Parity effects in nuclear collective and single particle motion

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Contents

Hamiltonian

$$H = H_{\text{rot}} + H_{\text{vib}} + H_{\text{sp}} + H_{\text{coriol}}$$

Assumptions

- The nucleus **oscillates simultaneously** with respect to axial **quadrupole β_2 and octupole β_3** deformation variables coupled through a **centrifugal interaction**
- **odd mass nuclei** $\rightarrow$ the single nucleon moves in a quadrupole-octupole **deformed potential** induced by the even-even core
- the core and the unpaired nucleon are coupled through the **Coriolis interaction** and the **good total parity** of the nucleus

Collective Hamilton

[N. M. et al, Phys. Rev. C **73**, 044315 (2006); **76**, 034324 (2007)]

$$H_{qo} = H_{rot} + H_{vib} + H_{coriol}$$

$$H_{qo} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{X(I,K,\pi a)}{2(d_2\beta_2^2 + d_3\beta_3^2)}$$

$$X(I,K,\pi a) = \frac{1}{2}\left[d_0 + I(I+1) - K^2 + \pi a\delta_{K,\frac{1}{2}}(-1)^{I+1/2}\left(I + \frac{1}{2}\right)\right]$$

$a \rightarrow$ **Coriolis decoupling factor**

Coherent quadrupole-octupole mode

$$C_2/B_2 = C_3/B_3 \equiv \omega^2 \qquad \beta_2 \to \eta\cos\phi\ , \quad \beta_3 \to \eta\sin\phi$$

$$E_{n,k}(I,K,\pi a) = \hbar\omega\left[2n+1+\sqrt{k^2+bX(I,K,\pi a)}\right],\ \ n = 0,1,2,\ldots$$

Core wave function: $\boxed{\Phi^{\pm}(\eta,\phi) = \psi(\eta)\varphi^{\pm}(\phi)}$

$\psi^{\mathbf{I}}_{\mathbf{n}}(\eta)$ $\rightarrow$ generalized Laguerre functions

$$\varphi^{+}(\phi) = \sqrt{2/\pi}\cos(\mathbf{k}\phi)\ , \qquad \mathbf{k} = 1,3,5,\ldots \rightarrow \pi = (+)$$
$$\varphi^{-}(\phi) = \sqrt{2/\pi}\sin(\mathbf{k}\phi)\ , \qquad \mathbf{k} = 2,4,6,\ldots \rightarrow \pi = (-)$$

$\Rightarrow$ **Parity effects in the collective and s.p. motion**

Axially deformed Woods-Saxon potential

[M. Brack et al, Rev. Mod. Phys. **44**, 320 (1972)]

$$H_{\text{sp}} = T + V_{\text{ws}} + V_{\text{so}} + V_{\text{Coul}}$$

$$V_{\text{ws}}(\mathbf{r}, \hat{\beta}) = V_0 \left[1 + \exp\left(\frac{\text{dist}_{\Sigma}(\mathbf{r}, \hat{\beta})}{a} \right) \right]^{-1}$$

$$R(\theta, \hat{\beta}) = c(\hat{\beta}) R_0 \left(1 + \sum_{\lambda=2,3,\ldots} \beta_\lambda Y_{\lambda 0}(\cos\theta) \right) , \quad \hat{\beta} \equiv (\beta_2, \beta_3, \beta_4, \beta_5, \beta_6)$$

[computer code by S. Cwiok et al., Comp. Phys. Comm. **46**, 379 (1987)]

Parity mixed s.p. states

S.P. wave function: $\mathcal{F}_\Omega = \sum_{Nn_z\Lambda} C^{\Omega}_{Nn_z\Lambda} |Nn_z\Lambda\Omega\rangle, \ \ (\Omega = K)$

$$\hat{\pi}_{\text{sp}}|Nn_z\Lambda\Omega\rangle = (-1)^N|Nn_z\Lambda\Omega\rangle \qquad \pi_{\text{sp}} = (-1)^N$$

