Light-Particle Emission from Deformed, Hot and Rotating Nuclei

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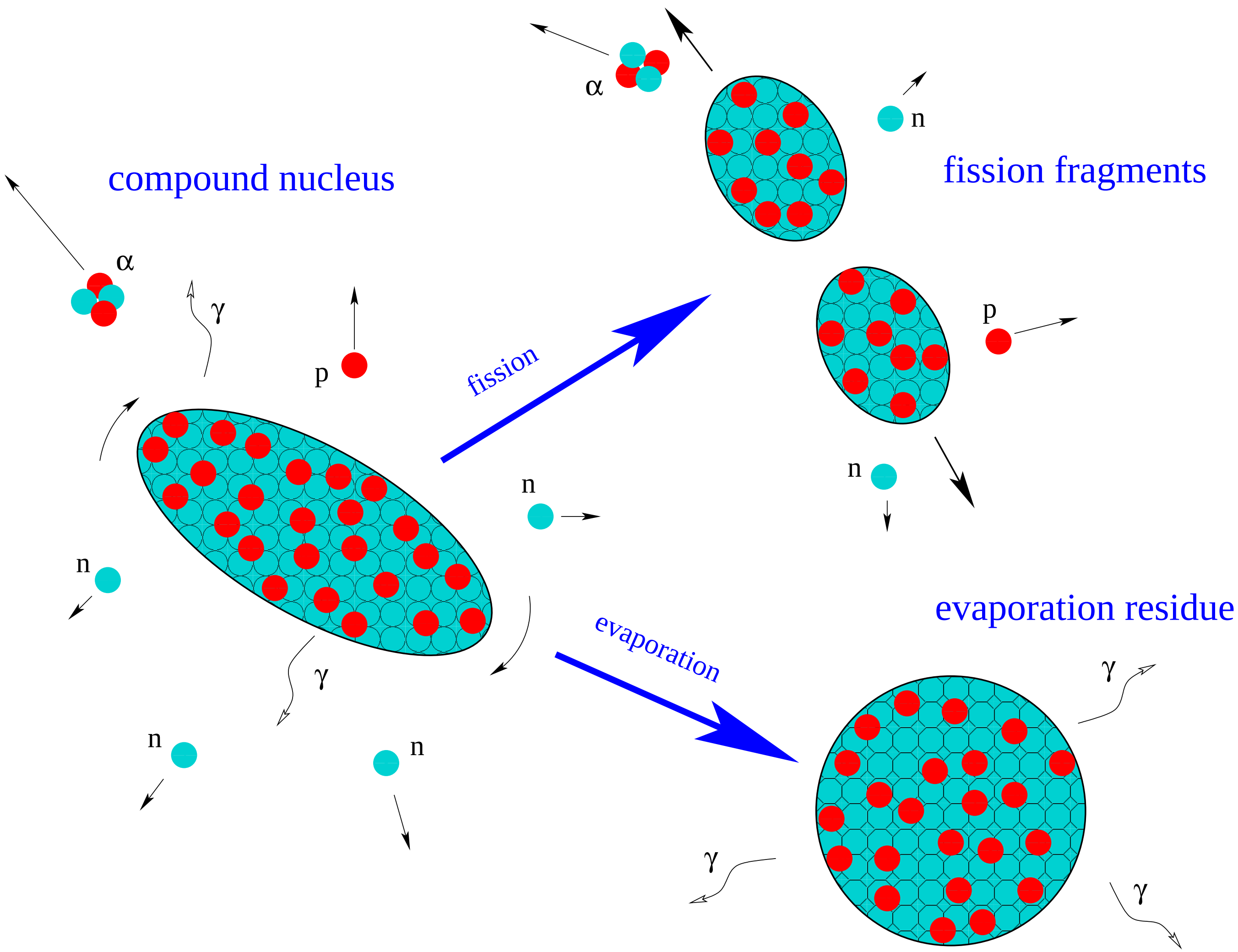
K a z i m i e r z W o r k s h o p, 28 September – 02 October 2011

Contents:

- **Introduction**
- **Fission dynamics $\Longleftrightarrow$ light-particle evaporation**
  - * Equations of motion,
  - * Particle emission – Weisskopf versus semiclassical approach,
  - * Integration of the coupled Langevin and Masters equations,
  - * Role of initial conditions,
- **Results:**
  - * Time dependence of multiplicities and fission events,
  - * Dependence of emission probabilities on deformation,
  - * Influence of friction on multiplicities of prescission particles,
  - * Spectral distribution of the emitted particles,
  - * Comparison of theoretical estimates $\Leftrightarrow$ experimental data.
- **Conclusions**

Fusion-evaporation-fission:



The Modified Funny-Hills shape parametrization:

$$\varrho_s^2\,(z) = \frac{R_0^2}{c\,f(a,B)}\,(1-u^2)\,(1-\gamma\alpha u)\,(1-B\,e^{-a^2(u-\alpha)^2})$$

where

$$f(a,B) = 1 + \frac{3\,B}{4\,a^2}\left[e^{-a^2} + \sqrt{\pi}(a - \frac{1}{2a})\,\mathrm{Erf}(a)\right]$$

<u>breaking axial symmetry</u>

supose ellipsoidal shape $\perp$ to z axis and introduce the non-axiality parameter

$$\eta = \frac{a_y - a_x}{a_y + a_x}$$

assume that η is independent of z

$$a_x(z) = \varrho_s(z)\left(\frac{1-\eta}{1+\eta}\right)^{1/2}\,, \qquad a_y(z) = \varrho_s(z)\left(\frac{1+\eta}{1-\eta}\right)^{1/2}$$

volume conservation then leads to

$$\tilde{\varrho}_s^2(z,\varphi) = \varrho_s^2(z)\,\frac{1-\eta^2}{1+\eta^2+2\eta\cos(\varphi)}$$

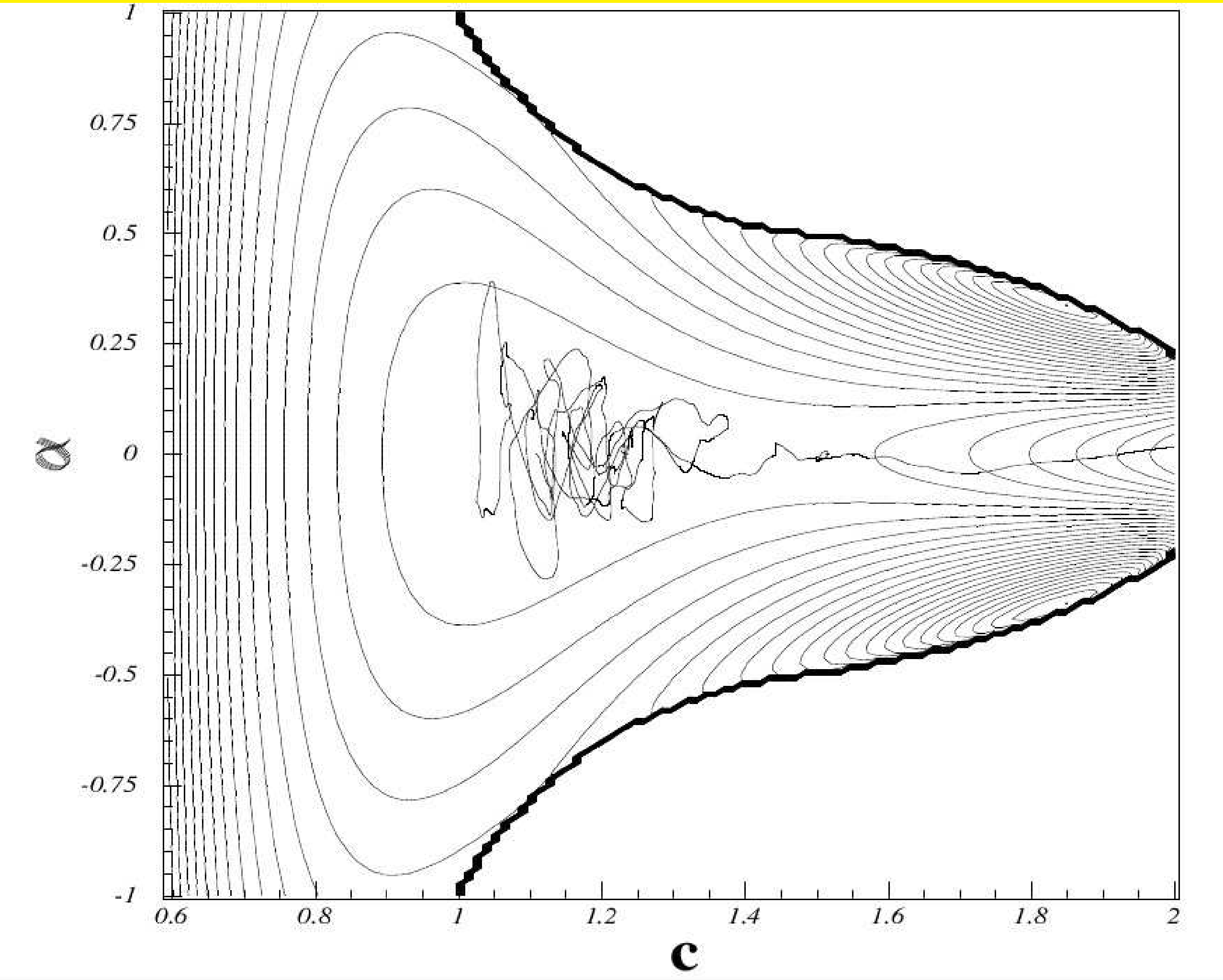
Equations of motion:

The Langevin equation in the collective coordinates space ($\{q_i\}, \quad i = 1, 2, ..., n$) is used to describe the fission dynamics of a highly excited compound nucleus:

$$
\begin{aligned}
\frac{dq_i}{dt} &= \sum_j [\mathcal{M}^{-1}(\vec{q}\,)]_{i\,j}\, p_j \\
\frac{dp_i}{dt} &= -\frac{1}{2}\sum_{j,k} \frac{d[\mathcal{M}^{-1}(\vec{q}\,)]_{jk}}{dq_i}\, p_j\, p_k - \frac{dV(\vec{q}\,)}{dq_i} \\
&\quad - \sum_{j,k} \gamma_{ij}(\vec{q}\,)\, [\mathcal{M}^{-1}(\vec{q}\,)]_{jk}\, p_k + F_i(t)\ ,
\end{aligned}
$$

Here, $\mathcal{M}_{jk}$ and V are the collective inertia tensor and the potential. $F_i(t) = \sum_j g_{ij} G_j(t)$ is the random force with G_j a Gaussian distributed random number. Strength of the random force is related to the diffusion tensor $\mathcal{D}_{ij} = \sum_k g_{ik}\, g_{jk}$, which is determined by the friction tensor γ via the Einstein relation $D_{ij} = \gamma_{ij} T$.

