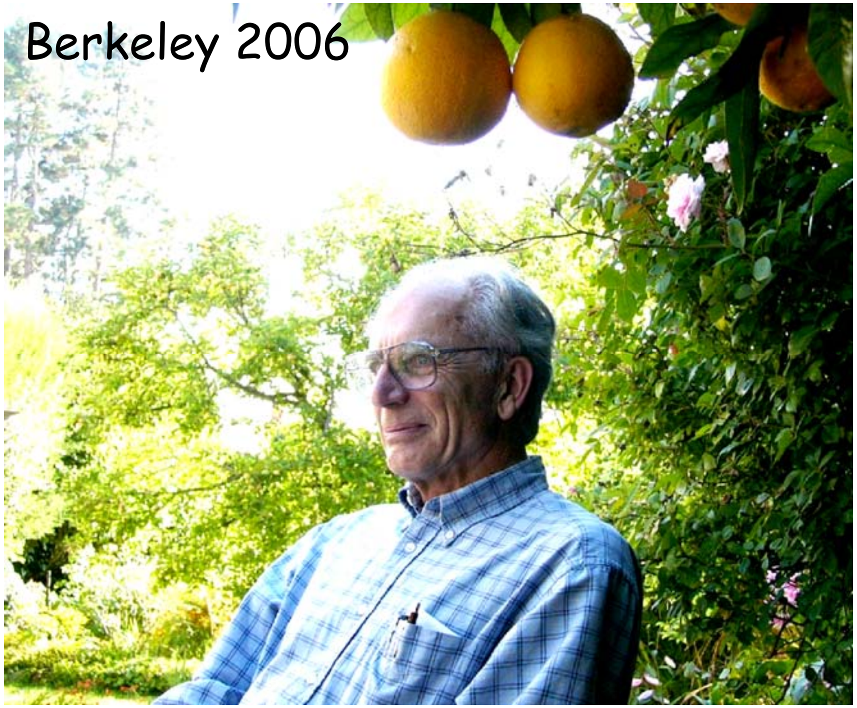
The „Fusion by Diffusion" model revisited



Dedicated to the memory of Władek our Mentor and Friend

Kazimierz, September 2010

2003 - formulation of the „Fusion by Diffusion model"
- FBD

W.J. Świątecki, K. Siwek-Wilczyńska, J. Wilczyński
Acta Physica Polonica 34, 2049 (2003) (53 citations)

2005 – version II of the FBD

W.J. Świątecki, K. Siwek-Wilczyńska, J. Wilczyński
Physical Review C 71, 014602 (2005) (61 citations)

……

last joint paper - January 2010

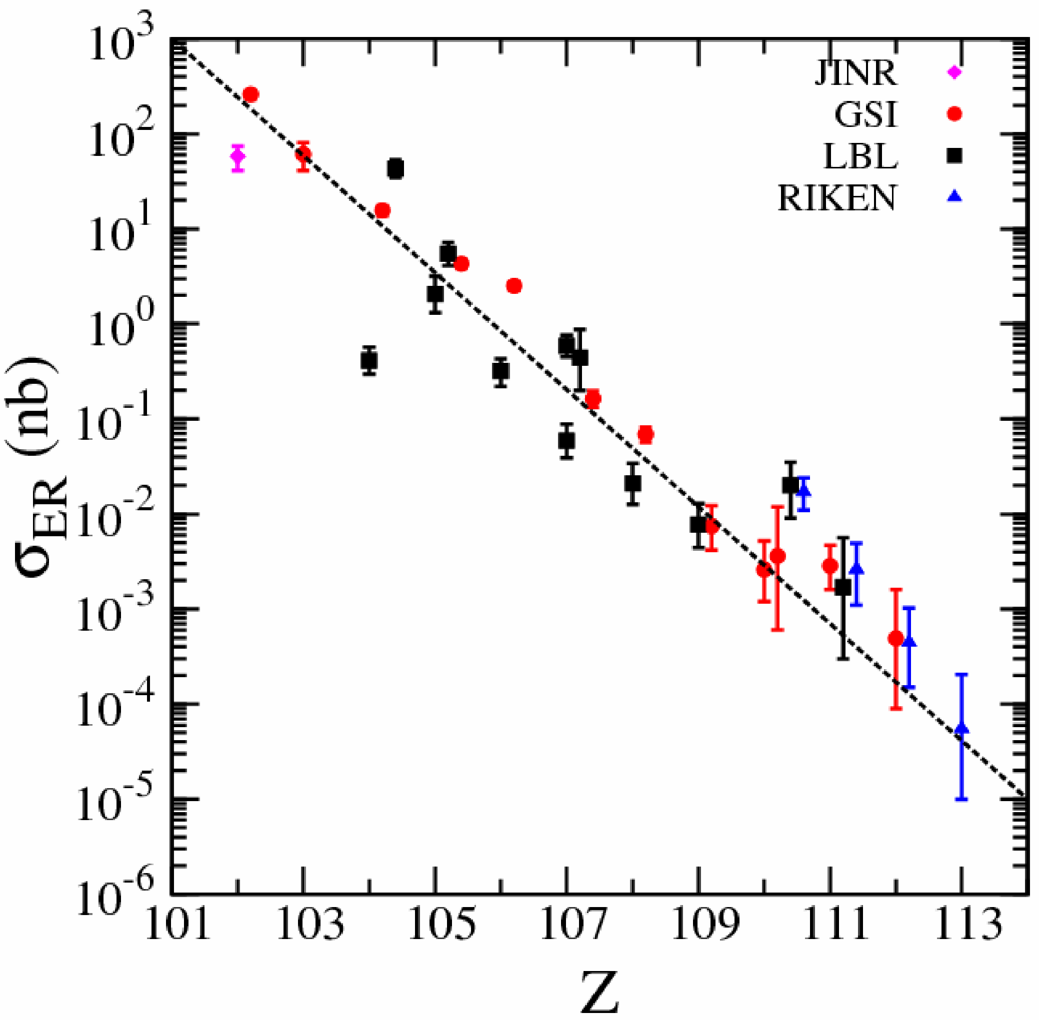
W.J. Świątecki, K. Siwek-Wilczyńska, J. Wilczynski
Physical Review C81, 019804 (2010)

Władek's legacy:

