EXOTIC NUCLEAR FORCES STUDIED WITHIN THE MEAN FIELD THEORY

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INTRODUCTION

- ⋆ Since the early days, the concept of **mean field** has been very succesful in nuclear structure physics.
- ⋆ We propose a method combining the non self-consistent mean-field part with self-consistent extra terms such as the **spin-orbit**, the **anti-symmetric spin-orbit**, the **tensor force**...
- ⋆ Our approach is based on a **minimal number of parameters** describing the form factors of the forces, in order to increase the **predictive power**.

GENERAL TWO-BODY INTERACTION

When looking for possible nucleon-nucleon interactions one usually adopts the following postulates (Eisenbud and Wigner; Okubo and Marshak) for the two-body potential :

- ⋆ Invariance with respect to particle exchange.
- ⋆ Invariance with respect to spatial translations.
- ⋆ Invariance with respect to spatial rotations.
- ⋆ Invariance with respect to rotations in isospace.
- ⋆ Galilean invariance.
- ⋆ Hermiticity.
- ⋆ Invariance with respect to inversions.
- ⋆ Invariance with respect to time-reversal.

SPIN-TENSOR DECOMPOSITION

- ⋆ In the fermionic spin-1/2 space, any operator can be expressed with the help of $\sigma_0 \equiv \mathbb{I}$ and the Pauli matrices σ_x, σ_y and σ_z.
- ⋆ Therefore, the space of two nucleons can be described by a set of $4 \times 4 = 16$ operators composed of the tensor product of the corresponding operators for each particle, as e.g. $\sigma_i^a \sigma_j^b$, with $i = 0, 1, 2, 3$ and $j = 0, 1, 2, 3$.
- ⋆ We require the interaction to be independent of the interchange between the two particles, and therefore we use the 6 irreducible tensors (Conze, Feldmeier, Manakos) :

$$S_1^{(0)} = 1, \qquad S_2^{(2)} = [\vec{\sigma}^a \times \vec{\sigma}^b]^{(0)}, \qquad S_3^{(1)} = \vec{\sigma}^a + \vec{\sigma}^b$$

$$S_4^{(2)} = [\vec{\sigma}^a \times \vec{\sigma}^b]^{(2)}, \qquad S_5^{(1)} = [\vec{\sigma}^a \times \vec{\sigma}^b]^{(1)}, \qquad S_6^{(1)} = \vec{\sigma}^a - \vec{\sigma}^b.$$

Advantage : These 6 tensors $S_\mu^{(k)}$ of rank k can immediately be coupled with a tensor operator of the same rank in configuration space $X_\mu^{(k)}$ to a scalar and the so obtained scalar functions finally summed to the general scalar (i.e. invariant with respect to spatial rotations) two-particle interaction ($P_{T=0}$ and $P_{T=1}$ are projectors on the states $T = 0$ and $T = 1$) :

$$V(a,b) = \sum_{\mu=1}^{6} \left\{ [X_\mu^{(k)} \times S_\mu^{(k)}]^{(0)} P_{T=0} + [Y_\mu^{(k)} \times S_\mu^{(k)}]^{(0)} P_{T=1} \right\}$$

SYMMETRY CONSIDERATIONS

- ⋆ We demand $V(a, b)$ to be **symmetric** with respect to particle permutation.
- ⋆ The combinations S_1, S_2, S_3, S_4 are **symmetric** with respect to the interchange of the spins of the particles, and therefore the corresponding tensors X_1, X_2, X_3, X_4 and Y_1, Y_2, Y_3, Y_4 will have to be **symmetric**.
- ⋆ The combinations S_5, S_6 are **anti-symmetric** with respect to the interchange of the spins of the particles, and therefore the corresponding tensors X_5, X_6 and Y_5, Y_6 will have to be **anti-symmetric**.

ANTI-SYMMETRIC SPIN-ORBIT

- ⋆ The last possibility corresponds to the **ALS (anti-symmetric spin-orbit)** part of the interaction.
- ⋆ It violates the principle of invariance of the interaction with respect to the relative parity of two nucleons, and is therefore in principle **not allowed**.
- ⋆ However, this is true for the **free interaction**, but not really necessary in **effective interactions** (Conze, Feldmeier, Manakos).

