Pairing effects on the Isospin-symmetry-breaking correction to Super-allowed nuclear β decay with HTDA

J. Le Bloas, L. Bonneau, P. Quentin
(CENBG–University of Bordeaux, France)

J. Bartel (IPHC–Louis Pasteur University, France)

17th Nuclear Physics Workshop
"Marie & Pierre Curie"
Kazimierz Dolny 2010

CNRS IN2P3 Les deux infinis

CENBG

UNIVERSITÉ BORDEAUX 1 Sciences Technologies

Introduction

Corrections to the matrix element of superallowed Fermi beta decay

$\rightarrow$ Selection rules: $(J^\pi = 0^+; T = 1) \rightarrow (J^\pi = 0^+; T = 1)$.

$$ft(1+\delta_R)(1+\delta_{NS}-\delta_C) \equiv \mathcal{F}t = \frac{K}{|M_F|_0^2 G_V^2 (1+\Delta_R)}$$

with...

- $K = \frac{2\pi^3 \hbar (\hbar c)^6 \ln(2)}{(m_e c^2)^5}$.
- $|M_F|_0^2 = |\langle f | \hat{T}_+ | i \rangle|^2 = 2$: square of nuclear matrix element between pure-$(T = 1)$ initial and final states.
- $G_V = V_{ud} G_F$: vector coupling constant expected to be a constant (CVC hypothesis).

... and some corrections,

- Δ_R: transition-independant part of the radiative correction.
- δ_R & δ_{NS}: transition-dependant part of the radiative correction (δ_R doesn't depend on nuclear structure, δ_{NS} does).
- ***δ_C: isospin-symmetry breaking (ISB) correction.***

If CVC hypothesis is verified, $\mathcal{F}t$ is a constant !

Outline

1. Theoretical framework
   - The HTDA method
   - A model description of β decay
2. Application to the β^+ decay of $^{50}\mathrm{Mn}$
   - Numerical framework
   - Spurious Isospin-Symmetry Breaking (ISB)
   - Impact of some theoretical ingredients
   - Comparison with other studies
3. Conclusion

Theoretical framework - The HTDA method

HTDA $\approx$ Highly-truncated Shell-Model based on a 'Mean-Field' solution !

Theoretical framework - The HTDA method

HTDA $\approx$ Highly-truncated Shell-Model based on a 'Mean-Field' solution !

Principle

- HF calculations $\Rightarrow$ one-body reduced density matrix.
  $\Rightarrow$ Lowest energy Slater determinant: $|\Phi_0\rangle$

N. Pillet, P. Quentin, and J. Libert, *Nucl. Phys.* **A697**, 141 (2002)

Theoretical framework - The HTDA method

HTDA $\approx$ Highly-truncated Shell-Model based on a 'Mean-Field' solution !

Principle

- HF calculations $\Rightarrow$ one-body reduced density matrix.
  $\Rightarrow$ Lowest energy Slater determinant: $|\Phi_0\rangle$
- Many body basis $\mathcal{B} \equiv \{|\Phi_i\rangle\}$: particle-hole excitations on $|\Phi_0\rangle$.
- Diagonalization of the HTDA hamiltonian in $\mathcal{B}$.
- Correlated states (including the GS $|\Psi_0\rangle$):

$$|\Psi_0\rangle = C_0^0|\Phi_0\rangle + \sum_{1p1h} C_1^0 \hat{a}_\lambda^\dagger \hat{a}_\ell |\Phi_0\rangle + \sum_{2p2h} C_2^0 \hat{a}_\lambda^\dagger \hat{a}_\mu^\dagger \hat{a}_m \hat{a}_\ell |\Phi_0\rangle + \cdots$$

$$|\Psi_n\rangle = C_0^n|\Phi_0\rangle + \sum_{1p1h} C_1^n \hat{a}_\lambda^\dagger \hat{a}_\ell |\Phi_0\rangle + \sum_{2p2h} C_2^n \hat{a}_\lambda^\dagger \hat{a}_\mu^\dagger \hat{a}_m \hat{a}_\ell |\Phi_0\rangle + \cdots$$

