Microscopic Corrections at Scission Configurations when Mass Symmetry Is Broken

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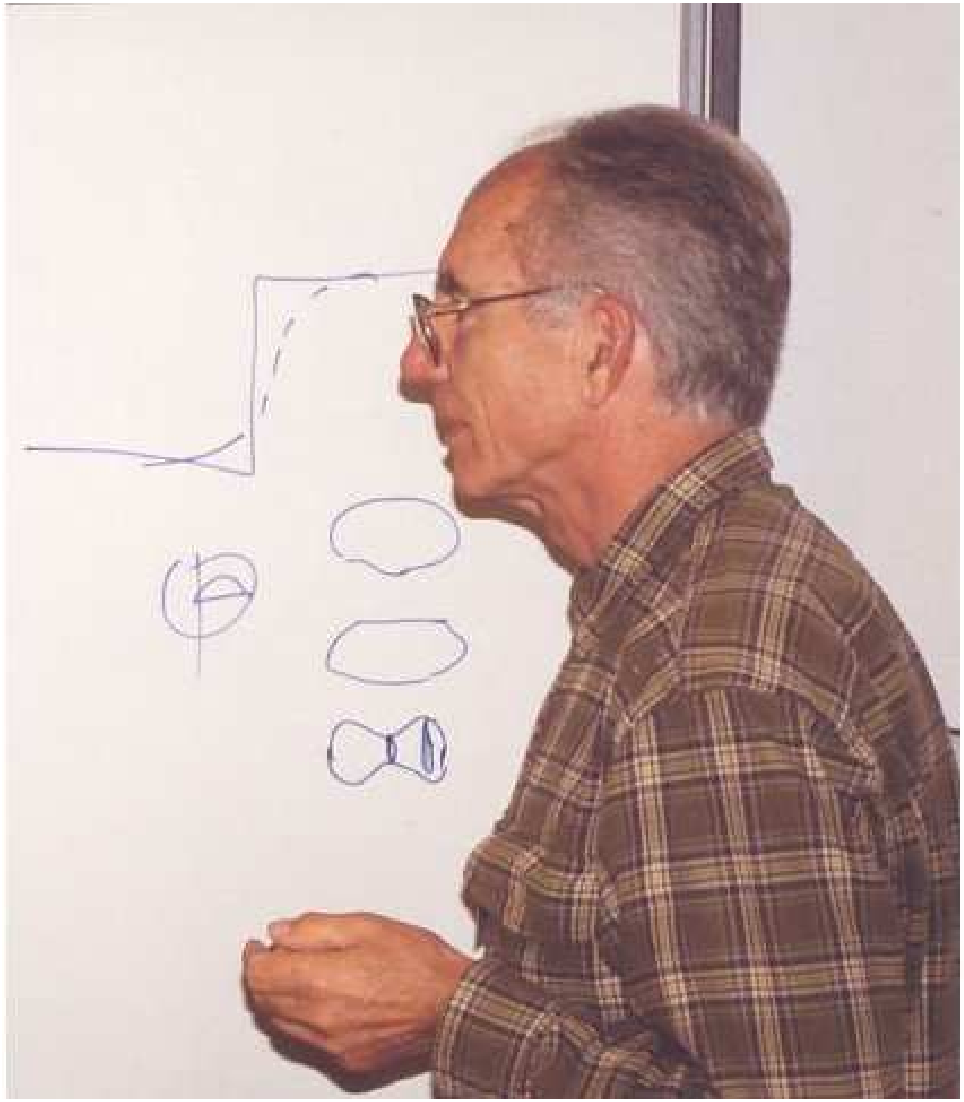
$17^{\rm th}$ Nuclear Physics Workshop

Kazimierz Dolny, September 22-26, 2010

During the 15^{th} Workshop in Kazimierz, 2008



Władysław Jan Świątecki (1926 - 2009)

My coworkers:

- Johann Bartel – IPHC and University of Strasbourg
- Artur Dobrowolski – University MCS, Lublin
- Fedir Ivaniuk – KINR, Kyiv
- Bożena Nerlo-Pomorska – University MCS, Lublin

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- Molecular structure of ^{232}Th in the 3^{rd} minimum.
- Macro–micro model with the Lublin–Strasbourg Drop.
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- Pairing correlations around scission configuration.
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- Topographical theorem of Świątecki and the saddle point masses and the barrier heights.
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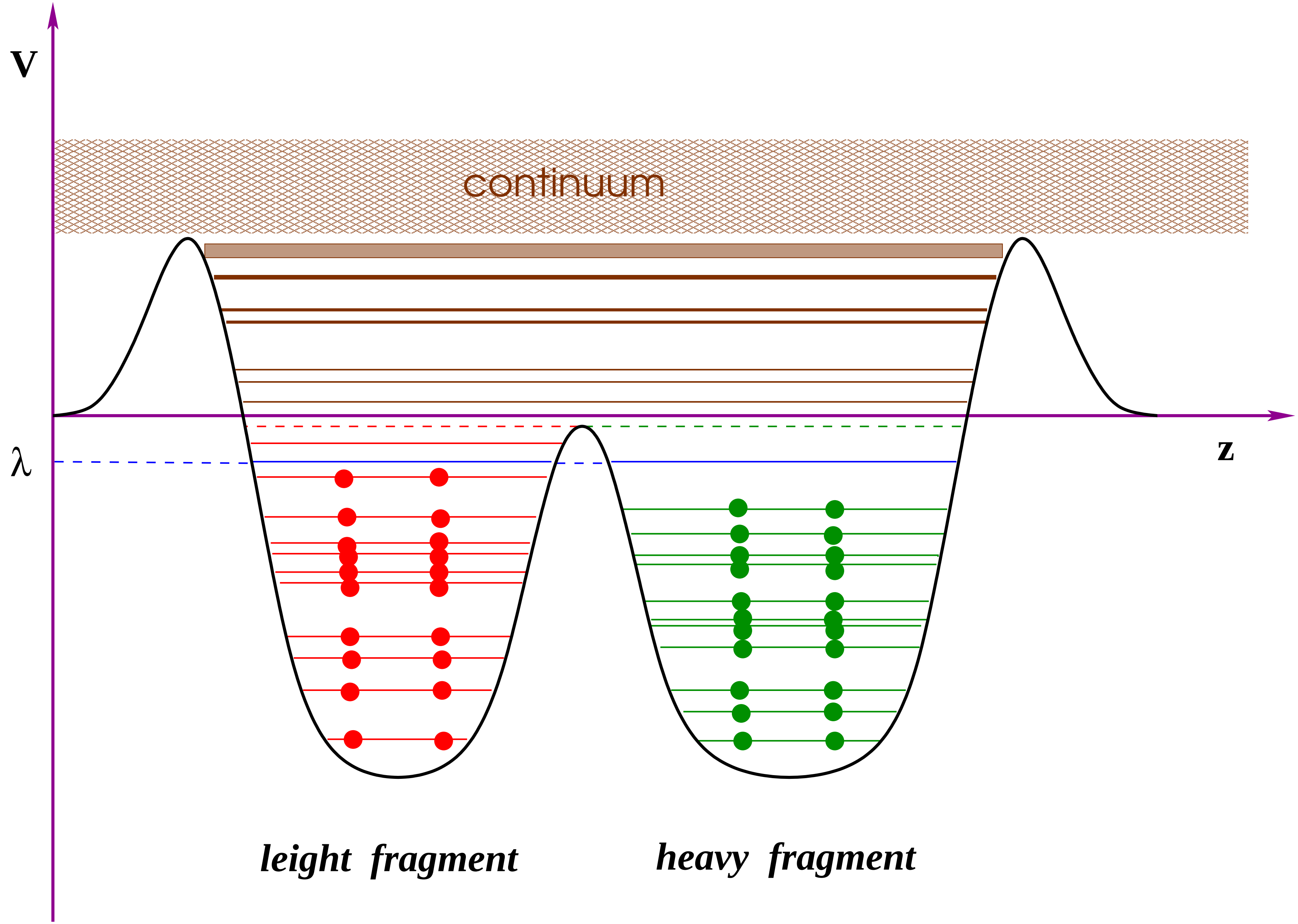
Single-particle levels in the nascent fragments