$\beta_3 = 0 \Rightarrow N$ =even $(\pi_{\text{sp}} = +)$ **or** N =odd $(\pi_{\text{sp}} = -)$
$\beta_3 \neq 0 \Rightarrow N$ =even **and** N =odd $\Rightarrow$ **parity mixed s.p. states**

$$\mathcal{F}_\Omega = \sum_{\pi_{\text{sp}}=\pm 1} \mathcal{F}_\Omega^{(\pi_{\text{sp}})} = \mathcal{F}_\Omega^{(+)} + \mathcal{F}_\Omega^{(-)}$$

$$\hat{\pi}_{\text{sp}}\mathcal{F}_\Omega^{(\pm)} = \pm\mathcal{F}_\Omega^{(\pm)} \quad \Rightarrow \quad \boxed{\hat{\pi}_{\text{sp}}\mathcal{F}_\Omega = \mathcal{F}_\Omega^{(+)} - \mathcal{F}_\Omega^{(-)}}$$

average parity: $\langle\hat{\pi}_{\text{sp}}\rangle = \langle\mathcal{F}_\Omega|\hat{\pi}_{\text{sp}}|\mathcal{F}_\Omega\rangle = \sum_{Nn_z\Lambda}(-1)^N(C^{\Omega}_{Nn_z\Lambda})^2$

Total Particle-Core wave function

$$\Psi^{\pi}_{IMK} \;=\; \frac{1}{2}\mathcal{N}(1+\mathcal{R}_1)D^{I}_{M\,K}(\boldsymbol{\theta})(1+\pi\hat{P})\Phi^{\pi_c}_{\text{core}}\mathcal{F}_K$$

$$\mathcal{R}_1 D^{I}_{M\,K} = (-1)^{I-K}D^{I}_{M\,-K} \qquad \hat{P} = \hat{\pi}_c \cdot \hat{\pi}_{\text{sp}}$$

$$\mathcal{R}_1\mathcal{F}_K = \overline{\mathcal{F}}_K = \mathcal{F}_{-K} \qquad \hat{\pi}_{\text{sp}}\mathcal{F}_K = \mathcal{F}^{(+)}_K - \mathcal{F}^{(-)}_K$$

$$\mathcal{R}_1\Phi^{\pi_c}_{\text{core}} = \hat{\pi}_c\Phi^{\pi_c}_{\text{core}} = \pm\Phi^{\pm}_{\text{core}}$$

$\pi_c \rightarrow$ **fixed**

$\pi_c = (+) \rightarrow \Phi^{+}_{\text{core}} \Rightarrow$ downwards shifted levels

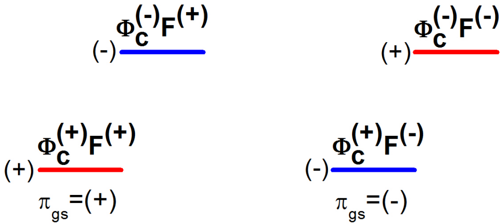
$\pi_c = (-) \rightarrow \Phi^{-}_{\text{core}} \Rightarrow$ upwards shifted energy sequence

$(1+\pi\hat{P}) \rightarrow$ projects out $\mathcal{F}^{(+)}_K$ or $\mathcal{F}^{(-)}_K \Rightarrow \pi = \pi_c \cdot (\pi_{\text{sp}})$

$\Rightarrow$ soft octupole shape in a *strong coupling limit*

Schematic levels

[N. Minkov, S. Drenska, M. Strecker and W. Scheid, J. Phys. G: Nucl. Part. Phys. **36**, 025108 (2009); **37**, 025103 (2010)]



Decoupling factors in parity mixed s.p. states

$$-\langle \Psi^{\pi}_{IM\frac{1}{2}} | \hat{I}_{-}\hat{j}_{+} | \Psi^{\pi}_{IM\frac{1}{2}} \rangle = \mathcal{N}^2 (-1)^{I+\frac{1}{2}} \left(I + \frac{1}{2} \right) \cdot \langle \text{decoupling factor} \rangle$$

$$\begin{aligned} a &= \frac{1}{2}\pi_c \left\{ \left\langle \mathcal{F}_{\frac{1}{2}} | \hat{j}_{+} | \mathcal{F}_{-\frac{1}{2}} \right\rangle + \pi\pi_c \left\langle \hat{\pi}_{\text{sp}} \mathcal{F}_{\frac{1}{2}} | \hat{j}_{+} | \mathcal{F}_{-\frac{1}{2}} \right\rangle \right\} \\ &= \frac{1}{2}\pi_c \left[(1+\pi\pi_c) a^{(+)} + (1-\pi\pi_c) a^{(-)} \right] \end{aligned}$$

