17th Nuclear Physics Workshop "Marie & Pierre Curie")

17 – 22 September 2010 at Kazimierz Dolny, Poland

SYMMETRY & SYMMETRY BREAKING IN NUCLEAR PHYSICS

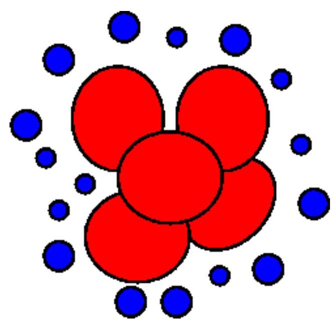
Spinodal phase separation in nuclear collisions

Jørgen Randrup, LBNL

1977: Mikołaiki

1995: Piaski

2010: Kazimierz



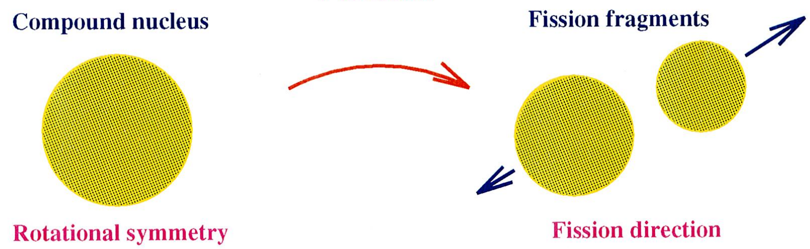
- dedicated to the memory of Władek Swiatecki

Simple examples of

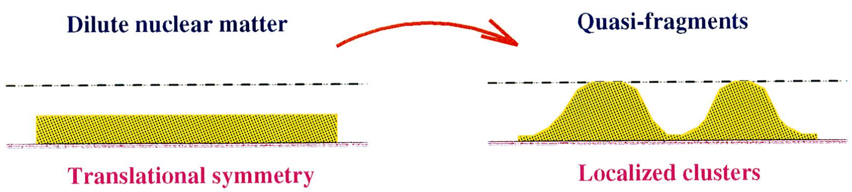
NUCLEAR CATASTROPHIES

- spontaneous symmetry breaking

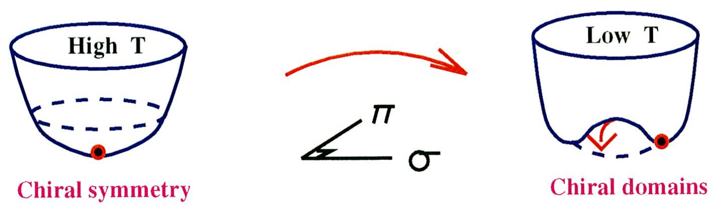
Fission:



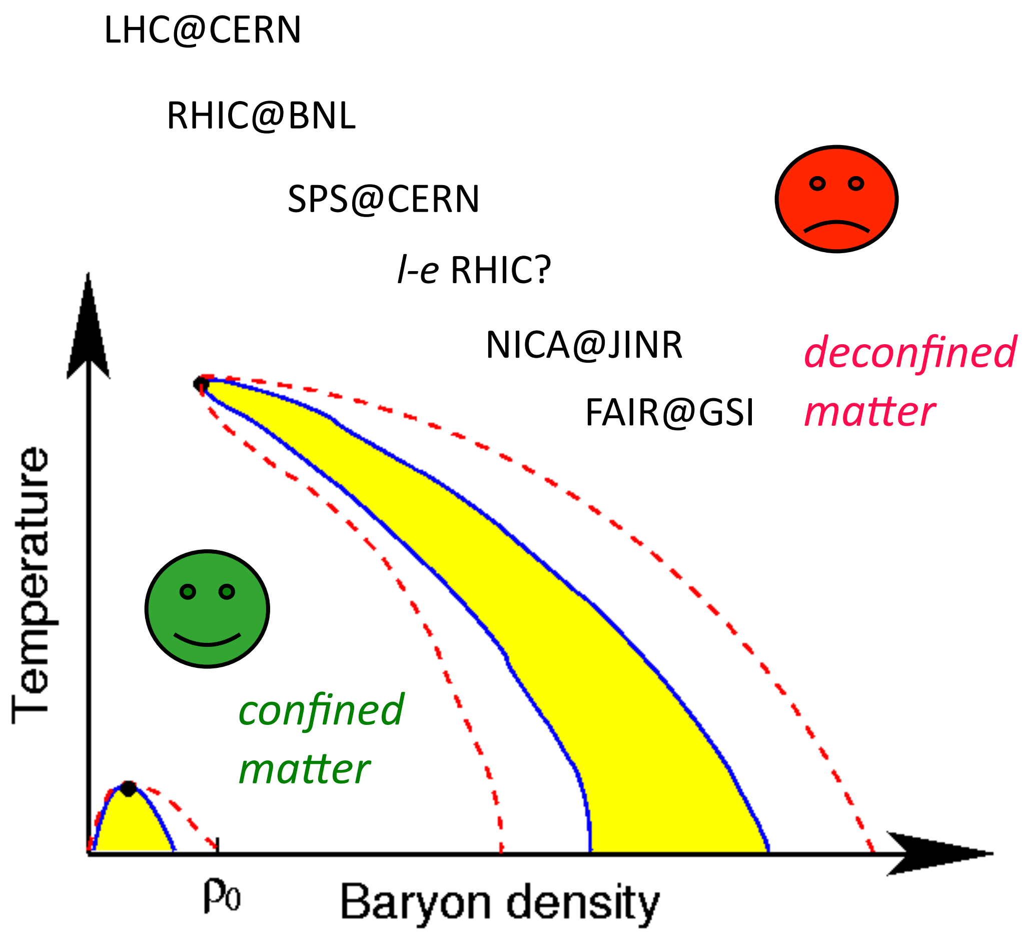
Multifragmentation:

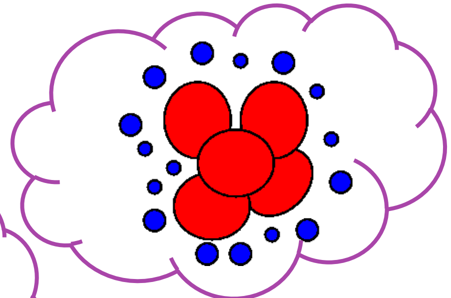


Disoriented Chiral Condensates:

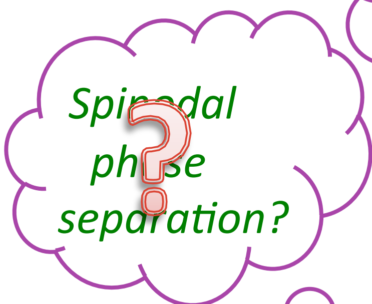


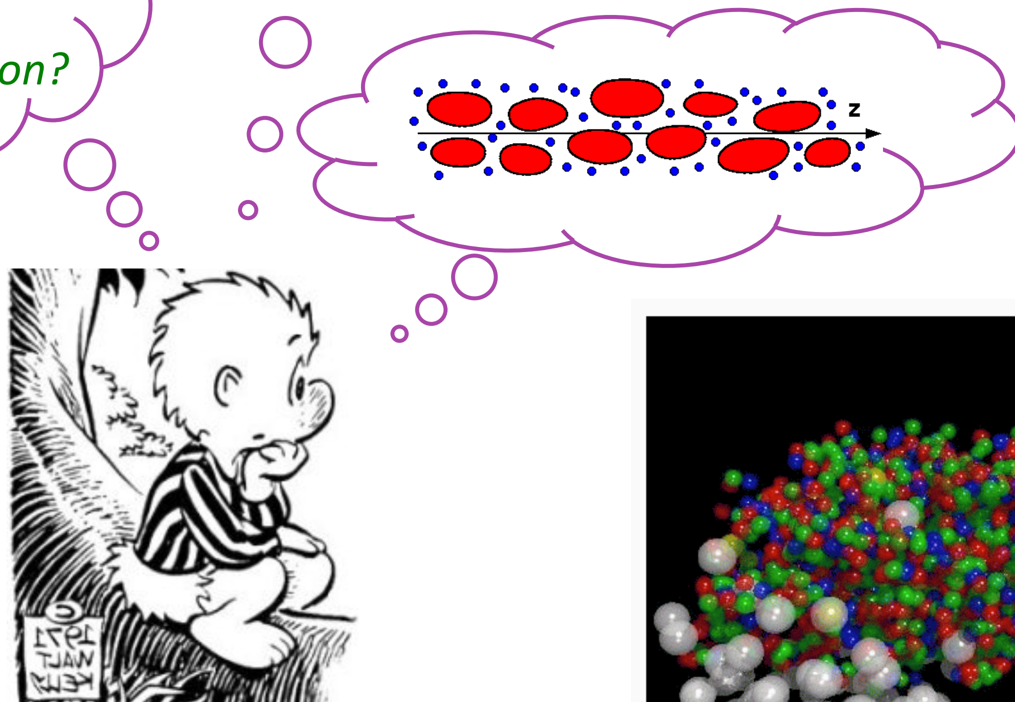
Schematic and simplified
phase diagram of strongly interacting matter

