sqrt{3}} \int d\vec{r} \;\; \hat{\vec{j}}_{nuc}(\vec{r}) \, [\frac{\sqrt{2}}{5} r^2 \vec{Y}_{12\mu} + (r^2 - < r^2 >_0) \vec{Y}_{10\mu}]$$

$$\hat{M}_{com}(E1\mu) = -\frac{i}{2c\sqrt{3}} \int d\vec{r} \;\; \hat{\vec{j}}_{nuc}(\vec{r}) \, [\frac{2\sqrt{2}}{5} r^2 \vec{Y}_{12\mu} - (r^2 - < r^2 >_0) \vec{Y}_{10\mu}]$$

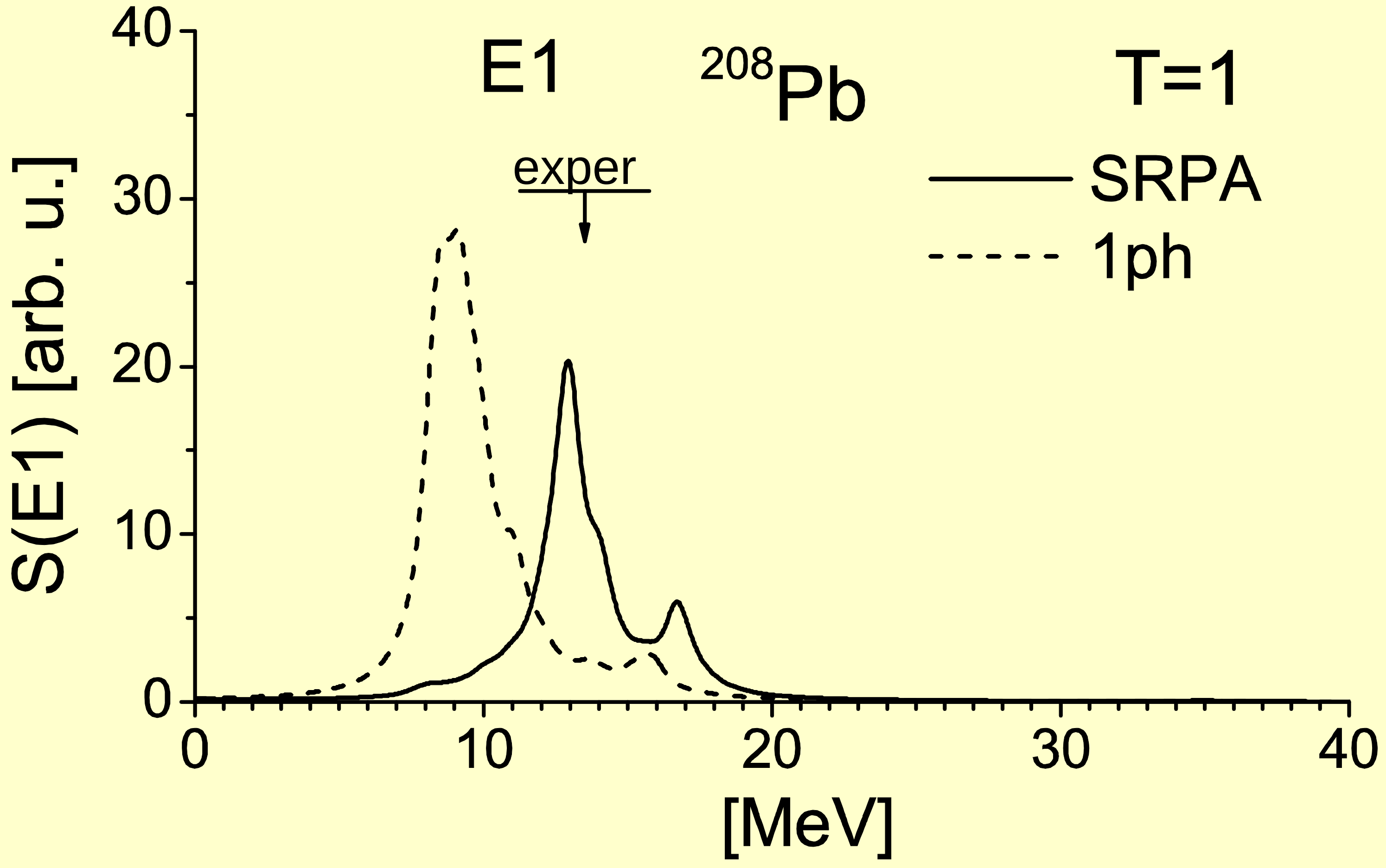
$$\hat{M}_{vor}(E1\mu) = -\frac{i}{5c} \sqrt{\frac{3}{2}} \int d\vec{r} \;\; \hat{\vec{j}}_{nuc}(\vec{r}) \, r^2 \vec{Y}_{12\mu}$$

- involves $\vec{Y}_{12\mu}$
- no c.m.c.