Langevin random trajectory:



Master equations:

The Langevin equation is coupled to Master equations for the number ($\mathcal{N}$) of evaporated light particles:

$$d\mathcal{N}_{\nu}^{\alpha\beta} = \Gamma_{\nu}^{\alpha\beta}(\vec{q}, T)\, dt \ .$$

The emission rate $\Gamma_{\nu}^{\alpha\beta}$ can be evaluated using the Weisskopf or microscopic-semiclassical theory.

The index ν denotes type of emitted particle $(n, p, \alpha, ...)$.

The energy e_{α} of a particle is discretized in bins which are marked by the index $\alpha = 1, 2, ..., n_{\max}$, where the width of a bin is $\Delta e = e_{\max}/n_{\max}$.

The angular momentum ℓ_{β} of the emitted particle $(\ell_1, \ell_2, ..., \ell_{\max})$ is denoted by β

Weisskopf theory*:

The partial decay rate for emission of a particle of type ν with energy e_α and orbital angular momentum ℓ_β from a compound nucleus with excitation energy E^* and angular momentum L is given by

$$\Gamma_\nu^{\alpha\beta}(E^*, L) = \frac{2S_\nu + 1}{2\pi\hbar\rho(E^*, L)} \sum_{L_R} \int_{e_\alpha - \frac{\Delta e_\nu}{2}}^{e_\alpha + \frac{\Delta e_\nu}{2}} w_\nu(e, \ell_\beta; \chi)\rho_R(E_R^*, L_R)de \, ,$$

where
$$\rho(E^*, L) = (2L+1)(\frac{\hbar^2}{2\mathcal{J}})^{3/2}\sqrt{a}\,\frac{e^{2\sqrt{aE^*}}}{12{E^*}^2}$$

is the level density in the emitting nucleus and $w_\nu(e, \ell; \chi)$ is the transmission coefficient for emitting a particle from deformed compound nucleus. a is the s.p. level density.

* V. Weisskopf, Phys. Rev. **52**, (1937) 295.

Single-particle level density*:

Using the Yukawa-folded single-particle potential we evaluate the free energy of a hot and deformed nucleus

$$F(Z,A;\vec{q},T) = E(Z,A;\vec{q},T) - T\cdot S(Z,A;\vec{q},T)\ ,$$

where $S = -\sum_\nu [n_\nu \log(n_\nu) + (1-n_\nu)\log(1-n_\nu)]$ and $E = \sum_\nu 2e_\nu n_\nu$ are the entropy and total s.p. energy, respectively and n_ν is the Fermi occupation number.

The s.p. level density parameter a, is related to the entropy by $S = 2aT^2$, and is evaluated from

$$a(Z,A;\vec{q}) = \left(\widetilde{F}(Z,A;\vec{q},T=0) - \widetilde{F}(Z,A;\vec{q},T)\right)/T^2$$

and approximated by the liquid-drop type expression:

$$\frac{a(Z,A;q)}{\mathrm{GeV}^{-1}} = 92A + 36A^{\frac{2}{3}}B_S(q) + 275A^{\frac{1}{3}}B_{cur}(q) - 1.46\frac{Z^2}{A^{\frac{1}{3}}}B_C(q)$$

*B. Nerlo-Pomorska, K. Pomorski and J. Bartel, Phys. Rev. **C74** (2006) 034327.

Semiclassical emission theory*:

Here, the particle emission rates are defined as

$$\Gamma_\nu^{\alpha\beta} = \frac{d^2 n_\nu}{d\varepsilon_\alpha d\ell_\beta} \Delta\varepsilon\, \Delta\ell\,,$$

The number n_ν of particles of type ν which are emitted per time unit through the surface Σ is given by

$$n_\nu = \int_\Sigma d\sigma \int d^3p'\, f_\nu(\vec{r}_0', \vec{p}\,')\, v_\perp'(\vec{r}_0')\, w_\nu(v_\perp'(\vec{r}_0'))\,,$$

where $v_\perp'$ is the velocity component perpendicular to the surface and the primed quantities refer to the body-fixed frame. The classical phase-space distribution function is:

$$f_\nu(\vec{r}\,', \vec{p}\,') = \frac{2}{h^3} \frac{\theta(\vec{r}\,')}{1 + \exp\left[\frac{1}{T}\left(\frac{p'^2}{2m} + V(\vec{r}\,) - \omega\ell' - \mu_\nu\right)\right]}\,.$$

* K. Dietrich,K. Pomorski and J. Richert, Z. Phys. **A351**, (1995) 397.

Effective emission rate*:

Emission rates averaged over all particle angular momenta:

$$\Gamma_\nu^\alpha(E^*,L) = \frac{2S_\nu+1}{2\pi\hbar\tilde{\rho}(E^*)} \int\limits_{e_\alpha-\Delta e_\alpha/2}^{e_\alpha+\Delta e_\alpha/2} w_\nu^{\mathrm{eff}}(e;\chi)\,\tilde{\rho}_R(E_R^*)\;d\,e \quad ,$$

where $\tilde{\rho}(E^*) = \rho(E^*,L)/(2L+1)$ is the angular momentum independent part of the density and

$$w_\nu^{\mathrm{eff}}(e;\chi) = \frac{1}{2L+1} \sum_{\ell_\beta=0}^{\ell_{\mathrm{max}}} \sum_{L_{\mathrm{R}}=|L-\ell_\beta|}^{L+\ell_\beta} (2L_R+1)\bar{w}_\nu(e,\ell_\beta;\chi) \quad .$$

is the effective transmission coefficient.

* K. Pomorski, J. Bartel, J. Richert, and K. Dietrich, Nucl. Phys. **A605**, (1996) 87.

Particle and total emission rates:

The total particle emission rate for evaporation of a particle of given type, irrespectively of its energy, is given by

$$\Gamma_\nu(E^*, L) = \frac{2S_\nu + 1}{2\pi\hbar\tilde{\rho}(E^*)} \int_0^{e_{\max}} w_\nu^{\text{eff}}(e;\chi)\tilde{\rho}_R(E_R^*)de = \sum_{\alpha=1}^{n} \Gamma_\nu^\alpha(E^*, L)\,.$$

The total emission rate Γ for emission of any kind of particles is the sum of the particle emission rates:

$$\Gamma = \Gamma_n + \Gamma_p + \Gamma_\alpha\,.$$

Evaluation of the transmission coefficients:

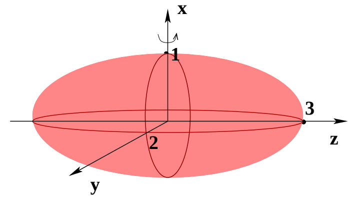
• Using the semiclassical and WKB approximation one evaluates the transmission coefficients at three points (i=1,2,3) in which the main body-fixed axes (x,y,z) cross the nuclear surface:

$$w_\nu(e,\ell,\ell_x;i) = (1 + exp[-2\pi(E_B - e)/(\hbar\omega_i)])^{-1} \quad ,$$

where $\omega_i = \sqrt{(d^2 V_{\rm tot}(i)/dr^2_{i_\perp})/m_\nu}$ and $V_{\rm tot} = V_{\rm nucl} + V_{\rm centr} + V_{\rm Coul} - \omega\ell_x$.

• One averages w_ν over all projections of ℓ

$$\tilde{w}_\nu(e,\ell;i) = \frac{1}{2\ell+1}\sum_{\ell_x=-\ell}^{\ell} w_\nu(e,\ell,\ell_x;i) \quad , \quad i = 1,2,3$$



• The transmission coefficients at arbitrary surface point (θ,φ) are obtained by interpolation:

$$\bar{w}_\nu(e,\ell;\theta,\varphi) = \sin^2\theta \cdot [\tilde{w}_\nu(e,\ell;1)\cos^2\varphi + \tilde{w}_\nu(e,\ell;2)\sin^2\varphi] + \tilde{w}_\nu(e,\ell;3)\cos^2\theta$$

• Finally, the average transmission coefficient is given by:

$$\widetilde{w}_\nu(e,\ell;\chi) = \int_\Sigma \bar{w}_\nu(e,\ell;\ \theta,\varphi)\, d\sigma \,/ \int_\Sigma d\sigma$$

Numerical algorithm for the emission of particles*:

- One first decides in every time interval [t, t + τ] whether a particle is emitted or not. The probability of the particle emission during a small enough time step τ, is given by

$$P(\tau) = 1 - e^{-\Gamma\,\tau} \approx \Gamma\,\tau\,.$$

One then draws a first random number η_1 in the interval $[0, 1]$. If $\eta_1 < P(\tau)$ the particle is emitted.

- If yes, one decides which type of particle is emitted by drawing a second random number $\eta_2 \in [0, 1]$ falling into one of three bins:

$$\{\,\Gamma_n/\Gamma \;+\; \Gamma_p/\Gamma \;+\; \Gamma_\alpha/\Gamma\,\} \;=\; 1$$

* K. Pomorski, J. Bartel, J. Richert, and K. Dietrich, Nucl. Phys. **A605**, (1996) 87.