- to improve the model,
- to involve young people into this project

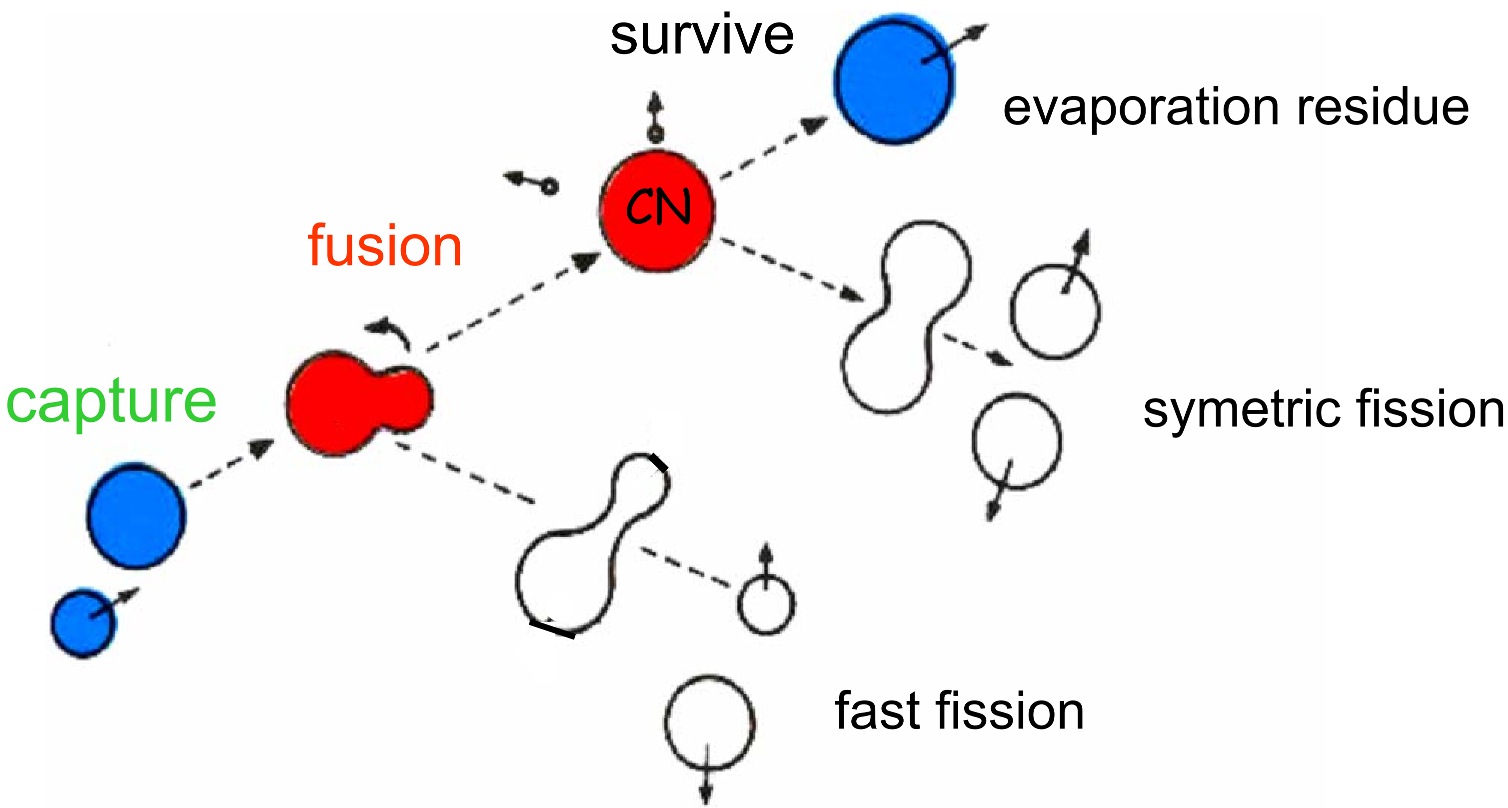
Tomek Cap, J. Wilczyński & KSW

Cold fusion reactions



- $102 < Z < 113$
- target: ^{208}Pb or ^{209}Bi
- projectile: $^{48}Ca \ldots ^{70}Zn$
- $E^* = 10 - 20$ MeV
- $10^2 < \sigma_{ER} < 10^{-5}$ nb

A collision of two nuclei leading to the formation of a super-heavy nucleus



$$\sigma(\text{synthesis}) = \sigma(\text{capture}) \times P(\text{fusion}) \times P(\text{survive})$$

W. J. Świątecki, K. Siwek-Wilczyńska, and J. Wilczyński, *Acta Phys. Pol.* 34, 2049 (2003).
W. J. Świątecki, K. Siwek-Wilczyńska, and J. Wilczyński, *Phys. Rev.* C 71, 014602 (2005).

Original version of FBD - for central collisions only

Extended FBD – L-dependence included

$$\sigma(\text{synthesis}) = \pi \lambda\!\!\!^{-2} \sum_{l=0}^{l\max} (2l+1) P_l(\text{fusion}) P_l(\text{survive})$$

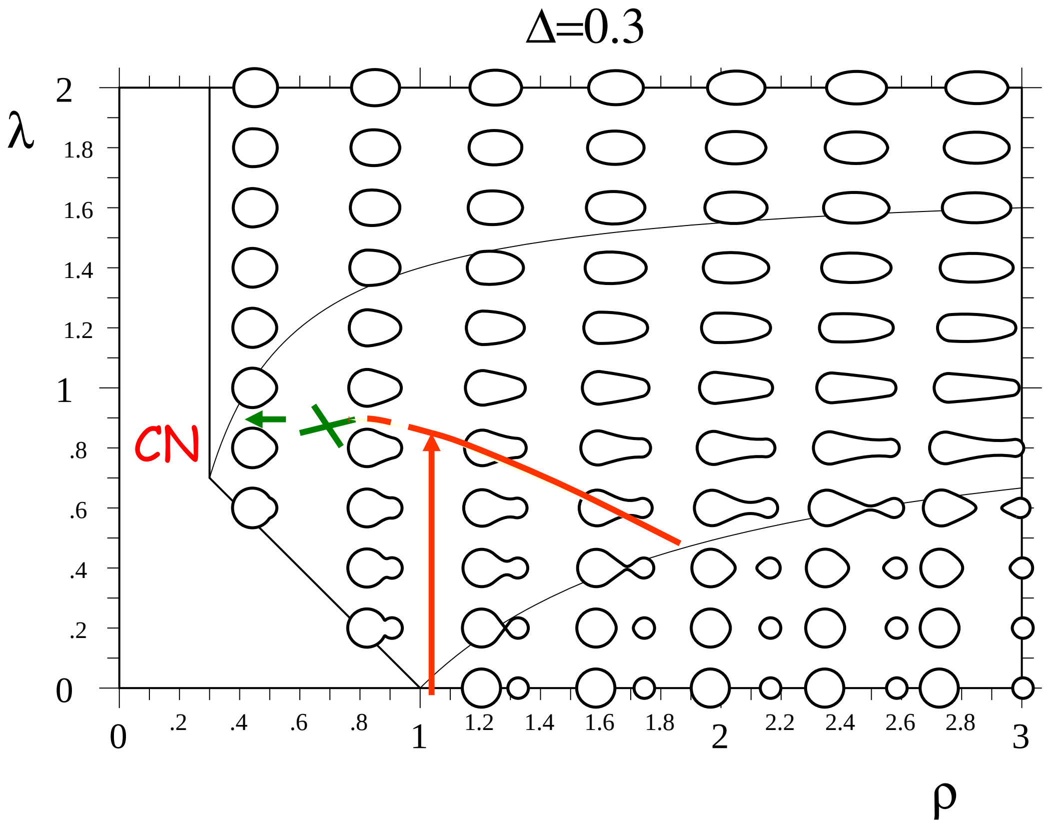
$l_{\max}$ – calculated from the capture cross section

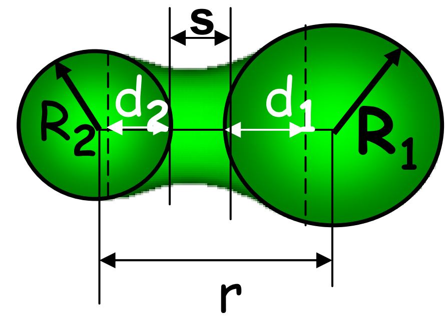
$$\sigma_{cap} = \pi \lambda\!\!\!^{-2} (l_{\max} + 1)^2$$

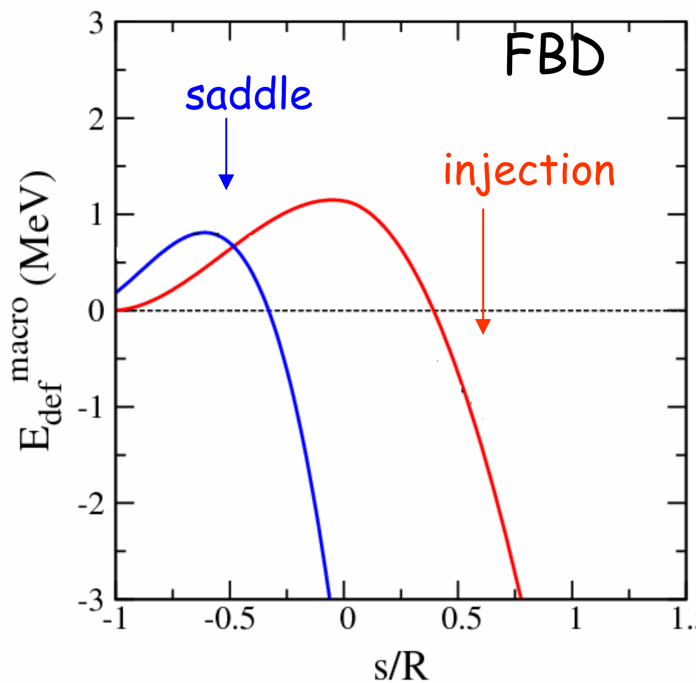
σ(capture) from the „diffused barrier formula"
(K. Siwek-Wilczyńska, J. Wilczyński Phys. Rev. C 69 (2004) 024611)
(fusion cross section parametrized assuming the Gaussian shape of the barrier distribution).

