Thomas-Fermi theory for pairing in finite Fermi systems: The weak coupling regime. Average properties in nuclei

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THE THOMAS-FERMI APPROXIMATION IN WEAK COUPLING

In the canonical basis

$$\hat{\rho}|n_c\rangle = v_n^2|n_c\rangle \quad \textit{and} \quad \hat{\kappa}|n_c\rangle = u_n v_n|n_c\rangle.$$

The gap equation reads

$$\Delta_{n_c} = -\sum_{n_c'} V_{n_c n_c'} \frac{\Delta_{n_c'}}{2E_{n_c'}},$$

where

$$V_{n_c n_c'} = \langle n_c \bar{n}_c|v|n_c' \bar{n}_c'\rangle \quad \textit{and} \quad E_n = [(\epsilon_n - \mu)^2 + \Delta_n^2]^{1/2},$$

are the pairing matrix element and the diagonal elements of the HFB energy matrix.

In the weak cooupling regime $\Delta/\mu << 1$. In this case the canonical basis can be replaced by the HF one: $H|n\rangle = \epsilon_n|n\rangle$, which in terms od the associated density matrix read as:

$$(H - \epsilon_n)\hat{\rho}_n = 0.$$

Taking the Wigner transform of this latter equation, we obtain in the $\hbar \to 0$ limit: $(H_{cl.} - \epsilon)f_\epsilon(\mathbf{R}, \mathbf{p}) = 0$, which solution is

$$f_E(\mathbf{R}, \mathbf{p}) = \frac{1}{g^{TF}(E)}\delta(E - H_{cl.}) + O(\hbar^2).$$

with

$$H_{cl.} = \frac{p^2}{2m^*(\mathbf{R})} + V(\mathbf{R}) \quad \textit{and} \quad g^{TF}(E) = \frac{1}{(2\pi\hbar)^3}\int d\mathbf{R}d\mathbf{p}\delta(E - H_{cl.}).$$

X. Viñas, P. Schuck, M. Farine and M. Centelles, Phys. Rev. **C67**, 054307 (2003).

The gap equation in semiclassical TF approximation reads:

$$\Delta(E) = \int_0^\infty dE' g^{TF}(E') V(E, E') \kappa(E'),$$

with

$$\kappa(E) = \frac{\Delta(E)}{2\sqrt{(E-\mu)^2 + \Delta^2(E)}}.$$

and

$$V(E, E') = \int \frac{d\mathbf{R}d\mathbf{p}}{(2\pi\hbar)^3} \int \frac{d\mathbf{R}'d\mathbf{p}'}{(2\pi\hbar)^3} f_E(\mathbf{R}, \mathbf{p}) f_{E'}(\mathbf{R}', \mathbf{p}') v(\mathbf{R}, \mathbf{p}; \mathbf{R}', \mathbf{p}'),$$

with $v(\mathbf{R}, \mathbf{p}; \mathbf{R}', \mathbf{p}')$ the double Wigner transform of $< \mathbf{r_1 r_2} | v | \mathbf{r}'_1 \mathbf{r}'_2 >$ what for a local translationally invariant force yields $v(\mathbf{R}, \mathbf{p}; \mathbf{R}', \mathbf{p}') = \delta(\mathbf{R} - \mathbf{R}') v(\mathbf{p} - \mathbf{p}')$

