

ODD-ODD NUCLEI AS THE CORE-PARTICLE-HOLE SYSTEMS AND CHIRALITY

Chirality seen from the laboratory frame

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- 1 Introduction: Chirality in nuclei
- 2 A description of odd-odd nuclei
 - The Core-Particle-Hole Coupling (CPHC) Model
 - Description of the even-even core
 - The single-particle proton and neutron bases
- 3 Results and conclusions
 - The calculations
 - Symmetries of the core and the valence particles
 - The α -symmetric cores
 - The α -asymmetric cores
 - The broken proton-neutron symmetry

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CHIRALITY

- Chirality in nuclei — the topic of the present session — is since a few years a hot topic in the nuclear structure physics of odd and odd-odd nuclei. It allows us to interpret the odd nuclei spectra in a simple way.

- Is it really the chirality?

The chirality or the handedness phenomenon is usually connected with the inversion of the three-dimensional physical space.

In nuclear physics we discuss the chirality which is connected with time reversal rather than the space inversion.

(Perhaps, the term spin-chirality would be plausible to distinguish the phenomena.)

- A model nuclear chiral system considered originally is: the odd proton, the odd neutron hole, and the rigid triaxial even-even core.

(in the quasi-classical picture: the three mutually perpendicular angular momenta \vec{j}_π , \vec{j}_ν , \vec{R} along the three intrinsic principal axes of the core)

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THE CHIRALITY SIGNATURES

Commonly considered main signatures of the chirality in nuclei:

- Appearance of a pair of the almost degenerate $\Delta I = 1$ bands of the same parity and similar electromagnetic properties (called chiral partner bands).
- Staggering of the intraband and interband M1 and $\Delta I = 1$ E2 transitions.

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Core-Particle-Hole Coupling

- The odd-odd nucleus is treated as the three-body system: the even-even core, the proton and the neutron-hole.
- States of the odd-odd nucleus with the proton and neutron numbers Z and N , respectively, are assumed in the following form:

$$|Z, N; iIM\rangle = \sum_{\rho, \sigma} \sum_{L, R, r} U_{il}(\rho, \sigma, L, R, r) \left[[a_{\pi\rho}^\dagger \times \tilde{a}_{\nu\sigma}]_L \times |Z-1, N+1; rR\rangle \right]_{IM}$$

- The quadrupole-quadrupole two-body interaction between the proton, the neutron and the core is assumed.
(The coupling constant $\chi_2 = 40\text{MeV}/b^2$, relatively strong, is taken in the calculations)
- Details in: K. Starosta et al, Phys. Rev. C **65**, 044328 (2002); Ch. Droste et al, Eur. Phys. J., **42**, 79 (2009).

The Bohr Hamiltonian

The core states $|Z - 1, N + 1; rRM_R\rangle$ are described by a version of the Bohr Hamiltonian in the following form:

$$\begin{aligned}
 H(\beta, \gamma, \Omega) \\
 = -\frac{1}{2B_{\beta\beta}} \frac{1}{\beta^4} \frac{\partial}{\partial \beta} \left(\beta^4 \frac{\partial}{\partial \beta} \right) - \frac{1}{2B(\gamma)} \frac{\Lambda^2(\gamma, \Omega)}{\beta^2} + V(\beta, \gamma)
 \end{aligned}$$

with the potential of the form

$$\begin{aligned}
 V(\beta, \gamma) = \frac{1}{2} V_C \beta^2 + (G + h_1 \cos 3\gamma + h_2 (\cos^2 3\gamma - 1)^\kappa) \\
 \times (\exp(-\beta^2/d^2) - 1)
 \end{aligned}$$

The Bohr Hamiltonian

- The seniority operator is

$$\Lambda^2(\gamma, \Omega) = \frac{1}{\sin 3\gamma} \frac{\partial}{\partial \gamma} \left(\sin 3\gamma \frac{\partial}{\partial \gamma} \right) - \sum_{k=1}^3 \frac{R_k^2(\Omega)}{\sin^2(\gamma - 2\pi k/3)}$$

- Variables β and γ are the Bohr deformation parameters, Ω stands for the three Euler angles of orientation of the body-fixed system and $R_k(\Omega)$ for $k = 1, 2, 3$ are the three (dimensionless) intrinsic components of angular momentum.