HARTREE-FOCK FORMALISM

- ⋆ Many-body hamiltonian :

$$\hat{H} = \sum_{\alpha\beta} \langle\alpha|\hat{t}|\beta\rangle a^\dagger_\alpha a_\beta + \frac{1}{2}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|\hat{V}|\gamma\delta\rangle a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma$$

- ⋆ Hartree-Fock ground state of the system of A particles :

$$|\Phi\rangle = \prod_{\mu=1}^{A} a^\dagger_\mu |0\rangle$$

- ⋆ Hartree-Fock equations :

$$\langle\alpha|\hat{h}_{HF}|\beta\rangle \equiv \langle\alpha|\hat{t} + \hat{U}_{HF}|\beta\rangle = \varepsilon_\alpha \delta_{\alpha\beta}$$

- ⋆ Hartree-Fock potential :

$$\langle\alpha|\hat{U}_{HF}|\beta\rangle \equiv \sum_{\mu=1}^{A} \langle\alpha\mu|\hat{V}|\widetilde{\beta\mu}\rangle = \sum_{\mu=1}^{A} \Big[\langle\alpha\mu|\hat{V}|\beta\mu\rangle - \langle\alpha\mu|\hat{V}|\mu\beta\rangle\Big]$$

INTEGRO-DIFFERENTIAL FORM

⋆ The HF equations can be written as a system of integro-differential equations

$$-\frac{\hbar^2}{2m}\Delta\phi_i(q) + \int dq' \sum_{\mu=1}^{A} \phi_\mu^*(q')\langle q; q'|\hat{V}|\widetilde{i;\mu}\rangle = \varepsilon_i\phi_i(q)$$

⋆ In the Hartree approximation one has

$$-\frac{\hbar^2}{2m}\Delta\phi_i(\vec{r}\sigma) + \langle\vec{r}\sigma|\hat{U}_H|i\rangle = \varepsilon_i\phi_i(\vec{r}\sigma)$$

where

$$\langle\vec{r}\sigma|\hat{U}_H|i\rangle = \int d^3\vec{r}\,' \sum_{\sigma'} \sum_{\mu=1}^{A} \phi_\mu^*(\vec{r}\,'\sigma')\langle\vec{r}\sigma; \vec{r}\,'\sigma'|\hat{V}|i;\mu\rangle$$

MATRIX FORM OF THE HARTREE EQUATIONS

- Introduce a single-particle basis $|i\rangle$, $|j\rangle$, $|k\rangle$, $|l\rangle$ ..., and the coefficients $c_i^\alpha \equiv \langle i|\alpha\rangle$.
- Introducing closure relations one gets the matrix relation :

$$\sum_k (H)_{ik} c_k^\alpha = \varepsilon_\alpha c_i^\alpha$$

- where :

$$(H)_{ik} \equiv \langle i|\hat{t}|k\rangle + \sum_{jl} \langle ij|\hat{V}|kl\rangle d_{jl}$$

- and :

$$d_{jl} \equiv \sum_{\mu \text{ occ.}} c_j^{\mu *} c_l^\mu$$

PARTICULARITY OF THE METHOD

- ⋆ The **particularity** of the method is that the interaction is split into a non self-consistent part (Woods-Saxon), and terms treated self-consistently (spin-orbit...).
- ⋆ **Advantage**: Woods-Saxon potential well under control, especially when it comes to extrapolations to large numbers of particles in the system.
- ⋆ **Goal**: compare non self-consistent and self-consistent treatment of “well known" interactions like the spin-orbit, and also more “exotic" terms like the anti-symmetric spin-orbit interaction.

CHOICE OF THE S.P. BASIS

⋆ Three-dimensional harmonic-oscillator eigenfunctions

$$\varphi_{n,m_s}(\vec{r}\sigma) \equiv \langle \vec{r}\sigma | n, m_s \rangle = \varphi_{n_x}(x)\, \varphi_{n_y}(y)\, \varphi_{n_z}(z)\, \chi_{m_s}(\sigma)$$

where

$$\varphi_{n_\mu}(x_\mu) = \langle x_\mu | n_\mu \rangle = \frac{\sqrt{\beta_\mu}}{\sqrt{2^{n_\mu} n_\mu! \sqrt{\pi}}} e^{-\frac{\beta_\mu^2 x_\mu{}^2}{2}} H_{n_\mu}(\beta_\mu x_\mu)$$

⋆ The actual basis used is :

$$|n+\rangle \equiv \sum_\sigma \alpha_{n\sigma}\, |n, \sigma\rangle \quad \text{and} \quad |n-\rangle \equiv \sum_\sigma \beta_{n\sigma}\, |n, \sigma\rangle$$

⋆ **Advantage**: hamiltonian matrix H_{ik} may be bloc-diagonal.

