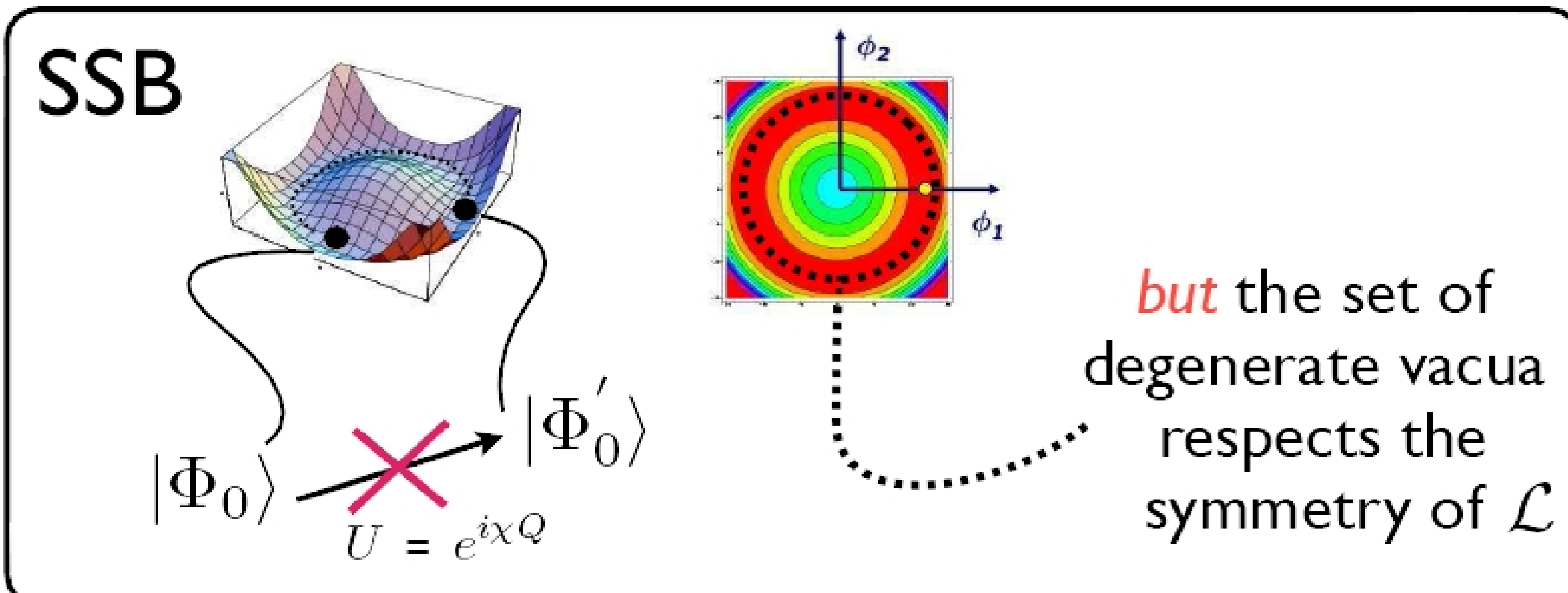
# Spontaneously broken symmetries in nuclear system - are there any?

Spontaneous symmetry breaking in QFT (global symmetry):