• One still has to determine the energy of the emitted particle. Introducing a quantity

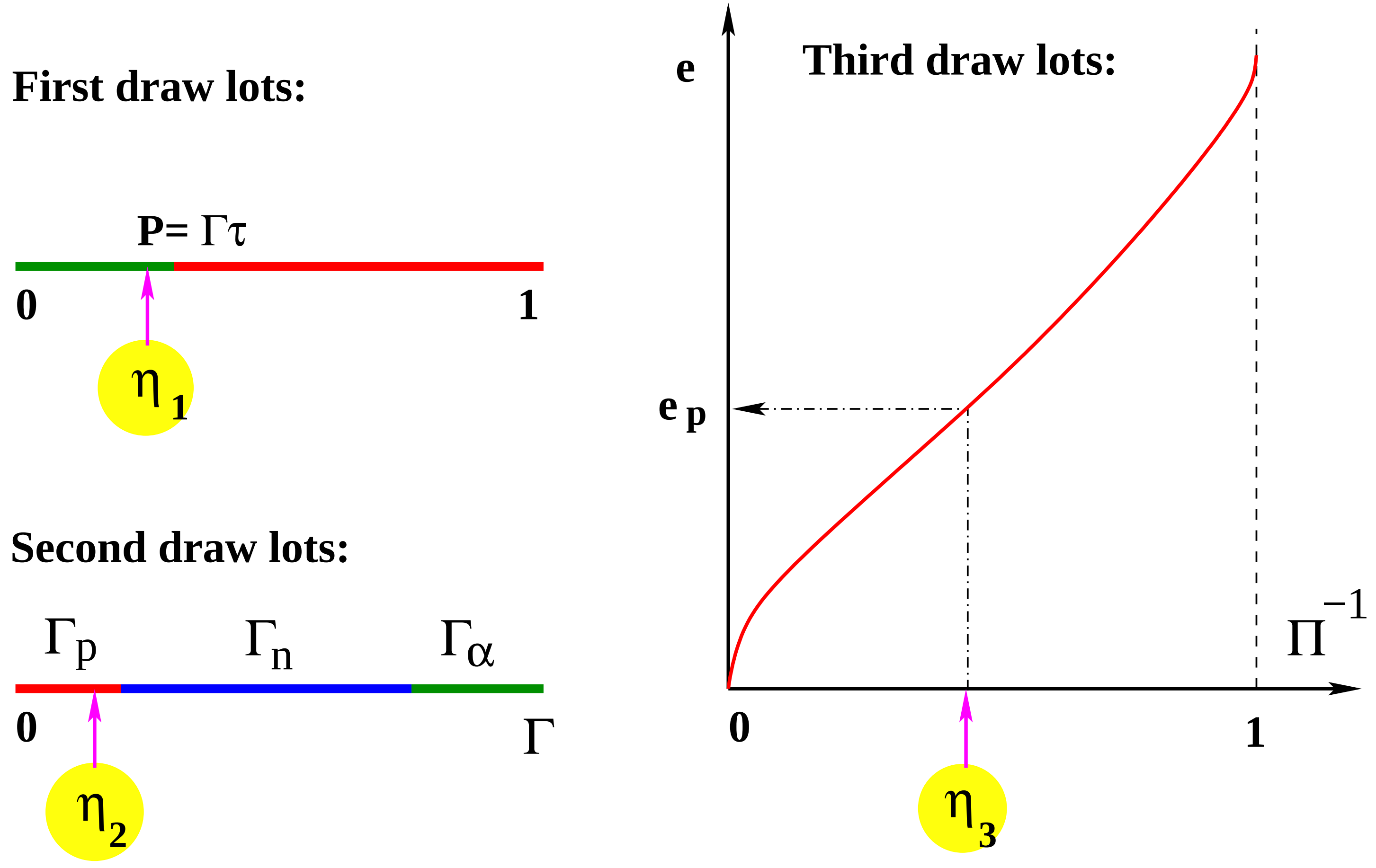
$$\Pi_\nu(e_\alpha) = \frac{1}{\Gamma_\nu} \sum_{e_\mu \le e_\alpha} \Gamma_\nu^\mu$$

which is the probability that the particle of type ν is emitted with energy smaller than e_α (obviously $\lim_{e\to\infty} \Pi_\nu(e) = 1$). Subdividing the interval $[0, 1]$ in a certain number of equal bins, and inverting the function $\Pi_\nu(e)$ one can decide with which energy the particle is emitted by drawing a third random number $\eta_3 \in [0, 1]$

$$\Pi_\nu^{-1}(\eta_3) \in [e_\alpha - \frac{\Delta e_\alpha}{2}, e_\alpha + \frac{\Delta e_\alpha}{2}]$$

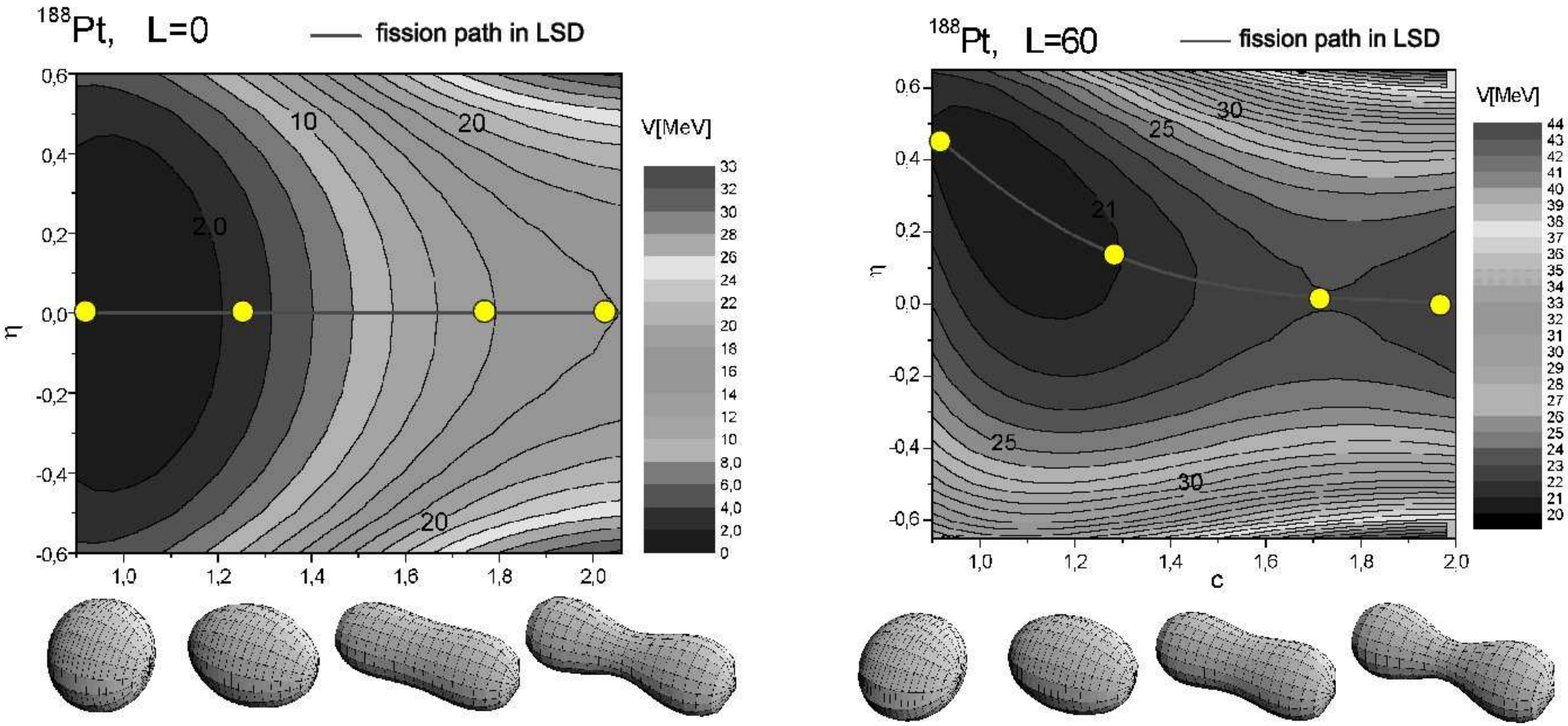
that selects one of the energy bins thus deciding on the energy of the particle.

In three steps one decides: whether a particle will be emitted, of which type, and with which energy :



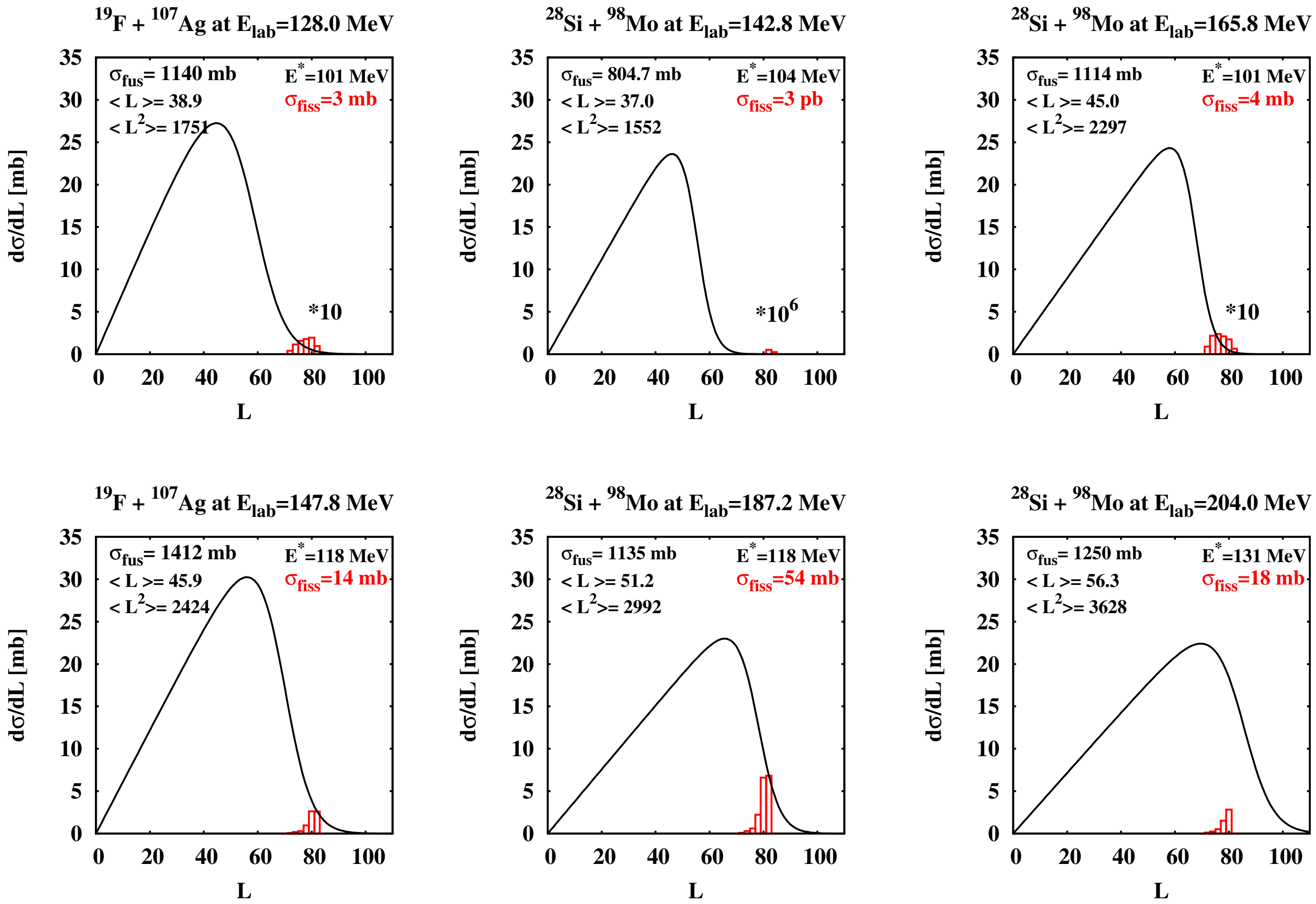
Here, η_1, η_2 and η_3 are the random numbers.