The Bohr Hamiltonian

- The rotational inertial function is

$$B(\gamma) = b_0 + b_1 \cos 3\gamma$$

- The parameters of the potential V_C , G , h_1 , h_2 , κ and d , and the kinetic energy b_0 and b_1 and $B_{\beta\beta}$ are selected in a way to obtain the values of energy of the lowest excited state $E(2_1^+)$ and reduced transition probability $B(E2; 2_1^+ \rightarrow 0_1^+)$ always close to the experimental values $E(2_1^+) = 354$ keV and $B(E2; 2_1^+ \rightarrow 0_1^+) = 0.282 e^2 b^2$ for $^{128}_{56}\text{Ba}$ ($A = 128$, $Z - 1 = 56$).
- Parameters h_2 and κ are responsible for the γ -softness of the potential.
- Parameters h_1 and b_1 are responsible for the γ -asymmetry of the Bohr Hamiltonian.

Proton and neutron states

- The proton and the neutron hole can occupy the single-particle states in the spherically symmetric potential well:
 $\pi\rho = \pi n l j$ and $\nu\sigma = \nu n l j$, respectively.
- The calculations are performed assuming that the proton and neutron bases contain **only one** orbital ρ and σ , respectively.
- For the proton $\rho = h_{11/2}$.
- For the neutron $\sigma = h_{11/2}$, or $g_{9/2}$.

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Principles of the calculations

- A fictitious nucleus with $Z = 57$, $N = 71$ (^{128}La ?).
- Variants of the calculation:
 - The α -symmetric cores: $h_1 = b_1 = 0$, and $h_2 = 20$ MeV [potential well (PW)], $h_2 = -8$ MeV [potential barrier (PB)], compared to $h_2 = 0$ [the Wilets-Jean (WJ) soft potential],
The single-particle states: $\rho = \sigma = h_{11/2} (\pi h \otimes \nu h)$.
 - The α -asymmetric cores: $h_2 = 0$, $b_1 = 0$, and $h_1 = 2$ MeV [$\langle \gamma \rangle \approx 21^\circ$],
 $h_1 = 8$ MeV [$\langle \gamma \rangle \approx 15^\circ$],
The single-particle states: $\rho = \sigma = h_{11/2}$.
 - The Wilets-Jean potential : $h_1 = h_2 = b_1 = 0$,
The different single-particle states: proton $\rho = h_{11/2}^-$, neutron $\sigma = g_{9/2}^+$
($\pi h \otimes \nu g$).

The α -parity of the core

- The laboratory quadrupole variables are related to β , γ , Ω as follows:

$$\alpha_{\mu}(\beta, \gamma, \Omega) = D_{\mu 0}^2(\Omega)\beta \cos \gamma + \frac{1}{\sqrt{2}} (D_{\mu 2}^2(\Omega) + D_{\mu -2}^2(\Omega)) \beta \sin \gamma$$

- The inversion in the five-dimensional space (the O(5) inversion) is:
 $\alpha_{2\mu} \rightarrow -\alpha_{2\mu}$
- A possible realisation of the inversion in the intrinsic variables: $\gamma \rightarrow \gamma \pm \pi$.
- If the Bohr Hamiltonian is invariant under the O(5) inversion (the α -symmetric) the core states possess the definite α -parity (the γ -parity introduced by Bés, 1959).
- In the present calculations
 - when $h_1 = b_1 = 0$ the core is the α -symmetric,
 - when $h_1 \neq 0$ (the α -asymmetric potential) and/or $b_1 \neq 0$ (the α -asymmetric kinetic energy) the α -symmetry is broken.