$$\mathcal{L}\left(\phi_i(x), \partial_\mu \phi_i(x)\right)$$ - Lagrangian

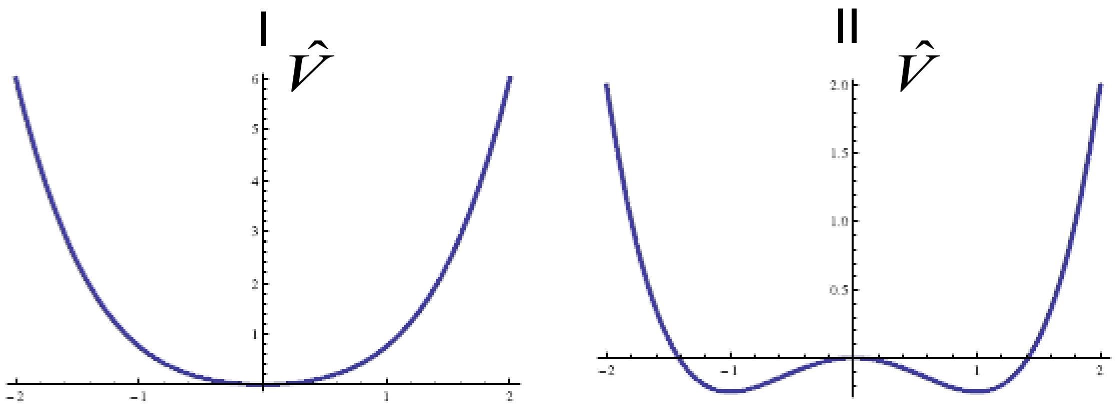
$$\begin{aligned} \phi_i(x) &\rightarrow V_{ij}(\xi_1, \xi_2, \ldots, \xi_n)\phi_j(x) \\ V &= e^{i\xi_k I_k}\ , \end{aligned}$$ - symmetry of Lagrangian

$$Q_k = \int J_k^0(\boldsymbol{x}) d^3x$$ - charge



## Quantum mechanics of a system with finite number of degrees of freedom

$$\hat{H} = \hat{T} + \hat{V}$$ - Hamiltonian




In both cases the exact ground state will be symmetric (ie. NO SSB)

If we perform some kind of approximation, eg. mean field:

- in the case I: ground state is symmetric
- in the case II: two nonsymmetric degenerate ground states: $|\Phi_1\rangle, |\Phi_2\rangle$

BUT

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_1\rangle \pm |\Phi_2\rangle\right) \quad *$$

are again symmetric!

In a system with finite number of degrees of freedom there is no mechanism which prevent of taking the solution in the form (*), since the tunneling probability is always nonzero!

## EXAMPLE: Harmonic one dimensional crystal

$$H=\sum_j \frac{p_j^2}{2m}+\frac{\kappa}{2}\sum_j (x_j-x_{j+1})^2$$ - Hamiltonian (N particles)

$$p_j = iC\sqrt{\frac{\hbar}{2}}(b_j^\dagger - b_j),$$

$$x_j = \frac{1}{C}\sqrt{\frac{\hbar}{2}}(b_j^\dagger + b_j),$$

$$H=\sqrt{\frac{\hbar^2\kappa}{2m}}\sum_k\left[A_k b_k^\dagger b_k + \frac{B_k}{2}(b_k^\dagger b_{-k}^\dagger + b_k b_{-k}) + 1\right],$$

$$A_k = 2-\cos(ka),\quad B_k = -\cos(ka).$$

Bogolubov transformation:

$$\beta_k = \cosh(u_k) b_{-k} + \sinh(u_k) b_k^\dagger$$

$$H = 2\hbar\sqrt{\frac{\kappa}{m}}\sum_k \sin\left|\frac{ka}{2}\right|\left[n_k+\frac{1}{2}\right]$$

where: $n_k = \beta_k^+ \beta_k$

## HOWEVER!

Bogolubov transformation $\beta_k = \cosh(u_k) b_{-k} + \sinh(u_k) b_k^\dagger$ is ill-defined for k=0

$$\sinh(u_0) \to -\text{infinity}$$
$$\cosh(u_0) \to \text{infinity}$$

Hence the case: k=0, has to be treated separately.

In terms of original operators it has the form:

$$H_{\text{coll}} = \frac{p_{\text{tot}}^2}{2Nm} + \text{const}$$ , where $p_{\text{tot}} \equiv \sum_j p_j$

Clearly: $[H_{coll}, H_{k \neq 0}] = 0$

The collective part represents the motion of the system as a whole.