Free potential energy landscape:



The Modiffied Funny-Hills nuclear shape parametrisation (c,h,α,η) of [A. Dobrowolski, K. Pomorski, J. Bartel, Phys. Rev. **C75** (2007) 024613] and the Lublin-Strasburg Drop formula of Ref. [K. Pomorski and J. Dudek, Phys. Rev. **C67** (2003) 044316] are used.

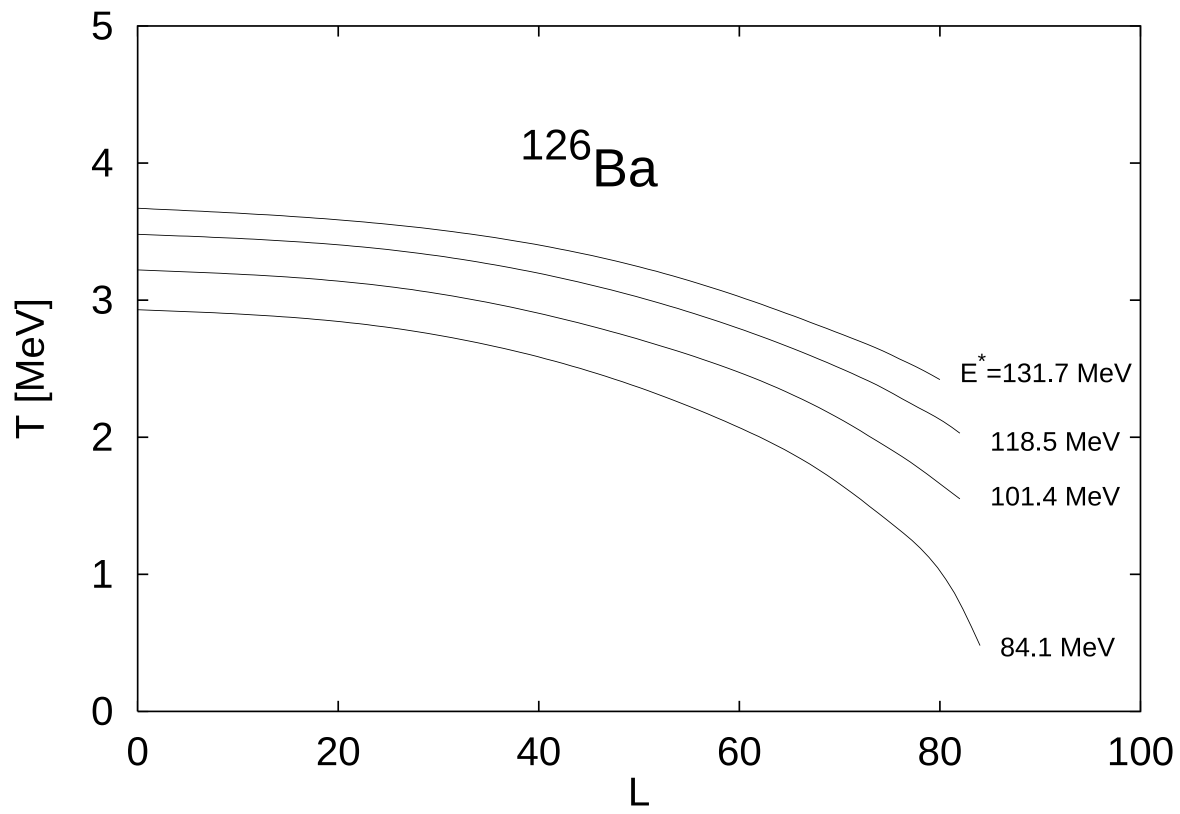
Fusion and fission cross-sections*:



* K. Pomorski et al., Nucl. Phys. **A679**, (2000) 25-53;

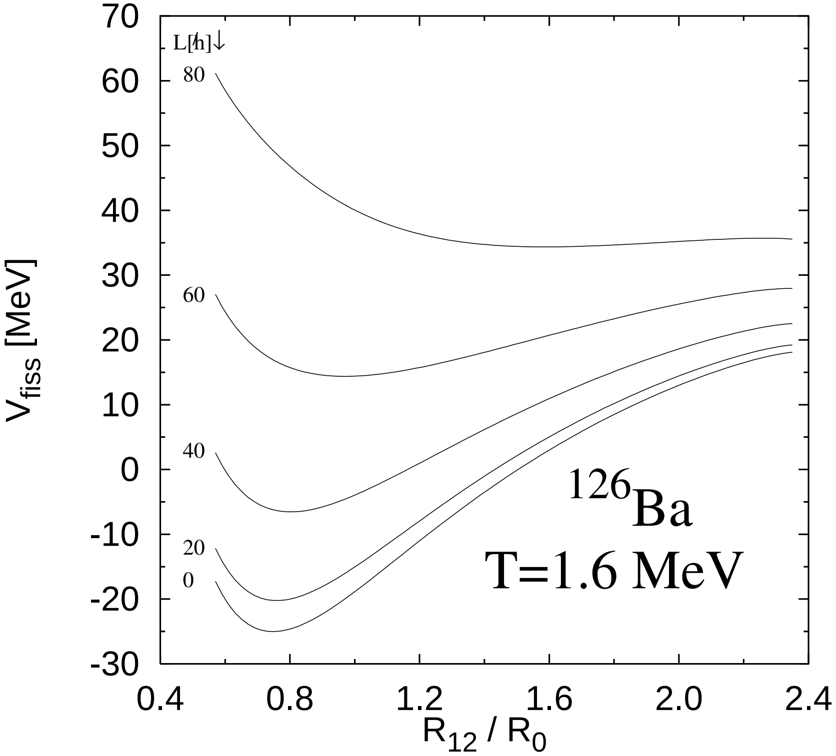
W. Przystupa, K. Pomorski, Nucl. Phys. **A572** (1994) 153-170.

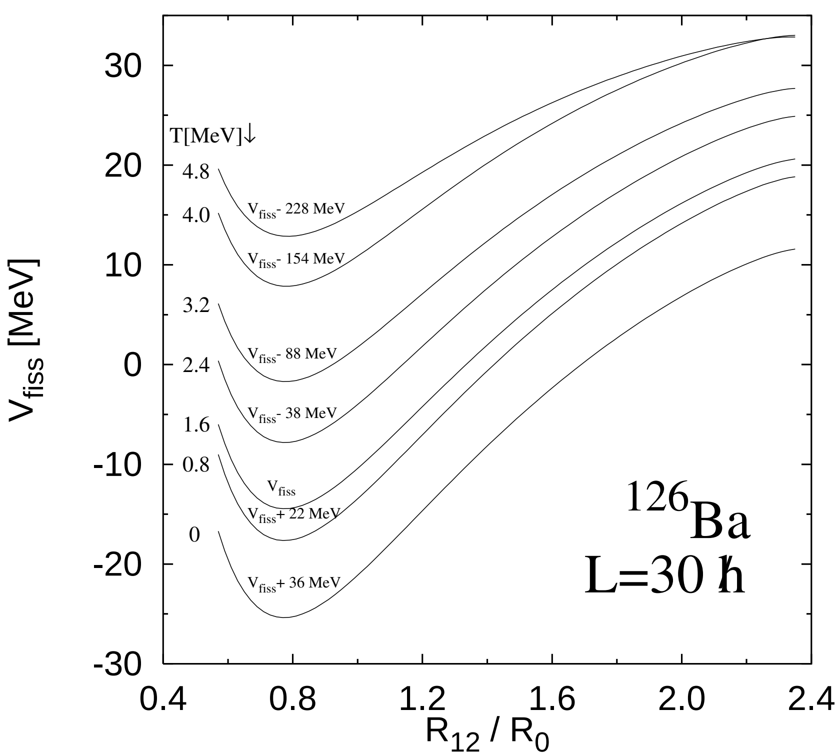
Temperature of fissioning system:



<u>Note</u>: A given excitation energy (E^*) is **shared** between the thermal excitation (temperature T) and the rotational energy of the compound nucleus (angular momentum L).

