ORDER VS CHAOS IN NUCLEI

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TO THE MEMORY OF WLADEK SWIATECKI

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Plan of talk

1. **INTRODUCTION.** **Order and Chaos: Quantum-classical correspondence in the collective dynamics**
2. **QUANTUM AND CLASSICAL DYNAMICS**
3. **RELATION TO THE SHELL EFFECTS**
4. **QUANTUM AND CLASSICAL CORRESPONDENCE**
5. **ORDER AND CHAOS THROUGH POINCARE SECTIONS**
6. **CONCLUSIONS**

1) WALL FORMULA, Swiatecki et al. (1977,78), Koonin, Randrup, Hatch (1977), Abrosimov et al. (1994), Ivanyuk et al. (1996) AVERAGING ⇒ LOCAL

2) CLASSICAL ⇐⇒ QUANTUM COLLECTIVE DYNAMICS, NON-ADIABATIC NON-LINEAR EFFECTS IN ONE-BODY DISSIPATION, ORDER vs CHAOS, POINCARE SECTIONS AND LYAPUNOV EXPONENTS, Swiatecki, Blocki, Skalski et al. (1995-1999)

4) QUANTUM RESULTS FOR EXCITATION ENERGY Magierski, Skalski, Blocki (1997)

5) SHELL AND DEFORMATION EFFECTS IN THE QUANTUM EXCITATION ENERGY FOR SLOW COLLECTIVE MOTION AS COMPARED TO THE CLASSICAL PICTURE

2. QUANTUM AND CLASSICAL DYNAMICS

$$i\hbar\frac{\partial}{\partial t}\psi = \mathcal{H}(t)\psi, \qquad \mathcal{H}(t) = \hat{T} + V\,(r,t)$$

$$V\,(r,t) = -\frac{V_0}{1+\exp\left\{\left[r - R(\theta,t)\right]/a\right\}}$$

$$R(\theta,t) = R_0\left[1 + \alpha_n(t)P_n\,(cos\theta) + \alpha_1(t)P_1\,(cos\theta)\right]/\lambda(t),$$

$$\alpha_n(t) = \alpha_{st} + \alpha_n^0 cos\,(\omega t)\,, \qquad \alpha_n^0 = \alpha\sqrt{(2n+1)/5}$$

$$\psi(t) = \sum_i C_i(t)\phi_i, \quad \Rightarrow \quad \Delta E(t) = E(t) - E(t=0)$$

CLASSICAL WALL FORMULA:

$$\frac{\Delta E_{wf}}{E_0} \propto \left(\tau + \frac{1}{5}\tau^2\right), \qquad \tau = \frac{3}{4}\,\alpha\eta\left[\omega t - \frac{1}{2}sin(2\omega t)\right]$$

ADIABATICITY PARAMETER:

$$\eta = \alpha\frac{\omega}{\Omega}, \qquad \Omega = \frac{v_F}{R_0}$$

3. RELATION TO THE SHELL EFFECTS

$$H_0\phi_i = \varepsilon_i\phi_i,$$

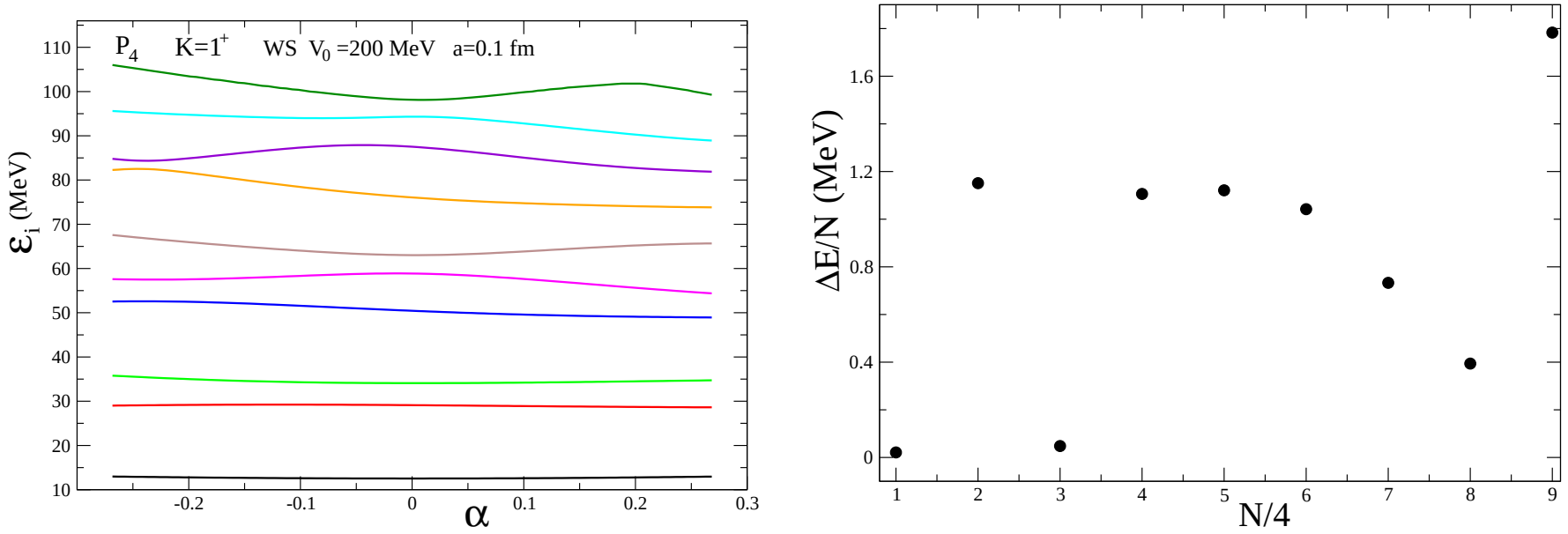


Fig.1. The spectrum of the s.p. levels ε_i as functions of the deformation α for the P_4 shape of the WS potential (left); excitation energy ΔE per particle vs the number of the occupied levels (the particle number N over the degeneracy equal to 4)