E1(T=1) GR

SLy6

$\Delta = 1\text{MeV}$



Center of mass corrections

$$\hat{O} = \sum_{k=1}^{A} o(\vec{r}_k) \quad \rightarrow \quad \hat{O} = \frac{1}{A}\sum_{k=1}^{A} z_k$$

$$\delta\langle\hat{O}\rangle = \int d\vec{r}\,\delta\rho(\vec{r})o(\vec{r})$$
$$= \int d\vec{r}\,\delta\vec{j}(\vec{r})\cdot\vec{\nabla}o(\vec{r}) = 0$$

translation invariance:
perturbation $\delta\rho$ **does not change**
z-coordinate of the c.m.

$$\vec{\nabla}o(\vec{r}) = \vec{\nabla}(rY_{10}) = \sqrt{3}\,\vec{Y}_{100}$$

$$\sum_{\nu}\langle 0|\hat{\vec{j}}(\vec{r})|\nu\rangle\langle 0|\hat{F}|\nu\rangle = \frac{1}{2mi}\rho_0(\vec{r})\vec{\nabla}f(\vec{r})$$

$$\sum_{\nu}\omega_\nu\langle 0|\hat{\rho}(\vec{r})|\nu\rangle\langle 0|\hat{F}|\nu\rangle = -\frac{1}{2m}\vec{\nabla}\cdot[\rho_0(\vec{r})\vec{\nabla}f(\vec{r})]$$

$$\delta\vec{j}_\nu(\vec{r}) = \langle 0|\hat{\vec{j}}(\vec{r})|\nu\rangle \propto \rho_0(\vec{r})\vec{\nabla}f(\vec{r}) \propto \rho_0(\vec{r})\vec{v}(\vec{r})$$

$$\delta\rho_\nu(\vec{r}) = \langle 0|\hat{\rho}(\vec{r})|\nu\rangle \propto \vec{\nabla}\cdot[\rho_0(\vec{r})\vec{\nabla}f(\vec{r})] \propto \vec{\nabla}\cdot[\rho_0(\vec{r})\vec{v}(\vec{r})]$$

$$\vec{v}_{vor} = r^2\vec{Y}_{12\mu} + \eta\vec{Y}_{10\mu}$$

$$\vec{v}_{tor} = \frac{\sqrt{2}}{5}r^2\vec{Y}_{12\mu} + (r^2-\eta)\vec{Y}_{10\mu}$$

$$\vec{v}_{com} = \frac{\sqrt{2}}{5}r^2\vec{Y}_{12\mu} - (r^2-\eta)\vec{Y}_{10\mu}$$