Effect of rotation and temperature on barrier heights





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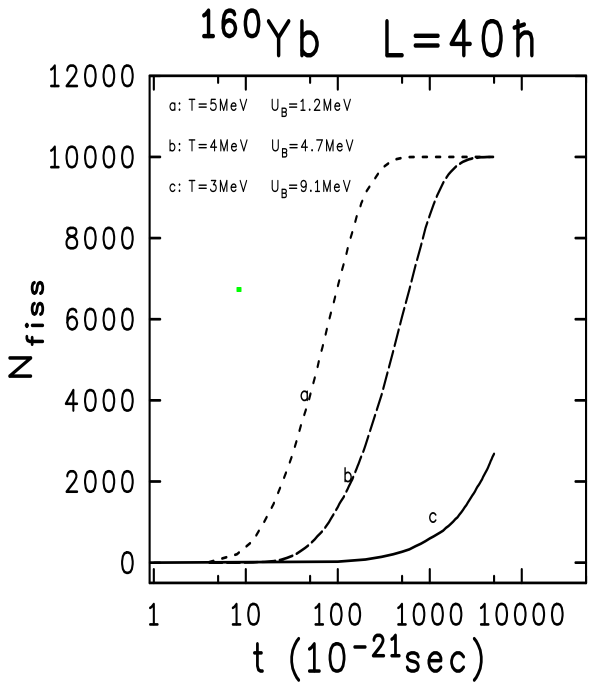
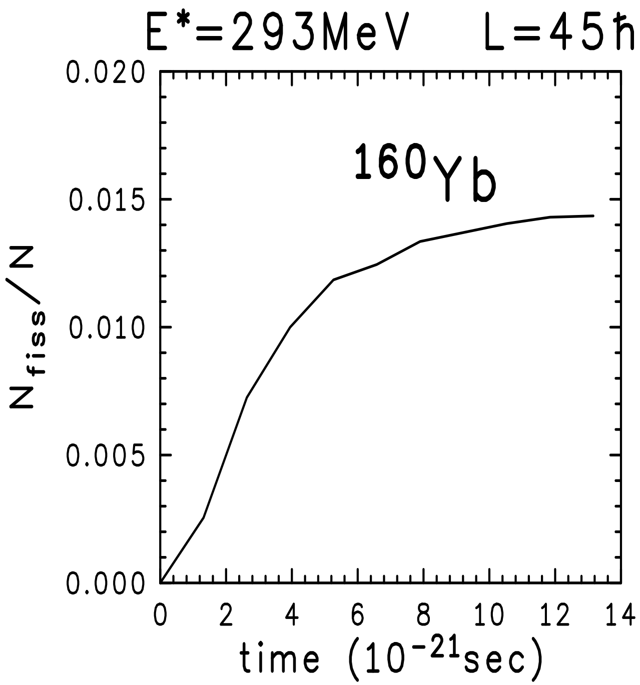
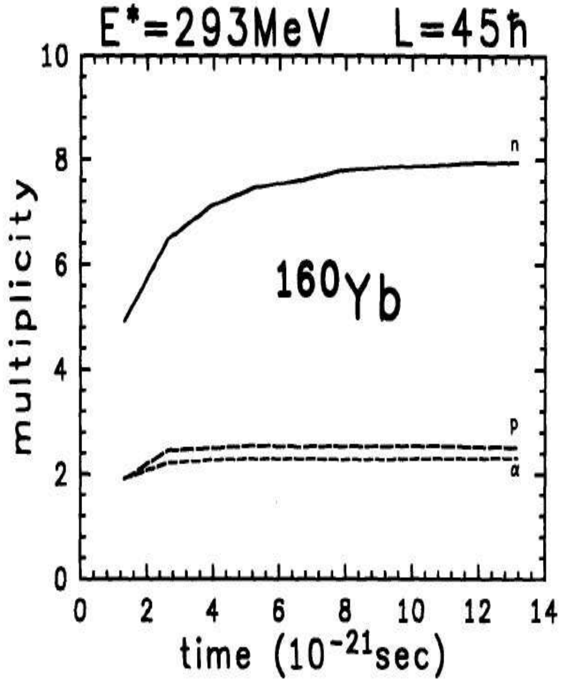
Example of the time dependence of the fission and the particle evaporation processes *:

Without particle evaporation, with evaporation , prefission particles



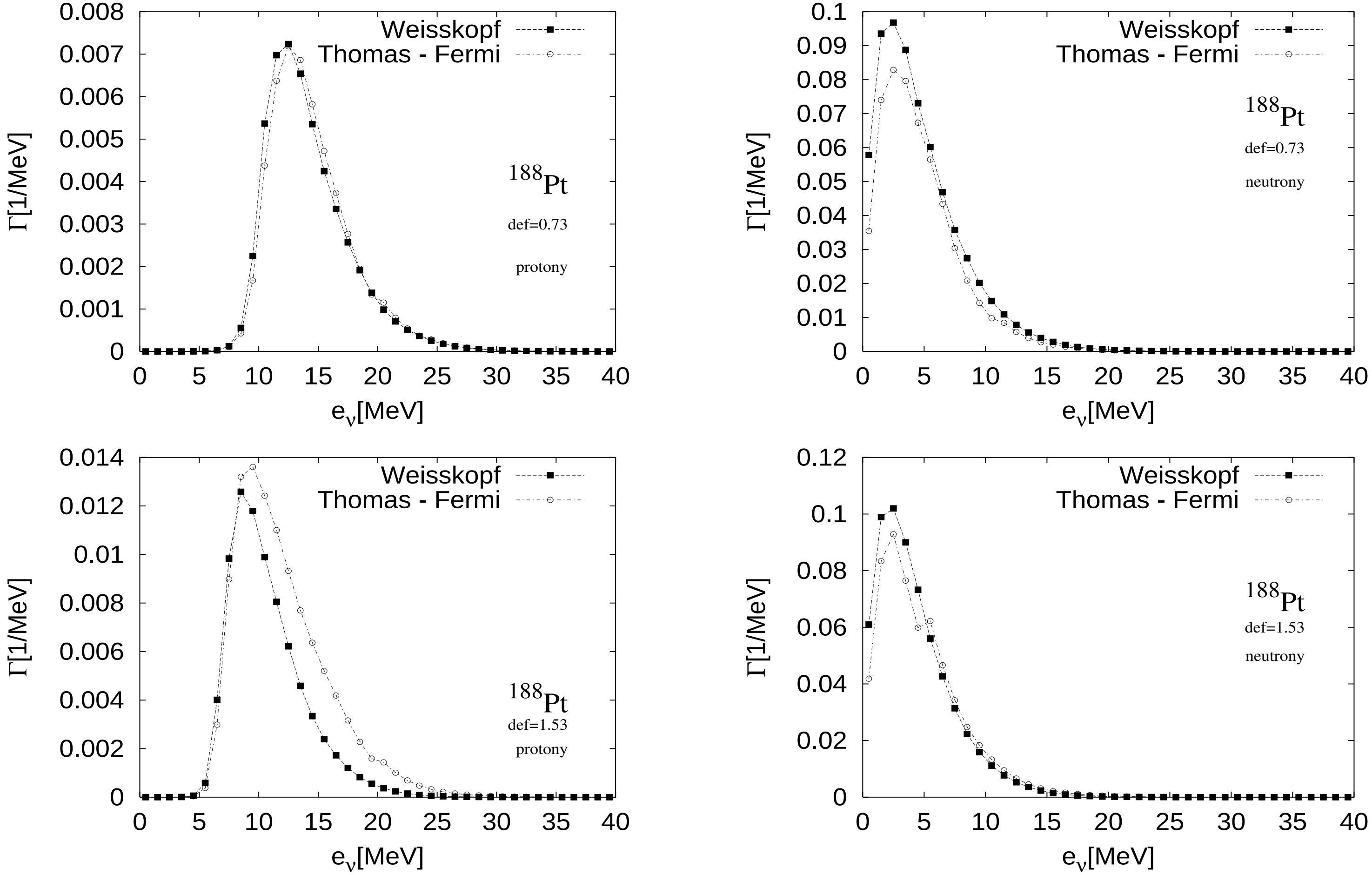




* K. Pomorski, J. Bartel, J. Richert, and K. Dietrich, Nucl. Phys. **A605**, (1996) 87.

Comparison of the Weisskopf and semiclassical rates*:

. neutrons protons



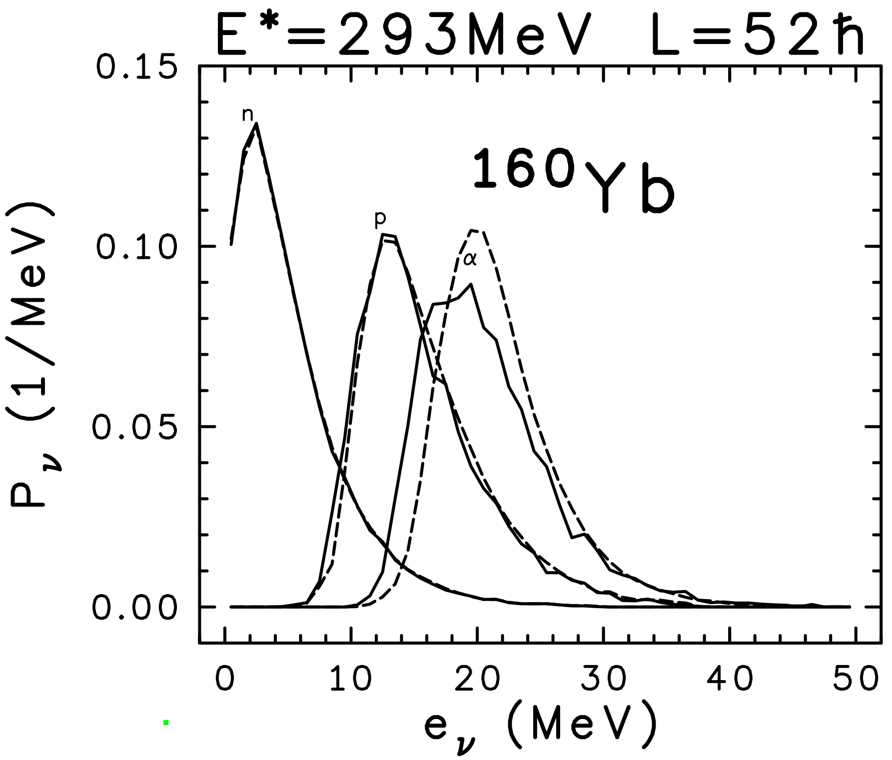
sph.

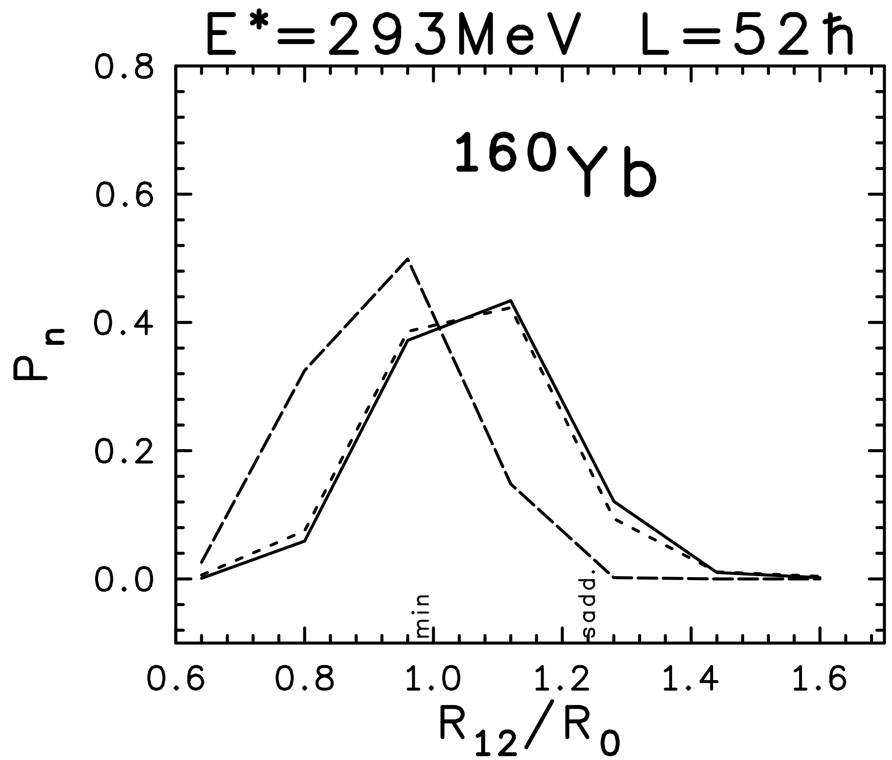
sadd.