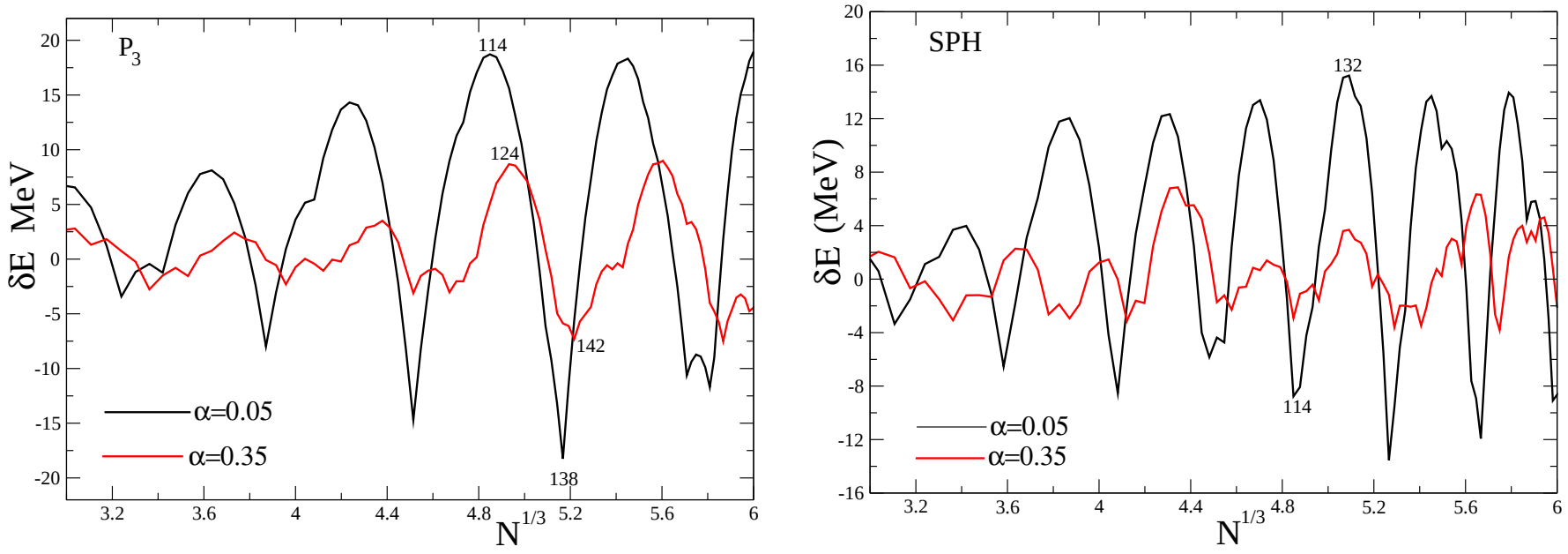


Fig.2. Energy shell corrections $\delta E = E_{s.p.} - \tilde{E}_{s.p.}$ ($E_{s.p.} = \sum_i n_i \varepsilon_i$) as functions of the particle number parameter $N^{1/3}$, $N = \sum_i n_i$ $= \sum_i \tilde{n}_i$, for deformations $\alpha_{st} = 0.05$ and 0.35 for P_3 (left) and those for spheroidal (right) shapes, $\delta E \sim \delta g(\varepsilon_F)$, $g(\varepsilon) = \sum_i (\varepsilon - \varepsilon_i)$ is the level density

3. QUANTUM AND CLASSICAL CORRESPONDENCE

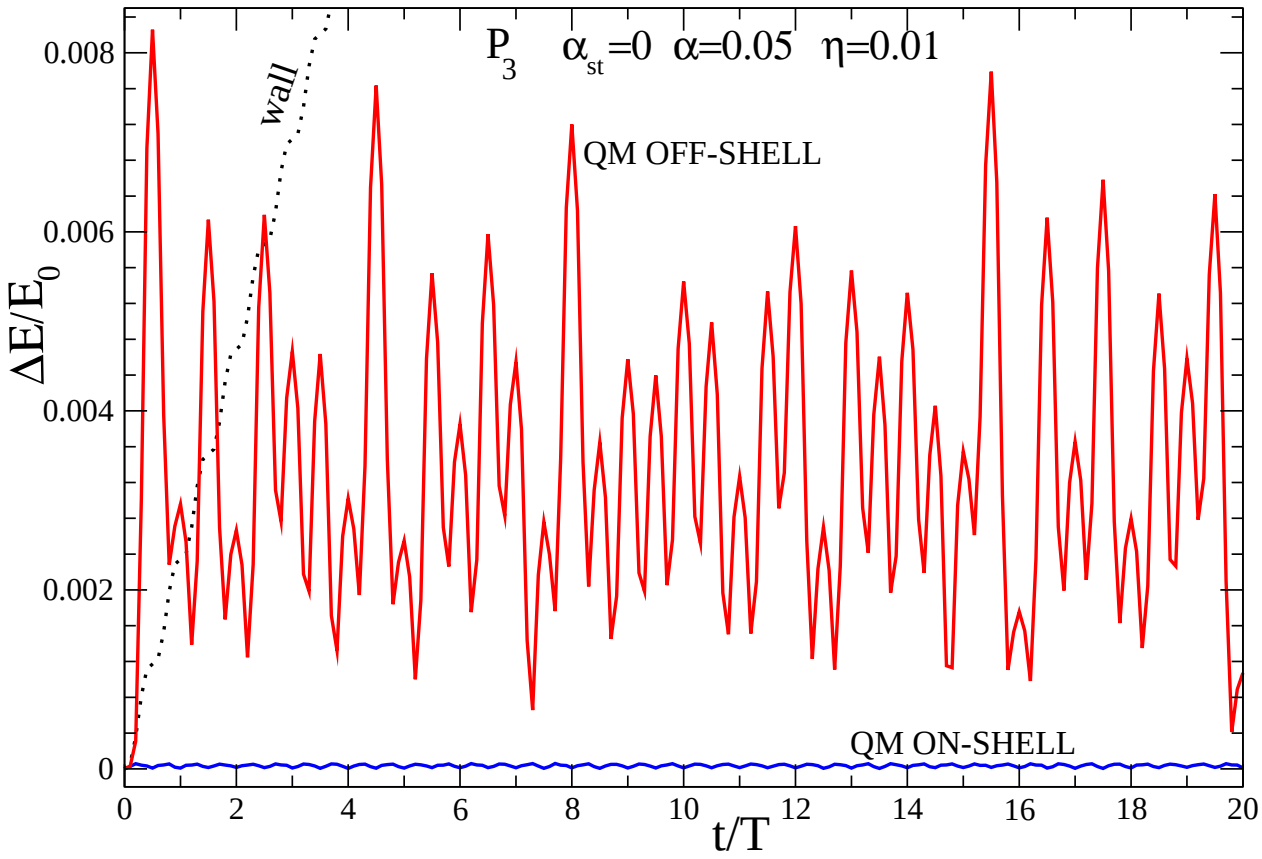


Fig.3. Excitation energy $\Delta E/E_0$ vs the time t in units of the period T for P_3 WS surface shapes; slow ($\omega/\Omega = 0.2$) vibrations of small amplitudes near the spherical shape; QM OFF-SHELL is the quantum results at maximum ($N = 114$) and QM ON-SHELL at minimum ($N = 138$) of δE (or $\delta g(\varepsilon_F)$); averages for QM ON-SHELL and QM OFF-SHELL are $3.6 \cdot 10^{-5}$ and $3.4 \cdot 10^{-3}$