The proton-neutron symmetry

- The symmetry with respect to the exchange of the proton and the neutron states:

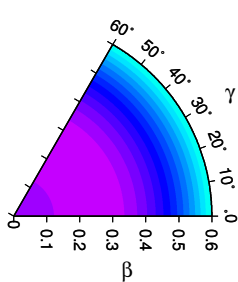
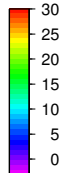
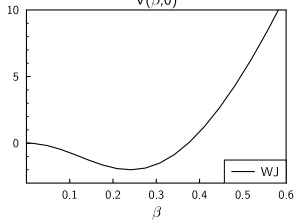
$$\pi\rho \rightarrow \pi\sigma, \nu\sigma \rightarrow \nu\rho$$

is (a bit misleading) called the proton-neutron symmetry.

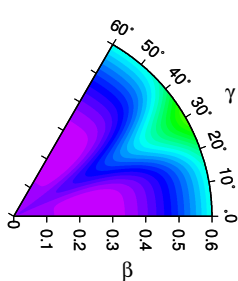
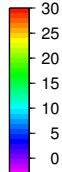
- It is not the particle-hole exchange (time reversal).
- In the present calculations
 - when $\rho = \sigma = h_{11/2}$ (the proton and the neutron hole on the same orbital) the proton-neutron symmetry is conserved,
 - when $\rho = h_{11/2}$ and $\sigma = g_{9/2}$ (the proton and the neutron hole on different orbitals) the proton-neutron symmetry is broken.

Collective α -symmetric potentials

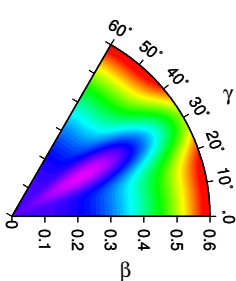
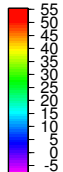
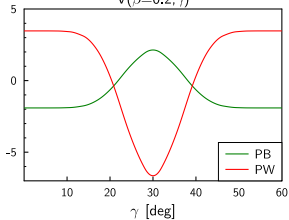
WJ

 $V(\beta, \gamma)$  $V(\beta, 0)$ 

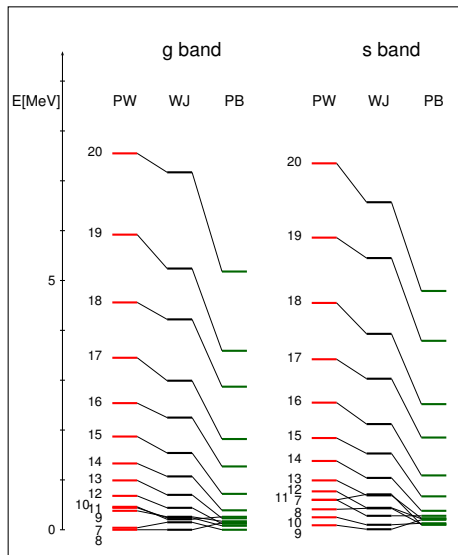
PB

 $V(\beta, \gamma)$ 

PW

 $V(\beta, \gamma)$  $V(\beta=0.2, \gamma)$ 

Partner bands



Ground (g) and side (s) partner bands at α -symmetric cores and $\pi h_{11/2} \otimes \nu h_{11/2}^{-1}$

Partner bands

- The partner bands appear for an arbitrary rigidity.
- The bands become more and more stretched when the core is more and more rigid.
- The bands are the most split and squeezed for the potential barrier in γ .

Magnetic dipole moments

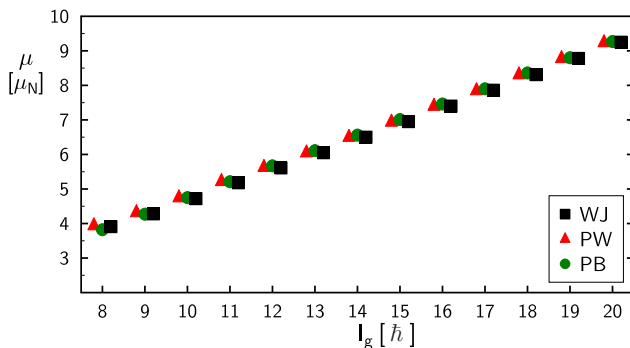


Figure: Magnetic dipole moments $\mu(I_g)$.

