



Joint Institute for Nuclear Research, Dubna, Russia

*Appearance of Nuclear Shell Effects and Initial
Charge (Mass) Asymmetry in Formation of
Products in Heavy Ion Collisions*

Avazbek Nasirov *

* Institute of Nuclear Physics Academy of Science of Uzbekistan

**17th Nuclear Physics Workshop
"Marie & Pierre Curie"
Kazimierz, 22-26 September, 2010**



In collaboration with colleagues:

**Prof. G. G. Giardina, Dr. G. Mandaglio,
Dr. M. Manganaro,
*INFN, Sezione di Catania, and
Dipartimento di Fisica dell'Università di Messina,
Messina, Italy***

**Prof. A.I. Muminov
* Institute of Nuclear Physics
Tashkent, Uzbekistan**

Content



- Introduction
- Main mechanisms of heavy ion collisions at low energies.
- Reason causing the hindrance of complete fusion
- Peculiarities of dinuclear system model
- The ways of searching optimal conditions for the synthesis of superheavy elements.
- Conclusions

Introduction



For light or medium-heavy systems, capture inside the Coulomb barrier leads invariably to fusion, so that the capture (or barrier-passing) cross-section coincides with the complete fusion cross-section. Total fusion implies the formation of the compound nucleus. However, for heavy systems capture inside the barrier, i.e. formation of a dinucleus, is not a sufficient condition for fusion. The dinucleus system may re-separate into two fragments before that full equilibration of all degrees of freedom has been reached. Consequently, a considerable part of the total capture cross-section goes to the quasi-fission channel. This phenomenon is experimentally observed as a hindrance to fusion.

For the light dinuclear system leading to compound nucleus with high fission barrier the measurement of the evaporation residues allows us to find directly fusion cross section:

Capture = Quasifission + Fusion-Fission + Fast-fission + Evaporation residues

Because for light system Capture = Fusion = Evaporation residues.

Achievements in synthesis of superheavy elements

Nuclear centers/ Elements	SHIP-GSI, Darmstadt	Flerov Lab. JINR-Dubna	RIKEN, Japan	Lawrence Berkeley Lab.
Z=110	Darmstadtium		Confirmed	
Z=111	Roentgenium		Confirmed	
Z=112	Copernicium		Confirmed	
Z=113			<u>Synthesized</u>	
Z=114		<u>Synthesized</u>		Confirmed
Z=115		<u>Synthesized</u>		
Z=116	Confirmed !	<u>Synthesized</u>		
Z=117		<u>Synthesized</u>		
Z=118		<u>Synthesized</u>		

Introduction

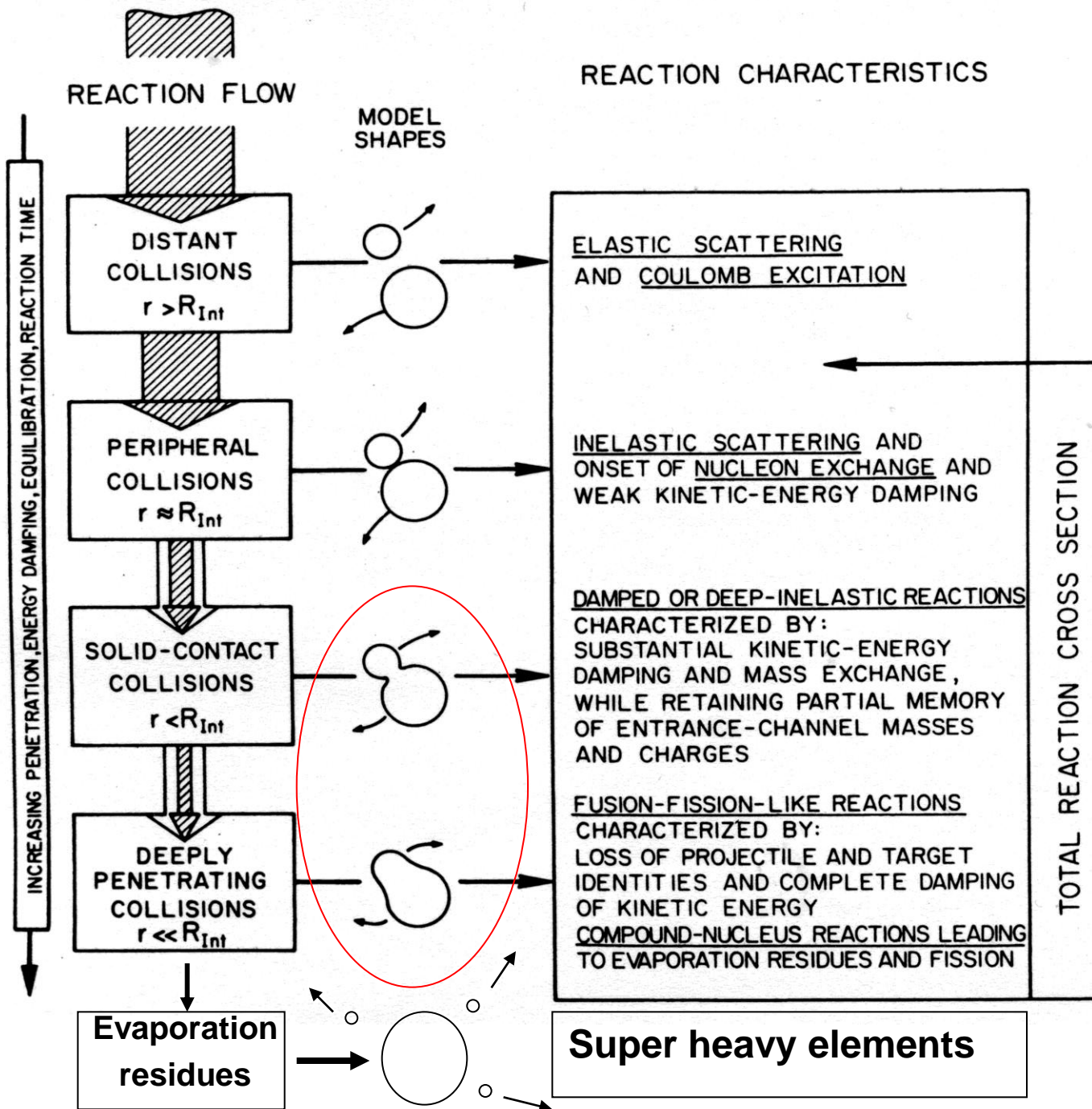


The experimental knowledge on fusion between light and medium-heavy nuclei at sub- and near-barrier energies has grown considerably in the last twenty years

1. M. Dasgupta, D.J. Hinde, N. Rowley, A.M. Stefanini, *Annu. Rev. Nucl. Part. Sci.* 48, 401 (1998).

2. A.B. Balantekin, N. Takigawa, *Rev. Mod. Phys.* 70, 77 (1998)..

The theoretical models are able to reproduce and predict the main features of such processes, but properly understanding the fusion dynamics for heavy systems requires many more ingredients. The need for more experimental data to disentangle various concurrent effects, is clearly felt. A full understanding of all steps of the reaction dynamics is very important for the challenging issue of superheavy elements production and new isotopes far from the valley of stability.



Description of the nucleus-nucleus collision at energies $< 10A \text{ MeV}$ as the 3 stage process.

$$\sigma_{\text{ER}}(E_{\text{Lab}}, L) = \langle \sigma_{\text{cap}}(E_{\text{Lab}}, L; \{\alpha_i\}) P_{\text{CN}}(E_{\text{Lab}}, L; \{\alpha_i\}) W_{\text{surv}}(E_{\text{Lab}}, L; \{\alpha_i\}) \rangle_{\{\alpha\}}$$

Deep – inelastic

quasifission

fission

+

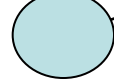
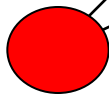
+

+

Capture

complete fusion

emission + ER



1

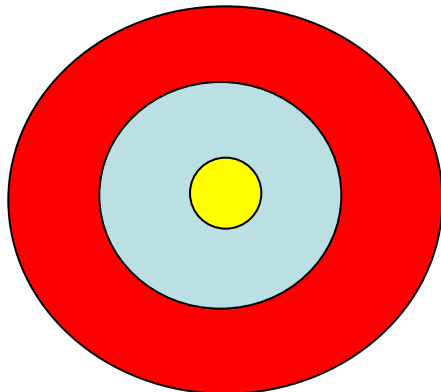
2

3

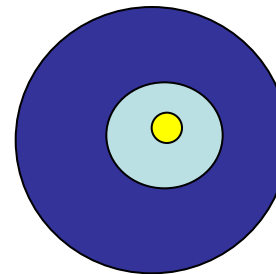
dinuclear system

compound nucleus

ER-evaporation residue



Hot fusion



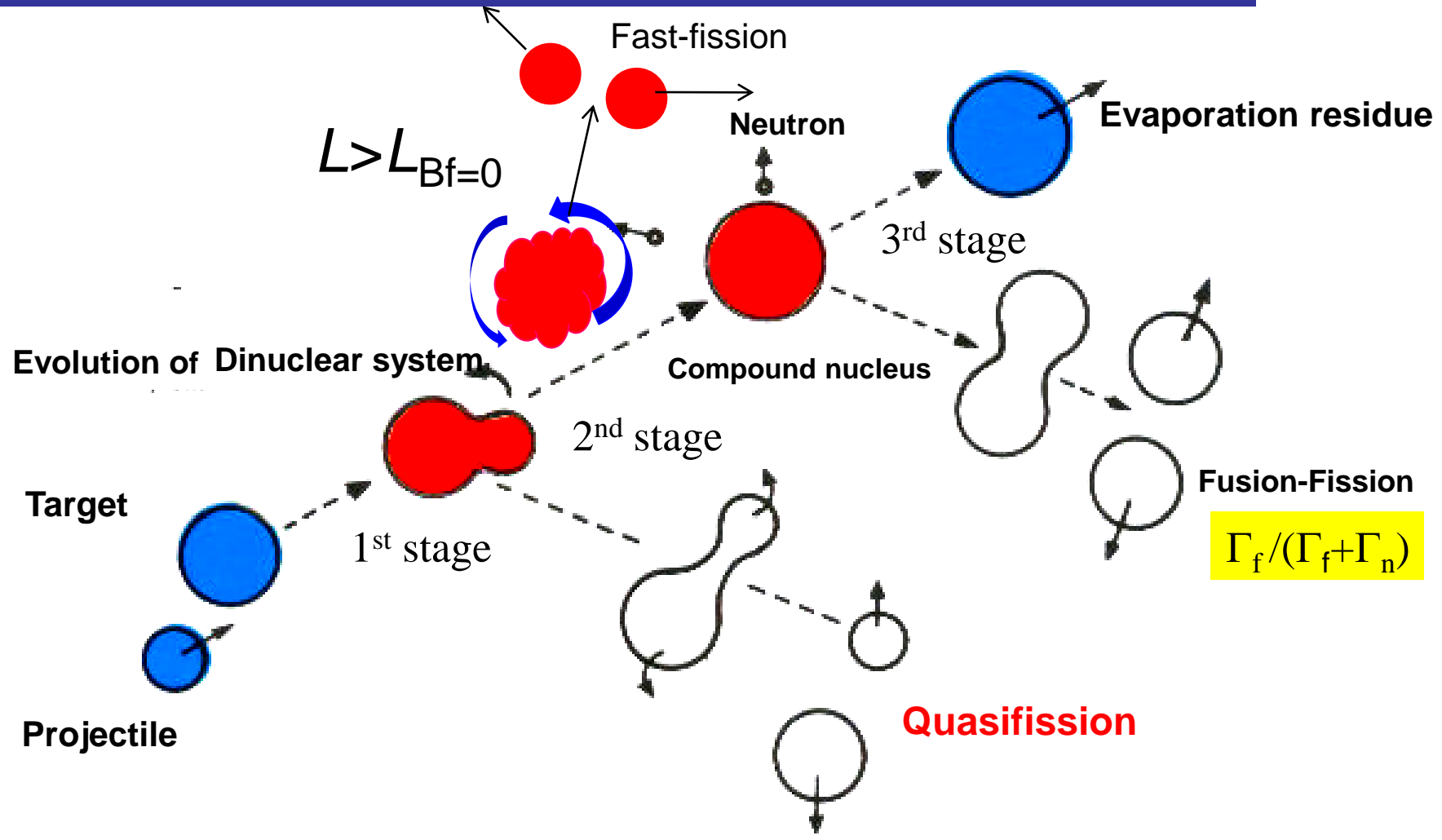
Cold fusion

The formation of dinuclear systems

- Dinuclear system is formed due to shell effects as the quantum states of the neutron and proton systems of nuclei.
- Shell effects is observed as cluster states in the large amplitude collective motions of nuclei.
- The observed cluster emission, mass-charge distribution of the quasifission fragments and spontaneous asymmetric fission of Th, U and Cf isotopes proved the strong role of shell structure.
- Reactions of heavy ion collisions and fission (spontaneous and induced) processes can be studied well using dinuclear system concept.