* K. Pomorski et al., Nucl. Phys. **A679**, (2000) 25;

A. Surowiec, K. Pomorski, C. Schmitt, J. Bartel, Acta Phys.Pol. **B33** (2002) 479.

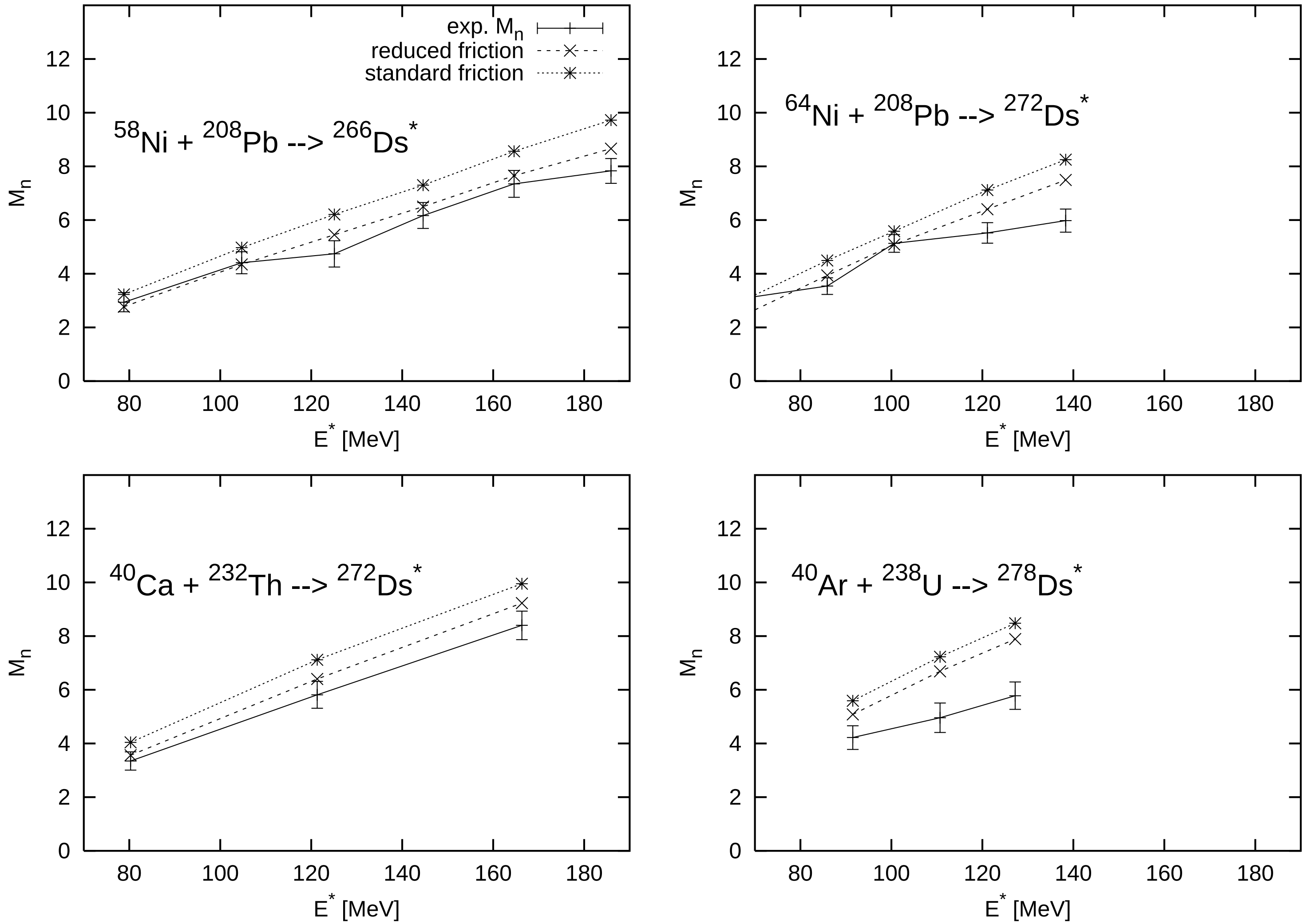
Spectra of evaporated particles and the deformation distribution of emitters:





Solid line corresponds to prefission particles and deformations of the emitters which are going to fission and dashed ones to the evaporation residua while the dotted line to the case when the α-emission is off.

Prefission neutron multiplicities for $^{266-278}$Ds*:



K. Pomorski et al., Nucl. Phys. **A679**, (2000) 25-53;

Estimates of the prefission neutron multiplicities:

Nucleus	Reaction	E^* [MeV]	MS-LD	LSD	exp
^{126}Ba	^{28}Si+^{98}Mo	131.7	1.50	2.48	2.52 ± 0.12
		118.5	1.32	2.02	2.01 ± 0.13
		101.4	0.38	1.36	1.32 ± 0.09
	^{19}F+^{107}Ag	118.5	1.32	2.08	1.85 ± 0.11
		101.5	1.00	1.23	1.31 ± 0.17
^{188}Pt	^{34}S+^{154}Sm	66.5	1.75	2.29	2.50 ± 0.7
		100.0	4.48	4.44	4.5 ± 0.7
	^{16}O+^{172}Yb	99.7	4.65	4.52	5.4 ± 0.7
^{266}Ds	^{58}Ni+^{208}Pb	78.8	2.72	2.06	2.94 ± 0.36
			3.18	2.49	
		104.7	4.27	3.17	4.41 ± 0.41
			4.91	3.67	
		125.1	5.52	4.03	4.74 ± 0.49
			6.20	4.59	
		144.6	6.61	4.85	6.17 ± 0.48
			7.38	5.43	
		164.6	7.66	5.61	7.35 ± 0.50
			8.46	6.18	
		185.9	8.76	6.42	7.83 ± 0.46
			9.64	6.99	

The estimates typed in black are obtained with 50% reduction of the ww friction*.

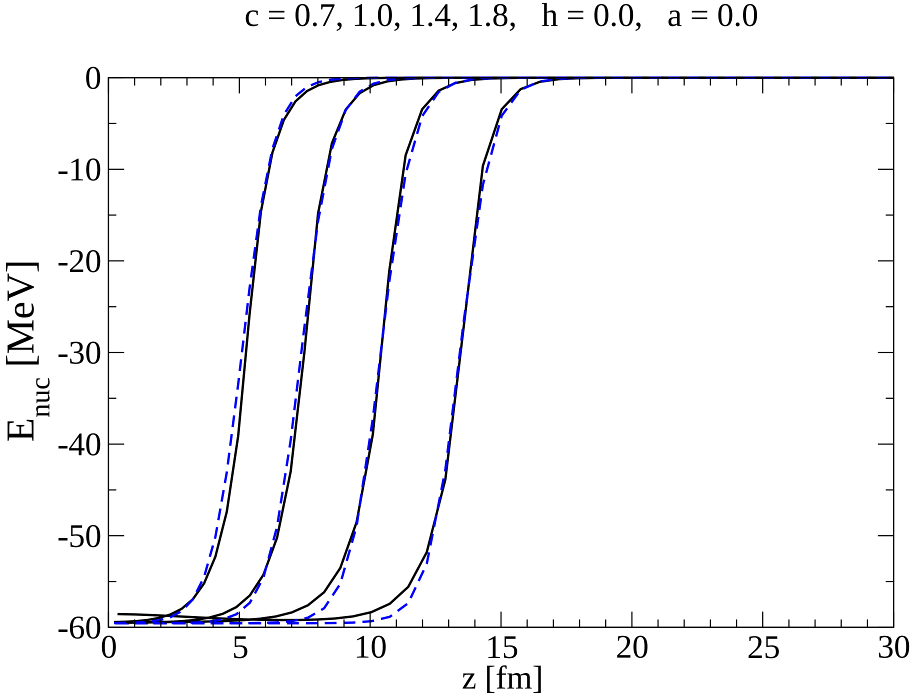
* J. Blocki, J. Randrup and W. J. Swiatecki, Ann. Phys. **105** (1977) 427.

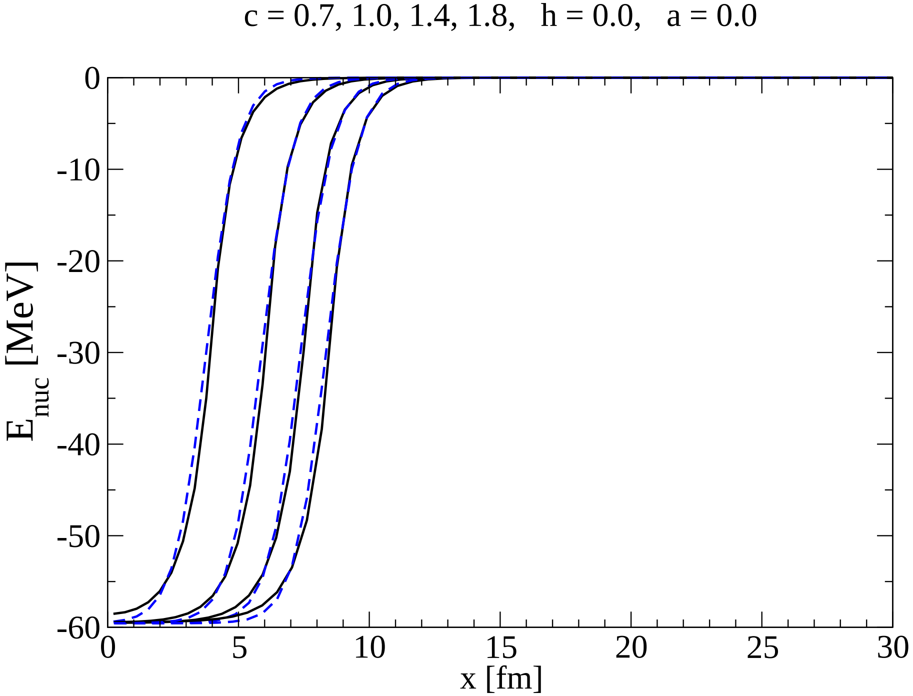
Find easy approximation of nuclear and Coulomb potential:

$$V_{nuc}(z) = \frac{V_o}{1 + e^{\frac{z-\tilde{z}_o}{\sigma_o}}} \quad \text{and} \quad V_{nuc}(x) = \frac{V_o}{1 + e^{\frac{x-\tilde{x}_o}{\sigma_o}}}$$