Electromagnetic moments

- The magnetic dipole moments in the s-band states are close to those in the g-band states.
- The magnetic dipole moments are independent practically on the rigidity of the core.
- The quadrupole electric moments are smaller than the single-particle estimates for a given spin ($|Q| < 0.1 \text{ eb}$) for the states of both bands at each rigidity of the core.

Stretched E2 transitions

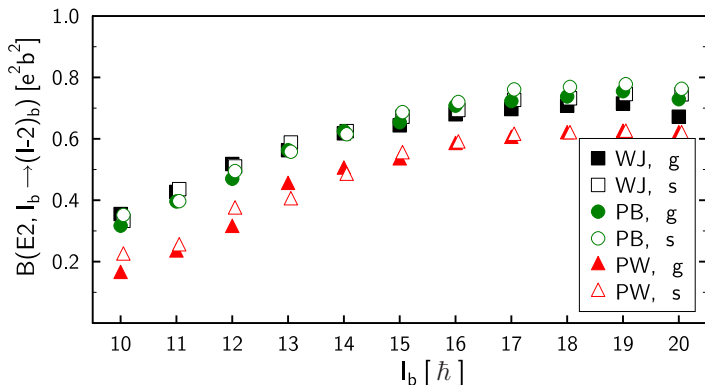


Figure: Reduced transition probabilities $B(E2; I_b \rightarrow (I-2)_b)$ of the stretched intra-band E2 transitions for the $b=g$ ground band and the $b=s$ side band.

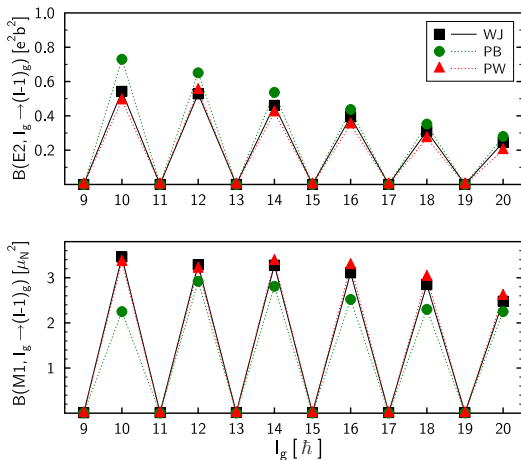
$\Delta I = 1$ intra-band transitions

Figure: Reduced transition probabilities of the $\Delta I = 1$ intra-band electromagnetic transitions for the ground band.

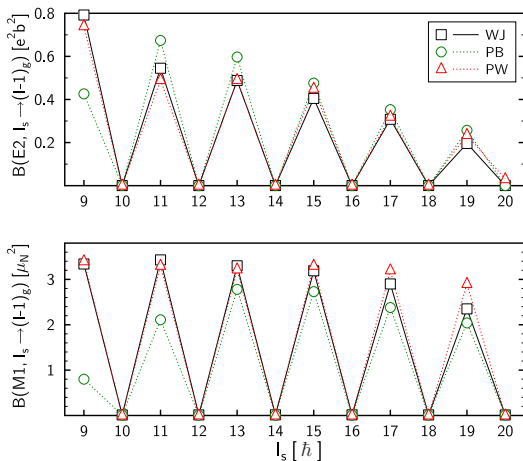
$\Delta I = 1$ inter-band transitions

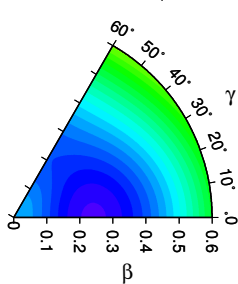
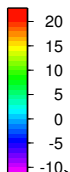
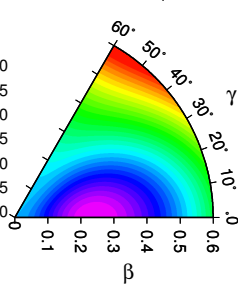
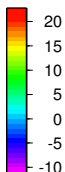
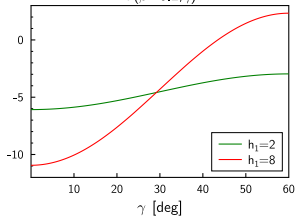
Figure: Reduced transition probabilities of the $\Delta I = 1$ inter-band electromagnetic transitions.