The eigenenergies of the collective Hamiltonian scale like 1/N – nearly degenerate in thermodynamic limit.

These states are thermodynamically invisible, since:

$$Z_{coll} \sim \sqrt{N}, \quad F_{coll} = -kT \log Z_{coll} \sim \log N$$

Let us consider a small (translational symmetry breaking) perturbation of the collective Hamiltonian:

$$H_{\rm coll}^{\rm SB} = \frac{p_{\rm tot}^2}{2Nm} + \frac{B}{2}x_{\rm tot}^2.$$

B – small perturbation

$$\psi_0(x_{\rm tot}) = \left(\frac{m\omega N}{\pi\hbar}\right)^{1/4} e^{-(m\omega N/2\hbar)x_{\rm tot}^2}, \quad \text{with} \quad \omega = \sqrt{B/mN}.$$

The new ground state is effectively made up of states with total momenta

$$p_{tot} < \sqrt{N}$$

The ground state wave packet becomes completely localized at $x_0$=0 in thermodynamic limit

$$\lim_{N\to\infty}\lim_{B\to 0} |\psi_0(x_{\rm tot})|^2 = \text{const},$$

$$\lim_{B\to 0}\lim_{N\to\infty} |\psi_0(x_{\rm tot})|^2 = \delta(x_{\rm tot}).$$

The sign of the existence of the Spontaneous Symmetry Breaking (SSB) effect is indicated by the noncommutativity of these limits.

The system is unstable (in the thermodynamic limit) with respect to an infinitely weak perturbation.

Order parameter: $\langle x_{\rm tot}^2\rangle / N.$

$$\lim_{N\to\infty}\lim_{B\to 0} \langle x_{\rm tot}^2\rangle/N = \infty$$

$$\lim_{B\to 0}\lim_{N\to\infty} \langle x_{\rm tot}^2\rangle/N = 0,$$

**Summarizing:**

- In quantum systems the SSB occurs as a result of instability of the system with respect to an infinitely small external perturbation, which break the symmetry of the original Hamiltonian.

- The SSB gives rise to the set of vacua, each defining its own Hilbert space. These spaces cannot be connected by any local operator. In our example each Hilbert space is associated with a particular localization of the whole crystal.

- Physically it means that the tunneling between vacua vanishes in the thermodynamic limit, since it behaves as: exp(-N), or exp(-V). Hence, one cannot restore the symmetry by taking linear combination of all vacua.