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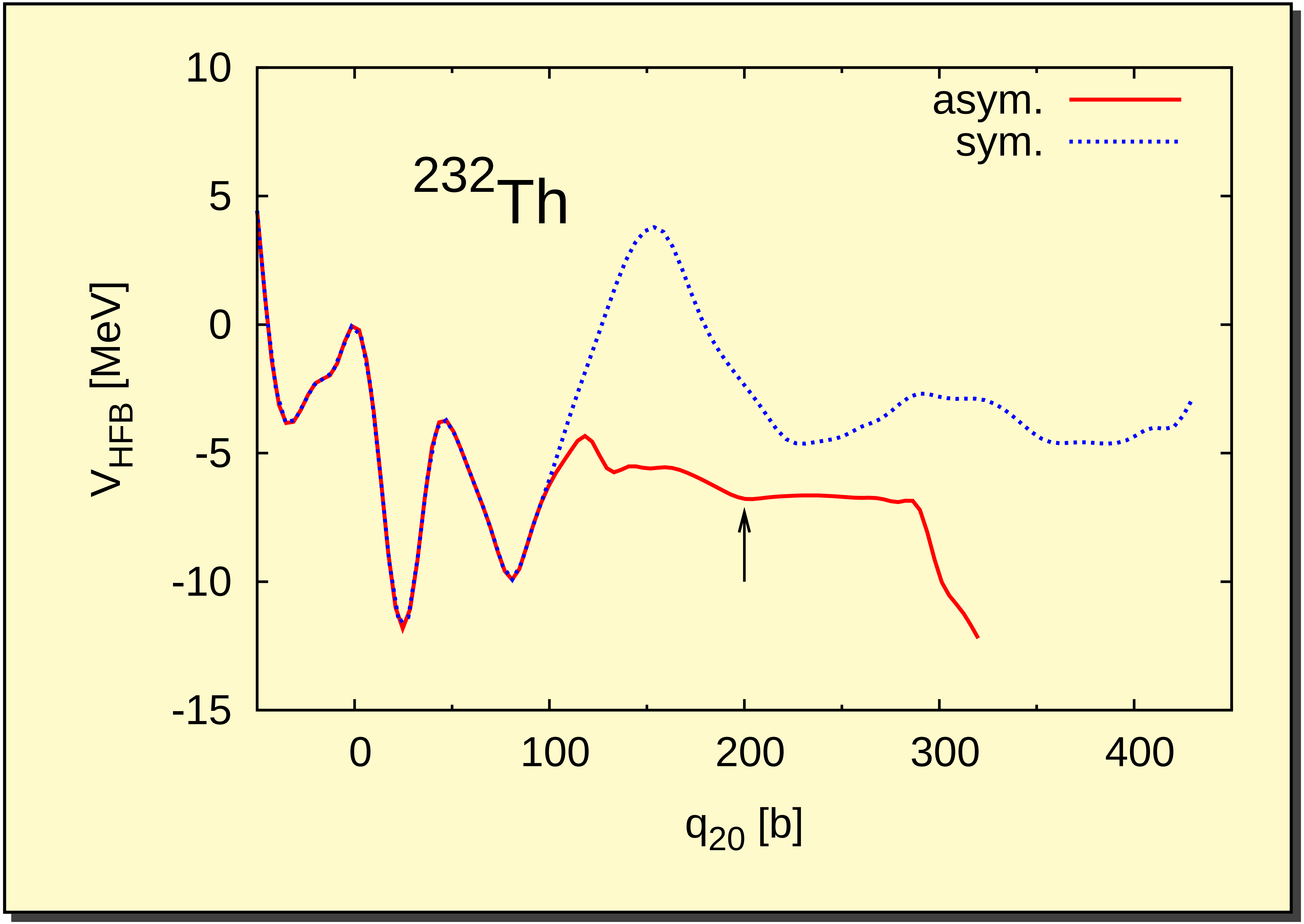
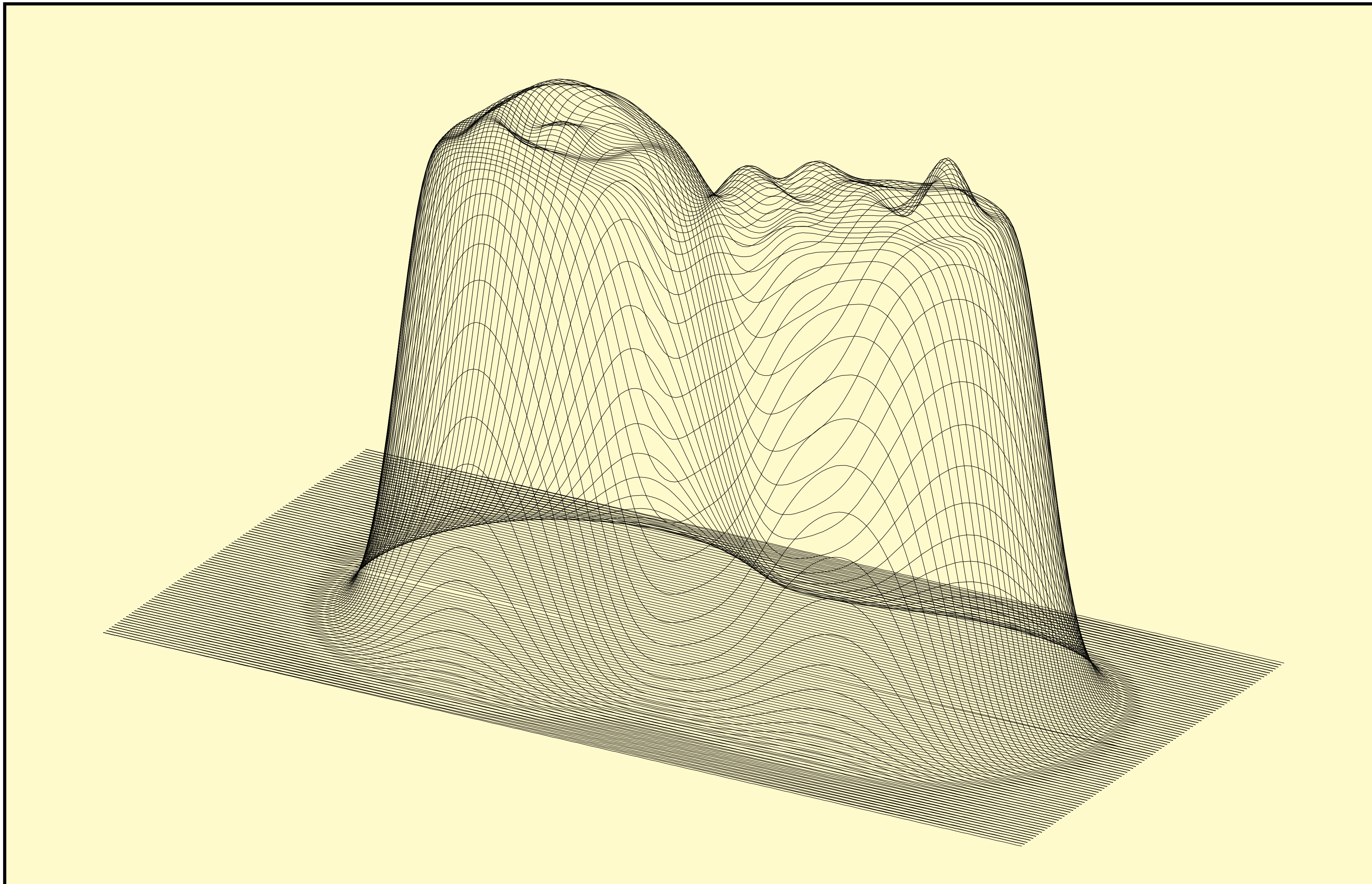
Fission barrier of ^{232}Th within the HFB+Gogny