P_I(fusion)

J. Błocki, W. J. Świątecki, Nuclear Deformation Energies, Report LBL 12811 (1982)







$\lambda = (d_1+d_2)/(R_1 + R_2)$ - neck variable

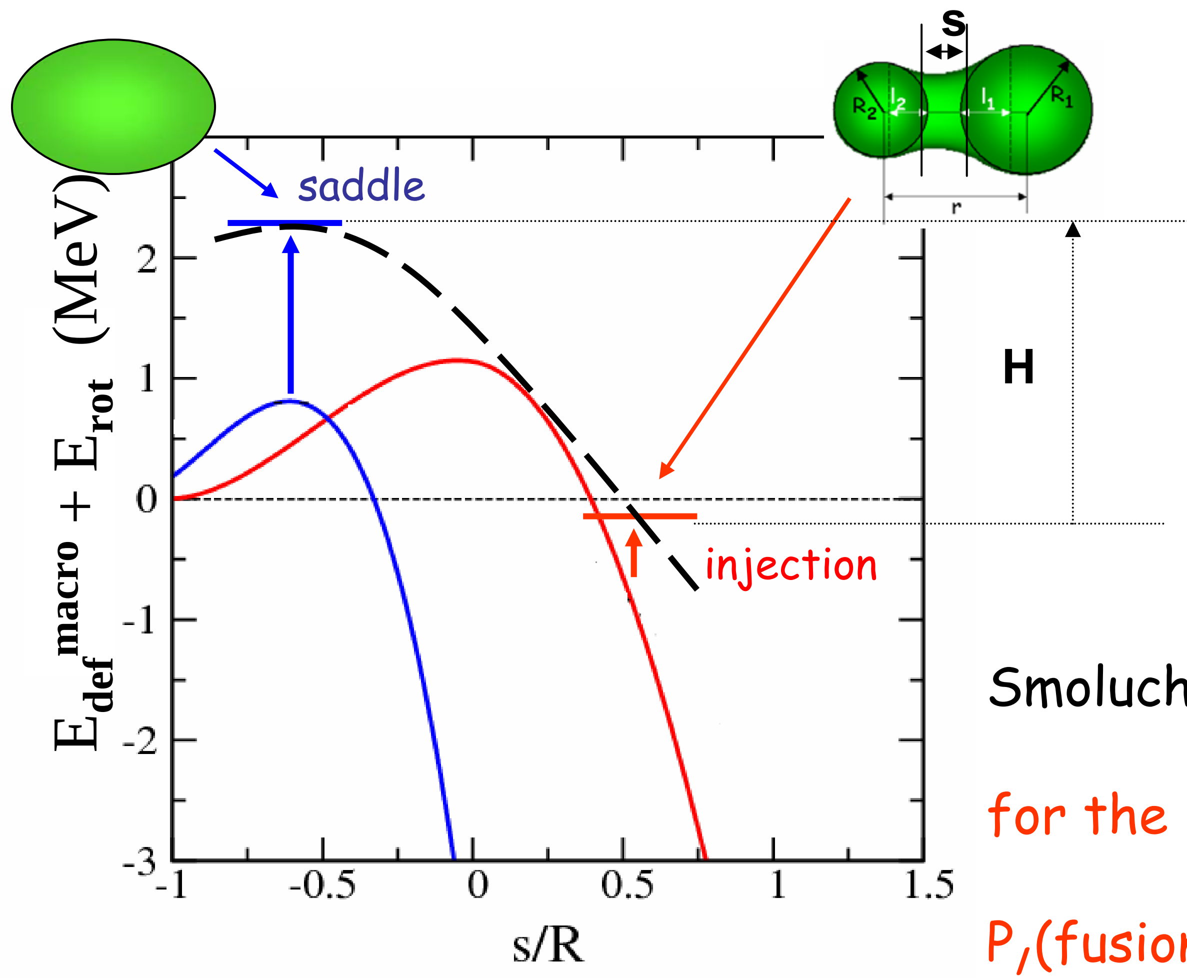
$\rho = r/(R_1 + R_2)$ – relative distance between centers of mass

$\Delta = (R_1 - R_2)/(R_1 + R_2)$ – assymmetry

Deformation energy for symmetric and asymmetric fission

P_l (fusion)

EFBD



Smoluchowski Diffusion equation

for the parabalic potenctial

$$P_l(\text{fusion}) = \tfrac{1}{2}\left(1 - \text{erf}\sqrt{H(l)/T}\right)$$

P_l (fusion) << 1   for heavy systems

$$P_l\text{ (survival)} = \frac{\Gamma_n}{\Gamma_n + \Gamma_f} \times P_<$$

Probability that after 1 n emission the excitation energy is less than the threshold for second chance fission or 2n emission.

Partial widths for emission of light particles – Weisskopf formula

$$\Gamma_i = \frac{m_i}{\pi^2 \hbar^2}(2s_i + 1)\int_0^{E_i^{\max}} \varepsilon_i \sigma_i \frac{\rho_i\left(E_i^{\max} - \varepsilon_i\right)}{\rho\left(E^*\right)} d\varepsilon_i$$

where: $E_i^{\max} = E^* - E_{rot}^i - B_i - V_i^C - P$ Upper limit of the final-state excitation energy after emission of a particle *i*

σ_i – cross section for the production of the compound nucleus in the inverse process

m_i, s_i, ε_i – mass, spin and kinetic energy of the emitted particle

ρ, ρ_i — level densities of the parent and daughter nuclei

The fission width (transition state method), E*< 40 MeV

$$\Gamma_{fiss} = \frac{1}{2\pi}\int_0^{E_f^{\max}} \frac{\rho_{fiss}\left(E_f^{\max} - K\right)}{\rho\left(E^*\right)} dK$$

$E_f^{\max} = E^*(saddle) - E_{rot}(saddle) - P$ Upper limit of the thermal excitation energy at the saddle

The level density is calculated using the Fermi-gas-model formula

$$\rho(E) \propto \exp\left(2\sqrt{aE}\right)$$

- Shell effects

included as proposed by Ignatyuk
(A.V. Ignatyuk et al., Sov. J. Nucl. Phys. 29 (1975) 255)

$$a = a_{macro}\left[1 + \frac{\delta_{shell}}{U}\left(1 - e^{-U/E_d}\right)\right]$$

where: U - excitation energy, Ed - damping parameter

δ_{shell} – shell correction energy, δ_{shell} (g.s.) (Möller et al., At. Data Nucl. Data Tables 59 (1995) 185),
δ_{shell} (saddle) ≈ 0

$$a_{macro} = 0.04543\, r_0^3 A + 0.1355\, r_0^2 A^{2/3} B_s^j + 0.1426\, r_0 A^{1/3} B_k^j$$