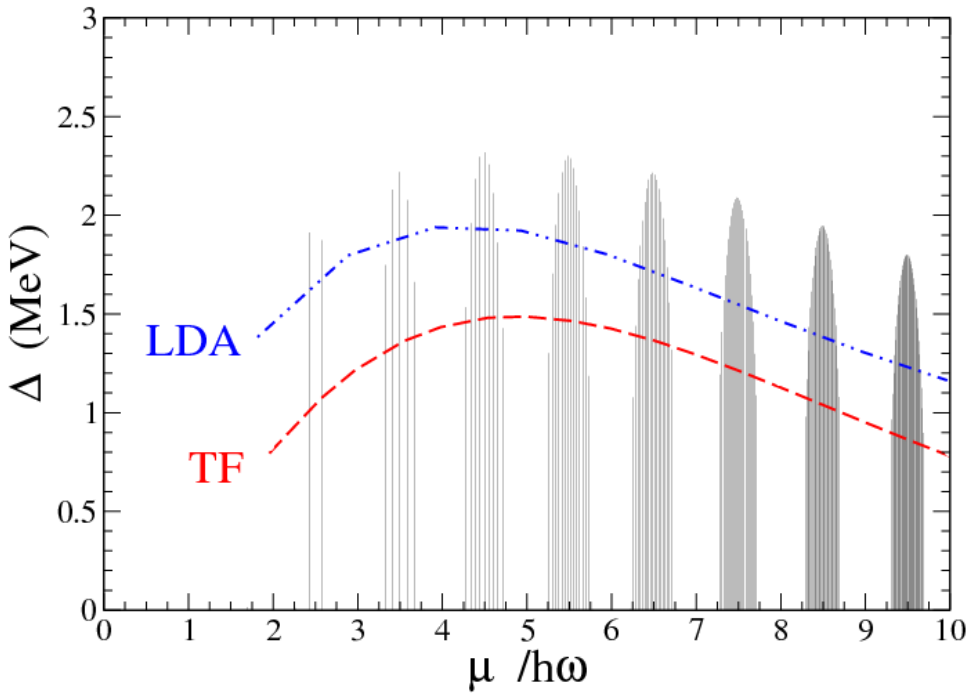
HARMONIC OSCILLATOR POTENTIAL

$\hbar\omega = 8.65$ MeV $m^*/m = 0.8$

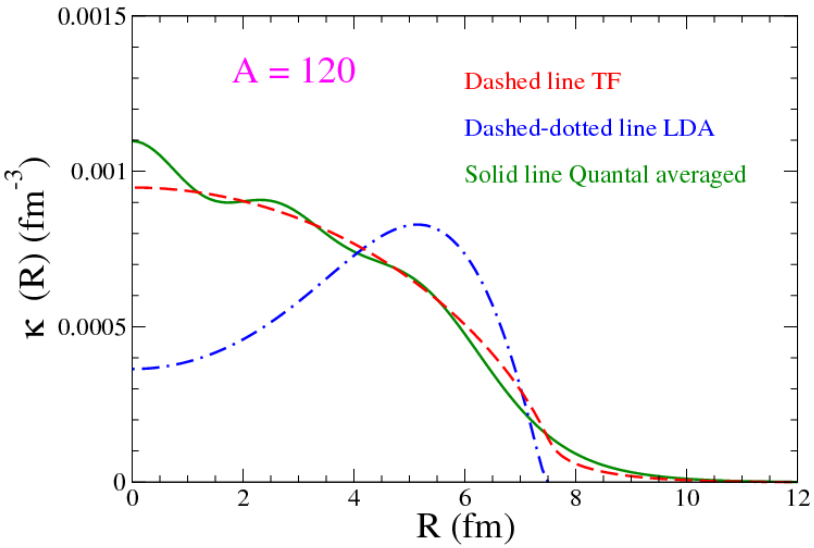
M. Prakash, S. Shlomo and V.A. Kolomietz, Nucl. Phys. **370**, 30 (1981).