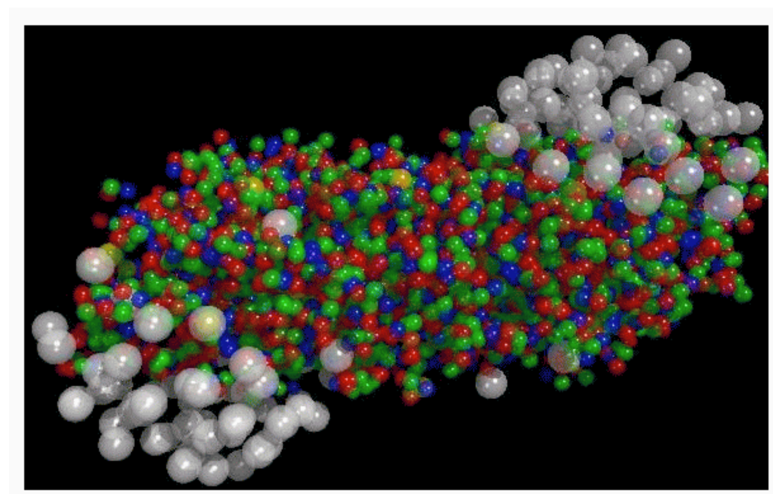




How would a first-order phase transition manifest itself in a nuclear collision?

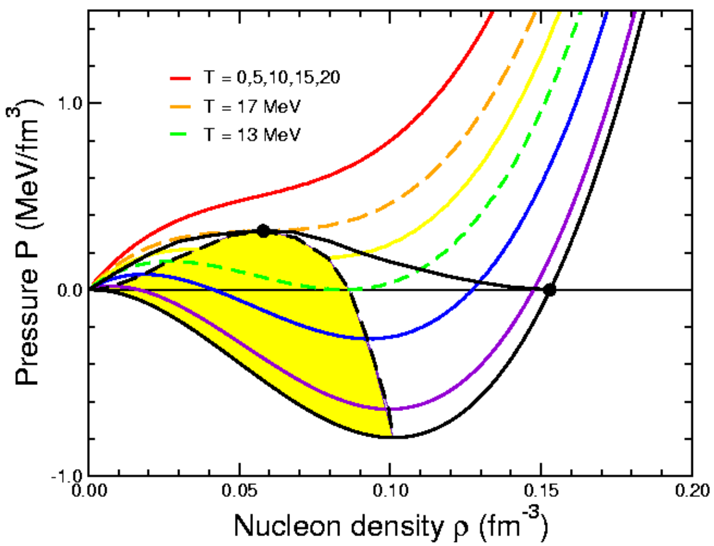




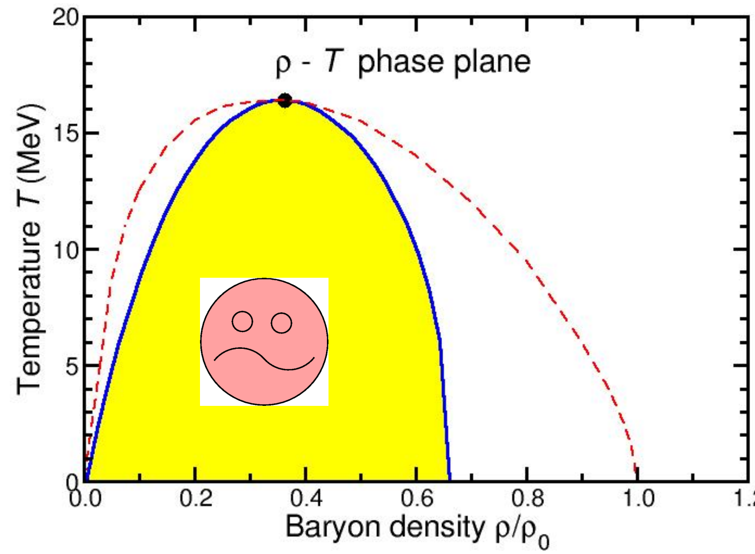


Nuclear matter

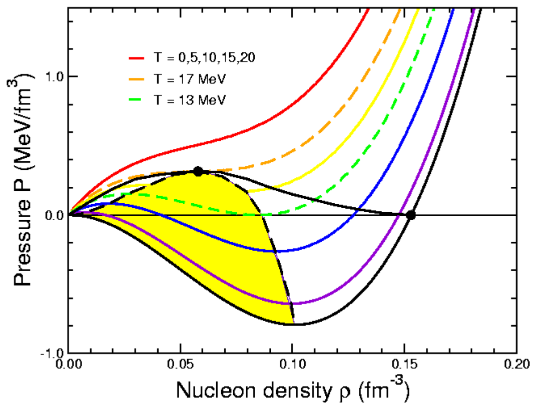
Equation of state: $p_T(\rho)$



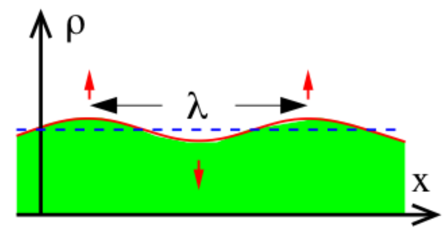
Phase diagram

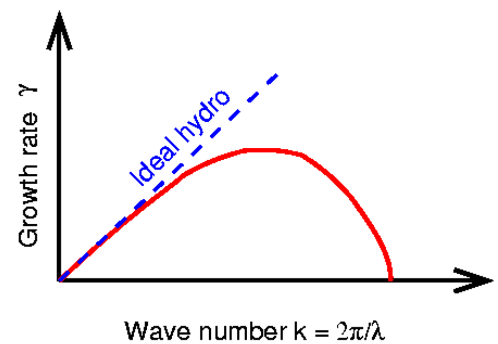


Spinodal pattern formation



Density undulations are amplified in the spinodal region:

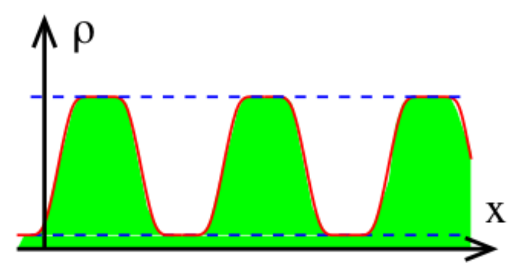




Long-wavelength distortions grow slowly (it takes time to relocate the matter)

Short-wavelength distortions grow slowly (they are hardly felt due to finite range)

There is an *optimal length scale* that grows faster than all others



Ph Chomaz, M Colonna, J Randrup
Nuclear Spinodal Fragmentation
Physics Reports 389 (2004) 263

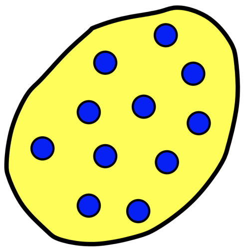
Spinodal decomposition occurs in many areas of science and technology, for example:

- Colloidal aggregation
- Polymer blends
- Binary fluid mixtures
- Binary metallic alloys
- Inorganic glasses
- Sticky emulsions

It also plays a role in nuclear *multifragmentation*!

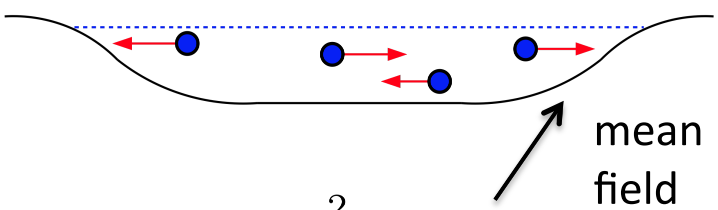
Does it play a role in the *hadronization* of the QGP?