The α -symmetry — conclusion

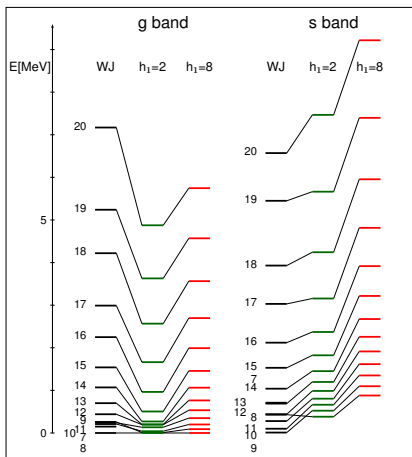
- Properties of the partner bands:
 - A small splitting of the bands.
 - The similar electromagnetic properties (moments and transitions).
 - The strong staggering of the intra- and the inter-band E2 and M1 transitions.
- The chirality signatures are manifested in all cases of the α -symmetric cores and the particle and the hole on the same orbital.

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Collective α -asymmetric potentials $h_1=2$  $V(\beta, \gamma)$  $h_1=8$  $V(\beta, \gamma)$  $V(\beta=0.2, \gamma)$ 

Partner bands



Ground (g) and side (s) (partner?) bands at α -asymmetric cores and $\pi h_{11/2} \otimes \nu h_{11/2}^{-1}$

Magnetic dipole moments

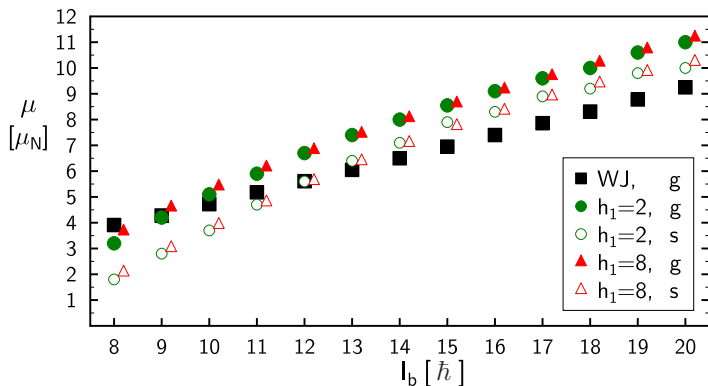


Figure: Magnetic dipole moments $\mu(I_b)$ for the ground (b=g) and the side (b=s) bands.

Electric quadrupole moments

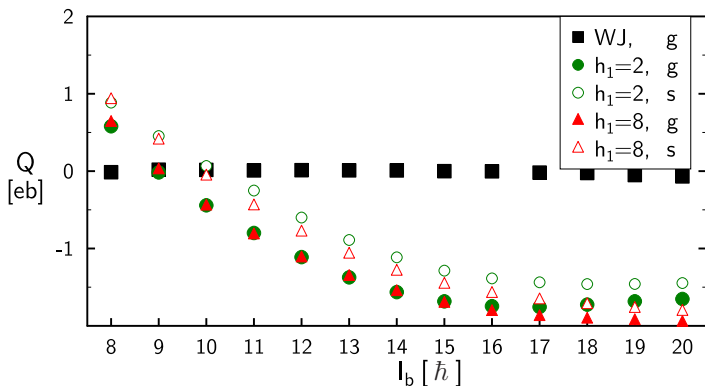


Figure: Electric quadrupole moments $Q(I_b)$ for the ground ($b=g$) and the side ($b=s$) bands.

Electromagnetic moments

- Small differences between the magnetic dipole moments of states in the ground and side bands.
- Considerable values of the electric quadrupole moments.
- Visible differences between the values of Q in the states of the ground and side bands.