Three final stages of Competition between complete fusion and quasifission



Capture = Quasifission + Fast-fission + Fusion-Fission + Evaporation residues

About description of the events of the synthesis of superheavy elements



The measured evaporation cross section can be described by the formula:

$$\sigma_{ER}(E^*) = \sum_{\ell=0}^{\ell=\ell_f} \sigma_{\text{cap}}(E_{\text{c.m.}}, \ell) P_{\text{CN}}(E^*, \ell) W_{\text{surv}}(E^*, \ell)$$

where

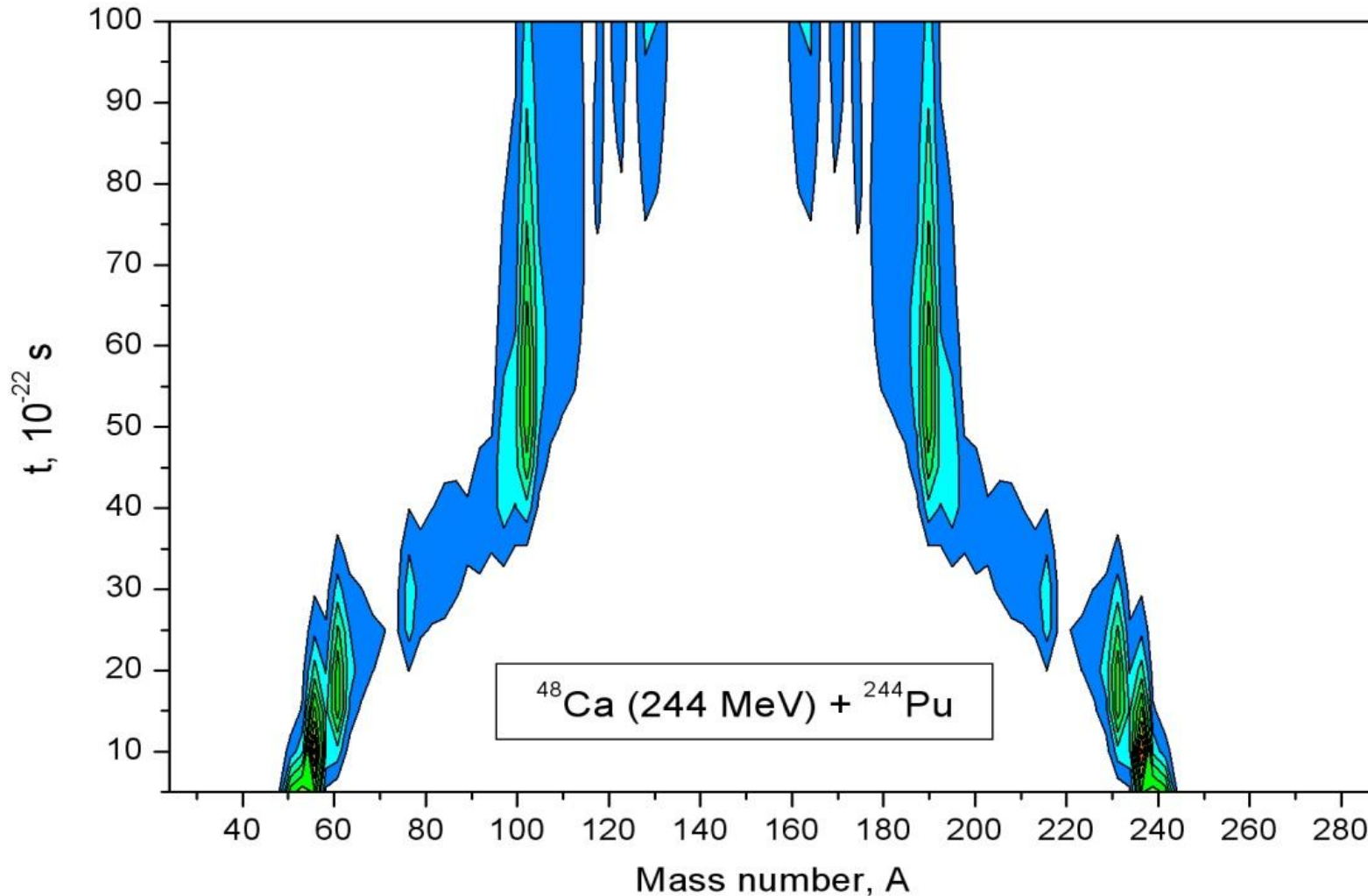
$$\sigma_{\text{fus}}(E_{\text{c.m.}}, \ell) = \sigma_{\text{cap}}(E_{\text{c.m.}}, \ell) P_{\text{CN}}(E^*, \ell)$$

is considered as the cross section of compound nucleus formation; W_{surv} is the survival probability of the heated and rotating nucleus. The smallness of P_{CN} means hindrance to fusion caused by huge contribution of quasifission process:

$$\sigma_{\text{qfis}}(E_{\text{c.m.}}, \ell) = \sigma_{\text{cap}}(E_{\text{c.m.}}, \ell) (1 - P_{\text{CN}}(E^*, \ell))$$



Evolution of mass distributions of the dinuclear system fragments $Y_z(t)$



9
e
7
:

Calculation of the competition between complete fusion and quasifission: $P_{cn}(E_{DNS}, L)$

$$P_{CN}(E_{DNS}^*, \ell) = \sum_{Z_{sym}}^{Z_{max}} Y_Z(E_{DNS}^*, \ell) P_{CN}^{(Z)}(E_{DNS}^*, \ell)$$

where

$$P_{CN}^{(Z)}(E_{DNS}^*, \ell) = \frac{\rho(E_{DNS}^*(Z) - B_{fus}^*(Z), \ell)}{\rho(E_{DNS}^*(Z) - B_{fus}^*(Z), \ell) + \rho(E_{DNS}^*(Z) - B_{qf}^*(Z), \ell) + \rho(E_{DNS}^*(Z) - B_{sym}^*(Z), \ell)}$$

$$\begin{aligned} \frac{\partial}{\partial t} Y_Z(E_Z^*, \ell, t) &= \Delta_{Z+1}^{(-)} Y_{Z+1}(E_Z^*, \ell, t) + \Delta_{Z-1}^{(+)} Y_{Z-1}(E_Z^*, \ell, t) \\ &\quad - (\Delta_Z^{(-)} + \Delta_Z^{(+)} + \Lambda_Z^{qf}) Y_Z(E_Z^*, \ell, t), \quad \text{for } Z = 2, 3, \dots, Z_{tot} - 2. \end{aligned} \quad (6)$$

Here, the transition coefficients of multinucleon transfer are calculated as in Ref. 18

$$\Delta_Z^{(\pm)} = \frac{1}{\Delta t} \sum_{P,T} |g_{PT}^{(Z)}|^2 n_{T,P}^{(Z)}(t) (1 - n_{P,T}^{(Z)}(t)) \frac{\sin^2(\Delta t(\tilde{\epsilon}_{PZ} - \tilde{\epsilon}_{TZ})/2\hbar)}{(\tilde{\epsilon}_{PZ} - \tilde{\epsilon}_{TZ})^2/4}, \quad (7)$$



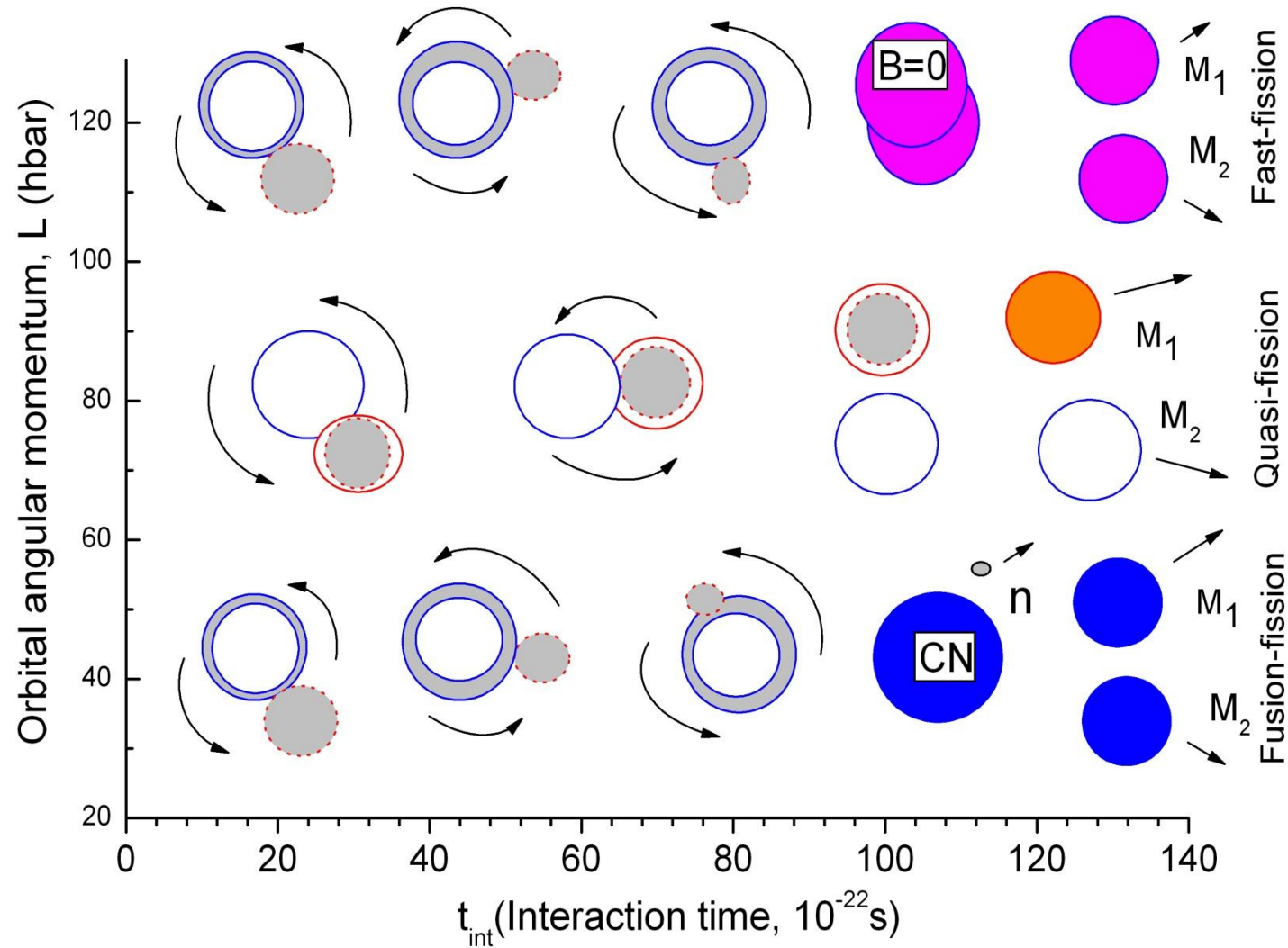
What we know about quasifission fragments?

- The mass distribution its fragments has a maximum usually near magic numbers $Z=20, 28, 50, 82$ and $N=20, 28, 50, 82$;
- Total kinetic energy distribution is very close to Viola systematics as for fusion-fission: $TKE=Z_1Z_2e^2/D(A_1,A_2)$;
- Angular distribution of fragments has more large anisotropy in comparison with that of fusion-fission.