$$r_0 = 1.153\, fm$$

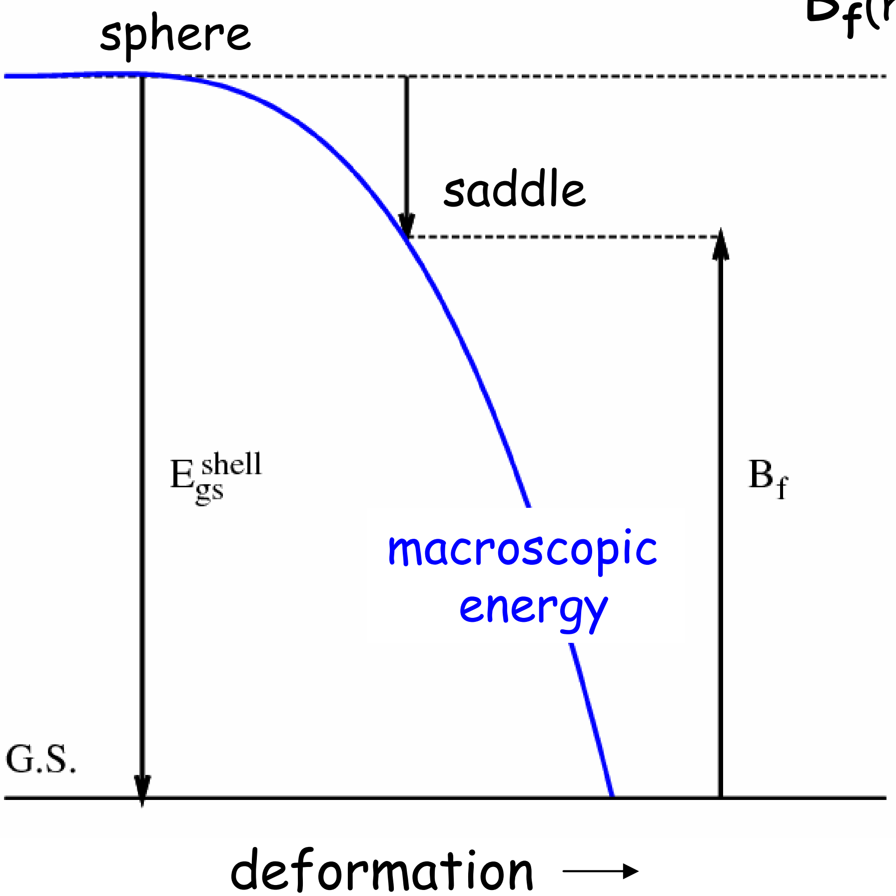
$E_d = 18.5\, MeV$ (W. Reisdorf, Z. Phys. A. – Atoms and Nuclei 300 (1981) 227)

B_s, B_k (W.D. Myers and W.J. Świątecki, Ann. Phys. 84 (1974) 186)

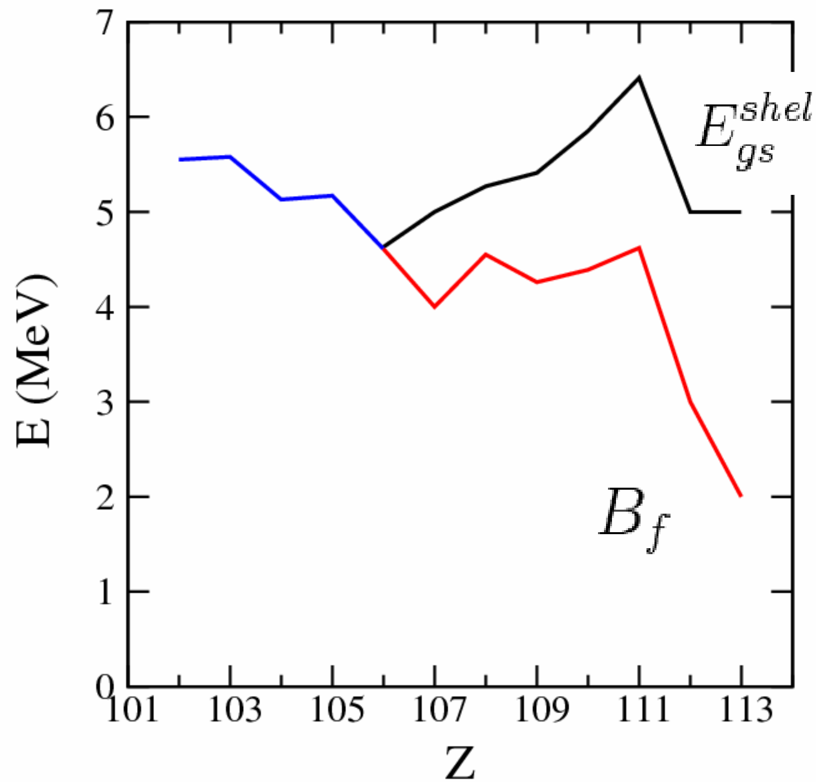
Fission barrier

$B_f = B_f(\text{macro}) - E_{gs}^{shell}$

$B_f(\text{macro}) = E_{saddle}(\text{macro}) - E_{sphere}(\text{macro})$



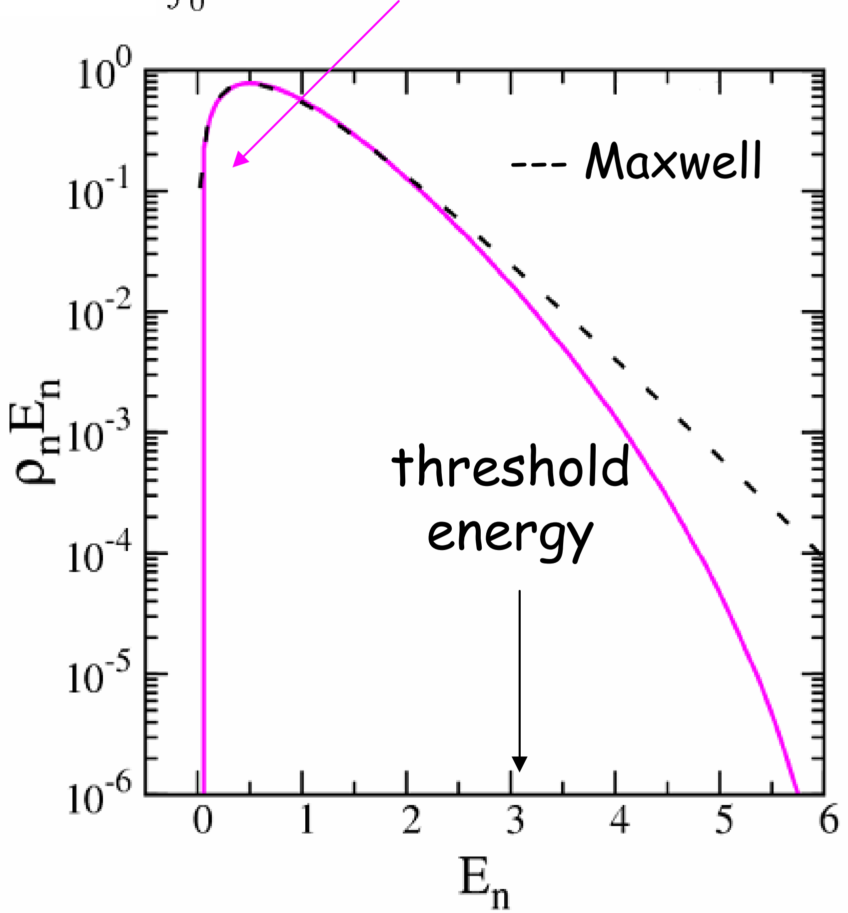
For nuclei with Z > 106 there is no macroscopic barrier

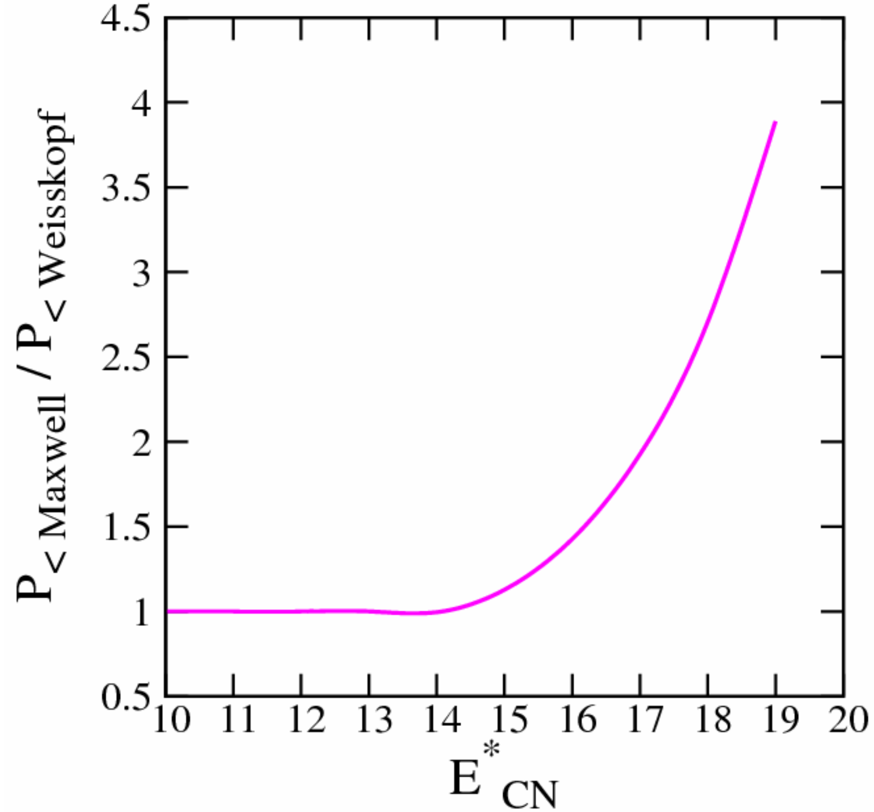


the macroscopic energy in saddle configuration → deformation at the saddle point (A. Sobiczewski et al. private communication)