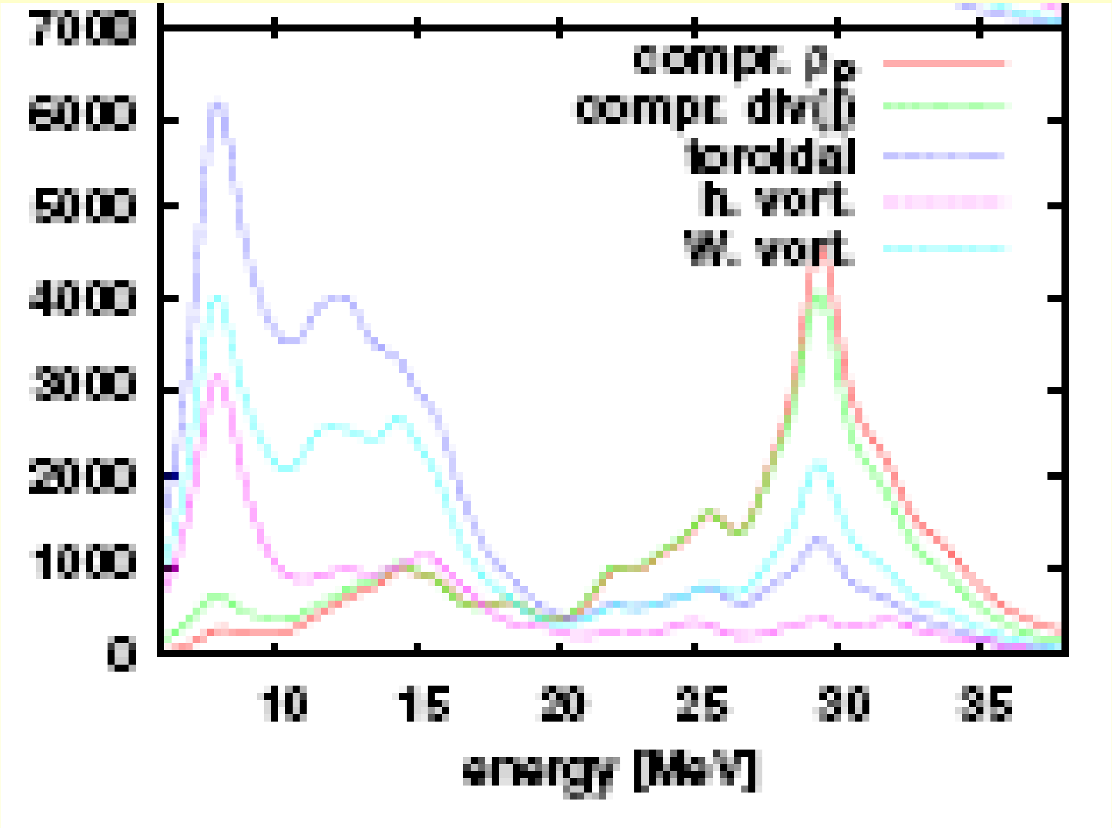
$\Rightarrow$

$$\eta_{vor} = 0$$

$$\eta_{tor} = \eta_{com} = \langle r^2\rangle_0$$

$$\eta'_{com} = \frac{5}{3}\langle r^2\rangle_0$$

Wambach vs HD vorticity -1



Toroidal, compressional, and vortical operators for $\lambda = 1$:

$$\hat{M}_{tor}(E1\mu) = -\frac{i}{2c}\sqrt{\frac{1}{3}}\int d\vec{r}\;\;\hat{\vec{j}}_{nuc}(\vec{r})\,[r^2(\frac{\sqrt{2}}{5}\vec{Y}_{12\mu} + \vec{Y}_{10\mu}) - \langle r^2 \rangle_0\,\vec{Y}_{10\mu}]$$

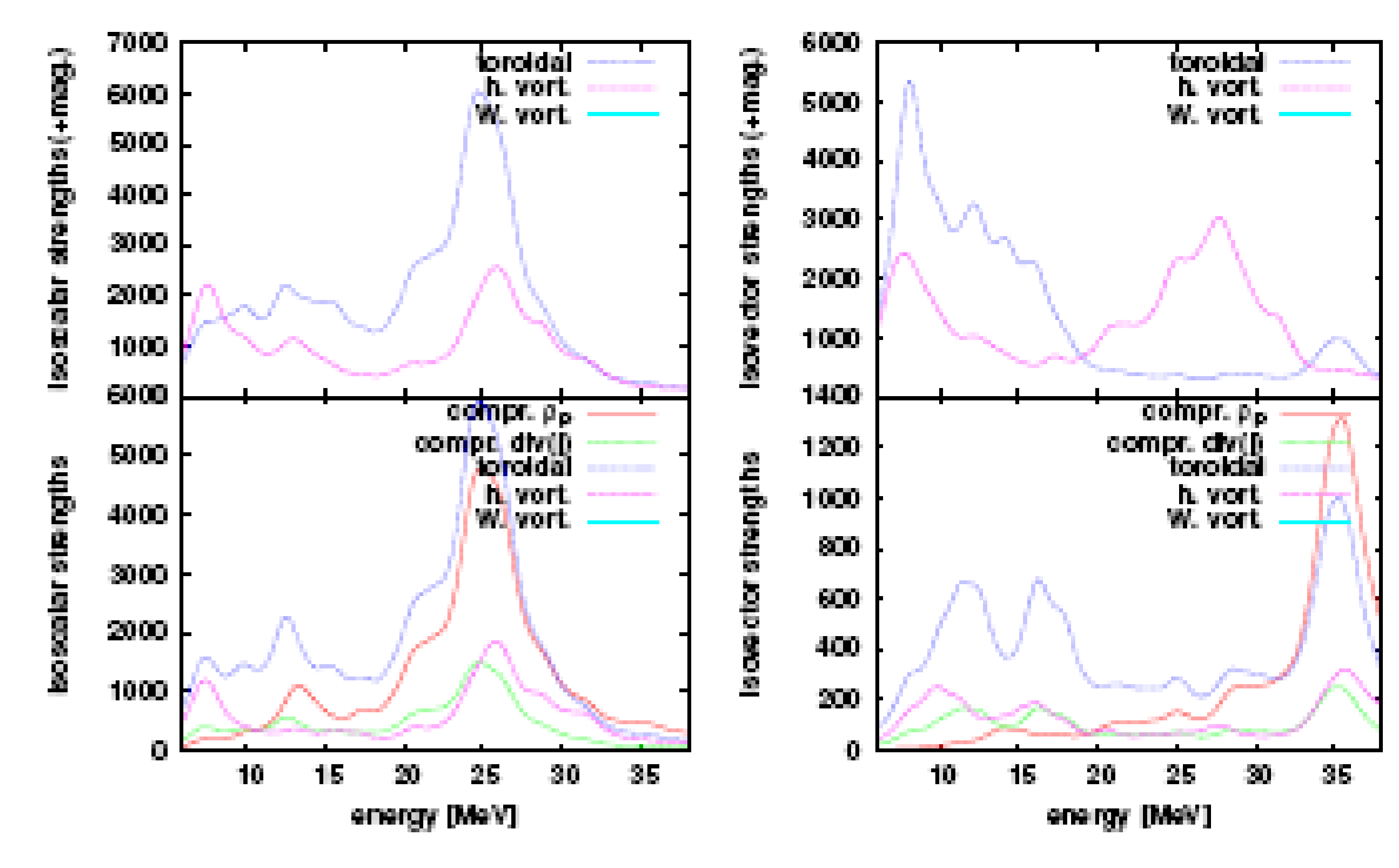
$$\hat{M}_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3 - \frac{5}{3}\langle r^2 \rangle_0\,r]Y_{1\mu}$$