We would like to stress that **angular distribution of quasifission fragments is mainly anisotropic but it may be isotropic and angular distribution of fusion-fission fragments may be isotropic in dependence on the reaction dynamics.**



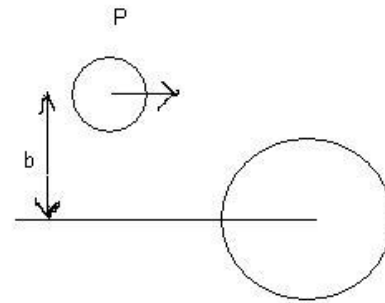
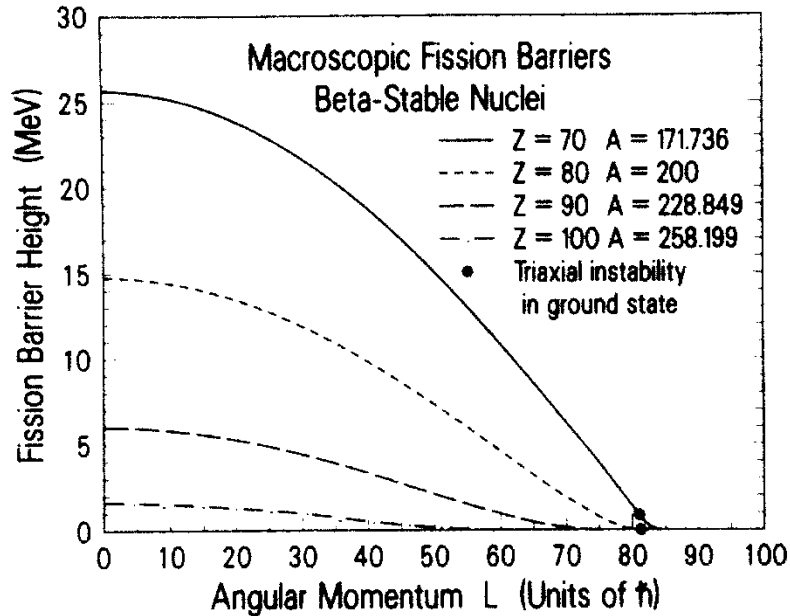
**Mechanisms of the reaction following capture:
Fusion-fission, quasifission and fast-fission.**



Fast-fission of the mononucleus

33

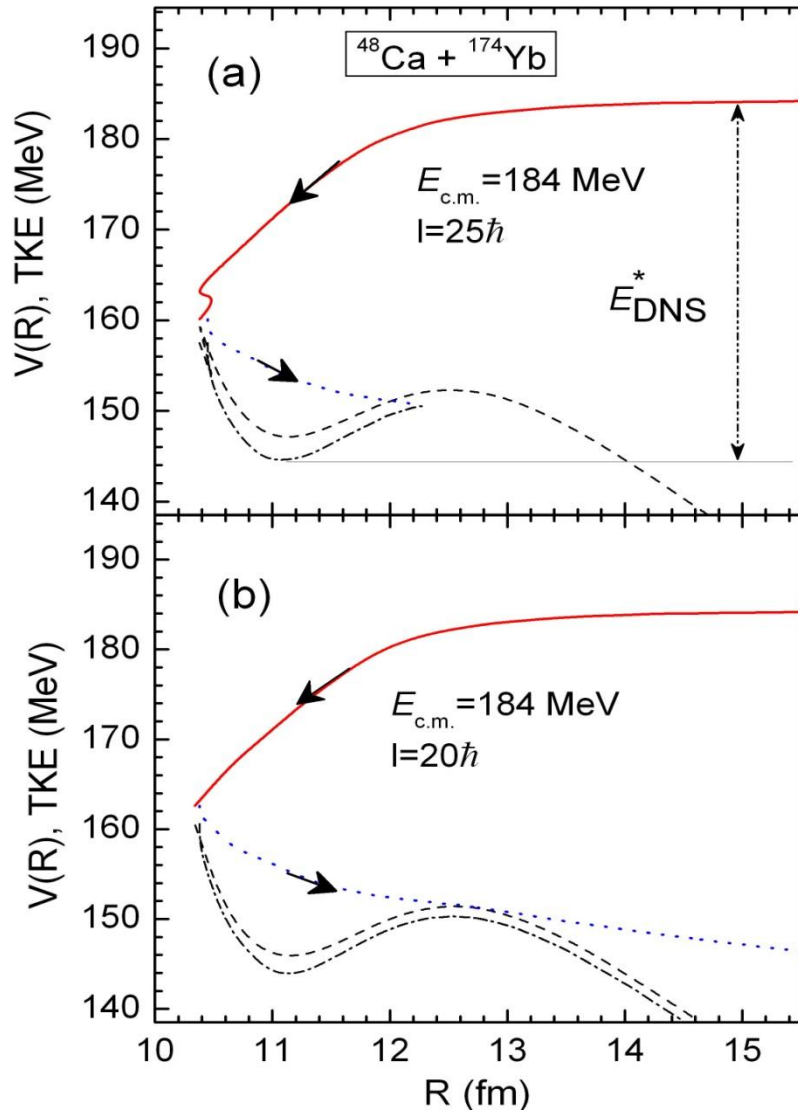
MACROSCOPIC MODEL (



$$L_{fus} > L > L_{fis.bar}$$

FIG. 10. Same as Fig. 9 for $Z=70$ to 100. There are no solid points for $Z=90$ and $Z=100$ since no triaxial ground states exist for these nuclei.

Difference between classical paths of the capture and deep inelastic collisions



TKE-total kinetic energy
 $V(R)$ – nucleus-nucleus potential

E_{DNS}^* – excitation energy of double nuclear system

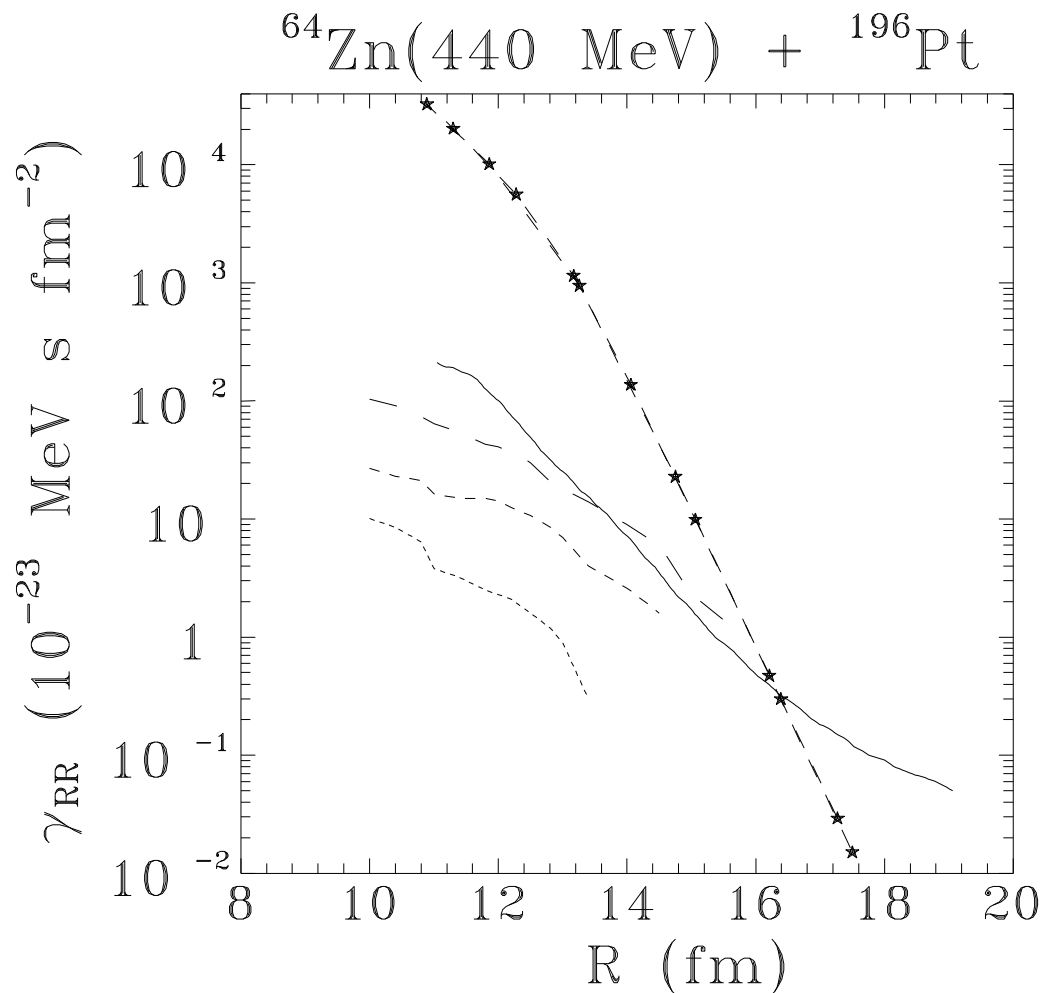
Full momentum transfer reactions

Capture=Fusion+Quasifission +Fast fission

Fusion=Fission+Evaporation residues

Fission >> Evaporation residues 17

Comparison of the friction coefficients, calculated by different methods



★ D. H. E. Gross and H. Kalinowski, Phys. Rep. **45**, (1978) 175.

Solid line – G.G. Adamian, et al. PRC56 (1997) 373

By Yamaji et al(microscopic):

Long dashed -- Temperatura= 2 MeV

Short dashed- - Temperatura= 1 MeV

Dotted - Temperatura= 0.5 MeV

S. Yamaji and A. Iwamoto, Z. Phys. A **313**, (1983) 161.

Equations of motion used to find capture of projectile by target

$$\mu(R)\ddot{R} + \gamma_R(R)\dot{R}(t) = -\frac{\partial V(R)}{\partial R} - \dot{R}^2 \frac{\partial \mu(R)}{\partial R}$$

$$\mu(R) = \delta\mu(R) + m_0 A_T A_P / A_{tot}$$

$$\times \left(1 - \frac{2}{A_{tot}} \int \frac{\rho_1^{(0)}(\mathbf{r} - \mathbf{r}_1)\rho_2^{(0)}(\mathbf{r} - \mathbf{r}_2)}{\rho_1^{(0)}(\mathbf{r} - \mathbf{r}_1) + \rho_2^{(0)}(\mathbf{r} - \mathbf{r}_2)} d^3\mathbf{r} \right),$$

$$\frac{dL}{dt} = \gamma_\theta(R)R(t) \left[\dot{\theta}R(t) - \dot{\theta}_1 R_{1eff} - \dot{\theta}_2 R_{2eff} \right]$$

$$L_0 = J_R \dot{\theta} + J_1 \dot{\theta}_1 + J_2 \dot{\theta}_2, \quad E_{rot} = \frac{J_R \theta^2}{2} + \frac{J_1 \theta_1^2}{2} + \frac{J_2 \theta_2^2}{2}$$

Nucleus-nucleus interaction potential

$$V_C(R, \alpha_1, \alpha_2) = \frac{Z_1 Z_2}{R} e^2 + \frac{Z_1 Z_2}{R^3} e^2 \left\{ \left(\frac{9}{20\pi} \right)^{1/2} \sum_{i=1}^2 R_{0i}^2 \beta_2^{(i)} P_2(\cos \alpha_i) + \frac{3}{7\pi} \sum_{i=1}^2 R_{0i}^2 \left[\beta_2^{(i)} P_2(\cos \alpha_i) \right]^2 \right\}$$

$$V_{nucl}(R, \alpha_1, \alpha_2) = \int \rho_1^{(0)}(\vec{r} - \vec{R}) f_{eff} \left[\rho_1^{(0)} + \rho_2^{(0)} \right] \rho_2^{(0)}(\vec{r}) d^3 \vec{r}$$