$P_<$ Shape of the kinetic energy spectrum

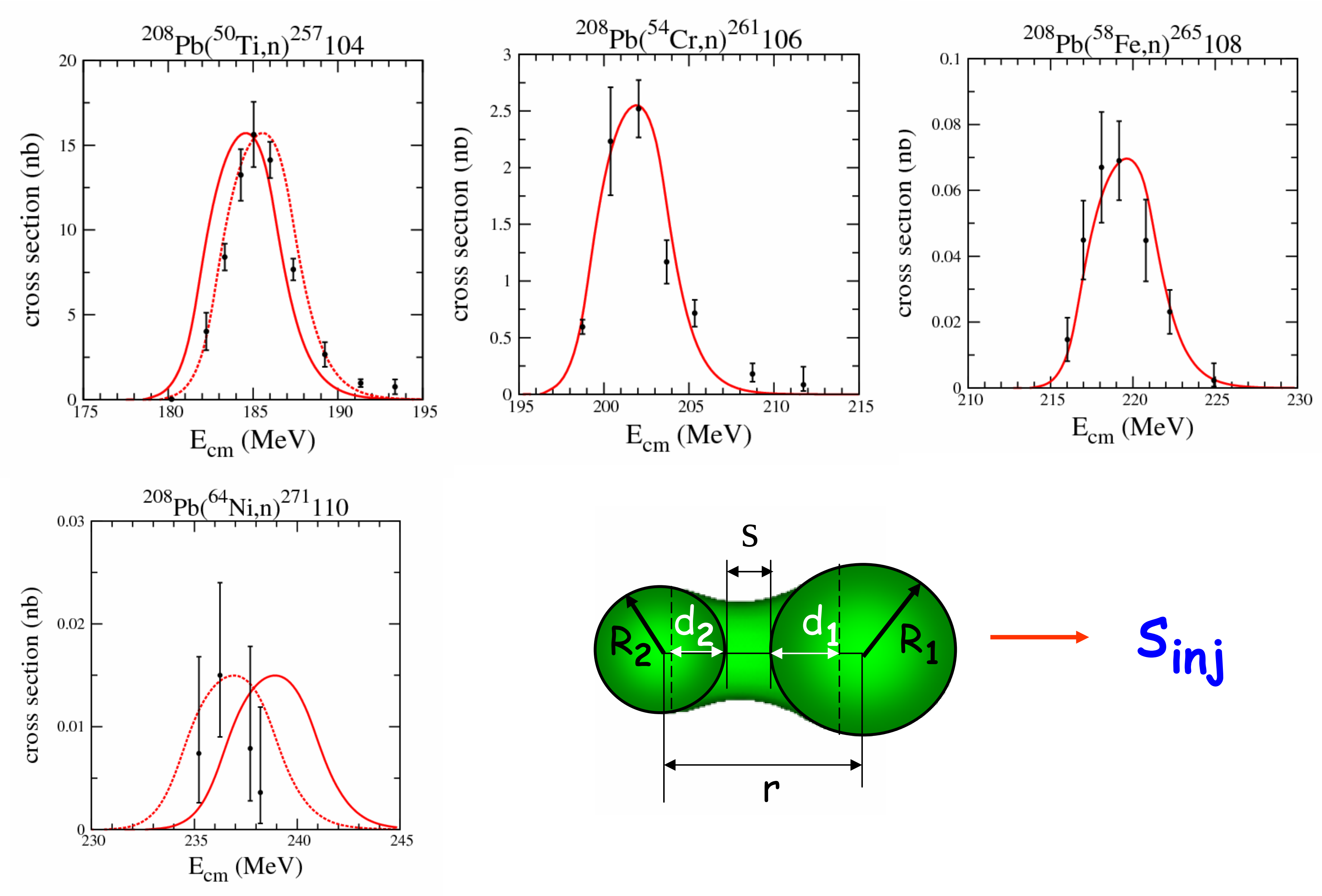
$$\int_0^{E_n^{max}} \rho_n(A-1,\epsilon^*)\,(E_n^{max}-\epsilon^*)\,d\epsilon^*$$