N. Pillet, P. Quentin, and J. Libert, *Nucl. Phys.* **A697**, 141 (2002)

Theoretical framework - The HTDA method

$$\hat{H} = \underbrace{\left(\hat{K} + \hat{V}_{\mathrm{HF}} - \langle\Phi_0|\hat{V}|\Phi_0\rangle + E_R\right)}_{\hat{H}_0} + \underbrace{\left(\hat{V} - \hat{V}_{\mathrm{HF}} + \langle\Phi_0|\hat{V}|\Phi_0\rangle - E_R\right)}_{\hat{V}_{\mathrm{res}}} .$$

Such as,

$$\langle\Phi_0|\hat{H}_0|\Phi_0\rangle = \langle\Phi_0|\hat{H}|\Phi_0\rangle \qquad \text{and} \qquad \langle\Phi_0|\hat{V}_{\mathrm{res}}|\Phi_0\rangle = 0 .$$

$\hat{V}$	nucleon–nucleon interaction (*NN*, *NNN*. . .),
$\hat{V}_{\mathrm{HF}}$	one-body reduction of $\hat{V}$ in $|\Phi_0\rangle$,
E_R	Rearrangement energy in relation to the possible density-dependence of $\hat{V}$.

- Choice for $\hat{H}_{\mathrm{HF}}$: Coulomb + Skyrme (SIII, SkM* or SLy4)
- Approximation for $\hat{V}_{\mathrm{res}}$: Coulomb + $\hat{\delta}$ or $\hat{QQ}$ interactions

Theoretical framework - A model description for β decays

Good treatment of the isospin symmetry

- if $N \neq Z$, HF breaks isospin: different HF fields for neutrons and protons even without any physical source of ISB.

Theoretical framework - A model description for β decays

Good treatment of the isospin symmetry

- if $N \neq Z$, HF breaks isospin: different HF fields for neutrons and protons even without any physical source of ISB.
- Spurious ISB always present when one truncates the sp-space.

- if $N \neq Z$, HF breaks isospin: different HF fields for neutrons and protons even without any physical source of ISB.
- Spurious ISB always present when one truncates the sp-space.
- A further source of Spurious ISB may results from an unsuitable truncation of the MB-space.

- if $N \neq Z$, HF breaks isospin: different HF fields for neutrons and protons even without any physical source of ISB.
- Spurious ISB always present when one truncates the sp-space.
- A further source of Spurious ISB may results from an unsuitable truncation of the MB-space.

Requirement *for $N = Z$ nuclei without Coulomb*

- ☞ Identical sp-spectrum for neutrons and protons.
- ☞ Truncated sp-space stable under $\hat{T}^2$.
- ☞ Idem for truncated MB-space *(satisfied for the subset of multiple nn, pp & np pairs)*.

- if $N \neq Z$, HF breaks isospin: different HF fields for neutrons and protons even without any physical source of ISB.
- Spurious ISB always present when one truncates the sp-space.
- A further source of Spurious ISB may results from an unsuitable truncation of the MB-space.

Requirement *for $N = Z$ nuclei without Coulomb*

- ☞ Identical sp-spectrum for neutrons and protons.
- ☞ Truncated sp-space stable under $\hat{\mathrm{T}}^2$.
- ☞ Idem for truncated MB-space *(satisfied for the subset of multiple nn, pp & np pairs)*.

Since we take the same $|\Phi_0\rangle$ for both initial and final nuclear states, the model hamiltonian is the same.

Cases under study: decays involving even-A nuclei only. A good description of the lowest $K^\pi = 0^+$ and $T = 1$ nuclear states is needed.

Theoretical framework - A model description for β decays

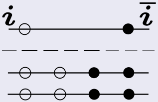
MB basis and Time-Reversal symmetry

Cases under study: decays involving even-A nuclei only. A good description of the lowest $K^\pi = 0^+$ and $T = 1$ nuclear states is needed.

Isospin and Time-reversal (TR) symmetries

Example of the lowest-energy unperturbed MB states (initial & final).

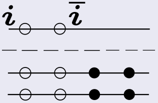
$|\Phi_i\rangle$



$$|\Phi_i^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_i\rangle \pm |\overline{\Phi_i}\rangle\right) \quad \text{such as} \quad \hat{\mathfrak{T}}|\Phi_i^{(\pm)}\rangle = \pm|\Phi_i^{(\pm)}\rangle$$

$$\hat{\mathrm{T}}^2|\Phi_i^{(+)}\rangle = 2|\Phi_i^{(+)}\rangle \quad \text{whereas} \quad \hat{\mathrm{T}}^2|\Phi_i^{(-)}\rangle = 0$$

$|\Phi_f\rangle$



$$|\Phi_f\rangle = |\overline{\Phi_f}\rangle \quad \text{and} \quad \hat{\mathrm{T}}^2|\Phi_f\rangle = 2|\Phi_f\rangle$$

(in the 'no-Coulomb limit')

Theoretical framework - A model description for β decays

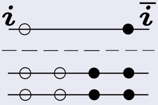
MB basis and Time-Reversal symmetry

Cases under study: decays involving even-A nuclei only. A good description of the lowest $K^\pi = 0^+$ and $T = 1$ nuclear states is needed.