to be shown later:

$$\hat{M}_{vor}(E1\mu) = -\frac{i}{5c}\sqrt{\frac{3}{2}}\int d\vec{r}\;\;\hat{\vec{j}}_{nuc}(\vec{r})\,r^2\vec{Y}_{12\mu}$$

- no c.m.c. for the vortical operator

Wambach vs HD vorticity -2



Motivation

Nuclei demonstrate both

- **irrotational flow (most of electric GR)** $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) = 0$
- **vortical flow (toroidal GR)** $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \neq 0$

Vorticity $\vec{w}(\vec{r})$ is a fundamental quantity:

- **does not contribute to the continuity equation,**
- **represents an independent part of charge-current distribution beyond the continuity equation.**

Vorticity is related to the exotic modes:

- **toroidal E1 mode (TM) ,**
- **compression E1 mode (CM),**

which are now of a keen interest .

different conclusions
on CM vorticity

Open points:

- **definition of nuclear vorticity (HD vs Wambach),**
- **the vortical mode (VM) and its operator field,**
- **relation between VM and TM/CM,**
- **IS (T=0) and IV(T=1) branches of the modes,**
- **role of magnetization (spin) nuclear current.**

J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, P. Vesely, subm. to PRC, arXiv: 1105.0837[nucl-th]

SRPA (1)

E. Lipparini and S. Stringari, NPA, 371, 430 (1981).

Time-dependent formulation:

$$E(J_\alpha(\vec{r},t)) = \langle \Psi | H | \Psi \rangle,$$

$$J_\alpha(\vec{r},t) \in \{\rho(\vec{r},t), \vec{j}(\vec{r},t), \ldots\} \qquad J_\alpha(\vec{r},t) = <\Psi | \hat{J}_\alpha | \Psi> \quad \leftarrow$$ T-even and T-odd densities

$$J_\alpha(\vec{r},t) = \bar{J}_\alpha(\vec{r}) + \delta J_\alpha(\vec{r},t)$$ ← Linear regime: small time-dependent perturbation

← Mean field hamiltonian: static g.s. + time-dependent response

$$h(\vec{r},t) = h_0(\vec{r}) + h_{res}(\vec{r},t)$$

$$= \sum_\alpha [\frac{\delta E}{\delta J_\alpha}]_{J=\bar{J}} \hat{J}_\alpha(\vec{r}) + \sum_{\alpha\alpha'} [\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}}]_{J=\bar{J}} \delta J_\alpha(\vec{r},t) \hat{J}_\alpha(\vec{r})$$

$$\delta J_\alpha(t) = \langle \Psi(t) | J_\alpha | \Psi(t) \rangle - \langle 0 | J_\alpha | 0 \rangle$$ ← **The only unknowns**

Now we have to specify the perturbed many-body wave function $\Psi(t)$

SRPA (2)

$$V_{res} \Rightarrow \sum_{k,k'=1}^{K} \{\kappa_{kk'}\hat{X}_k \hat{X}_{k'} + \eta_{kk'}\hat{Y}_k \hat{Y}_k\}$$

Macroscopic step:

Perturbed w.f. via scaling: $\Psi(t) = \prod_{k=1}^{K} \exp\{-q_k(t)\hat{P}_k\}\exp\{-p_k(t)\hat{Q}_k\}|0\rangle,$

both $\Psi(t), |0\rangle$ **are Slater determinants**

$$\hat{Q}_k = \hat{Q}_k^+, \quad \hat{T}\hat{Q}_k\hat{T}^{-1} = \hat{Q}_k$$
$$\hat{P}_k = i[\hat{H},\hat{Q}_k]_{ph} = \hat{P}_k^+, \quad \hat{T}\hat{P}_k\hat{T}^{-1} = -\hat{P}_k$$

$$q_k(t) = \overline{q}_k \cos(\omega t)$$
$$p_k(t) = \overline{p}_k \sin(\omega t)$$

$$\hat{h}_{res}(t) = \sum_k \{-q_k(t)\hat{X}_k + p_k(t)\hat{Y}_k\} \overset{kk'}{=} 1/2\sum_{kk'}\{\kappa_{kk'}\delta\hat{X}_k(t)\hat{X}_k + \eta_{kk'}\delta\hat{Y}_k(t)\hat{Y}_k\}$$

Microscopic step:

Perturbed w.f. via Thouless theorem: $\Psi_{Th}(t) = \{1 + \sum_{ph} c_{ph}(t)\hat{A}_{ph}^+\}|0\rangle$

$$c_{ph}(t) = c_{ph}^+ e^{i\omega t} + c_{ph}^- e^{-i\omega t}$$

Merging step:

Both scaling and Thouless w.f. $\Psi(t)$ **must give equal variations:**

$$\delta\hat{X}_k(t)|_{sc} = \delta\hat{X}_k(t)|_{Th}, \quad \delta\hat{Y}_k(t)|_{sc} = \delta\hat{Y}_k(t)|_{Th}$$

SRPA (3)

Time-dependent HF equation $$i\hbar \frac{d}{dt}\Psi(t) = (h_0 + h_{res}(t))\Psi(t)$$

Perturbed w. f. by Thouless theorem $$\Psi(t) = (1 + \sum_{ph} c_{ph}(t)\hat{A}^{+}_{ph})\Psi_0$$

Response Hamiltonian $$\hat{h}_{res}(t) = \sum_k \{-q_k(t)\hat{X}_k + p_k(t)\hat{Y}_k\}$$

Harmonic oscillations
$$\begin{cases} c_{ph}(t) = c^{+}_{ph}e^{i\omega t} + c^{-}_{ph}e^{-i\omega t}, \\ q_k(t) = \overline{q}_k \cos(\omega t) = \frac{1}{2}\overline{q}_k(e^{i\omega t} + e^{-i\omega t}), \\ p_k(t) = \overline{p}_k \cos(\omega t) = \frac{1}{2i}\overline{p}_k(e^{i\omega t} - e^{-i\omega t}) \end{cases}$$

TDHF gives the coupling

$c^{\pm}_{ph} \leftrightarrow \overline{q}_k, \overline{p}_k$ ⟹

$$c^{\pm}_{ph} = -\frac{1}{2}\frac{\sum_k \{\overline{q}_k \langle ph | \hat{X}_k | 0\rangle \mp i\,\overline{p}_k \langle ph | \hat{Y}_k | 0\rangle\}}{\varepsilon_{ph} \pm \omega}$$

which allows to reduce a large set of 1ph apmlitudes $c^{\pm}_{ph}$ to a few unknowns $\overline{q}_k, \overline{p}_k$

So high-rank RPA problem can be reduced to low-rank one!

However, $\overline{q}_k, \overline{p}_k$ still remain to be unknown

SRPA (4)

Physical requirement:

Variations of operators of the residual interaction must be the same for both macroscopic (scaling) and microscopic (Thouless) many-body wave functions.

$$\delta \hat{X}_k(t)|_{sc} = \delta \hat{X}_k(t)|_{Th}, \quad \delta \hat{Y}_k(t)|_{sc} = \delta \hat{Y}_k(t)|_{Th}$$

$$\delta \hat{X}_k(t)|_{sc} = \sum_{k'} q_{k'}(t) \;\; \kappa^{-1}_{k k'}$$

$$\delta \hat{X}_k(t)|_{Th} = \sum_{ph} (c_{ph}(t)^* < ph | \hat{X}_k | 0 > + c_{ph}(t) < 0 | \hat{X}_k | ph >^*)$$

$$\sum_k \{\bar{q}_k (d_{kk'}(XX) - \kappa^{-1}_{kk'}) + \bar{p}_k d_{kk'}(XY)\} = 0$$
$$\sum_k \{\bar{q}_k d_{kk'}(YX) + \bar{p}_k (d_{kk'}(YY) - \eta^{-1}_{kk'})\} = 0$$

final RPA equations for

$$\bar{q}_k, \bar{p}_k$$

$$d_{kk'}(XY) = \sum_{ph} [\frac{\langle ph | \hat{X}_k | 0 \rangle^* \langle ph | \hat{Y}_{k'} | 0 \rangle}{(\varepsilon_{ph} - \omega)} + \frac{\langle ph | \hat{X}_k | 0 \rangle \langle ph | \hat{Y}_{k'} | 0 \rangle^*}{(\varepsilon_{ph} + \omega)}]$$

SRPA (3)

Final RPA equations:

$$\sum_k \{\bar{q}_k (d_{kk'}(XX) - \kappa^{-1}_{kk'}) + \bar{p}_k d_{kk'}(XY)\} = 0$$

$$\sum_k \{\bar{q}_k d_{kk'}(YX) + \bar{p}_k (d_{kk'}(YY) - \eta^{-1}_{kk'})\} = 0$$