$\tilde{z}_o = c\,R_0 - 0.2\,fm$

$\tilde{x}_o = \rho_s(0,0) - 0.2\,fm$





and for the Coulomb potential:

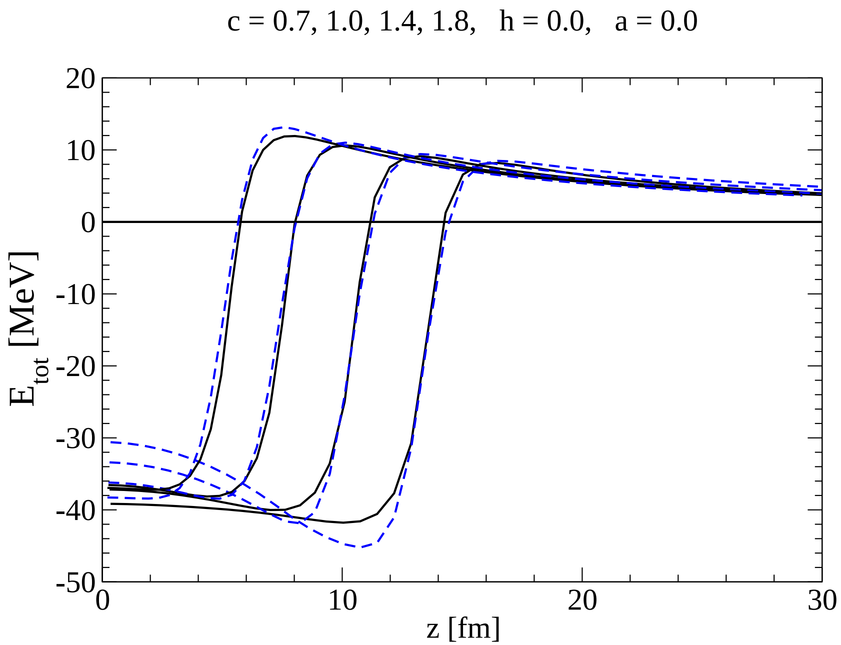
$$V_{Coul}^{(\text{fit})}(z) = f_{\text{fit}}(c,h,\alpha,\eta)\; V_{Coul}^{(\text{o})}(z)$$

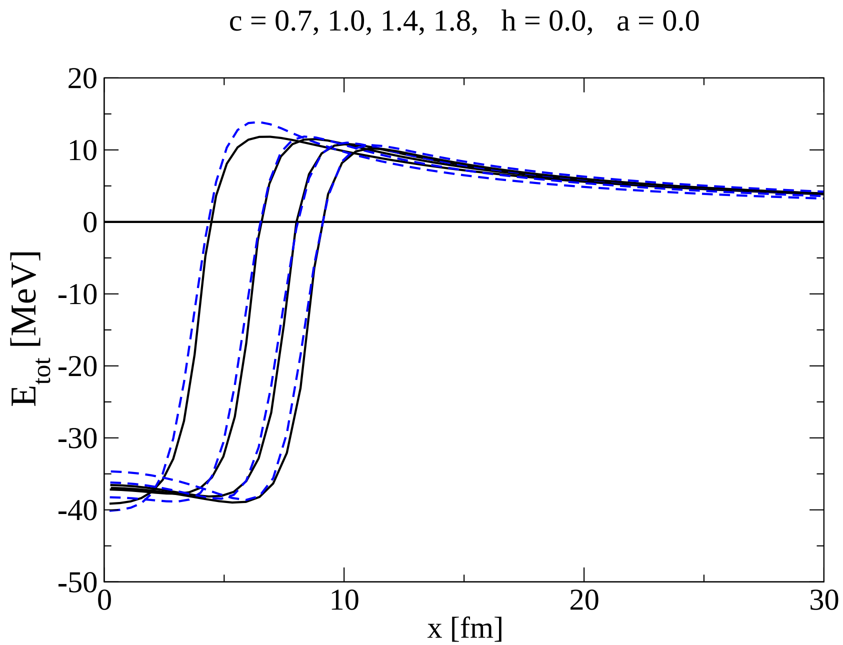
ξ_c	ξ_h	ξ_α	ξ_η
0.30	-0.04	-0.10	—

κ_c	κ_h	κ_α	κ_η
-0.22	0.03	0.07	—

with

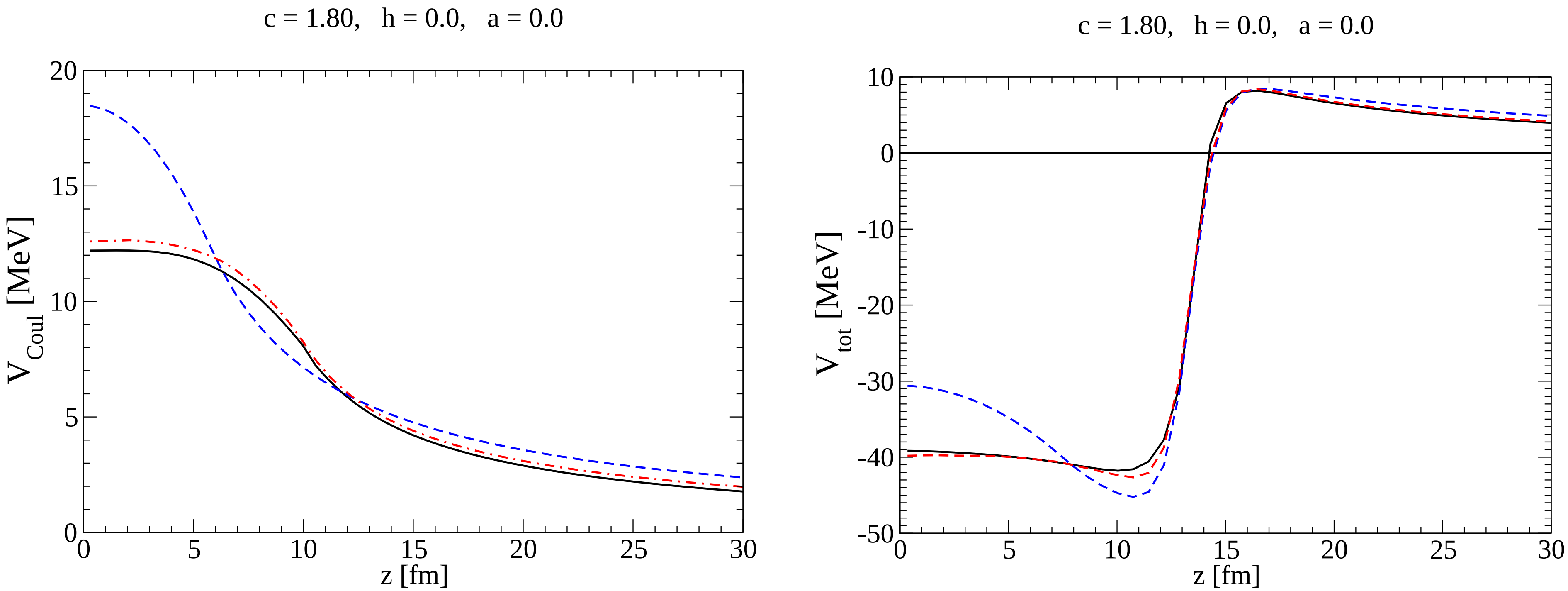
$$f_{\text{fit}}(c,h,\alpha,\eta) = \Big[1+\xi_c\,(c-1)\Big]\;\Big[1+\xi_h\,h\Big]\;\Big[1+\xi_\alpha\,\alpha\Big]\;\Big[1+\xi_\eta\,\eta\Big]$$





but this is pretty bad

Use the Coulomb potential of spheroidal charge distribution



Summary and conclusions:

- Particle evaporation determines the time-scale of the fission process of compound nuclei.
- Deformation enhances the emission process of charged particles.
- Energy spectra of charged prefission particles are shifted towards lower energies with respect to the spectra of particles emitted by evaporation residua.
- The magnitude of the wall+window friction should be reduced by app. 50% in order to reproduce the data on the prefission neutron multiplicities of low-excited compound nuclei while the ww-friction gives satisfactory results at higher excitations.
- Lublin-Strasburg Drop model reproduces nicely the fission barrier heights of nuclei from different mass regions.
- Full four-dimensional calculations in the (c, h, α, η) deformation space are in preparation.

Emission of α-particle in the semiclassical model:

The Wigners function for α-particle is defined as follows:

$$f_\alpha(\vec{R},\vec{P}) = \frac{1}{h^3}\int I_{n_1} I_{n_2} I_{p_1} I_{p_2} d\sigma_{n_1} d\sigma_{n_2} d\sigma_{p_1} d\sigma_{p_2} \mathcal{P}\left(\sigma_\alpha\right) \; .$$

where σ_{ν_i} is spin of nucleon ν_i, $\mathcal{P}(\sigma_\alpha)$ is the projection operator for the spin of α-particle and