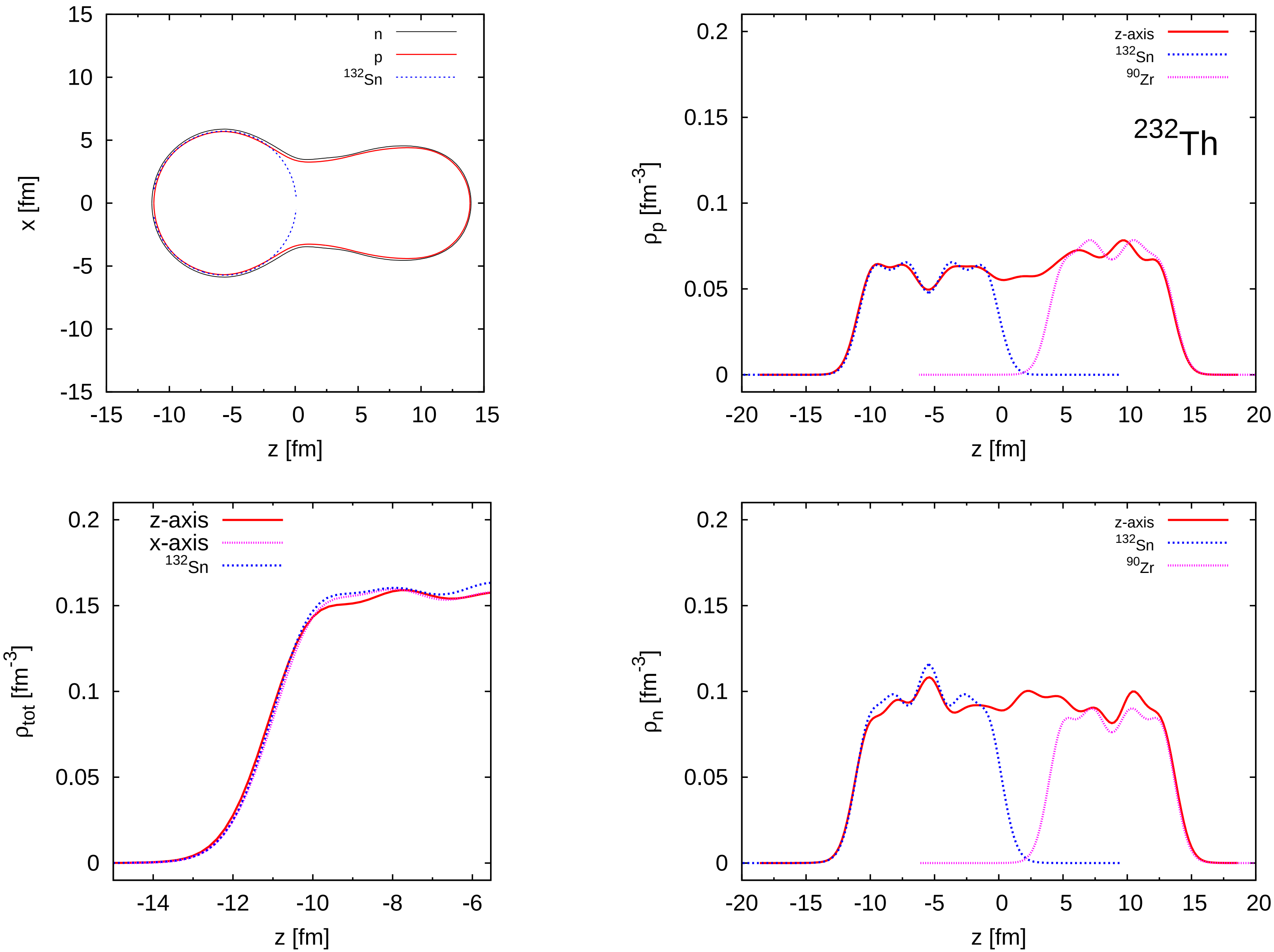
J.F. Berger and K. Pomorski, Phys. Rev. Lett. **85** (1999) 30.

Density of ^{232}Th in the 3^{rd} minimum



Densities evaluated within the HFB theory by J.F. Berger et al.

Molecular structure of ^{232}Th in the 3^{rd} minimum?



Macroscopic – Microscopic Model*:

$$M(Z,N;\mathrm{def}) = Z M_{\mathrm{H}} + N M_{\mathrm{n}} - b_{\mathrm{elec}}\, Z^{2.3}$$

$$+\, b_{\mathrm{vol}}\ (1 - \kappa_{\mathrm{vol}}\, I^2\,)\, A$$

$$+\, b_{\mathrm{surf}}\, (1 - \kappa_{\mathrm{surf}} I^2\,)\, A^{2/3} B_{\mathrm{surf}}(\mathrm{def})$$

$$+\, b_{\mathrm{cur}}\ (1 - \kappa_{\mathrm{cur}}\, I^2\,)\, A^{1/3} B_{\mathrm{cur}}(\mathrm{def})$$

$$+\, \frac{3}{5}\, \frac{e^2 Z^2}{r_0^{ch} A^{1/3}}\, B_{\mathrm{Coul}}(\mathrm{def}) -\ C_4\, \frac{Z^2}{A}$$

$$+\, E_{\mathrm{micr}}(Z,N;\mathrm{def}) + E_{\mathrm{cong}}(Z,N)$$

*W.D. Myers and W.J. Świątecki, Nucl. Phys. **81**, 1 1966.

Shell and Pairing Corrections:

where $$E_{\rm micr} = \delta E_{\rm shell} + \delta E_{\rm pair} \ ,$$

$$\delta E_{\rm shell} = \sum_{occ} 2e_\nu - \langle \sum_{occ} 2e_\nu \rangle_{\rm Strut} \ .$$

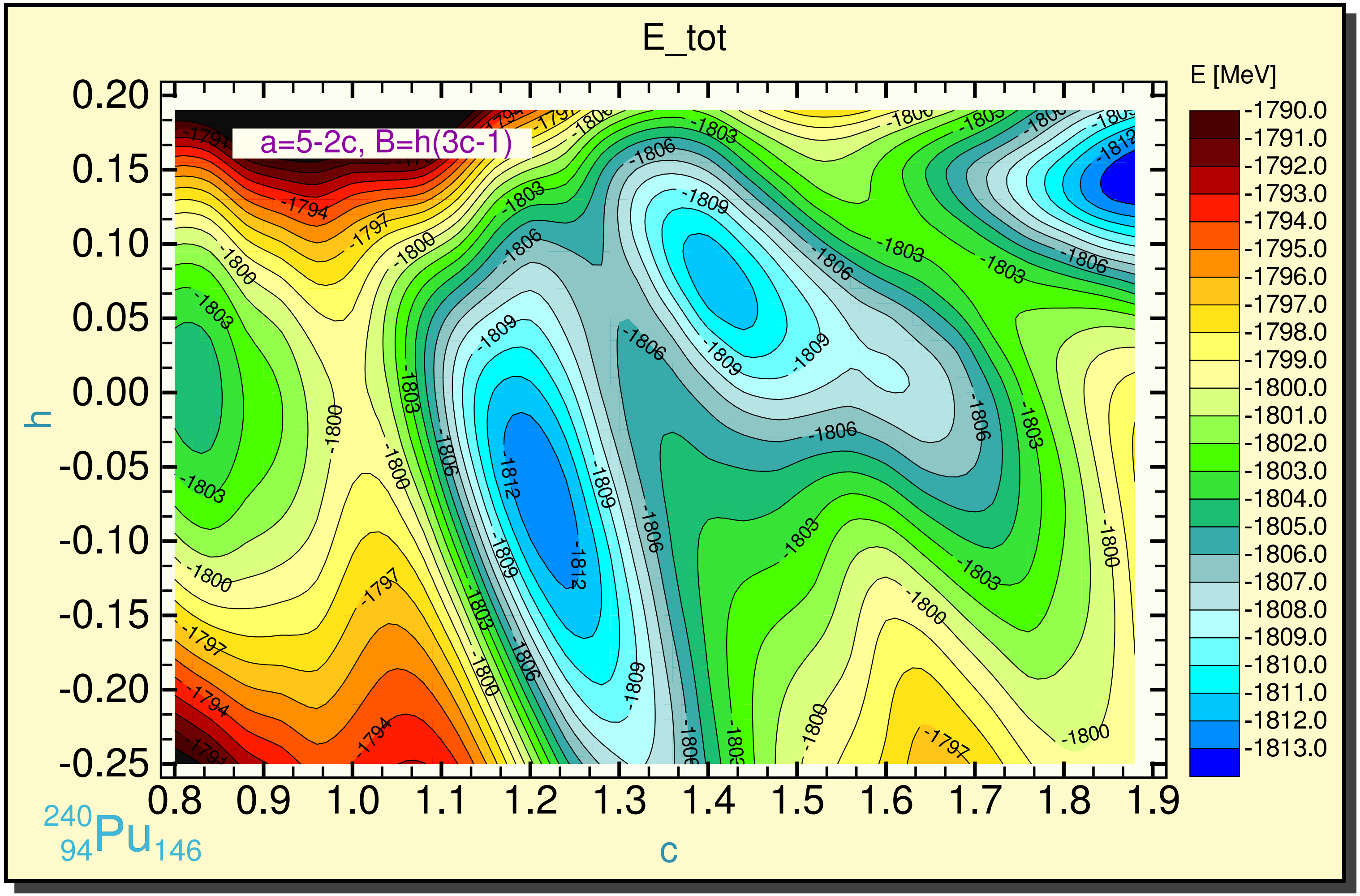
$$\delta E_{\rm pair} = E_{\rm pair} - \langle \, E_{\rm pair} \, \rangle \ ,$$

with $$E_{\rm pair} = E_{\rm BCS} - \sum_{occ} 2e_\nu \ ,$$

and

$$E_{\rm BCS} = \sum_\nu 2v_\nu^2 e_\nu - G \sum_\nu u_\nu v_\nu - G \sum_\nu v_\nu^4$$