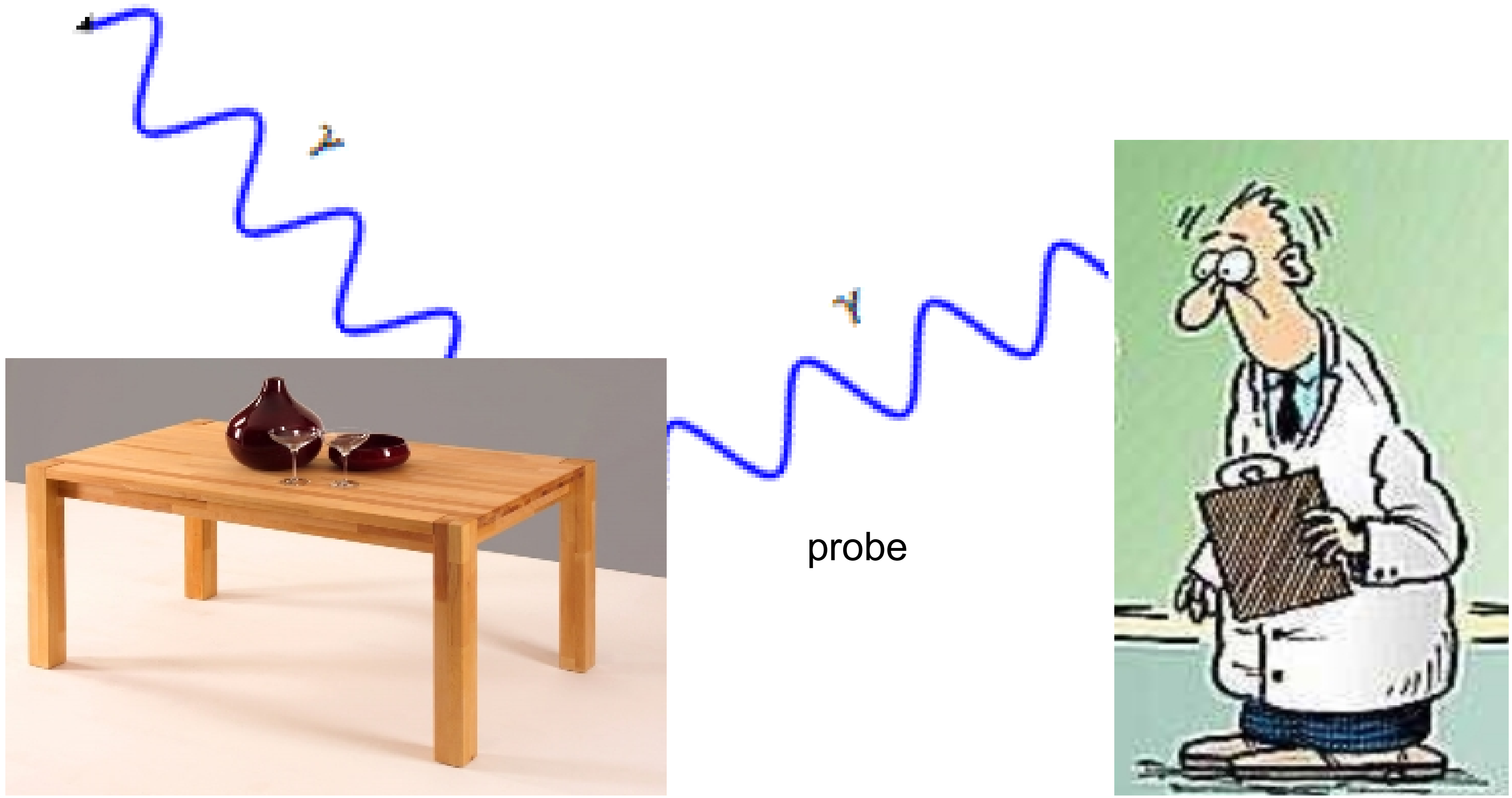
- In other words, this is due to an infinite „stiffness" of the many-body wave function in the thermodynamic limit. For example the overlap between a slightly perturbed Slater determinants describing N fermions behaves like:

$$\langle \psi(0) | \psi(\delta) \rangle \sim N^{-|\delta|}$$ (orthogonality catastrophe)

This effect gives rise to the stability of the SSB configuration

**Spontaneous Symmetry Breaking and quantum decoherence (Can we have SSB in finite systems?)**

The possible source of the symmetry breaking field which causes the SSB can be associated with the measurement process.




Quantum object in the SSB state

observer

Let us denote the states of the quantum object as: $\varphi_i^O$

Let us denote the states of the probe as: $\varphi_i^P$ $\quad i=1,2$

The total state of the system initially in the state:

$$\left( a\varphi_1^O + b\varphi_2^O \right) \otimes \varphi_{initial}^P$$

Due to the measurement process the coherence between states 1 and 2 is lost:

$$\rho = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix} \longrightarrow \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}$$

Before measurement $\qquad$ After measurement

The measurement process destroys the coherence between various SSB vacua and picks only one.

**Conditions:** - the time of the measurement (interaction of the probe with the quantum system) is much shorter than the tunneling rate between various vacua.
- the resolution of the probe (e.g. wavelength of the gamma ray) should allow to distinguish between various SSB vacua.

