Shape evolution in even-even Mo isotopes studied via Coulomb excitation

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Outline

- Motivation.
- Quadrupole Sum Rules method – practical application to the stable Mo isotopes.
- Quadrupole deformation parameters of the heaviest stable Mo isotopes.
- Comparison of the experimentally obtained results with the General Bohr Hamiltonian calculations.
- Conclusions

Motivation

- Transitional nuclei (A~100) challenging for nuclear structure theories: competition of single-particle and collective excitation modes.
- In the Sr and Zr isotopic chain a dramatic change of the ground state structure is observed at N = 58, 60.
- This effect is less pronounced in Mo isotopes, but still the rapidity of shape change gives rise to shape coexistence in these nuclei.

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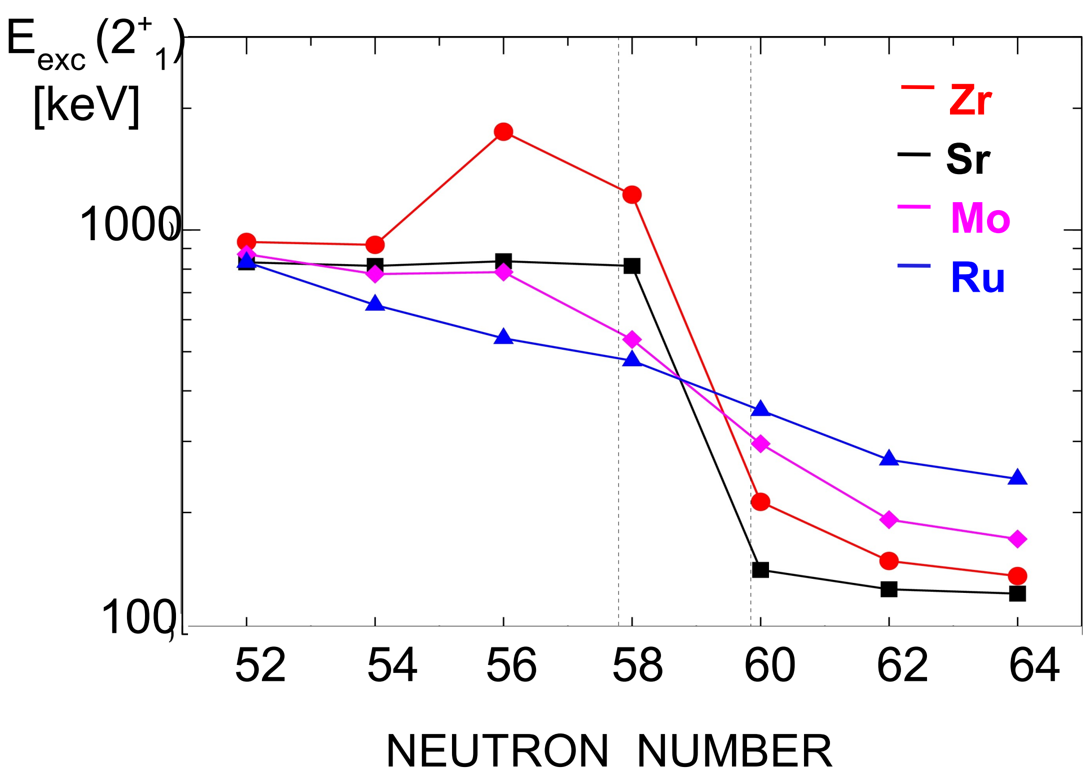