T-even T-odd

$$H = h_0 + 1/2 \sum_{kk'} \{\kappa_{kk'} \overbrace{\hat{X}_k \hat{X}_{k'}} + \eta_{kk'} \overbrace{\hat{Y}_k \hat{Y}_k}\}$$

$$\det[\omega_j] = 0 \quad \Longrightarrow \quad \textbf{RPA spectrum}$$

where e.g. $d_{kk'}(XY) = \sum_{ph} \left[\frac{\langle ph | \hat{X}_k | 0 \rangle^* \langle ph | \hat{Y}_{k'} | 0 \rangle}{(\varepsilon_{ph} - \omega)} + \frac{\langle ph | \hat{X}_k | 0 \rangle \langle ph | \hat{Y}_{k'} | 0 \rangle^*}{(\varepsilon_{ph} + \omega)} \right]$

$$\hat{X}_k(\vec{r}) = i \sum_{\alpha\alpha'} \left[\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}} \right] \langle 0 | [J_\alpha, \hat{P}_k] | 0 \rangle \hat{J}_{\alpha'}$$

$$\hat{Y}_k(\vec{r}) = i \sum_{\alpha\alpha'} \left[\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}} \right] \langle 0 | [J_\alpha, \hat{Q}_k] | 0 \rangle \hat{J}_{\alpha'}$$

$$\kappa^{-1}_{kk'} = i \langle 0 | [\hat{X}_k, \hat{P}_{k'}] | 0 \rangle$$

$$\eta^{-1}_{kk'} = i \langle 0 | [\hat{Y}_k, \hat{Q}_{k'}] | 0 \rangle$$

$$C^+ = \sum_{ph} [c^-_{ph} a^+_p a_h - c^+_{ph} a^+_h a_p]$$

RPA phonon

$$c^{\pm}_{ph} = -\frac{1}{2} \frac{\sum_k \{\bar{q}_k \langle ph | \hat{X}_k | 0 \rangle \mp i \bar{p}_k \langle ph | \hat{Y}_k | 0 \rangle\}}{\varepsilon_{ph} \pm \omega}$$

- Rank of RPA matrix is 4K.
For giant resonances usually K=2 is enough.

Very low rank!

SRPA (5): detailed expressions with isospin indices **s={n,p}**

$$h_{res}(\vec{r},t) = \sum_{ss'} \sum_{\alpha\alpha'} [\frac{\delta^2 E}{\delta J_{s\alpha} \delta J_{s'\alpha'}}]_{J=\bar{J}} \delta J_{s\alpha}(\vec{r},t) \hat{J}_{s'\alpha'}(\vec{r})$$

$$= \sum_{ss'} \sum_{k} \{q_{qs}(t) \hat{X}^{s'}_{sk} + p_{qs}(t) \hat{Y}^{s'}_{sk}\} = \sum_{ss'} \sum_{kk'} \{\kappa^{s'k'}_{sk} \delta \hat{X}_{sk}(t) \hat{X}_{s'k'} + \eta^{s'k'}_{sk} \delta \hat{Y}_{sk}(t) \hat{Y}_{s'k'}\}$$

$$\delta J_{s\alpha}(t) = i \sum_{k} \{q_{s\alpha}(t) \langle [\hat{P}_{sk}, J_{s\alpha}] \rangle + p_{s\alpha}(t) \langle [\hat{Q}_{sk}, J_{s\alpha}] \rangle\}$$

$$\hat{X}_{sk} = \sum_{s'} \hat{X}^{s'}_{sk} = i \sum_{s'\alpha\alpha'} [\frac{\delta^2 E}{\delta J_{s\alpha} \delta J_{s'\alpha'}}] \langle [\hat{P}_{sk}, \hat{J}_{s\alpha}] \rangle \hat{J}_{s'\alpha'}$$

$$\hat{Y}_{sk} = \sum_{s'} \hat{Y}^{s'}_{sk} = i \sum_{s'\alpha\alpha'} [\frac{\delta^2 E}{\delta J_{s\alpha} \delta J_{s'\alpha'}}] \langle [\hat{Q}_{sk}, \hat{J}_{s\alpha}] \rangle \hat{J}_{s'\alpha'}$$

$$[\kappa^{-1}]^{s'k'}_{sk} = [\kappa^{-1}]^{sk}_{s'k'} = -i \langle [\hat{P}_{s'k'}, \hat{X}^{s'}_{sk}] \rangle$$

$$[\eta^{-1}]^{s'k'}_{sk} = [\eta^{-1}]^{sk}_{s'k'} = -i \langle [\hat{Q}_{s'k'}, \hat{Y}^{s'}_{sk}] \rangle$$