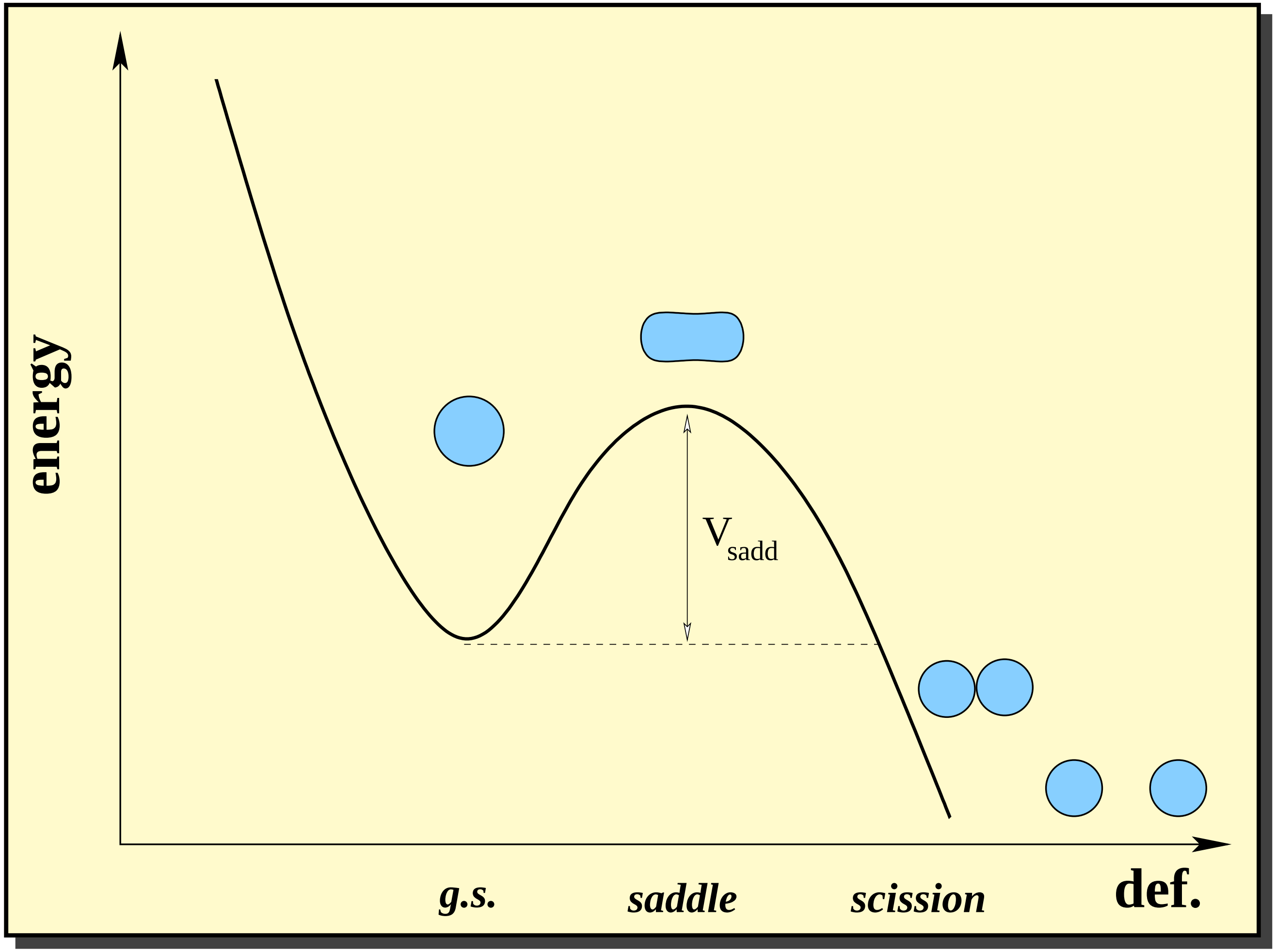
PES of ^{240}Pu on the (c,h) plane for $\alpha=0$, $\eta=0$:



LSD with shell and pairing corrections are obtained with the Yukawa-folded single-particle potential.

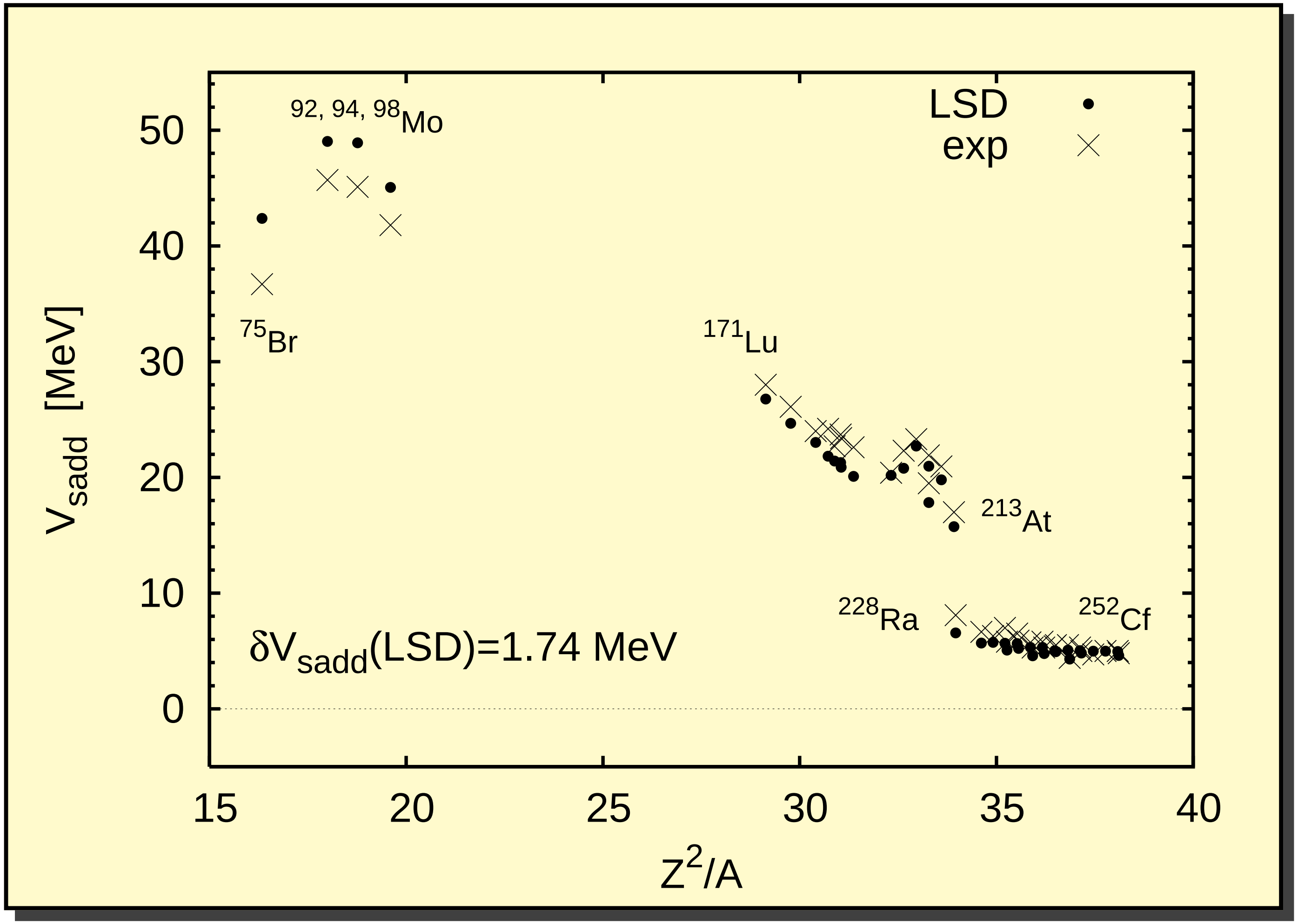
A. Dobrowolski, K. Pomorski, J. Bartel, Phys. Rev. **C75**(2007) 024613.

Fission barrier:



$$V_B = M_{\mathbf{sadd}} - M_{\mathbf{g.s.}}$$

Fission barrier heights:



Deformation dependent congruence energy term is included here according to:
W.D. Myers, W.J. Świątecki, Nucl. Phys. A612 (1997) 249.
K. Pomorski, J. Dudek, Int. Journ. Mod. Phys. E13 (2004) 107.