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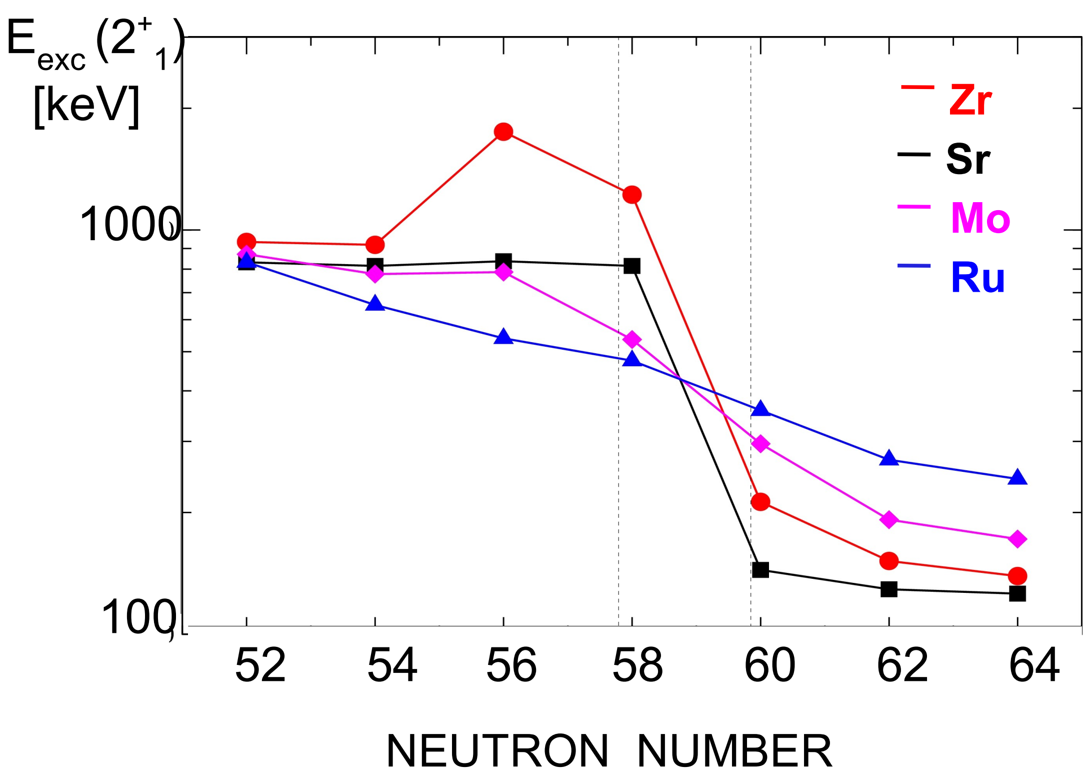
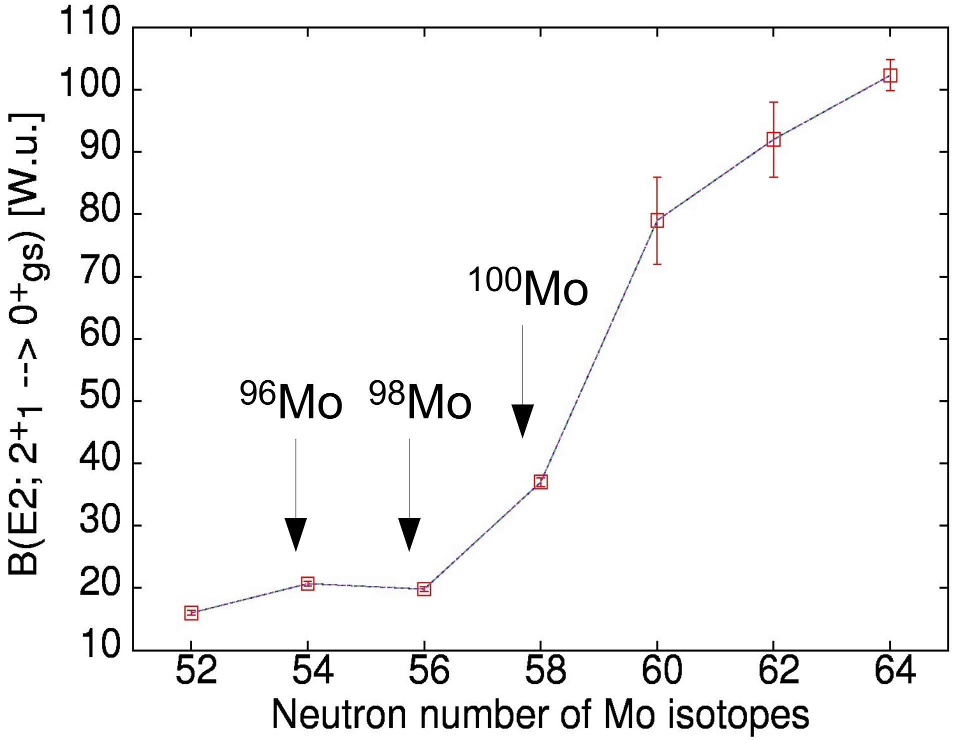
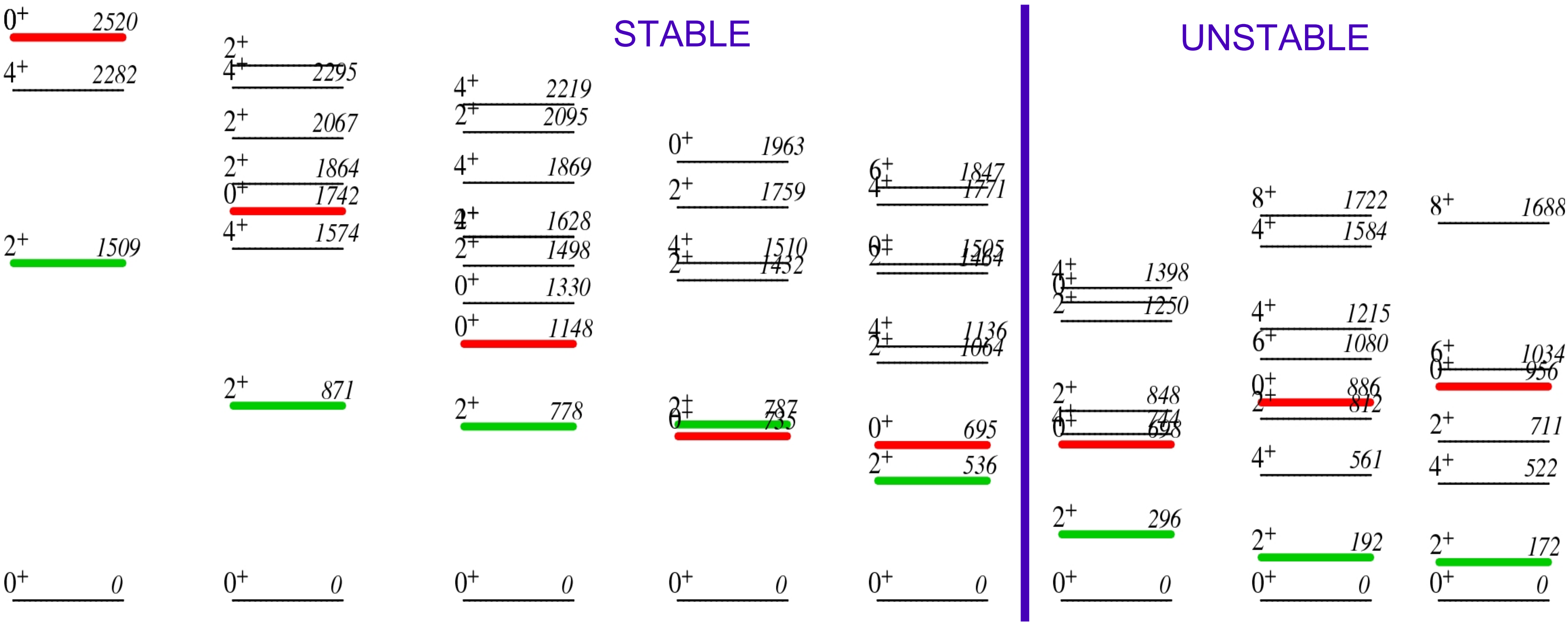




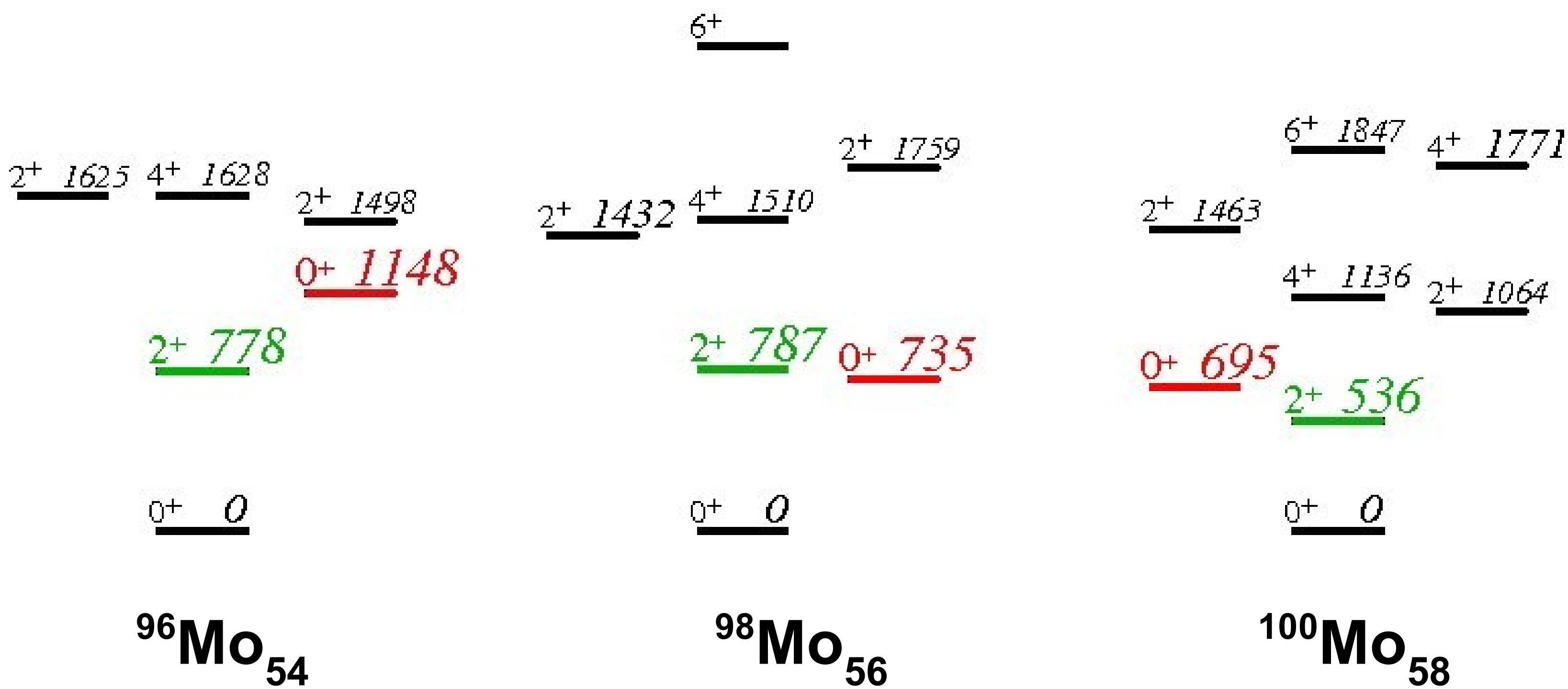
even-even Mo isotopes

Low-lying 0^+ states characteristic for transitional nuclei (A~100 around N = 60)

In extreme cases the second 0^+ state appears to be the first excited state.