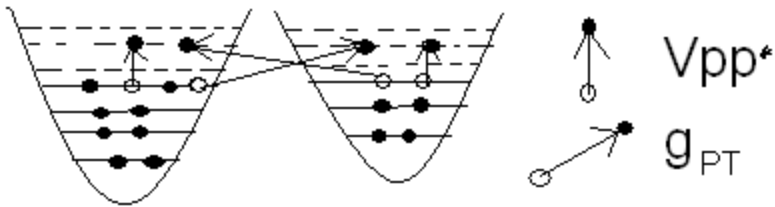
$$\rho_i^{(0)}(\vec{r}, \vec{R}_i, \alpha_i, \theta_i, \beta_2^{(i)}) = \left\{ 1 + \exp \left[\frac{|\vec{r} - \vec{R}_i(t)| - R_{0i} (1 + \beta_2^{(i)} Y_{20}(\theta_i, \alpha_i))}{a} \right] \right\}^{-1}.$$

$$V_{rot} = \hbar^2 \frac{l(l+1)}{2\mu [R(\alpha_1, \alpha_2)]^2 + J_1 + J_2}$$

Friction coefficients

$$\gamma_\lambda = \frac{2}{i\hbar^2 D_\lambda} \sum_{i,i',j,k} (n_j^{(i)} - n_k^{(i')}) \left| \frac{\partial V_{jk}(R, \beta_\lambda)}{\partial \beta_\lambda} \right|^2 \int_{t_0}^t dt' (t-t') \exp\left(\frac{t-t'}{\tau_{jk}}\right) \sin\left[(\varepsilon_j - \varepsilon_k)(t-t')/\hbar\right]$$

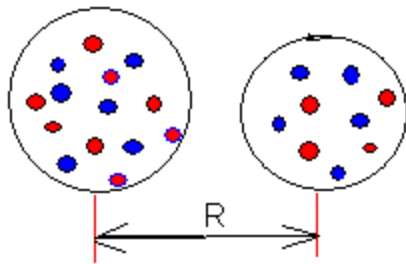
$$\frac{1}{\tau_i^{(\alpha\alpha)}} = \frac{\sqrt{2}\pi}{32\hbar\varepsilon_{F_K}^{(\alpha\alpha)}} \left[(f_K - g)^2 + \frac{1}{2}(f_K + g)^2 \right] \left[(\pi\pi_K)^2 + (\varepsilon_i - \lambda_K^{(\alpha\alpha)})^2 \right] \left[1 + \exp\left(\frac{\lambda_K^{(\alpha\alpha)} - \varepsilon_i}{T_K}\right) \right]^{-1}$$



ε_j and ε_k are single particle energies of nucleons in dinuclear system;

$$\Gamma_j = \hbar / \tau_j$$

Decay width of the single-particle excitations of nucleons caused by residual forces .



G.G. Adamian, et al. Phys. Rev. C56 No.2, (1997) p.373-380

Hamiltonian for calculation of the transport coefficients

The macroscopic motion of nucleus and microscopic motion of nucleons must be calculated simultaneously.

$$H = H_{\text{coll}} + H_{\text{micr}} + \delta V \quad (1)$$

where

$$H_{\text{coll}} = \frac{P^2}{2\mu} + U(R) \text{ - for the relative motion of nuclei;} \quad (2)$$

$$H_{\text{micr}} = \sum_{i_P} \varepsilon_{i_P} \hat{a}_{i_P}^+ \hat{a}_{i_P} + \sum_{i_T} \varepsilon_{i_T} \hat{a}_{i_T}^+ \hat{a}_{i_T} \text{ - for nucleons of nuclei;} \quad (3)$$

$$\begin{aligned} \delta V = & \sum_{i_P, j_T} g_{i_P j_T}(R) (\hat{a}_{i_P}^+ \hat{a}_{j_T} + \hat{a}_{j_T}^+ \hat{a}_{i_P}) + \sum_{i_P, j_T} \kappa_{i_P j_T}(R) (\hat{a}_{i_P}^+ \hat{a}_{j_T} + \hat{a}_{j_T}^+ \hat{a}_{i_P}) \\ & + \sum_{i_P, j_P} \Lambda_{i_P j_P}^{(T)}(R) \hat{a}_{i_P}^+ \hat{a}_{j_P} + \sum_{i_T, j_T} \Lambda_{i_T j_T}^{(P)}(R) \hat{a}_{i_T}^+ \hat{a}_{j_T} \text{ -- nucleon exchange between nuclei and} \\ & \text{particle - hole excitations in nuclei;} \end{aligned} \quad (4)$$

$g_{i_P j_T}$, $\kappa_{i_P j_T}$ and $\Lambda_{i_T j_T}^{(P)}$ – matrix elements of nucleon exchange between nuclei and particle – hole excitations in them caused by meanfield of partner nucleus.

Master equations for the nucleon occupation numbers and
Equation of motion for the relative distance

$$i\hbar \frac{\partial \hat{n}(t)}{\partial t} = [H(R(t), \hat{n}(t))], \quad (5) \quad n_i(t) = a_i^\dagger a_i \quad i=P,T$$

$$i\hbar \frac{\partial \hat{P}(t)}{\partial t} = [H(R(t), \hat{P}(t))], \quad (6) \quad P \equiv (n_P, j_P, l_P, m_P)$$

$$T \equiv (n_T, j_T, l_T, m_T)$$

$$i\hbar \frac{\partial \tilde{n}_i(t)}{\partial t} = [H, n_i(t)] - \frac{i\hbar}{\tau_i} [n_i(t) - n_{i^{eq}}(R(t))] \quad (7)$$

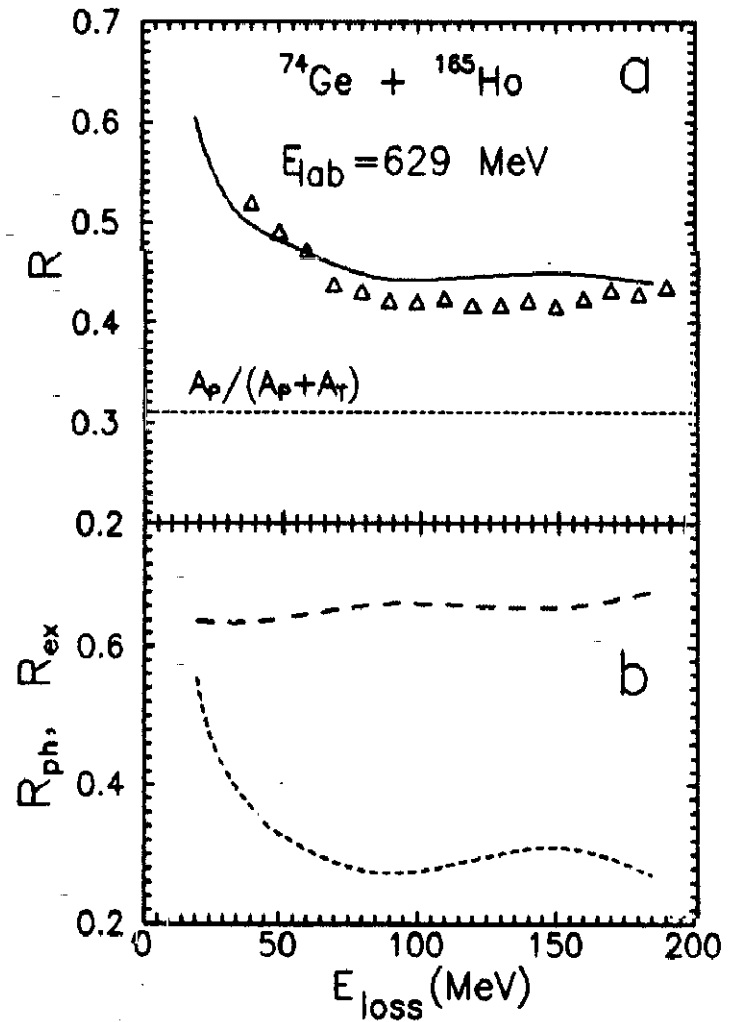
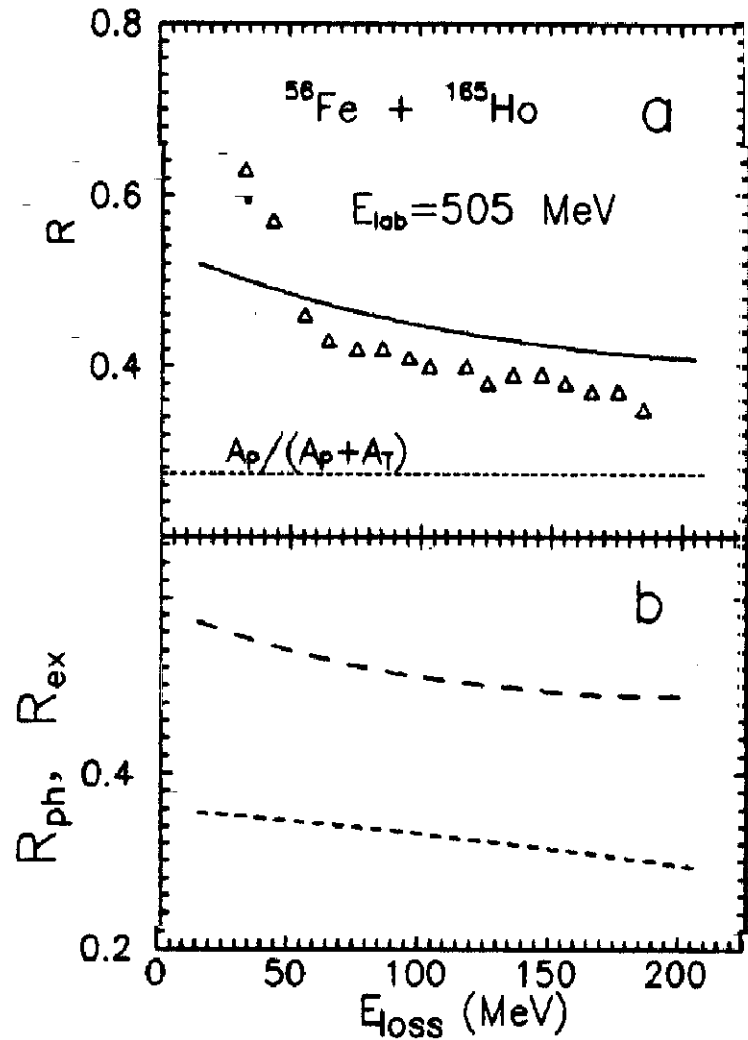
$$\tilde{n}_i = \tilde{n}_i^{eq}(R(t)) \left[1 - \exp\left(\frac{-\Delta t}{\tau_i}\right) \right] + n_i(t) \exp\left(\frac{-\Delta t}{\tau_i}\right)$$

$$n_i(t) = \tilde{n}_i(t - \Delta t) + \sum_k \bar{W}_{ik}(R(t), \Delta t) [\tilde{n}_k(t - \Delta t) - \tilde{n}_i(t - \Delta t)] \quad (8)$$

$$\bar{W}_{ik}(R(t), \Delta t) = |V_{ik}(R(t))|^2, \quad V_{ik}(R) = \langle i | V(R) | k \rangle \quad (9)$$

G.G. Adamian, et al. Phys. Rev. C **53**, (1996) p.871-879
R.V. Jolos et al., Eur. Phys. J. A **8**, 115–124 (2000)

Non-equilibrium sharing of excitation energy in the deep-inelastic collisions is explained by nuclear shell structure.