If $\hat{A}^{\dagger} = \hat{A}, \quad \hat{T}^{-1} \hat{A}_{\pm} \hat{T} = \pm \hat{A}_{\pm}$ **then**

$$\langle [\hat{A}_{+}, \hat{B}_{+}] \rangle = \langle [\hat{A}_{-}, \hat{B}_{-}] \rangle = 0$$

$$\langle [\hat{A}_{+}, \hat{B}_{-}] \rangle \neq 0$$

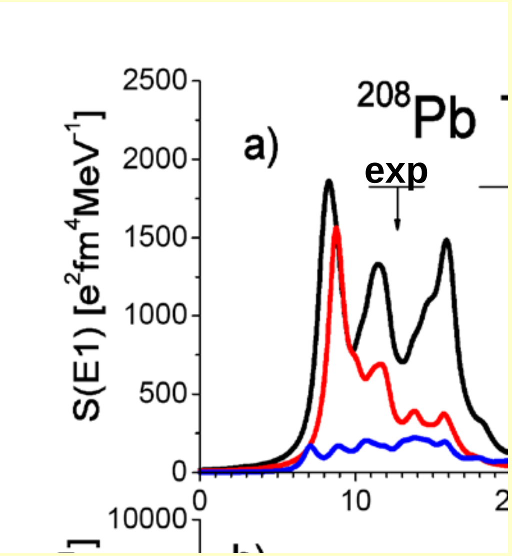
Calculation of average commutators via s-p matrix elements →

$$\langle [\hat{A}, \hat{B}] \rangle = \sum_{ph} \{ \langle | \hat{A} | ph \rangle \langle ph | \hat{B} | \rangle - \langle | \hat{B} | ph \rangle \langle ph | \hat{A} | \rangle \}$$

SRPA (6): strength function

$$S_L(D_{X\lambda\mu},\omega) = \sum_{\nu} \omega_{\nu}^{L} < \nu \,|\, \hat{D}_{X\lambda\mu} \,|\, 0 >^2 \; \xi(\omega-\omega_{\nu}) =$$

Comparison of VM, TM, and CM



exp

- Broad low-energy (LE) and high-energy (HE) bumps for VM, TM, and CM.
- LE strength is dominated by VM and TM
- HE strength is dominated by VM and CM

- General agreement for TM and CM with previous studies.
- Poor agreement with exper. of Ichida (like in previous studies).

Uchida et al., 2003:

$$E_1 = 12.7\,\text{MeV}, \quad \Gamma_1 = 3.5\,\text{MeV}$$
$$E_2 = 23.0\,\text{MeV}, \quad \Gamma_2 = 10.3\,\text{MeV}$$

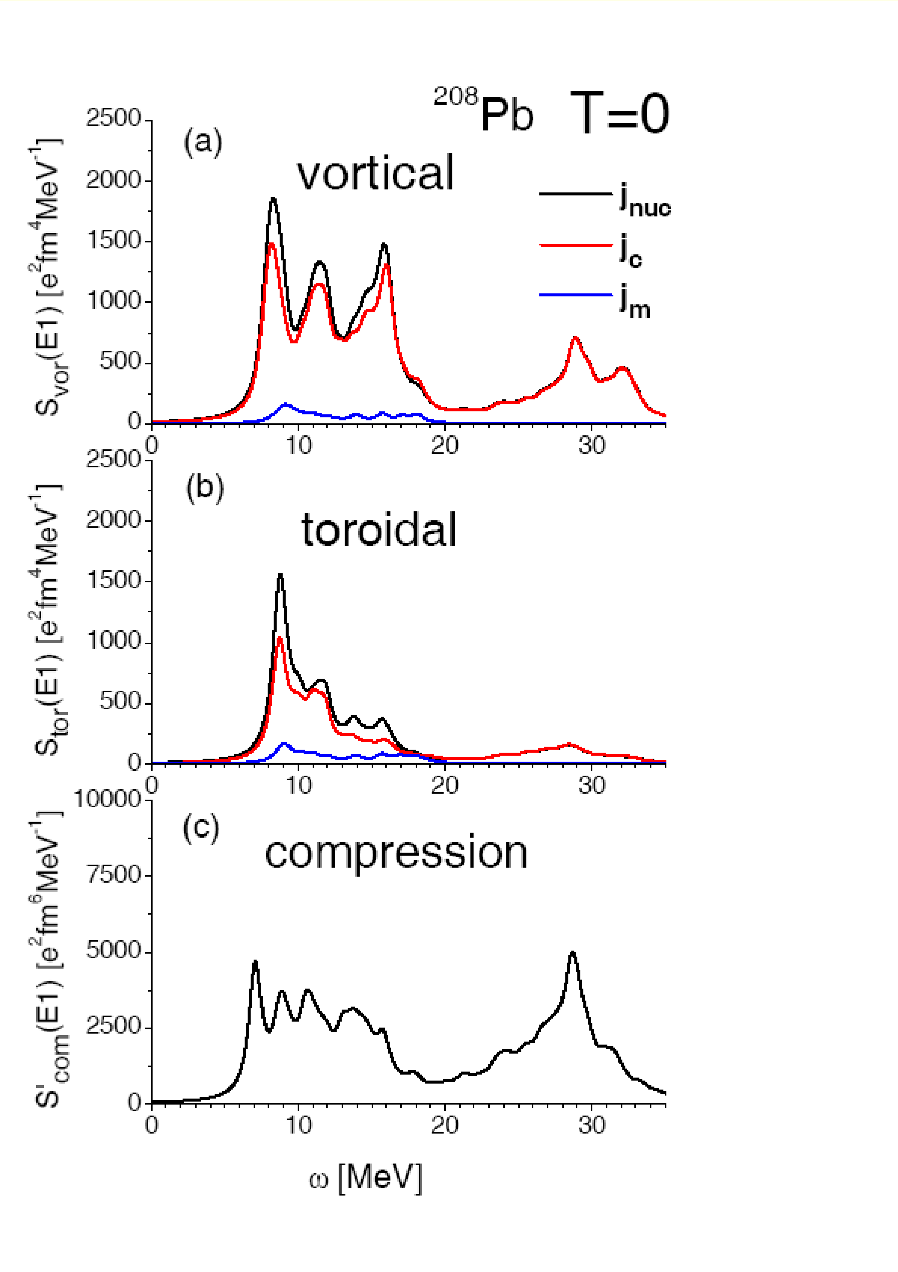
- Purely vortical VM does not coincide with partly vortical TM, especially at HE.