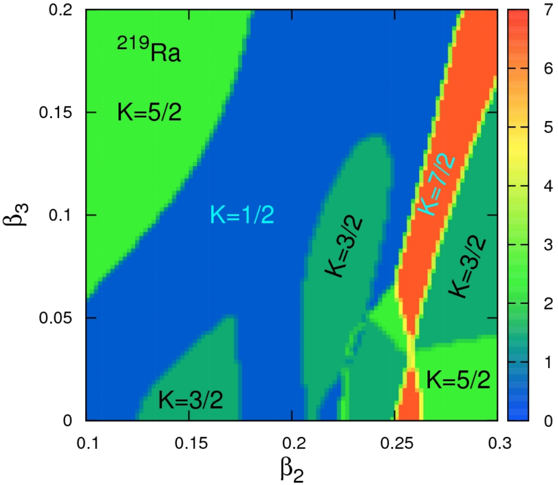
$$a^{(+)} = \left\langle \mathcal{F}^{(+)}_{1/2} | \hat{j}_{+} | \mathcal{F}^{(+)}_{-1/2} \right\rangle \qquad a^{(-)} = \left\langle \mathcal{F}^{(-)}_{1/2} | \hat{j}_{+} | \mathcal{F}^{(-)}_{-1/2} \right\rangle$$

$$\pi_{\text{gs}} = (+) \Rightarrow a = a^{(+)}$$
$$\pi_{\text{gs}} = (-) \Rightarrow a = -a^{(-)}$$

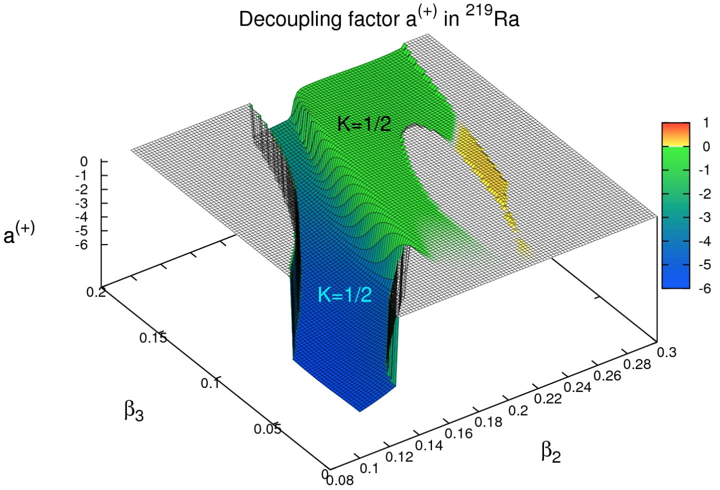
Global deformed shell model calculations in the (β_2, β_3)- plane

- Dependence of the s.p. angular momentum projection K and the s.p. parity mixing on quadrupole-octupole deformations
- Behaviour of the Coriolis decoupling factor in the (β_2, β_3)-plane
- Effects on the collective parity-doublet spectra
- Favourable deformation regions and quadrupole-octupole collectivity

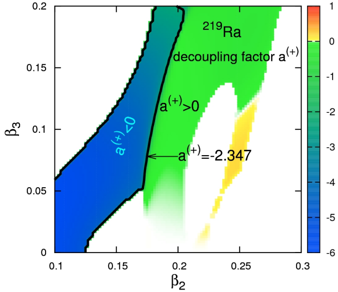
K-values for the odd nucleon of ^{219}Ra in the (β_2, β_3) plane

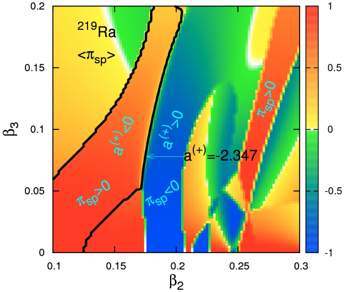


Decoupling factor a^+ in ^{219}Ra (3D plot)