Effect of shell structure on energy dissipation in heavy-ion collisions

R.V. Jolos¹, A.K. Nasirov^{1,2,a}, G.G. Adamian^{2,3}, and A.I. Muminov²

¹ Joint Institute for Nuclear Research, 141980, Dubna, Russia

² Heavy Ion Physics Department, Institute of Nuclear Physics, 702132 Ulugbek, Tashkent, Uzbekistan

³ Institut für Theoretische Physik der Justus-Liebig-Universität, D-35392 Giessen, Germany

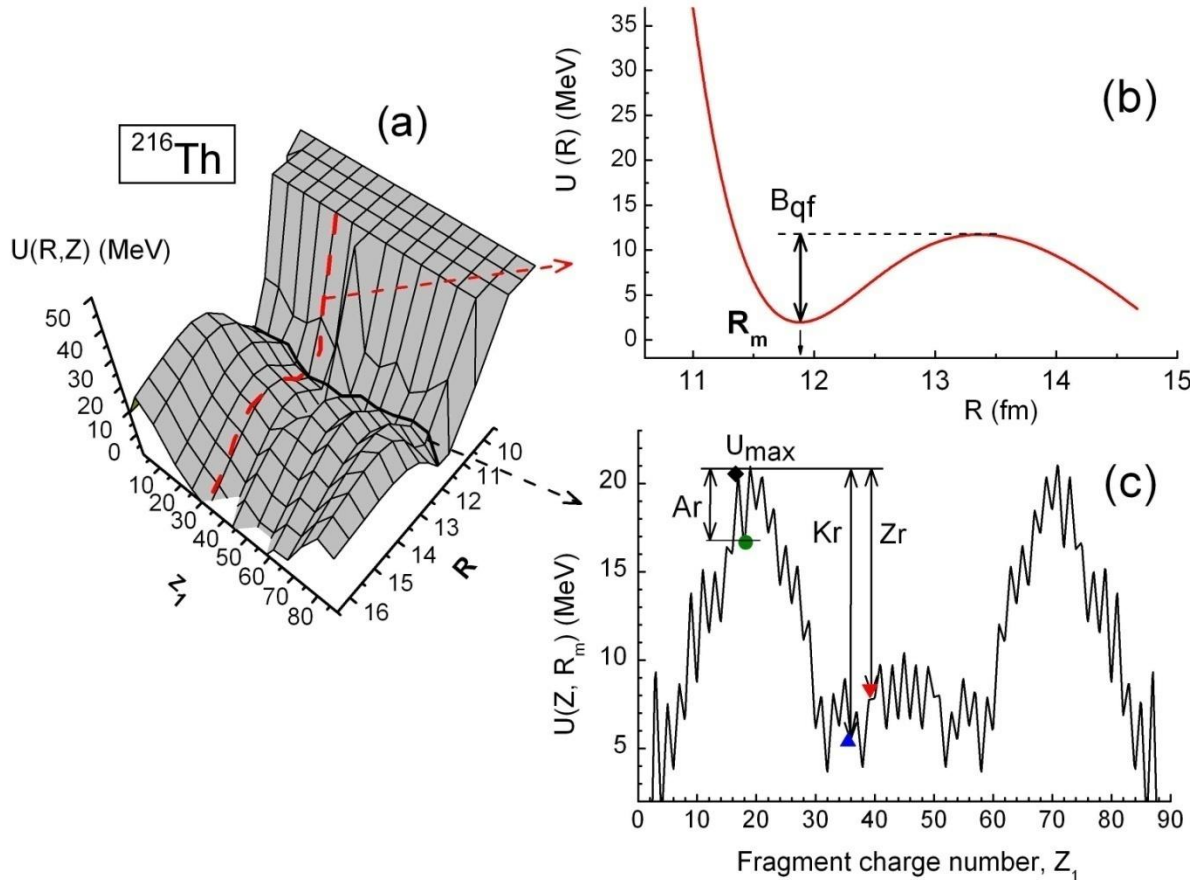
$$\begin{aligned}
 E_{P(T)}^*(t + \Delta t) &= E_{P(T)}^*(t) \\
 &+ \sum_{i_{P(j_T)}} \left[\tilde{\varepsilon}_{i_{P(j_T)}}(\mathbf{R}(t)) - \lambda_{P(T)}(\mathbf{R}(t)) \right] \\
 &\times \left[\tilde{n}_{i_{P(j_T)}}(t + \Delta t) - \tilde{n}_{i_{P(j_T)}}(t) \right]. \quad (28)
 \end{aligned}$$



Application of the dinuclear system model
to the study of synthesis of superheavy elements.



Driving potential $U_{\text{driving}}(c)$ for reactions $^{40}\text{Ar}+^{172}\text{Hf}$, $^{86}\text{Kr}+^{130}\text{Xe}$, $^{124}\text{Sn}+^{92}\text{Zr}$ leading to formation of compound nucleus ^{216}Th :

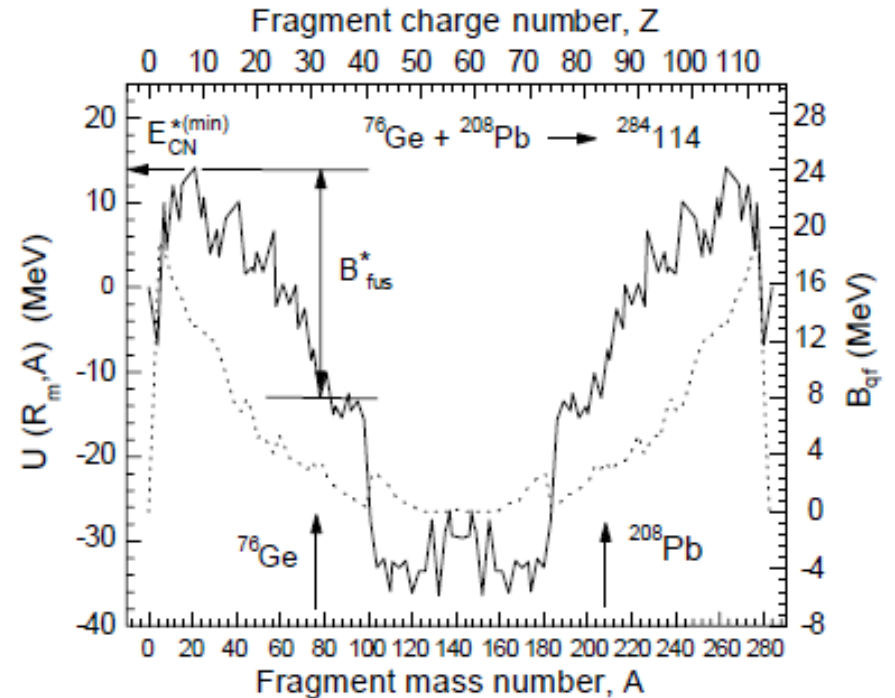
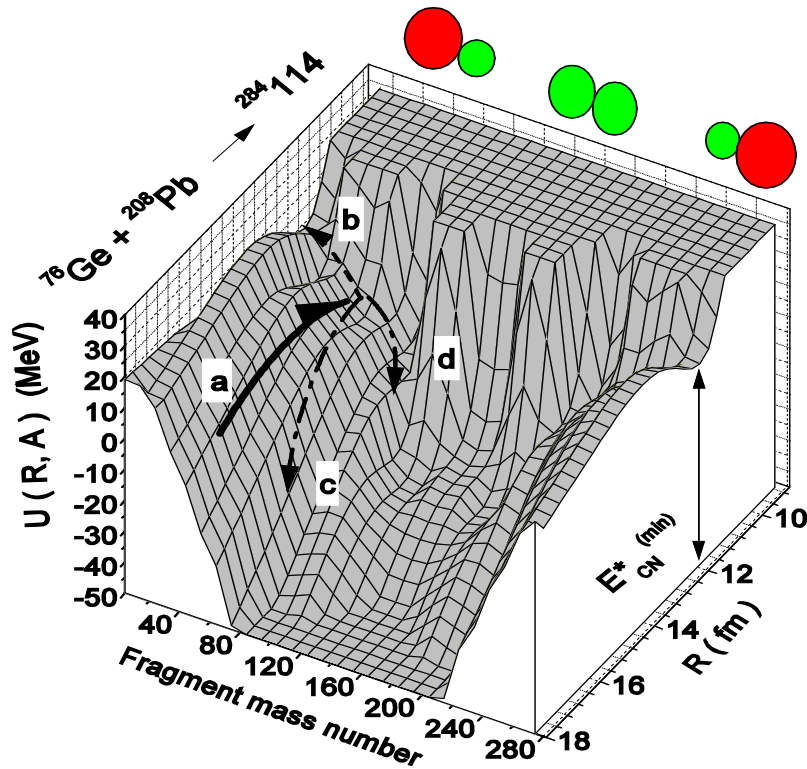


Due to peculiarities of shell structure
 $B_{\text{fus}}(\text{Kr}) > B_{\text{fus}}(\text{Kr})$
 and, consequently,

$$\sigma_{\text{fus}}(\text{Kr}+\text{Xe}) < \sigma_{\text{fus}}(\text{Zr}+\text{Sn})$$

$$U_{\text{driving}} = B_1 + B_2 - B_{(1+2)} + V(R)$$

Potential energy surface of dinuclear system

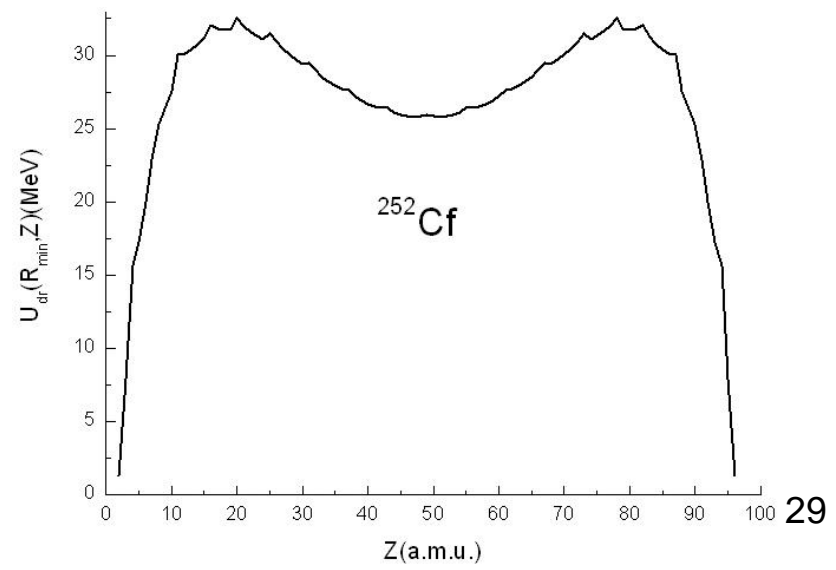
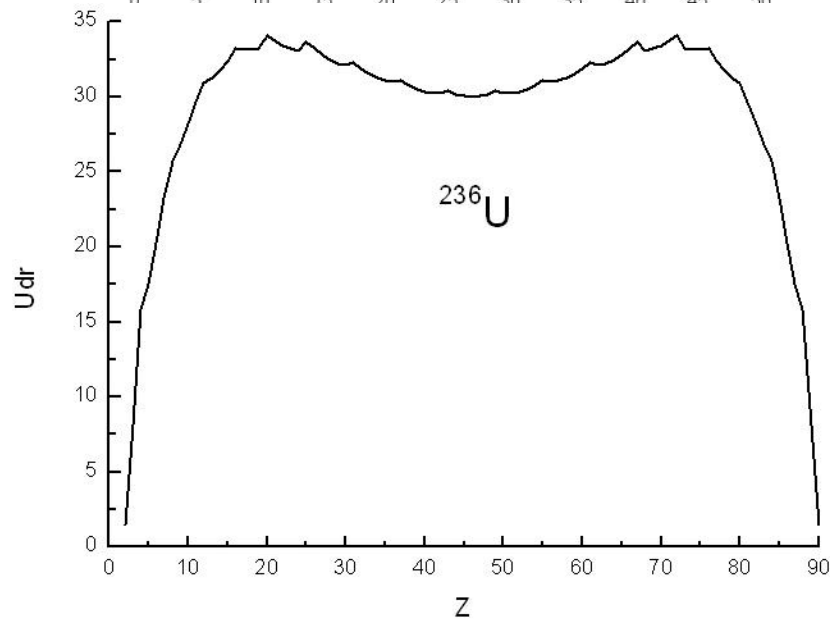
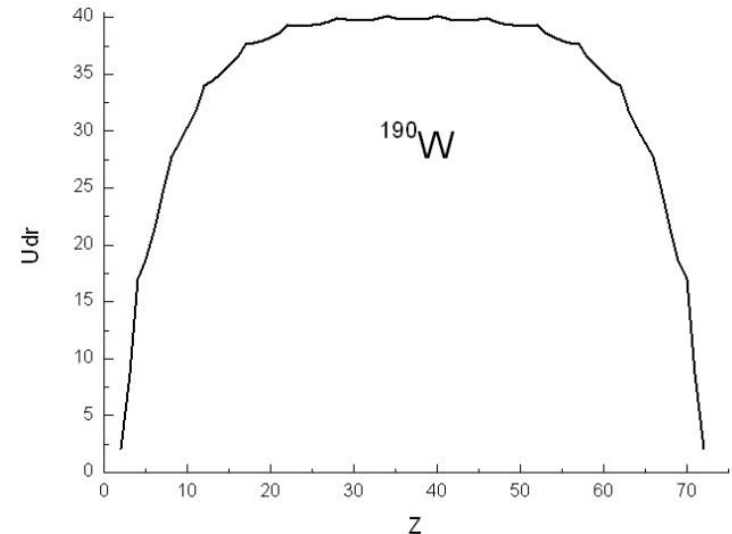
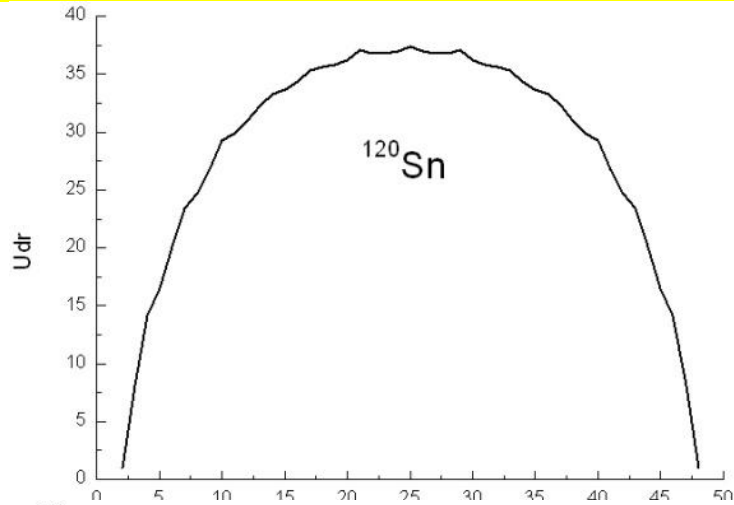