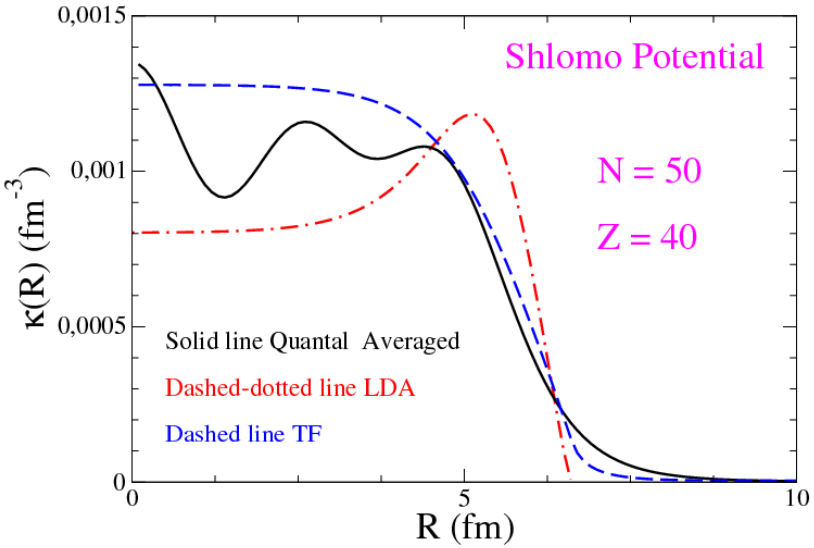
$$\kappa(\mathbf{R},\mathbf{p}) = \sum_N \kappa_N f_N(H_{cl.}) = \sum_N u_N v_N f_N(H_{cl.})$$

$$f_N(H_{cl.}) = [\sum_{nlm} \varphi_{nlm}(\mathbf{r})\varphi_{nlm}(\mathbf{r}')]_W = 8(-1)^N e^{-\frac{2H_{cl}}{\hbar\omega}} L_N^{(2)}(\tfrac{4H_{cl}}{\hbar\omega})$$



COMPARISON BETWEEN TF AND LDA

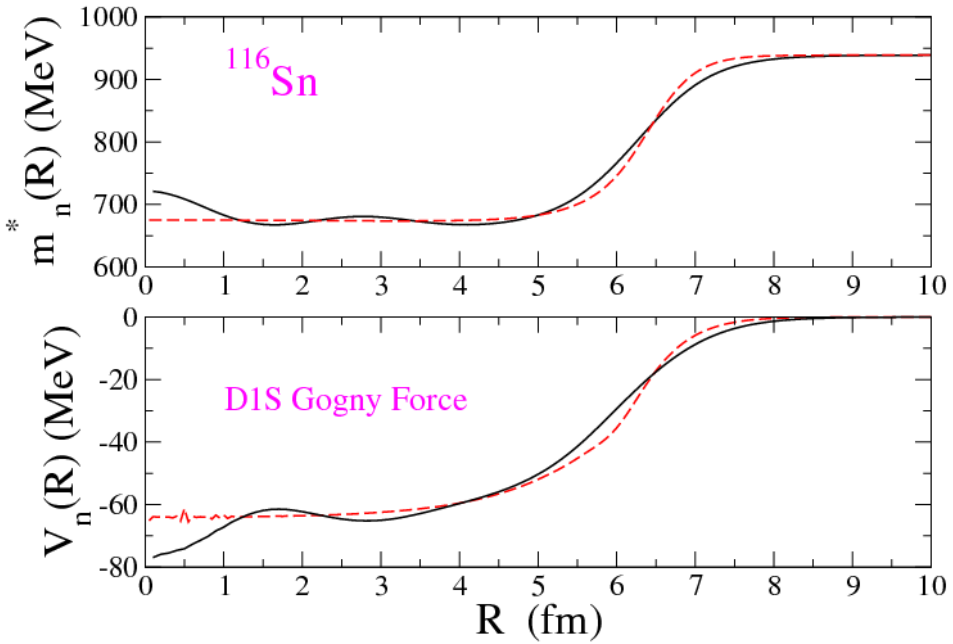




$$\Delta(\mathbf{R},\mathbf{p}) = \int d\mathbf{p}' v(\mathbf{p}-\mathbf{p}')$$

$$\kappa^{LDA}(\mathbf{R},\mathbf{p}) = \frac{\Delta(\mathbf{R},\mathbf{p})}{2\sqrt{((p^2 - p_F^2(\mathbf{R}))/2m^*)^2 + (\Delta(\mathbf{R},\mathbf{p}))^2}}$$