$^{92}Mo_{50}$	$^{94}Mo_{52}$	$^{96}Mo_{54}$	$^{98}Mo_{56}$	$^{100}Mo_{58}$	$^{102}Mo_{60}$	$^{104}Mo_{62}$	$^{106}Mo_{64}$
8.4 W.u.	16 W.u.	21 W.u.	20 W.u.	**37 W.u.**	**74 W.u.**	92 W.u.	87 W.u.

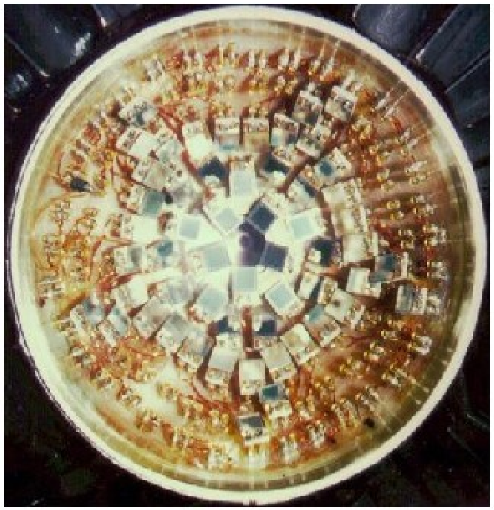


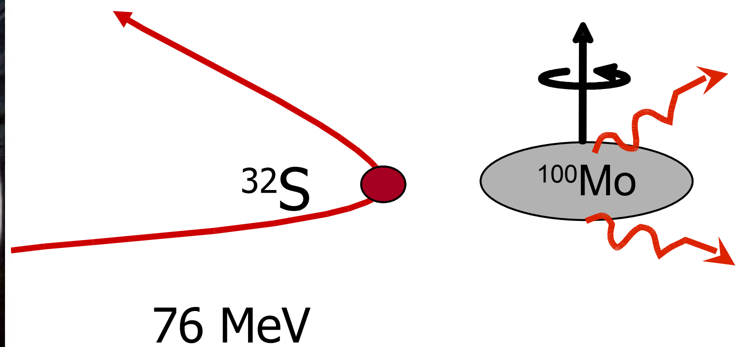
- Such a rare structure - first experimental indication of **shape coexistence** phenomenon.

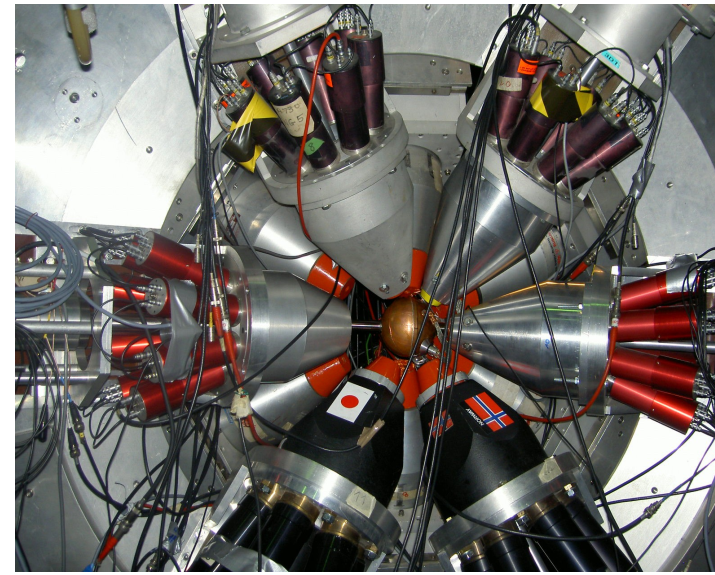
- What is the shape of the two lowest 0^+ states in the heaviest stable ^{96,98,100}Mo isotopes?

- Low-energy Coulomb excitation – the only experimental technique that can distinguish between **prolate** and **oblate** character of the deformation of the investigated nucleus **for each level**.

Coulomb excitation experiments @ HIL in Warsaw







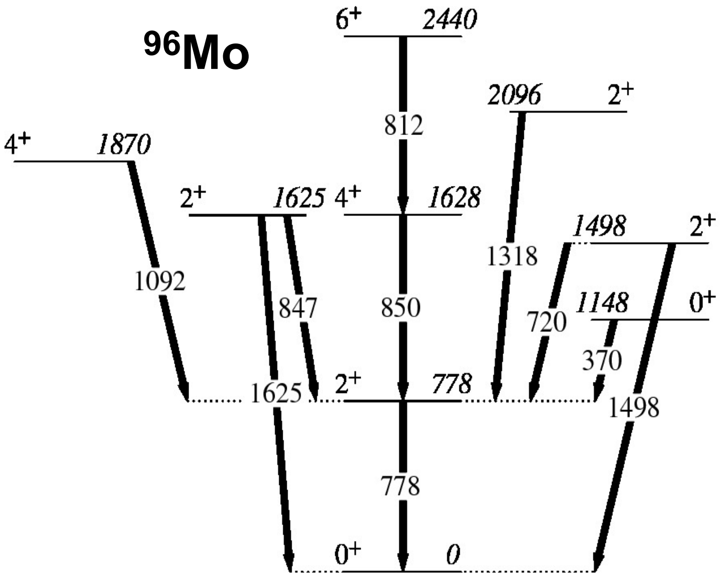
- particle detection system: 44 PiN diodes placed at backward angles ($\theta_{LAB} = 112^{O} – 152^{O}$)

- Gamma rays detection array: OSIRIS-II spectrometer (12HPGe detectors)

- The deexcitation γ rays **mesured in coincidence** with backscattered particles.

- A complete set of E2 matrix elements, **including signs and magnitudes**, can be extracted for low-lying states in nuclei.