Strutinsky smoothed energy

In this method one evaluates first the smooth s.p. particle level density $\tilde{g}(e)$ by folding the discrete spectrum of s.p. energies e_ν

$$g(e) = \sum_\nu \delta(e - e_\nu) \quad \longrightarrow \quad \tilde{g}(e) = \frac{1}{\gamma_S} \sum_\nu j_n\left(\frac{e - e_\nu}{\gamma_S}\right) \ ,$$

where for $n = 6$ $j_6(x) = \frac{1}{\sqrt{\pi}} e^{-x^2} (\frac{35}{16} - \frac{35}{8} x^2 + \frac{7}{4} x^4 - x^6)$.

According to Strutinsky the smoothed s.p. energy is given by

$$\tilde{E}_{\rm Str} = \int\limits_{-\infty}^{\tilde{\lambda}} 2\, e\, \tilde{g}(e)\, de \ , \qquad \mathcal{N} = \int\limits_{-\infty}^{\tilde{\lambda}} 2\, \tilde{g}(e)\, de \ .$$

where $\tilde{\lambda}$ is the Fermi energy in a system without the shell structure.

Additive property of the shell corrections:

Let us consider two separated fission fragments with the s.p. spectra $\{e\} \equiv \{e^l, e^h\}$ and having the same average Fermi energies as the mother system: $\tilde{\lambda}^l = \tilde{\lambda}^h = \tilde{\lambda}$ One can easy show that for such a systems the following relations are hold:

$$\tilde{E}_{\mathrm{Str}} = \tilde{E}^l_{\mathrm{Str}} + \tilde{E}^h_{\mathrm{Str}} \quad \textbf{and} \quad \mathcal{N} = \mathcal{N}^l + \mathcal{N}^h$$

what means that

$$E_{shell} = \sum_{occ} 2e_\nu - \tilde{E}_{\mathrm{Str}} = \sum_{occ} 2e^l_\nu + \sum_{occ} 2e^r_\nu - \tilde{E}^l_{\mathrm{Str}} - \tilde{E}^h_{\mathrm{Str}}$$

and

$$\boxed{E_{shell} = E^l_{shell} + E^r_{shell}}$$

The same is not true for the monopole pairing energy as the average pairing gaps could be different in the both fragments.

Pairing correlations in almost separated systems:

Let us consider two separated fission fragments with the s.p. spectra $\{e\} \equiv \{e^l, e^h\}$ described by the following Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{H}_{pair} = \sum_{\nu} e_\nu \left(a_\nu^+ a_\nu + a_{\bar{\nu}}^+ a_{\bar{\nu}}\right) - G \sum_{\nu,\mu} a_\nu^+ a_{\bar{\nu}}^+ a_{\bar{\mu}} a_\mu$$

which can be rewritten as

$$\hat{H}_0 = \hat{H}_0^l + \hat{H}_0^h - G_l \hat{P}_l^+ \hat{P}_l - G_h \hat{P}_h^+ \hat{P}_h - G_{lh} \left(\hat{P}_l^+ \hat{P}_h + P_h^+ \hat{P}_l\right.$$

Here $\hat{P}_l^+ = (\hat{P}_l)^+$, $\hat{P}_l = \sum_\nu a_{\bar{\nu}}^l a_\nu^l$ and $\hat{P}_h = \sum_\nu a_{\bar{\nu}}^h a_\nu^h$.

It is obvious that in case of the separated fragments the approximation $G_l = G_h = G_{lh} = G$ is not valid any more and it rather holds $G_l \geq G_h > G$ and $G_{lh} \approx 0$.

Pairing correlations in almost separated systems:

In such approximation the BCS equations takes the following form:

$$\frac{2}{G} = \sum_{\nu} \frac{1}{E_{\nu}} \neq \sum_{\nu} \frac{1}{E^l_{\nu}} + \sum_{\nu} \frac{1}{E^h_{\nu}} = \frac{2}{G_l} + \frac{2}{G_h}$$

and

$$N = \sum_{\nu} 2v_{\nu}^2 = \sum_{\nu} 2(v^l_{\nu})^2 + \sum_{\nu} 2(v^h_{\nu})^2 = N_l + N_h \ ,$$

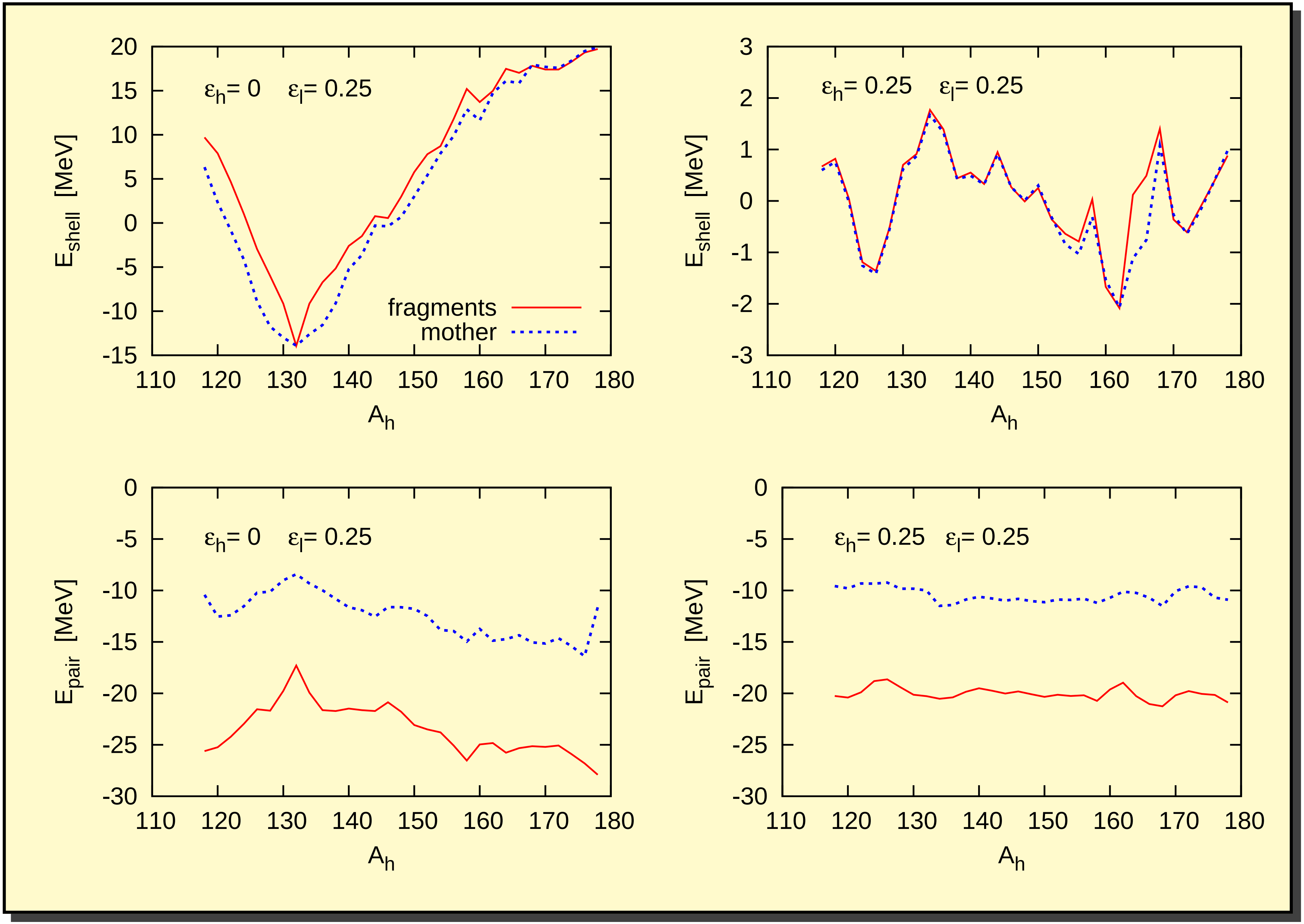
where

$$E^l_{\nu} = \sqrt{(e^l_{\nu} - \lambda_l)^2 + \Delta_l^2} \ ,$$

$$(v^l_{\nu})^2 = \frac{1}{2}\left(1 - \frac{e^l_i - \lambda_l}{E^l_i}\right) \ , \quad \text{etc.}$$