Isospin and Time-reversal (TR) symmetries

Example of the lowest-energy unperturbed MB states (initial & final).

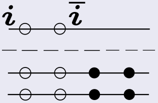
$|\Phi_i\rangle$



$$|\Phi_i^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\left(|\Phi_i\rangle \pm |\overline{\Phi_i}\rangle\right) \quad \text{such as} \quad \hat{\mathfrak{T}}|\Phi_i^{(\pm)}\rangle = \pm|\Phi_i^{(\pm)}\rangle$$

$$\hat{\mathrm{T}}^2|\Phi_i^{(+)}\rangle = 2|\Phi_i^{(+)}\rangle \quad \text{whereas} \quad \hat{\mathrm{T}}^2|\Phi_i^{(-)}\rangle = 0$$

$|\Phi_f\rangle$



$$|\Phi_f\rangle = |\overline{\Phi_f}\rangle \quad \text{and} \quad \hat{\mathrm{T}}^2|\Phi_f\rangle = 2|\Phi_f\rangle$$

(in the 'no-Coulomb limit')

Equal-filling for the odd-odd is unsuitable with regard to isospin ***!***

Outline

1. Theoretical framework
   - The HTDA method
   - A model description of β decay
2. Application to the β^+ decay of $^{50}\mathrm{Mn}$
   - Numerical framework
   - Spurious Isospin-Symmetry Breaking (ISB)
   - Impact of some theoretical ingredients
   - Comparison with other studies
3. Conclusion

Application to $^{50}\mathrm{Mn} \rightarrow {}^{50}\mathrm{Cr}\,(\beta^+)$ - Numerical framework

In practice...

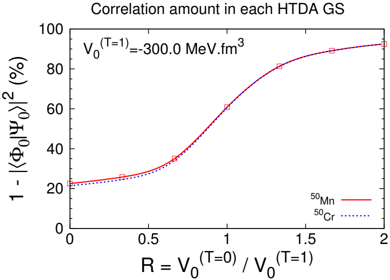
- Nuclear force for HF calculations: ***SIII*** *(+ Coulomb)*
- δ residual interaction to simulate the ***pairing***.
- Fixed strength of the $T = 1$ channel of $\hat{V}_{\mathrm{res}}$:
  $V_0^{(T=1)} = -300.0$ MeV.fm^2
- Sensitivity study as a function of the $T = 0$ strength of $\hat{V}_{\mathrm{res}}$.
- ***sp-space***: **7** hole levels and **8** particle levels.
- $|^{50}\mathrm{Mn}\rangle$: $2p0h$, $4p2h$ & $6p4h$ 'excitations' on $|^{48}\mathrm{Cr}\rangle$.
- $|^{50}\mathrm{Cr}\rangle$: $2p0h$, $4p2h$ & $6p4h$ 'excitations' on $|^{48}\mathrm{Cr}\rangle$ with a $\Delta T_z = 1$ shift.

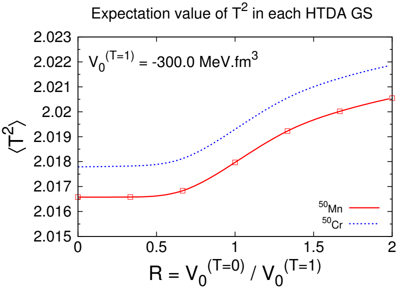
Main advantage of the model

$\Rightarrow$ *Minimizes spurious ISB effects.*

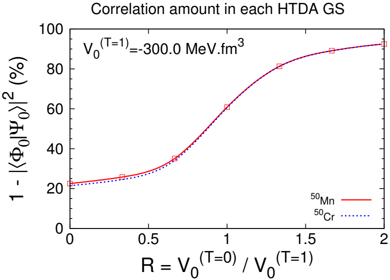
$\Rightarrow$ *Expects to provide a lower limit of the δ_c correction.*