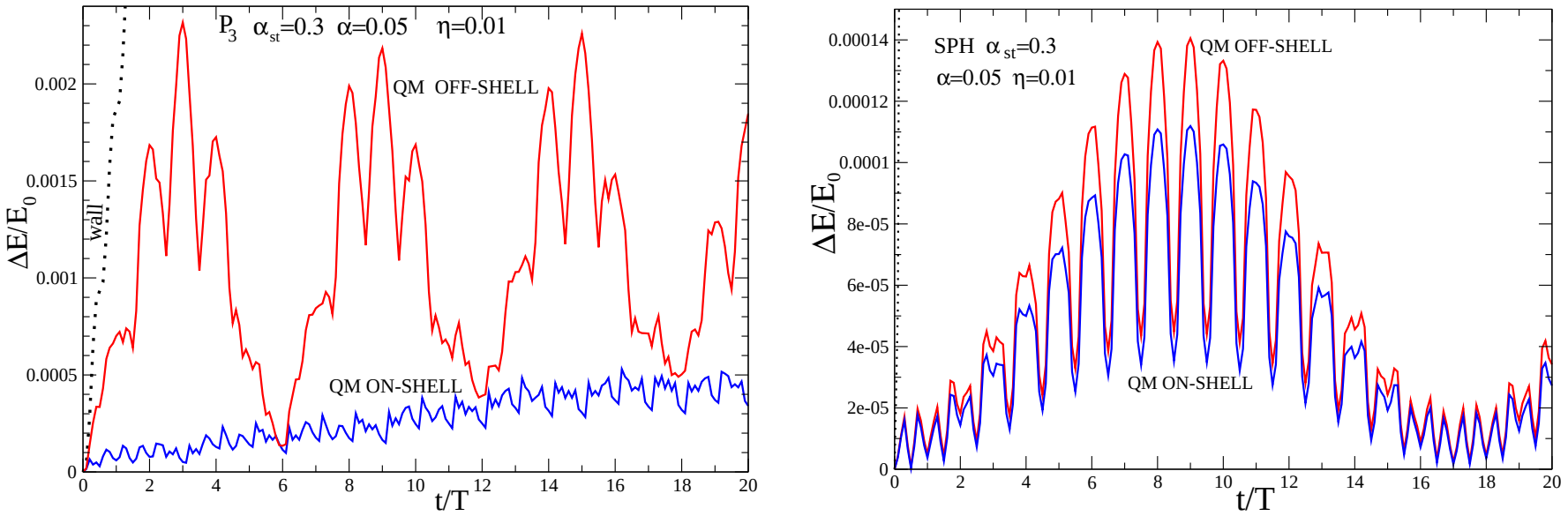


Fig.4. The same as in Fig.3 but for deformed equilibrium $\alpha_{st} = 0.3$ of P_3 (left) and spheroidal (right) shapes; QM OFF-SHELL is the quantum results at maximum ($N = 124$) and QM ON-SHELL at minimum ($N = 142$) of δE

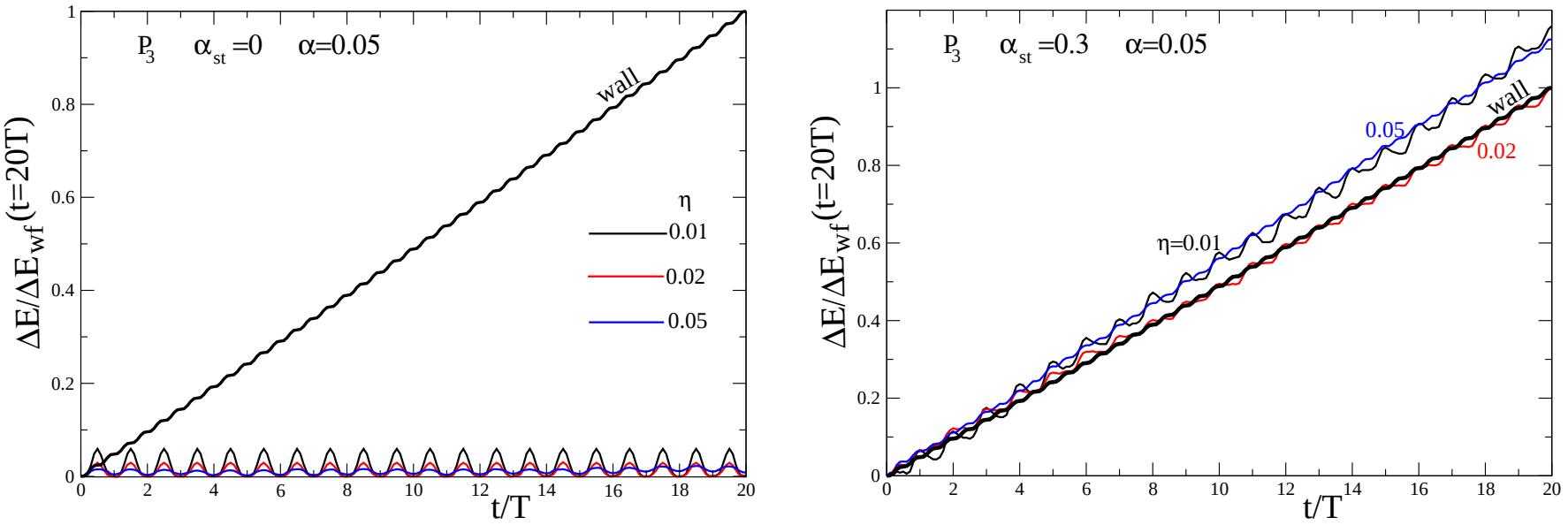


Fig.6. The classical dynamical excitations ΔE in units of the wall formula value $\Delta E_{wf}(t = 20T)$ as functions of the time t in units of the period $T = 2\pi\alpha/\eta$ for vibrations near the spherical ($\alpha_{st} = 0$, left) and deformed ($\alpha_{st} = 0.3$, right) shapes of the cavity