REALISTIC APPROACH TO AVERAGE PAIRING GAPS

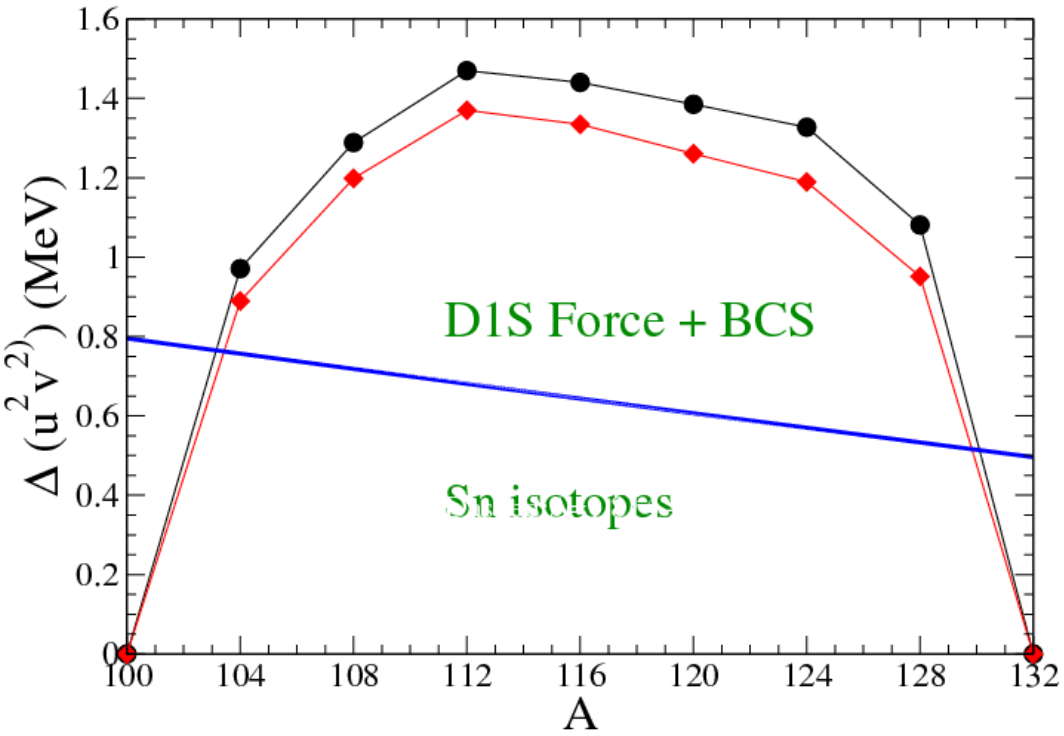


The main ingredient to solve the TF gap equation is

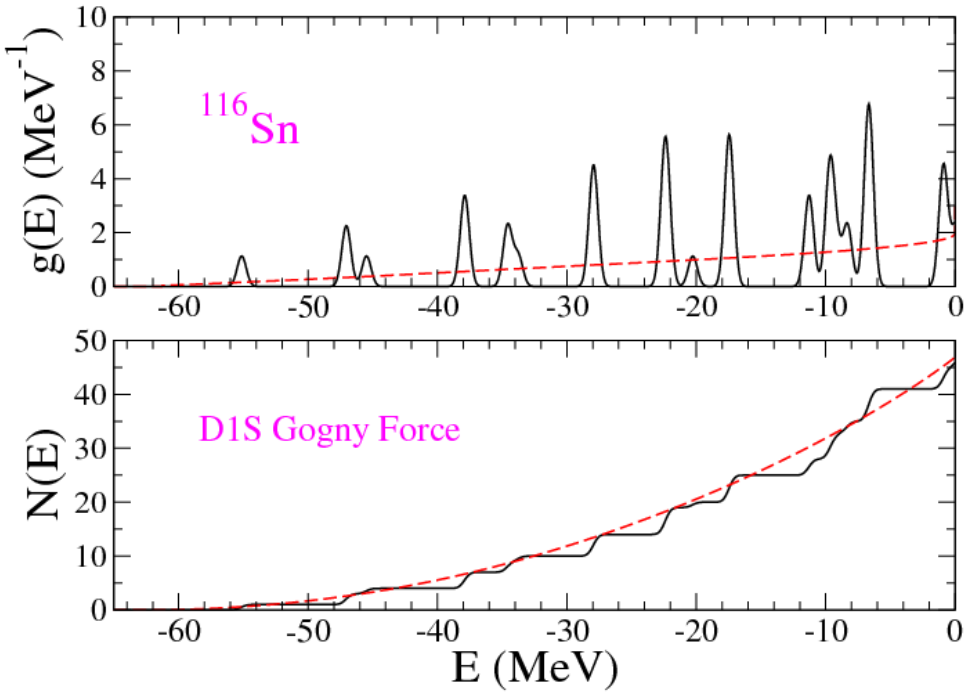
$$f_E(\mathbf{R},\mathbf{p}) \quad with \quad H_{cl} = \frac{p^2}{2m^*(\mathbf{R})} + V(\mathbf{R})$$

Nucl. Phys.**A665**, 291 (2000); Phys. Rev.**C67**, 014324 (2003);
Phys. Rev.**C74**, 064310 (2006)