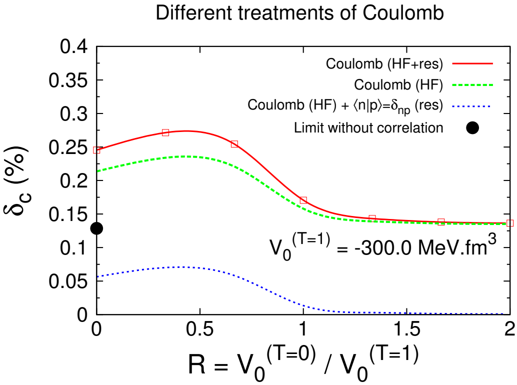
Application to $^{50}\mathrm{Mn} \rightarrow {}^{50}\mathrm{Cr}\,(\beta^+)$ - Spurious ISB

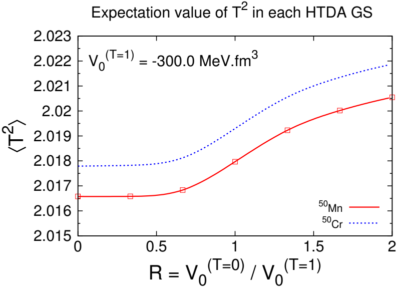




Application to $^{50}\mathrm{Mn} \rightarrow {}^{50}\mathrm{Cr}\,(\beta^+)$ - Spurious ISB

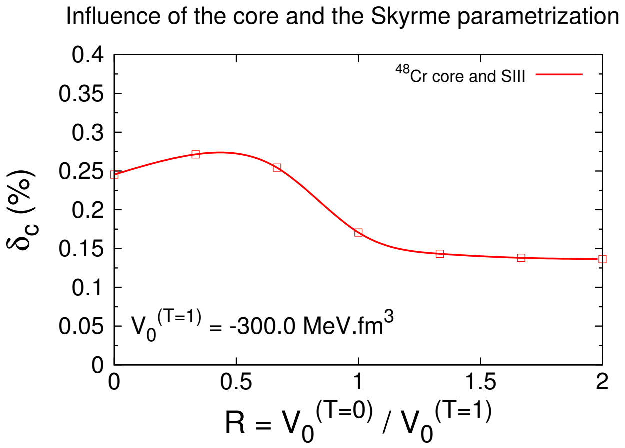






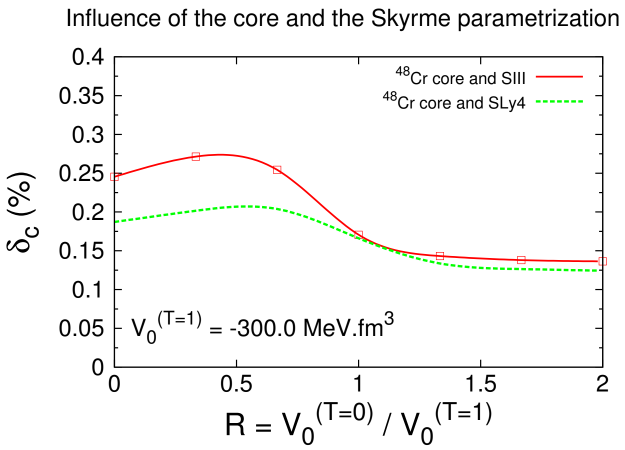
- Two regimes $R \lessgtr 1$.
- Differencies of n & p wave-functions play an important role.
- Correlations explain the patern for $R < 1$.
- Coulomb in $\hat{V}_{\mathrm{res}}$ plays a significant role.

Application to $^{50}\mathrm{Mn} \rightarrow {}^{50}\mathrm{Cr}\,(\beta^+)$ - Impact of ingredients



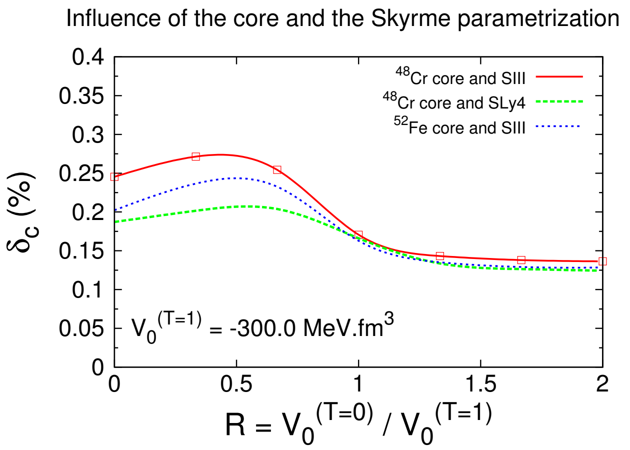
- The model is significantly (but not drastically) sensitive to ingredients.
- Beyond $R = 1$, no impact of ingredients.