3. ORDER VS CHAOS AND POINCARE SECTIONS

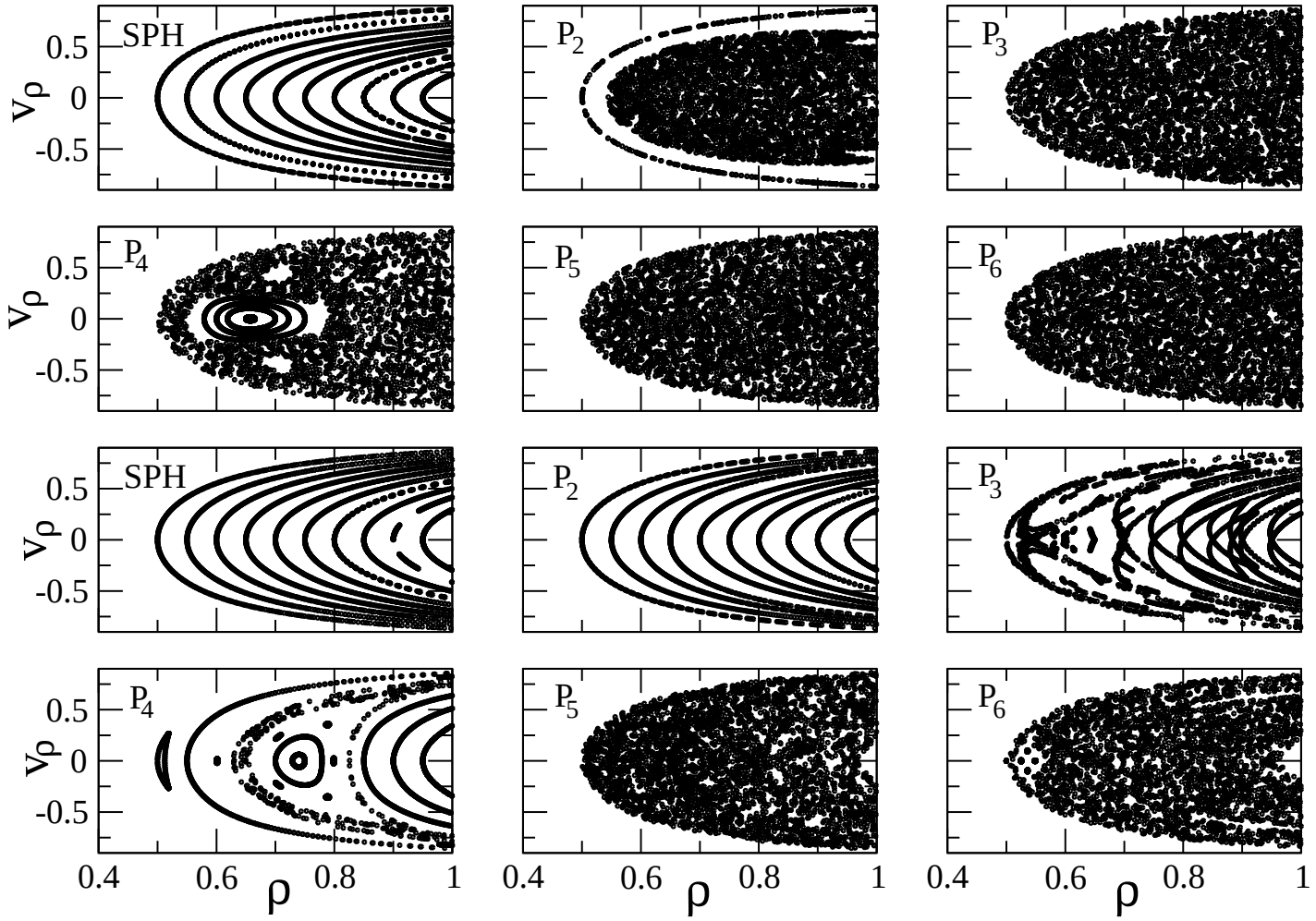


Fig.7. Poincare sections for six shapes at the large deformation $\alpha = 0.4$ (two upper rows) and small deformation $\alpha = 0.05$ (two lower rows) for the projections of the angular momentum on the symmetry axis $K = 0.5$ with respect to the maximal value

CONCLUSIONS

- We found the correlations between the quantum excitation energies and shell effects at slow collective motion, the one-body dissipative energies become larger with increasing the level density near the Fermi surface
- The excitation energy in the classical picture as a function of time becomes closer to the wall formula with increasing the equilibrium deformation of the collective vibrations, in contrast to the quantum dynamics
- Looking at the Poincare sections, this property can be explained as increasing of a chaoticity with the equilibrium deformation, the bigger the larger deformation
- As perspectives, our quantum and classical results might be helpful for better understanding the dissipative energy in a slow and fast collective dynamics with different deformed shapes more systematically and analytically for nuclear fission and heavy-ion collision physics

THANKS TO WLADEK SWIATECKI!