Reduced matrix elements in $^{96,98,100}Mo$

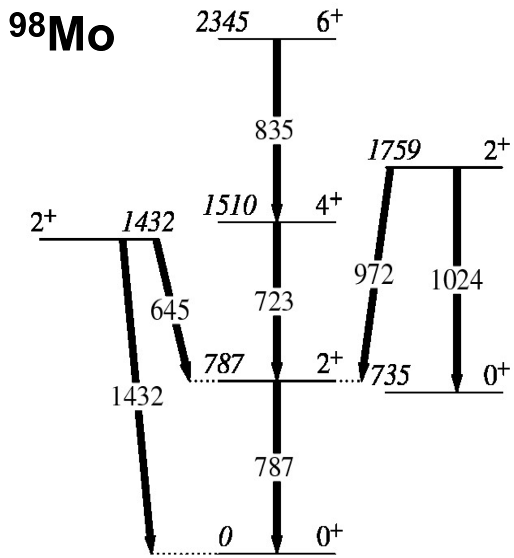


HIL (Warsaw), JAERI (Tokai)

- **26** reduced E2 and M1 matrix elements:

20 transitional E2 matrix elements

3 transitional M1 matrix elements



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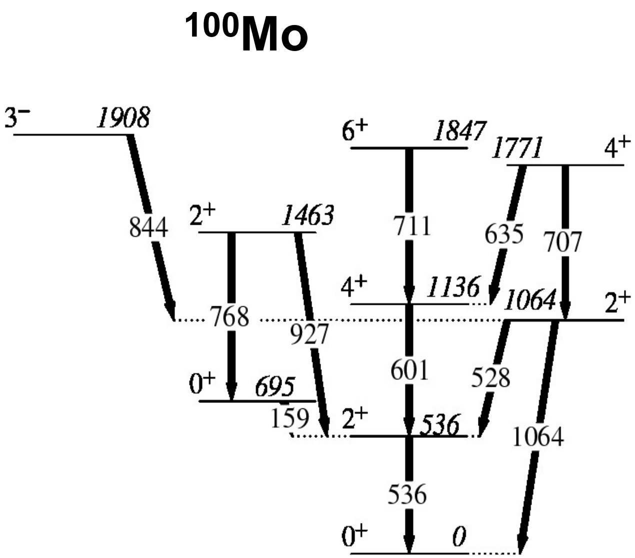

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- **26** reduced E2, E1, E3, M1 matrix elements:

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2 transitional E1 and E3 matrix elements

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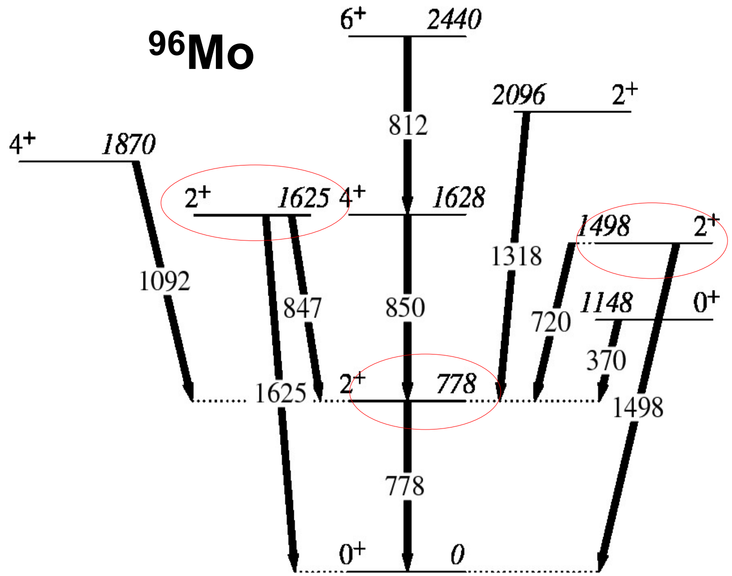

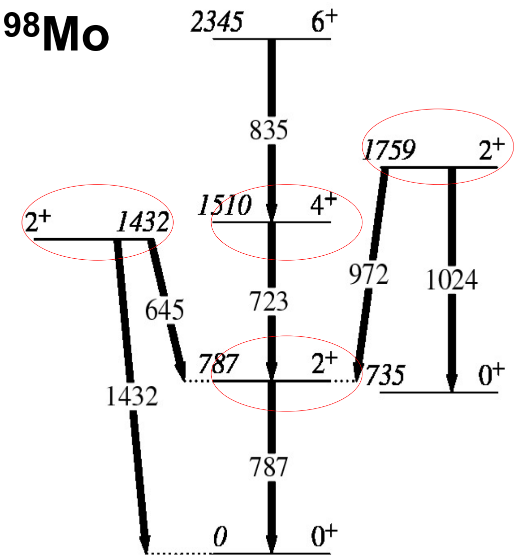

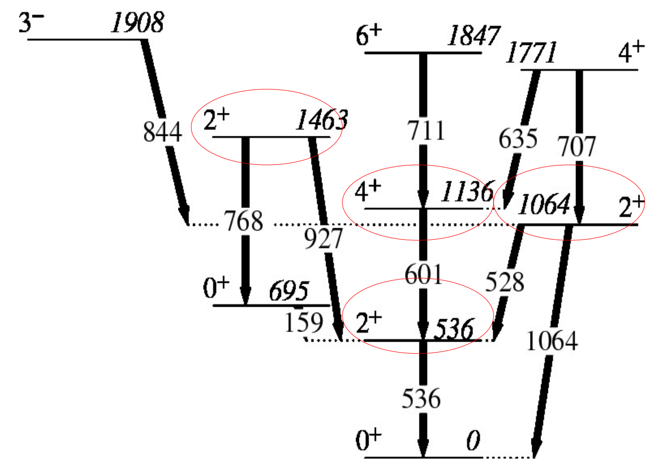

HIL (Warsaw), JAERI (Tokai)

- **26** reduced E2 and M1 matrix elements:

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HIL (Warsaw), JAERI (Tokai)

- **19** reduced E2 and M1 matrix elements:

4 diagonal E2 matrix elements

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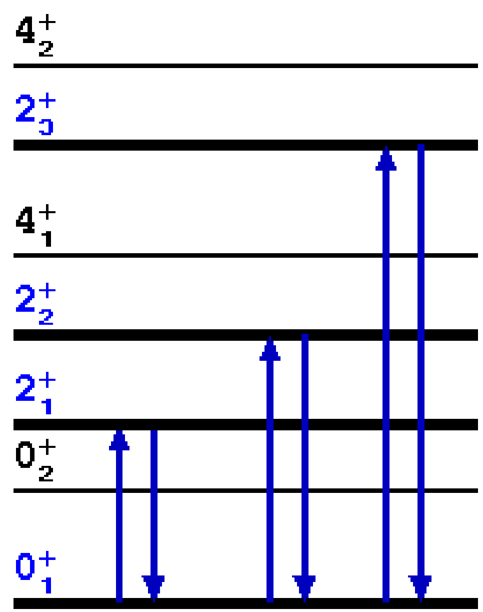
2 transitional E1 and E3 matrix elements

Rotation invariant <Q^2>

Quadrupole Sum Rules Method (D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683)

For E2 transitions from the 0^+ states one can form quadrupole tensor products coupled to angular momentum 0 – i.e. **rotation invariant**

$$\frac{\langle Q^2\rangle}{\sqrt{5}} = \langle i\,|[E2\times E2]_0|\,i\rangle = \frac{1}{\sqrt{(2I_i+1)}}\sum_t \langle i||\mathrm{E2}||t\rangle\langle t||\mathrm{E2}||i\rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_i & I_t \end{Bmatrix}$$