Application to $^{50}\mathrm{Mn} \to {}^{50}\mathrm{Cr}\,(\beta^+)$ - Impact of ingredients



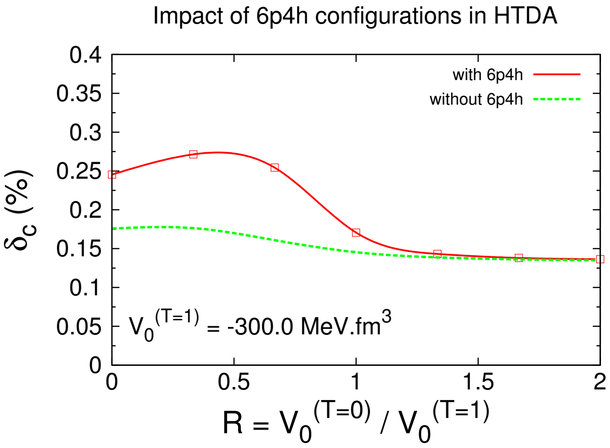
- The model is significantly (but not drastically) sensitive to ingredients.
- Beyond $R = 1$, no impact of ingredients.

Application to $^{50}\mathrm{Mn} \to {}^{50}\mathrm{Cr}\,(\beta^+)$ - Impact of ingredients



- The model is significantly (but not drastically) sensitive to ingredients.
- Beyond $R = 1$, no impact of ingredients.

Application to $^{50}\mathrm{Mn} \rightarrow {}^{50}\mathrm{Cr}\,(\beta^+)$ - Impact of ingredients



- Without 6p4h, no influence of the $T = 0$ pairing on δ_C.
- 6p4h are necessary to explain the patern for $R < 1$.

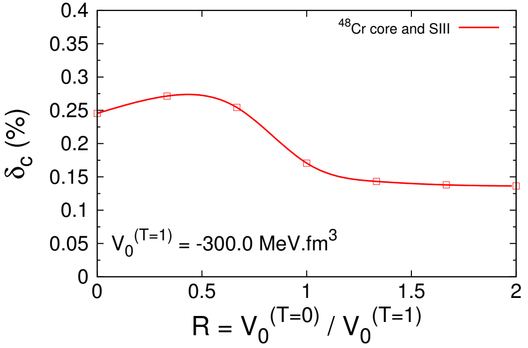
Application to $^{50}\mathrm{Mn} \rightarrow {}^{50}\mathrm{Cr}\,(\beta^{+})$ - Comparison with other studies

To compare...

δ_C (%)			
Damgaard	SM-SW	SM-HF	IVMR
0.550	0.655	0.620	0.122

- **Damgaard:** J. Damgaard, *Nucl. Phys.* **A130**, 233 (1969).
- **SM-SW:** I.S. Towner and J.C. Hardy, *Phys. Rev.* **C 66**, 035501 (2002).
- **SM-HF:** W.E. Ormand and B.A. Brown, *Phys.Rev.* **C 52**, 2455 (1995).
- **IVMR:** N. Auerbach, Phys. Rev. C 79, 035502 (2009).

Influence of the core and the Skyrme parametrization



Outline

Conclusions & perspectives

Conclusions

- We constructed a 'HTDA-based' model to describe β decays involving even-A nuclei.
- Main advantage: minimizes spurious ISB inherent to HF and HTDA.
- Better understanding about how pairing correlations impacts M_F and δ_C.
- δ_C obtained is expected to be a lowest bound !

Conclusions & perspectives

Conclusions

- We constructed a 'HTDA-based' model to describe β decays involving even-A nuclei.
- Main advantage: minimizes spurious ISB inherent to HF and HTDA.
- Better understanding about how pairing correlations impacts M_F and δ_C.
- δ_C obtained is expected to be a lowest bound !