MATRIX ELEMENTS OF THE INTERACTION

For instance we have

$$\langle n+, n'+|\hat{V}|m+, m'+\rangle$$

$$= \sum_{\sigma,\sigma',\kappa,\kappa'} {\alpha_{n\sigma}}^* {\alpha_{n'\sigma'}}^* \langle n\sigma; n'\sigma'|\hat{V}|m\kappa; m'\kappa'\rangle \alpha_{m\kappa}\alpha_{m'\kappa'}$$

and therefore the problem consists in evaluating the bracket

$$\langle n\sigma; n'\sigma'|\hat{V}|m\kappa; m'\kappa'\rangle$$

MATRIX ELEMENTS OF THE INTERACTION

The last matrix element is in turn evaluated by the introduction of the closure relation

$$\mathbb{I} = \int\int d^3\vec{r}d^3\vec{r}\,' \sum_{S}\sum_{S'} |\vec{r}S;\vec{r}\,'S'\rangle\langle\vec{r}S;\vec{r}\,'S'|$$

wherefrom finally

$$\langle n\sigma; n'\sigma'|\hat{V}|m\kappa; m'\kappa'\rangle$$

$$= \int\int d^3\vec{r}d^3\vec{r}\,' \sum_{S}\sum_{S'} \langle n\sigma; n'\sigma'|\vec{r}S;\vec{r}\,'S'\rangle\langle\vec{r}S;\vec{r}\,'S'|\hat{V}|m\kappa; m'\kappa'\rangle$$

THE CASE OF THE SPIN-ORBIT

- ⋆ Has been introduced in the mean-field to reproduce the correct magic numbers.
- ⋆ Origin has been intensively discussed.
- ⋆ There was early evidence that it should stem from the nucleon-nucleon spin-orbit part (Brueckner, Lockett, Rotenberg 1961; Barrett 1967)
- ⋆ Relativistic effect (Thomas term) too small by an order of magnitude.
- ⋆ Correct spin-orbit term in the s.p. potential can be obtained from a relativistic HF calculation with OBEP (Brockmann 1978); $\longrightarrow$ exchange of ω-bosons.
- ⋆ Easy description within the context of RMF theory.

FORM FACTOR OF THE SPIN-ORBIT

- ⋆ In the context of mean-field calculations (Woods-Saxon...) sometimes very intuitive arguments are given to justify the form factors used.
- ⋆ Bohr-Mottelson: *The spin-orbit coupling is of necessity a surface term since, in a region of constant density, the only direction with local significance is that of the particle motion and, thus, it is impossible to define a pseudovector that can be coupled to the nuclear spin. In the surface region, however, the density gradient defines the radial direction and makes it possible to introduce a local potential of the form*

$$V_{LS} \propto \nabla\rho(r) \wedge \vec{p}\cdot\vec{s} = \hbar^{-1}(\vec{l}\cdot\vec{s})\frac{1}{r}\frac{\partial\rho(r)}{\partial r}$$

- ⋆ A justification for the form factor can be found in the textbook by W. Horniak 1975.

SPIN-ORBIT POTENTIAL

- ⋆ Consider the nucleon-nucleon spin-orbit interaction

$$\hat{V}^{SO} = J(|\vec{r} - \vec{r}\,'|)\,(\vec{r} - \vec{r}\,') \wedge (\vec{p} - \vec{p}\,') \cdot (\vec{\sigma} + \vec{\sigma}\,')$$

- ⋆ In the Hartree equations one has to evaluate

$$\langle \vec{r}\sigma | \hat{U}_H^{\mathrm{SO}} | i \rangle = \int d^3\vec{r}\,' \sum_{\sigma'} \sum_{\mu=1}^{A} \phi_\mu^*(\vec{r}\,'\sigma') \langle \vec{r}\sigma; \vec{r}\,'\sigma' | \hat{V}^{\mathrm{SO}} | i; \mu \rangle$$

SPIN-ORBIT POTENTIAL

This can be done for all the 8 following terms separately :