Nuclear dynamics at $E_{coll} \approx E_{Fermi}$



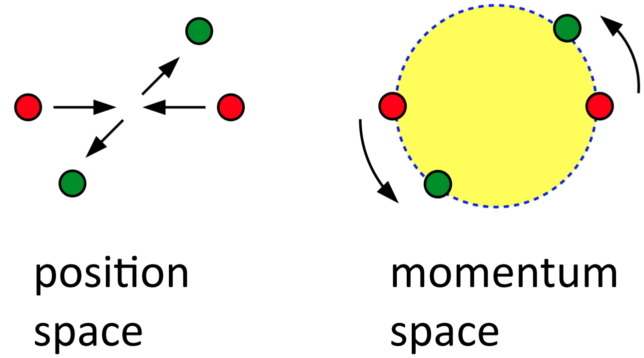
Individual nucleons move in common one-body field while occasionally experiencing Pauli-suppressed binary collisions

One-particle Hamiltonian



$$h[f](\boldsymbol{r},\boldsymbol{p}) = \frac{p^2}{2m^*} + U[\rho](\boldsymbol{r})$$

Two-body collisions



The state of the system is characterized by its reduced one-particle phase-space density:

$$f(r,p)$$

Individual nucleons move in common one-body field while occasionally experiencing Pauli-suppressed binary collisions

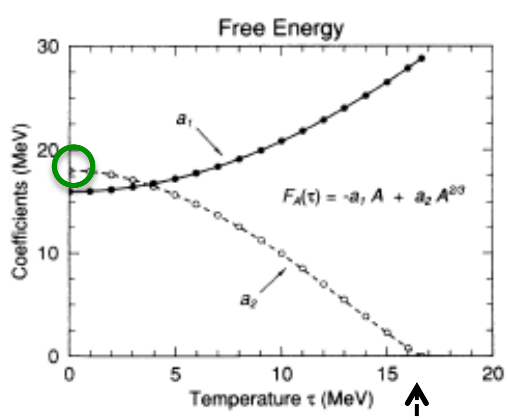
STATICS

DYNAMICS

Thermal nuclear properties

$F_A(T) = -a_1 A + a_2 A^{2/3}$

Medeiros & Randrup: PRC45 (1992) 372

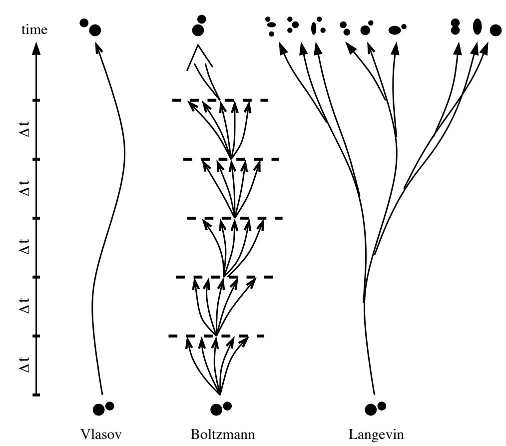


T_{crit}

WJS: *The Nuclear Surface Energy*, Proc. Phys. Soc. **A** 64 (1951) 226

Nuclear collision dynamics

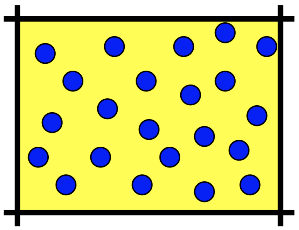
$$\dot{f} \equiv \partial_t f - \{h[f], f\} \doteq C[f] = \bar{C}[f] + \delta C[f]$$



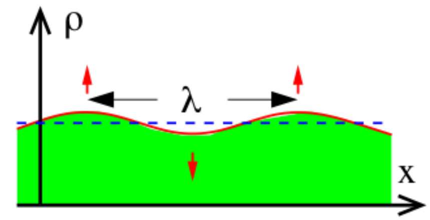
$C = 0$ $\quad C = \bar{C}$ $\quad C = \bar{C} + \delta C$

Vlasov ≈HF

Instabilities in Fermi liquids: Nuclear matter



$$\delta f(\boldsymbol{r},\boldsymbol{p},t) = f(\boldsymbol{r},\boldsymbol{p},t) - f_0(\boldsymbol{p})$$
$$\delta f(\boldsymbol{r},\boldsymbol{p},t) = \sum_{\boldsymbol{k}} f_{\boldsymbol{k}}(\boldsymbol{p},t)\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$



$$h[f](\boldsymbol{r},\boldsymbol{p}) = \frac{p^2}{2m^*} + U(\rho(\boldsymbol{r})) \qquad \frac{\partial}{\partial t}\delta f + \boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{r}}\delta f - \frac{\partial f_0}{\partial \boldsymbol{p}}\cdot\left(\frac{\partial U}{\partial \rho}\frac{\partial}{\partial \boldsymbol{r}}\delta\rho\right) = 0 \qquad \boldsymbol{v} = \frac{\partial h}{\partial \boldsymbol{p}} = \frac{\boldsymbol{p}}{m^*}$$

$$\delta\rho(\boldsymbol{r},t) = g\int\frac{d^3\boldsymbol{p}}{h^3}\,\delta f(\boldsymbol{r},\boldsymbol{p},t) \quad \Rightarrow \quad \rho_{\boldsymbol{k}}(t) = g\int\frac{d^3\boldsymbol{p}}{h^3}\,f_{\boldsymbol{k}}(\boldsymbol{p},t)$$

$$f_{\boldsymbol{k}}(\boldsymbol{p},t) = f_{\boldsymbol{k}}(\boldsymbol{p})\,\mathrm{e}^{-i\omega_k t} \quad \Rightarrow \quad (-\omega_k + \boldsymbol{v}\cdot\boldsymbol{k})f_{\boldsymbol{k}}(\boldsymbol{p}) = \boldsymbol{v}\cdot\boldsymbol{k}\frac{\partial f_0}{\partial \epsilon}\frac{\partial U}{\partial \rho}\rho_{\boldsymbol{k}}$$

$$1 \doteq \frac{\partial U}{\partial \rho}g\int\frac{d^3\boldsymbol{p}}{h^3}\frac{\boldsymbol{v}\cdot\boldsymbol{k}}{\boldsymbol{v}\cdot\boldsymbol{k}-\omega_k}\frac{\partial f_0}{\partial \epsilon} = \frac{\partial U}{\partial \rho}g\int\frac{d^3\boldsymbol{p}}{h^3}\frac{(\boldsymbol{v}\cdot\boldsymbol{k})^2}{(\boldsymbol{v}\cdot\boldsymbol{k})^2-\omega_k^2}\frac{\partial f_0}{\partial \epsilon}$$

Finite range: $\tilde{g}(r_{12}) : \ \tilde{U} = \tilde{g} * U : \ \partial_\rho U \ \rightarrow \ \tilde{g}_k \partial_\rho U$

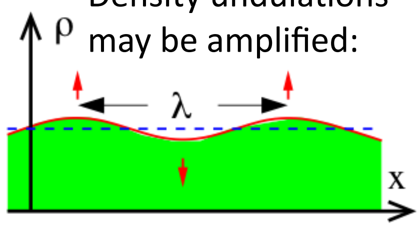
Landau parameter $F_0 = \dfrac{\partial h}{\partial \epsilon_F}$

ω is imaginary when $F_0 < -1$

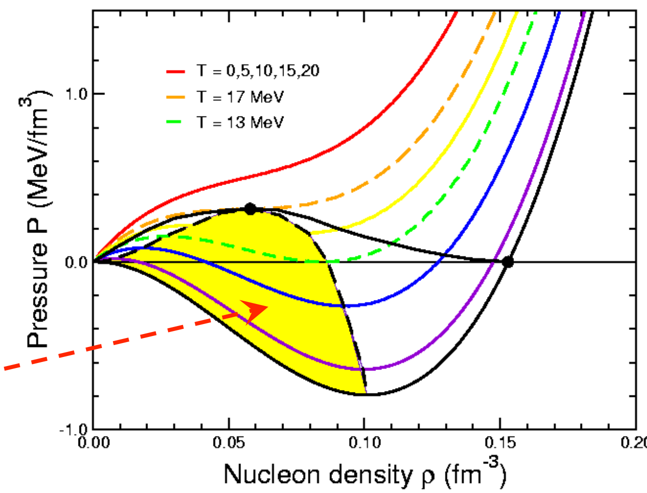
Nuclear spinodal instabilities

Spinodal region: $F_0 < -1$
Matter is thermodynamically and mechanically unstable

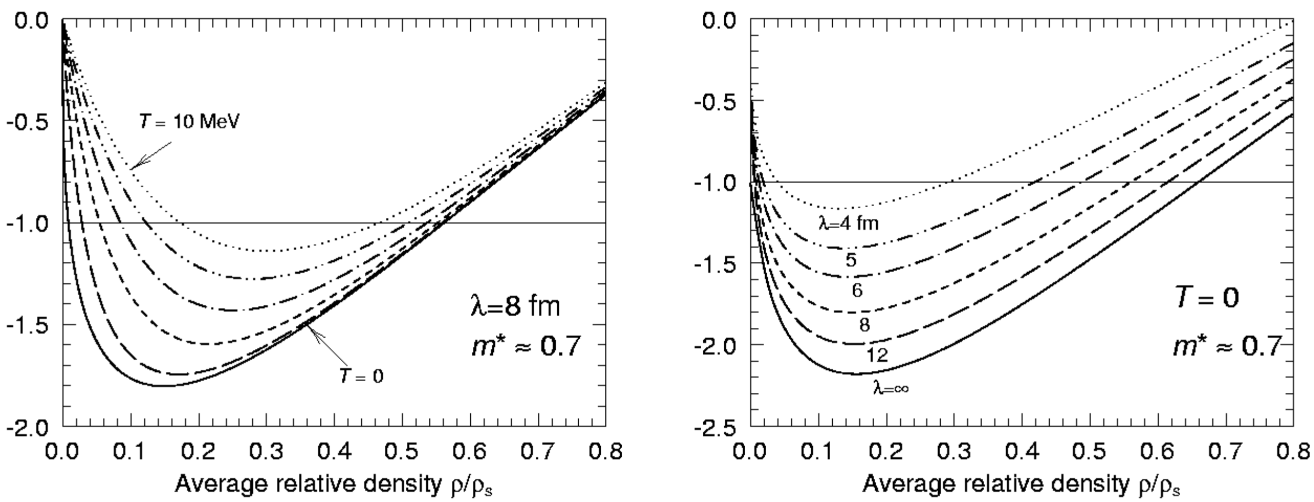
Density undulations may be amplified:



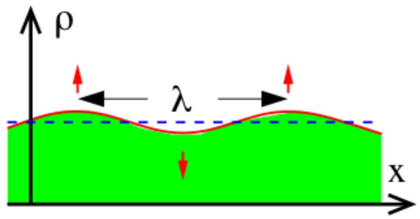
Nuclear Matter Equation of State:



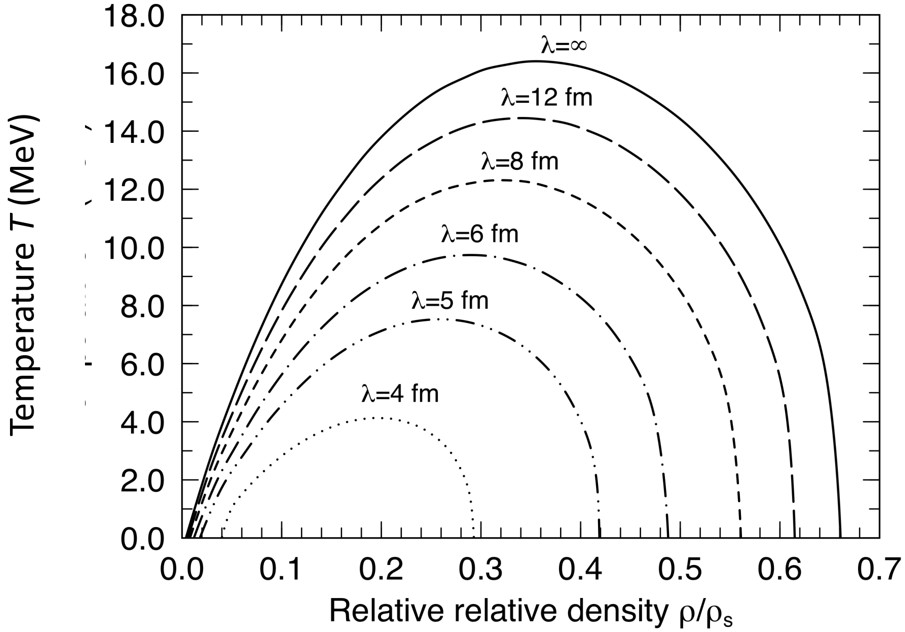
The Landau parameter F_0 depends on ρ, T, λ:



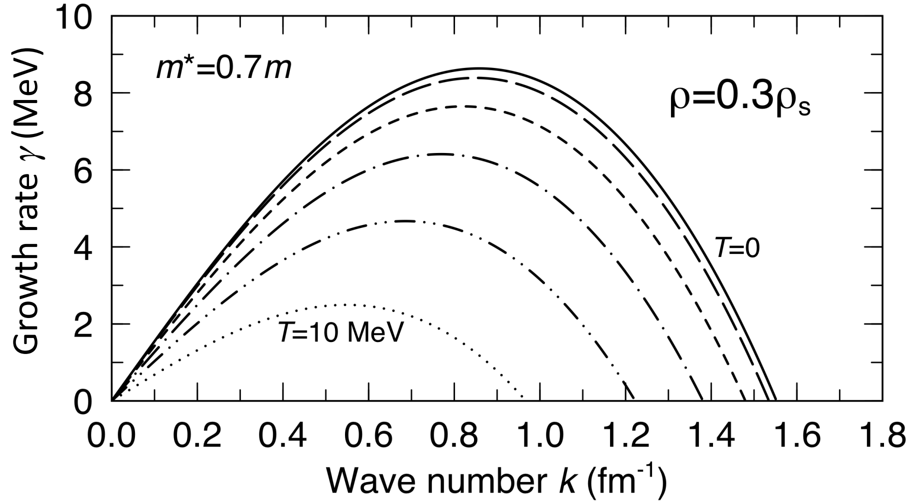
Spinodal instability in nuclear matter



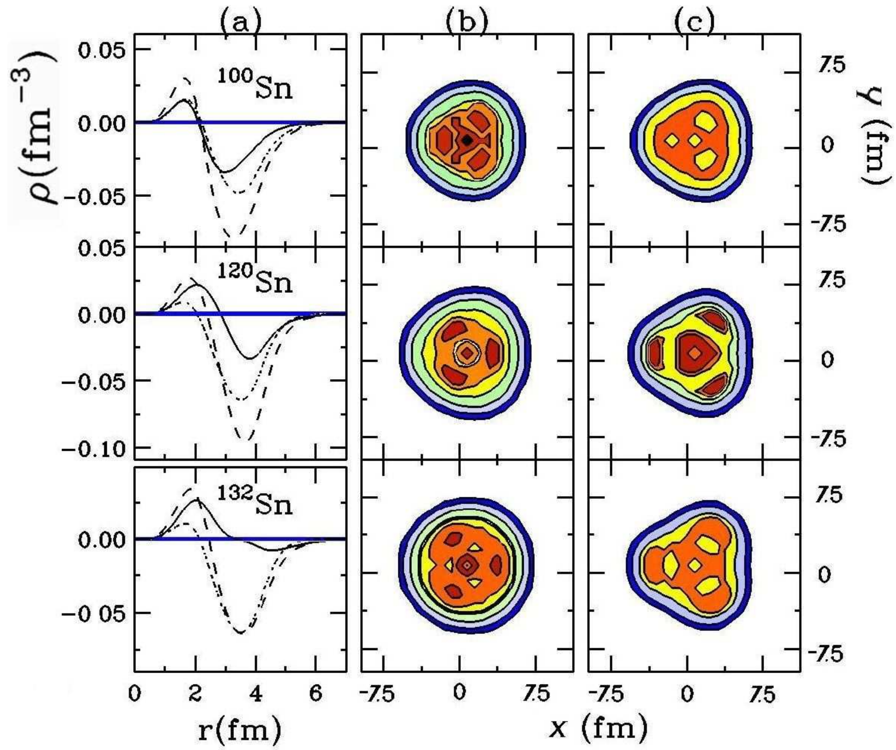
The spinodal boundary
depends on the wave length λ:



Growth rates γ_k for $\rho = 0.3\ \rho_s$
for various temperatures *T:*



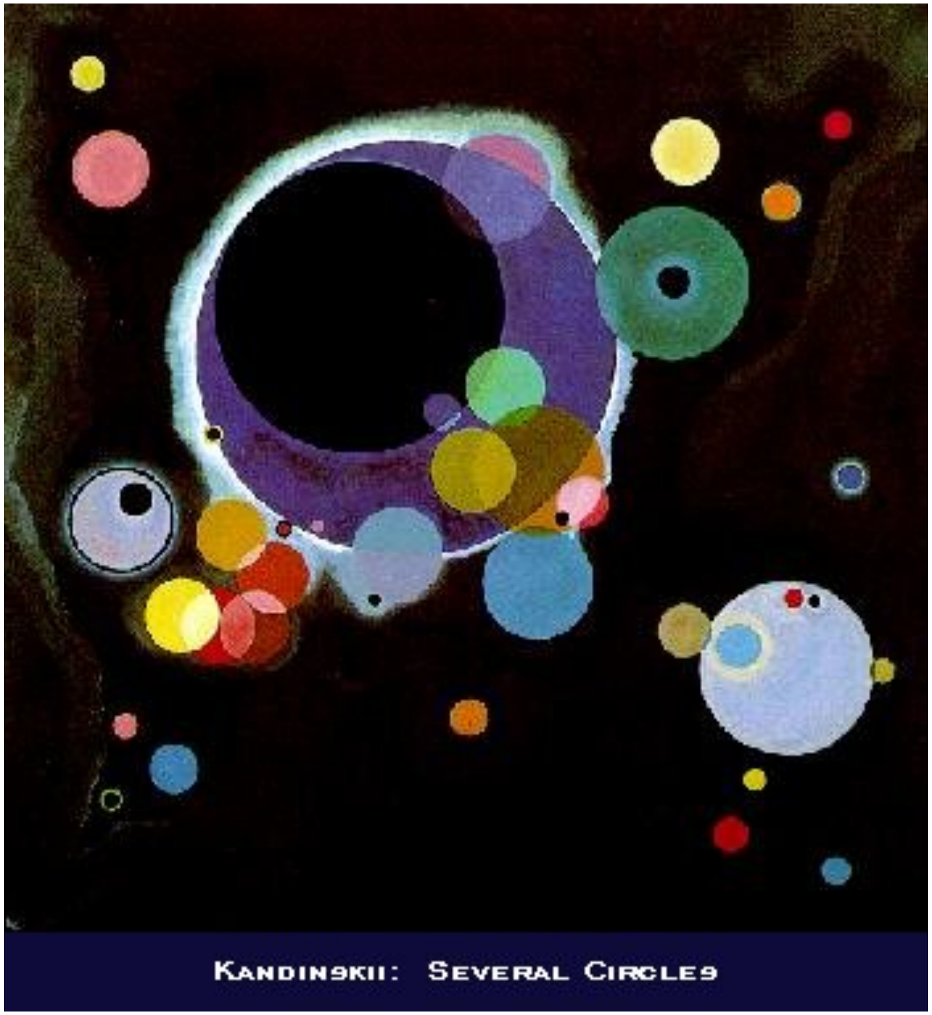
Spinodal instability in finite nuclear systems