## Enhanced stability of the SSB vacuum due to the measurement process

Suppose we have two level system with eigenstates: $|\Phi_{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_1\rangle \pm |\Phi_2\rangle\right)$

separated by the energy: $\hbar\omega$

Let us assume we perform s measurement at time: $\tau = 0$ picking either the state 1 or 2. Subsequently the density matrix will evolve:

$$\exp(-iH\tau/\hbar)\rho\exp(iH\tau/\hbar) =$$

$$= \frac{1}{2}\begin{pmatrix} 1+\delta_0\cos\omega\tau & i\delta_0\sin\omega\tau \\ -i\delta_0\sin\omega\tau & 1-\delta_0\cos\omega\tau \end{pmatrix}$$

where $\delta_0 = \pm 1$

Hence the state of the system will loose the original property after the time $T \sim \omega^{-1}$

*T* is supposed to be very large. It is infinite in thermodynamical limit but finite for any system with finite number of degrees of freedom.

Suppose now we conduct a series of ***n*** measurements during the time interval ***t:***

$$\tau_k = k\tau\,,\quad \tau << T$$
$$k = 0,1,2,\ldots,n$$

The density matrix after *n-th* measurement will be of the form:

$$\rho = \frac{1}{2}\begin{pmatrix} 1+\delta & 0 \\ 0 & 1-\delta \end{pmatrix} \quad \textit{where} \quad \delta = \delta_0 \prod_{k=0}^{n} \cos(\omega\tau_k)$$

in the limit: $\omega\tau_k << 1$, ie. $\tau_k << T$

$$\delta \approx \delta_0 \exp\left(-\omega^2 n\tau^2/2\right) = \delta_0 \exp\left(-\omega^2\tau t/2\right) = \delta_0 \exp\left(-\lambda t\right)$$

Where the relaxation time:

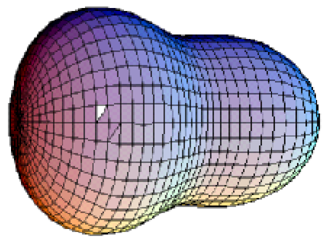
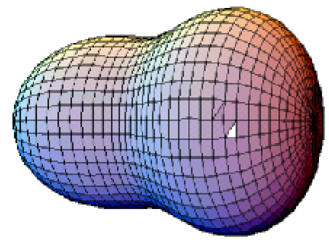
$$\lambda^{-1} = \left(\omega^2\tau/2\right)^{-1} \sim T^2/\tau >> T$$

The effect of blocking of the metastable configuration corresponding eg. to the SSB state in a finite quantum system (mean-field configuration) can be due to the interaction between the probe and the system during the acts of measurements

M. Simonius, PRL40, 980(1978)

## How this concept fits to atomic nuclei?

Let us consider as an example the parity breaking by the nuclear mean-field:

$$|\pm\rangle = \frac{1}{\sqrt{2}}\Big($$  $$\pm$$  $$\Big)$$

Energy difference: $\sim 1MeV$

Hence the SSB state flips between the two orientations with a frequency $\omega \sim 10^{21} s^{-1}$

In order to pin down a particular SSB mean-field configuration we would need to probe the nucleus with much higher frequency.
(Compare to the fastest e-m transitions in nuclei which are of the order of femtoseconds.)

The resolution of the probe (gamma or electrons) require energies at least of the order of few hundred MeV.

**Summary**

**The concept of the spontaneous symmetry breaking (SSB) in atomic nuclei has a vague meaning and strictly speaking SSB does not occur due to the tunneling between various SSB configurations.**

**Due to the finiteness of the nuclear system the nuclear SSB configurations (eg. mean-field solutions) form metastable states and their lifetimes are too short to be visible by an external probe (eg. photons or electrons).**

**Still the concept of SSB is sometimes useful as it helps to explain in an approximate way the structural changes of nuclear states (similarly to the concept of phase transition).**