AVERAGE PAIRING GAPS ALONG SN ISOTOPIC CHAIN

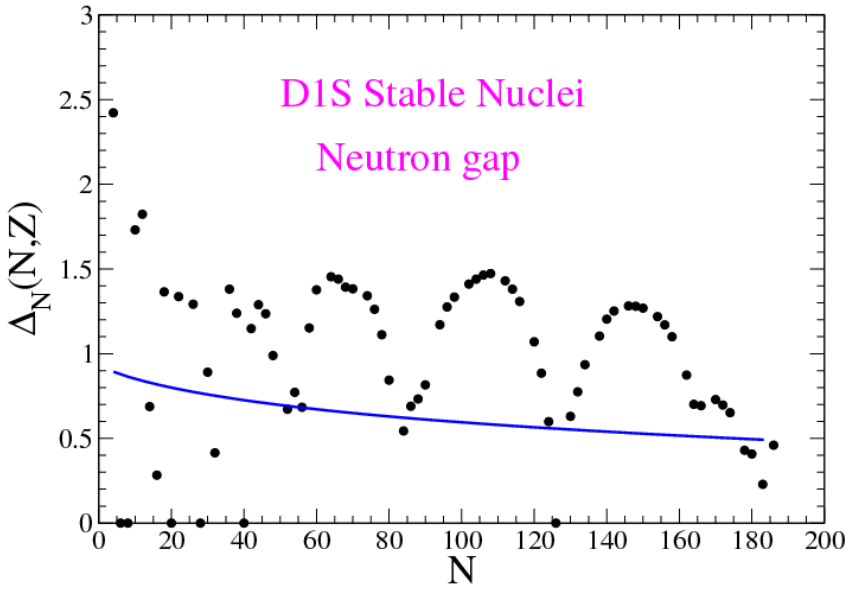


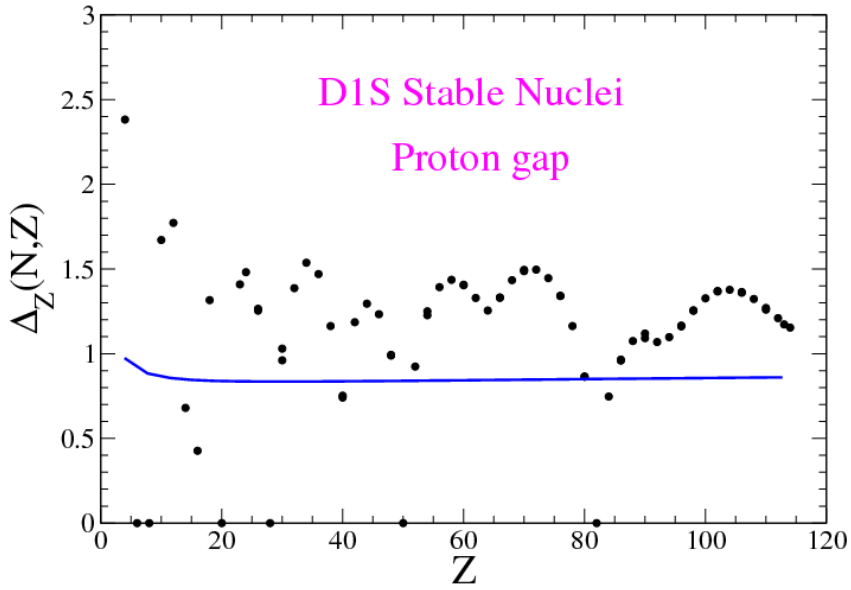
RECOVERING THE ARCH STRUCTURE