RPA calculations for unstable octupole modes in Sn isotopes:
(a) radial dependence of the form factor at the dilution D = 1:5 for neutrons (solid), protons (dotted), and nucleons (dashed);
(b) contour plots of the perturbed neutron density;
(c) contour plots of the perturbed proton density.

M. Colonna, Ph. Chomaz, S. Ayik, Phys. Rev. Lett. 88 (2002) 122701

Statistical multifragmentation: ⟷ Spinodal fragmentation:

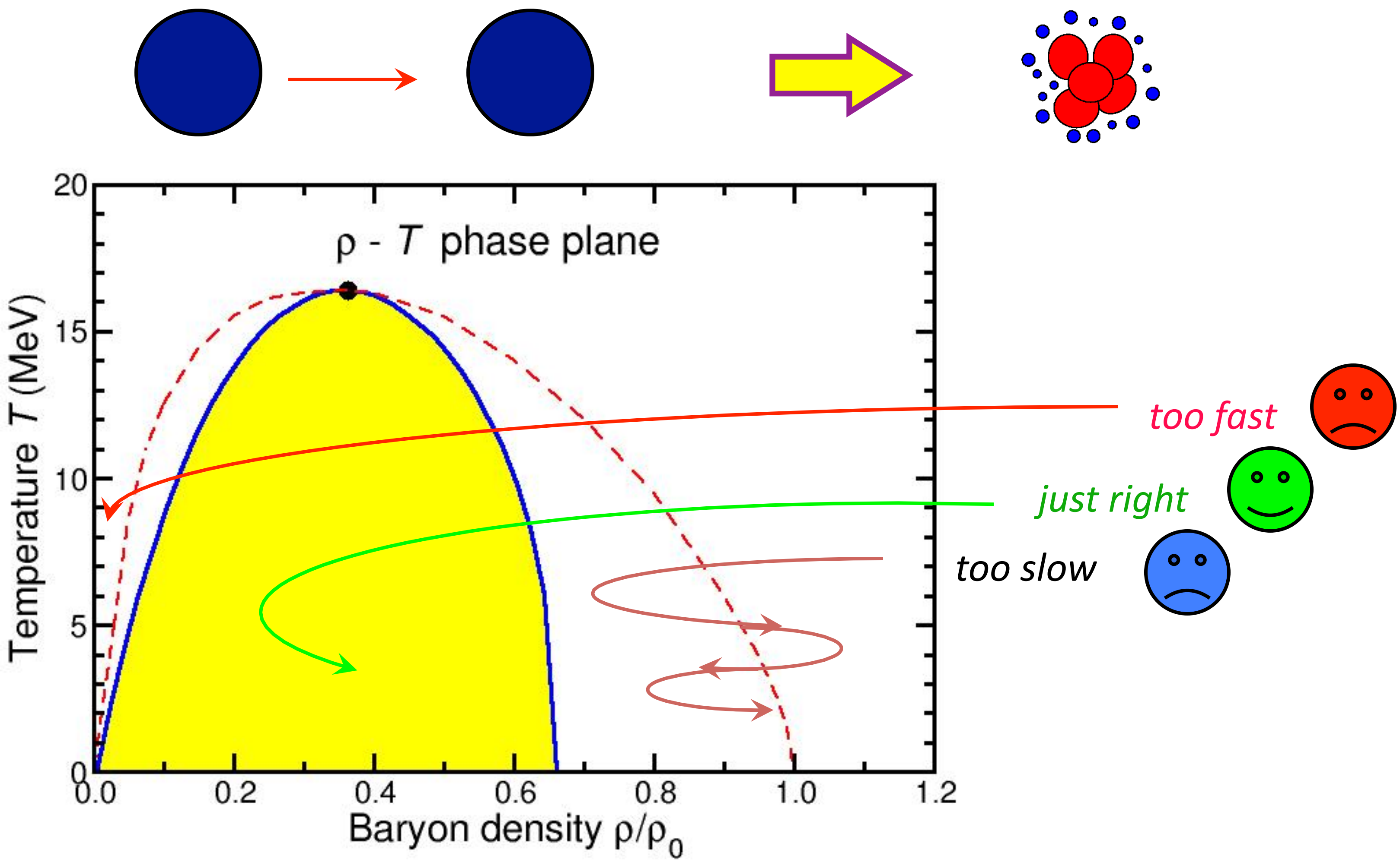


=> *Different* fragment sizes

=> *Equal* sizes

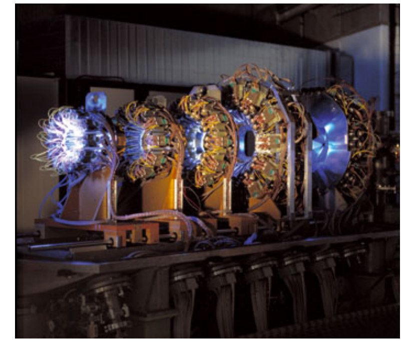
(Igor Mishustin)

Optimal collision energy



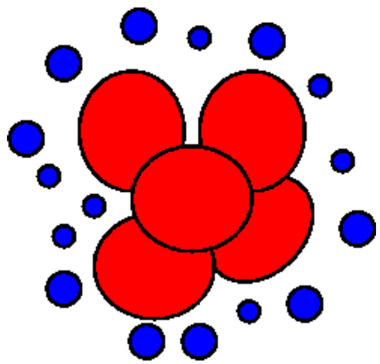
Experiment: *INDRA @ GANIL*

B. Borderie *et al*, Phys. Rev. Lett. 86 (2001) 3252



INDRA

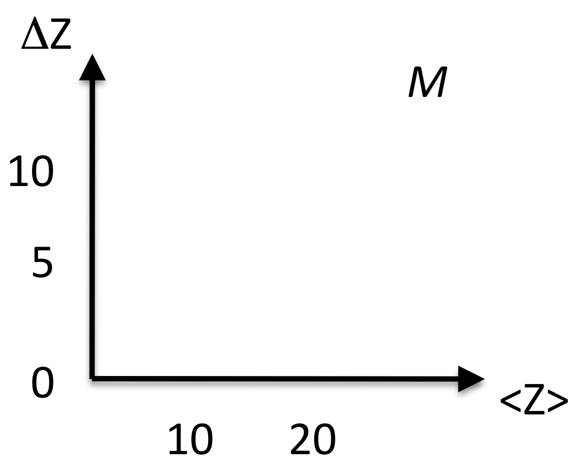
32 MeV/A Xe + Sn (b≈0)



Analysis:

For each event having *M* IMFs, calculate mean IMF charge <Z> and IMF charge dispersion ΔZ.
(L.G. Moretto)

Make LEGO plot of (<Z>, ΔZ):