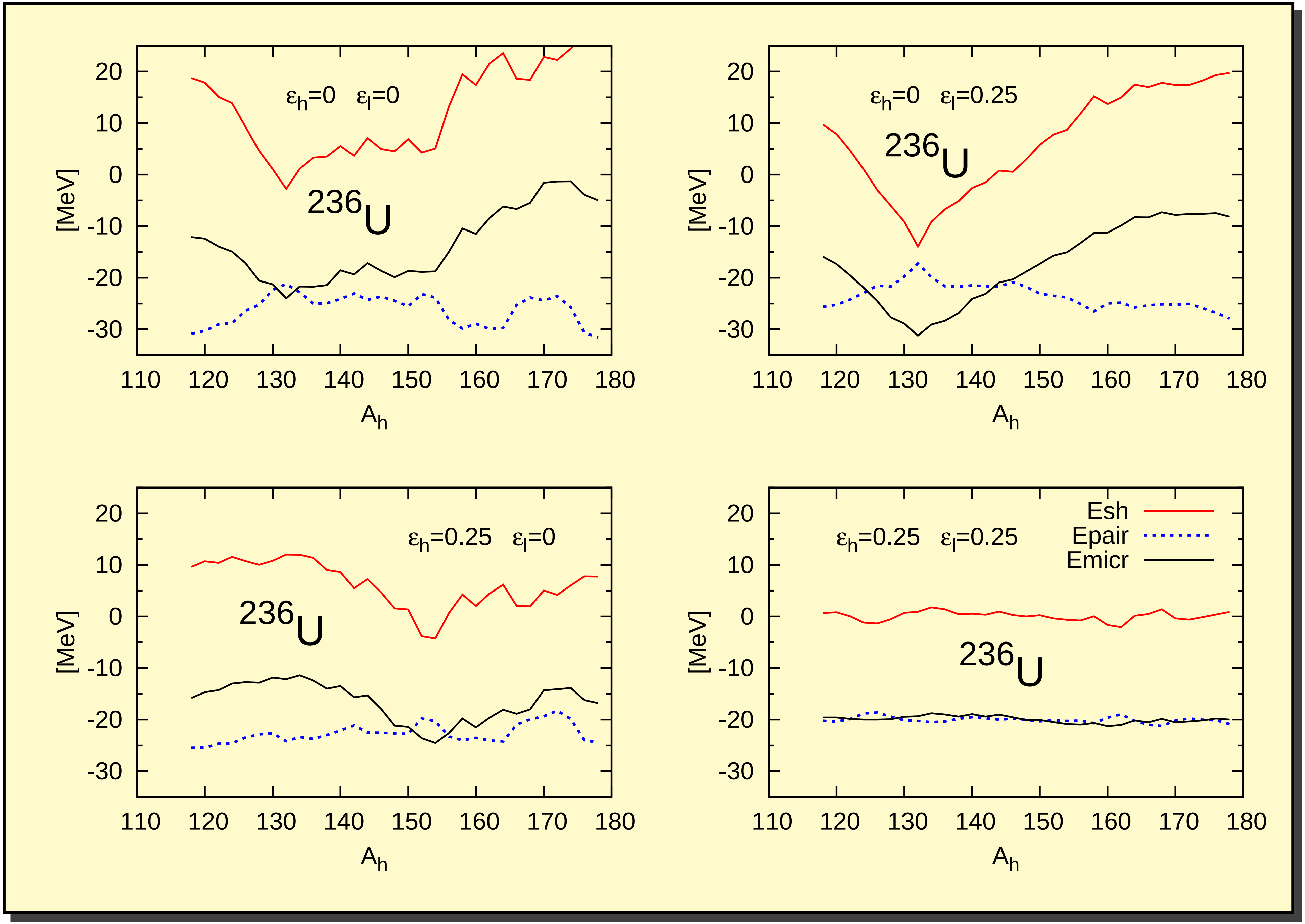
with $\Delta \neq \Delta_l \neq \Delta_h$ and $\lambda \approx \lambda_l \approx \lambda_h$.

Shell and pairing energy of fragments and mother nucleus:



Two Nilsson wells are used here.

Shell and pairing energy of two separated fragments:



Average pairing energy:

$$E_{\text{pair}} = \sum_{\nu} 2v_{\nu}^{2}e_{\nu} - G\sum_{\nu} u_{\nu}v_{\nu} - G\sum_{\nu} v_{\nu}^{4} - \sum_{\text{occ}} 2e_{\nu}\ ,$$

After replacing sums by integrals and assuming $\Omega \gg \Delta$

$$\sum_{\nu}(...) \to \frac{1}{2}\int_{\lambda-\Omega}^{\lambda+\Omega}(...)\tilde{g}(e)de$$

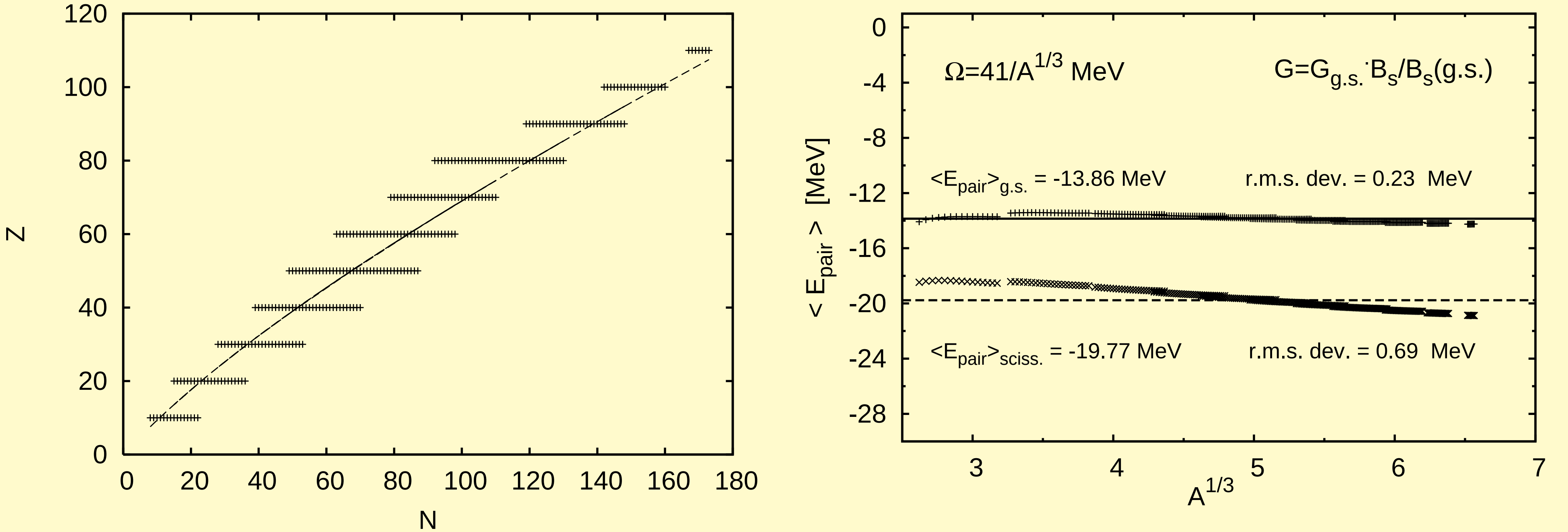
the average pairing energy becomes:

$$\widetilde{E}_{\text{pair}} \approx -\frac{1}{4}\tilde{g}(\lambda)\Delta^{2} - G\,\frac{N}{2} + \ ....$$

The average gap equation takes the following form:

$$\Delta = G\sum_{\nu} u_{\nu}v_{\nu} \quad \to \quad \frac{2}{G} \approx g(\lambda)\ln\left(\frac{2\Omega}{\Delta}\right)$$

Average pairing energy*:



The pairing strength was evaluated using the average experimental gaps

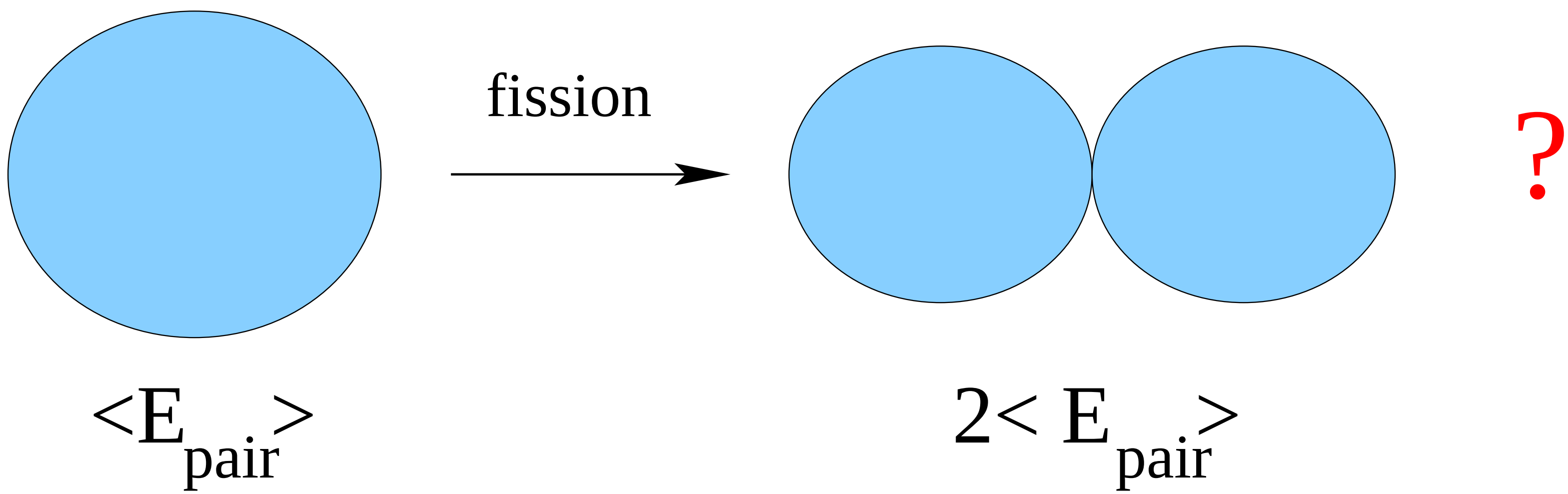
$$\bar{\Delta}_{\rm exp}^{(p)} = \frac{4.8\,B_s}{Z^{1/3}}{\rm MeV}\;; \qquad \bar{\Delta}_{\rm exp}^{(n)} = \frac{4.8\,B_s}{N^{1/3}}{\rm MeV}$$