$$
\begin{aligned}
J(|\vec{r}-\vec{r}\,'|)\,(\vec{r}-\vec{r}\,')\wedge(\vec{p}-\vec{p}\,')\cdot(\vec{\sigma}+\vec{\sigma}\,') \quad = \quad & J(|\vec{r}-\vec{r}\,'|)\,(\vec{r}\wedge\vec{p})\cdot\vec{\sigma} && \longrightarrow \hat{T}_1 \\
- \quad & J(|\vec{r}-\vec{r}\,'|)\,(\vec{r}\,'\wedge\vec{p})\cdot\vec{\sigma} && \longrightarrow \hat{T}_2 \\
- \quad & J(|\vec{r}-\vec{r}\,'|)\,(\vec{r}\wedge\vec{p}\,')\cdot\vec{\sigma} && \longrightarrow \hat{T}_3 \\
+ \quad & J(|\vec{r}-\vec{r}\,'|)\,(\vec{r}\,'\wedge\vec{p}\,')\cdot\vec{\sigma} && \longrightarrow \hat{T}_4 \\
+ \quad & J(|\vec{r}-\vec{r}\,'|)\,(\vec{r}\wedge\vec{p})\cdot\vec{\sigma}\,' && \longrightarrow \hat{T}_5 \\
- \quad & J(|\vec{r}-\vec{r}\,'|)\,(\vec{r}\,'\wedge\vec{p})\cdot\vec{\sigma}\,' && \longrightarrow \hat{T}_6 \\
- \quad & J(|\vec{r}-\vec{r}\,'|)\,(\vec{r}\wedge\vec{p}\,')\cdot\vec{\sigma}\,' && \longrightarrow \hat{T}_7 \\
+ \quad & J(|\vec{r}-\vec{r}\,'|)\,(\vec{r}\,'\wedge\vec{p}\,')\cdot\vec{\sigma}\,' && \longrightarrow \hat{T}_8 .
\end{aligned}
$$

EXPLICIT EXPRESSIONS

$$\langle \vec{r}\sigma|\hat{T}_1|i\rangle = \Big[\int d^3\vec{r}\,' J(|\vec{r}-\vec{r}\,'|)\rho(\vec{r}\,')\Big]\,\vec{l}\cdot\vec{\sigma}\,\phi_i(\vec{r}\sigma)$$

$$\langle \vec{r}\sigma|\hat{T}_2|i\rangle = \Big[\int d^3\vec{r}\,' J(|\vec{r}-\vec{r}\,'|)(-\vec{r}\,')\sum_{\sigma'}\sum_{\mu=1}^{A}\phi^*_\mu(\vec{r}\,'\sigma')\phi_\mu(\vec{r}\,'\sigma')\Big]\wedge\vec{p}\cdot\vec{\sigma}\,\phi_i(\vec{r}\sigma)$$

$$\langle \vec{r}\sigma|\hat{T}_3|i\rangle = \Big[\int d^3\vec{r}\,' J(|\vec{r}-\vec{r}\,'|)\sum_{\sigma'}\sum_{\mu=1}^{A}\phi^*_\mu(\vec{r}\,'\sigma')(-\vec{p}\,')\,\phi_\mu(\vec{r}\,'\sigma')\Big]\cdot(\vec{r}\wedge\vec{\sigma})\,\phi_i(\vec{r}\sigma)$$

$$\langle \vec{r}\sigma|\hat{T}_4|i\rangle = \Big[\int d^3\vec{r}\,' J(|\vec{r}-\vec{r}\,'|)\sum_{\sigma'}\sum_{\mu=1}^{A}\phi^*_\mu(\vec{r}\,'\sigma')\,\vec{l}\,'\,\phi_\mu(\vec{r}\,'\sigma')\Big]\cdot\vec{\sigma}\,\phi_i(\vec{r}\sigma)$$

EXPLICIT EXPRESSIONS

$$\langle\vec{r}\sigma|\hat{T}_5|i\rangle = \Big[\int d^3\vec{r}\,'J(|\vec{r}-\vec{r}\,'|)\sum_{\sigma'}\sum_{\mu=1}^{A}\phi^*_\mu(\vec{r}\,'\sigma')\,\vec{\sigma}\,'\,\phi_\mu(\vec{r}\,'\sigma')\Big]\cdot\vec{l}\,\phi_i(\vec{r}\sigma)$$

$$\langle\vec{r}\sigma|\hat{T}_6|i\rangle = \Big[\int d^3\vec{r}\,'J(|\vec{r}-\vec{r}\,'|)\sum_{\sigma'}\sum_{\mu=1}^{A}\phi^*_\mu(\vec{r}\,'\sigma')(-\vec{r}\,'\wedge\vec{\sigma}\,')\phi_\mu(\vec{r}\,'\sigma')\Big]\cdot\vec{p}\,\phi_i(\vec{r}\sigma)$$

$$\langle\vec{r}\sigma|\hat{T}_7|i\rangle = \Big[\int d^3\vec{r}\,'J(|\vec{r}-\vec{r}\,'|)\sum_{\sigma'}\sum_{\mu=1}^{A}\phi^*_\mu(\vec{r}\,'\sigma')(-\vec{p}\,'\wedge\vec{\sigma}\,')\,\phi_\mu(\vec{r}\,'\sigma')\Big]\cdot\vec{r}\,\phi_i(\vec{r}\sigma)$$

$$\langle\vec{r}\sigma|\hat{T}_8|i\rangle = \Big[\int d^3\vec{r}\,'J(|\vec{r}-\vec{r}\,'|)\sum_{\sigma'}\sum_{\mu=1}^{A}\phi^*_\mu(\vec{r}\,'\sigma')\,\vec{l}\,'\cdot\vec{\sigma}\,'\,\phi_\mu(\vec{r}\,'\sigma')\Big]\phi_i(\vec{r}\sigma)$$