TM was previously considered as a typical example of the vortical flow.

Vortical, toroidal, and compressional T=0 strength

SLy6

$\Delta = 1\,\text{MeV}$



- dominant contribution of j_{con} to VM and TM

-no j_{mag} contribution to:

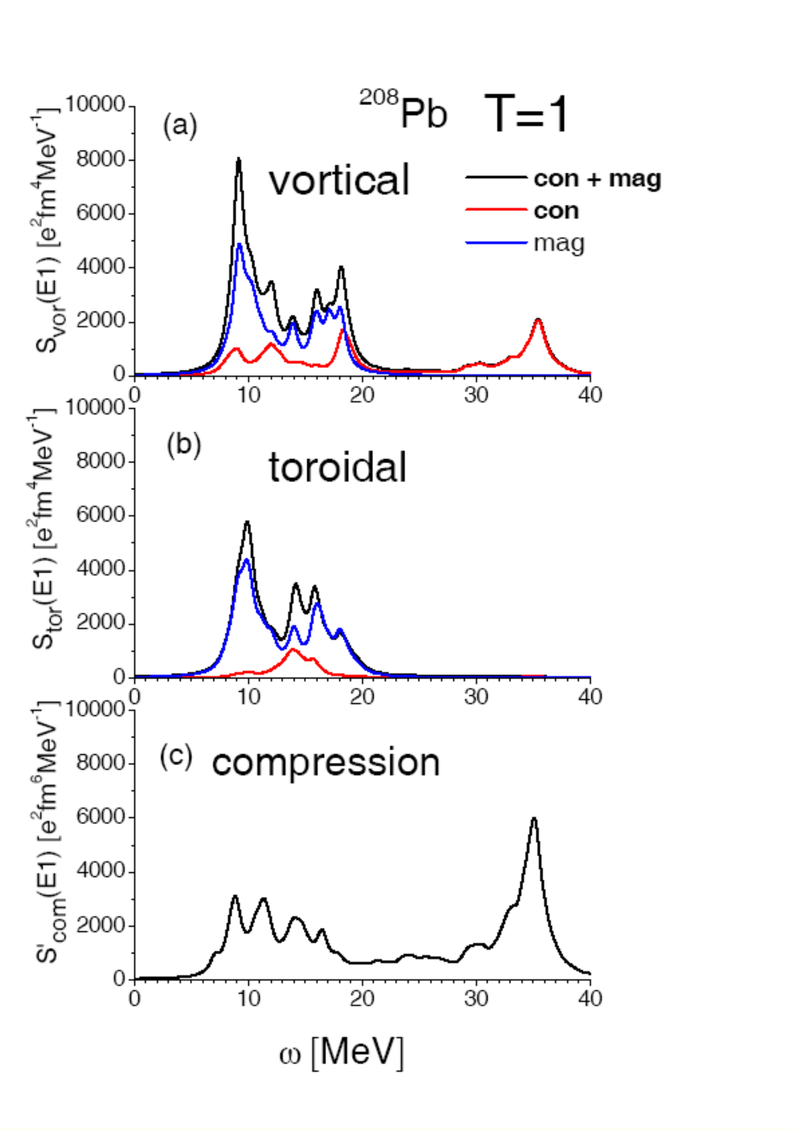
- CM
- HE strength

$$g_s^p = 5.58\varsigma, \quad g_s^n = -3.82\varsigma$$

$$g_s^{T=0} = \frac{1}{2}(g_s^p + g_s^n) = 0.88\varsigma$$

Small T=0 g-factors!

Vortical, toroidal, and compressional T=1 strength



SLy6

$\Delta = 1\text{MeV}$

VM and TM:

- dominant contribution of j_{mag} !!

$$g_s^p = 5.58\varsigma, \quad g_s^n = -3.82\varsigma,$$

$$g_s^{T=1} = \frac{1}{2}(g_s^p - g_s^n) = 4.7\varsigma$$

Large T=1 g-factors!

Vortical and toroidal modes in the T=1 channel are suitable to see the effect of j_{mag} in electric modes.