taken from Ref. {P. Möller and J.R. Nix, Nucl. Phys. A536 (1992) 61 }.

*K. Pomorski, F. Ivanyuk, Int. Journ. Mod. Phys. **E18** (2009) 900.

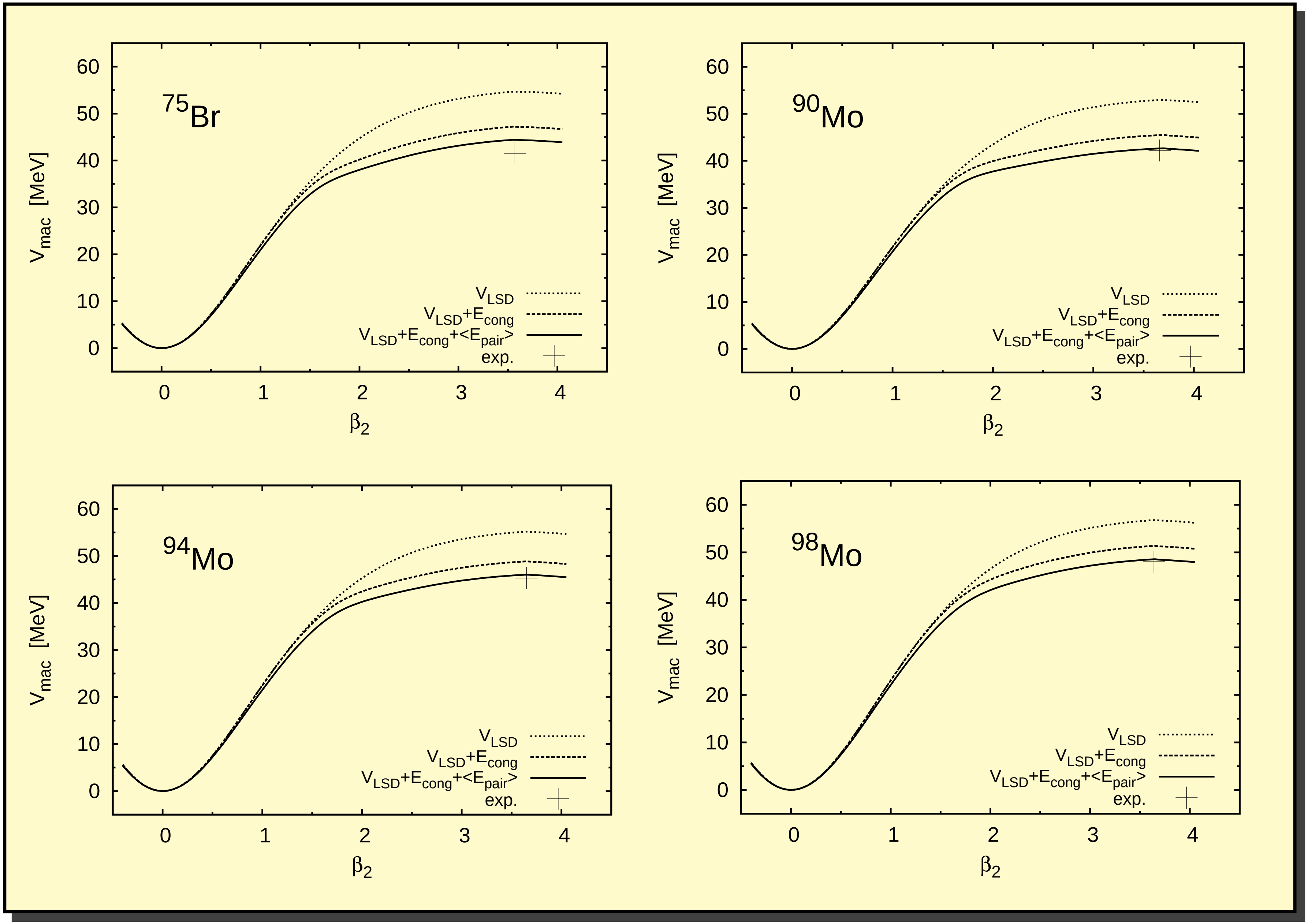
Average pairing energy is A almost independent !!

- **What does it mean?**
- **What will happen with the pairing energy when nucleus fission into two fragments?**



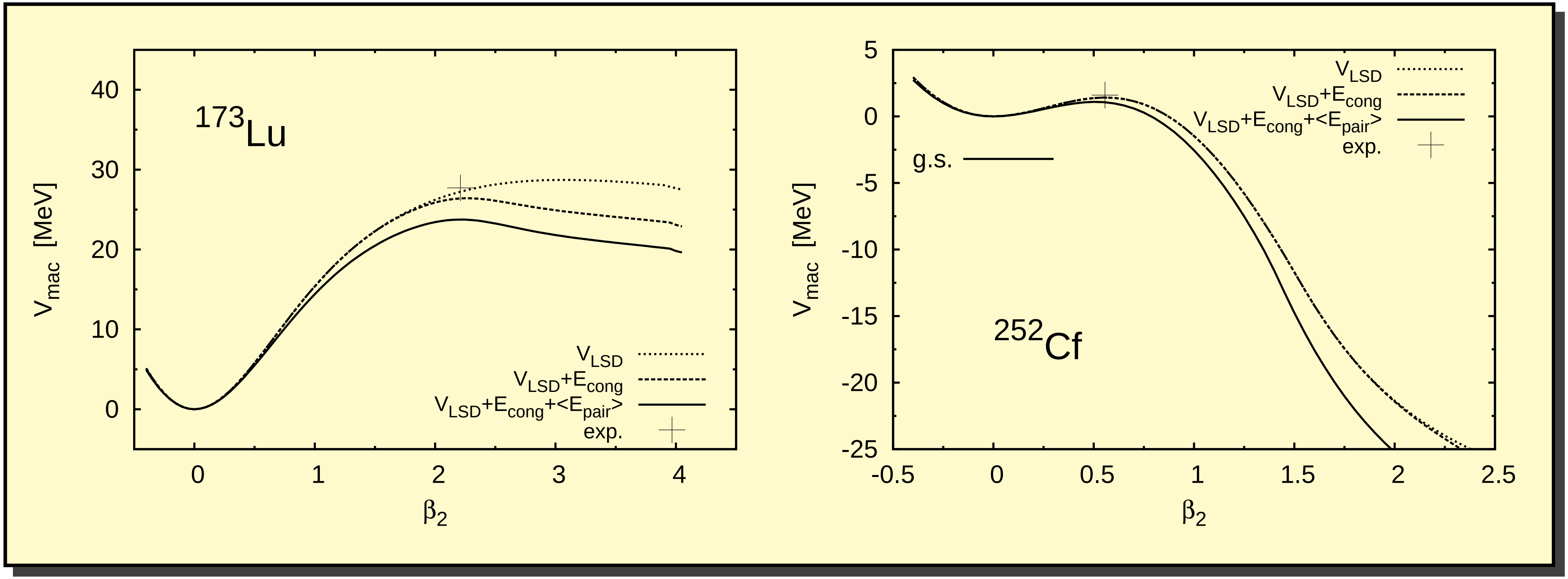
- **Should the pairing strength depend on deformation of fissioning nucleus?**

Effect of the congruence and the average pairing*:



*K. Pomorski, F. Ivanyuk, Int. Journ. Mod. Phys. **E18** (2009) 900.