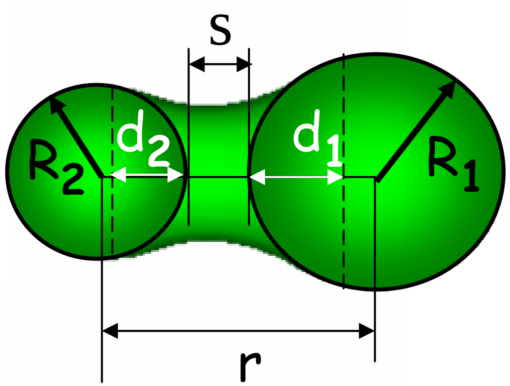


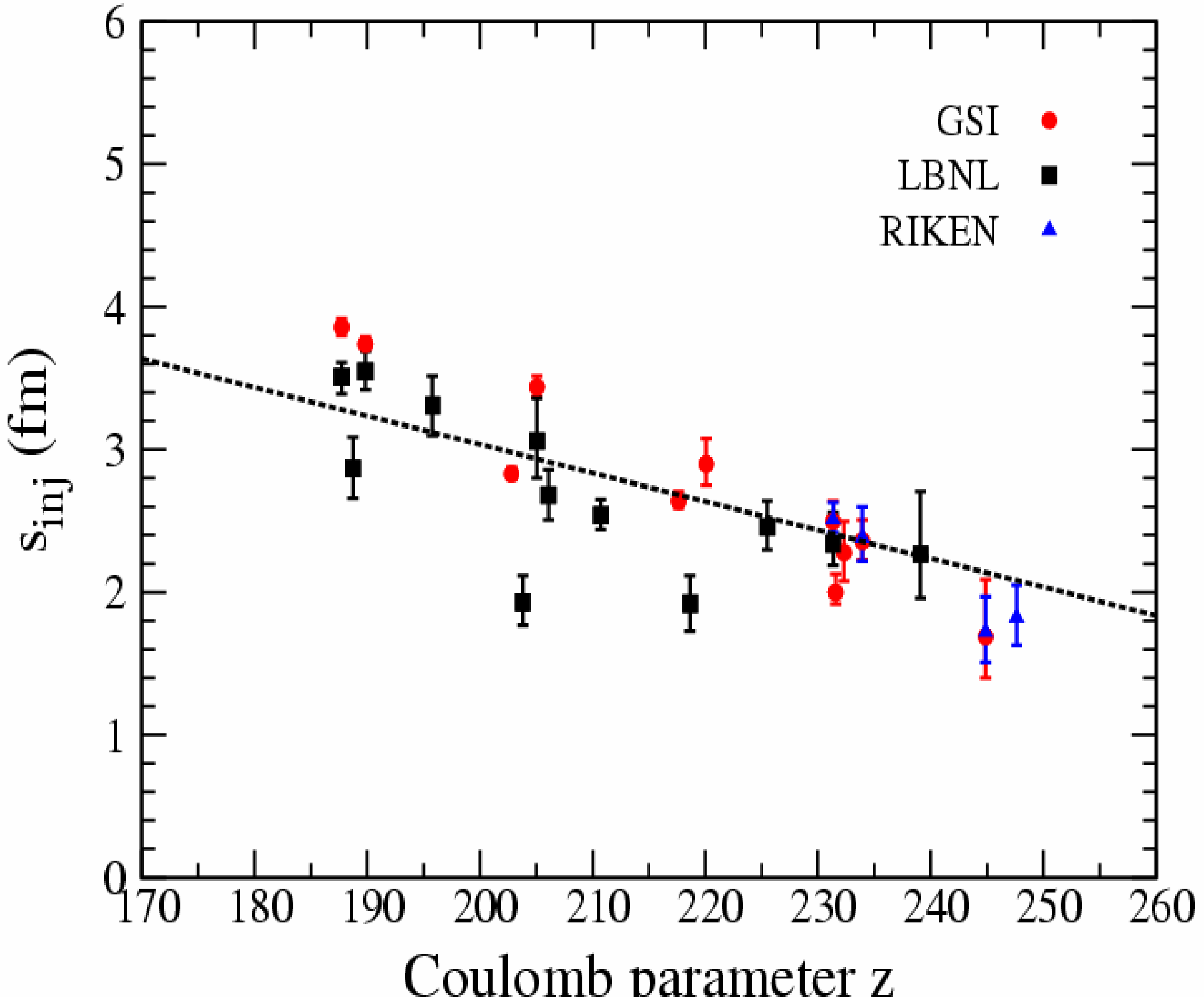
Probability $P_<$

Fit to 27 excitation functions



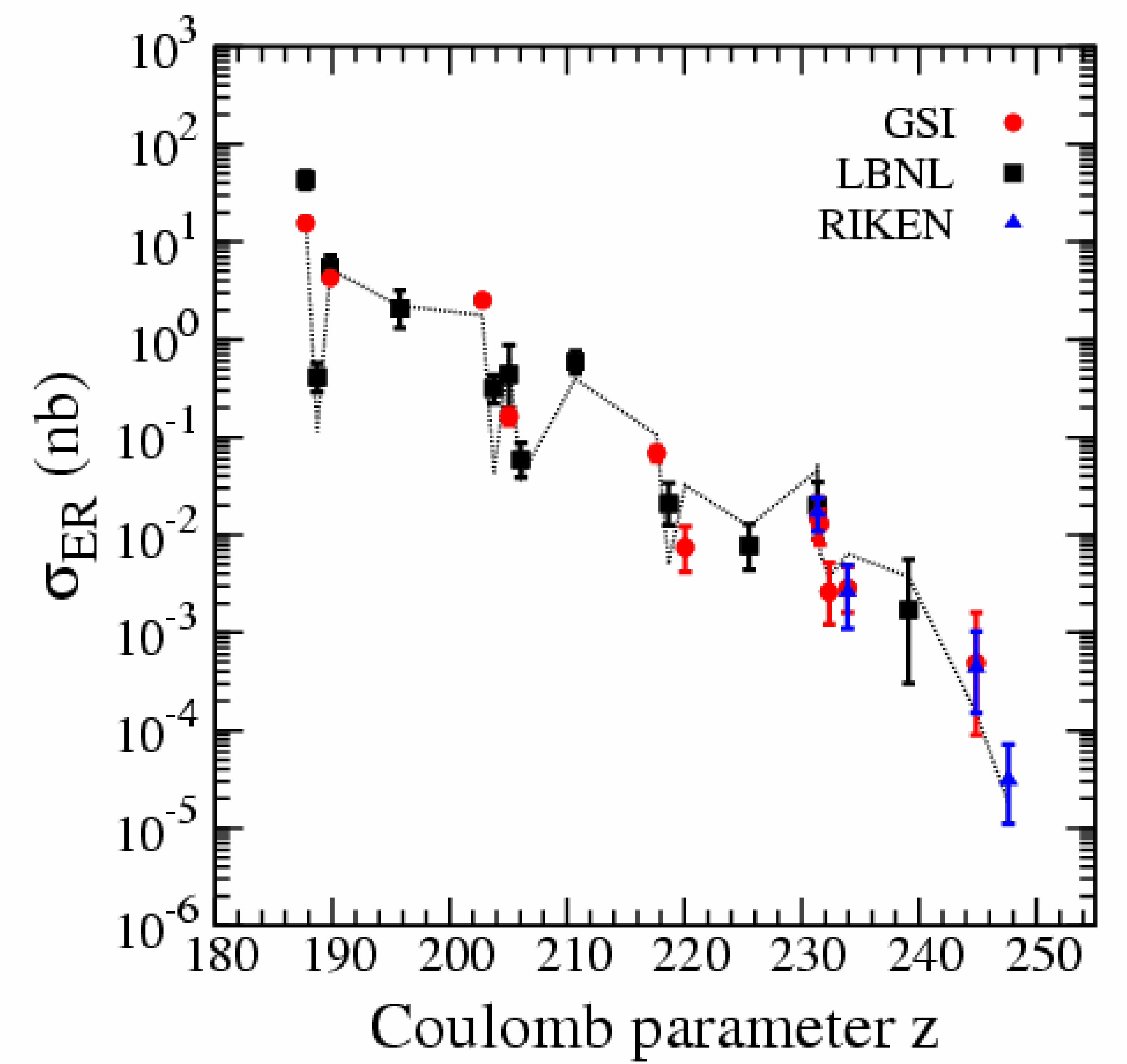
Parametrization of s_{inj}



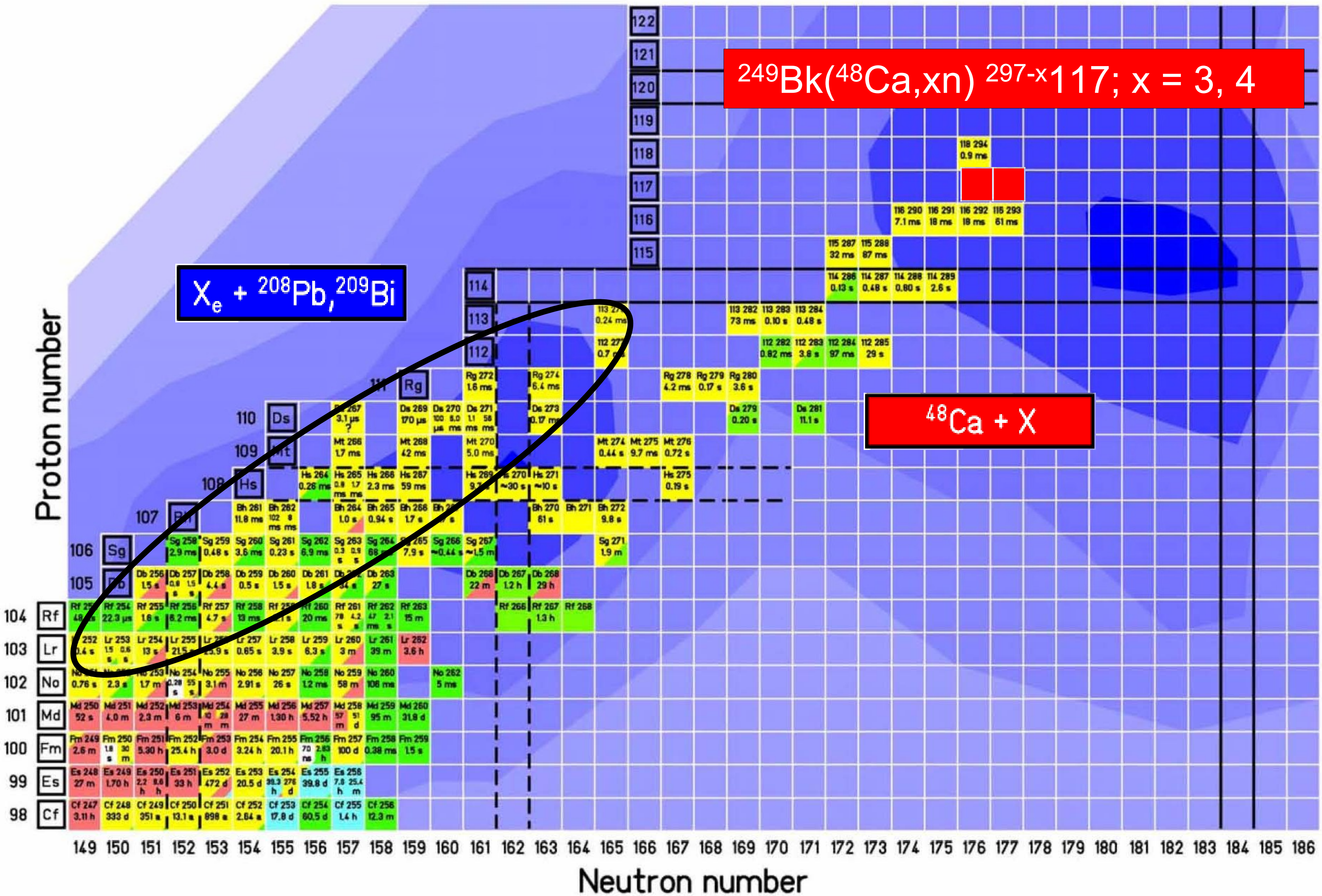


$$z = \frac{Z_p Z_T}{A_P^{1/3} + A_T^{1/3}}$$

results



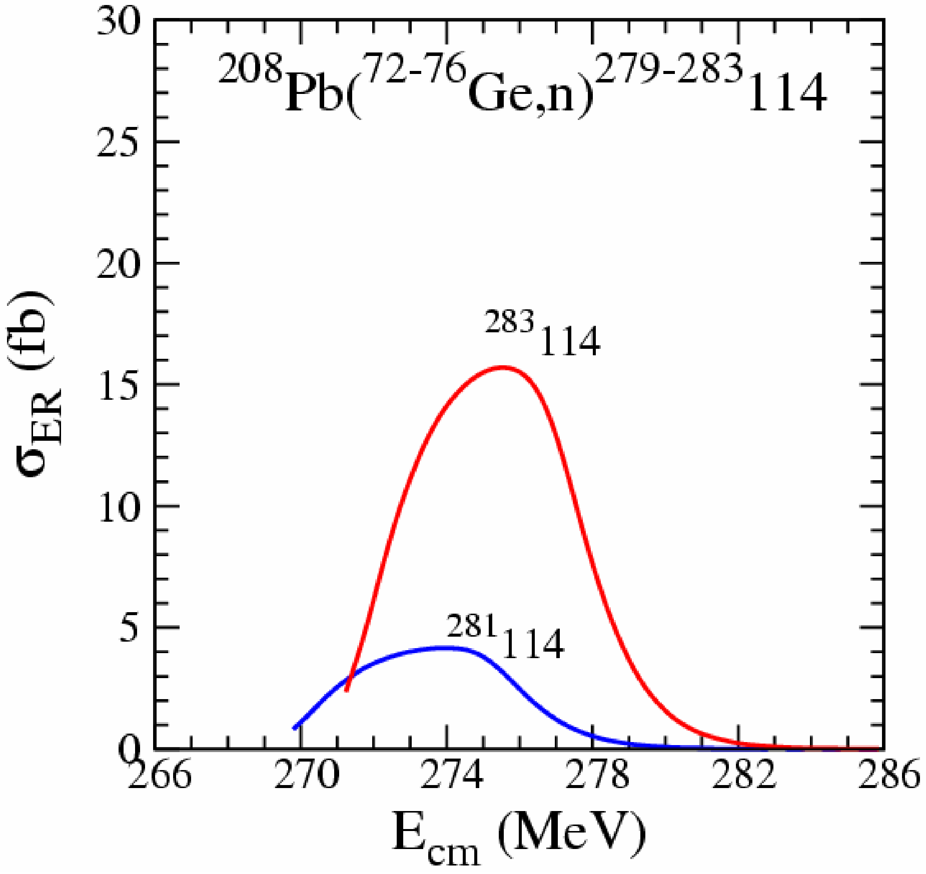
GSI
LBNL
RIKEN
σ_{ER} (nb)
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