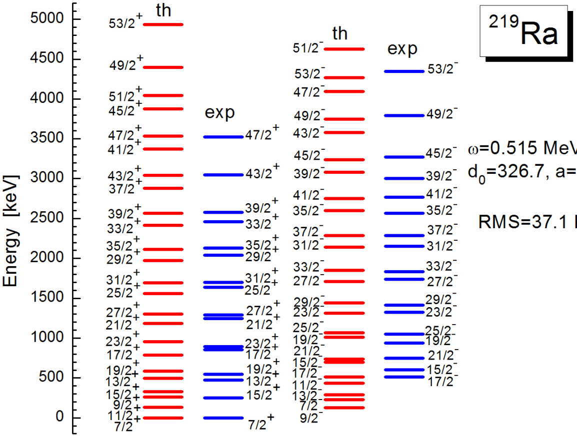


Decoupling factor a^+ and average parity $\langle \pi_{sp} \rangle$ in ^{219}Ra





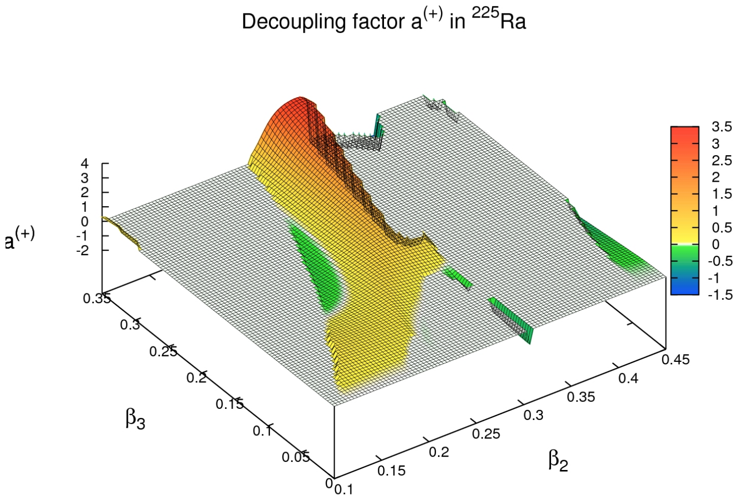
Split parity-doublet spectrum in ^{219}Ra



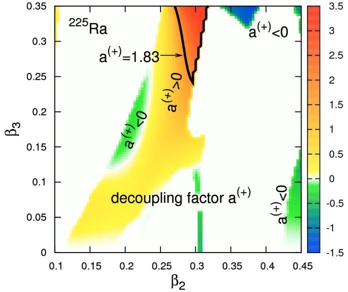
ω=0.515 MeV, b=0.785, d_0=326.7, a=-2.347

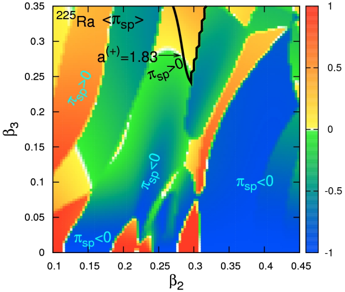
RMS=37.1 KeV

Decoupling factor a^+ in ^{225}Ra (3D plot)