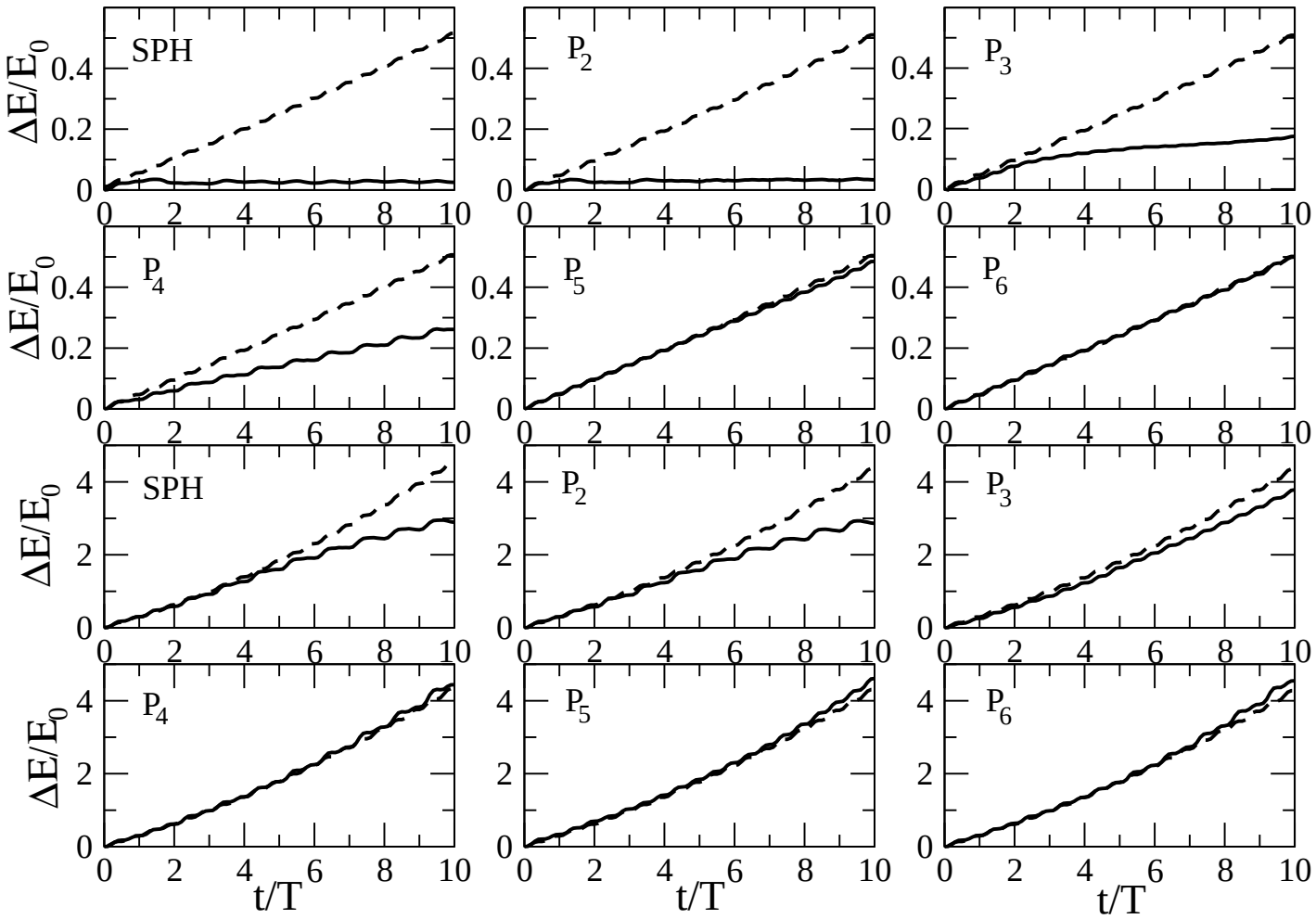


Fig.1.Relative excitation of a gas of particles δE during 10 periods of oscillations around a spherical shape for six shape deformations. The amplitude of vibrations is $\alpha = 0.1$ and the adibatic parameter $\eta = 0.1$ (upper two rows) and $\eta = 0.6$ (low two tows). Solid lines indicate results of the computer simulations and dashed ones are wall predictions

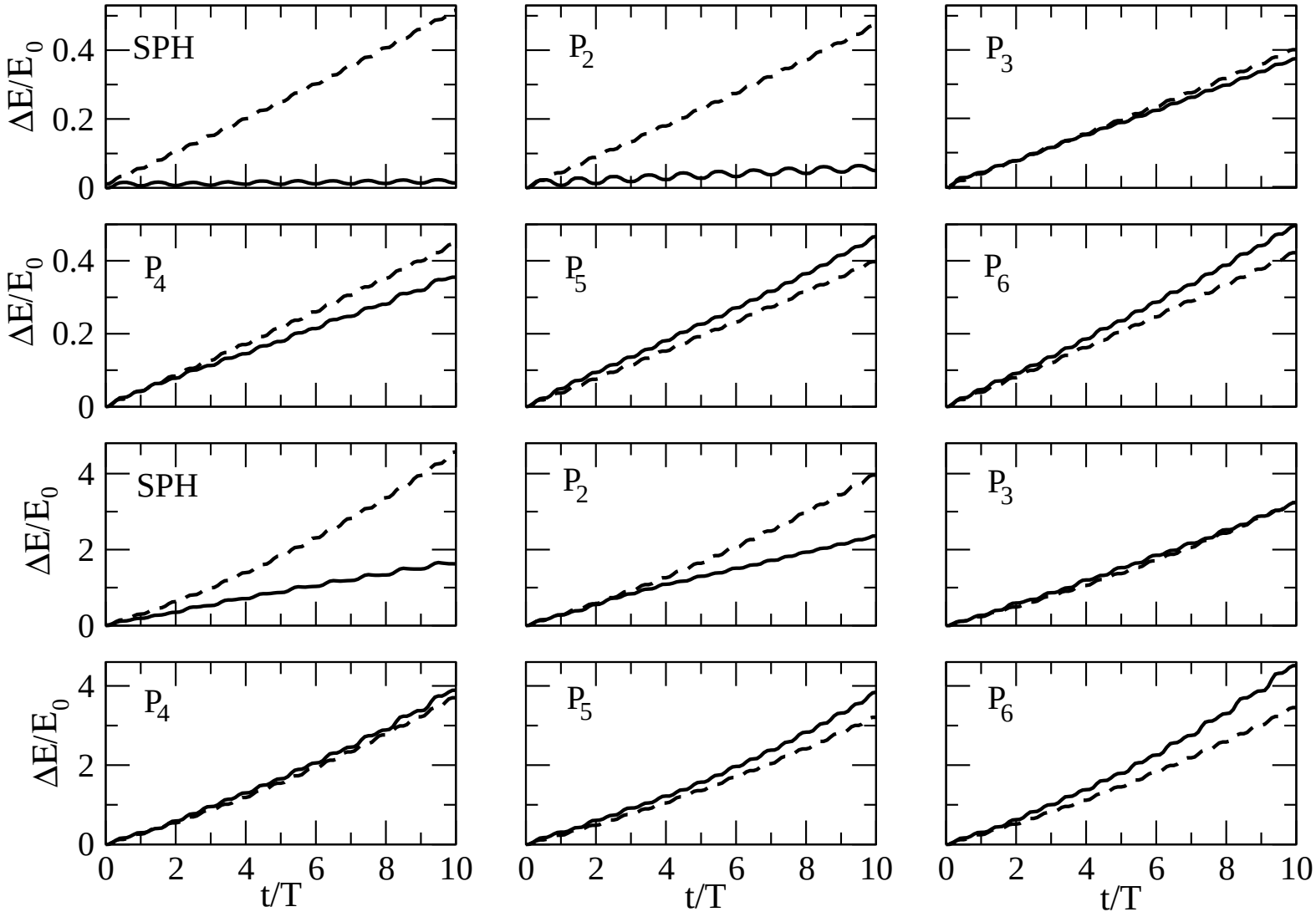


Fig.2. The same as in Fig.1 but for the vibrations around a deformed shape $\alpha = 0.3$