For events with ΔZ=0, all *M* IMFs have the same charge

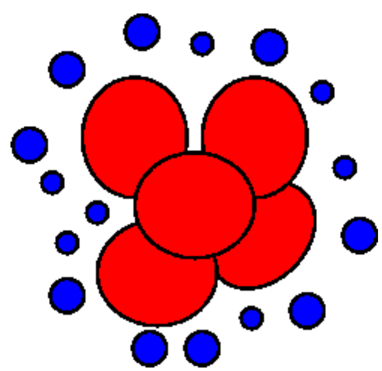
Transport calculations

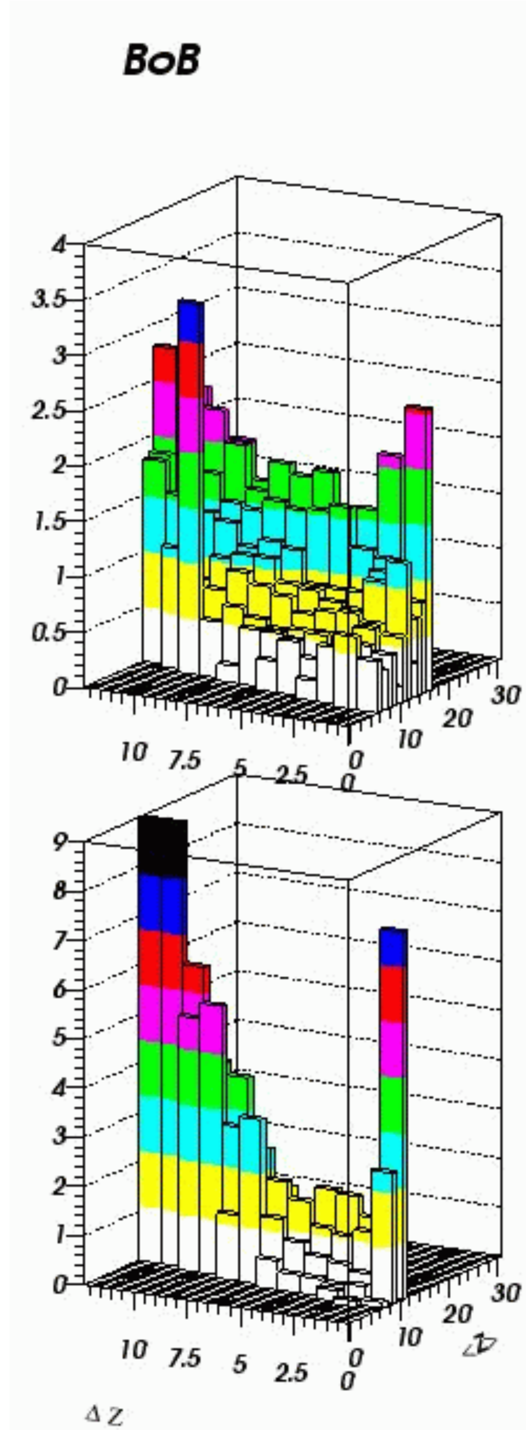
... suggest a visible spinodal signal:

Brownian One-Body dynamics *)
≈ Boltzmann-Langevin

$$\delta K[f] \;\rightarrow\; -\delta \boldsymbol{F} \cdot \frac{\partial f}{\partial \boldsymbol{p}}$$

32 MeV/A Xe + Sn (b=0):





M = 4

M = 6

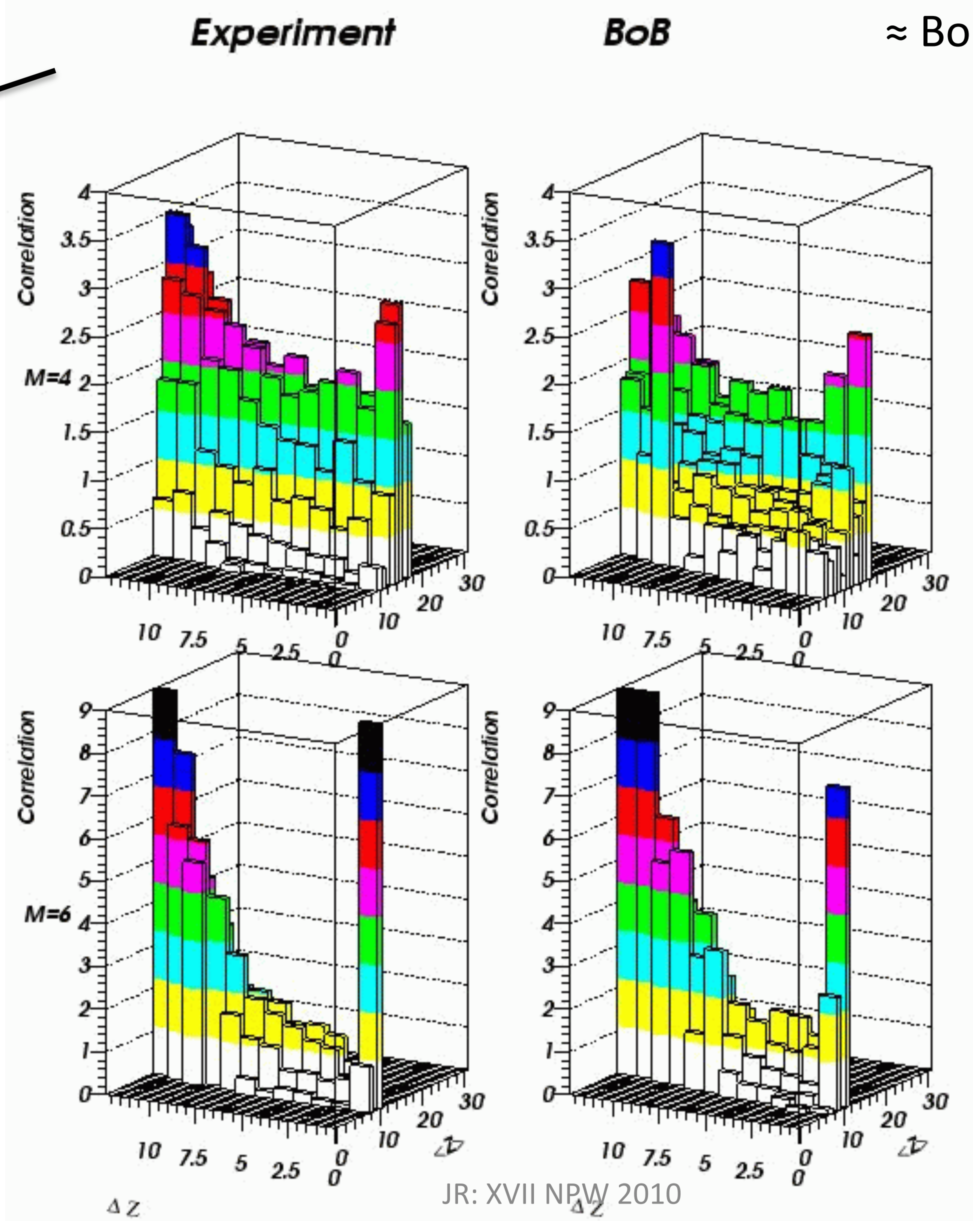
*) Ph. Chomaz, M. Colonna, A. Guarnera, J. Randrup, Physical Review Letters 73 (1994) 3512

Experiment: *INDRA @ GANIL*

$$\delta K[f] \rightarrow -\delta \boldsymbol{F} \cdot \frac{\partial f}{\partial \boldsymbol{p}}$$