$\langle Q^2\rangle$ is an **overall quadrupole deformation parameter**

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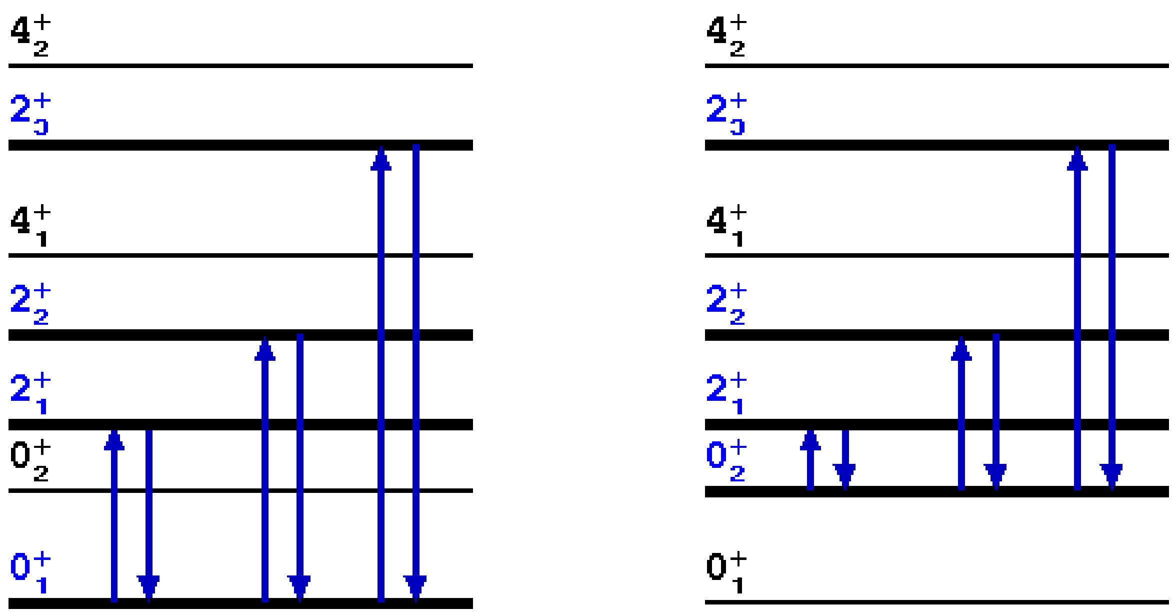
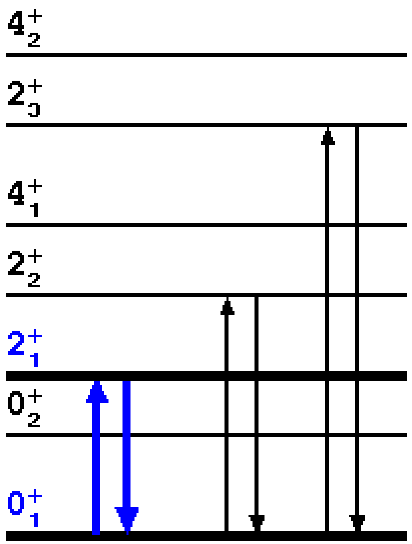


$\langle Q^2\rangle$ is an **overall quadrupole deformation parameter**

Determination of the overall quadrupole deformation of ^{100}Mo

state	the component E2×E2	contributions to the $\langle Q^2 \rangle$ [e^2b^2]
0_1^+	$\langle 0_1^+ \| E2 \| 2_1^+ \rangle \langle 2_1^+ \| E2 \| 0_1^+ \rangle$	0.46
	$\langle 0_1^+ \| E2 \| 2_2^+ \rangle \langle 2_2^+ \| E2 \| 0_1^+ \rangle$	0.01
	$\langle 0_1^+ \| E2 \| 2_3^+ \rangle \langle 2_3^+ \| E2 \| 0_1^+ \rangle$	0.0002
	Total	0.48



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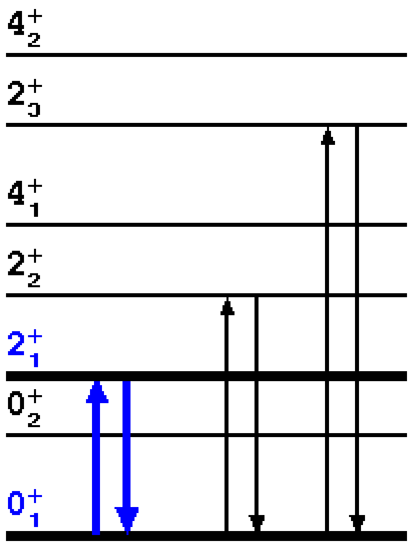
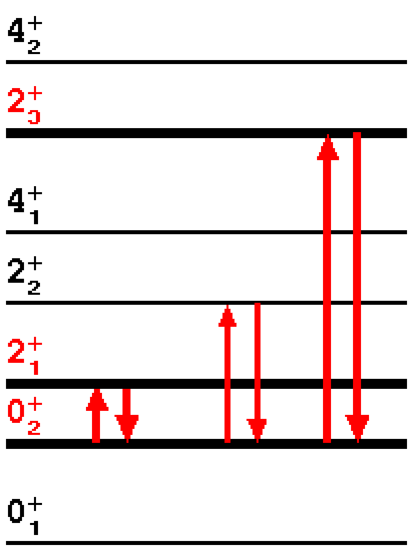
	Total	0.48
0_2^+	$\langle 0_2^+ \| E2 \| 2_1^+ \rangle \langle 2_1^+ \| E2 \| 0_2^+ \rangle$	0.26
	$\langle 0_2^+ \| E2 \| 2_2^+ \rangle \langle 2_2^+ \| E2 \| 0_2^+ \rangle$	0.10
	$\langle 0_2^+ \| E2 \| 2_3^+ \rangle \langle 2_3^+ \| E2 \| 0_2^+ \rangle$	0.25
	Total	0.62





Rotation invariant <Q^{3}cos(3δ)>

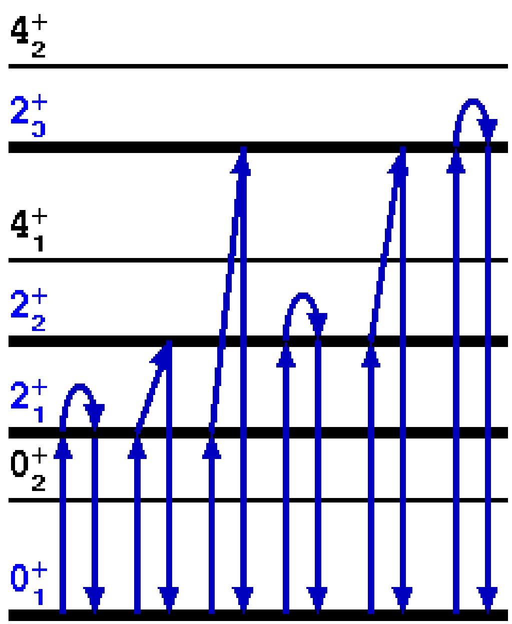
Quadrupole Sum Rules Method (D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683)

To get information on triaxiality the higher order invariant is needed:

$$\sqrt{\frac{2}{35}}\langle Q^3 \cos 3\delta\rangle = \langle i\,|[E2\times E2]_2 \times E2]_0|\,i\rangle =$$

$$= \pm\frac{1}{(2I_i+1)}\sum_{t,u}\langle i||\mathrm{E2}||u\rangle\langle u||\mathrm{E2}||t\rangle\langle t||\mathrm{E2}||i\rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_u & I_t \end{Bmatrix}$$