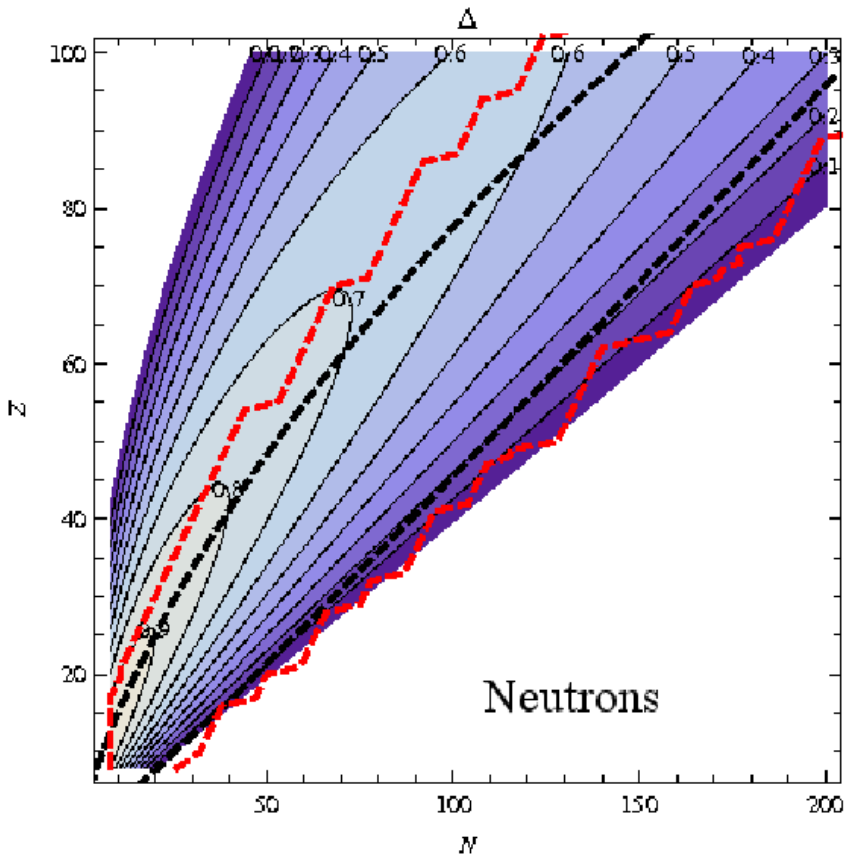
$$\tilde{g}(E) = \sum_{i=1}^{n_{tot}} g_{0,i} e^{-(\frac{E-\varepsilon_i}{\sigma})^2} \quad with \quad \sigma = 0.5$$