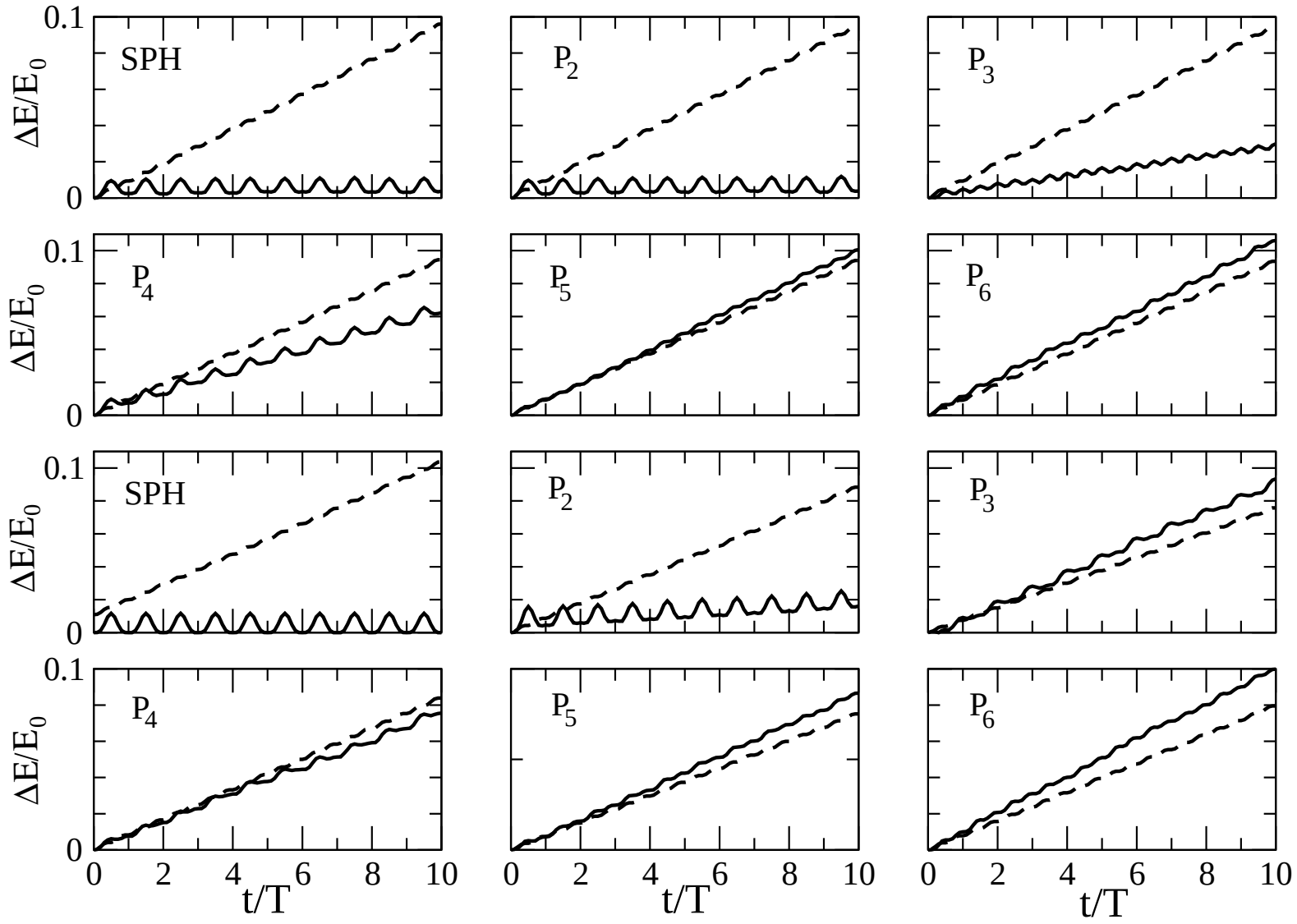


Fig.3. The same as in Figs.1,2 but around a spherical shape (two upper rows) and deformed shape $\alpha_{st} = 0.3$ (two lower rows). The amplitudes of the vibrations is $\alpha = 0.1$ and the adiabaticity parameter $\eta = 0.02$. Solid lines indicate results of the computer simulations and dashed ones are the wall formula predictions

3.