180 190 200 210 220 230 240 250
Coulomb parameter z



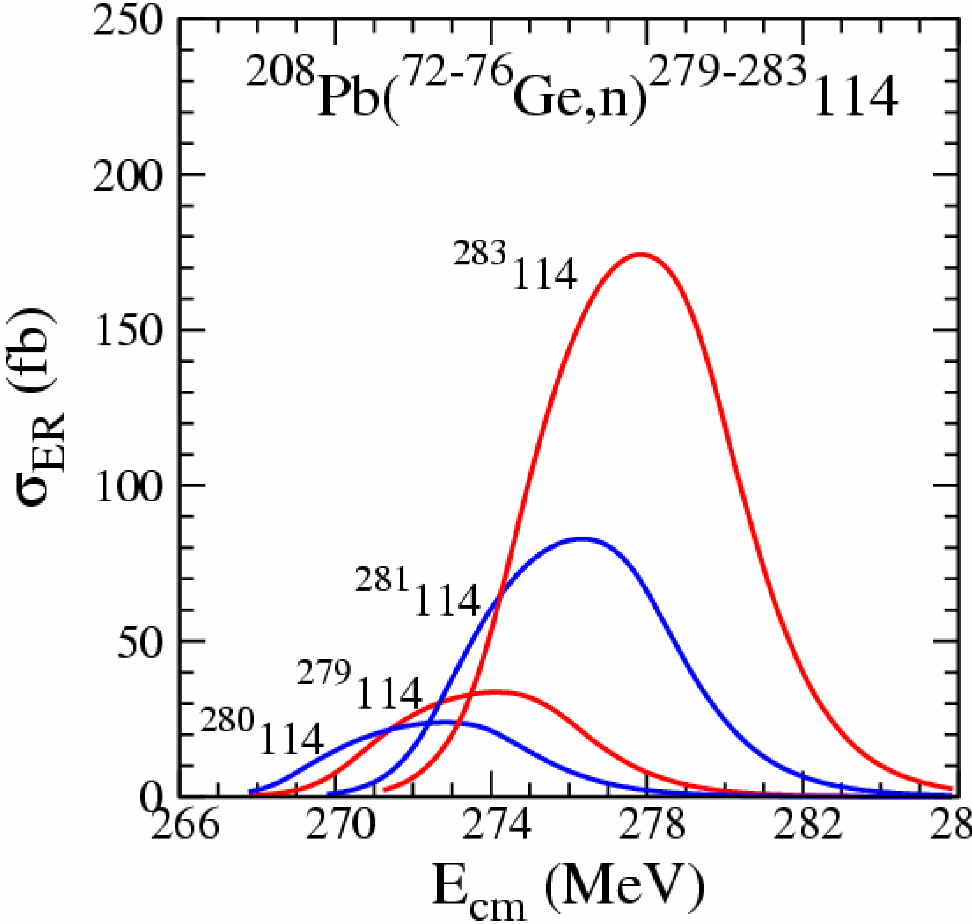
$^{249}Bk(^{48}Ca,xn)\ ^{297-x}117$; x = 3, 4
$X_e + {}^{208}Pb, {}^{209}Bi$
$^{48}Ca + X$
Proton number
Neutron number
122
121
120
119
118
117
116
115
114
113
112
111 Rg
110 Ds
109 Mt
108 Hs
107 Bh
106 Sg
105 Db
104 Rf
103 Lr
102 No
101 Md
100 Fm
99 Es
98 Cf
149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186