AVERAGE ALONG THE STABILITY LINE





TF NEUTRON GAP. GLOBAL FIT

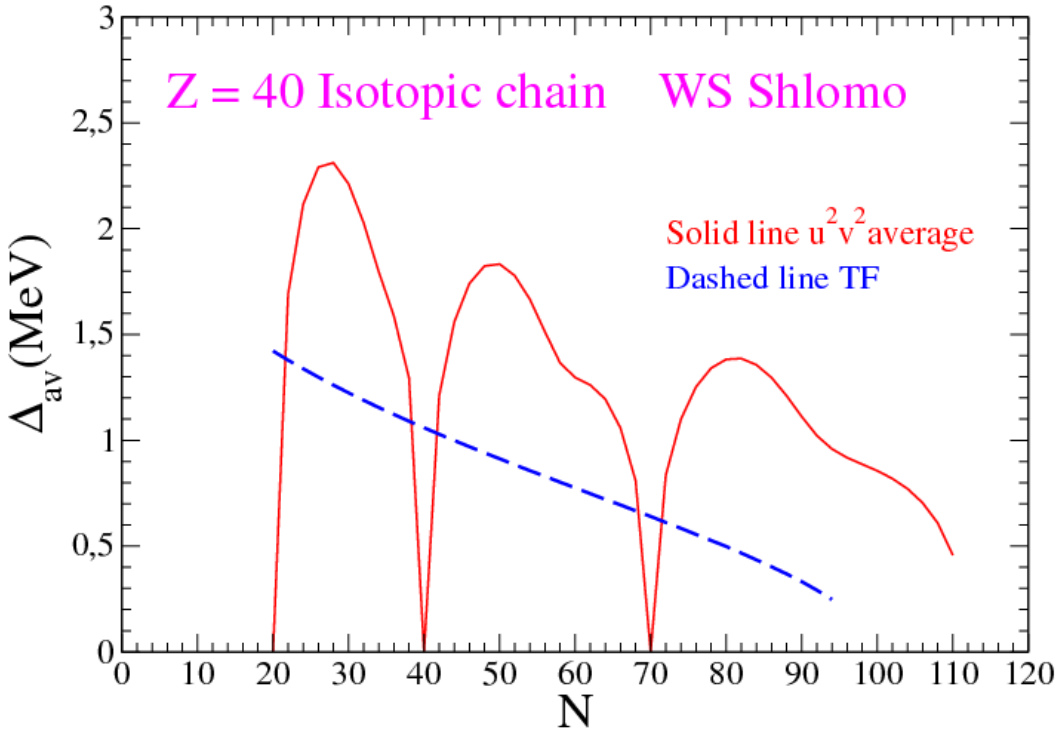


$$\Delta_N(N,Z) = a_0(A) + a_1(A)\frac{N-Z}{A} + a_2(A)\left(\frac{N-Z}{A}\right)^2.$$