a- entrance channel;
b- fusion channel;
c and **d** are quasifission channels

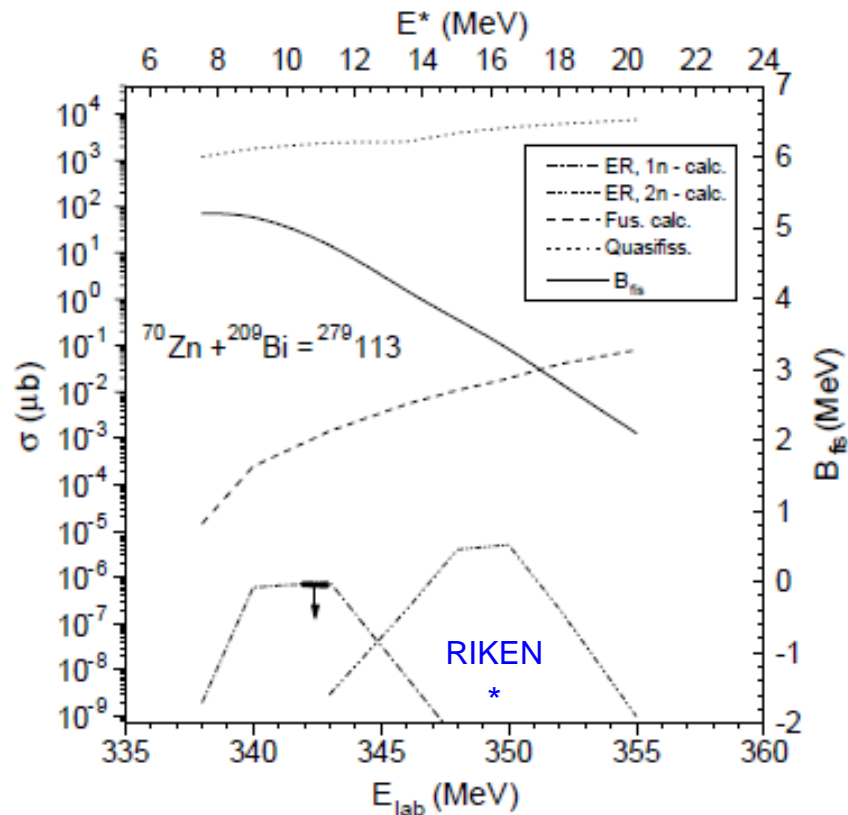
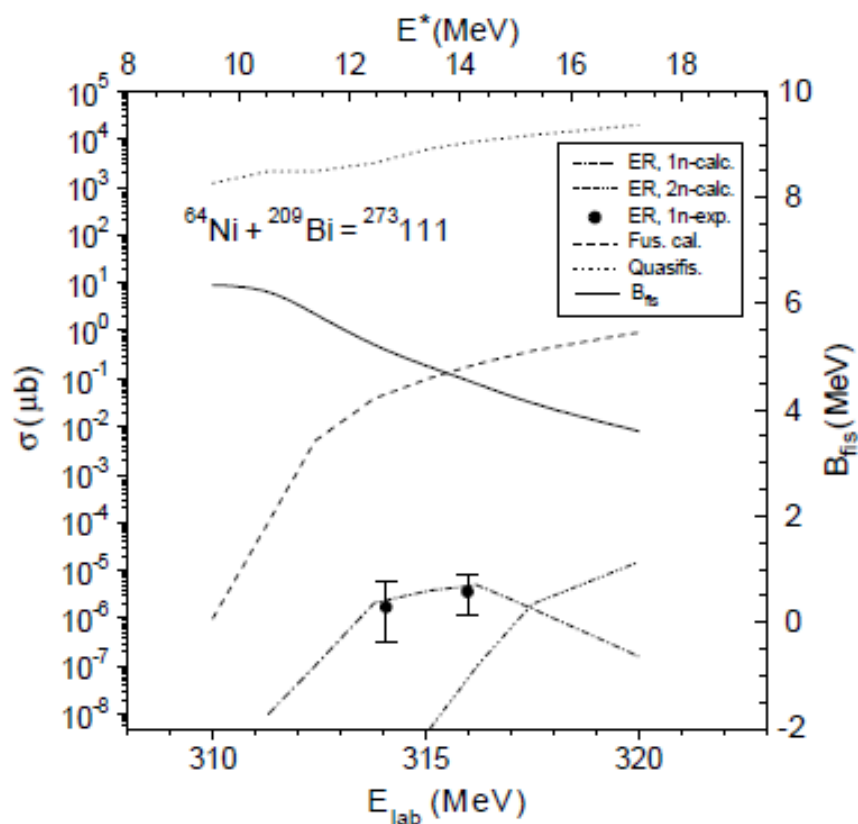
G. Giardina, S. Hofmann, A.I. Muminov, and A.K. Nasirov,
 Eur. Phys. J. A **8**, 205–216 (2000)

$$U_{dr}(A, Z, \beta_1, \beta_2) = B_1 + B_2 + V(A, Z, \beta_1; \beta_2; R) - B_{CN} - V_{CN}(L)$$

The change of driving potential by increase of the mass and charge of compound nucleus.



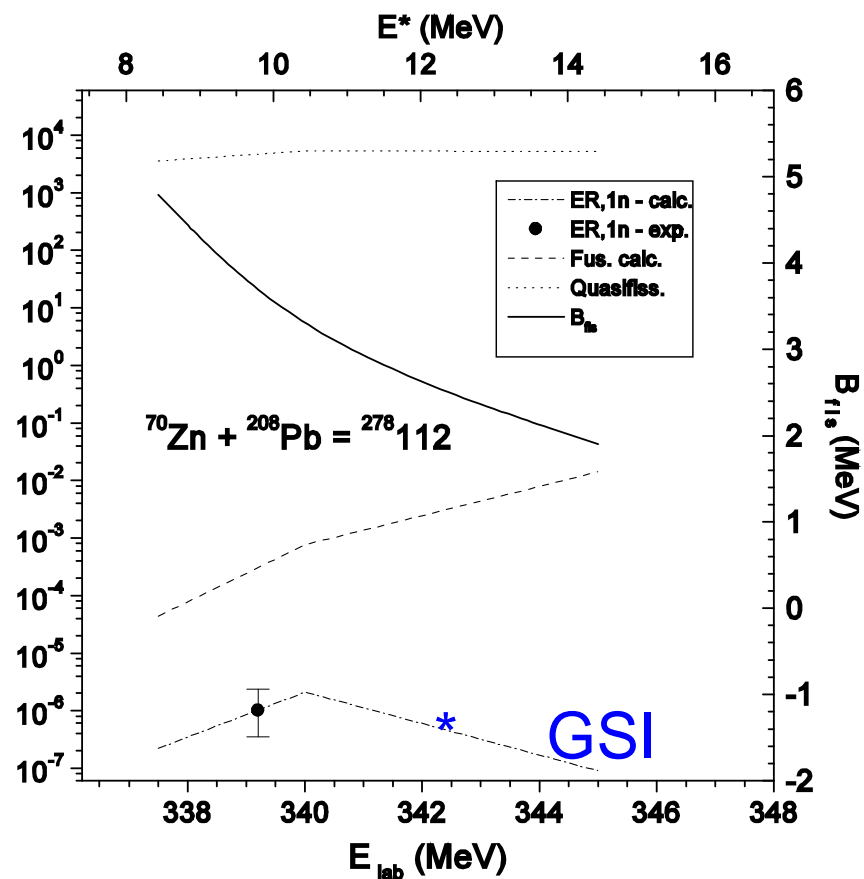
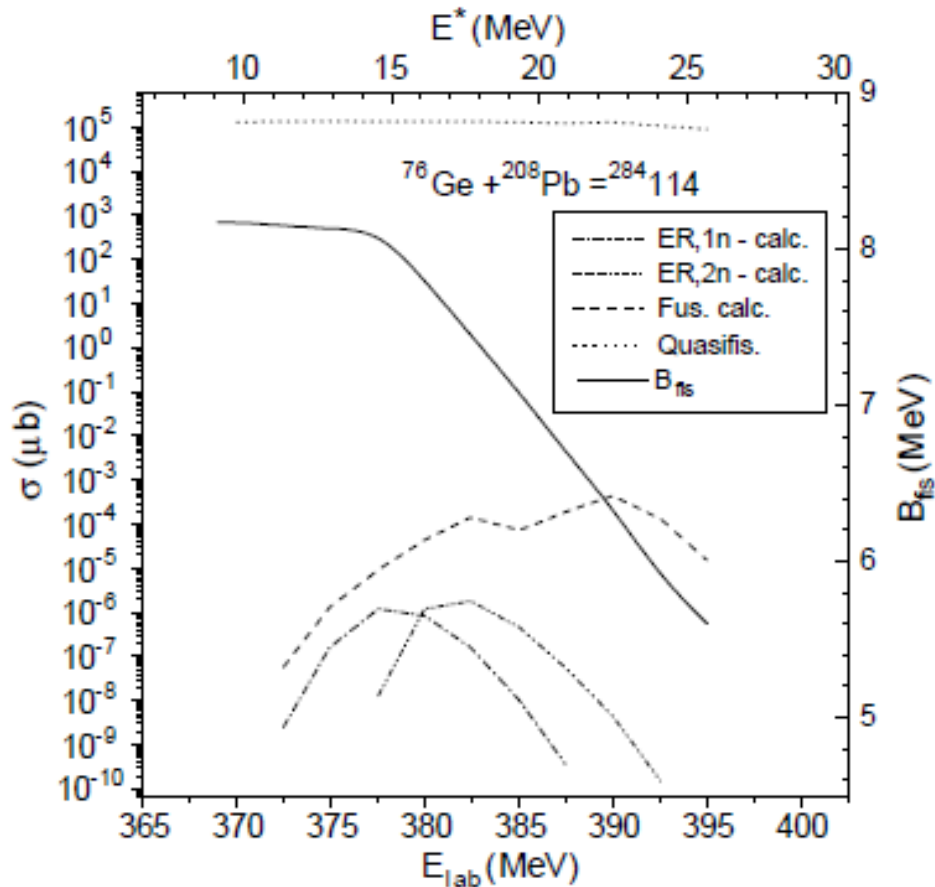
Theoretical results of capture, fusion and evaporation residues cross sections and comparison of them with the experimental data for the “cold” $^{64}\text{Ni}+^{209}\text{Bi}$ and $^{70}\text{Zn}+^{209}\text{Bi}$ reactions.