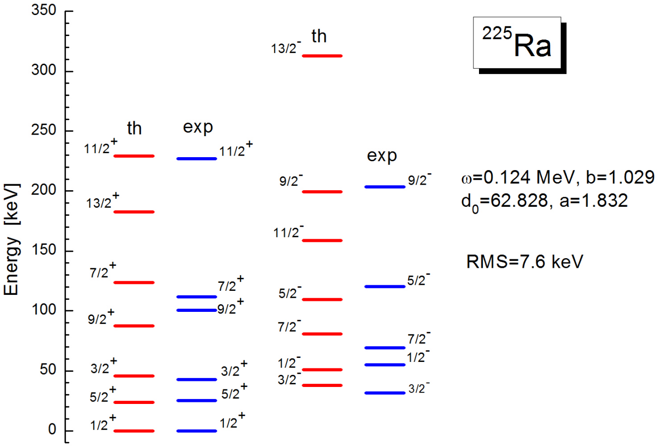


Decoupling factor a^+ and average parity $\langle \pi_{sp} \rangle$ in ^{225}Ra

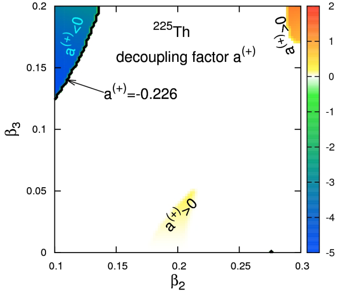


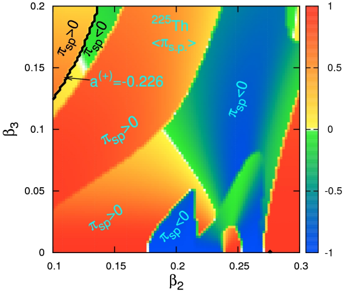


Split parity-doublet spectrum in ^{225}Ra

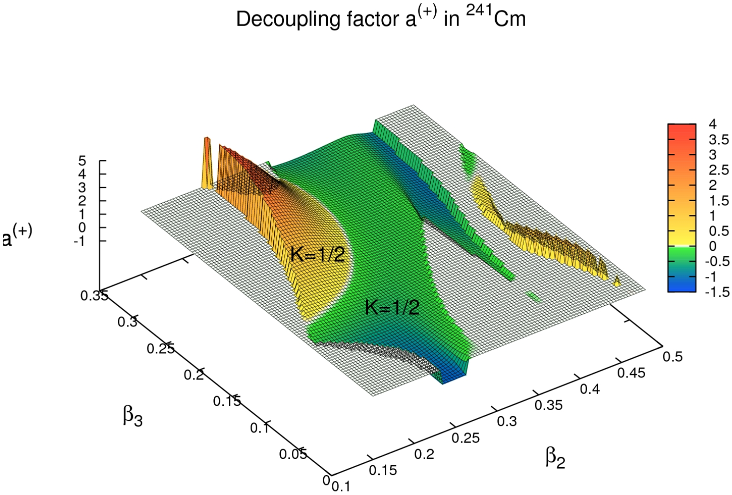


Decoupling factor a^+ and average parity $\langle\pi_{sp}\rangle$ in ^{225}Th

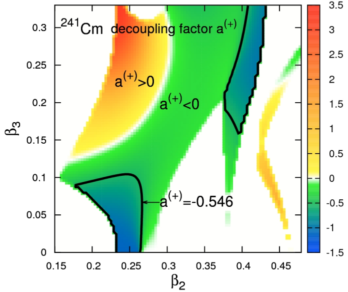


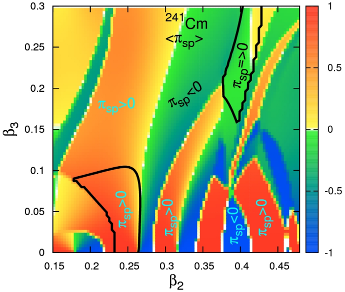


Decoupling factor a^+ in ^{241}Cm (3D plot)



Decoupling factor a^+ and average parity $\langle \pi_{sp} \rangle$ in ^{241}Cm





CONCLUSION

- Model of quadrupole-octupole vibrating and rotating core plus particle $\rightarrow$ new coupling scheme with parity mixed s.p. states
- Parity mixing $\rightarrow$ complex behaviour of $\langle \hat{\pi}_{\mathrm{sp}} \rangle$ in the $(\boldsymbol{\beta}_2, \boldsymbol{\beta}_3)$ plane without saturation to $\langle \hat{\pi}_{\mathrm{sp}} \rangle = 0$ with increasing β_3 $\rightarrow$ (wide ranges of approximately good "dominant" parity)
- Coriolis decoupling factor $\rightarrow$ strong dependence on β_3 and β_2 deformations; comparison to the collective model $\rightarrow$ regions of physically reasonable deformations in the $(\boldsymbol{\beta}_2, \boldsymbol{\beta}_3)$ plane
- Consistent collective and microscopic model description of the split parity-doublet spectra in odd-mass nuclei