$\langle \cos 3\delta \rangle$ is a **triaxiality** parameter



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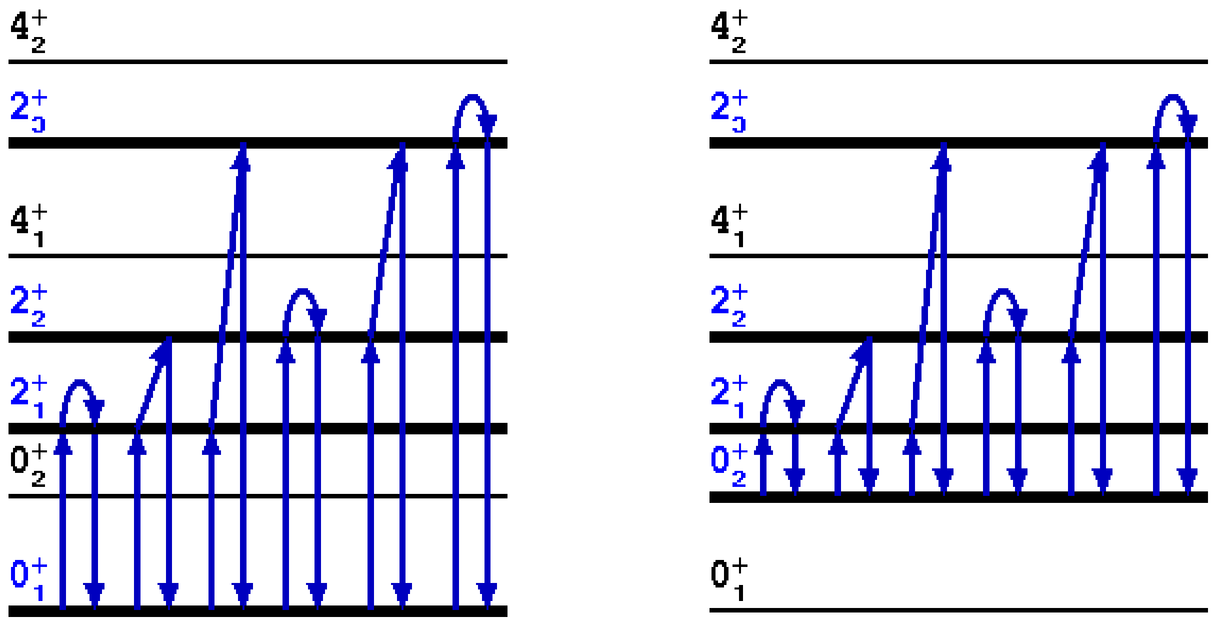
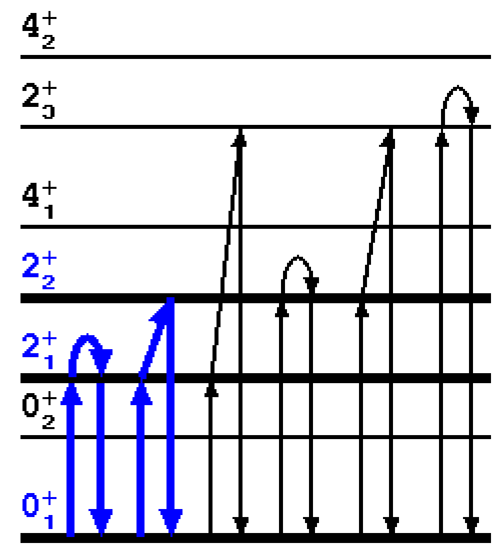


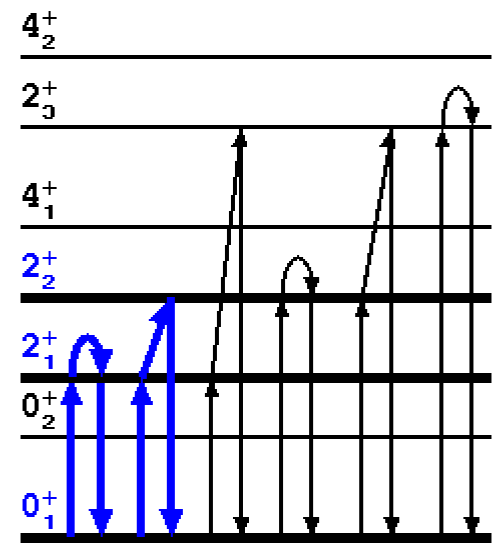
Determination of the triaxiality of ^{100}Mo

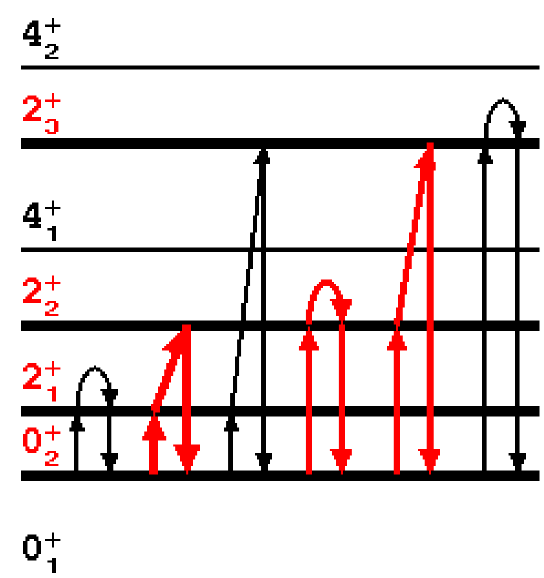
state	the component E2×E2×E2	contributions to $\langle Q^3 cos3\delta \rangle$
0_1^+	$\langle 0_1^+ \| E2 \| 2_1^+ \rangle \langle 2_1^+ \| E2 \| 2_1^+ \rangle \langle 2_1^+ \| E2 \| 0_1^+ \rangle$	-0.154
	$\langle 0_1^+ \| E2 \| 2_1^+ \rangle \langle 2_1^+ \| E2 \| 2_2^+ \rangle \langle 2_2^+ \| E2 \| 0_1^+ \rangle$	0.132
	$\langle 0_1^+ \| E2 \| 2_1^+ \rangle \langle 2_1^+ \| E2 \| 2_3^+ \rangle \langle 2_3^+ \| E2 \| 0_1^+ \rangle$	0.002
	$\langle 0_1^+ \| E2 \| 2_2^+ \rangle \langle 2_2^+ \| E2 \| 2_2^+ \rangle \langle 2_2^+ \| E2 \| 0_1^+ \rangle$	0.013
	$\langle 0_1^+ \| E2 \| 2_2^+ \rangle \langle 2_2^+ \| E2 \| 2_3^+ \rangle \langle 2_3^+ \| E2 \| 0_1^+ \rangle$	-0.001
	$\langle 0_1^+ \| E2 \| 2_3^+ \rangle \langle 2_3^+ \| E2 \| 2_3^+ \rangle \langle 2_3^+ \| E2 \| 0_1^+ \rangle$	-0.0001
	Total	-0.008