Brownian One-Body dynamics
≈ Boltzmann-Langevin

B. Borderie *et al*, Phys. Rev. Lett. 86 (2001) 3252

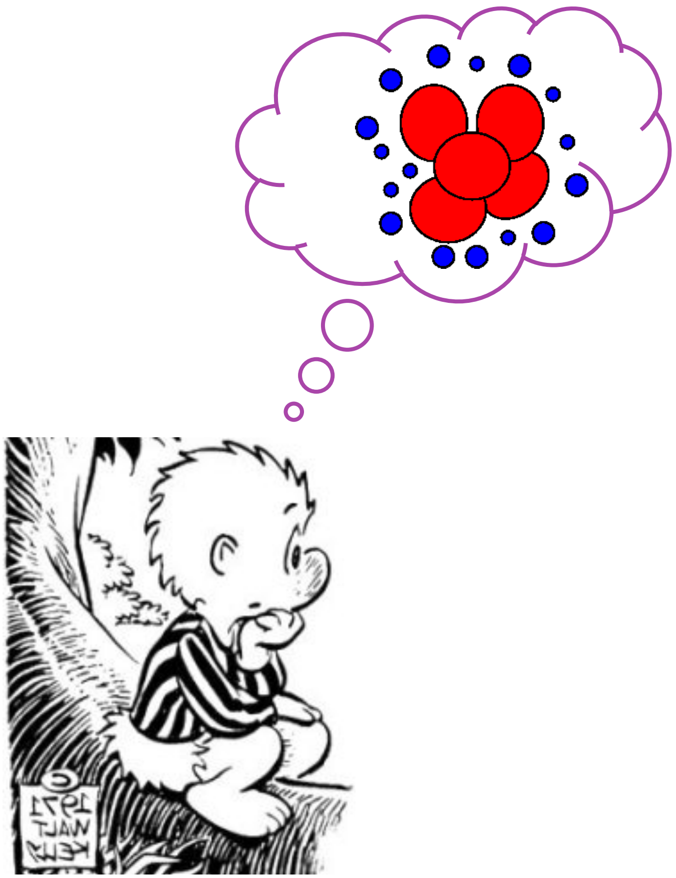


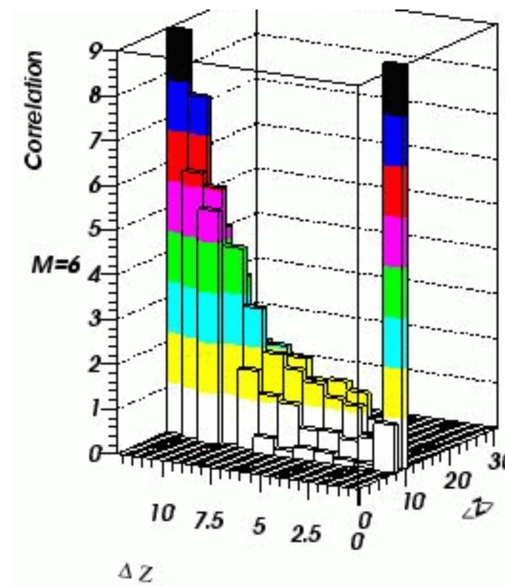
M = 4

M = 6

Ph. Chomaz *et al*, Phys. Rev. Lett. 73 (1994) 3512

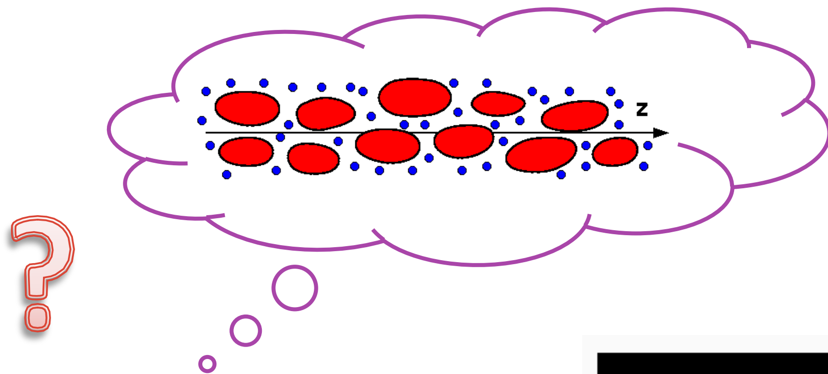
Spinodal phase separation does occur for the liquid-gas transition:

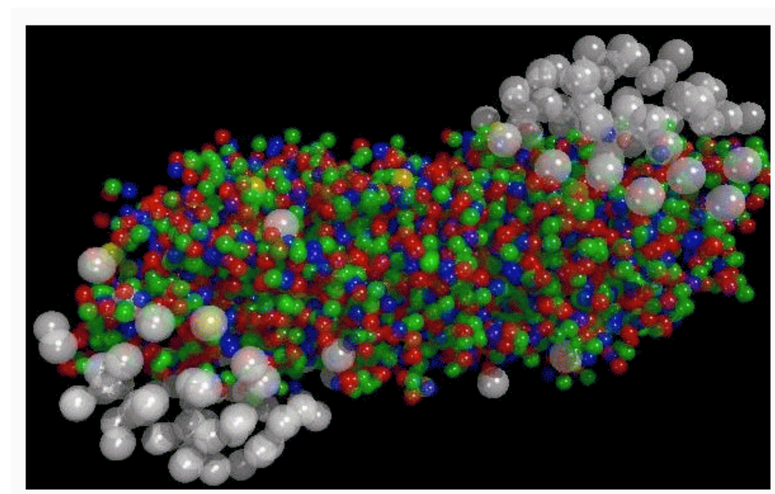




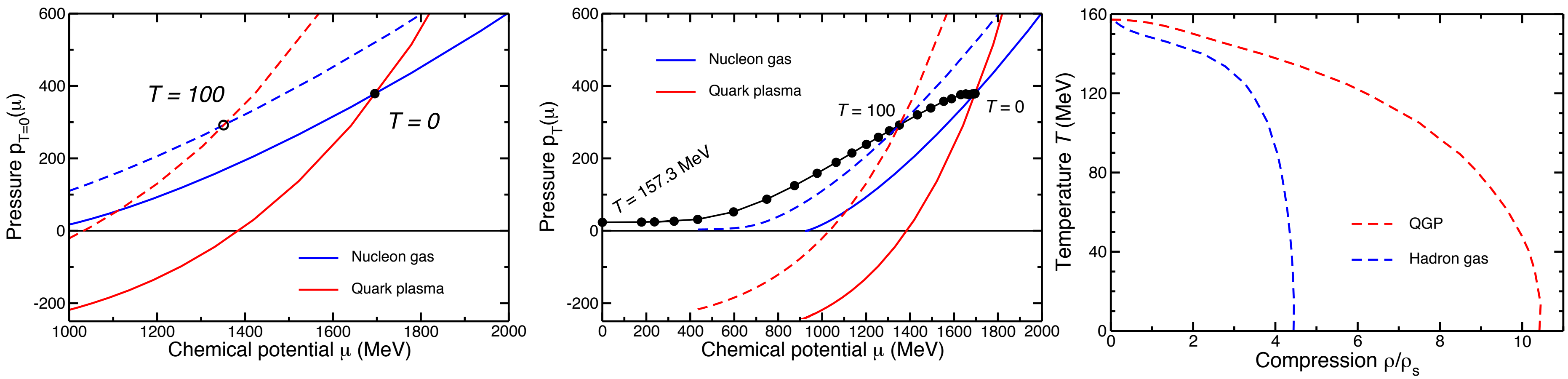
B. Borderie *et al:* PRL 86 (2001) 3252

Does spinodal phase separation occur for the confinement transition?





Idealized equations of state: HG -> QGP



Hadron Gas:

$$p^H = p_\pi + p_N + p_{\bar{N}} + p_w$$

$$p_\pi(T) = -g_\pi \int_{m_\pi}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 - \mathrm{e}^{-\beta\epsilon}]$$

$$p_N(T,\mu_0) = g_N \int_{m_N}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon-\mu_0)}]$$

$$p_{\bar{N}}(T,\mu_0) = g_N \int_{m_N}^{\infty} \frac{p\epsilon d\epsilon}{2\pi^2} \ln[1 + \mathrm{e}^{-\beta(\epsilon+\mu_0)}]$$

$$p_w(\rho) = \rho\partial_\rho w(\rho) - w(\rho)$$

$$w(\rho) = \left[-A\left(\frac{\rho}{\rho_s}\right)^\alpha + B\left(\frac{\rho}{\rho_s}\right)^\beta\right]\rho$$

Quark-Gluon Plasma:

$$p^Q = p_g + p_q + p_{\bar{q}} - B$$

$$p_g = g_g \frac{\pi^2}{90} T^4$$

$$p_q + p_{\bar{q}} = g_q \left[\frac{7\pi^2}{360} T^4 + \frac{1}{12}\mu_q^2 T^2 + \frac{1}{24\pi^2}\mu_q^4\right]$$

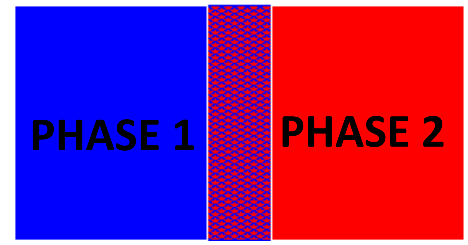
$$\mu = \mu_0 + \partial_\rho w = 3\mu_q$$

Interface equilibrium

finite-range correction

Free-energy density:

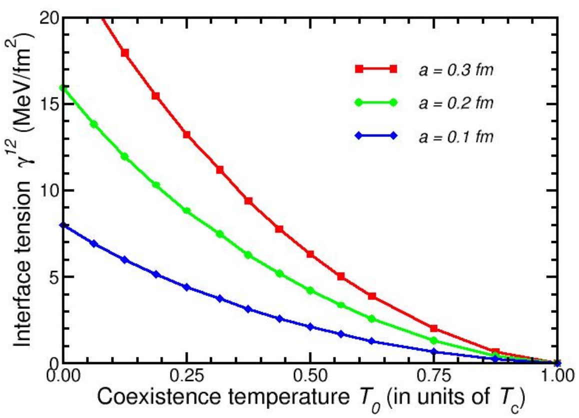
$$\tilde{f}_T(\boldsymbol{r}) = f_T(\tilde{\rho}(\boldsymbol{r})) + \boxed{\tfrac{1}{2}C(\boldsymbol{\nabla}\tilde{\rho}(\boldsymbol{r}))^2}$$

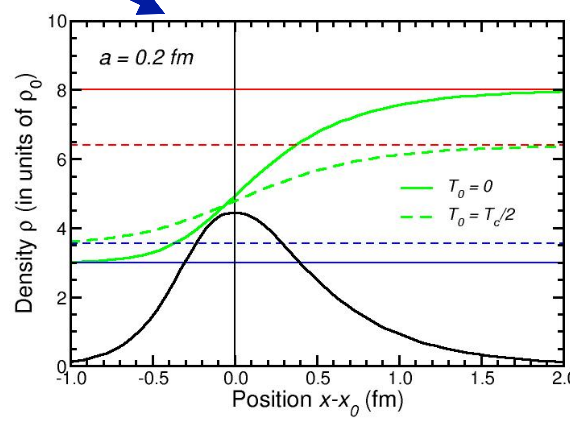


W.J. Swiatecki, Phys. Rev. 98 (1954) 203:
Nuclear Surface Energy and the Diffuseness of the Nuclear Surface

... we have found reasons to believe that the nuclear surface <u>energy</u> is intimately related to the <u>diffuseness</u> of the nuclear surface ...