Perspectives

- Axially deformed treatment $\Rightarrow$ Projection on J is required.
- Exact projection on isospin $\Rightarrow$ More fundamental understanding of mixing mechanisms.
- Variational HTDA $\Rightarrow$ Improves the description of sp-states.

Exact calculation of the Coulomb force

An analytical calculation of Coulomb matrix elements

Exact evaluation via the axial HO basis using a Moshinsky transformation.

$$\langle ij|\frac{1}{|\overrightarrow{r_{12}}|}|kl\rangle = \sqrt{\frac{2}{\pi}}\,\beta_0^3 \sum_n f^n \sum_p \mathcal{C}(n,n',p)\mathcal{A}(p)\times$$

$$\sum_a \sum_b g^{a,b}\,\delta_{a-b,a'-b'}\,\frac{(a+b')!}{\sqrt{a!\,a'!\,b!\,b'!}} \sum_k \frac{C_b^k C_{b'}^k}{C_{a+b'}^k}\,\mathcal{J}^{\beta_z,\beta_\perp}_{p/2,|a-b|,k,a+b'+1-k}$$

where,

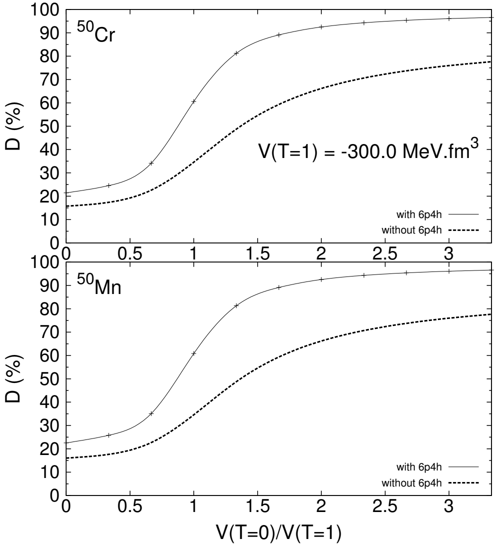
$$\mathcal{J}^{\beta_z,\beta_\perp}_{q,\ell,m,n} = \int_0^{+\infty} \frac{\left(\beta_\perp^2\sigma^2\right)^\ell \left(\beta_\perp^2\sigma^2-1\right)^m \sigma\, d\sigma}{\left(1+\beta_\perp^2\sigma^2\right)^n \sqrt{\left(1+\beta_z^2\sigma^2\right)^{2q+1}}}\,.$$

★ Exactly Coulomb exchange contribution to $\hat{V}_{\mathrm{HF}}$ (instead of Slater approx).

P. Quentin, *J. Phys. France*, **33**, 457-463 (1972)

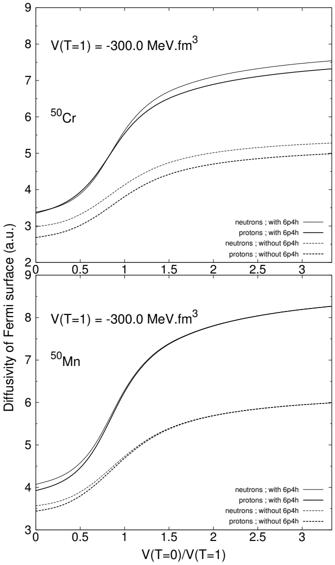
More plots - Influence of 6*p*4*h* configurations