Stretched E2 transitions

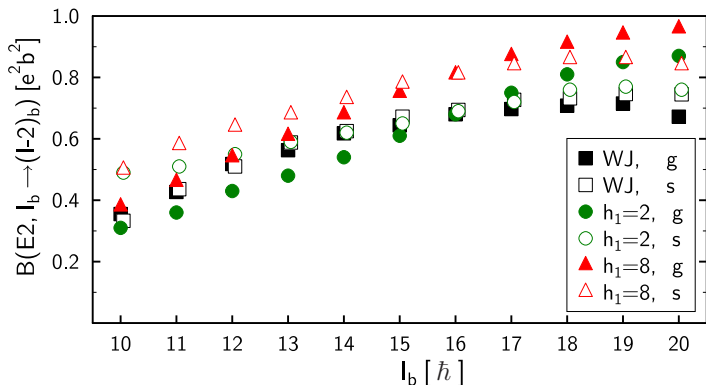


Figure: Reduced transition probabilities $B(E2; I_b \rightarrow (I-2)_b)$ of the stretched intra-band E2 transitions for the $b=g$ ground band and the $b=s$ side band.

$\Delta I = 1$ electromagnetic intra-band transitions

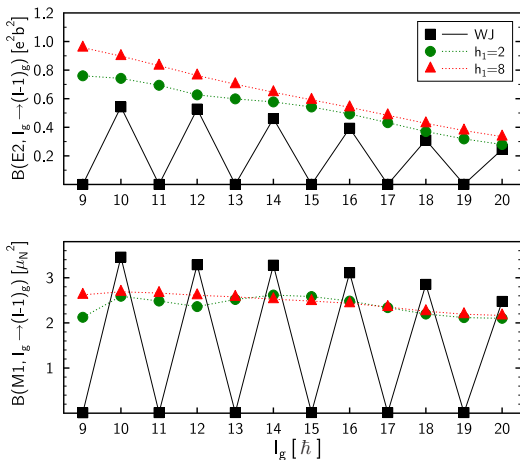


Figure: Reduced transition probabilities of the $\Delta I = 1$ intra-band electromagnetic transitions for the ground band.

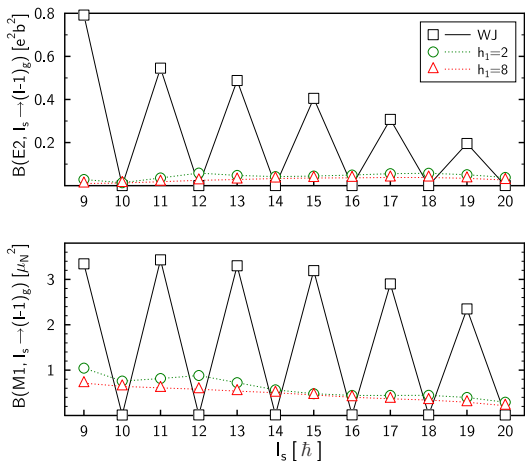
$\Delta I = 1$ electromagnetic inter-band transitions

Figure: Reduced transition probabilities of the $\Delta I = 1$ inter-band electromagnetic transitions.

The α -asymmetry — conclusion

- Properties of the partner bands:
 - A strong splitting of the partner bands.
 - Not quite big differences between the electromagnetic properties of the partner bands.
 - The immediate disappearance of staggering in the intra- and the inter-band $\Delta I = 1$ electromagnetic transitions with the appearance of the asymmetry.
- The signatures of chirality vanish in the cases of the α -asymmetry of the core.
- A similar conclusion could be drawn in the case of the α -asymmetry in the collective kinetic energy ($b_1 \neq 0$).

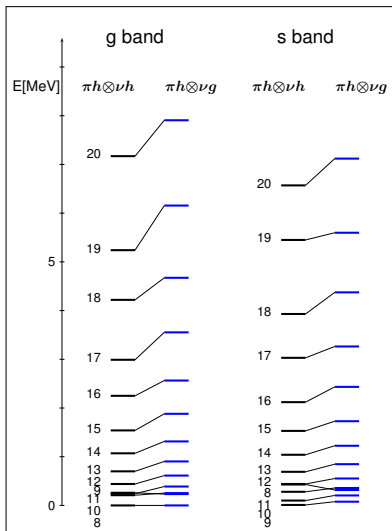
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Stretched E2 transitions