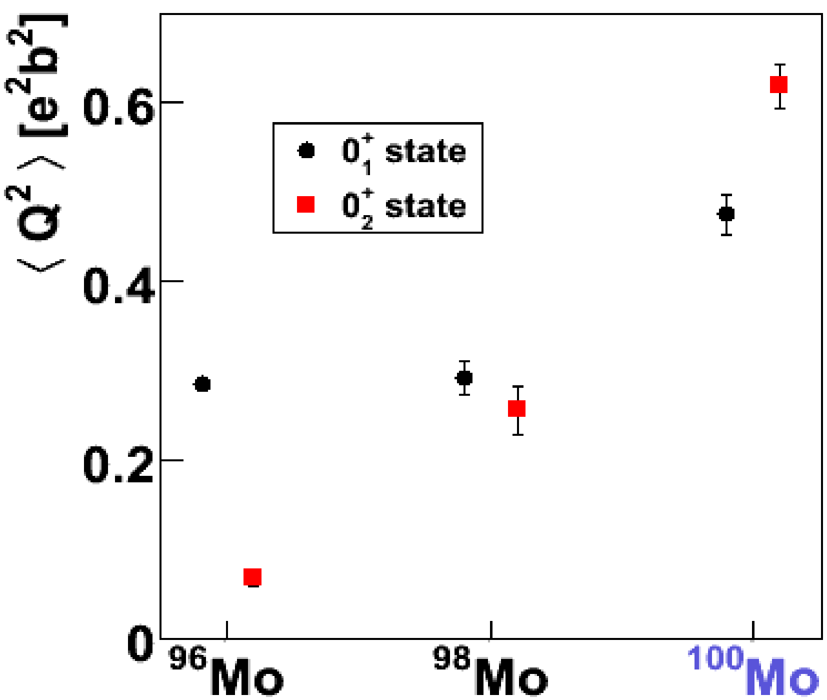
Determination of the triaxiality of ^{100}Mo

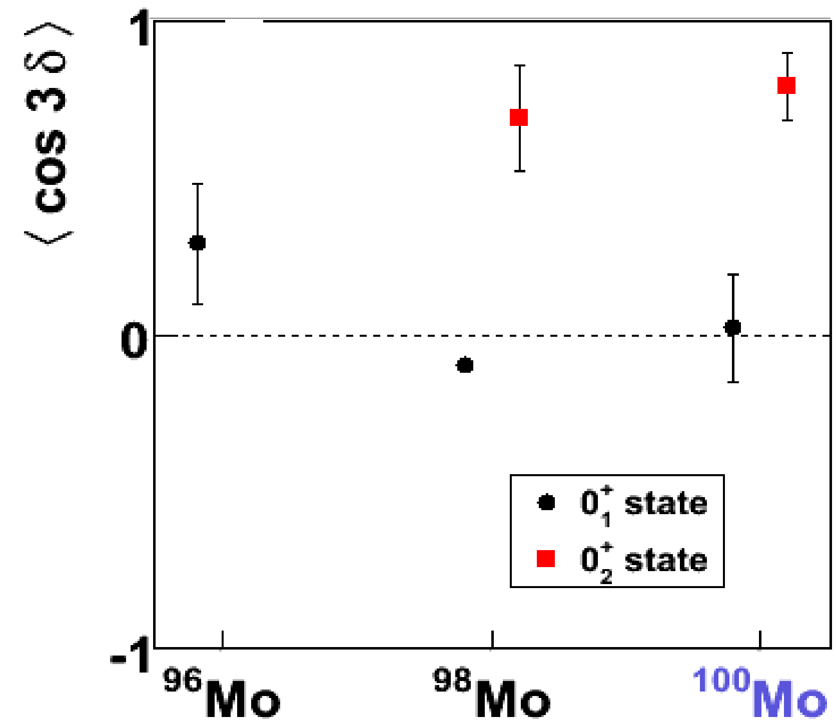
state	the component E2×E2×E2	contributions to $\langle Q^3 cos3\delta \rangle$
0^+_1	$\langle 0^+_1 \| E2 \| 2^+_1 \rangle \langle 2^+_1 \| E2 \| 2^+_1 \rangle \langle 2^+_1 \| E2 \| 0^+_1 \rangle$	-0.154
	$\langle 0^+_1 \| E2 \| 2^+_1 \rangle \langle 2^+_1 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| E2 \| 0^+_1 \rangle$	0.132
	$\langle 0^+_1 \| E2 \| 2^+_1 \rangle \langle 2^+_1 \| E2 \| 2^+_3 \rangle \langle 2^+_3 \| E2 \| 0^+_1 \rangle$	0.002
	$\langle 0^+_1 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| E2 \| 0^+_1 \rangle$	0.013
	$\langle 0^+_1 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| E2 \| 2^+_3 \rangle \langle 2^+_3 \| E2 \| 0^+_1 \rangle$	-0.001
	$\langle 0^+_1 \| E2 \| 2^+_3 \rangle \langle 2^+_3 \| E2 \| 2^+_3 \rangle \langle 2^+_3 \| E2 \| 0^+_1 \rangle$	-0.0001
	Total	-0.008
0^+_2	$\langle 0^+_2 \| E2 \| 2^+_1 \rangle \langle 2^+_1 \| E2 \| 2^+_1 \rangle \langle 0^+_2 \| E2 \| 2^+_1 \rangle$	-0.09
	$\langle 0^+_2 \| E2 \| 2^+_1 \rangle \langle 2^+_1 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| E2 \| 0^+_2 \rangle$	-0.31
	$\langle 0^+_2 \| E2 \| 2^+_1 \rangle \langle 2^+_1 \| E2 \| 2^+_3 \rangle \langle 2^+_3 \| E2 \| 0^+_2 \rangle$	-0.04
	$\langle 0^+_2 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| E2 \| 0^+_2 \rangle$	0.12
	$\langle 0^+_2 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| E2 \| 2^+_3 \rangle \langle 2^+_3 \| E2 \| 0^+_2 \rangle$	-0.13
	$\langle 0^+_2 \| E2 \| 2^+_3 \rangle \langle 2^+_3 \| E2 \| 2^+_3 \rangle \langle 2^+_3 \| E2 \| 0^+_2 \rangle$	-0.06
	Total	-0.51