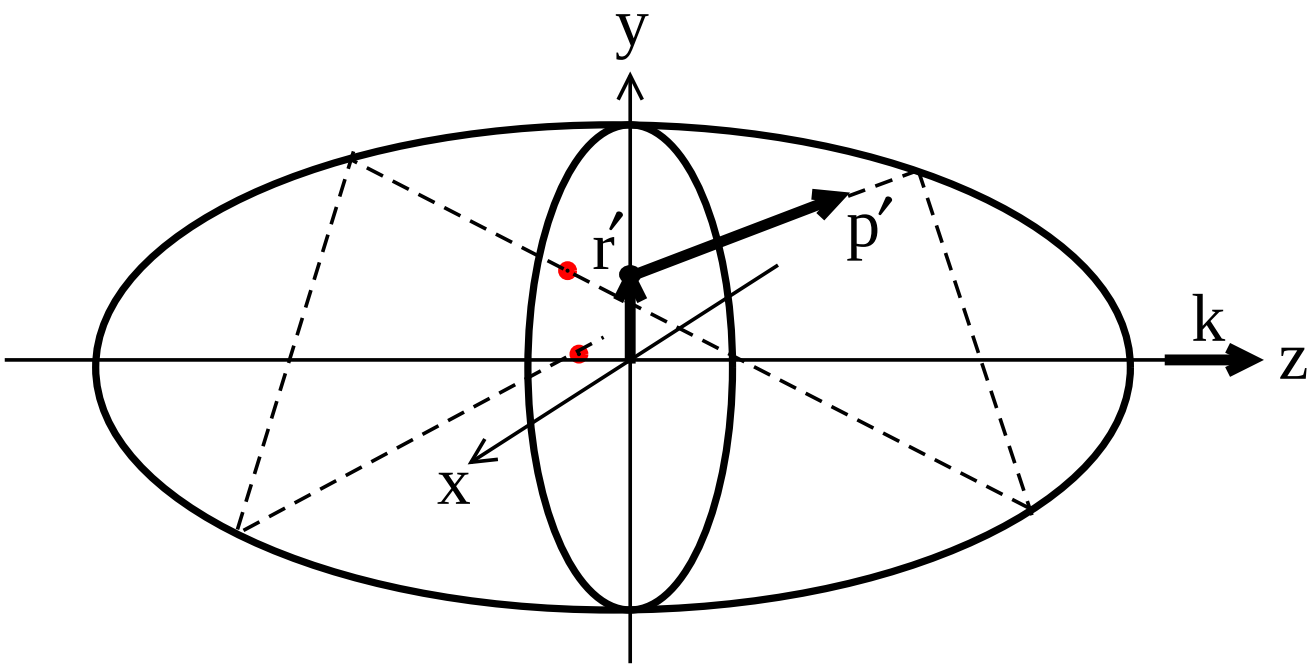


Fig. 4. Poincare section, $K = (r' \times p') \cdot k$

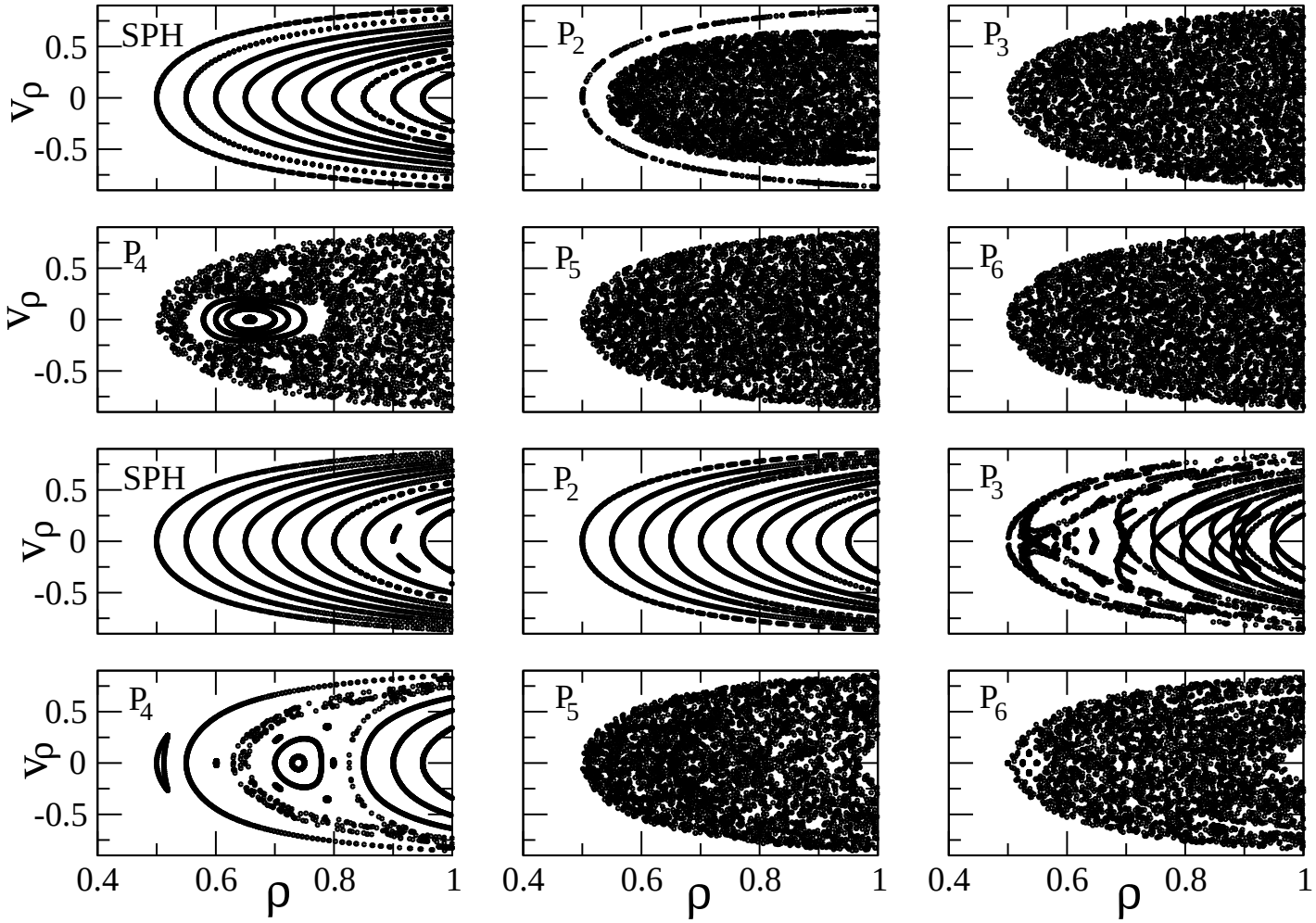


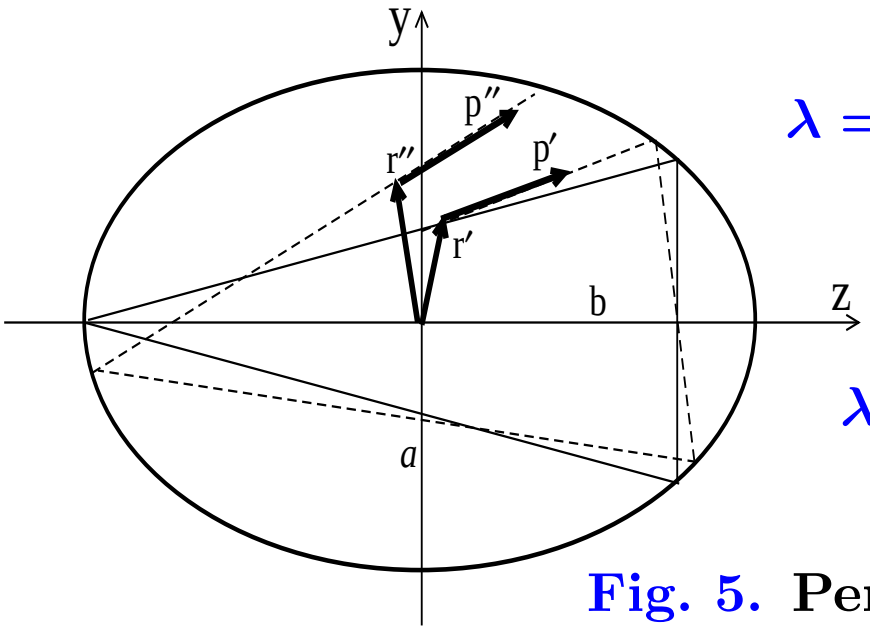
Fig.4. Poincare sections for six shapes at the deformation $\alpha = 0.05$ (two lower rows) and deformation $\alpha = 0.4$ (two upper rows) for the projections of the angular momentum on the symmetry axis $K = 0.5$ with respect to the maximal value

LYAPUNOV EXPONENTS: $\lambda(T)$

MONODROMY MATRIX $\mathcal{M}$ (propagator): $\xi'' = \mathcal{M}(T)\xi'$

$$\xi = \{\delta r, \delta p\}, \quad \mathcal{M} = \begin{pmatrix} \mathcal{M}_\perp & , & \dots \\ \dots & , & \mathcal{I} \end{pmatrix}, \quad \mathcal{I} = \begin{pmatrix} 1 & , & 0 \\ 0 & , & 1 \end{pmatrix}$$

$tr\mathcal{M}(T) = 1+1+a(T)+1/a(T), \quad a(T) = \exp[\lambda(T)]$ with period T



$\lambda = i\zeta$ stable *p.o.*, $\lambda = \zeta$ unstable *p.o.*

$\lambda = \pm 1$ parabolic *case*, ζ is *real*

Fig. 5. Periodic-orbit trajectory (p.o., solid) and its variation (dashed) in phase space, $\{r', p'\} \to \{r'', p''\}$