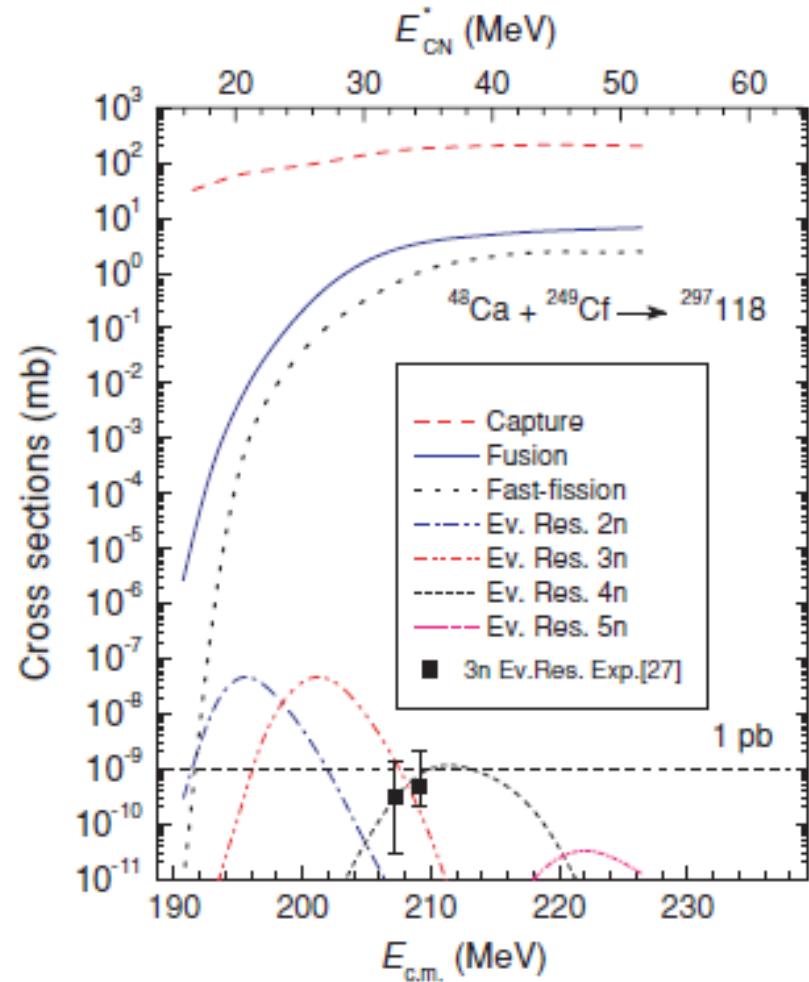
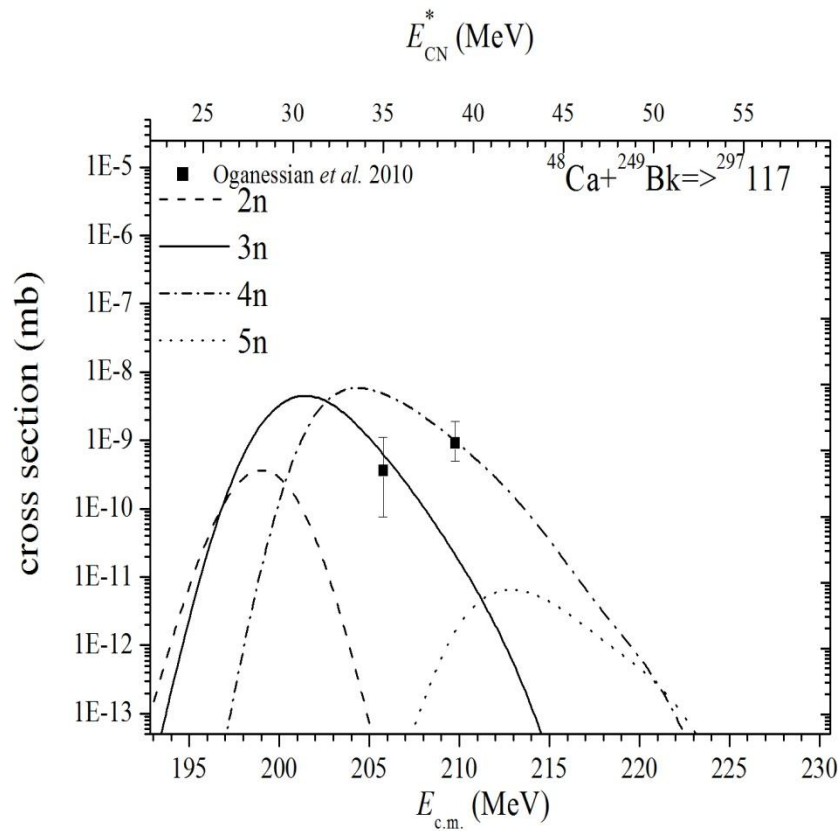
G. Giardina, S. Hofmann, A.I. Muminov, and A.K. Nasirov,
 Eur. Phys. J. A **8**, 205–216 (2000)

Results of calculation and comparison of them with the experimental data for the “cold” $^{76}\text{Ge}+^{208}\text{Pb}$ and $^{70}\text{Zn}+^{208}\text{Pb}$ reactions.

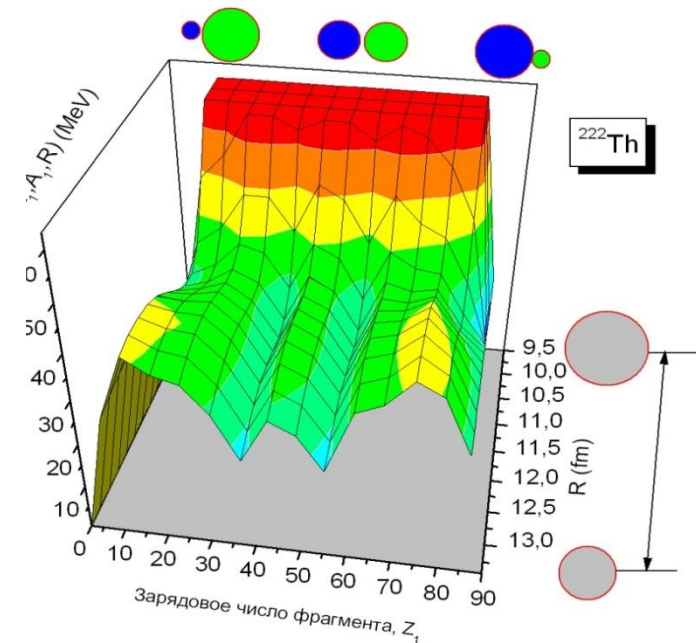
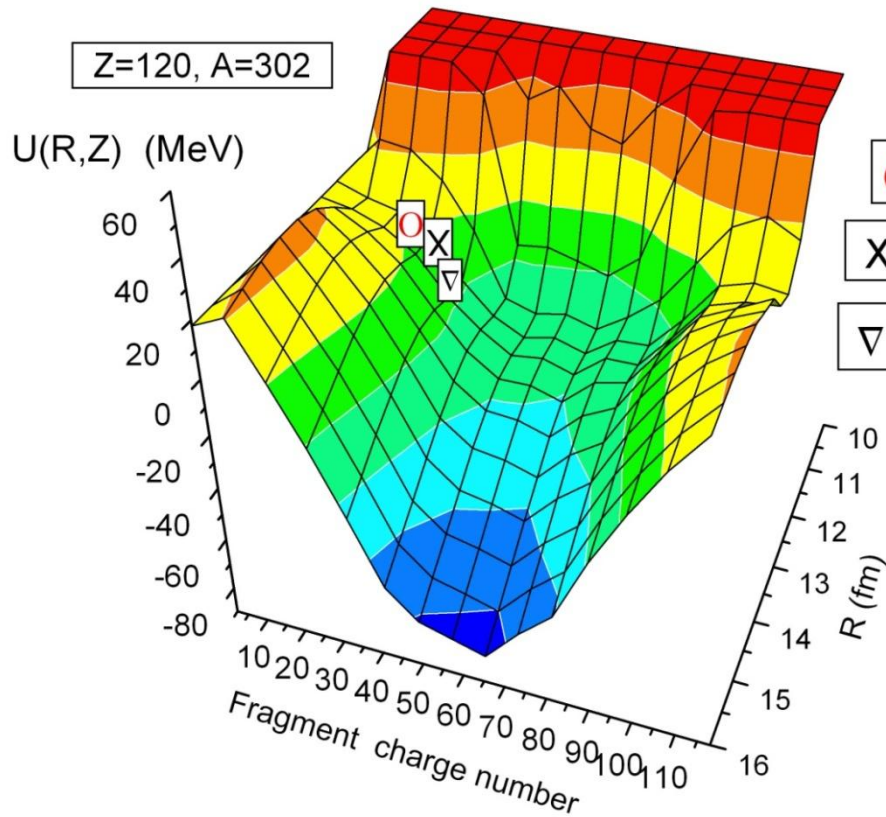
G. Giardina, S. Hofmann, A.I. Muminov, and A.K. Nasirov,
 Eur. Phys. J. A 8, 205–216 (2000)



Synthesis of superheavy elements in hot fusion reactions.

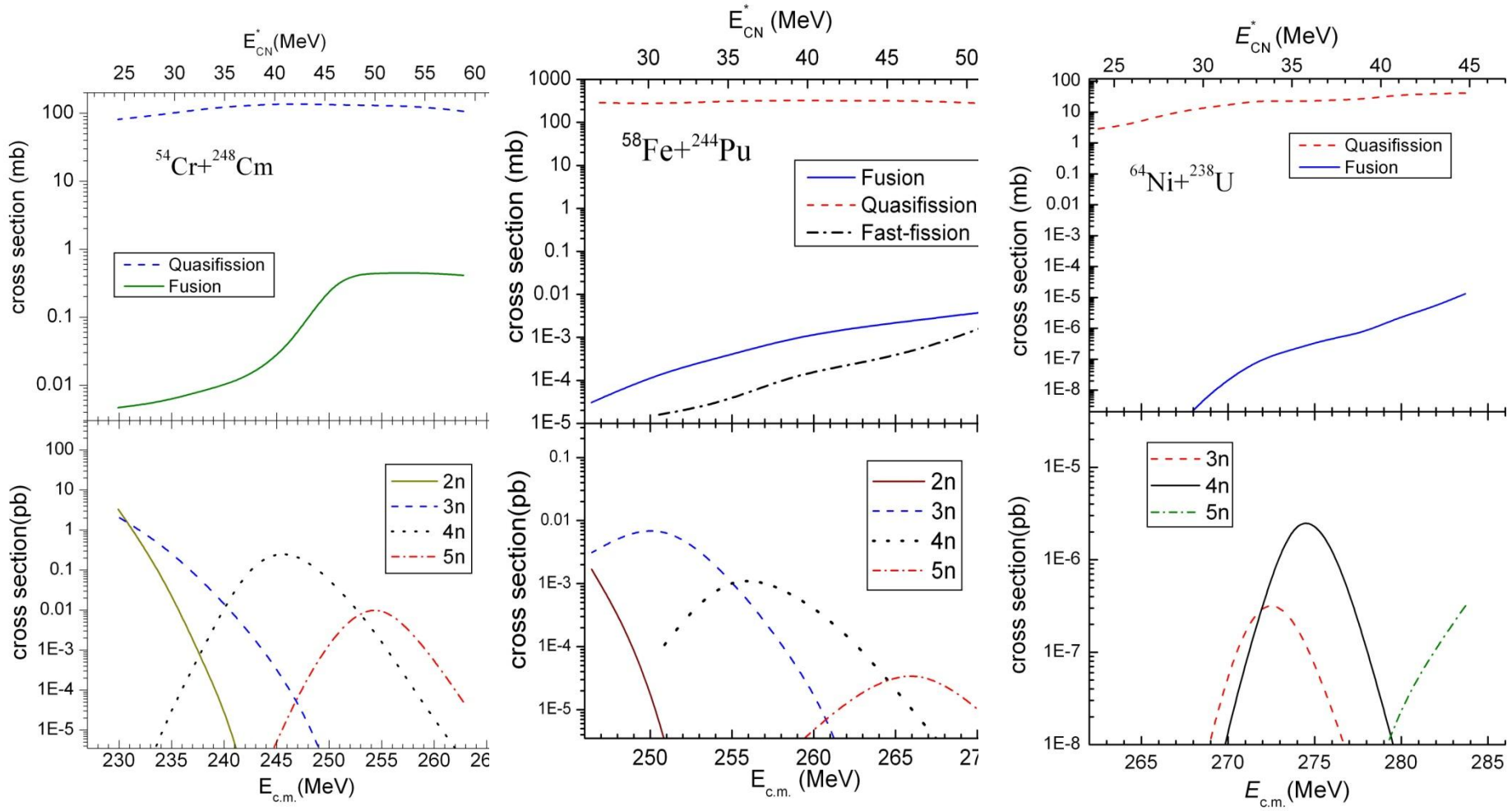


Potential energy surface of dinuclear system



$$U_{dr}(A, Z, \beta_1, \beta_2) = B_1 + B_2 + V(A, Z, \beta_1; \beta_2; R) - B_{CN} - V_{CN}(L)$$