Predictions for Z = 114

Deformed nuclei

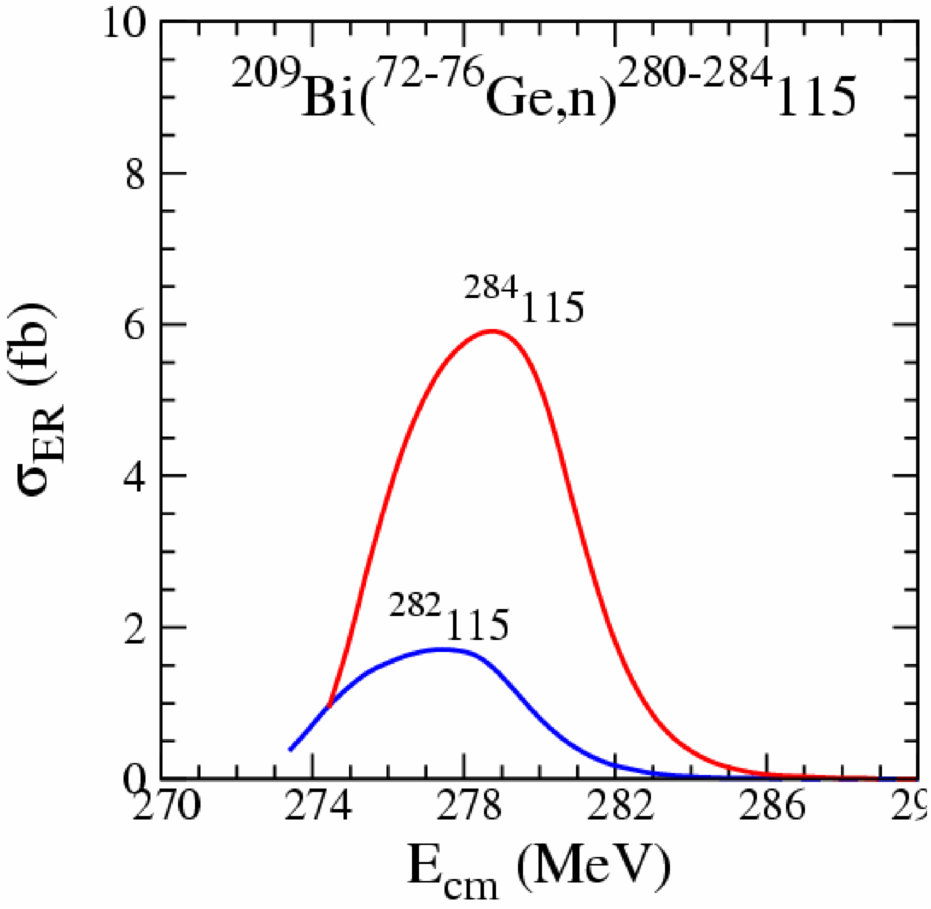


Spherical nuclei

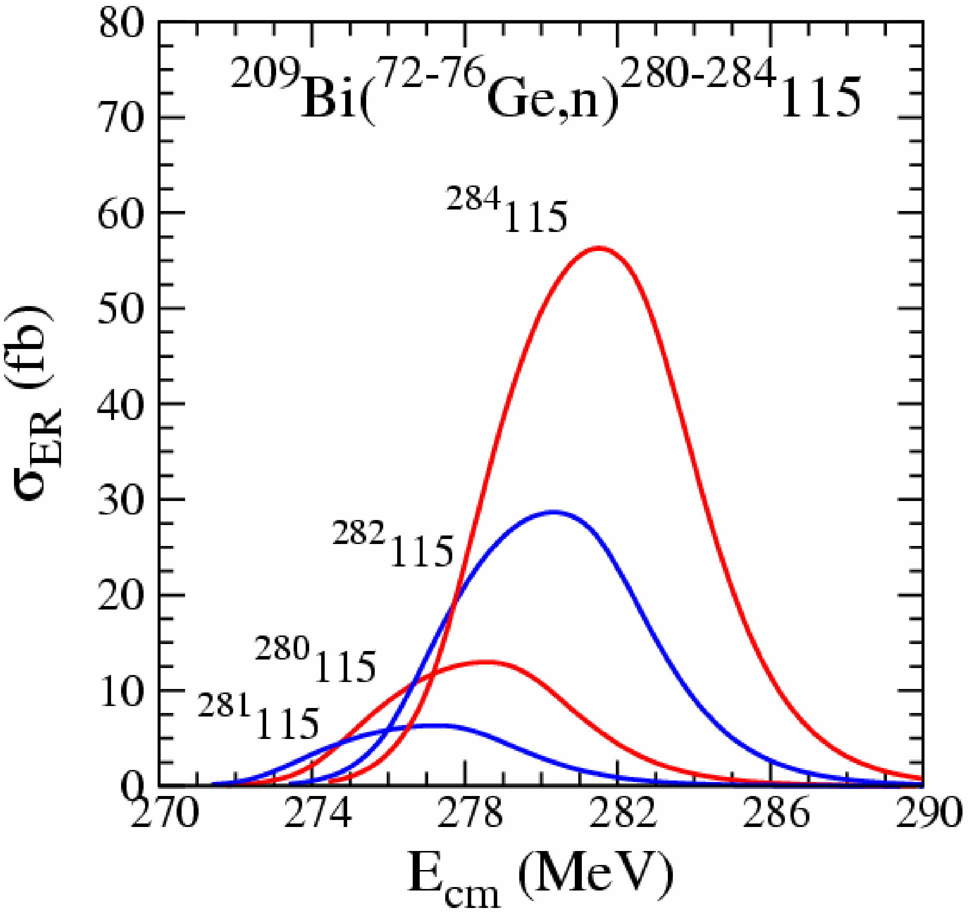


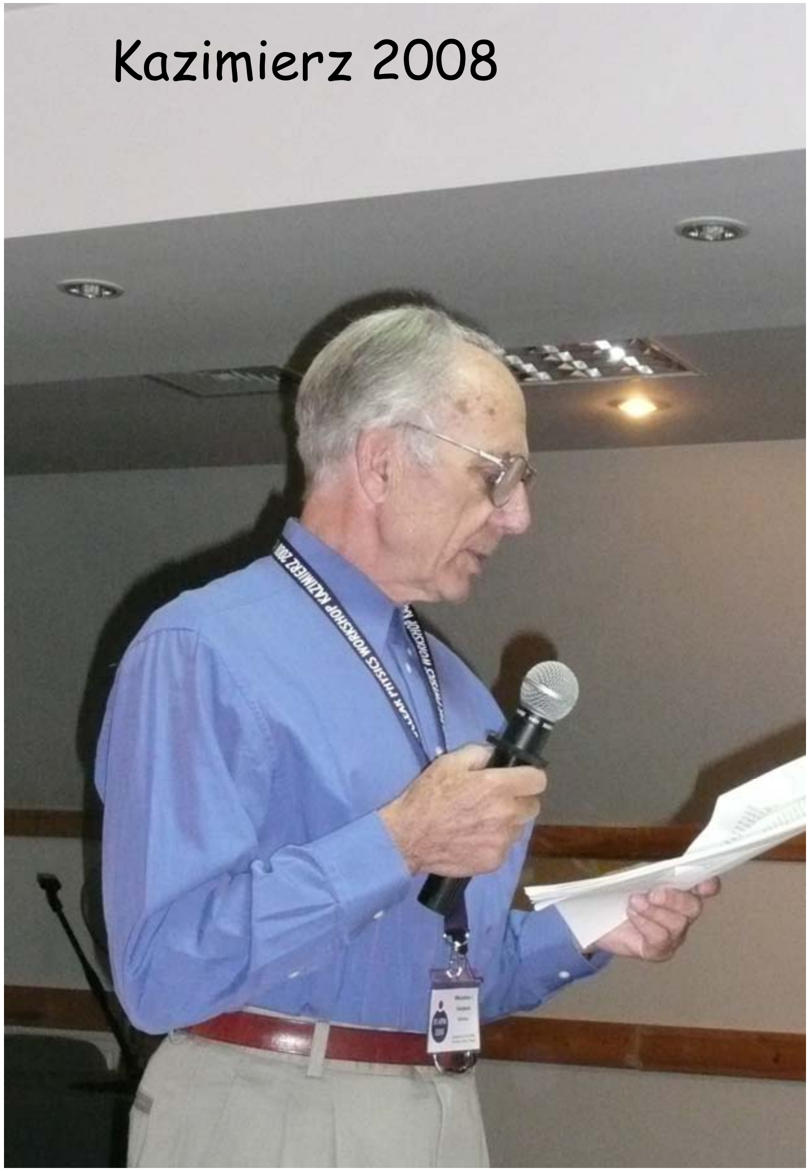
Predictions for Z = 115

Deformed nuclei



Spherical nuclei





Summary

- Extended version of the FBD model was presented.
- Good agreement with existing experimental data (entrance channel effects reproduced).
- Predictions for the production of new isotopes of elements 114 and 115 were presented.

Kazimierz 2010

September 10, 2010