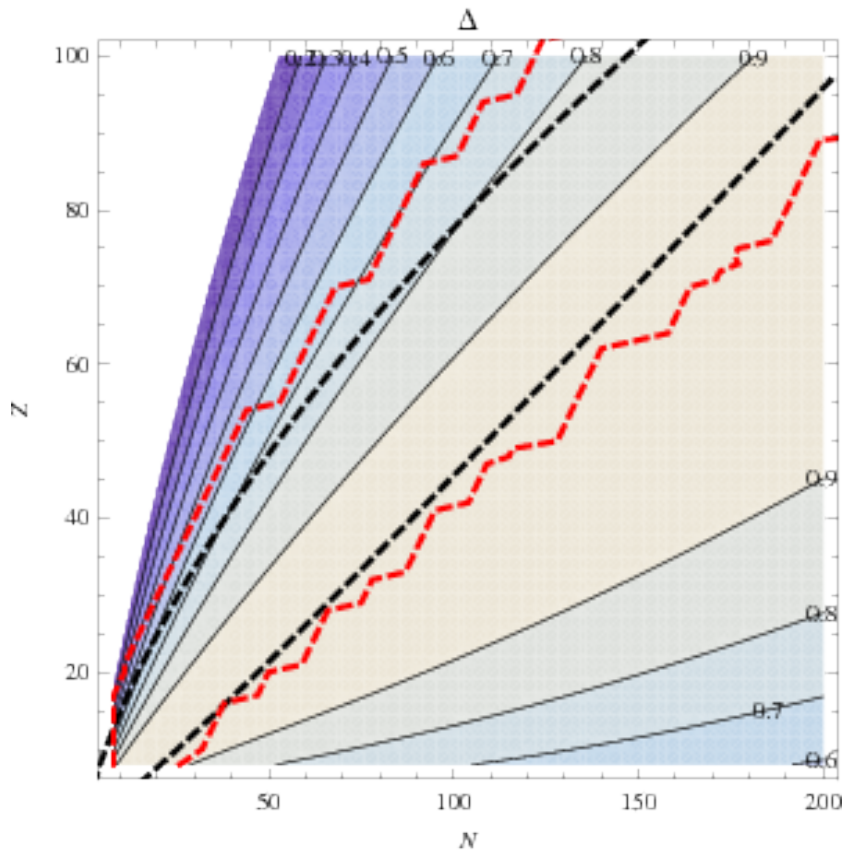
$$a_0(A) = 0.92419 - 0.0016122A,\ a_1(A) = -1.1858 + 0.0099181A,$$
$$a_2(A) = -2.0835 - 0.015077A.$$

Zr ISOTOPIC CHAIN. SHLOMO WS POTENTIAL

S. Shlomo, Nucl. Phys. **539**, 30 (1992).



TF PROTON GAP. GLOBAL FIT



$$\Delta_Z(N,Z) = b_0(A) + b_1(A)\frac{N-Z}{A} + b_2(A)\left(\frac{N-Z}{A}\right)^2.$$

$$b_0(A) = (1.60515 + 0.758392A - 0.000691949A^2)/A,$$
$$b_1(A) = (3.17766 + 0.938999A + 0.00256937A^2)/A,$$
$$b_2(A) = -(19.3033 + 0.96452A + 0.0028438A^2)/A.$$

CONCLUSIONS

- We have presented here a Thomas-Fermi theory for pairing in finite Fermi systems for weak coupling situations with $\Delta/\varepsilon_F << 1$.
- This Thomas-Fermi theory differs from the usual Local Density Approximation. This essentially stems from the fact that we approximate the gap equation in configuration space and, thus, keep the size dependence of the matrix elements of the pairing force. This is not the case in LDA where the matrix elements of the force are always evaluated in plane wave basis.
- This semiclassical approach to pairing is only based on the usual validity criterion of Thomas-Fermi theory, namely that the Fermi wave length is smaller than the oscillator length. At no point the Local Density Approxiamtion condition that the extension of the Cooper pairs (coherence length) must be smaller than the oscillator length enters the theory. Thus, the present Thomas-Fermi approach yields for all pairing quantities the same quality as Thomas-Fermi theory does for quantities in the normal fluid state.

- The gap values obtained represent very well the mass number dependence in N and Z of the average gap for the D1S Gogny force employed in this work. Essentially the obtained gap values correspond to nuclei where the discrete quantal single particle level density has been replaced by a Thomas-Fermi smoothed continuous level density.
- We presented the full chart of the N, Z dependence of the average gap at the Fermi surface using, as mentioned, the D1S force for the pairing field and also for the mean field and effective mass, as obtained from the D1S force using extended TF theory.
- An interesting feature of our study is that the average gap breaks down going to the drip line. This surprising result is confirmed by quantal calculations, though strongly masked by shell fluctuations.