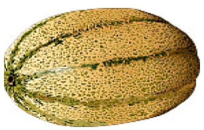




Quadrupole shape parameters of $^{96,98,100}Mo$







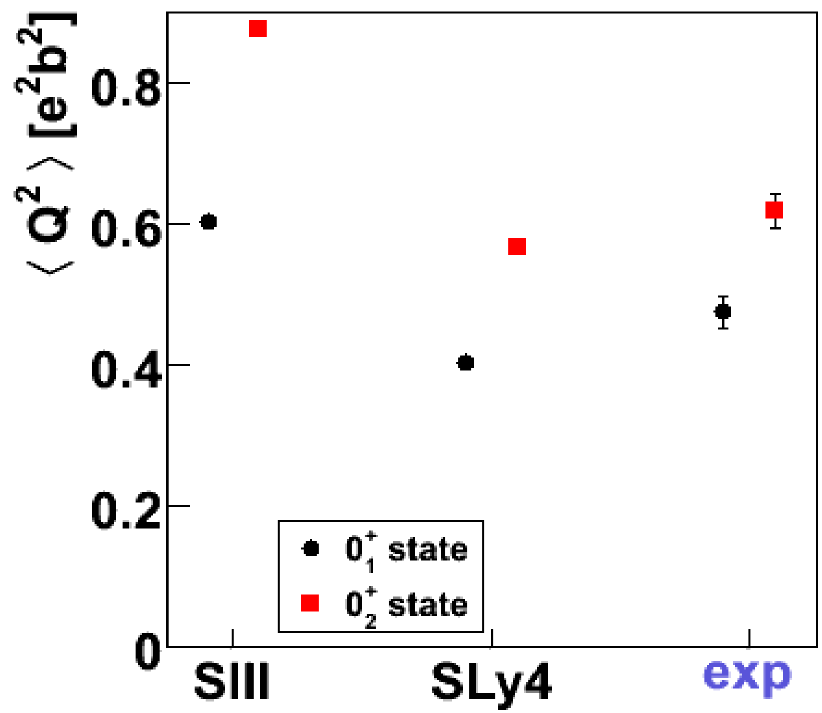
triaxial

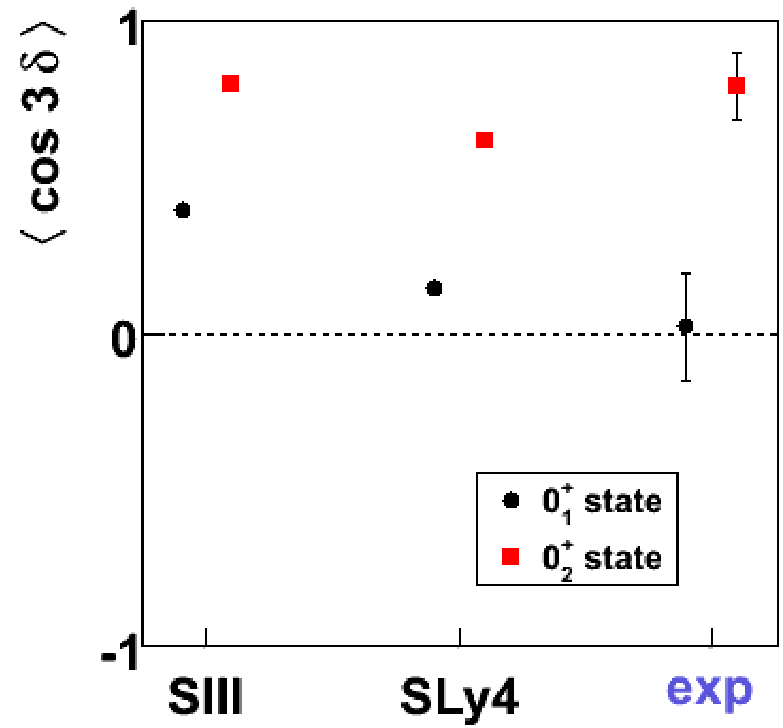
$\langle Q^2 \rangle$ is a measure of an overall deformation (analogous to Bohr's β)

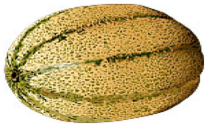
$\langle \cos(3\delta) \rangle$ is a measure of triaxiality (analogous to Bohr's γ)

Quadrupole deformation parameters of ^{100}Mo: exp vs theory

General quadrupole collective **B**ohr **H**amiltonian calculations
(L. Próchniak Int. J. Mod. Phys. E19 (2010) 705,
L. Próchniak, S. G. Rohoziński, J. Phys. G: Nucl. Part. 36 (2009) 123101)





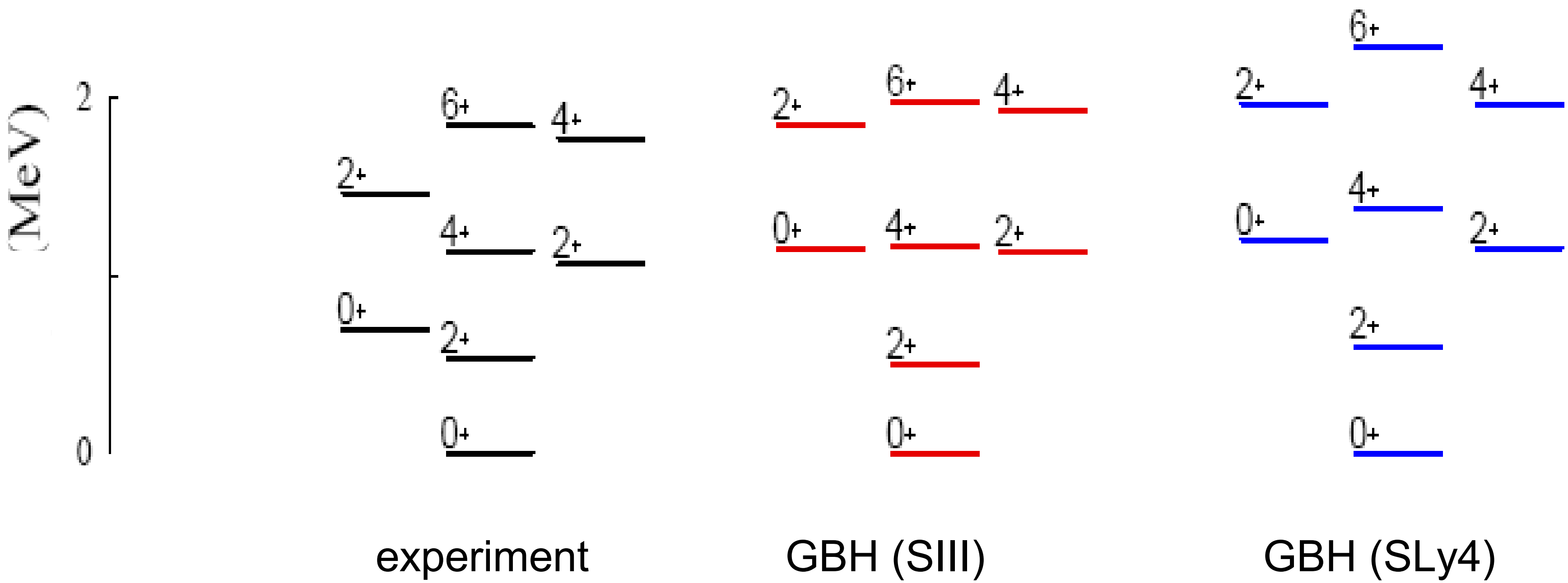


triaxial

GBH calculations with the **SLy4** variant of Skyrme interaction indicate **better agreement** with experimentally obtained quadrupole deformation parameters.

Level structure of ^{100}Mo: exp vs theory

L. Próchniak Int. J. Mod. Phys. E19 (2010) 705



Conclusions:

- Having the experimental magnitudes and signs of the E2 matrix elements, the expectation values of quadrupole **invariants <Q^2> and <$Q^3\cos(3\delta)$> for a given state may by calculated**.

- Quadrupole shape parameters of the two lowest 0^+ states in ^{96,98,100}Mo nuclei were inferred from Coulomb excitation data in a **nuclear model independent way** (Quadrupole Sum Rules method).

- The overall deformation of ^{96,98,100}Mo in the 0^+ states increases with the neutron number. The overall deformation of ^{100}Mo in the 0^+ states shows inverse trend than in ^{96}Mo – 0^+_{gs} state of ^{100}Mo is less deformed than 0^+_{exc} one. The shape of $^{96-100}$Mo changes **from triaxial** (the 0^+_{gs} state) to the **prolate** one (the 0^+_{exc} state).

- **GBH** calculations with the **SLy4** variant of Skyrme interaction lead to **better agreement** with experimentally obtained quadrupole deformation parameters.

FUTURE PLANS:

- What is the evolution of the deformation of low-lying 0^+ states in more neutron-rich Mo isotopes ?

Collaboration:

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