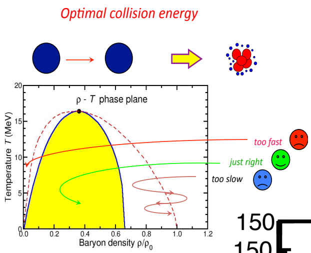
$\gamma_{12} \sim a$

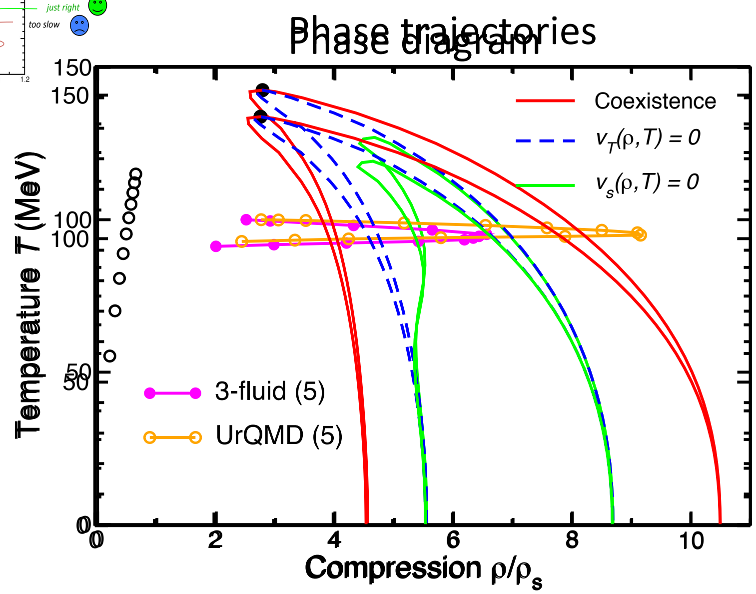


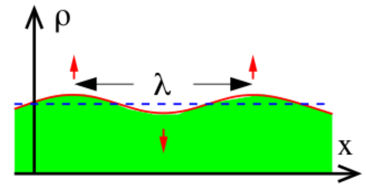


JR: Phys Rev C79 (2009) 054911

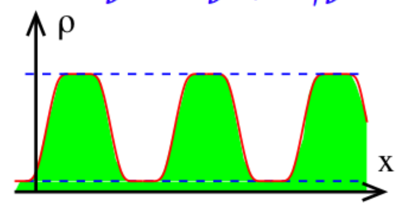
Spinodal phase separation







$$\omega_\nu = \epsilon_\nu + i\gamma_\nu$$



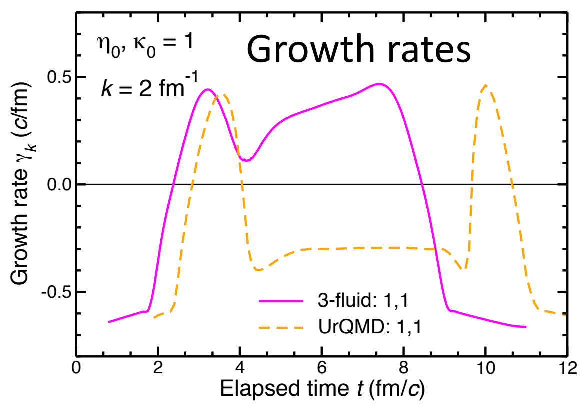
Irregularities are amplified

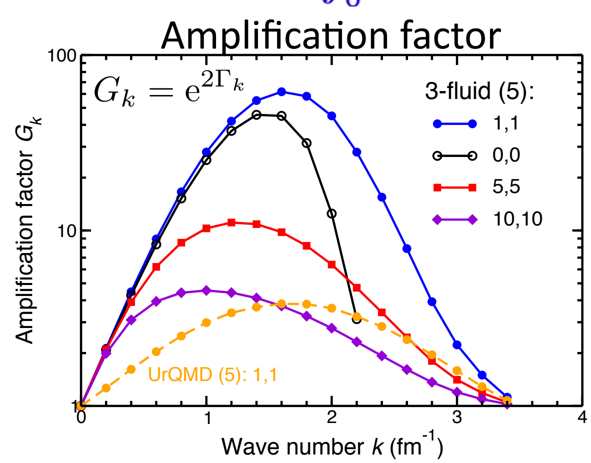
Dispersion relation

$$\omega^2 \doteq v_T^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 + \frac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v}k^2$$

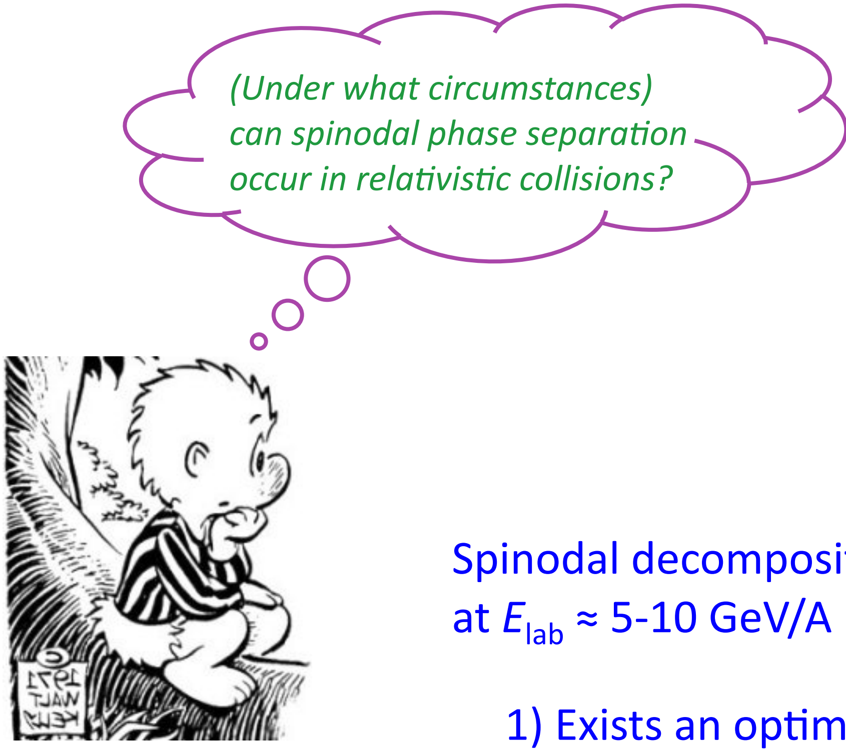
$$\Gamma_\nu(t) \equiv \int_0^t \gamma_\nu(t')dt'$$

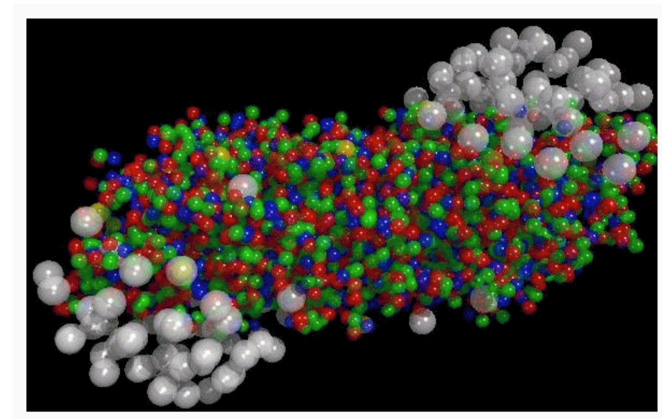
Amplification coefficient



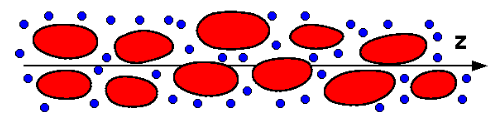


JR: Phys Rev C82 (2010) 034902





Spinodal decomposition *may* occur
at $E_{\text{lab}} \approx$ 5-10 GeV/A (FAIR & NICA):



1) Exists an optimal *E* range

2) Significant amplification

- but:

Full dynamical calculations are required!

No suitable transport model has yet been developed!!

17th Nuclear Physics Workshop "Marie & Pierre Curie")

17 – 22 September 2010 at Kazimierz Dolny, Poland

SYMMETRY & SYMMETRY BREAKING IN NUCLEAR PHYSICS

Spinodal phase separation in nuclear collisions

Jørgen Randrup, LBNL