RECOVERING STANDARD RESULTS...

- ⋆ It is easily seen that term $\hat{T}_1$ can be brought into the familiar form

$$\langle \vec{r}\sigma | \hat{T}_1 | i \rangle = F(\vec{r})\, \vec{l} \cdot \vec{\sigma}\, \phi_i(\vec{r}\sigma)$$

with

$$F(\vec{r}) \equiv \int d^3\vec{r}\,' J(|\vec{r} - \vec{r}\,'|)\rho(\vec{r}\,').$$

- ⋆ Term $\hat{T}_2$ can be written as

$$\langle \vec{r}\sigma | \hat{T}_2 | i \rangle = \vec{G}(\vec{r}) \wedge \vec{p} \cdot \vec{\sigma}\, \phi_i(\vec{r}\sigma)$$

where

$$\textbf{with} \qquad \vec{g}(\vec{r}, \vec{r}\,') \equiv J(|\vec{r} - \vec{r}\,'|)(-\vec{r}\,')\rho(\vec{r}\,').$$

RECOVERING STANDARD RESULTS...

Now, if the symmetries of the problem are such that the vector $\vec{G}(\vec{r})$ is proportionnal to the position vector $\vec{r}$, one can write (Horniak 1975)

$$\langle \vec{r}\sigma | T_2^{\vec{G}\|\vec{r}} | i \rangle = F'(\vec{r})\, \vec{l} \cdot \vec{\sigma}\, \phi_i(\vec{r}\sigma)$$

with

$$F'(\vec{r}) = \int d^3\vec{r}\,' \frac{\vec{g}(\vec{r},\vec{r}\,') \cdot \vec{r}}{r^2}.$$

... BUT WHAT ABOUT THE OTHER TERMS ?

- ⋆ First of all, the form factors can be calculated **explicitely**, avoiding the “standard" expression implying the gradient of the density.

- ⋆ Secondly, the term $\hat{T}_2$ should be used in its **general form**.

- ⋆ And what about the other **6** remaining terms ?

PRACTICAL IMPLEMENTATION

We will opt for the **iterative diagonalization** procedure of the hamiltonian matrix

$$(H)_{ik} \equiv \langle i|\hat{t}|k\rangle + \sum_{jl} \langle ij|\hat{V}|kl\rangle d_{jl}$$

This will for instance require calculating terms like

$$\langle n\sigma; n'\sigma'|\hat{T}_1|m\kappa; m'\kappa'\rangle$$

$$= \int\int d^3\vec{r} d^3\vec{r}\,' \varphi_n^*(\vec{r})\varphi_{n'}^*(\vec{r}\,')\langle\vec{r}\sigma; \vec{r}\,'\sigma'|\hat{T}_1|m\kappa; m'\kappa'\rangle$$

$$= \delta_{\kappa'\sigma'} \sum_{k=x,y,z} \langle\sigma|\hat{\sigma}_k|\kappa\rangle \int d^3\vec{r}\varphi_n^*(\vec{r})\Big[\int d^3\vec{r}\,'\varphi_{n'}^*(\vec{r}\,')J(|\vec{r}-\vec{r}\,'|)\varphi_{m'}(\vec{r}\,')\Big]\hat{l}_k\varphi_m(\vec{r})$$

CONCLUSIONS AND OUTLOOK

- ⋆ We propose a **direct way** to treat “standard terms" (spin-orbit...) as well as more “exotic" ones (anti-symmetric spin-orbit...) in the framework of the mean-field with a **minimal number of parameters**.
- ⋆ These terms correspond to those in the nucleon-nucleon interactions **a priori allowed** by symmetry considerations.
- ⋆ They are treated **self-consistently** in the mean-field approach.
- ⋆ The **Hartree** approximation is examined first; **Fock** (exchange) will follow.
- ⋆ **Symmetry-violating** terms can be studied with a certain freedom (spontaneous symmetry breaking and restoration; projection techniques...)