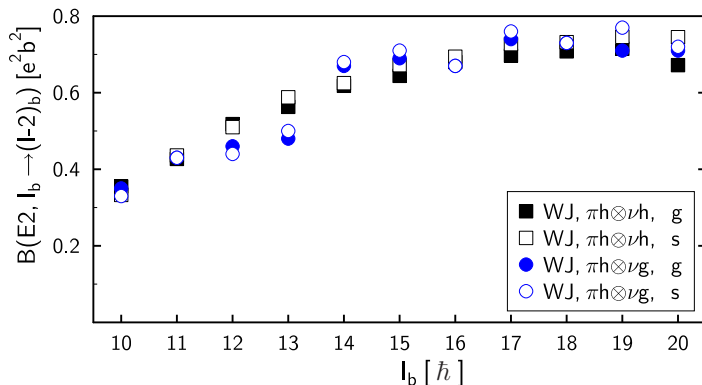


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$\Delta I = 1$ electromagnetic transitions

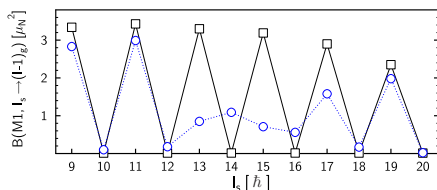
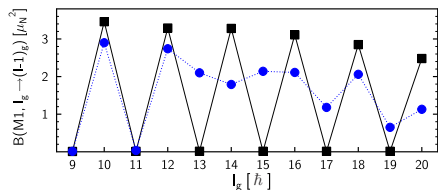
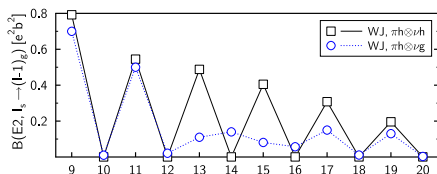
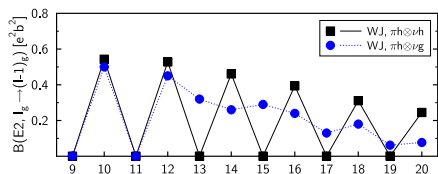


Figure: Reduced transition probabilities of the $\Delta I = 1$ intra-band (left) and inter-band (right) electromagnetic transitions.

The broken proton-neutron symmetry — conclusion

- Properties of the partner bands:
 - A little stronger splitting the partner bands than that in the case of the same orbitals.
 - Weak differences between the electromagnetic properties of the ground and side bands.
 - A weaker and irregular staggering of the intra- and the inter-band electromagnetic transitions.
- The signatures of chirality are obscure.

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Summary

- The use of the laboratory frame of reference:
 - The body-fixed frame of reference is eventually introduced in the description of the core structure
 - The properties of the core enter the description of the odd-odd nucleus only through the energies of the collective levels and the E2 matrix elements within the collective states.
 - No need and will to introduce an intrinsic frame of reference in the calculations for the odd-odd nucleus.
 - **No assumptions of the chiral symmetry were made.**
- Sufficient conditions for the odd-odd nucleus to manifest **the chirality signatures** are:
 - The odd-odd nucleus can be described as **the three-body system**: the even-even core, the proton and the neutron hole.
 - The core is **the α -symmetric** regardless of its rigidity.
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- Sufficient conditions for the odd-odd nucleus to manifest **the chirality signatures** are:
 - The odd-odd nucleus can be described as **the three-body system**: the even-even core, the proton and the neutron hole.
 - The core is **the α -symmetric** regardless of its rigidity.
 - **The proton-neutron symmetry** is conserved.

Summary

- The use of the laboratory frame of reference:
 - The body-fixed frame of reference is eventually introduced in the description of the core structure
 - The properties of the core enter the description of the odd-odd nucleus only through the energies of the collective levels and the E2 matrix elements within the collective states.
 - No need and will to introduce an intrinsic frame of reference in the calculations for the odd-odd nucleus.
 - **No assumptions of the chiral symmetry were made.**
- Sufficient conditions for the odd-odd nucleus to manifest **the chirality signatures** are:
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