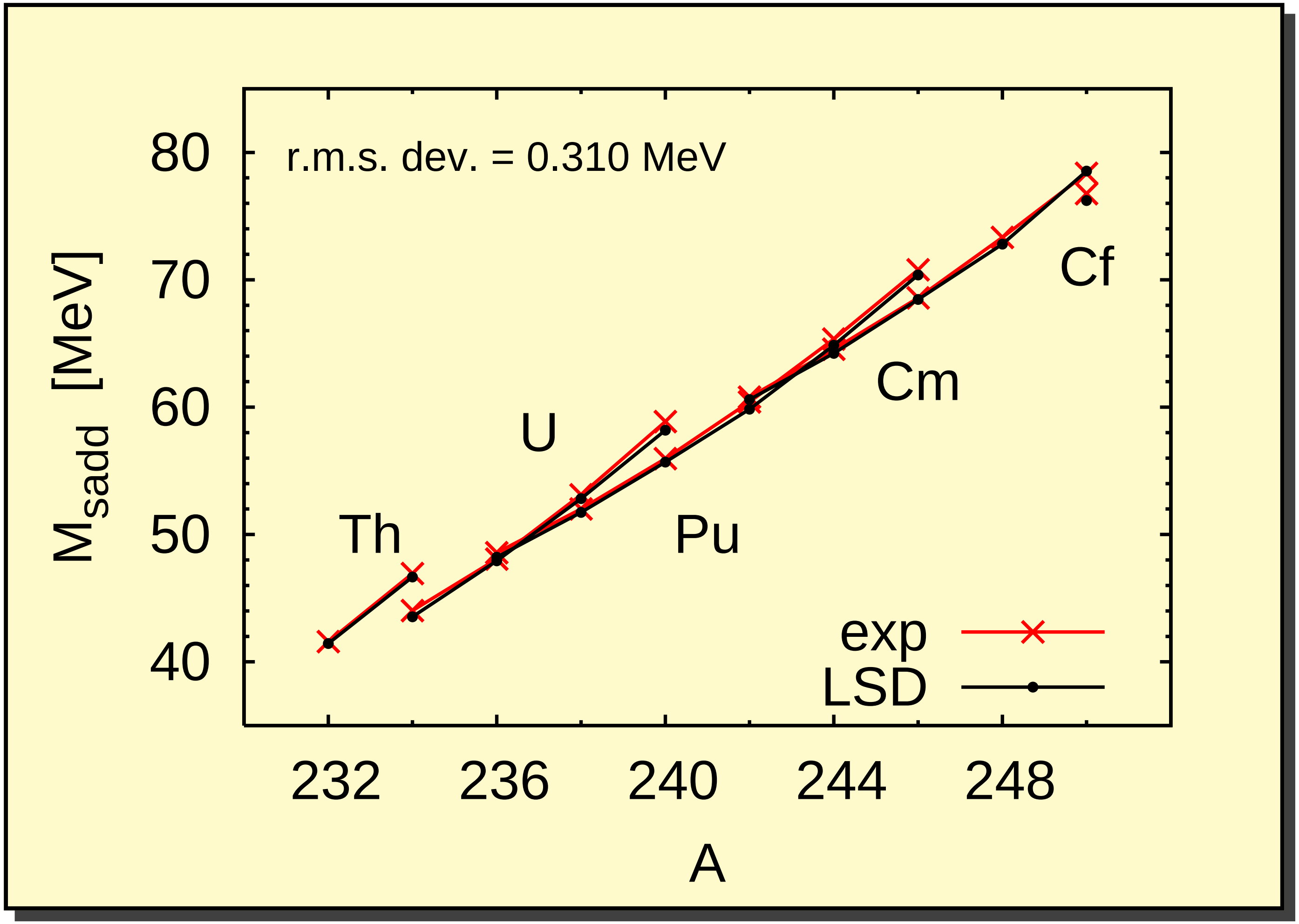
Effect of the congruence and the average pairing*:



*K. Pomorski, F. Ivanyuk, Int. Journ. Mod. Phys. **E18** (2009) 900.

Topographical theorem of Swiatecki

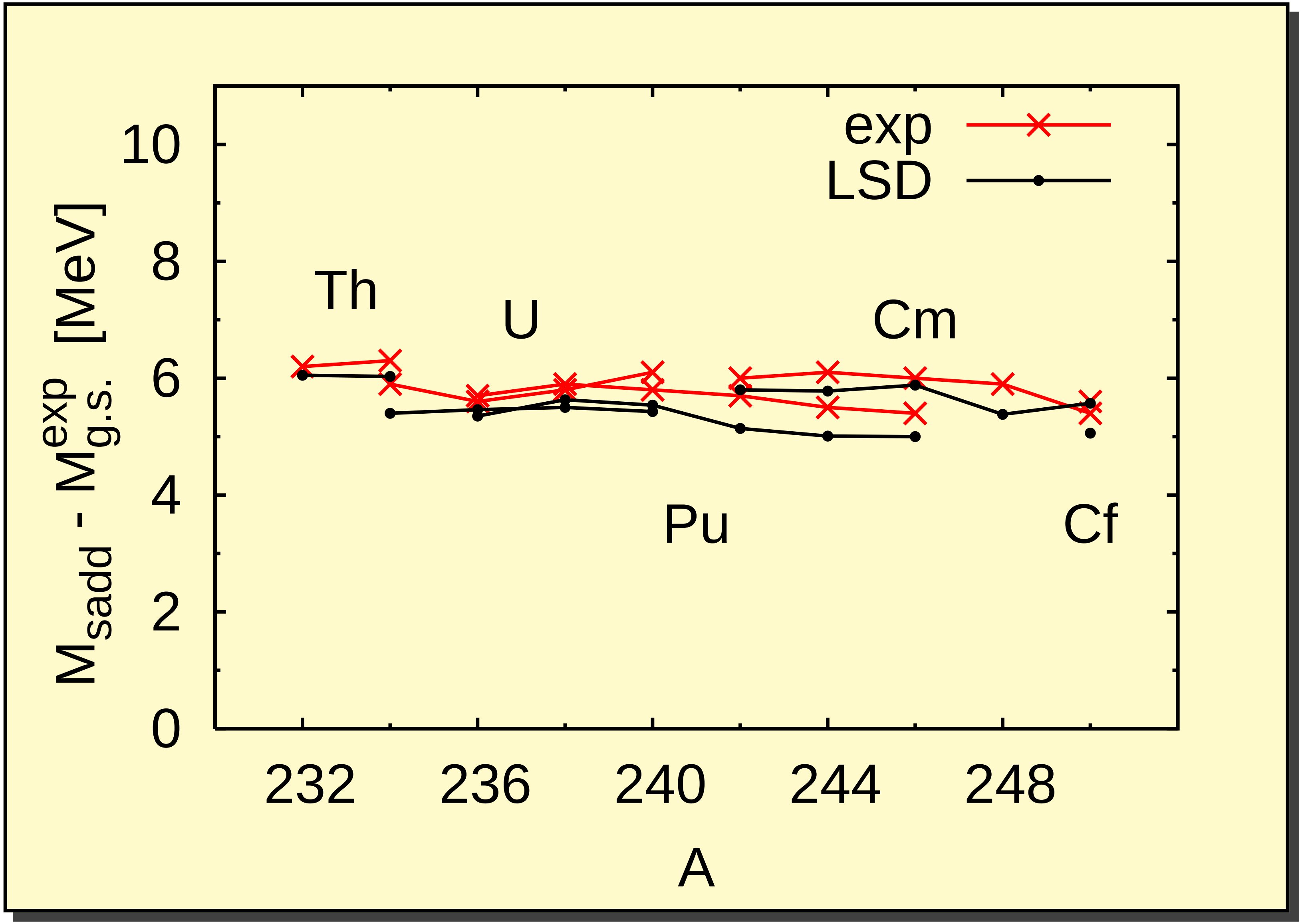
Pure LSD saddle point masses of heavy nuclei:



W. D. Myers, W.J. Świątecki, Nucl.Phys. **A612** (1997) 249. ⟵ Topographical theorem

J. Bartel, A. Dobrowolski, and K. Pomorski, IJMP **E16**, 459 (2007)

LSD barriers according to the topographical theorem



Summary:

- Binuclear structure of fissioning nuclei is manifested already at deformations corresponding to the third minimum.
- Shell energies of the fission fragments are additive and their sum is close to the shell energy of the common system.
- Pairing energies could be different in each nascent fragment and their sum differs from the pairing energy evaluated for the common system.
- Inclusion of the deformation dependent congruence (Wigner) energy and taking the pairing strength proportional to the surface area improves significantly the estimates of the barrier heights of the light nuclei.
- **L**ublin **S**trasburg **D**rop describes well masses of the known isotopes both in the ground state and saddle points.