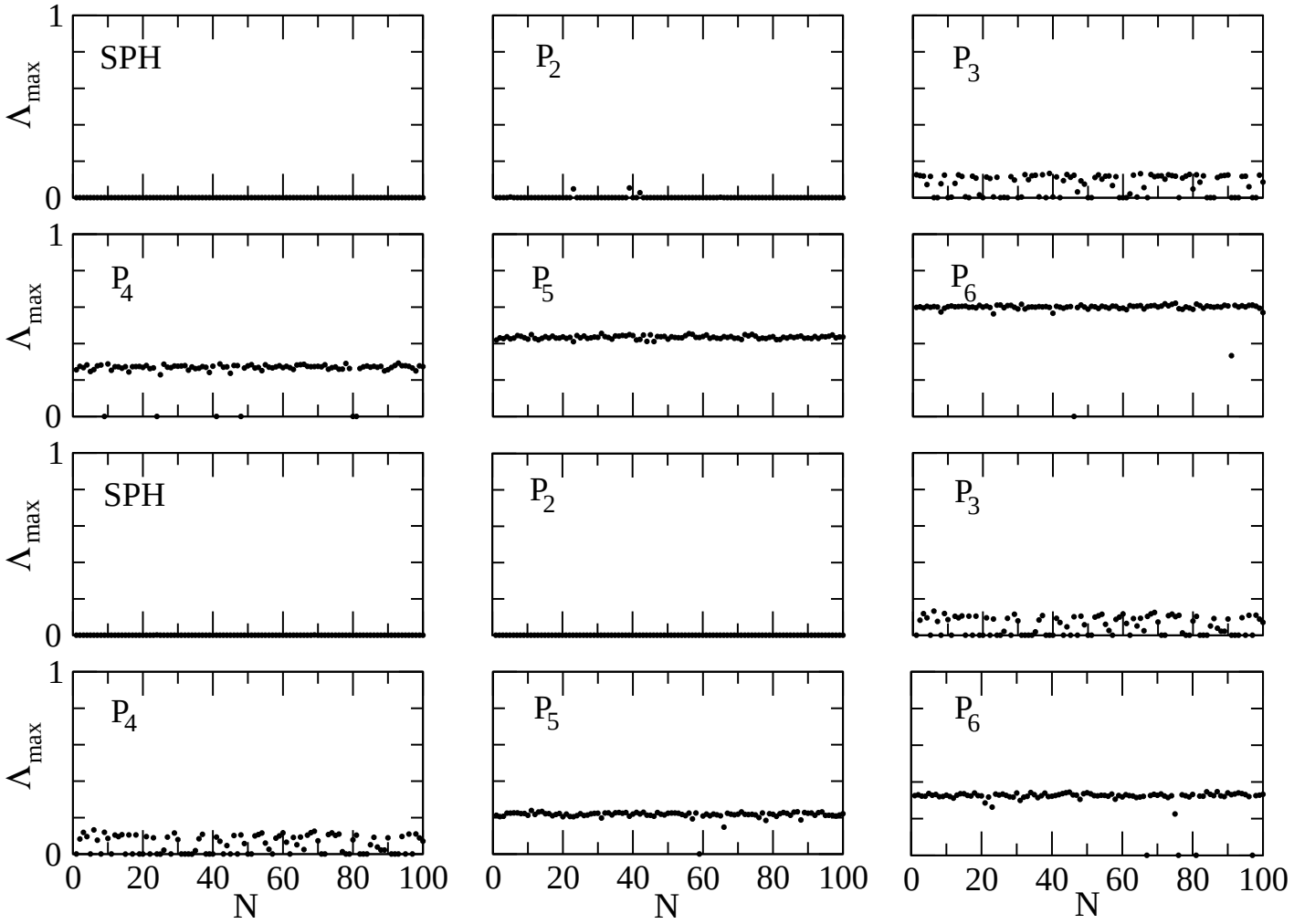


Fig.5. Lyapunov exponents Λ_{max} for six shapes at deformations $\alpha = 0.05$ (lower two rows) and $\alpha = 0.4$ (upper two rows); the projection of the angular momentum on to the symmetry axis $K = 0$

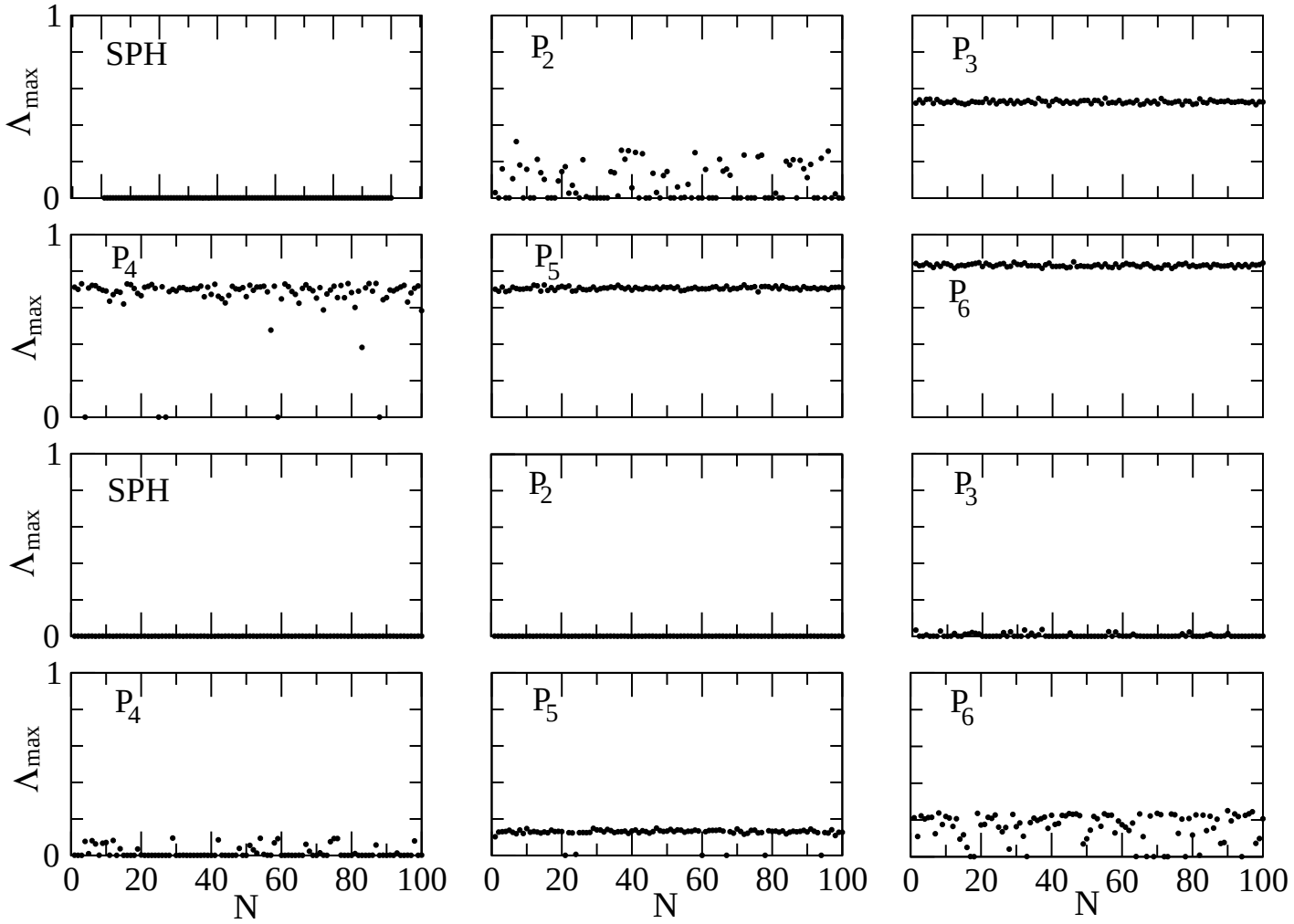


Fig.6. The same as in Fig.5 but for the projection of the angular momentum on to the symmetry axis $K = 0.5$ of the maximal projection