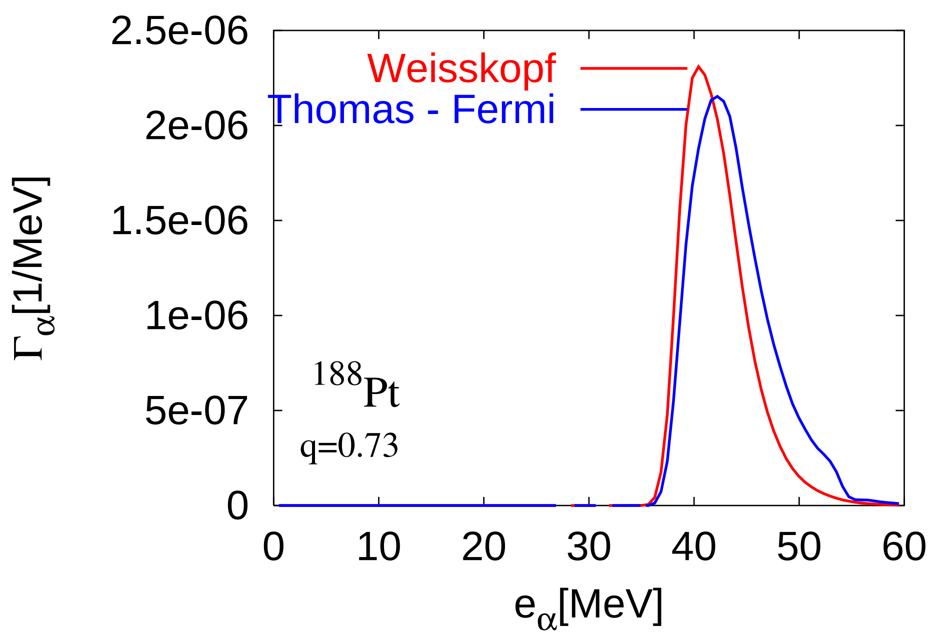
$$I_k = \frac{1}{2\pi ab}\int \tilde{f}_k\left(\vec{r_1},\vec{p_1}\right) e^{-\frac{\left(\vec{r_1}-\vec{R}\right)^2}{2a^2}} e^{-\frac{\left(\vec{p_1}-\vec{P}/4\right)^2}{2b^2}} d^3 r_1 d^3 p_1$$

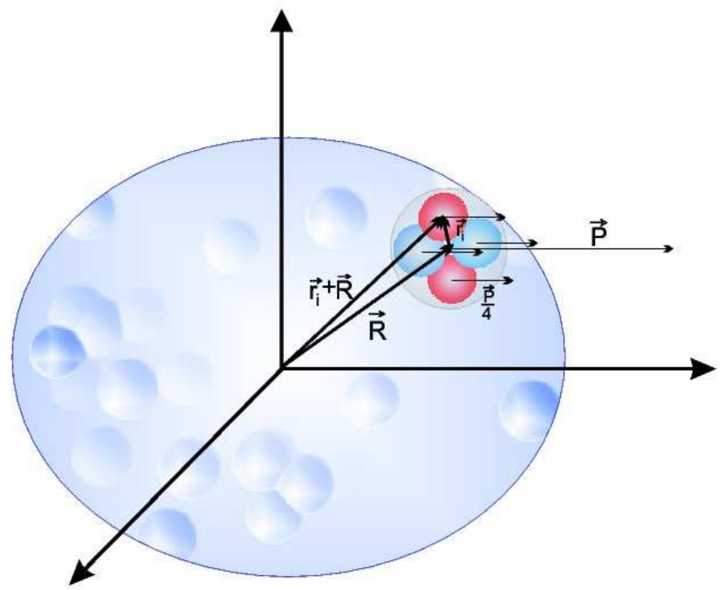
Here, f stays for the Fermi distribution function

$$\tilde{f}_n(\vec{r},\vec{p}) = \left\{1+\exp\left[\frac{1}{T}\left(\frac{p^2}{2m} - U\left(\vec{r}\right) - \omega l_x - \mu_n\right)\right]\right\}^{-1} \; .$$

* A. Surowiec, K. Pomorski, C. Schmitt, J. Bartel, Acta Phys.Pol. **B33** (2002) 479.

Comparison of the Weisskopf and semiclassical (T-F) rates for the emission of α-particles *:

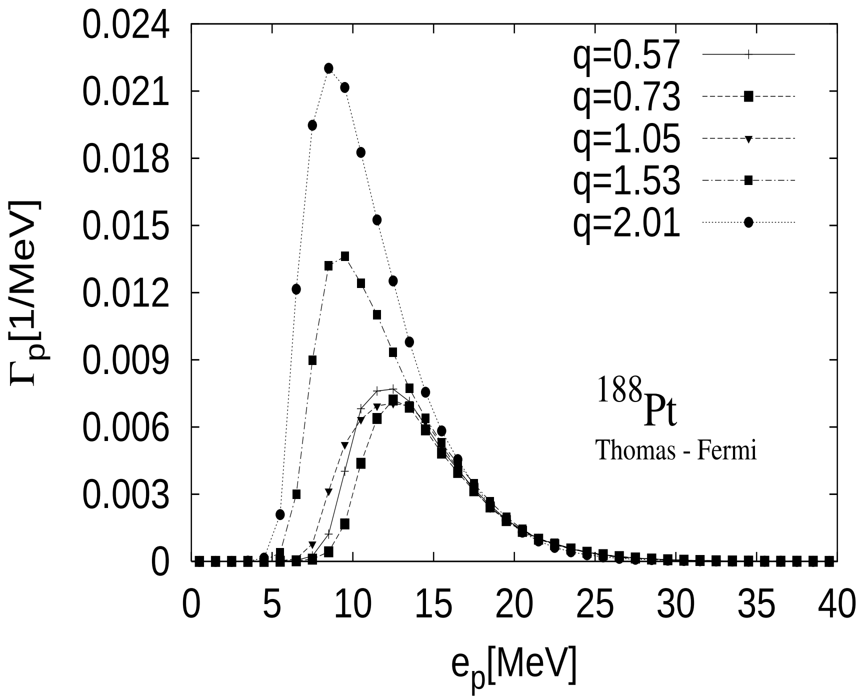


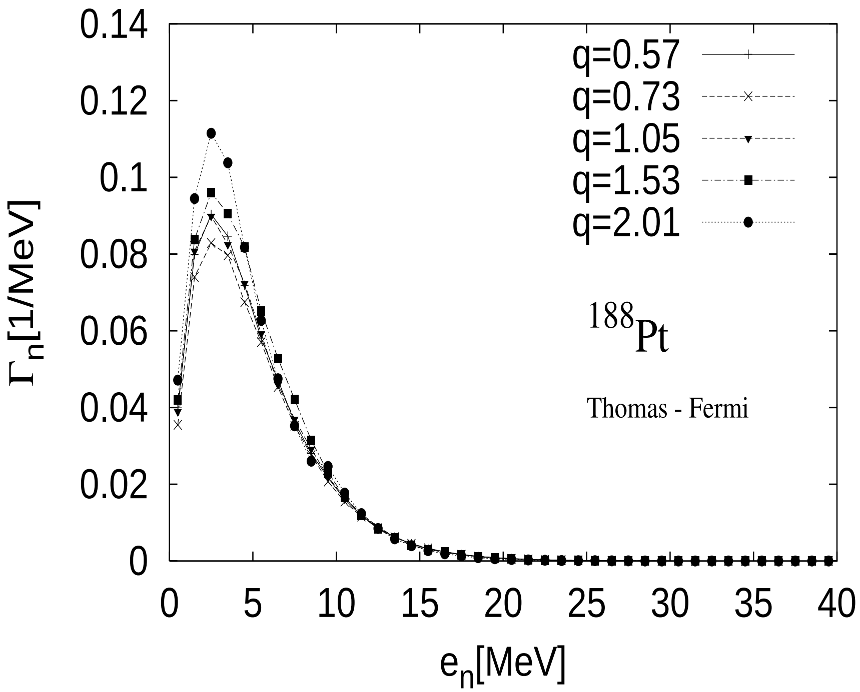


We have assumed here that the α-particle is described by a distribution function built from two correlated proton and neutron pairs.

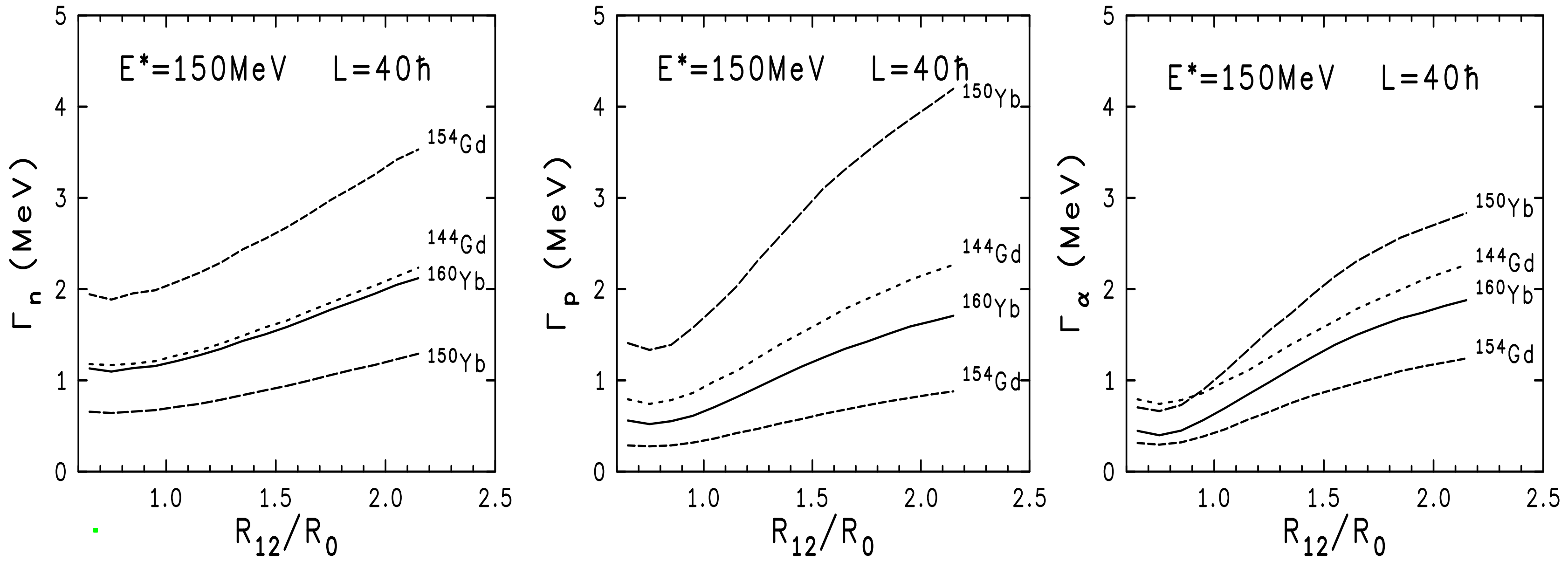
* A. Surowiec, K. Pomorski, C. Schmitt, J. Bartel, Acta Phys.Pol. **B33** (2002) 479.

Energy spectra for different deformations:





Deformation dependence of the total emission rates*:



* K. Pomorski, J. Bartel, J. Richert, and K. Dietrich, Nucl. Phys. **A605**, (1996) 87.