$$1 - |\langle|\Phi_0\rangle|\Psi_0\rangle|^2 \ (\%)$$

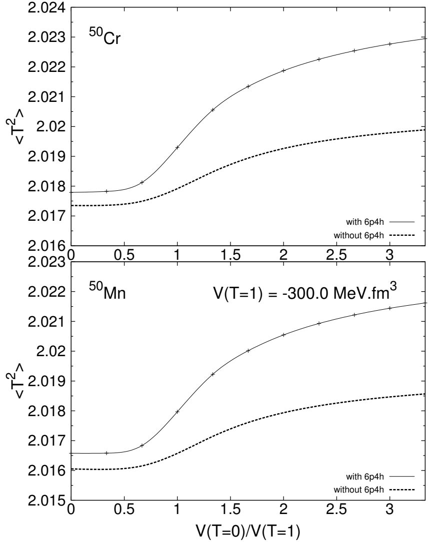


More plots - Influence of 6*p*4*h* configurations

Diffusivity of Fermi surfaces

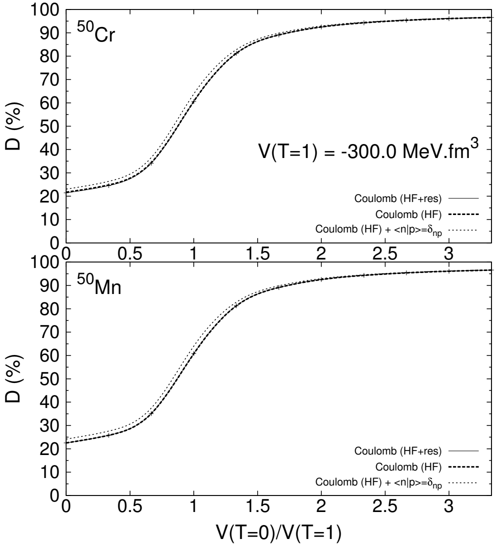


More plots - Influence of 6*p*4*h* configurations



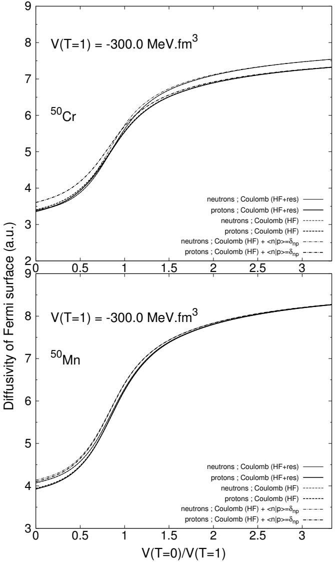
More plots - Treatments of Coulomb

$$1 - |\langle|\Phi_0\rangle|\Psi_0\rangle|^2\ (\%)$$

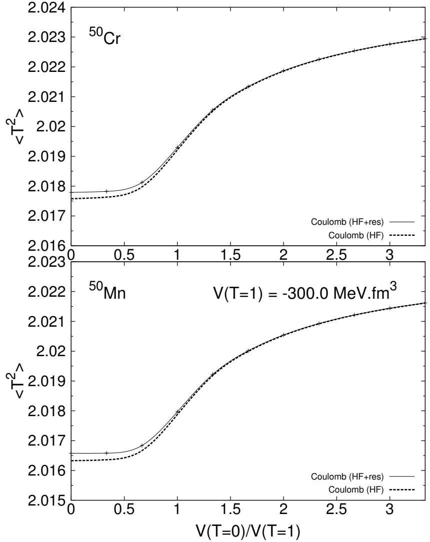


More plots - Treatments of Coulomb

Diffusivity of Fermi surfaces

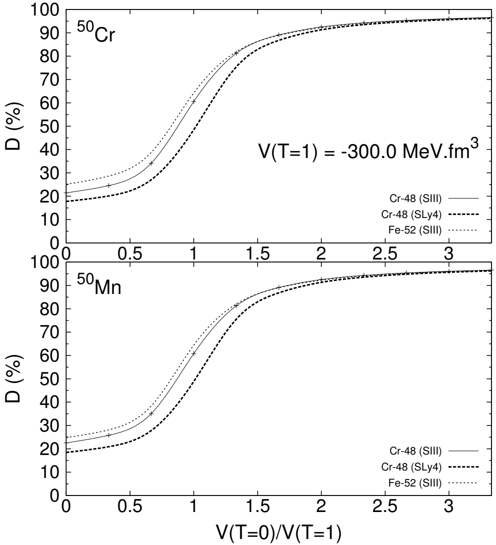


More plots - Treatments of Coulomb



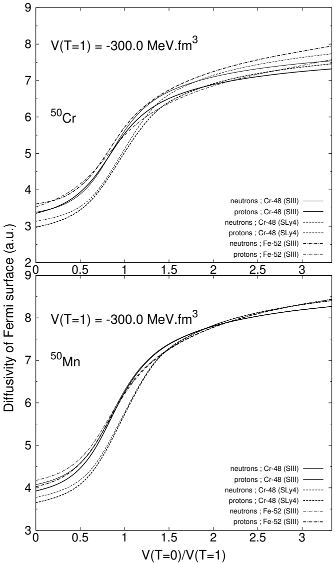
More plots - Influence of some ingredients

$$1 - |\langle|\Phi_0\rangle|\Psi_0\rangle|^2 \ (\%)$$



More plots - Influence of some ingredients

Diffusivity of Fermi surfaces



More plots - Influence of some ingredients