Comparisons of cross sections for complete fusion and formation evaporation residues



Fission barriers calculated by macroscopic-microscopic model:
M. Kowal, P. Jachimowicz, and A. Sobiczewski, Phys. Rev. C **82**, 014303 (2010)

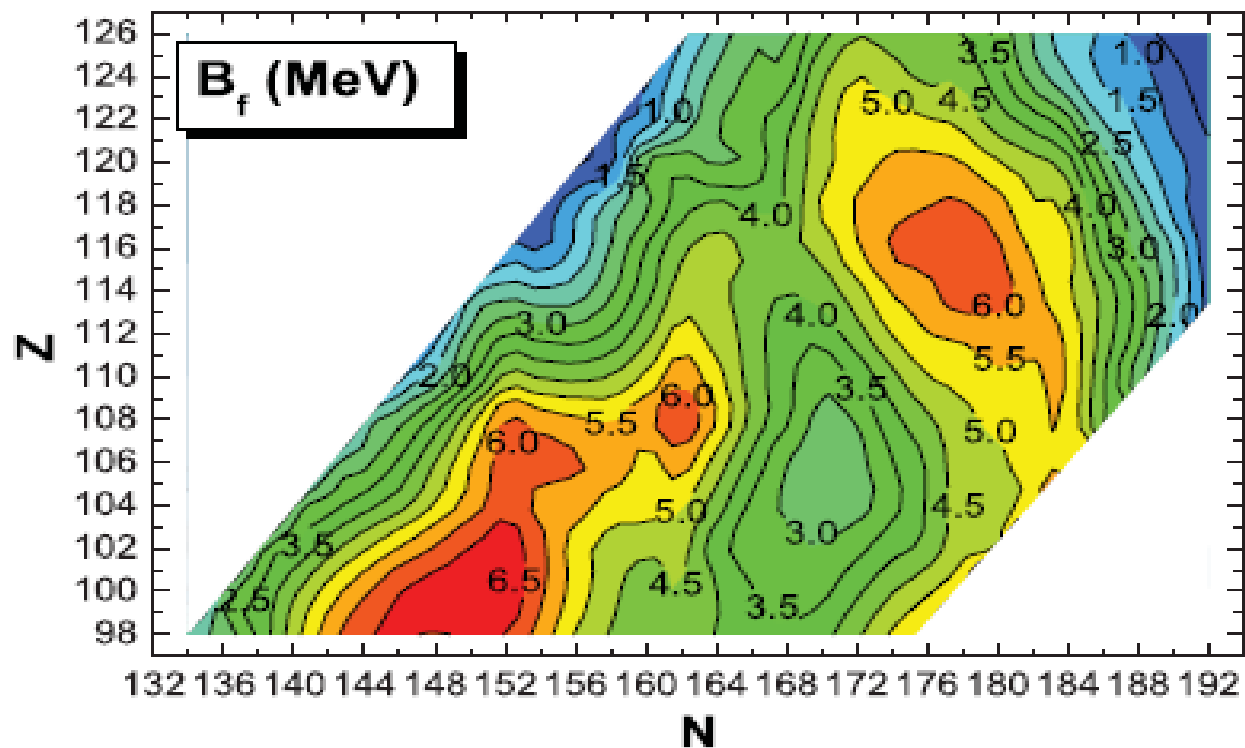
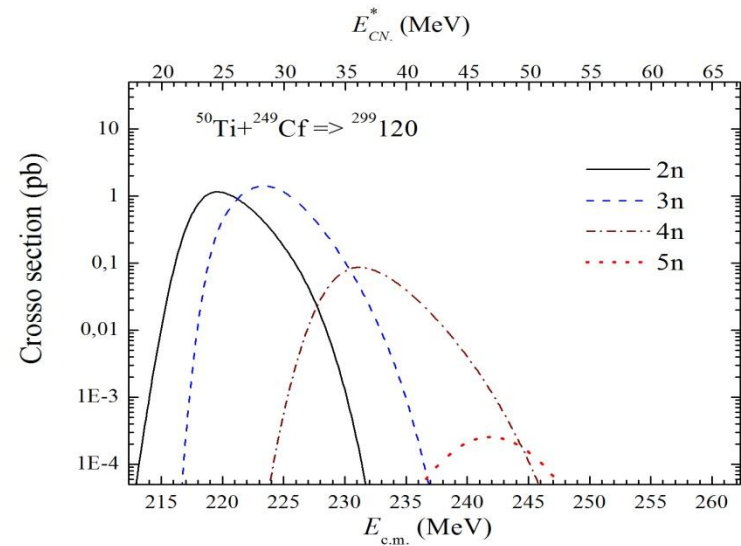
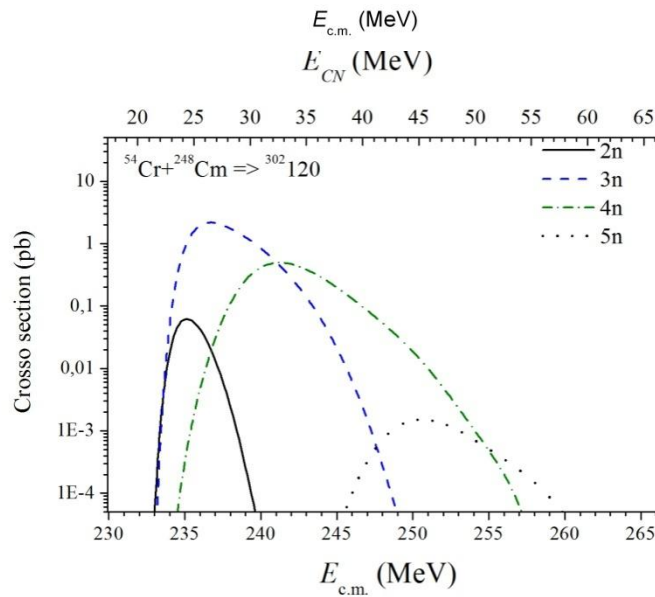
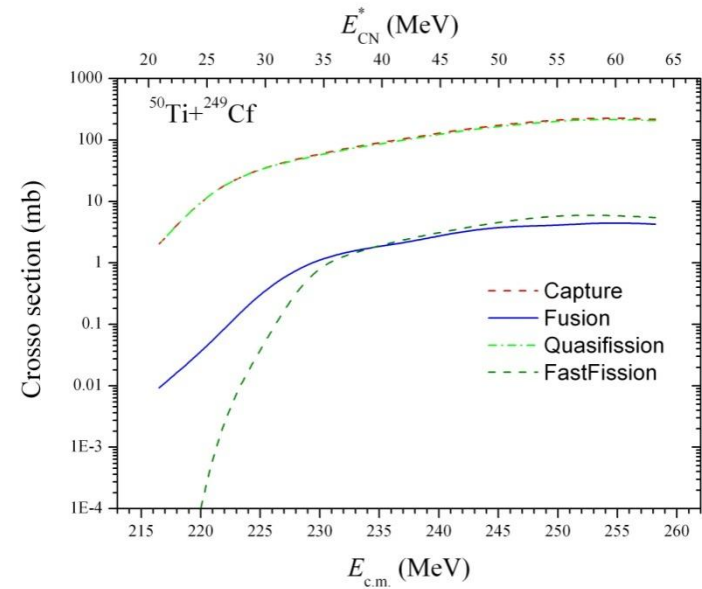
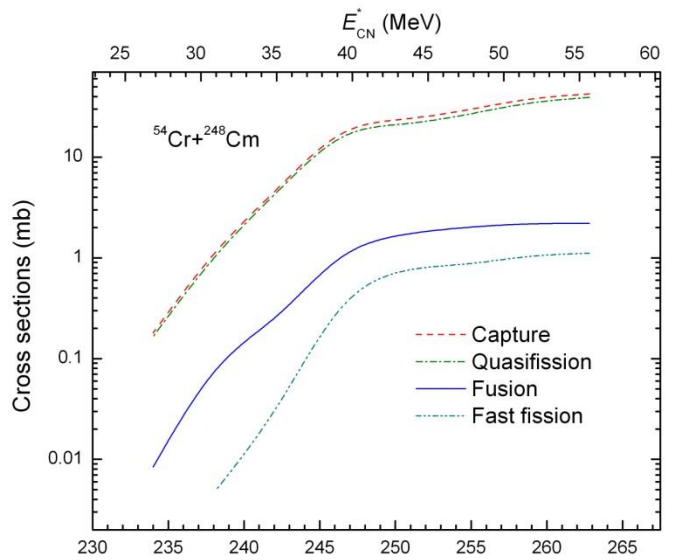


FIG. 6. (Color online) Contour map of calculated fission barrier heights B_f for even-even superheavy nuclei.

Synthesis of superheavy elements in hot fusion reactions.



Dependence of the fission barrier on the excitation energy and angular momentum of compound nucleus.

$$B_{\text{fis}}(J, T) = c B_{\text{fis}}^{\text{m}}(J) - h(T) q(J) \delta W,$$

with

$$h(T) = \begin{cases} 1, & T \leq 1.65 \text{ MeV}, \\ k \exp(-mT), & T > 1.65 \text{ MeV}, \end{cases}$$

and

$$q(J) = \{1 + \exp[(J - J_{1/2})/\Delta J]\}^{-1},$$

where $B_{\text{fis}}^{\text{m}}(J)$ is the parameterized macroscopic fission barrier [15] depending on angular momentum J , $\delta W = \delta W_{\text{sad}} - \delta W_{\text{gs}} \simeq -\delta W_{\text{gs}}$ is the microscopic (shell) correction to the fission barrier taken from the tables [8] and the constants for the macroscopic fission barrier scaling, temperature and angular momentum dependencies of the microscopic correction are chosen to be as follows: $c = 1.0$, $k = 5.809$, $m = 1.066 \text{ MeV}^{-1}$, $\Delta J = 3\hbar$; for nuclei with $Z > 102$ we use $J_{1/2} = 20\hbar$. This procedure let the shell corrections become dynamical quantities, too.

Conclusions

The complete fusion mechanism in the heavy ion collisions strongly depends on the entrance channel peculiarities: mass (charge) asymmetry, shell structure of interacting nuclei, beam energy and angular momentum (impact parameter of collision).

The calculation of the optimal beam energy to reach the maximal cross section of evaporation residues needs the values of fission barrier and binding energy of the being formed compound nucleus.